Stability Assessment of Power Systems Based on a Robust Sum-Of-Squares Optimization Approach

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Abstract—Secure operation of the evolving power systems, characterised by more renewable energy sources and increasingly variable consumption, will require enhanced monitoring and more automatic control actions. One example is the need for fast detection and control actions to avoid loss of synchronism or grid islanding caused by transient instability. In this paper, we present a method suitable for on-line transient stability assessment of power systems, based on Lyapunov's second method for stability analysis of dynamical systems. The method uses Sum-Of-Squares optimization to algorithmically construct a Polynomial Lyapunov Function and estimate the Region-Of-Attraction for a given stable operating state. The main benefit of the method is that it obviates the painstaking process of finding a suitable Lyapunov function. Our approach includes a robust handling of the truncation error in the Taylor series expansion of the system model, and thereby ensures that the estimate of the region of attraction around an operating point is inside the actual region of attraction. Using a single-machine-infinite-bus system, we demonstrate the application of the method in this paper.

Keywords—Direct Methods for Stability, Polynomial Lyapunov Function, Region of Attraction, Sum-Of-Squares Optimization, Transient Stability Analysis.

I. Introduction

Today's power systems are characterised by a mix of different renewable energy sources and increasingly variable consumption. These characteristics will even be more prominent in future power systems. The natural variability of renewable generation and active loads results in a highly dynamic system with large variations in power flows. Maintaining the power balance and system stability at all times becomes an increasing challenge for power system operators. The complexity and variability of the systems call for more automatic monitoring and control actions in order to ensure secure system operation. In a more complex system it is likely that transient stability phenomena, resulting in loss of synchronism and grid islanding, becomes a greater risk. On the other hand, with more distributed generation, it is also more likely (or technically possible) that separate grid islands can continue to operate if the separation is properly controlled. However, this demands that functions in today's Energy Management and Supervisory Control And Data Acquisition (SCADA) systems are extended to take into account on-line Transient Security Analysis (TSA). Information about critical system stability properties of the current operating state, and a set of credible contingencies, must be readily available. Based on this information and fast detection of a problem, automatic control actions can be taken that minimise the loss of load. Such control schemes require several breakthroughs in measurement systems, computation methods, and control schemes.

At present, stability analysis programs routinely used in utilities around the world are based mostly on offline step-by-step numerical integration of power system stability models used to simulate system dynamic behaviour [1]. This analysis is not suitable for on-line TSA as it requires long computation times, involving studies of several contingencies, and does not provide information regarding the degree of stability or instability.

An alternative approach to TSA is Lyapunov's second method also known as Lyapunov's direct method. This approach employs a Lyapunov function to estimate the Region-Of-attraction (ROA) [1], [2]. The Lyapunov function approach to transient stability analysis, however, has been traditionally considered very difficult due to the lack of a systematic methodology for constructing Lyapunov functions.

The approach using Sum-Of-Squares (SOS) decomposition and Polynomial Lyapunov Functions (PLF) has been proposed to determine wider estimates of the ROA for non-linear dynamical models [3], [4], [5], [6], [7], [8]. The PLF approach uses SOS optimization technique to progressively obtain estimates of the ROA. The main advantage of this approach lies in its ability to algorithmically synthesize the Lyapunov function, which is a key element in stability assessment. The key challenge in PLF based approaches, however, is that most of the proposed methods have numerical problems when higher order Lyapunov functions are used. Another problem is that the approach requires the system model to be polynomial, and of finite degree, whereas dynamic models for power systems normally involve trigonometric functions. This has normally been handled using Taylor series expansion of the trigonometric functions [4]. In this paper, a method suitable for on-line Transient Stability Assessment using a Polynomial-Lyapunov–Function to obtain an estimate of the ROA is proposed. The method is a further development of the approach in [4], whose main benefit is that it obviates the painstaking process of finding a suitable Lyapunov function. The method allows for the use higher order Lyapunov functions, which leads to wider estimates of the ROA. In the paper, we show how to robustly account for the approximation error when using a finite order Taylor series approximation, thereby ensuring that the estimate of the region of attraction around an operating point is inside the actual region of attraction. In contrast, the method in [4] could result in an exaggerated region of stability estimate. This paper is organised in six sections. Section I provides the background for transient stability assessment of power systems and the main contribution of the paper. In Section II, the general stability features of Lyapunov's second method for analysing stability of dynamical systems is briefly discussed and then in Section III, the SOS/PLF method is presented in detail. Power system modelling aspects are discussed in Section IV and then in Section V, results from some case studies showing the performance of the proposed approach are presented. Finally, the main conclusions are presented in Section VI.

