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Institutional Spending Policies: Implications for Future Asset Values and Spending

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Abstract University endowments, sovereign wealth funds, and foundations all support spending. This paper analyzes how different spending policies affect future values of the assets under management and future spending opportunities. We show that the covariance between the asset returns and the spending rate implied by the spending policy is important. Many of the spending policies used in practice aim to smooth the spending level by letting current spending be a function of both current asset values and past spending levels. One feature of these types of spending policies is that asset volatility increases and future spending volatility increases. A second feature is that the funds can be depleted. Depleted funds cannot support spending.

Keywords Endowments · sovereign wealth funds · endowment spending-policies

JEL Classification G11 · G23

1 Introduction

Universities, charitable foundations, and many countries accumulate large asset portfolios in the form of endowments and sovereign wealth funds (SWFs). A common goal when managing such portfolios is to maintain their real value in the long run.[1] To help achieve this goal, the above-mentioned institutions design a *spending policy* that builds on a target rate of withdrawal from the asset portfolio. Reflecting the long horizons of the institutions, the target spending rate is set at or below the expected real portfolio returns. Because the institutions discussed here take risk in their asset portfolios, spending (no more

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[1] See, e.g., Tobin (1974) and Hansmann (1990) for a discussion of this goal in the context of university endowments, and the volume of Bolton et al. (2011) for a discussion of why many SWFs aim to preserve their purchasing power in the long run.

than) the expected real return does not guarantee that the spending will be the same forever. Rather, the implied level of real spending can only be *expected* to be sustained forever. According to Hansmann (1990), most major universities in the United States employ a target spending rate roughly equal to the (expected) real return on the endowment. Likewise, the fiscal rule associated with Norway's SWF (which we return to in Subsection 4.2) calls for a spending rate that, on average, equals the expected real return on the fund. The Government of Singapore supports the budget in a sustainable way by including investment returns from their SWFs.[2]

While the spending policy's main objective may be long-run viability, many spending policies also include elements of short-run smoothing in the spending level. For example, leading universities in the US employ an endowment spending policy that sets current spending equal to a weighted average of target spending based on the current value of the endowment and real spending in the last period. The latter, backward-looking part of this policy is motivated by a desire to avoid large shifts in the amount spent as asset markets fluctuate (Hansmann (1990)). Universities that employ this type of spending rule include Harvard, MIT, Stanford, and Yale (see Campbell (2012), Alexander and Herring (2008), Mehrling et al. (2005), and Swensen (2009)). Similarly, SWFs use policies that aim to balance sustainability (that is, to spend the permanent income of their wealth) with the need for flexibility in case of economic shocks (see, e.g., Ang (2014)).

The present paper analyzes the effect different spending policies has on future values of assets under management and spending levels. To this end, we construct four stylized spending policies that resemble policies used in real life. We find that the joint stochastic properties of the spending rate and asset returns can be important for future fund values and spending levels. For instance, we illustrate that if spending from an SWF is used in a Keynesian fashion, the *average* spending rate can be set higher than expected fund returns and still have sustainable spending levels. We also show that spending policies with backward-looking elements, such as the one mentioned above that is often used by universities, produce short-term spending with low volatility. The disadvantage of this type of short-term spending smoothing is that expected future spending can decrease. Another side effect can be more volatile future asset values and spending levels. Backward-looking elements may also increase the probability of fund depletion. These are important features that should be acknowledged and taken into account when spending policies are formed.

Merton (1993) applied the classical normative theory of consumption and portfolio choice developed by Merton (1969) and Samuelson (1969) to analyze university endowment management. This theory dictates that decisions about spending and investment allocation should be made jointly. In practice, endowment managers do not follow this advice. The most common arrangement is for asset management to be delegated to professional asset managers, while other agents (such as politicians or a board of trustees) decide the spending

[2] See e.g., http://www.gic.com.sg/faq.

policy. Blume (2010) argued that because many institutions utilize the same type of spending rule, it is likely that their spending policy and their investment policy are set independently. Therefore, in the present analysis we take the asset allocation as given and model the returns on the investment opportunities by an exogenously given stochastic process. We then demonstrate that different spending policies give rise to large differences in the stochastic properties of future fund values and spending levels. The effects of different spending policies on future asset value distributions and spending opportunities are so extensive that we believe the results presented here should be of interest for anyone involved in forming spending policies.

There is a sizeable literature analyzing economic aspects of endowments. One strand of the literature analyzes normative behavior for endowments. This literature includes the works by Tobin (1974), Merton (1993), and Dybvig (1999). Another strand of the literature empirically analyzes endowment behavior. Some recent contributions to this strand of the literature include Brown and Tiu (2013) and Brown et al. (2014). Our paper falls between these two strands of literature. We focus on the consequences of adopting different spending policies, not on what policy is optimal to adopt. Our focus is neither on explaining why a given spending policy is chosen over other policies. Our paper is closest related to Blume (2010). He analyzed the long-term interaction between investment strategies and spending policies. He researched the effect of this interaction on future spending, wealth, and depletion risk. Our work is also related to Brown and Scholz (2017).

2 Asset Value Dynamics

Let F_t be the time t value of the assets under management (AUM) of the endowment, charity, or SWF. We assume that the assets are invested in a portfolio with risky returns. Funds are continuously withdrawn from the asset portfolio to support spending. We assume that the value dynamics of the AUM are given by

$$\begin{aligned} dF_t &= \mu F_t dt + \sigma F_t dB_t - S_t dt \\ &= (\mu - s_t) F_t dt + \sigma F_t dB_t, \end{aligned} \tag{1}$$

where μ is the constant instantaneous expected portfolio return, σ the accompanying standard deviation, and B is a standard Brownian motion. Here, S_t is the spending level at time t, while $s_t \equiv S_t/F_t$ is the corresponding rate of withdrawal (or spending rate for short). We do not permit the owner to finance spending by being a net borrower at any time t. Formally, we assume that $F_t = 0$ is an absorbing barrier for the process in expression (1), and that permissible spending policies $S_t(F_t)$ fulfills $S_t(0) = 0$. Let $F_0 \geq 0$ be the current value of AUM. As seen in the Appendix,

$$F_t = F_0 e^{R_t - Z_t}, \tag{2}$$

where

$$R_t = (\mu - \frac{1}{2}\sigma^2)t + \sigma B_t$$

is the cumulative log-return on the risky portfolio over the time interval $[0, t]$ and

$$Z_t = \int_0^t s_v dv \tag{3}$$

is the cumulative spending rate over the same interval. From expression (2) we see that the realized value of AUM at time t is equal to the current AUM-value F_0 if $R_t = Z_t$.

We are interested in characterizing the distribution of future asset values (expectation and variance/standard deviation). To this end, we start by calculating the expected future AUM value in expression (2):

$$E[F_t] = F_0\Big(E[e^{R_t}]E[e^{-Z_t}] + \text{cov}(e^{R_t}, e^{-Z_t})\Big). \tag{4}$$

The covariance between returns and the spending rate is key for understanding how spending policies affect AUM in the long run. Expression (4) shows that the expected value of AUM increases in $\text{cov}(e^{R_t}, e^{-Z_t})$. The economic meaning of this covariance can be understood by looking at how R_t and Z_t are affected by the "return shock" B_t. Since R_t is strictly increasing in B_t, so too is e^{R_t}. Therefore, a sufficient condition for $\text{cov}(e^{R_t}, e^{-Z_t}) > 0$ is that Z_t decreases monotonically in B_t. This condition is not as restrictive as it may seem at first. Positive cumulative returns R increase asset values. A lower cumulative spending rate, Z, can be accompanied by a higher spending level because the value of the assets we draw funds from to support spending is higher because of the positive returns.

