Optimal imperfect maintenance cost analysis of a two-component system with failure interactions

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Abstract

This paper considers a two-component system with failure interactions. Component 1 is repairable and component 2 is non repairable and is subject to an increasing degradation. One considers two different shock models. In model 1, component 1 failure causes random gradual damages to component 2 and increases its degradation level. In model 2, component 1 failure may cause the failure of component 2 with a given probability while the failure of component 2 is catastrophic and induces the failure of the whole system. For each model, three maintenance policies are proposed. In each policy, component 1 undergoes imperfect corrective maintenance actions and component 2 is perfectly repaired. An explicit expression of the long run average maintenance cost is developed and the existence of the optimal policy is discussed. Numerical examples are given to illustrate the effectiveness of the proposed models.

Keywords: two-component systems, failure interaction, imperfect repair, virtual age method, long-run average cost optimization.

Notation

	General notations
$F(\cdot)(f(\cdot))$	the lifetime distribution (density) function of component 1
L	the failure threshold of component 2
$Y(t)$	the degradation level of component 2 at time t

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σ_L	the failure time of component 2
G_{σ_L}	the distribution function of σ_L, $\bar{G}_{\sigma_L}(t) = 1 - G_{\sigma_L}(t)$
$N(t)$	the number of component 1 failure by time t
$\mathbb{E}(N(t))$	the expectation of $N(t)$
$p_n(t)$	the probability that the number of component 1 failures at time t is n
X_i	the inter-maintenance time between the $(i-1)$st and the ith repair of component 1
a	the age-reducing factor of component 1
$B_i(t)$	the effective age of component 1 after the ith repair by time t
$V_i(t)(v_i(t))$	the cumulative distribution (density) function of $B_i(t)$
c_1	the repair cost of component 1
c_2 (c_3)	the preventive (corrective) replacement cost of the system
T policy	the maintenance policy under which the system is renewed at its failure or when the age of component 2 reaches T, which occurs first
N policy	the maintenance policy under which the system is renewed at its failure or at the Nth failure of component 1, which occurs first
(N,T) policy	the maintenance policy under which the system is renewed at its failure, or at the Nth failure of component 1 or at the age T of component 2, which occurs first
	Notations with respect to type 3 failure interaction
Z_i	the damages caused to component 2 by the ith failure of component 1, Z_i , $i = 1, 2, \cdots$
$H(\cdot)$	the distribution function of Z_i, $i = 1, 2, \cdots$
$H^{*(n)}(\cdot)$	the n-fold convolution of $H(\cdot)$ with itself
$F_s(t)$	the lifetime distribution of component 2 by time t
$C_\infty(T)$	the long run average maintenance cost of the system under T policy
$C_\infty(N)$	the long run average maintenance cost of the system under N policy
$C_\infty(N,T)$	the long run average maintenance cost of the system under (N,T) policy

	Notations with respect to type 1 failure interaction
r	probability that the failure of component 1 has no effect on component 2
$\bar{r}$	probability that the failure of component 1 causes the instantaneous failure of component 2, $\bar{r} = 1 - r$
$F_{sI}(t)$	the lifetime distribution of component 2 by time t
$h_1(t)$	the failure rate of component 1 when it is minimally repaired at failure
$C_{\infty I}(T)$	the long run average maintenance cost of the system under T policy
$C_{\infty I}(N)$	the long run average maintenance cost of the system under N policy
$C_{\infty I}(N, T)$	the long run average maintenance cost of the system under (N, T) policy
	Notations in the numerical example
λ, b	the parameters of Weibull distribution used to describe the lifetime of component 1
μ	the parameter of Exponential distribution used to describe the damages caused to component 2
α, β	the parameters of Gamma process used to describe the degradation process of component 2

1. Introduction

In multi-component systems, for the sake of simplification, very often it is assumed that the failure of a component has no effect on failures of the other components [1, 2, 3]. However, in practice this assumption rarely holds. In most of multi-component systems, the failure of one component may affect the operational components by increasing their failure rates or their degradation levels. This is often the case for mechanical or electrical systems, when one component failure causes unexpected vibrations, frictions, overheating. For example, the non-uniform load sharing between mesh gear pairs may result in the degradation or even failure of multiple mesh gear systems [4]; in a mining operation, the conveyor belt containing numerous rollers is exerted to transport ore. The failure of one roller may increase the failure rate of the consecutive rollers [5]. These dependencies among components can significantly affect the system availability, maintenance costs, customer decisions, etc. Therefore, it is crucial to take them into account in system reliability analysis and in the development of maintenance policy.

In reliability engineering, stochastic dependence in multi-component systems was firstly introduced by Murthy et al. ([6, 7]). They considered two types of dependencies for a two-component system called type I and type II failure interactions. In type I failure interaction model, the failure of a component may induce the instantaneous failure of the other component with a given probability. In type II failure interaction model, the failure of a component only affects the failure rate of the other one. Later, Nakagawa et al. [8] proposed a Type III failure interaction model also known as shock damage interaction where the failure of one component induced random accumulated damage to the other component. Since then, an extensive literature has been developed to formalize the failure dependence, to propose relevant models and to assess their impacts on the system reliability [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

It is worth mentioning that in the existing literature for maintenance planning of two-component systems with failure interactions, the following hypotheses can be observed:

- the failure processes of all components are modeled by either their failure rates or their degradation processes [9, 10, 11, 12, 13, 16, 17, 19];
- at least one component is not ageing with time. It turns into the failure state when its damage induced by random shocks exceeds a pro-determined level[8, 14, 18];
- maintenance actions are either minimal repair or perfect repair[8, 9, 10, 11, 14, 16, 17, 18, 19].

Although these hypotheses may facilitate the evaluation of the system reliability as well as the optimization of maintenance policies, they are unrealistic in practical applications. First, for mechanical or electrical systems, it is normal to monitor some fatal components instead of the entire system. In such scenario, the failure processes of the monitored components may be described by degradation processes, while the rest can be modeled by their failure rates. Secondly, the natural ageing of a component cannot be ignored. Thirdly, in addition to minimal repair and perfect repair, imperfect repair is generally implemented in practical application.

To overcome these limitations, we consider here a two-component system with:

- one component ageing on its own and described by a lifetime distribution with non constant failure rate (component 1),
- one component ageing due to its own described by a degradation model and also due to the failure interactions (component 2),
- imperfect maintenance (Virtual age method) is carried out to component 1.

The main objective is to assess the impact of failure interactions on the reliability of component 2, and to demonstrate how we can control them with maintenance actions.

In a nutshell, the main goal of this paper is to propose a modeling framework for maintenance optimization of two-component systems with non-symmetrical failure interactions. In order to be generic and cover different practical situations, we consider:

- two types of failure interactions,
- three preventive maintenance policies. In each maintenance policy, the system is correctively replaced and component 1 undergoes imperfect corrective repairs whenever it fails.

The paper is organized as follows. In Section 2, the system with type III failure interaction between components is presented and the assumptions for maintenance actions are stated. Three maintenance policies are presented and their optimizations with respect to long run expected cost are discussed. In Section 3, similar results are calculated with type I failure interaction between components. Numerical examples illustrating the maintenance policies are given in Section 4. Finally, some conclusions and perspectives are given in Section 5.