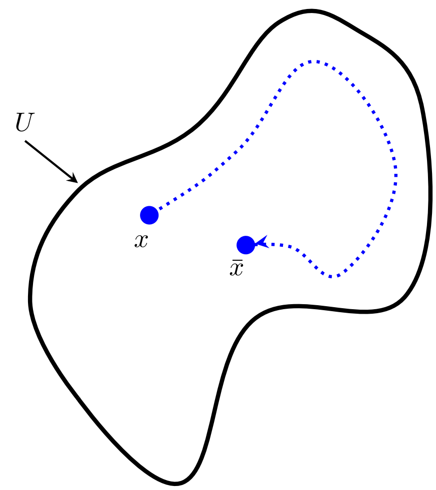

Fig. 1: An illustration of the ROA

II. Lyapunov's second method for stability

According to Lyapunov's second method for stability of dynamical systems, an operating point, $\bar{x} = 0$, is an equilibrium point of the system

$$\dot{x} = f(x) \tag{1}$$

if there exists a function $V(x)$, which is continuously differentiable in a neighbourhood, U, of $\bar{x} = 0$, such that:

$$\begin{aligned} &V(0) = 0; \\ &V(x) > 0 \quad \forall x \in U;\, x \neq 0 \\ &\dot{V}(x) \leq 0 \quad \forall x \in U;\, x \neq 0 \end{aligned} \tag{2}$$

In addition, the neighborhood U must be positive invariant, i.e., a state originating in U must remain in U for all future times. If such a function $V(x)$ exists, it is called a Lyapunov function for the system (1) in the neighbourhood U. In this work, an estimate of the ROA is based on a level set of the Lyapunov function, i.e., a set $\{x|V(x) \leq \gamma\}$, with γ some positive scalar. Level sets of Lyapunov functions are known to be positive invariant.

As shown in Fig. 1, the domain, U, of all states, x, from which the system converges to the equilibrium point, $\bar{x}$, without leaving the domain, is called the region of attraction (ROA) of the equilibrium point $\bar{x}$. For a perturbed system, the initial state x in Fig. 1 is the system state at the end of the disturbance. It should be noted that during the fault, the system is controlled by fault-on dynamics, which are different from (1). After the fault has been cleared, however, the trajectory of the system state x is governed by (1), starting from the state at the time of clearing the fault. If this state at the time of clearing the fault lies inside the ROA, conditions (2) can assert, without numerically integrating the post-fault trajectory that the system will eventually converge to its post-fault equilibrium $\bar{x}$. The knowledge of a Lyapunov function $V(x)$ and a scalar γ defining an ROA estimate can therefore allow for very quick assessment of system stability. ROA estimates based on Lyapunov function level sets are always conservative, i.e. there may be initial states outside the level set for which the state conveges to the equilibrium point, but there cannot be any state inside the level set for which the state does not converge to the level set (under the assumption that the dynamics are governed by (1), i.e., that no further fault occurs).

III. SOS/PLF Approach

A. Obtaining the ROA estimate

The starting point for our approach is the ROA analysis method proposed by [4]. The method iteratively improves feasible solutions to the equation set

$$V(x) - \epsilon_1 x^T x \text{ is SOS} \tag{3}$$

$$-\frac{d}{dx}V(x)f(x) - \epsilon_2 x^T x - s_1(x)(\gamma - V(x)) \text{ is SOS} \tag{4}$$

$$(\gamma - V(x)) - s_2(x)(\beta - p(x)) \text{ is SOS} \tag{5}$$

Here $V(x)$ is the PLF, and the first equation above, equation (3), ensures that this is strictly positive everywhere except at the origin. We only search for SOS polynomials $V(x)$ with zero constant term (as otherwise $V(0) \neq 0$) and zero first order terms (as otherwise $x = 0$ would not be the minimum of $V(x)$). ϵ_1 and ϵ_2 are small positive scalars. The second equation above, equation (4), ensures that the time derivative of $V(x)$ is negative for all $\{x|V(x) < \gamma\}$. The polynomial $s_1(x)$ is an SOS polynomial. In the third equation, $s_2(x)$ is also an SOS polynomial, and the equation ensures that the set $\{x|\gamma - V(x) \geq 0\}$ includes the set $\{x|\beta - p(x) \geq 0\}$. This is used to ensure that, as the iteration progresses, each new ROA estimate includes the ROA estimate from preceding iterations.

The equation set (3 - 5) contains several bilinear terms. In the second equation s_1 multiplies γ and $V(x)$, and in the third equation s_2 multiplies β. Optimization solvers for bilinear systems is an active research field (which we do not attempt to cover). Instead, in our work, we have resorted to the more common approach of iteratively solving linear sub-problems obtained by setting some of the variables constant. The iteration is initialized using a quadratic Lyapunov function obtained by solving a Linear Quadratic (LQ) problem for the linearised system.