Here, we consider a spending policy that aspires to preserve the value of the AUM in the long run as one in which $E[F_t] = F_0$. We can think of the policy as dictating the maximum sustainable spending level. Clearly, many of the spending policies we can think of do not preserve asset values. However, from expression (2) and Jensen's inequality, we can say something about the maximum sustainable cumulative spending rate. We note that F_t is convex in the difference $R_t - Z_t$. Therefore, Jensen's inequality implies that

$$E[F_t] \geq F_0 e^{E[R_t - Z_t]}. \tag{5}$$

From this inequality we can see that

$$E[Z_t] \leq E[R_t] = (\mu - \frac{1}{2}\sigma^2)t \tag{6}$$

is a *sufficient* condition for $E[F_t] \geq F_0$. Any spending policy in which the expected cumulative spending rate is less than or equal to the expected cumulative log-returns secures asset preservation. We emphasize that the condition in inequality (6) is not *necessary*. Later in this paper we show examples of spending policies with $E[F_t] \geq F_0$ that have cumulative spending rates higher than the right-hand side of inequality (6).

As mentioned in the Introduction, one goal for endowments and SWFs is to preserve the real value of the AUM. We do not explicitly include inflation in our model, but we can think of all relevant quantities to be expressed in real terms. Alternatively, we could include a stochastic inflation process and use it to deflate the value of AUM and spending levels. For simplicity, we abstract from modelling of inflation. We also abstract from explicitly modelling inflow of funds.

3 Taxonomy of Spending Policies

Among endowments, foundations, and SWFs, there is a wide range of different spending policies that are tailored to meet the beneficiaries' financial needs. We analyze four stylized spending policies. The first is a policy with a fixed spending rate (labeled *fixed rate*). In the second policy, the spending rate follows a stochastic, mean-reverting process (labeled *mean reverting*). The third policy is backward-looking in that the spending level is the same as is used to be, it is held constant (labeled *constant*). The fourth policy combines the fixed spending rate with a backward-looking element that looks at historical spending levels (labeled *hybrid*).

Endowed universities draw on different types of spending policies to determine their spending levels. In an analysis of these spending policies, Brown and Tiu (2013) sorted the policies into seven main categories (see their appendix for details). In category 1, the board of the endowment sets the appropriate spending rate on a yearly basis. With this policy, the board can take into account the endowment's current financial situation and the current financial needs of the university. In category 2, the spending level changes by a given percent of last year's spending. If the real spending is held constant or the percentage change is zero, this policy is captured by our policy with a constant spending level. In the third category, the spending level is given as a fixed percent of a moving average of AUM. This policy can reduce the (short-term) spending volatility, compared to our fixed-rate policy. If only the current value is used to calculate the "average" value, it coincides with our fixed-rate policy. In the fourth category, the board decides to spend a given fraction of the income generated by the AUM. In category 5, the spending level is a given percent of the AUM-value and coincides with our fixed-rate policy. The sixth category contains hybrid policies, which combine policies from other categories (note that their hybrid policies can differ from our hybrid policy). This category contains the Yale-policy we mentioned in the Introduction. We capture some of the policies in this category by our hybrid policy. Finally, their category 7 contains policies, which do not fit into the first six categories, for instance our mean-reverting policy. In Table 1 we summarize how these real-world spending policies map onto our stylized spending policies.

The spending rate for SWFs is typically set by politicians. They can use the fund to balance budgets and to finance bills with political support. Politicians may be inclined to increase spending from the fund when the economy is slow

Table 1 The table shows a mapping between the spending-policy classifications in Brown and Tiu (2013) and the stylized policies in this paper. 1 is decide on an appropriate rate annually, 2 is increase prior year's spending by a percentage, 3 is to spend a percentage of a moving average of market values, 4 is to spend a percentage of current yield, 5 is to spend a percentage of assets under management, 6 is hybrid rules, and 7 is other payout rules.

	Fixed rate	Mean reverting	Constant	Hybrid
1				
2			✓	
3	✓			
4				
5	✓			
6				✓
7		✓		

(a high spending rate) and to reduce spending when the economy is booming (a low spending rate). We try to capture this spending pattern with our mean-reverting spending policy.

In our model, we have for simplicity assumed that the expected return on AUM is constant and equal to μ. Cochrane (2011) argued that expected returns vary over time and across assets. Varying expected returns have implications for sustainable spending rates. Brown and Scholz (2017) showed that by relating the spending rate to expected returns (discount rates), the variation in year-to-year spending levels is significantly reduced, compared to using a constant spending rate. In the special case where there is no variation in the expected return (as we assume), their empirical spending policy is similar to our fixed-rate policy with a constant spending rate.

4 Spending Rate based Policies

We use the above framework to analyze our four different spending policies. Although the policies are stylized, we believe they capture the essence of rules that are used in practice and give qualitative insights into the effect on both future spending levels and AUM, cf. the discussion in section 3. Some of the policies will lead to fund depletion for some realizations of the asset returns. As fund depletion becomes more likely, it is less likely that the current spending policy will be maintained. In practice, spending policies are frequently replaced with new ones or the spending rates are revised. Brown and Tiu (2013) found that every year about 25 percent of university endowments change their spending rate or adopt a new type of spending policy. Nevertheless, we believe that maintaining a given policy and spending rate until the fund is bankrupt is a good way to analyze the properties of a given spending policy. If we allow for switching from one policy to another or lowering the spending rate as fund depletion becomes more likely, it becomes difficult to disentangle the effects from the current spending policy and the new policy on future values of AUM and spending levels.

4.1 Fixed Spending Rate

We start by analyzing a policy in which the owner sticks to his target rate of withdrawal under all market conditions; he uses a fixed spending rate. This policy resembles the optimal policy that Merton (1971) derived in his classical analysis of savings and portfolio choice with constant investment opportunities. In particular, we assume that the spending rate is equal to the instantaneous expected return μ on the asset portfolio. This rate does not necessarily result in the optimal proportion according to Merton's model, but it is the *maximum* fixed spending rate that gives a sustainable spending level. As such, it serves as a benchmark against which other policies can be compared.

When $s_t = \mu$ for all t, it follows immediately from expression (1) that

$$dF_t^f = \sigma F_t^f dB_t,$$

where superscript f denotes the fixed spending rate. In this case, the value of AUM follows a geometric Brownian motion with zero drift, while the volatility is equal to the volatility of underlying portfolio returns. The asset value at time $t > 0$ is lognormally distributed with expectation

$$E[F_t^f] = F_0$$

and variance

$$\mathrm{var}(F_t^f) = F_0^2 \left(e^{\sigma^2 t} - 1 \right). \tag{7}$$

We note that while the expected value is constant, the variance increases monotonically in t. The further into the future we look, the higher the uncertainty about the future value of AUM.

For this fixed-rate policy, we find from expression (3) that $Z_t^f = \mu t$. We note that the benchmark policy is an example of a sustainable policy in which $E[Z_t] > E[R_t]$; that is, inequality (6) does not hold, but we have asset preservation. The key here is that when $s_t = \mu$ at every instant, not only on average, we have $e^{-Z_t^f} = e^{-\mu t}$ and $\mathrm{cov}(e^{R_t}, e^{-Z_t^f}) = 0$. Expression (4) then simplifies to $E[F_t^f] = F_0$. The economic interpretation of this observation is that, at every instant, we spend the expected return on the portfolio.

Under the fixed-rate policy, the spending level is proportional to the portfolio value. More precisely, $S_t^f = \mu F_t^f$. Therefore, the fixed-rate spending follows the stochastic process

$$dS_t^f = \sigma S_t^f dB_t. \tag{8}$$

The instantaneous growth rate of spending, dS_t^f / S_t^f, has an expectation of zero and the same volatility σ as the value of the asset portfolio. The future level of spending follows a lognormal distribution with expectation $E[S_t^b] = S_0^b$ at all horizons, and with variance proportional to the variance of the fund value:

$$\mathrm{var}(S_t^f) = \mu^2 \,\mathrm{var}(F_t^f).$$

Empirically, riskier asset portfolios tend to have higher expected returns. Consequently, higher risk taking is accompanied by a higher expected spending level, but also a more volatile spending level.