2. Type III failure interaction

2.1. System description

- The system has two components.

- For a brand-new component 1, in absence of maintenance action, it has a lifetime distribution $F(\cdot)$, $F(0) = 0$ and density function $f(\cdot)$. Component 1 is repairable.
- Component 2 is non-repairable and it fails when its degradation level exceeds a pre-determined threshold L.
- Whenever component 1 fails, a random amount of damage is induced to component 2. We assume the damages Z_j $(j = 1, 2, ...)$ are additive and identical, independent random variables with distribution $H(\cdot)$.
- The failure of component 2 induces the failure of component 1.
- In absence of damages caused by component 1, let $\{Y(t), t \geq 0\}$ be the natural degradation level of component 2 at time t and σ_L be the time at which the degradation level reaches or first exceeds L, $L > 0$. Then its distribution function is

$$G_{\sigma_L}(t) = \mathbb{P}(\sigma_L \leq t) = \mathbb{P}(Y(t) \geq L),\ t \geq 0.$$

Let $p_k(t)$ be the probability that the number of component 1 failure is k by time t. Let $F_s(t)$ be the lifetime distribution function of component 2, we have the following equation:

$$F_s(t) = \sum_{k=0}^{\infty} p_k(t) \int_0^{\infty} G_{\sigma_{L-z}}(t) \mathrm{d}H^{*(k)}(z). \tag{1}$$

See Appendix B for more detail on this expression.

2.2. Maintenance model

In order to avoid failure and to extend the lifetime of the system, we propose to plan corrective and preventive maintenance actions. The following maintenance actions are carried out.

2.2.1. Corrective maintenance

Corrective maintenance is carried out at component-level and system-level respectively according to the following policy:

- the whole system is replaced when component 2 fails;
- component 1 undergoes Kijima model 1 imperfect repair when failed;
- the repair time and system renewal time are negligible.

Regarding Kijima model 1:

- let us note X_i and a, respectively the inter-maintenance time between the $(i-1)$st and the ith repair and the age-reducing factor of component 1. The effective age of component 1 after the ith repair is $B_i = B_{i-1} + aX_i$, $i = 1, 2, \cdots$, where the initial age $B_0 = 0$ and the reduction degree $0 \leq a \leq 1$.

- denoted by $V_n(\cdot)$ the distribution function of virtual age B_n, $N(t)$ the number of component 1 failures by time t and $p_n(t) = \mathbb{P}\{N(t) = n\}$. As shown in [20], it is easy to verify that the probability mass function of $p_n(t)$ is:

$$p_n(t) = \int_0^{at} \frac{\bar{F}(y + \frac{at-y}{a})}{\bar{F}(y)} v_n(y)\mathrm{d}y,$$

where $\bar{F}(\cdot) = 1 - F(\cdot)$, $v_n(x) = \frac{\mathrm{d}}{\mathrm{d}x} V_n(x)$, $v_1(x) = \frac{1}{a} f(\frac{x}{a})$ and $v_{n+1}(x) = \frac{1}{a} \int_0^x \frac{f(y + \frac{x-y}{a})}{\bar{F}(y)} v_n(y)\mathrm{d}y$ for $n \geq 1$.

For more details on virtual age models, refer to Appendix A and [20].

At last, in the following, we will denote c_1 the repair cost of component 1, c_2 and c_3 the preventive replacement cost and the corrective maintenance cost of the system respectively, with $c_3 \geq c_2 > c_1$.

2.2.2. Preventive maintenance

Three following preventive maintenance policies are considered:

1. Age-based policy, called T policy, under which the system is replaced either at its failure or when component 2 reaches age T, whichever comes first.

2. Failure-number-based policy, called N policy, under which the system is replaced at the Nth failure of component 1 or at the system failure whichever occurs first.
3. Mixture policy, denoted (N, T) policy, under which the system is replaced at age T of component 2, or at the Nth failure of component 1, or at the system failure time whichever occurs first.

It is noted that these maintenance strategies are consistent with the stochastic dependencies existing between components: the impact of component 1 failures on component 2 is modeled by either damages or by shocks (this is a stationary phenomena); however, their occurrence times are time dependent (this is a non-stationary phenomena). Hence the age of component 2 and the number of failures experienced by component 1 are then reasonable health indicators for decision-making.

We provide results on the system long run average maintenance cost under different maintenance policies.

2.3. Maintenance cost derivation

In this part, the average long run maintenance costs are derived considering the renewal reward theorem. Indeed, since the system is replaced as good as new after the failure of component 2, the system replacement intervals are independent and identically distributed. Therefore, the interval Δ between two replacements can be considered as a renewal cycle and the long run average maintenance cost $\mathbb{E}C_\infty$ can be obtained considering the average maintenance cost on a renewal cycle $\mathbb{E}(C(\Delta))$ as follows: $\mathbb{E}C_\infty = \frac{\mathbb{E}(C(\Delta))}{\mathbb{E}(\Delta)}$.

2.3.1. T policy

Let us consider the age-based policy, under which the system is replaced either at its failure or when component 2 reaches age T, whichever comes first. Let $C_\infty(T)$ be the long run average maintenance cost associated to the age-based policy. Its expression can be obtained as follows:

$$C_\infty(T) = \frac{c_3 - (c_3 - c_2)\bar{F}_s(T) + c_1 \mathbb{E}N(T)\bar{F}_s(T) + c_1 \int_0^T \mathbb{E}N(t) \mathrm{d}F_s(t)}{T\bar{F}_s(T) + \int_0^T t \mathrm{d}F_s(t)}, \quad (2)$$

where $\mathbb{E}N(t) = \sum_{n=0}^{\infty} n p_n(t)$ is the expected number of component 1 failures by time t, $F_s(t)$ is the component 2 lifetime distribution function given in equation (1). See Appendix C.1 for detailed calculations leading to this result.

2.3.2. N policy

Under this policy, we replace the whole system at the Nth failure of component 1 or at the failure of component 2 whichever occurs first. Let $C_\infty(N)$ be the average long run maintenance cost. It is calculated as follows:

$$C_\infty(N) = \frac{c_3 - (c_3 - c_2)\int_0^\infty R_{N-1}(t)\mathrm{d}V_N(at) + c_1 \sum_{k=1}^{N-1}\int_0^\infty R_{k-1}(t)\mathrm{d}V_k(at)}{\int_0^\infty \sum_{k=0}^{N-1} p_k(t)R_k(t)\mathrm{d}t}, \quad (3)$$

where $R_i(t) = \int_0^L (1-G_{\sigma_{L-z}}(t))\mathrm{d}H^{*(i)}(z)$, $i = 1, 2, \cdots, N$, $R_0(t) = 1-G_{\sigma_L}(t)$, $G_{\sigma_L}(t)$ is the probability distribution function of the first hitting time of level L by the component 2 degradation, $H(\cdot)$ is the cumulative distribution function of the total damages caused by component 1 to component 2, $H^{*(n)}(\cdot)$ is the n-fold convolution of $H(t)$ with itself, $p_k(t)$ is the probability that the number of component 1 failures occur in $[0, t]$ is k, $V_k(t)$ is the distribution function of the virtual age after the kth repair. See Appendix C.2 for detailed calculations leading to this result.