B. Transforming non-polynomial model into a set of polynomial differential algebraic equations

Conditions (3 - 5) and SOS programming cannot be directly applied to study the stability of power systems because their models contain trigonometric non-linearities and are, therefore, non-polynomial. Hence, the first step is to transform the original non-polynomial system model (1) into a set of polynomial differential algebraic equations. This is achieved by applying a Taylor series expansion based procedure to obtain an estimate of the trigonometric functions in the model, as was shown in [4]. In this paper, however, the procedure in [4] is extended to robustly account for the approximation error when using a finite order Taylor series approximation.

In standard engineering mathematics textbooks (e.g. [9]) we find that, for an $(n+1)$ times differentiable function $g(z)$, with $z = a+v$ and a a known scalar, the Taylor series expansion of $g(z)$ around a is given by

$$g(a+v) = g(a) + vg'(a) + \frac{v^2}{2}g''(a) + \cdots + \frac{v^n}{n!}g^{(n)}(a) + \frac{v^{n+1}}{(n+1)!}g^{(n+1)}(a+\xi) \tag{6}$$

where $0 \le \xi \le v$. In power systems, the functions $g(z)$ in consideration are commonly sine or cosine functions, and their derivatives of all orders are therefore also sine or cosine functions. Therefore we can bound the final (error) term in the expansion by defining $\Delta = g^{(n+1)}(a+\xi)$, and we observe that $-1 \le \Delta \le 1$. Consequently, provided each trigonometric term in (1) enters affinely, we may for each term introduce the Taylor series expansion (6), and a corresponding bounded uncertain term Δ_i. If, for all possible combinations of extreme values of the Δ_is (from Taylor series expansion of different trigonometric terms), (4) is fulfilled, it will necessarily also be fulfilled for the actual value of $f(x)$ in (1). This follows from the observation that the Δ_is enter affinely in (4). The actual value of the this equation can therefore be found by linear interpolation between the values at the extreme values of Δ_i. A non-negative linear combination of sums of squares is clearly also a sum of squares. Note that the same $V(x)$ has to be used for all combinations of Δ_is, whereas the polynomials $s_1(x)$ may be different for each i provided all the $s_{1,i}(x)$ are SOS polynomials.

C. Estimating the ROA

Our approach to estimating the ROA follows closely that of [4]:

1) To initialize the calculations, the model in (1) is linearized at the operating point under study, yielding
$$\dot{x} = Ax \tag{7}$$
Then a quadratic Lyapunov function is found for the linearized system by solving the Lyapunov equation
$$A^T P + PA + Q = 0 \tag{8}$$
where Q is a positive definite matrix and P defines the corresponding quadratic Lyapunov function $V(x) = x^T Px$.
2) Then an ROA estimate based on the quadratic Lyapunov function is found by holding $V(x)$ fixed while maximizing γ in (4), with $s_1(x)$ as a degree of freedom in the optimization[1]. In accordance with the description above of the robust approach to handling the truncation error from the Taylor series expansion, one version of (4) is used for each combination of extreme values of the truncation errors Δ_i. The same $V(x)$ and γ are used in each of these equations, whereas different versions $s_{1,i}$ may be used for the polynomial $s_1(x)$. Due to the fact that $s_{1,i}$ multiplies γ, the maximal value of γ is found using bisection.
3) Setting β equal to the value of γ obtained in Step 1, $p(x)$ equal to $V(x)$ from Step 1, and keeping the $s_{1,i}$ from Step 1, γ is maximized for equations (3 - 5) using $V(x)$ and $s_2(x)$ as degrees of freedom[2].
4) Keeping γ and $V(x)$ fixed, $s_{1,i}$ is optimized in (4). This is a semidefinite feasibility problem, and since the interior point solvers generally return a solution in the analytic center of the feasible region [10], this provides opportunities for further optimization in the subsequent steps.
5) Keeping β, $p(x)$ and $s_{1,i}$ fixed, γ is maximized in (3 - 5) using $V(x)$ and $s_2(x)$ as degrees of freedom. At this point the initialization procedure is finished, and the next three points are iterated until the estimated ROA no longer increases in size, or until feasible solutions no longer can be found.
6) Perform Step 4 with the new values of γ and $V(x)$.
7) Set $p(x)$ equal to $V(x)$ in the previous step.
8) For fixed $p(x)$ and $s_{1,i}$, maximize β with γ, $V(x)$, and $s_2(x)$ as degrees of freedom and (3 - 5) as constraints. β multiplies $s_2(x)$ in (5), and therefore this maximization is performed using bisection on β. Note that in this step we attempt to maximize the size of the level set of $p(x)$ (the Lyapunov function from the previous iteration) that fits inside the ROA estimate for the present iteration.