4.2 A Mean Reverting Spending Rate

Normative analyses of spending policies for SWFs typically recommend spending the fund's permanent income in "normal times", but also encourages deviation from this level when the macroeconomic situation of the nation calls for it. Ang (2012) argued that SWFs should have spending policies to clarify the situation in normal times where the fund's capital should not be withdrawn, and that this rule should be flexible to meet negative shocks to a country's economy. The spending policy of Norway's SWF explicitly relates spending to the fund's expected returns and to the domestic macroeconomic conditions. This policy aims for a spending rate that, on average, corresponds to the expected real rate of return on the fund's AUM, although the policy emphasizes that the rate can be higher (lower) in periods of high (low) unemployment.[3] The estimated (ex-ante) real return has been 4 percent annually, but in February 2017 the ruling government proposed to reduce the rate to 3 percent. These return estimates work as financial constraints for the politicians. The Economic and Social Stabilization Fund in Chile is another example of an SWF that is used to stabilize fiscal spending.[4] The endowment spending policy at the University of Chicago provides a similar example of a desire to use a countercyclical spending rate. Here, the endowment's board of trustees sets a spending rate within the range of 4.5 to 5.5 percent. The intention is to lower the spending rate during periods of "market appreciation" and to increase it during periods of decline.[5]

Such policies imply that the spending rate fluctuates over time, but there is a "normal" rate $\overline{s}$ to which s_t^m tends to revert (superscript m refers to the mean-reverting policy). We model this behavior by assuming that s_t^m follows the Ornstein-Uhlenbeck process

$$ds_t^m = \kappa(\overline{s} - s_t^m)dt + \sigma_s dB_t^m, \tag{9}$$

where the constants $\kappa \geq 0$ and σ_s are the speed of reversion and the instantaneous standard deviation of the spending rate, respectively. Moreover, B^m is a standard Brownian motion and $dBdB^m = \rho dt$, where ρ is the instantaneous correlation between the spending rate and asset returns. The last term on the right-hand side of the equation tells us that the changes in the spending rate evolve randomly over time. The first term shows that when the current spending rate deviates from the normal spending rate, we can expect future spending rates to be pulled back to the normal rate.

The expression for s^m is presented in the appendix (expression (17)). The value s_t^m follows a normal distribution and can therefore be negative. States where $s_t^m < 0$ effectively have net inflow of funds to the asset portfolio. We must then think of S_t^m as *net* spending (gross spending minus inflows). For

[3] See, for example, the Norwegian Government's National Budget for 2014 (www.statsbudsjettet.no/Upload/Statsbudsjett_2014/dokumenter/pdf/national_budget_2014.pdf).

[4] See www.hacienda.cl/english/sovereign-wealth-funds/economic-and-social-stabilization-fund.html.

[5] See investments.uchicago.edu/page/endowment-spending.

an SWF, the spending level, which determines the spending rate, can be the residual budget item in a political negotiation process. Inflow of funds can be financed by politicians running the state budget with a surplus. It is reasonable to expect negative spending to coincide with a well performing economy and high asset values after a period of positive returns. Such surpluses built the foundations for the Norwegian SWF. For a period of time during the Clinton administration, surpluses were projected to result in a significant investment portfolio for the US government (see e.g., Greenspan (2007)). Thus, negative spending rates are simply the consequence of a larger inflow of funds (for instance budget surpluses for SWF or donations for university endowments) than the withdrawal of funds used for spending.

The advantage of using the Ornstein-Uhlenbeck process over more general mean reversion processes with non-negative values is that it allows a closed-form solution of the distribution of AUM (F_t^m). Given that the purpose of the present paper is to gain insight into how spending policies affect asset values, it is valuable to study a case in which this relationship can be inspected analytically. In addition, for the parameter values we use to exemplify this spending policy, the probability of observing a negative spending rate is low.

The future value of AUM is now given by

$$F_t^m = F_0 e^{R_t - Z_t^m}. \tag{10}$$

The expressions for R_t and Z_t^m (the expression for Z_t^m is presented in the appendix) imply that F_t^m has a lognormal distribution. Let $M_t = E[R_t - Z_t^m]$ and $V_t = \mathrm{var}(R_t - Z_t^m)$ be the mean and variance, respectively, of the difference between the cumulative log-return on AUM and the cumulative spending rate over the time interval $[0, t]$. The expressions for M_t and V_t are presented in the appendix.

Look first at the expected fund value. We use the lognormal property of F_t^m in expression (10) to calculate

$$E[F_t^m] = F_0 e^{M_t + \frac{1}{2} V_t}. \tag{11}$$

Recall that, under the fixed-rate policy, the expected asset value is F_0 at all horizons. Thus, the expected value of AUM at time t is higher under the mean-reverting policy if and only if $M_t + \frac{1}{2} V_t > 0$. Let us assume that the target rate of withdrawal $\bar{s}$ is equal under the two policies and that the initial spending rate s_0^m is equal to the target rate; that is, $\bar{s} = \mu = s_0^m$. Inserting for this parameter configuration in the expressions for M_t and V_t, we find that

$$M_t + \frac{1}{2} V_t = -\frac{2\rho\sigma\sigma_s}{\kappa^2} \underbrace{(\kappa t + e^{-\kappa t} - 1)}_{\geq 0} + \frac{\sigma_s^2}{2\kappa^3} \underbrace{(2\kappa t - 3 + 4e^{-\kappa t} - e^{-2\kappa t})}_{\geq 0}. \tag{12}$$

The last term of this expression is the variance of Z_t^m and is positive. The first term is the covariance between R_t and Z_t^m and its sign is determined by the sign of ρ. There is a critical

$$\rho^* = \frac{\sigma_s}{4\kappa\sigma} \frac{2\kappa T - 3 + 4e^{-\kappa T} - e^{-2\kappa T}}{\kappa T + e^{-\kappa T} - 1} > 0$$

such that $M_t + \frac{1}{2}V_t > 0$ for $\rho < \rho^*$. Thus, when $\rho < \rho^*$ we see from expression (11) that $E[F_t^m] > F_0$. We note in particular that this case includes the parameter value $\rho = 0$. A noisy mean-reverting spending rate gives a higher expected AUM, even if it is uncorrelated with asset returns. We show in the appendix that expected future values of AUM increase in the variance of the difference $R_t - Z_t$.

We note from expression (12) that $M_t + \frac{1}{2}V_t$ is strictly decreasing in ρ. Thus, a high value of ρ is associated with a low expected future fund value, and vice versa for a low value of ρ. As indicated by our discussion at the start of this section, the purpose of mean-reverting policies is for them to be countercyclical. This property clearly points to $\rho < 0$; counter cyclicality would tend to set a high spending rate when returns are low and vice versa. When $\rho < 0$ the spending level tends to be lower than in the fixed-rate case when returns, and thereby asset values, are higher.

The variance of the future value of AUM is

$$\mathrm{var}(F_t^m) = F_0^2 e^{2M_t + V_t}(e^{V_t} - 1).$$

Expected values of AUM with standard deviations are plotted in Figure 1. First, we observe that even though the expected spending rate is equal to the fixed spending rate, the expected value of AUM increases in time. The cost is a higher standard deviation of future AUM-values. The model predicts a lower expected spending level for the first 25 years. It is interesting to observe that the standard deviation of future spending is lower for spending in years 3-12. Here we have assumed a low volatility of the spending rate ($\sigma_s = 0.01$) and a high force of mean reversion ($\kappa = 0.5$). Still, the effect from the stochastic spending rate on future spending and AUM is clearly significant.

In Figure 2 we plot expected future fund values for different parameter combinations. The values are sensitive to many of the parameter values. In particular, observe the sensitivity to σ_s and ρ, parameters determining the statistical properties of the spending rate and the statistical relationship between the spending rate and returns.