2.3.3. (N, T) policy

In the mixture policy called (N, T) policy, the system is replaced at age T of component 2, or at the Nth failure of component 1, or at the time of component 2 failure whichever occurs first. Assume that the long run average maintenance cost under this circumstance is $C_\infty(N, T)$. Then we have:

$$\begin{aligned} C_\infty(N,T) &= \frac{c_3 - (c_3 - c_2)\Big(\int_0^T R_{N-1}(t)\mathrm{d}V_N(at) + \sum_{k=0}^{N-1} p_k(T)R_k(T)\Big)}{\int_0^\infty \sum_{k=0}^{N-1} p_k(t)R_k(t)\mathrm{d}t} \\ &\quad + \frac{c_1 \sum_{k=1}^{N-1}\int_0^T R_{k-1}(t)\mathrm{d}V_k(at)}{\int_0^\infty \sum_{k=0}^{N-1} p_k(t)R_k(t)\mathrm{d}t}, \end{aligned}$$

where $R_i(t) = \int_0^L (1 - G_{\sigma_{L-z}}(t))\mathrm{d}H^{*(i)}(z)$, $i = 1, 2, \cdots, N$, $R_0(t) = 1 - G_{\sigma_L}(t)$, $G_{\sigma_L}(t)$ is the probability distribution function of the first hitting time of level L by the component 2 total degradation at time t, $H(t)$ is the lifetime distribution of damages caused by component 1 to component 2 until time t, $H^{*(n)}(t)$ is the n-fold convolution of $H(t)$ with itself, $p_k(t)$ is the probability that the number of component 1 failures occur in $[0, t]$ is k, $V_k(t)$ is the distribution function of the component 1 virtual age after the kth repair. Refer to Appendix C.3 for detailed calculations leading to this result.

2.4. Existence of the optimal maintenance policy

In this paragraph, the existences of optimal T^* and N^* minimizing $C_\infty(T)$ and $C_\infty(N)$ respectively are discussed. Sufficient conditions regarding the existence of the optimal maintenance actions under T policy and N policy are derived respectively.

2.4.1. Existence of the optimal age-based policy

Under the age-based policy, the optimal and unique age T^* which minimizes the long run average maintenance cost exists if the following assumptions are satisfied:

$$\lim_{T\to\infty} \frac{\mathrm{d}\mathbb{E}N(T)}{\mathrm{d}T} \int_0^T \bar{F}_s(t)\mathrm{d}t - \mathbb{E}N(T)\bar{F}_s(T) - \int_0^T \mathbb{E}N(t)\mathrm{d}F_s(t) > \frac{c_3}{c_1}, \tag{4}$$

and

$$\frac{\mathrm{d}^2\mathbb{E}N(t)}{\mathrm{d}^2 t} > 0, t > 0. \tag{5}$$

Particularly, when $c_3 = c_2$ and component 1 has a Weibull lifetime distribution $F(t) = 1 - exp(-\lambda t^b)$ with minimal repair ($a = 1$), the optimal $T^* = \infty$ when $b \leq 1$ which means there is no preventive maintenance in the optimal policy. See the Appendix D.1 for detailed calculations leading to results.

2.4.2. Existence of the optimal failure number-based policy

Under the failure-number-based maintenance policy, when $c_2 = c_3$, the optimal N^* exists if the following assumptions are satisfied:

$$\lim_{N\to\infty} \frac{d_1(N)\sum_{k=1}^{N-1} d_2(k)}{d_2(N)} - \sum_{k=1}^{N-1} d_1(k) > \frac{c_2}{c_1}, \tag{6}$$

and

$$\frac{d_1(N)}{d_2(N)} \text{ is a convex function with respect to } N, \tag{7}$$

where $d_1(k) = \int_0^\infty R_{k-1}\mathrm{d}V_k(at)$ and $d_2(k) = \int_0^\infty p_k(t)R_k(t)\mathrm{d}t$ where $R_i(t) = \int_0^L (1 - G_{\sigma_{L-z}}(t))\mathrm{d}H^{*(i)}(z)$, $i = 1, 2, \cdots, N$, $R_0(t) = 1 - G_{\sigma_L}(t)$.

Refer to Appendix D.2 for detailed calculations leading to results.

3. Type I failure interaction: description and maintenance models

3.1. System description and maintenance policy

Let us consider another scenario: whenever component 1 failure occurs, either it has no impact on component 2 with probability r or it induces the instantaneous failure of component 2 with probability $\bar{r} = 1 - r$.

Then the failure interaction between the two components can be seen as a type I failure interaction given by [6]. In this framework, when the conditions of section 2 hold, similar optimality results are derived. Proofs are presented briefly since the same methods as in section 2 are adopted. Let $F_{sI}(t)$ be the lifetime distribution of component 2, $\bar{F}_{sI}(t) = 1 - F_{sI}(t)$. It is easily seen that

$$\bar{F}_{sI}(t) = \sum_{k=0}^{\infty} r^k p_k(t) \bar{G}_{\sigma_L}(t),$$

where $\bar{G}_{\sigma_L}(t) = 1 - G_{\sigma_L}(t)$. Hence

$$F_{sI}(t) = 1 - \sum_{k=0}^{\infty} r^k p_k(t) \bar{G}_{\sigma_L}(t). \tag{8}$$

3.2. Cost calculation for the age-based policy

Denote by $C_{\infty I}(T)$ the average long run cost when it undergoes age-based policy. It can be proved that:

$$C_{\infty I}(T) = \frac{c_3 - (c_3 - c_2)\sum_{n=0}^{\infty} r^n p_n(T)\bar{G}_{\sigma_L}(T) + \sum_{n=1}^{\infty}(n-1)c_1 r^{n-1}\bar{r}\int_0^T \bar{G}_{\sigma_L}(t)\mathrm{d}V_n(at)}{\int_0^T \bar{F}_{sI}(t)\mathrm{d}t} + \frac{\sum_{n=0}^{\infty} n r^n c_1 (\int_0^T p_n(t)\mathrm{d}G_{\sigma_L}(t) + p_n(T)\bar{G}_{\sigma_L}(T))}{\int_0^T \bar{F}_{sI}(t)\mathrm{d}t}, \tag{9}$$

where $F_{sI}(t)$ is given in equation (8), other notations have been given in Section 2. See Appendix E.1 for calculations leading to this result.

3.3. Cost calculation for the failure number-based policy

Suppose that $C_{\infty I}(N)$ is the system average long run cost under failure-number-policy. The following expression is obtained:

$$C_{\infty I}(N) = \frac{c_3 - (c_3 - c_2)\int_0^\infty r^{N-1}\bar{G}_{\sigma_L}(t)\mathrm{d}V_N(at)}{\int_0^\infty \sum_{k=0}^{N-1} r^k p_k(t)\bar{G}_{\sigma_L}(t)\mathrm{d}t} + \frac{\sum_{n=1}^{N-1}(n-1)c_1 r^{n-1}\bar{r}\int_0^\infty \bar{G}_{\sigma_L}(t)\mathrm{d}V_n(at)}{\int_0^\infty \sum_{k=0}^{N-1} r^k p_k(t)\bar{G}_{\sigma_L}(t)\mathrm{d}t} + \frac{\sum_{n=0}^{N-1} nr^n c_1\int_0^\infty p_n(t)\mathrm{d}G_{\sigma_L}(t) + \int_0^\infty r^{N-1}(N-1)c_1\bar{G}_{\sigma_L}(t)\mathrm{d}V_N(at)}{\int_0^\infty \sum_{k=0}^{N-1} r^k p_k(t)\bar{G}_{\sigma_L}(t)\mathrm{d}t}. \tag{10}$$

See Appendix E.2 for the proof.