It would be possible to arrange the calculations in steps 6 - 8 such that the computationally demanding bisection in step 8 could be avoided. However, in our experience the calculations proceeded much more reliably when optimizing β and $s_2(x)$ simultaneously.

[1] More precisely, it is the coefficients of the polynomial $s_1(x)$ that are degrees of freedom in the optimization.

[2] Now $V(x)$ is of full degree, and it is no longer constrained to be a quadratic function.

IV. Power system modelling

The dynamics of power system components are represented by mathematical models with several levels of difficulty, depending on the intended purpose for the model. A synchronous generator is represented by differential equations which account for the machine speed deviations and changes in rotor angle during disturbances. Additionally, the manner in which the armature flux gradually penetrates into the rotor, during system disturbances, is quantified by differential equations. These additional equations sufficiently quantify the effects of generator components such as field and damper windings. For TSA, a third-order model, which is presented as a transient voltage behind the direct axis transient reactance, is generally considered to be sufficient [11]. The 3^{rd} order dynamic model of the i^{th} generator is given as:

$$\Delta\dot{w}_i = \frac{1}{M_i}(P_{mi} - P_{ei} - D_i\Delta w_i) \tag{9a}$$

$$\dot{\delta}_i = \omega_0 \Delta w_i \tag{9b}$$

$$\dot{E}'_{qi} = \frac{1}{T'_{d0i}}\Big(E_{fi} - E'_{qi} + I_{di}(x_{di} - x'_{di})\Big) \tag{9c}$$

where, $i = 1, 2, 3, \ldots, N$; $M_i = 2H_i$, H_i is the inertia constant of the i^{th} generator; Δw_i is the speed deviation of the i^{th} generator; P_{mi} is the mechanical input power to the i^{th} generator; P_{ei} is the active electrical power injection from the i^{th} generator; D_i is the damping constant of the i^{th} generator. δ_i is the power angle of i^{th} generator; T'_{d0i} is the direct-axis open circuit transient time constant for the i^{th} generator; E'_{qi} is the quadrature-axis transient voltage of the i^{th} generator; E_{fi} is the field voltage for the i^{th} generator; I_{di} is the direct-axis current of the i^{th} generator; x_{di} and x'_{di} are the direct-axis synchronous and transient reactances of the i^{th} generator, respectively.

$$I_{di} = -E'_{qi}Y_{ii}sin\theta_{ii} + \sum_{\substack{j=1\\j\neq i}}^{N} Y_{ij}E'_{qj}sin(\delta_i - \delta_j - \theta_{ij}); \tag{10}$$

$$P_{ei} = E'^2_{qi}Y_{ii}cos\theta_{ii} + E'_{qi}\sum_{\substack{j=1\\j\neq i}}^{N} Y_{ij}E'_{qj}cos(\delta_i - \delta_j - \theta_{ij}); \tag{11}$$

where, Y_{ij} are the magnitudes of elements of the bus admittance matrix of the power system and θ_{ij} are the corresponding phase angles.

To obtain a dynamical model in polynomial form, the model (9a) - (9c) is expanded using Taylor series expansion of the sine and cosine functions. The truncation error in the Taylor series expansion is handled as described in Section III-B.

V. Case Study: Single-Machine-Infinite-Bus system

A. Model description

In order to evaluate the consistency and performance of the proposed method in estimating the ROA, a Single Machine connected to large external system (SMIB system) was used. The model is represented by equations (12), where $x_1 = \delta$ (in radians), $x_2 = \Delta\omega$ (in per unit) and $x_3 = E'_q$ (per unit). The model has multiple equilibrium points. However, in this study, the the ROA was estimated for the equilibrium point $\bar{x} = \{0.3398, 0, 1\}$.

$$\begin{aligned} \dot{x_1} &= 0.0750 - 0.2250x_3\sin(x_2) - x_1 \\ \dot{x_2} &= 314x_1 \\ \dot{x_3} &= 0.7361 - 1.1251x_3 + 0.4126\cos(x_2) \end{aligned} \tag{12}$$

Using this nominal equilibrium point, the model (12) is translated to the origin, yielding the dynamical model in (13).

$$\begin{aligned} \dot{x_1} &= 0.0750 - 0.22121x_3\sin(x_2) - 0.0750x_3\cos(x_2) - x_1 \\ &\quad - 0.2121\sin(x_2) - 0.0750\cos(x_2) \\ \dot{x_2} &= 314x_1 \\ \dot{x_3} &= -0.389 - 1.1251x_3 + 0.389\cos(x_2) - 0.1375\sin(x_2) \end{aligned} \tag{13}$$

The model (13) is then converted to a polynomial representation using Taylor series expansion of the sine and cosine functions. The degree of the Taylor approximation is decided taking into account the trade-off between model accuracy and computational overhead. In this study, a sixth order Taylor series approximation was used.