It is not clear what is to be understood by a "sustainable spending rate" in the current setting when the rate follows a stochastic process. By our definition, the requirement is that $E[F_t] = F_0$, implying that $M_t = -\frac{1}{2}V_t$. If we use expressions (18) and (19), it is clear that both the long-term level of the spending rate, $\bar{s}$, and the initial spending rate, s_0^m, can only give a sustainable spending level for a given time horizon t. To illustrate, we calculate the target spending rate $\bar{s}$ that gives (expected) asset preservation for $t = 25$. We then calculated the expected asset value for different ts with this level of $\bar{s}$. We performed the same calculations for the s_0^m that gives expected asset preservation for $t = 25$. The results are plotted in the first panel of Figure 3. Although the graph for the sustainable $\bar{s}$ seems to be independent of t, a higher σ_s shows that the graph is not flat. In the lower panel of Figure 3 we used the same levels of $\bar{s}$ and s_0^m to calculate expected "annual" cumulative spending rate.

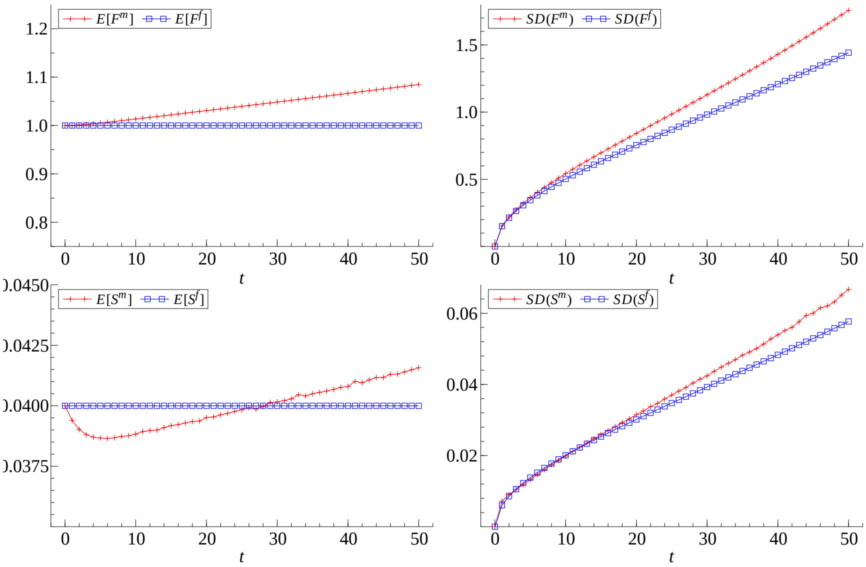

Fig. 1 The upper two panels show expected fund values with standard deviations for the policy with a mean-reverting spending rate ($E[F^m]$ and $SD(F^m)$) and expected fund values with standard deviations for the fixed-rate policy ($E[F^f]$ and $SD(F^f)$). The lower two panels show the corresponding expectations and standard deviations for the future spending levels ($E[S^m]$, $E[S^f]$, $SD(S^m)$, and $SD(S^f)$). The moments $E[S^m]$ and $SD(S^m)$ are estimated using Monte Carlo simulations (1,000,000 simulation runs). Parameter values are: $F_0 = 1$, $\sigma_S = 0.15$, $\mu = s_0^m = \bar{s} = 0.04$, $\sigma_s = 0.01$, $\kappa = 0.5$, $\rho = -0.5$.

5 Spending Level based Policies

Many spending policies used in practice have mechanisms that smooth the spending level over time. As discussed in the Introduction, endowments and charities often adjust the spending level gradually by including a backward-looking element in their spending policy. We now discuss how such backward-looking spending rules affect fund value dynamics and the spending opportunities in the long run.

5.1 A Constant Spending Level

Acharya and Dimson (2007) described how policies of endowments at the Universities at Oxford and Cambridge traditionally targeted a certain fixed *level* of real spending. Ameriks and Jaconetti (2006) also mentioned constant real spending over time as a possible policy for endowments and foundations. In household finance, the influential retirement planning model of Bengen (1994) proposes withdrawing a fixed proportion (four percent) of wealth at the start of retirement, and then holding spending fixed in real terms at this level. A

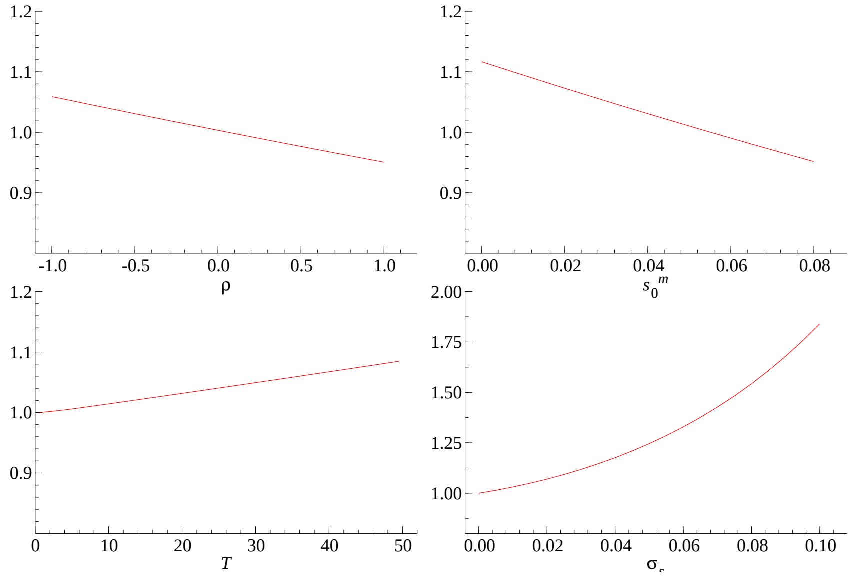

Fig. 2 The plots show expected fund values. Base-case parameter values are: $F_0 = 1$, $\mu = s_0^m = \bar{s} = 0.04$, $\sigma = 0.15$, $\sigma_s = 0.1$, $\kappa = 0.5$, $\rho = -0.5$, $T = 10$.

policy that attempts to keep spending fixed at a predetermined level is a basic example of a backward-looking policy. As recognized by Bengen, even if one manages to keep real spending constant, the spending *rate* will fluctuate with asset markets. In addition, strict adherence to this policy implies that the endowment or fund may be depleted.[6]

Let S_0^c be current (real) spending, where superscript c denotes constant. With a constant spending level, fund dynamics are

$$\begin{aligned} dF_t^c &= \mu F_t^c dt + \sigma F_t^c dB_t - S_0^c dt \\ &= (\mu - s_t^c) F_t^c dt + \sigma F_t^c dB_t. \end{aligned} \tag{13}$$

The instantaneous change in fund value is equal to the change in a lognormally distributed variable less a constant withdrawal $S_0^c dt = s_t^c F_t dt$.

With risky returns, a constant spending level implies a real possibility of fund depletion for any positive initial spending rate s_0^c. From the fund dynamics in expression (13), we calculate that the depletion time τ is the (first) time where

$$e^{(\mu - \frac{1}{2}\sigma^2)\tau + \sigma B_\tau} = s_0^c \tau.$$

[6] Using the revenue from the book *Sophie's World*, the author (Jostein Gaarder) set up the Sophie Prize. The goal was to spend USD 100,000 each year on prizes for environment and development, as long as the fund backing the prize was not depleted. The first price was awarded in 1998 and the last in 2013, after which the fund was depleted.