3.4. Cost calculation for the mixture policy

Suppose that $C_{\infty I}(N,T)$ is the average long run system cost under (N,T) policy, then

$$C_{\infty I}(N,T) = \frac{c_3 - (c_3 - c_2)\left(\int_0^T r^{N-1}\bar{G}_{\sigma_L}(t)\mathrm{d}V_N(at) + \sum_{k=0}^{N-1} r^k p_k(T)\bar{G}_{\sigma_L}(T)\right)}{\int_0^T \sum_{k=0}^{N-1} r^k p_k(t)\bar{G}_{\sigma_L}(t)\mathrm{d}t} + \frac{\sum_{n=1}^{N-1}(n-1)c_1 r^{n-1}\bar{r}\int_0^T \bar{G}_{\sigma_L}(t)\mathrm{d}V_n(at) + \sum_{n=0}^{N-1} nr^n c_1\int_0^T p_n(t)\mathrm{d}G_{\sigma_L}(t)}{\int_0^T \sum_{k=0}^{N-1} r^k p_k(t)\bar{G}_{\sigma_L}(t)\mathrm{d}t} + \frac{\int_0^T r^{N-1}(N-1)c_1\bar{G}_{\sigma_L}(t)\mathrm{d}V_N(at) + c_1\sum_{k=0}^{N-1} kr^k p_k(T)\bar{G}_{\sigma_L}(T)}{\int_0^T \sum_{k=0}^{N-1} r^k p_k(t)\bar{G}_{\sigma_L}(t)\mathrm{d}t}.$$

Since the result is a combination of results obtained in sections 3.2 and 3.3, detailed calculations are omitted.

3.4.1. Particular case of nonhomogeneous Poisson process

More particularly, when component 1 failure occurs according to a nonhomogeneous Poisson process, we have the following result.

Suppose that the failure rate of component 1 at t is $h_1(t)$. When component 1 undergoes minimal repair ($a = 1$), the optimal and unique age T^* which minimizes $C_{\infty I}(T)$ exists if

$$\lim_{T\to\infty} h_1(T)\int_0^T \bar{F}_{sI}(t)\mathrm{d}t - \int_0^T \bar{F}_{sI}(t)h(t)\mathrm{d}t > \frac{c_3}{rc_1}, \tag{11}$$

and

$$\frac{\mathrm{d}h_1(t)}{\mathrm{d}t} > 0, t > 0. \tag{12}$$

Besides, when $c_2 = c_3$ and component 1 has Weibull lifetime $F(t) = 1 - \exp(-\lambda t^b)$, the optimal $T^* = \infty$ when $b \leq 1$ which means no preventive maintenance is the optimal policy. Otherwise the optimal T^* can be decided by only equation (11). See Appendix E.3 for detailed calculations.

4. Numerical examples

In this section, first, several quantities and the long run average maintenance cost calculated by both their exact expressions and Monte Carlo simulation are presented to validate our results. Afterward the optimization of the long run average maintenance cost and the sensitivity analysis under different maintenance policies are explored.

4.1. Illustrative example

Here we assume component 1 has Weibull cumulative distribution function $F(t) = 1 - e^{-\lambda t^b}, t > 0$. The amount of damages is exponentially distributed with expectation μ. It is assumed that $\mu = 0$ when the failure processes of components 1 and 2 are independent, which means that no damages are induced to component 2 due to the failure of component 1. The deterioration of component 2 follows a homogeneous Gamma process which has been widely used and successfully data-fitted in describing system degradation on the account of erosion, corrosion, crack growth, etc. [21, 22, 23]. Its density function is as follows:

$$g_{\alpha t,\beta}(u) = \frac{\beta^{\alpha t} u^{\alpha t - 1} e^{-\beta u}}{\Gamma(\alpha t)}, \quad \alpha > 0, \beta > 0,$$

where

$$\Gamma(\alpha) = \int_0^\infty u^{\alpha-1} e^{-u} \mathrm{d}u.$$

By [24], the first hitting time has the following distribution function:

$$G_{\sigma_L}(t) = \frac{\Gamma(\alpha t, L\beta)}{\Gamma(\alpha t)}, t \geq 0,$$

where the lower incomplete Gamma function is defined as follows:

$$\Gamma(\alpha, x) = \int_x^{\infty} z^{\alpha-1} e^{-z} \mathrm{d}z.$$

To begin with, assume that $\lambda = 0.01, b = 2, a = 0.6, T = 10, N = 2, L = 20, \lambda = 0.01, b = 2, \alpha = 4, \beta = 2, c_1 = 50, c_2 = 250, c_3 = 300$. Table 1 shows some quantities obtained by their formulas and Monte Carlo simulation with 10^5 histories and 95% confidence intervals. All results are coherent.

	Formula	Simulation	95% confidence interval
$p_0(T)$	0.3679	0.3670	[0.3576,0.3764]
$p_1(T)$	0.4211	0.4234	[0.4137,0.4331]
$p_2(T)$	0.1648	0.1626	[0.1554, 0.1689]
$F_s(T)$	0.5820	0.5800	[0.5770, 0.5831]
$C_\infty(T, N)$	34.2762	34.2715	[34.1976, 34.3454]

Table 1: Calculations of various quantities by their formulas and the Monte Carlo simulations ($N = 10^5$) respectively

4.2. Sensitivity analysis

In this paragraph, the baseline parameters are chosen according to the results obtained in the previous paragraph ($\lambda = 0.1, b = 2, a = 0.6, L = 20, \lambda = 0.1, b = 2, \alpha = 4, \beta = 2, \mu = 1$). For the optimization of the long run average maintenance cost, as it is shown in equation (4) , the existence of the optimal value depends on the system parameters and the cost ratio: $\frac{c_3}{c_1}$. The smaller is $\frac{c_3}{c_1}$, the higher is the possibility of the existence of a optimum. Therefore, under the constraint $c_3 > c_1$, we chose close cost values for c_1 and c_3 in order to assure the existence of the optimum. It should be mentioned that this is an example presenting the system properties and optimal maintenance cost rates under different maintenance policies. In reality, parameters selections are based on the real data and the parameter estimation etc.

4.2.1. Sensitivity analysis with type III failure interaction between units

We set $c_1 = 50, c_2 = 60, c_3 = 80$. In the following scenario, one parameter is changed to evaluate the variation of the average cost while other parameters remain unchanged.

Figure 1 and Table 2 show the long run average maintenance cost $C_\infty(T)$ with different parameters setting. The following behaviors are pointed out.