B. Stability assessment - Actual ROA

To obtain the actual ROA, a time-domain simulation of the model was carried out to obtain the ROA as shown in Fig. (2). The nominal equilibrium point is indicated by a plus sign in this figure. While the time-domain simulation gives the actual ROA, this process is tedious and, therefore, not suitable for on-line stability assessment. There are also no rules for determining how closely the initial conditions of the simulations have to be spaced in order to reliably determine the ROA. In contrast,

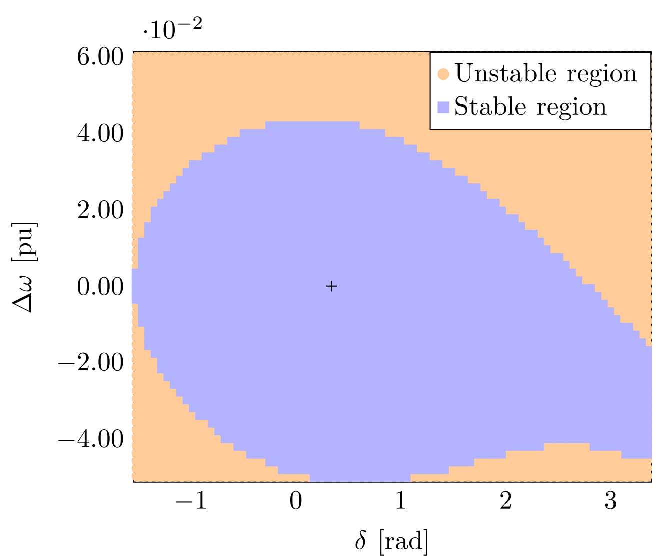

Fig. 2: Actual ROA - Time-domain simulation of the model

Lyapunov-based analysis, albeit conservative, gives an explicit region inside which the system is guaranteed to be stable.

C. Stability assessment - Estimated ROA

Using the proposed SOS/PLF algorithm, with a Taylor series approximation of order 6, the ROA was estimated as shown in Fig. 3. In this Figure, the estimated ROA is compared with the actual ROA obtained from time-domain simulations. From the Figure, it can be observed that the estimated ROA is a relatively accurate estimate of the actual ROA. As can further be observed from Fig. 4, the algorithm progressively estimates the ROA from an initial estimate to increasingly accurate estimates over successive iterations. Throughout this work, Yalmip [12] with the optimization solver MOSEK has been used for solving the SOS problems.