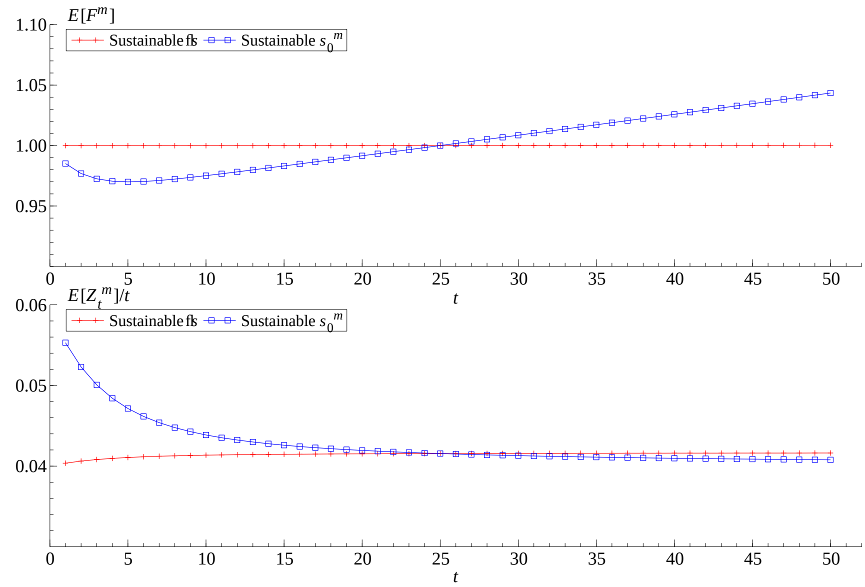

Fig. 3 The upper panel shows expected fund values. The target spending rate $\bar{s}$ is expected to be sustainable for $t = 25$. The sustainable initial spending rate s_0^m is expected to sustainable for $t = 25$. The lower panel shows expected "annual" cumulative spending rate for the sustainable levels of $\bar{s}$ and s_0^m used in the first panel. Base-case parameter values are: $F_0 = 1$, $\mu = s_0^m = \bar{s} = 0.04$, $\sigma = 0.15$, $\sigma_s = 0.01$, $\kappa = 0.5$, $\rho = -0.5$.

Although this policy aspires to keep real spending constant over time, the spending level is a binary variable following a (one-) jump process; it is either S_0^c or 0. This spending process contrasts with the spending from the fixed-rate policy discussed in Subsection 4.1, where spending follows the dynamics in expression (8). Two paths for the fund value in which the fund is eventually depleted are plotted in Figure 4. Observe how the volatility vanishes as the fund is about to be depleted; that is, as $F_t^c \to 0$. Thus, the volatility term in the dynamics in expression (13) also approaches zero while the spending *rate* approaches infinity.[7]

We also note that although the level of spending is constant as long as $F_t^c > 0$, the spending rate, $s_t^c = S_0^c / F_t^c$, is clearly stochastic. Using Ito's lemma, we see that

$$ds_t^c = \left((s_t^c)^2 - \mu s_t^c + \sigma^2 S_0^c\right) dt - \sigma s_t^c dB_t.$$

The instantaneous covariance between changes in the spending rate and the asset value is $-\sigma^2 s_t^c F_t^c = -\sigma^2 S_0^c$, and thus negative. Because the spending level is fixed for $F_t^c > 0$, a higher fund value leads to a lower spending rate. The

[7] Although the spending rate $s_t \to \infty$ as $F_t \to 0$, the drift term $\mu F_t - S_0$ satisfies Lipschitz and the growth condition (see e.g., Duffie (2001) pp. 340-341) and a unique solution of the SDE for the fund value in (13) exists.

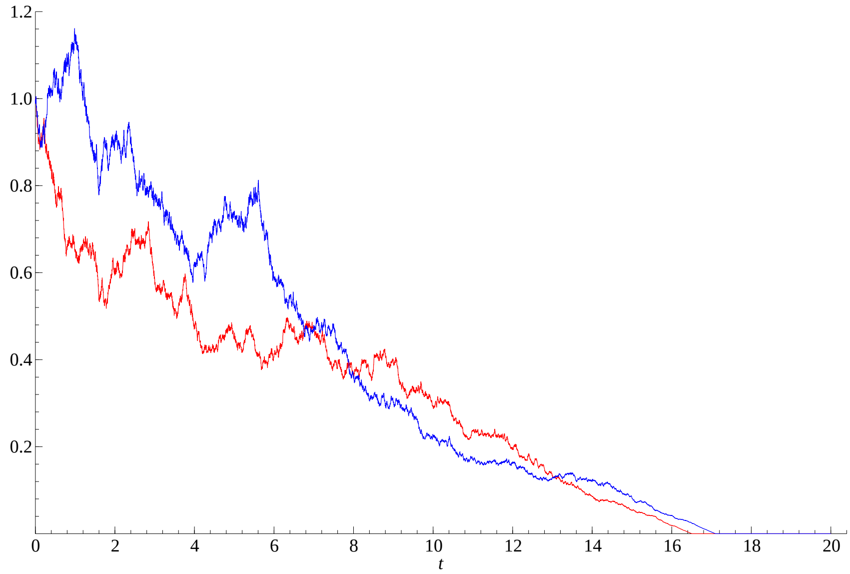

Fig. 4 The plots show two paths where the fund is depleted. Base-case parameter values are: $F_0 = 1$, $\mu = S_0 = 0.04$, $\sigma = 0.15$.

net effect of a negative volatility term in the spending rate is often a reduction in the (instantaneous) volatility of the spending level. In the case we have analyzed in this subsection, the net effect is to have a constant spending level (if the fund can support the spending level).

In the right-hand panels of Figure 5 we plot the distributions of the asset value at a 20-year horizon for the fixed-rate and the constant spending policy. There is no analytical solution for the distribution of future AUM-values under the constant spending policy, so we approximate it numerically. We set $s_0 = \mu = 0.04$, $\sigma = 0.15$ (at annual basis), $F_0 = 1$, and $t = 20$. It is evident that this policy results in rather different distribution of F_t than the fixed-rate policy. The constant spending policy can result in asset depletion. This fact is reflected in the distribution for AUM. It has pronounced left-hand tail risk and a higher probability of low asset values. The lower-right panel shows that, given our parameter values, there is a probability of approximately $\frac{2}{3}$ that the fund value is lower with the constant spending policy than with the fixed-rate policy. The binary nature of the future spending level under the constant policy is illustrated in the left-hand panels of Figure 5. The lower-left panel shows that this policy results in fund depletion within 20 years, with a probability of approximately 10 percent.

In Figure 6 we plot the expected value and standard deviation of spending and of AUM value for different time horizons. The lower-left panel shows that, in the short run (less than 10 years with the current parameter configuration),

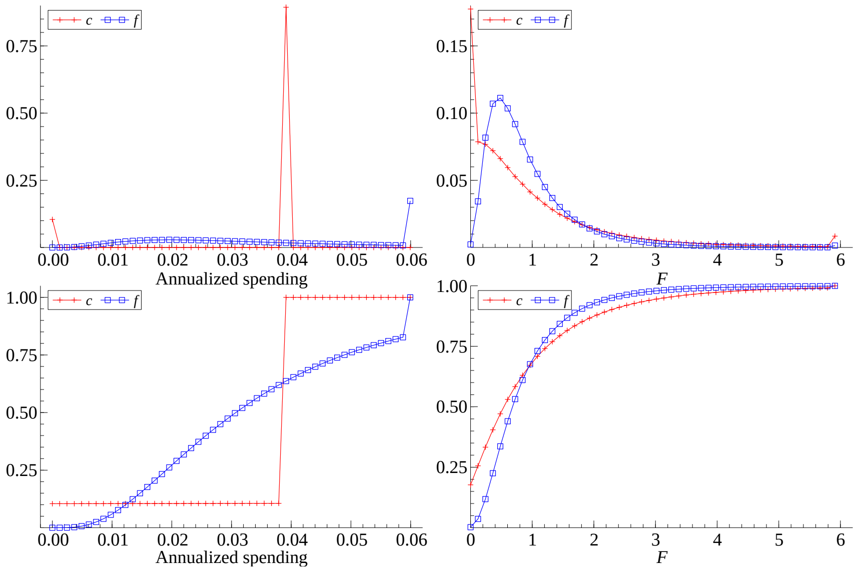

Fig. 5 The plots in the upper panels show the distribution of monthly spending and the distribution of fund value within 20 years. The plots in the lower panels show the corresponding cumulative probability distributions. Parameter values are: $F_0 = 1$, $\mu = s_0 = 0.04$, and $\sigma = 0.15$. The distributions are estimated with Monte Carlo simulations with 1,000,000 runs.

