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OADE- Integration of Plug-in Electrical Vehicles in the Software Ecosystem of Smart Grids

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Summary

The market share of electric vehicles is currently skyrocketing. This causes an increased demand for electric power. It is therefore crucial to consider how to efficiently charge electric vehicles without overloading the power grid. With technology continuously improving and affecting more and more aspects of our life, the concept of smart power grids has started to appear. A smart grid's ability to exchange information between entities will allow better decisions to be made for power balancing. As the added load of charging a large fleet of electric vehicles is substantial, it is important to define a process that ensures charging in an optimal fashion for both consumer and grid.

Therefore, this thesis has a dual focus: The first is to introduce a new approach to electric vehicle charging optimization and the second is to develop a core system for computing optimal charging patterns of electric vehicles connected to a smart grid. This system is intended as a base for future research as well as a base for easy integration in future smart grids. It consists of multiple different models for optimizing charging patterns. These models are the central, the decentral and a new hybrid model. A hybrid model is in this thesis defined as a model in between a central and a decentral approach. Aspects of mathematical optimization that were of interest to allow the definition of the hybrid model were The Lagrangian relaxation and The Benders dual decomposition. In all the models, the electric vehicles are modeled through an agent-based model. This approach allows for each vehicle to make decisions on their own, based on local information. The central and the decentral model achieve positive, expected results, but unfortunately the hybrid model has an unresolved implementation issue which limits the usefulness of its results. Despite this, the theoretical foundation for hybrid models is solid, and a working implementation of one should still be pursued.

Even though the implementation of the new model was not a complete success, the developed system resulted in a valuable core that can be utilized by any interested party; There are few technical prerequisites, and the system is implemented to be adaptable to any situation.

Sammendrag

Markedsandelen av elektriske biler stiger raskt, og dette skaper et økt behov for strømenergi. Det er derfor essensielt å vurdere hvordan disse bilene kan lades på en effektiv måte, uten at strømmettet overbelastes. Med økt teknologisk kunnskap har konseptet smarte strømmnett begynt å utvikle seg. Et smart strømmnetts evne til å utveksle informasjon mellom enheter muliggjør derfor bedre beslutninger for strømbalanseringen. Ettersom den ekstra belastningen med å lade en stor flåte med elektriske kjøretøy er betydelig, er det viktig å definere en prosess som sikrer ladning på optimal måte for både forbruker og strømmettet. Dermed har denne oppgaven et todelt fokus: Det første er å introdusere en ny metode innenfor ladingsoptimalisering av el-biler, mens det andre er å utvikle et kjernesystem for å beregne optimale ladningsmønstre for elektriske kjøretøyer som er koblet til et smart strømmnett.

Dette systemet er tenkt som en base for fremtidig forskning i tillegg til å fungere som en base for enkel integrering i fremtidige smarte strømmnett. Systemet består av flere forskjellige modeller for ladingsoptimalisering. Disse modellene er den sentrale, den desentrale og en ny hybrid modell. En hybrid modell er i denne oppgaven definert som en modell i mellom den sentrale og den desentrale tilnærmingen. Aspekter ved matematisk optimalisering som muliggjorde den hybride tilnærmingen var «The Lagrangian relaxation» og «The Benders dual decomposition». I alle de tre modellene er de elektriske bilene modellert gjennom en agent basert modell (agent based model). En slik tilnærming gjør det mulig for hvert kjøretøy å ta beslutninger alene basert på lokal informasjon. Den sentrale og den desentrale modellen oppnår positive, forventede resultater, men dessverre har den hybride modellen et uløst implementeringsproblem som begrenser nytten av resultatene dens. Til tross for dette er det teoretiske grunnlaget for den hybride modellen solid, og en fungerende implementering bør derfor utvikles videre.

Selv om implementeringen av den nye, hybride modellen ikke var en fullstendig suksess, resulterte den hybride modellen i et verdifullt kjernesystem som kan benyttes videre av andre. Det er få tekniske forutsetninger ved dette systemet, og det er implementert for å kunne tilpasses enhver situasjon.

Preface

This master's thesis concludes my Master of Science in Computer Science at the Norwegian University of Science and Technology (NTNU). The thesis was conducted during the spring semester 2018 in the Department of Computer Science. It is written with guidance from Salman Zaferanlouei at the Department of Electric Power Engineering. The inspiration for this project originated from discussions with my supervisor Salman Zaferanlouei. I would therefore like to thank Salman for excellent guidance and motivational discussions. I truly appreciate the time he was able to spend helping me and the project forward. His feedback and knowledge regarding the electric power aspects of the thesis was invaluable. Additionally, I would also thank Diego I. Hidalgo-Rodríguez for attempting to help with solving the implementation error we were unable to locate. Furthermore, I would like to thank everyone else at the Department of Electric Power Engineering for valuable discussion regarding the project scope. Finally, I would especially like to thank my partner, Ragnhild Sophie, for not only the time she spent reading the thesis and supplying valuable feedback for improvements, but also for her encouragement and sticking with me through a long period of my thesis work while looking after our son. Truly, thank you!