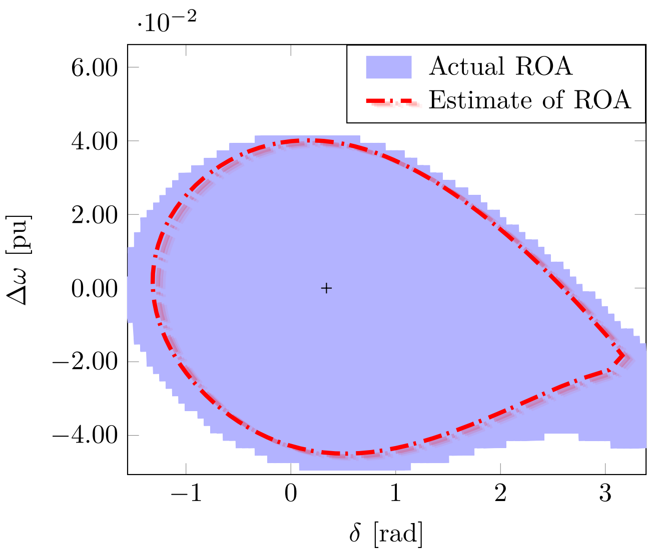

Fig. 3: Actual ROA and Estimated ROA

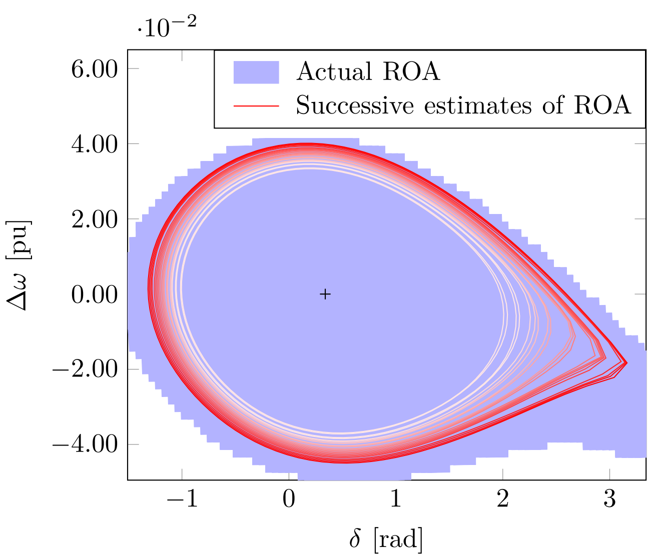

Fig. 4: Time-domain simulation results and successive estimates of the ROA

D. Validation of Estimated ROA - Step change in speed

To validate the estimated ROA further, the model (12) was constructed in Matlab/Simulink. The model was initialised by introducing a step change in the state variable x_2 (generator speed). Fig. 5 shows the system trajectory following a step change $\Delta x_2 = 0.039$. As can be seen from the Figure, the system converges to the equilibrium point following this disturbance. The initial point following the disturbance lies within the estimated ROA. Therefore, the estimated ROA is fairly accurate. Stability is also confirmed by plotting the variation of the Lyapunov function as shown in Fig. 6, where it can be observed that the value of the Lyapunov function remains below a pre-set threshold.

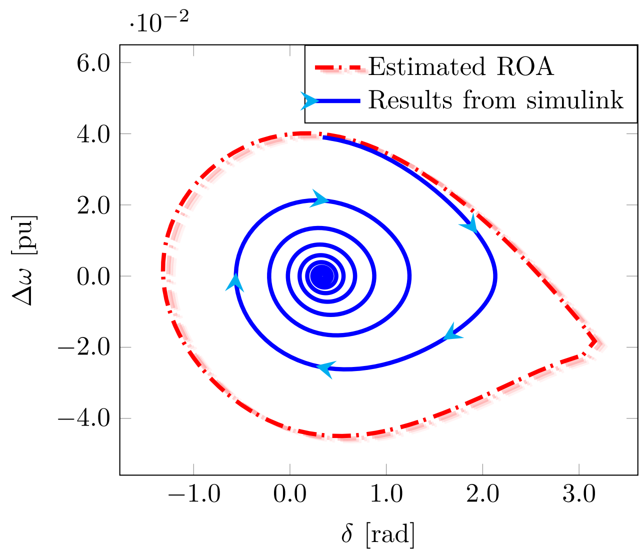

Fig. 5: Estimated ROA and simulation results from Simulink

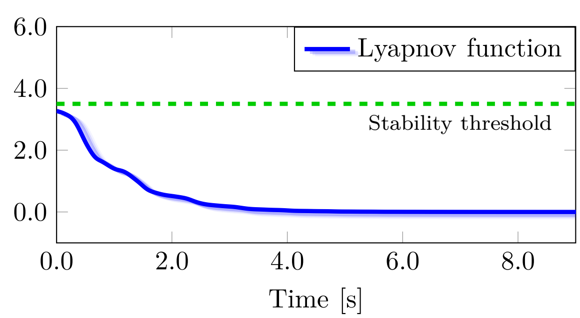

Fig. 6: Lyapunov function - stable point

When the system is perturbed by a change in speed $\Delta x_2 = 0.045$, the ensuing system trajectory does not converge to the equilibrium point as shown in Fig. 7. This agrees with the observation that the initial starting point following the disturbance lies outside the estimated

ROA. This observation reinforces the conclusion that the estimated ROA is fairly accurate. The evolution of the Lyapunov function is depicted in Fig. 8, where it is clear that the stability threshold is exceeded.