there is no risk with respect to the level of spending. The standard deviation starts to increase from years 10-12, but it always stays low compared to the case with a fixed spending rate. From the upper-left panel we see that the expected spending level decreases from about year 12. This behavior of the expectation and the standard deviation is explained by the fact that it takes about 10 years before we observe any simulated asset-value paths with depletion and thereby zero spending. Thus, one of the costs of having a low risk on the spending level is that the expected spending level decreases. The lower-right panel illustrates another cost associated with the constant spending policy. The standard deviation of the future AUM value is significantly higher than for the fixed-rate policy. The upper-right panel in Figure 6 shows that the expected asset value increases in t under the constant spending policy. This happens because we truncated the distribution of F at zero, while the right-hand tail is unbounded. Paths with high realizations of asset returns will then tend to drive expected AUM upwards as we look far into the future. While this policy reduces short-term spending risk, the risk does not disappear. The risk is transferred to higher asset risk and the risk of fund depletion and, thus, no more spending. It is clearly impossible to remove all spending risk as long as spending is supported by a risky asset portfolio.

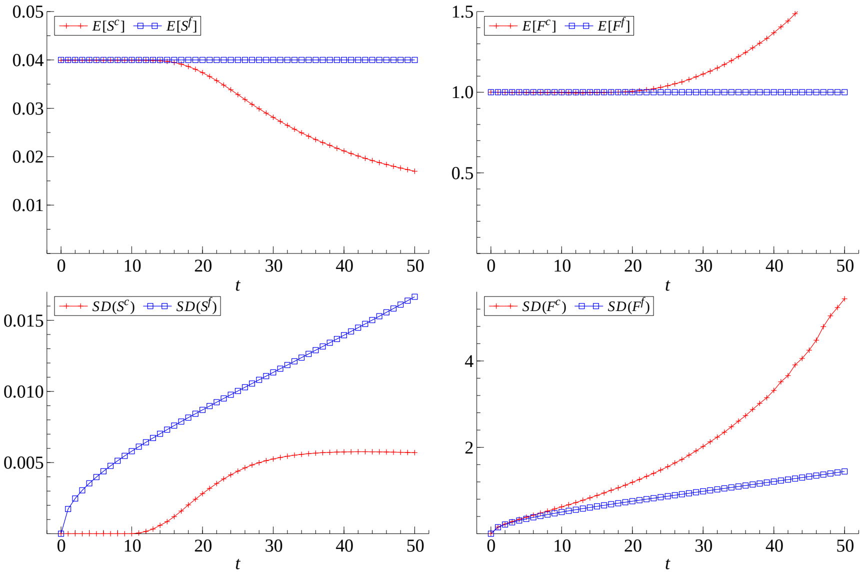

Fig. 6 The plots show expected spending and AUM value ($E[\cdot]$) and corresponding standard deviations ($SD(\cdot)$) for different time horizons t. Parameter values are: $F_0 = 1$, $\mu = s_0 = 0.04$, $\sigma = 0.15$. t is measured in years. For the constant spending policy, the expectations and standard deviations are estimated by Monte Carlo simulations with 100,000 simulation runs.

5.2 A Hybrid Policy

A common spending policy among American university endowments is to combine the fixed-rate policy discussed in Subsection 4.1 with a backward-looking element intended to dampen the impact of capital market fluctuations on current spending. In practice, this policy sets annual real spending equal to some fraction $\alpha \in [0, 1]$ times last year's spending plus $1 - \alpha$ times target spending out of the current asset value. The weight α on short-term smoothing is typically in the $60 - 80$ percent range among elite universities.

As before, we assume that the target spending rate out of the current asset value is μ. The time t spending level under the hybrid spending policy can then be formulated as (top script h indicates hybrid policy)

$$S_t^h = \alpha X_t + (1 - \alpha)\mu F_t^h, \tag{14}$$

where X_t determines how past spending influences the spending level at time t. This level of spending is conditional on the endowment having sufficient funds to support $S_t^h \geq 0$. In addition, the fund can be depleted in this case. A spending policy that always gives some weight to (recent) historical spending levels will effectively be influenced by all former spending levels. We try to capture this influence by assuming that the backward-looking element X_t is a weighted average of past spending with the weights declining exponentially into the past:

$$X_t = X_0 e^{-\beta t} + \beta \int_0^t S_\tau^h e^{-\beta(t-\tau)} d\tau. \tag{15}$$

In expression (15), $\beta \geq 0$ is a parameter that determines the relative weight of spending in earlier time periods. The larger the β, the more important spending in the recent past is in determining the backward-looking part of current spending. We note that the special case $\alpha = 1, \beta = 0$ corresponds to the constant spending policy, while $\alpha = 0$ takes us back to the fixed-rate policy.

By inserting for the spending level in expression (14) into the dynamics in expression (1), we find that the asset value dynamics under the hybrid policy are

$$dF_t^h = \alpha(\mu F_t^h - X_t)dt + \sigma F_t^h dB_t.$$

The asset value distribution is a function of the state variable X_t. The distribution of this state variable cannot be found analytically and we simulate paths for the evolution of F^m. We note that the mean growth rate of assets under management, $E[\frac{dF_t^h}{F_t^h}]$, is state-dependent: it is positive at time t if the instantaneous expected portfolio return μ is larger than the ratio of time-weighted past spending to current asset value, $\frac{X_t}{F_t^h}$, and negative if $\mu < \frac{X_t}{F_t^h}$.

From expression (15) we find that

$$dX_t = \beta(S_t^h - X_t)dt. \tag{16}$$

Thus, the backward-looking part X_t responds linearly to past spending. Using this expression for dX_t, we find from expression (14) that

$$dS_t^h = \alpha\beta(S_t^h - X_t)dt + (1-\alpha)\mu dF_t^h.$$

If current spending is higher than the time-weighted average of past spending, $S_t^h > X_t$, the spending level tends to further increase, and vice versa for $S_t^h < X_t$.

The spending that can be supported by the endowment under the fixed-rate policy can experience sharp changes as asset values fluctuate. Universities and others use this hybrid policy, or something similar, to reduce short-term spending volatility. Sharp changes in the spending level can be costly and inconvenient. For instance, research projects may have to be prematurely aborted when the fund value is low. Restarting projects when the fund value is high can incur high start-up costs. Therefore, there are compelling arguments that support the idea of reducing spending volatility. However, there are costs associated with short-term smoothing of the spending level. The value of the asset portfolio immediately becomes more volatile and this increased asset volatility projects onto future spending volatility. In addition, the smoothing results in a positive probability of fund depletion.

In Figure 7, we plot the spending and asset value distributions at a 20-year horizon for the fixed-rate and the university policy. We use the same parameter values as in Subsection 5.1, with the addition of setting $\alpha = 0.75$ and $\beta = 0.2$ for the hybrid policy. The difference between the distributions of these two policies is less pronounced than the difference between the fixed-rate and the constant spending policy. The distributions for the hybrid policy inherit the shapes of the fixed-rate policy and the constant spending policy. The hybrid policy has a higher left-hand tail risk of both spending and asset value. Close inspection of the lower-right panel reveals that the university policy implies an asset depletion probability of approximately five percent at the 20-year horizon, given our parameters. Similar calculations for different values of α show that the depletion risk is lower when the α is lower. The lower-right panel also shows that there is an approximately 70 percent probability that the fund value is lower under the hybrid policy than under the fixed-rate policy.