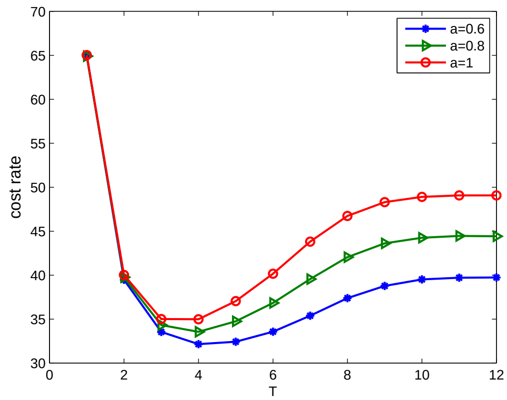

Figure 1: $C_\infty(T)$ with different a

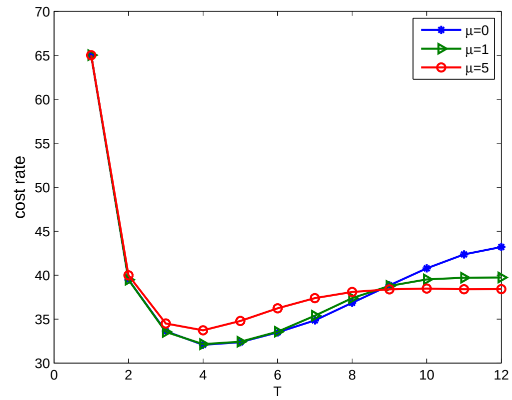

Figure 2: $C_\infty(T)$ with different μ

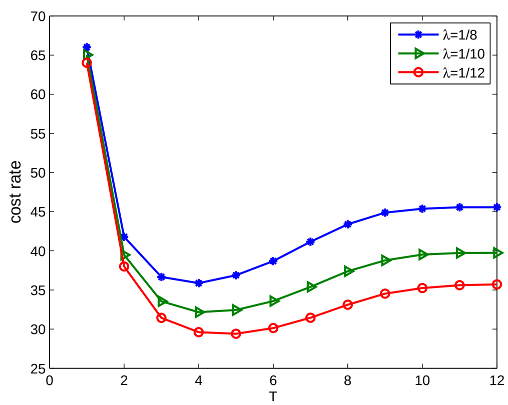

Figure 3: $C_\infty(T)$ with different λ

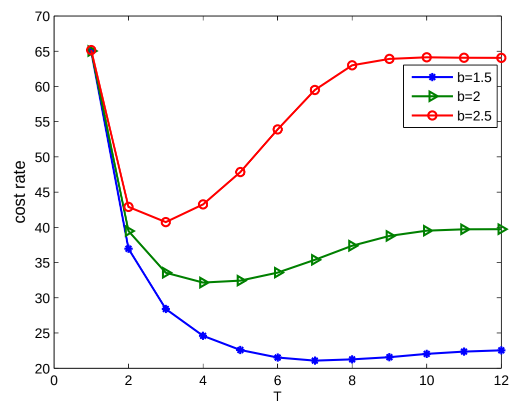

Figure 4: $C_\infty(T)$ with different b

- The optimal expected cost $C_\infty(T)$ increases with the age-reducing factor a and the expectation of the damages μ. The larger a is, the worse is the repair. Therefore, there are more damages caused to component 2 which induces the system failure. As μ represents the expectation of damages, for large values of a and μ the system fails more often. It can be observed in Figure 2 that the expected maintenance cost reaches its minimum when the failure processes of components 1 and 2 are independent ($\mu = 0$). It indicates that the failure dependence between components has a non-negligible impact on the maintenance cost. Under these parameters setting, the optimal cost is less sensitive to the variations of the expectation of the damages μ. Because in this case, the failure of component 2 is mostly due to its deterioration in comparison to damages caused by component 1.

- From Figures 3-4, it can be observed that the optimal expected maintenance cost $C_\infty(T)$ decreases with the expected lifetime of component 1. The larger the Weibull parameters λ and b are, the smaller is the lifetime of component 1. Therefore more component 1 failures as well as the system failure may occur which lead to an increase in the maintenance cost.

- It can be noticed in Table 2, the optimal average cost $C_\infty(T)$ increases with respect to the shape parameter α of the Gamma process used to describe the degradation of component 2 and the maintenance cost units (c_1, c_2 and c_3). In this scenario, the deterioration of component 2 is faster and the maintenance are more costly. On the contrary, there is a decreasing tendency of the cost rate with respect to the scale parameter of the Gamma process b. Because the smaller is b, the slower is the deterioration of component 2 and so the smaller is its probability of failure.

GP parameters		Cost units			Optimal cost rate	
α	β	c_1	c_2	c_3	$C_\infty(T^*)$	T^*
4	2	50	60	80	32.1556	4
4	1	50	60	80	33.6287	4
2	2	50	60	80	32.0982	4
4	2	50	60	80	32.1556	4
4	2	30	60	80	24.2826	5
4	2	50	70	80	34.4246	5
4	2	50	60	90	32.1280	4

Table 2: The optimal average cost rate under age-T-based policy with different gamma process parameters and different repair costs

Figure 5 shows the long run average maintenance cost under N policy $C_\infty(N)$ with different parameters setting. The repair cost of component 1 is set to $c_1 = 10$ and other conditions are as in T policy. As expected, it can be noticed that under N policy the average cost rate increases with the lifetime

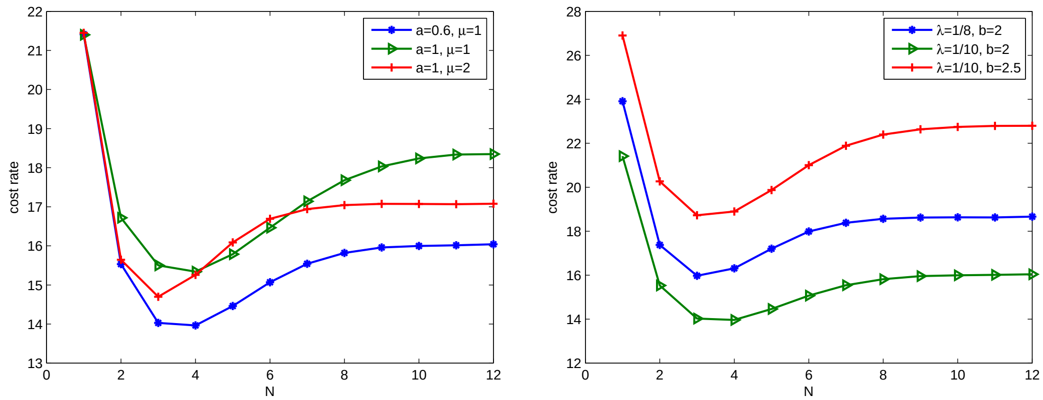

Figure 5: $C_\infty(N)$ with different parameters

of component 1 and the expectation of the amount of damages μ. A similar behavior as in the T policy can be pointed out.

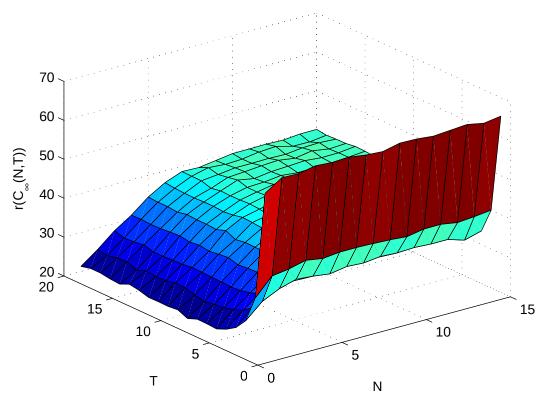

Figure 6: $C_\infty(T, N)$ with different parameters policy

Figure 6 shows the cost rate $C_\infty(N, T)$ under (N, T) policy with the parameters given as in the T policy. The cost rate shows a decreasing tendency with respect to the number of component 1 failure N and is a convex function with respect to T, the age of component 2. It reaches a minimum with $C_\infty(1, 6) = 20.9874$.