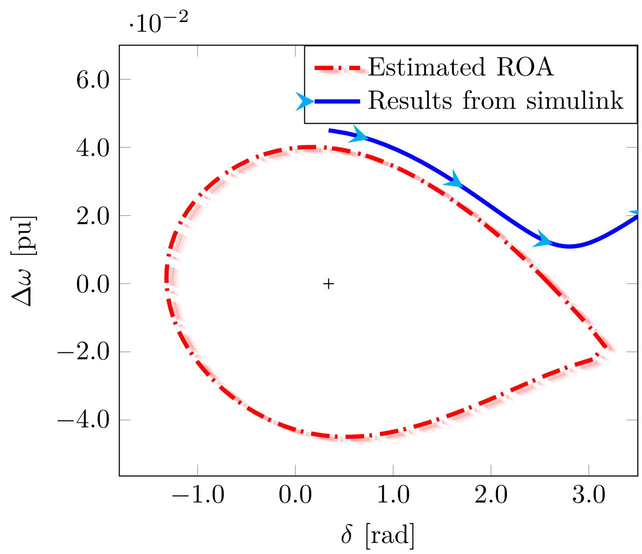

Fig. 7: Estimated ROA and simulation results from Simulink

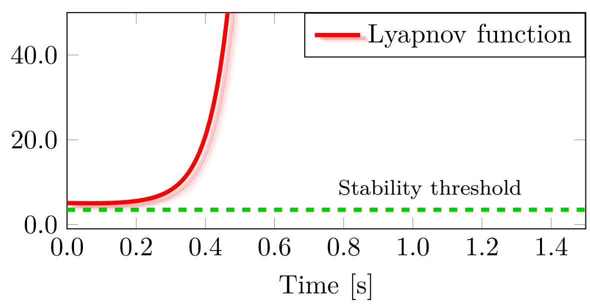

Fig. 8: Lyapunov function - unstable initial point

E. Validation of Estimated ROA - Simulating a fault

To further validate the estimated ROA, a fault was simulated. The system was constructed in Matlab/simulink, but it can be visualized as the SMIB system shown in Fig. 9. A short-circuit was applied on the line as shown in the Figure. The fault clearing time was adjusted so as to determined the critical fault clearing time that puts the system on the verge of instability. For this system,

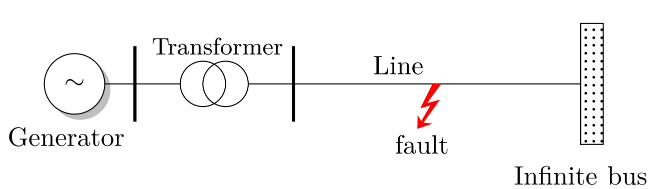

Fig. 9: Single-Machine-Infinite-Bus (SMIB) power system

the critical fault clearing time was obtained as 325 milliseconds. The trajectory of the rotor angle versus change in speed was then compared with the estimated ROA as shown in Fig 10. According to asymptotic Lyapunov stability, the system is stable if the system trajectory remains within the ROA. From the Figure, it is evident that the system trajectory lies within the estimated ROA for this fault clearing time. Therefore, the estimated ROA accurately captures the system dynamics. The evolution of the Lyapunov function is depicted in Fig. 11, where it is clear that the post-fault value of the Lyapnov function stays below the stability threshold.

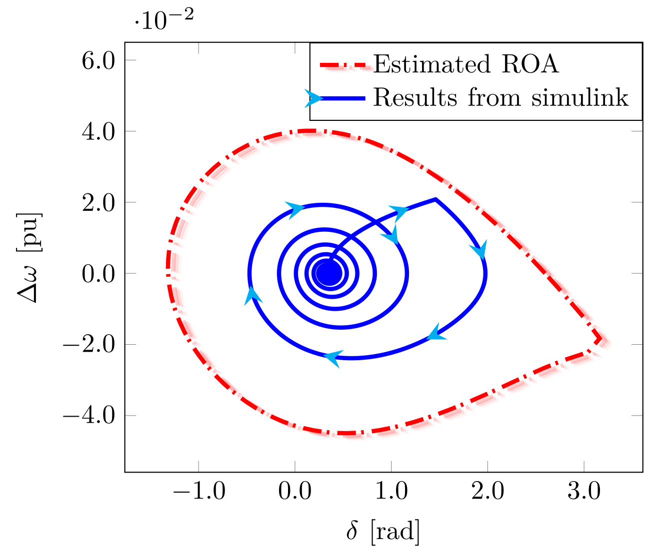

Fig. 10: Estimated ROA and simulation results from Simulink

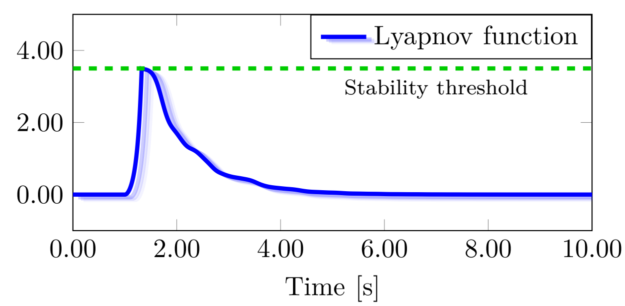

Fig. 11: Lyapunov function - unstable point

When the fault clearing time is increased to 362 milliseconds, the system becomes unstable and the system trajectory does not converge to the equilibrium point in the estimated ROA as shown in Fig. 12. From Fig. 13, it is also clear that the value of the Lyapunov function exceeds the set stability threshold. Therefore, the system is unstable.