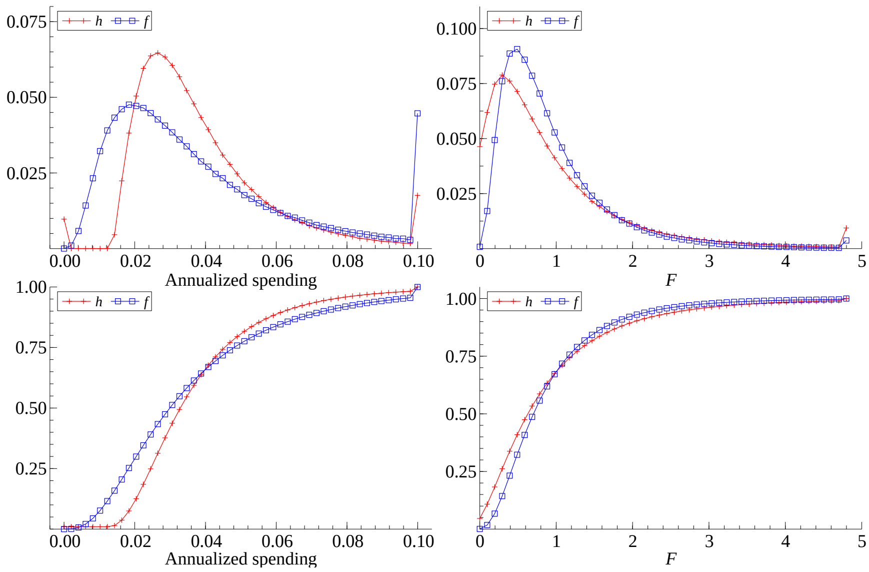

Fig. 7 The plots in the upper panels show the distribution of annualized monthly spending and the distribution of fund value in 20 years for the fixed-rate and the hybrid policy. The plots in the lower panels show the corresponding cumulative probability distributions. Parameter values are: $F_0 = 1$, $\mu = s_0 = 0.04$, $\sigma = 0.15$, $\alpha = 0.75$, $\beta = 0.2$, and $X_0 = 0.04 \cdot \frac{1}{12}$. The distributions are estimated with Monte Carlo simulations with 1,000,000 runs and 12 time steps per year.

Figure 8 plots the expected value and standard deviation of spending and asset value for different time horizons, again comparing the hybrid policy to the policy with a fixed spending rate. The upper panels show that expected future spending levels and fund values for the fixed-rate policy and the hybrid policy are indistinguishable. The lower-right panel shows that future asset values are more volatile under the hybrid policy for all time horizons; this illustrates one of the costs of smoothing of short-term spending, as discussed above. The lower-left panel illustrates that the reduction in spending volatility can be large. The panel also illustrates how the increased asset volatility eventually leads to higher spending volatility than the fixed-rate policy. With the current parameters, it takes approximately 40 years before the spending volatility for the hybrid policy exceeds the volatility under the fixed-rate policy.

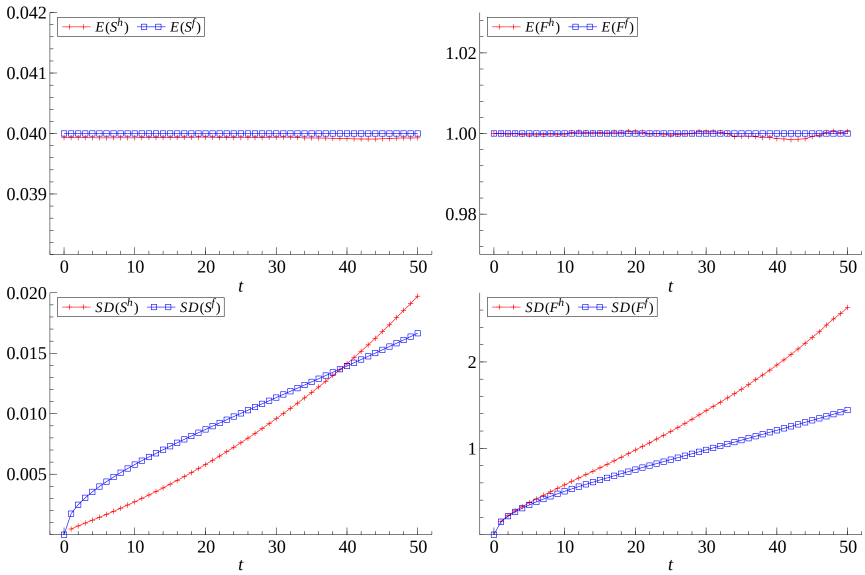

Fig. 8 The plots in the upper panels show expected spending and fund value for different points in time. The plots in the lower panels show the corresponding standard deviations. Parameter values are: $F_0 = 1$, $\mu = s_0 = 0.04$, $\sigma = 0.15$, $\alpha = 0.75$, $\beta = 0.2$, and $X_0 = 0.04 \cdot \frac{1}{12}$. The distributions are estimated with Monte Carlo simulations with 1,000,000 runs and 12 time steps per year.

6 Implications and Validity

From the probability distributions for future values of AUM and the distributions for future spending levels, we cannot say that one spending policy stochastically dominates any of the other policies. This fact is illustrated in Figure 9 for 1. order stochastic dominance for the value of AUM in 20 years. When one spending policy is chosen over others, there can be many factors influencing the choice. Short-term smoothing of the spending level can for some endowments be more important than for others. For endowments with strong interest groups as claimants, spending policies with little or no discretion with respect to the spending level can be important to secure sustainable spending levels. In practice, the most popular spending policy among universities is the moving-average policy. Between 65-75 percent of the universities in the NACUBO database use this spending policy (see Table 3 in Brown and Tiu (2013)). This policy is easy to implement, it gives little discretion for the spending level, and it gives less variation in short-term spending than our policy with a fixed spending rate. These factors may explain why many universities prefer this type of spending policy.

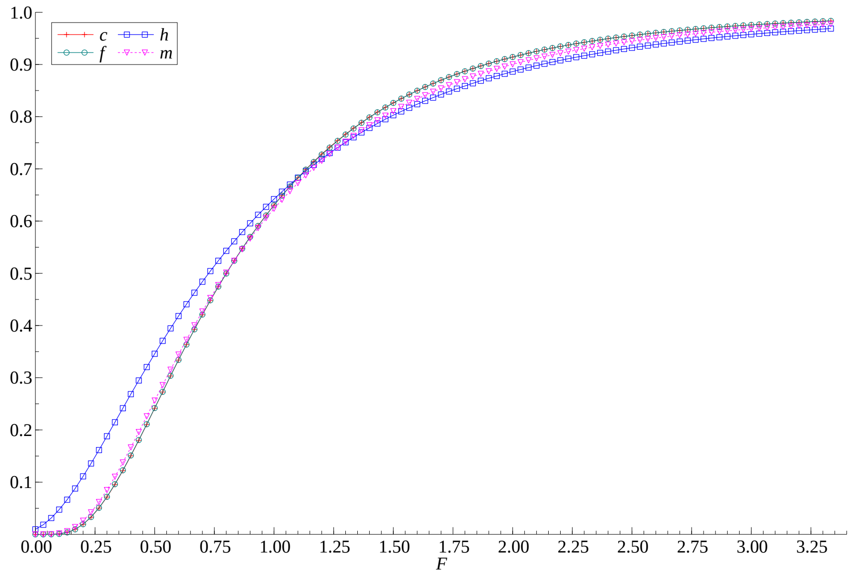

Fig. 9 The graphs show the cumulative probability distribution for the value of AUM in 20 years for our four stylized spending policies. Parameter values are: $F_0 = 1$, $\mu = s_0 = \bar{s} = 0.04$, $\sigma = 0.15$, $\alpha = 0.75$, $\beta = 0.2$, $X_0 = 0.04 \cdot \frac{1}{12}$, $\sigma_s = 0.015$, $\kappa = 0.5$, and $\rho = -0.5$. The distributions are estimated with Monte Carlo simulations with 1,000,000 runs. For policy with a constant spending level and for hybrid policy, we use 12 time steps per year.