4.2.2. Numerical analysis with type I failure interaction between units

Here we adopt the original parameters as shown in section 4.2.1 ($\alpha = 4, \beta = 2, \lambda = 0.1, b = 2, a = 0.6, L = 20, c_1 = 50, c_2 = 60, c_3 = 80$) and set $r = 0.8$.

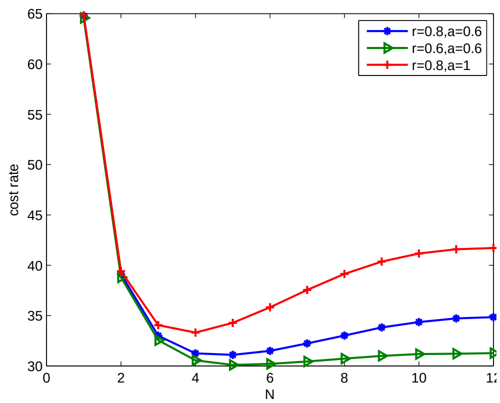

Figure 7: $C_{\infty I}(T)$ with different r and a

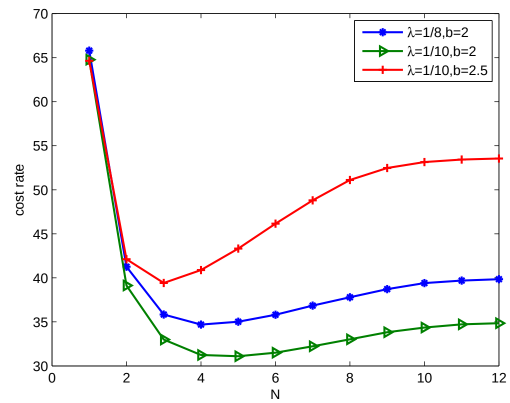

Figure 8: $C_{\infty I}(T)$ with different λ

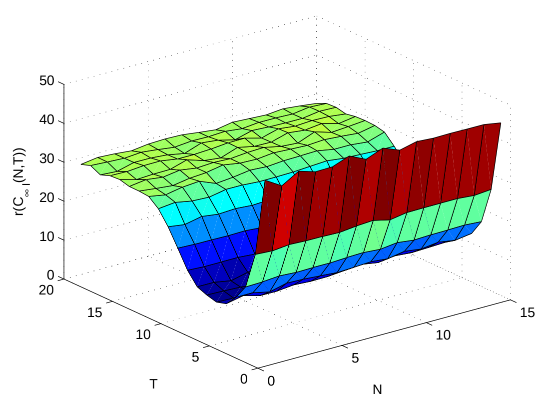

Figure 9: $C_{\infty I}(N, T)$ with different N and T

Figures 7 and 8 show the cost rate $C_{\infty I}(T)$ when we change only one parameter. Similarly, the variation of the expected cost rate under the (N, T) policy is illustrated in Figure 9.

The following results can be pointed out.

- The optimal expected cost $C_{\infty I}(T)$ increases with r which is the probability that the failure of component 1 has no impact on component 2. Because the greater r is, more imperfect repairs are carried out. However, the imperfect repair is not effective as it slightly reduces the age of component 1 ($a = 0.6$) with a high cost ($c_1 = 50$) compared to the system replacement costs ($c_2 = 60, c_3 = 80$). Similarly, the optimal expected cost $C_{\infty I}(T)$ increases with the age-reducing factor of component 1 a. The larger a is, the worse is the repair. Therefore more maintenance actions due to the failure of component 1 are implemented.

- The optimal $C_{\infty I}(N, T)$ initially decreases with T, the age of component 2, but turns to increase after reaching a minimum. It is not very sensitive with N.

5. Conclusions

In this study, we proposed different preventive maintenance policies for a two component system with two types of failure interactions. The system is successively supposed to be preventively replaced i) at age T of component 2, ii) at the Nth failure of component 1, iii) at the age T of component 2 or the Nth failure of component 1 which occurs first. Two types of component interactions are considered. In type III failure interaction, component 1 failure causes a random amount of damage to component 2 while in type I failure interaction, component 1 failure induces component 2 failure with probability $r, 0 < r < 1$. Component 2 failure is always lethal which results in the system failure under both type III and type I failure interaction. The average maintenance costs on the long time horizon are formulated and the optimizations are discussed. It is shown that the failure interaction between components has significant effect on the system maintenance cost. The neglect of dependencies between components may lead to bias in the evaluations of system reliability and maintenance cost. It is therefore necessary to take the stochastic dependence among components into consideration in the product reliability analysis. In our future work, it may be more interesting and challenging to generalize the two-component system to more complex systems and analyse the impact of failure dependence on the system reliability as well as the maintenance cost optimization problem.

Appendix A. Introduction of virtual age models

In the past several decades, imperfect repair has been extensively investigated [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. The virtual age methods are proposed by Kijima et al.[20, 36, 37]. Instead of considering the system age as the time elapsed since it was new, they assumed the virtual age (or effective age) as the real condition of the system which is reduced after the repair. They developed two imperfect maintenance models. Let B_n, A_n, X_n be the component virtual age after the nth repair, the repair degree of the nth repair and the time between the $(n-1)$th and nth repair. In Kijima model 1, it is assumed that the repair can reduce the damage emerged only during the last survival period which yields $B_n = B_{n-1} + A_n X_n, B_0 = 0$. While in Kijima model 2, the maintenance effect decreases all damages before the nth repair which yields: $B_n = A_n(B_{n-1} + X_n), B_0 = 0$. In our study, we take into consideration Kijima model 1 to describe the imperfect corrective maintenance of component 1. To reduce the calculation complexity, we suppose the repair degree is a constant and depends neither on time nor on the failure number n which means: $A_n = a$, $0 \leq a \leq 1$. The repair becomes minimal if $a = 1$ and perfect if $a = 0$. Extended studies of the virtual age method can be seen in [30, 32, 31, 33] etc.