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Abbreviations

ABM	=	Agent based model
ABO	=	Agent based optimization
CCU	=	Central control unit
EV	=	Electric vehicle
OADE	=	Open and Autonomous Digital Ecosystems
PEV	=	Plug-in electric vehicle
PHEV	=	Plug-in hybrid electric vehicle
SoC	=	State of charge
V2G	=	Vehicle to grid
V2H	=	Vehicle to home

Introduction

In recent years, the market share of electric vehicles (EVs) has drastically increased. Norway is one of the leading countries in this regard with a 39.2%¹ market share in 2017 [3, 4]. According to the international energy agency, the number of electric vehicles in Norway are expected to reach roughly 2 million by the year 2030 [5]. This is broadly accepted as a positive trend as it greatly reduces our dependency on fossil fuels in the transportation sector. Despite this, a large-scale adoption of EVs adds a new large strain on the power grid that has not existed before. Most modern power grids are in theory capable of providing the total amount of energy needed to support a large fleet of EVs. Still, challenges can arise in cases where the main load from charging these EVs occurs at times with an already high demand for power (peak hours) [6]. To reduce the stress on power grids in these peak hours, it is crucial to define a protocol for EV charging which can smooth out the load over time, while still ensuring that all EVs get the charge they need. Several works have been published on this topic in recent years discussing a variety of different approaches [7, 8, 9].

Today's power grids are restricted to one-way communication. A consequence of this is that the power market is purely based on predictions regarding future demand [8]. If situations of imbalance between supply and demand occur, the options are either to reduce the supply or increase the demand. As technology is currently becoming more prevalent in all areas of society, so-called smart grids are in the process of being defined and implemented [1, 10]. These networks enable two-way communication between the supply side and the demand side. Such an ability allows for a cooperation in regard to matching the supply and demand in an optimal manner for the grid. A more in-depth description of smart grids is given in section 2.1.

The cooperation between the supply side and the demand side enables a new set of approaches to control the charging patterns of EVs and thus the total power flow of a grid. An increasingly popular area within artificial intelligence is agent-based modeling (ABMs). These models allow the representation of systems where there are several entities working either independently or together to achieve certain goals [11]. This is described in more detail in section 2.2. This kind of models fits perfectly in a system of EVs, where the EVs are the agents of the system. ABMs are often-times utilized for simulations and visualization of these. Another possibility for these ABMs is to include them in an optimization process, which we call agent-based optimization (ABO), an example is defined by Leung et al. [12]. The main idea is to combine the mathematical optimization of an objective function with the agent based representation as building blocks for the objective function. This allows the formulation of a function, that when optimized, includes the behavioral pattern of the agents associated with the optimal objective value. In this way, the charging pattern of EVs in a power grid is described in a mathematical form. This formalization as a mathematical problem simplifies the process of researching new approaches to EV charging.

1.1 Goals

With this thesis we wish to define a new set of tools and ideas which can be utilized in the development of smart grids. Additionally, the tools and concepts introduced will hopefully work as a core base for future research in this area.

¹Including both PEVs and PHEVs

The world is in the early phases of moving towards a transportation network where most vehicles will be powered by electricity. Supplying the field with a core for future developments should be a helpful resource. From this, two main goals are defined for this thesis: The first is to introduce a new and innovative approach to charging pattern optimization for PEVs within a smart grid, based on an agent-based model and state of the art research. This type of optimizer will optimize charging in such a way as to simultaneously minimize cost and congestion in the system for both grid and vehicles. The resulting effects of the new optimization model will be shown and discussed in comparison to three other models later in the thesis. The second goal of this thesis is to ensure that the resulting system can be a useful resource for both practical problems and continued research. The system shall be structured in such a way that it can be incorporated with ease; it shall not be model dependent, but data dependent², different optimization models can be defined and tested within the same system and there should be no need for a complex background within the field to understand and use the system.

1.2 Outline

The structure of the thesis is as follows: Chapter 2 presents the core theoretical foundation relevant for the project. Chapter 3 describes state of the art research where most the inspiration for this thesis originated. Next, in chapter 4, the method for the project is detailed. This includes the new optimization model introduced, the environment in which the system is created and definitions revolving the generation of results. Following the method are the results achieved in chapter 5. Finally, these results are analyzed and discussed in chapter 6. Future work is then described in chapter 7 and a conclusion to the thesis is presented in chapter 8.