F. Validation of Estimated ROA - Simulation from PowerFactory

To further validate the ROA estimate, the SMIB system, Fig. 9, was modelled in the power system simulation

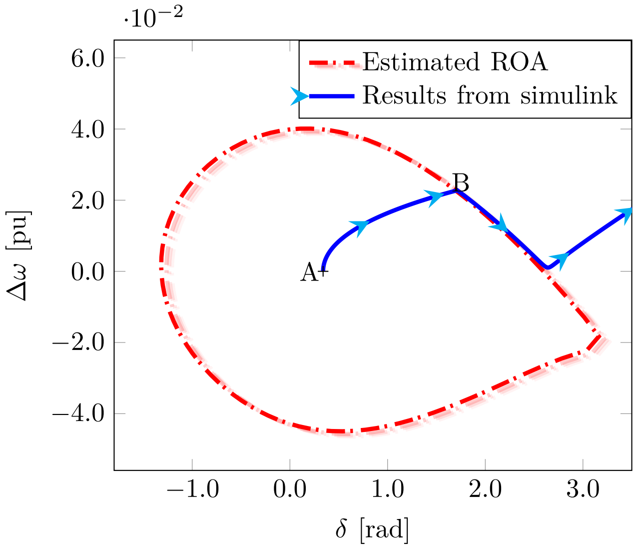

Fig. 12: Estimated ROA and simulation results from Simulink - unstable. Note: the trajectory A-B is due to the simulation of the fault, and thus does not follow the dynamics used when calculating the Lyapunov function. From point B, the trajectory follows the system dynamics as given by (1)

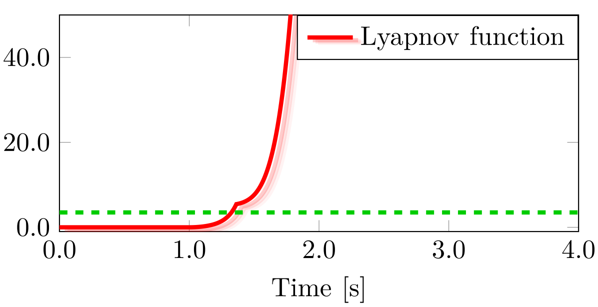

Fig. 13: Lyapunov function - unstable point

software DIgSILENT PowerFactory, with the generator represented by a detailed 5^{th} order model. The system parameter values used are provided in Tables (I) and (II), in the appendix. The critical fault clearing time was obtained as 320ms. The system response is depicted in Fig. 14. Again, it is observed that the system trajectory remains within the estimated ROA at all times. The estimate of the ROA is, therefore, fairly accurate.

VI. Conclusion

n this paper, a method suitable for on-line Transient Stability Assessment using a Polynomial-Lyapunov–Function to obtain an estimate of the ROA was presented. Starting with an initial estimate of the ROA, the algorithm rapidly expands the ROA estimate and then slowly converges as the boundary for asymptotic stability is approached. Robust truncation of trigonometric functions in the power system model has also been demonstrated in the paper. With this, we avoid dangerous over-approximation of the stable region, i.e., there are no unstable trajectories which would be incorrectly labelled as stable

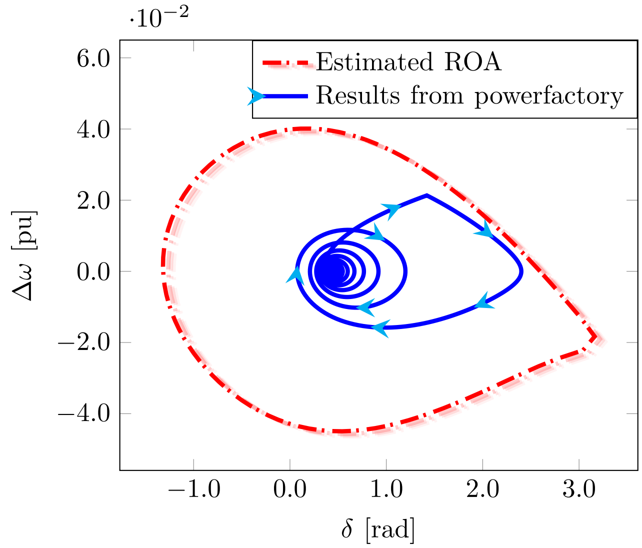

Fig. 14: Estimated ROA and simulation results from PowerFactory

by the ROA estimate. A simple Single-Machine-Infinity-Bus power system model was used in this paper. The method still needs to be further tested in a realistic power system, with multiple operational areas. However, the real challenge in the method is the size of the resulting semi-definite optimization problems, in particular for higher-order Taylor series approximations. Current research on exploiting sparsity patterns in SOS calculations bear the promise of allowing larger systems to be handled [13].

Appendix A
Parameters for the SMIB Model in Power Factory

TABLE I: Generator Parameters

Parameter	H	$x_d = x_q$	x'_d	$x''_d = x''_q$	T'_d	$T''_d = T''_q$
Value	4	1.25	0.528	0.01	1.5	0.03

TABLE II: Transformer and Line Parameters

Parameter	R	X
Transformer	0	0.022
Line	0	0.5

References

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