Appendix B. Calculations leading to equation (1)

To obtain equation (1), let T_i, $i = 1, 2, \cdots$, be the ith system renewal time. It is obvious that they are identically and independently distributed. Then

$$\begin{aligned}
\mathbb{P}\{T_1 \leq t\} &= p_0(t)\mathbb{P}(Y(t) > L) + \sum_{k=1}^{\infty} \mathbb{P}(Y(t) + \sum_{i=1}^{N(t)} Z_i > L \mid N(t) = k)\mathbb{P}(N(t) = k) \\
&= p_0(t)\mathbb{P}(Y(t) > L) + \sum_{k=1}^{\infty} p_k(t) \int_0^{\infty} \mathbb{P}(Y(t) > (L - z))\mathrm{d}H^{*(k)}(z) \\
&= p_0(t)G_{\sigma_L}(t) + \sum_{k=1}^{\infty} p_k(t) \int_0^{\infty} G_{\sigma_{L-z}}(t)\mathrm{d}H^{*(k)}(z)
\end{aligned}$$

where $G_{\sigma_x}(t) = 1$ when $x < 0$. $H^{*(n)}(t)$ is the n-fold convolution of $H(t)$ with itself. Therefore

$$F_s(t) = p_0(t)G_{\sigma_L}(t) + \sum_{k=1}^{\infty} p_k(t) \int_0^{\infty} G_{\sigma_{L-z}}(t)\mathrm{d}H^{*(k)}(z).$$

$\diamond$

Appendix C. Calculations leading to the cost expression

Appendix C.1. Calculations for results in paragraph 2.3.1

Let Δ_i^T, U_i^T be respectively the length of the ith replacement cycle and the cost incurred during this period, $i = 1, 2, \cdots$. Then $\{\Delta_i^T, U_i^T\}$ constitutes a renewal reward process which yields

$$C_\infty(T) = \frac{\mathbb{E}(U_1^T)}{\mathbb{E}(\Delta_1^T)}.$$

As

$$\begin{aligned} E(U_1^T) &= \mathbb{P}(T_1 > T)(c_2 + c_1 \mathbb{E}N(T)) + \int_0^T (c_3 + c_1 \mathbb{E}N(t)) \mathrm{d}\mathbb{P}(T_1 \leq t) \\ E(\Delta_1^T) &= T\mathbb{P}(T_1 > T) + \int_0^T t \mathrm{d}\mathbb{P}(T_1 \leq t) \end{aligned}$$

where $\mathbb{P}(T_1 \leq T) = F_s(T)$. Therefore equation (2) is straightforward. $\diamond$

Appendix C.2. Calculations for results in paragraph 2.3.2

Let S_n^1 be the nth failure time of component 1 which is $S_n^1 = \sum_{i=0}^n X_i$. One shall denote $\varphi(t)$ the degradation level of unit 2 at time t, Δ_1^N the time elapsed in one replacement cycle, U_1^N the total cost in one replacement cycle and $\mathbb{E}(\Delta_1^N)$, $\mathbb{E}(U_1^N)$ be the expectations of Δ_1^N and U_1^N respectively. Let be $R_i(t) = \int_0^L (1 - G_{\sigma_{L-z}}(t)) \mathrm{d}H^{*(i)}(z)$ for $i = 1, 2, \cdots, N$ and $R_0(t) = \int_0^L (1 - G_{\sigma_L}(t))$. The following equality is verified

$$\begin{aligned} \mathbb{P}(\Delta_1^N > t) &= \mathbb{P}(S_N^1 > t, \varphi(t) < L) \\ &= \mathbb{P}(N(t) < N)\mathbb{P}(\varphi(t) < L | N(t) < N) \\ &= \sum_{k=0}^{N-1} p_k(t) \mathbb{P}(Y(t) + \sum_{i=0}^{k} Z_i < L) \\ &= \sum_{k=0}^{N-1} p_k(t) R_k(t). \end{aligned}$$

Thus:

$$\mathbb{E}(\Delta_1^N) = \sum_{k=0}^{N-1} \int_0^\infty p_k(t) R_k(t) \mathrm{d}t. \tag{C.1}$$

Note that the expected number of component 1 imperfect repairs is

$$\sum_{k=1}^{N-1} \int_0^\infty R_{k-1}(t) \mathrm{d}V_k(at),$$

Therefore,

$$\mathbb{E}(U_1^N) = c_3 - (c_3 - c_2) \int_0^\infty R_{N-1}(t) \mathrm{d}V_N(at) + c_1 \sum_{k=1}^{N-1} \int_0^\infty R_{k-1}(t) \mathrm{d}V_k(at). \tag{C.2}$$

By the renewal theory $C_\infty(N) = \frac{\mathbb{E}(U_1^N)}{\mathbb{E}(\Delta_1^N)}$, and considering equations (C.1) and (C.2) the result is obtained.

Appendix C.3. Calculations for results in paragraph 2.3.3

Denote Δ_1^{NT}, U_1^{NT} be the length and the cost of one replacement cycle respectively, then

$$\Delta_1^{NT} = \begin{cases} T, & \Delta_1^N > T, \\ \Delta_1^N, & \Delta_1^N \leq T, \end{cases}$$

where Δ_1^N is the renewal cycle under N policy defined by equation (C.1). Therefore,

$$\begin{aligned} \mathbb{E}(\Delta_1^{NT}) &= \int_0^T t \mathrm{d}\mathbb{P}(\Delta_1^N \leq t) + T\mathbb{P}(\Delta_1^N > T) = \int_0^T \mathbb{P}(\Delta_1^N > t) \mathrm{d}t \\ &= \sum_{k=0}^{N-1} \int_0^T p_k(t) R_k(t) \mathrm{d}t. \end{aligned}$$

As the probability that the system is preventively replaced is as follows:

$$\int_0^T R_{N-1}(t) \mathrm{d}V_N(at) + \sum_{k=0}^{N-1} p_k(T) R_k(T),$$

hence the expected maintenance cost during the period U^{NT} can be derived:

$$\begin{aligned}\mathbb{E}(U_1^{NT}) &= c_3 - (c_3 - c_2)\Big(\int_0^T R_{N-1}(t)\mathrm{d}V_N(at) + \sum_{k=0}^{N-1} p_k(T)R_k(T)\Big) \\ &\quad + c_1 \sum_{k=1}^{N-1} \int_0^T R_{k-1}(t)\mathrm{d}V_k(at).\end{aligned}$$

The expected long run maintenance cost is derived from the expressions of $\mathbb{E}(\Delta_1^{NT})$ and $\mathbb{E}(U_1^{NT})$ and the renewal reward theorem.

Appendix D. Optimality conditions

Appendix D.1. Calculations for results in paragraph 2.4.1

By differentiating $C_\infty(T)$ with respect to T and setting it to zero we obtain:

$$\frac{\mathrm{d}\mathbb{E}N(T)}{\mathrm{d}T}\int_0^T \bar{F}_s(t)\mathrm{d}t - \int_0^T \bar{F}_s(t)\mathrm{d}\mathbb{E}N(t) = \frac{c_3 - (c_3 - c_2)\bar{F}_s(T)}{c_1} - \frac{(c_3 - c_2)f_s(T)\int_0^T \bar{F}_s(t)\mathrm{d}t}{c_1\bar{F}_s(T)}$$

where $f_s(\cdot)$ is the density function related to F_s. Denote the left hand side as $G_{au1}(T)$. Since

$$\frac{\mathrm{d}G_{au1}(T)}{\mathrm{d}T} = \frac{\mathrm{d}^2\mathbb{E}N(T)}{\mathrm{d}^2T}\int_0^T \bar{F}_s(t)\mathrm{d}t,$$

$G_{au1}(T)$ is an increasing function of T if $\frac{\mathrm{d}^2\mathbb{E}N(T)}{\mathrm{d}^2T} > 0$. Note that $G_{au1}(0) = 0$, as a result, if also $\lim\limits_{T\to\infty} G_{au1}(T) > \frac{c_3}{c_1}$, then there is a finite and unique T^* which minimizes the average long run cost $C_\infty(T)$.