A sound implementation of the spending policies requires good estimates of both expected returns on the AUM and a good understanding of the return risk. Estimates of the expected return are important to determine how much spending the fund will support. Our results show that the joint stochastic properties of the returns and the spending rates are important. Thus, a good grasp on the return risk can help the decision makers in the process of forming an expedient spending policy. The empirical fact that 25 percent of university endowments make changes to their spending rate or spending policy each year (see Brown and Tiu (2013)), indicates that this process is important and possibly challenging.

For two of our stylized spending policies, many of the results we present are based on Monte Carlo simulations. These results are based on a given set of parameter values and other sets of parameters can give other results. We have used parameter values that we think are reasonably close to what can be empirically estimated in financial markets. As robustness tests, we have also tried to alter the parameter values without observing significant changes to how the spending policies work (not reported in the paper). We therefore think our numerical results shed light on important issues associated with different spending policies. These are issues that can be important for decision makers to take into account when choosing between different spending policies.

7 Conclusions

In this paper we have analyzed four stylized spending policies that resemble policies that are widely used in practice. With a given exogenous return process for the risky asset portfolio, we have focused on how the spending policies affect future distributions for fund values and spending levels. First, we showed that the stochastic properties of the cumulative spending rate and the joint stochastic properties of the cumulative spending rate and the asset returns are important for future expected fund values. The first policy we analyzed dictates a constant spending rate. For the second policy we analyzed, the spending rate follows an exogenously specified mean-reverting stochastic process that may be correlated with the return process for the risky assets. This policy clearly shows the importance of the correlation between the spending rate and the asset returns for expected future values of AUM. The fund is never depleted under these two policies. The third policy dictates a constant spending level, resulting in a high spending rate when the fund value is low and a low spending rate when the fund value is high. This policy leads to an expected decline in the spending level as time passes, and an increase in the expected fund value. The fourth policy is one that is often used by university endowments. It dictates a spending level that is (close to being) a weighted combination of the first policy and the third policy. To reduce short-term fluctuations in the spending level, it puts positive weight on previous spending levels. A consequence of this backward-looking behavior of these last two policies is that the fund can be depleted. Short-term smoothing in the spending level leads to more volatile asset values and, eventually, increasing spending volatility or decreasing expected spending level. While a well-crafted spending policy can transform the financial risk in the asset portfolio to a certain extent to better protect the spending level, it cannot remove the financial risk. Table 2 summarizes the paper's main findings for the four stylized spending policies.

Table 2 The table summarizes the main findings of the paper. For each of the four spending policies, the table shows expected future value of AUM ($E[F_t]$), the variance of AUM ($\text{var}(F_t)$), xpected future spending level ($E[S_t]$), the variance of the spending level ($\text{var}(S_t)$), if the policy leads to short-term spending smoothing (relative to the fixed-rate policy), and the default probability ($\Pr(F_t = 0)$).

	Fixed rate	Mean reverting	Constant spending	Hybrid
$E[F_t]$	F_0	$F_0 e^{M_t + \frac{1}{2} V_t}$	$> F_0$	$\approx F_0$
$\text{var}(F_t)$	$F_0^2 \left(e^{\sigma^2 t} - 1\right)$	$\text{var}(F_t^m) = F_0^2 e^{2M_t + V_t}(e^{V_t} - 1)$	$> F_0^2 \left(e^{\sigma^2 t} - 1\right)$	$> F_0^2 \left(e^{\sigma^2 t} - 1\right)$
$E[S_t]$	S_0	$\leq S_0$	$< S_0$	$\approx S_0$
$\text{var}(S_t)$	$\mu^2 F_0^2 \left(e^{\sigma^2 t} - 1\right)$	$\leq \mu^2 F_0^2 \left(e^{\sigma^2 t} - 1\right)$	$< \mu^2 F_0^2 \left(e^{\sigma^2 t} - 1\right)$	$\leq \mu^2 F_0^2 \left(e^{\sigma^2 t} - 1\right)$
Short-term spending smoothing	No	$\rho < -\frac{\sigma_s}{2\mu\sigma}$	Yes	Yes
$\Pr(F_t = 0)$	0	0	> 0	> 0

We did not find that any of the policies stochastically dominate any of the other policies.

A Future Fund Value

We have that

$$dF_t = (\mu - s_t)F_t dt + \sigma F_t dB_t.$$

Consider $Y_t = \ln F_t$. Itô's lemma gives that

$$\begin{aligned} dY_t &= \frac{1}{F_t} dF_t - \frac{1}{2}\frac{1}{F_t^2}(dF_t)^2 \\ &= (\mu - \frac{1}{2}\sigma^2 - s_t)dt + \sigma dB_t. \end{aligned}$$

Integrating both sides from 0 to T, we get

$$Y_T = Y_0 + \int_0^T (\mu - \frac{1}{2}\sigma^2 - s_t)dt + \int_0^T \sigma dB_t.$$

Thus,

$$e^{Y_T} = F_T = F_0 e^{R_T - Z_T},$$

where R_T and Z_T is defined in Section 2.

B Z_T when s_t is mean reverting

Here we show how to find Z_T.

The SDE for the spending rate is

$$ds_t = \kappa(\bar{s} - s_t)dt + \sigma_s dB_t.$$

Multiply both sides by the integrating factor e^{kt} to get

$$e^{\kappa t} ds_t + \kappa e^{\kappa t} s_t dt = \kappa e^{\kappa t}\bar{s} dt + e^{\kappa t}\sigma_s dB_t.$$

Integrate both sides from 0 to t, add both sides by s_0, and multiply by $e^{-\kappa t}$ to get

$$s_t = s_0 e^{-\kappa t} + \bar{s}(1 - e^{-\kappa t}) + \int_0^t \sigma_s e^{-\kappa(t-v)} dB_v. \tag{17}$$

We now integrate both sides of equation (17) with respect to t from 0 to T and get[8]

$$Z_T = \frac{1}{\kappa}(1 - e^{-\kappa T})(s_0 - \bar{s}) + sT + \frac{\sigma_s}{\kappa}\int_0^T (1 - e^{-\kappa(T-v)})dB_v.$$

C M_t and V_t when s_t is mean reverting

From the expression for Z_t, we calculate that

$$M_t = (\mu - \bar{s} - \frac{1}{2}\sigma^2)t - \frac{1}{\kappa}(1 - e^{-\kappa t})(s_0^m - \bar{s}) \tag{18}$$

and

$$\begin{aligned} V_t =& \sigma^2 t - \frac{2\rho\sigma\sigma_s}{\kappa^2}(\kappa t + e^{-\kappa t} - 1) \\ &+ \frac{\sigma_s^2}{2\kappa^3}(2\kappa t - 3 + 4e^{-\kappa t} - e^{-2\kappa t}). \end{aligned} \tag{19}$$

[8] Note that $\int_0^T \int_0^t \sigma_s e^{-\kappa(t-v)} dB_v dt = \int_0^T \int_v^T \sigma_s e^{-\kappa(t-v)} dt dB_v$.

D The effect of $\mathrm{var}(R_t - Z_t)$ on $E[F_t]$

We can write the future value of AUM as $F_t = f(R_t - Z_t)$, where the function f is convex. Assume $E[R_t - Z_t] = 0$. By using a Taylor expansion of F_t around the point $R_t - Z_t = 0$ and taking expectations, we get

$$E[F_t] = F_0 + \frac{1}{2} f''(0)\,\mathrm{var}(R_t - Z_t) + \sum_{n=4}^{\infty} \frac{f^{(n)}(0)}{n!} E[(R_t - Z_t)^n], \tag{20}$$

where the ns are even numbers and $f^{(n)}(0) > 0$. As $\mathrm{var}(R_t - Z_t)$ is strictly increasing in σ_s, we also see from the Taylor expansion in (20) that $E[F_t]$ is increasing in σ_s. Intuitively, a wider distribution for $R_t - Z_t$ with the same mean increases the expected value of F_t because F is a convex function.

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