Additionally, when $c_2 = c_3$ and unit 1 has a Weibull lifetime distribution $F(t) = 1 - exp(-\lambda t^b)$, it is obvious that $\mathbb{E}N(t) = \lambda t^b$ which yields $\frac{\mathrm{d}^2\mathbb{E}N(t)}{\mathrm{d}^2t} = \lambda b(b-1)t^{b-2}$. Therefore $\frac{\mathrm{d}^2\mathbb{E}N(t)}{\mathrm{d}^2t} \leq 0$ for $b \leq 1$ which implies G_{au1} is a decreasing function of T. Hence $T^* = \infty$. On the other hand, for $b > 1$, $\frac{\mathrm{d}^2\mathbb{E}N(t)}{\mathrm{d}^2t} > 0$ and the condition in equation (5) is always hold.

Appendix D.2. Calculations for results in paragraph 2.4.2

Let us suppose $C_\infty(N+1) - C_\infty(N) \geq 0$. This yields to:

$$\frac{d_1(N)\sum_{k=1}^{N-1} d_2(k)}{d_2(N)} - \sum_{k=1}^{N-1} d_1(k) > \frac{c_2}{c_1}, \tag{D.1}$$

where $d_1(k) = \int_0^\infty R_{k-1}\mathrm{d}V_k(at)$ and $d_2(k) = \int_0^\infty p_k(t)R_k(t)\mathrm{d}t$ where $R_i(t) = \int_0^L (1 - G_{\sigma_{L-z}}(t))\mathrm{d}H^{*(i)}(z)$, $i = 1, 2, \cdots, N$, $R_0(t) = 1 - G_{\sigma_L}(t)$.

Denoted by $G_{au2}(N)$ the left hand side of equation (D.1), then

$$G_{au2}(N+1) - G_{au2}(N) = \sum_{k=1}^{N} d_2(k)\left(\frac{d_1(N+1)}{d_2(N+1)} - \frac{d_1(N)}{d_2(N)}\right).$$

Therefore, as we have discussed in paragraph 2.4.1, the unique and optimal N^* exists when equation (6) and (7) hold.

Appendix E. Cost calculation under type I failure interaction

Appendix E.1. Calculations for results in paragraph 3.2

Let $P_1^{cm}(T)$ and $P_2^{cm}(T)$ be the probabilities that the system is replaced before T due to the shock caused by component 1 and due to the natural degradation of component 2 respectively. Let $P^{pm}(T)$ be the probability that the system is replaced at time T. These probabilities are given as follows:

$$\begin{aligned}
P_1^{cm}(T) &= \int_0^T \sum_{n=1}^{\infty} r^{n-1}\bar{r}\bar{G}_{\sigma_L}(t)\mathrm{d}V_n(at), \\
P_2^{cm}(T) &= \int_0^T \sum_{n=0}^{\infty} r^n p_n(t)\mathrm{d}G_{\sigma_L}(t), \\
P^{pm}(T) &= \sum_{n=0}^{\infty} r^n p_n(T)\bar{G}_{\sigma_L}(T).
\end{aligned}$$

As a result, the expected cycle cost is calculated as follows:

$$c_3 - (c_3 - c_2)P^{pm}(T) + \sum_{n=1}^{\infty}(n-1)c_1 r^{n-1}\bar{r}\int_0^T \bar{G}_{\sigma_L}(t)\mathrm{d}V_n(at)$$
$$+\sum_{n=0}^{\infty} n r^n c_1\left(\int_0^T p_n(t)\mathrm{d}G_{\sigma_L}(t) + p_n(T)\bar{G}_{\sigma_L}(T)\right).$$

As the expected length of one replacement cycle is

$$T\bar{F}_{sI}(T) + \int_0^T t\mathrm{d}F_{sI}(t) = \int_0^T \bar{F}_{sI}(t)\mathrm{d}t,$$

So the average long run cost under age-based policy is obtained by the renewal reward theorem.

Appendix E.2. Calculations for results in paragraph 3.3

Let $P_1^{cm}(N)$ and $P_2^{cm}(N)$ be the probabilities that the system is correctively replaced due to the system failure induced by component 1 or due to the natural system failure respectively. Let $P^{pm}(N)$ be the probability that the system is replaced at the Nth unit 1 failure. They are given by

$$\begin{aligned}
P_1^{cm}(N) &= \int_0^\infty \sum_{n=1}^{N-1} r^{n-1}\bar{r}\bar{G}_{\sigma_L}(t)\mathrm{d}V_n(at), \\
P_2^{cm}(N) &= \int_0^\infty \sum_{n=0}^{N-1} r^n p_n(t)\mathrm{d}G_{\sigma_L}(t), \\
P^{pm}(N) &= \int_0^\infty r^{N-1}\bar{G}_{\sigma_L}(t)\mathrm{d}V_N(at).
\end{aligned}$$

So the expected cost over a replacement cycle is given by:

$$\begin{aligned}
\mathbb{E}(C(\Delta_I)) &= c_3 - (c_3 - c_2)P^{pm}(N) + \sum_{n=1}^{N-1}(n-1)c_1 r^{n-1}\bar{r}\int_0^\infty \bar{G}_{\sigma_L}(t)\mathrm{d}V_n(at) \\
&\quad + \sum_{n=0}^{N-1} n r^n c_1 \int_0^T p_n(t)\mathrm{d}G_{\sigma_L}(t) + \int_0^\infty r^{N-1}(N-1)c_1\bar{G}_{\sigma_L}(t))(t)\mathrm{d}V_N(at).
\end{aligned}$$

As the total time elapsed in one cycle Δ_I satisfies:

$$\mathbb{P}(\Delta_I > t) = \sum_{k=0}^{N-1} r^k p_k(t)\bar{G}_{\sigma_L}(t),$$

which implies

$$E(\Delta_I) = \int_0^\infty \sum_{k=0}^{N-1} r^k p_k(t)\bar{G}_{\sigma_L}(t)\mathrm{d}t.$$

From the renewal reward theory, the calculation of $C_{\infty I}(N)$ is straightforward.

Appendix E.3. Calculations for results in paragraph 3.4

In the following, we call component 1 failure as minor failure if it has no effect on component 2 and major failure otherwise. Component 1 failure occurs according to a non-homogeneous Poisson process when it undergoes minimal repair while the system is supposed to be replaced when component 2 fails. Assume $h_1(t)$ to be the component 1 failure rate at time t, from the decomposition property of the Poisson process, the minor failure of component 1 occurs according to a non-homogeneous Poisson process with intensity rate $rh_1(t)$. Denoted by S_n^M the nth minor failure time of component 1, hence

$$\mathbb{P}(S_n^M \leq t) = \sum_{i=n}^{\infty} \frac{(rH_1(t))^i \exp(-rH_1(t))}{i!},$$

where $H_1(t) = \int_0^t h_1(\theta)\mathrm{d}\theta$. By the similar method as we mentioned in previous paragraph 3.2, $C_{\infty I}(T)$ can be rewritten as

$$C_{\infty I}(T) = \frac{c_3 - (c_3 - c_2)\bar{F}_{sI}(T) + c_1\int_0^T \bar{F}_{sI}(t)rh_1(t)\mathrm{d}t}{\int_0^T \bar{F}_{sI}(t)\mathrm{d}t}.$$

Thus the optimal condition of T^* is derived by adopting the method in paragraph 3.2.

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