²Instead of a predefined model description, the system builds a model from externally supplied data

Background

2.1 Smart grids

The electric power industry is currently undergoing a fast-paced advancement. Power grids are increasingly taking advantage of recent technological innovations and this has been called the next-generation of power grids, or smart grids [1]. There has recently been a large focus on formulating and promoting a definition of what exactly a smart grid encompasses. The existing definitions are still only partial, and the concept is still evolving [10]. Despite this, certain aspects are already clear; the main goal is to bring power grids into the 21st century technology wise by taking advantage of state-of-the-art technologies. The research done by Amin et al. [10] state that with a common digitized platform, smart grids will enable increased flexibility in control, operation, and expansion. They will allow for embedded intelligence, essentially foster the resilience and sustainability of the grids; and eventually benefit the customers with lower costs, improved services, and increased convenience. The key differences between existing and smart grids are shown in Table 2.1.

Smart grids are expected to bring a revolution in the interaction between power suppliers and customers [8]. This interaction is for the time being limited, and one-directional. Customers buy power when it is needed, for the price at the given time. Since there is no communication to the supplier, it is hard for the supplier to know how much power will be needed at any point in time. This is a challenge as electric power is a good that must be consumed the moment it has been produced [13]. Not supplying the correct amount can cause instability in the grid. This challenge is one that smart grids are expected to alleviate as they introduce the ability for two-way communication between suppliers and customers. It is this feature EV charging systems can take advantage of for improved charging patterns. As EVs share information and wishes, charging optimization can then be performed in a multitude of ways.

Existing grid	Smart grid
One-way communication	Two-way communication
Centralized Generation	Distributed Generation
Hierarchical	Network
Few Sensors	Sensors Throughout
Blind	Self-Monitoring
Manual Restoration	Self-Healing
Failures and Blackouts	Adaptive and Islanding
Manual Check/Test	Remote Check/Test
Limited Control	Pervasive Control
Few Customer Choices	Many Customer Choices

Table 2.1: The existing grid compared with the smart grid [1]

Privacy of information

Despite the positives of implementing smart grids as a power grid standard, just as with other areas where technology is prevalent, there are concerns related to both the privacy and security of the data utilized in such a system. According to research by Siddiqui et al. [14], potential privacy concerns of Smart Grid consumers include: the way the required information is going to be collected, used and disclosed, how customer information is expected to be safeguarded, how permissions will be granted for the collected data to be shared with multiple agencies and the liabilities related to any breaches of consumer information. Another important aspect is to what degree a smart grid will recognize separate people or households. It is theoretically possible to determine something as specific as the model brand of appliances in a home, based on the energy fluctuation pattern. And finally, a large enough sample of data that individually seem harmless, can quickly become a threat to privacy as third parties analyze them together [14].

Furthermore, Siddiqui et al. [14] state that even when the data about electricity consumption is not collected at regular intervals, information can still be collected at a slower rate through the persistent monitoring of energy consumption. A result of this is that information, which can be argued should not be accessible by the power grid privacy and security wise, can be inferred. Examples of this are that the number of people residing in a home can be inferred and that even the routine of these people can be inferred (since specific appliances can be detected, when they are detected enables this inferral). With the connection of personal PEVs to this grid, even more private information, now based on travel patterns, can be inferred. This is why the amount and type of information exchanged between PEVs and the grid must be in focus when developing charging pattern models in a smart grid.

2.2 Agent based modeling

Agent based modeling is a sub-field of artificial intelligence with roots in several areas of science. These include areas like social sciences and economics. Agents are capable of making personal decisions based on facts that are known to them. Each agent can have its own set of rules it follows to achieve a certain behaviour or reach a certain goal [15]. Their individual behaviour can, and often does, lead to a system which seems organized from the outside, even though it isn't. This phenomenon is known as emerging intelligence [16]. The main focus of the field of agent based modeling is exploring the different interactions and behaviours that emerge in this way.

It is important to note that different agents can have very different knowledge and goals. One must therefore consider the possible interactions that can happen when agents are mostly heterogeneous or homogeneous [8]. In some situations, agents might benefit from cooperation towards their goals, while in other cases they might be more inclined to be competitive and work individually. Both the facts known by agents and their goals can affect this behaviour. It is these aspect that must be considered if one attempts to achieve some specific behaviour. Predicting the behaviour of these agent-based models correctly is hard by nature, which is why some trial and error will always be necessary in this process.

2.3 Optimization theory

Optimization theory is the field of optimizing functions. Optimizations are usually either maximizations or minimizations of an objective. More often than not, these functions are not straight forward to optimize because they are subject to constraints on their variables. An example is illustrated in Equation 2.1. The main challenge then arises in solving optimizations, while remaining within these constraints.

$$\begin{aligned} \min_{x,y} \quad & f(x,y) = x^2 - 2y \\ \text{s.t.} \quad & x > 0, \quad y < 100 \end{aligned} \tag{2.1}$$

The constraints defined in such a manner as in Equation 2.1 are called hard constraints, because if they can not be fulfilled the optimization is deemed infeasible. An example of such an optimization problem with infeasibility is shown in Equation 2.2, where logically, both constraints can't be satisfied simultaneously.

$$\begin{aligned} \min_{x,y} \quad & f(x,y) = x + y \\ \text{s.t.} \quad & x + y = 0, \quad x + y = 10 \end{aligned} \tag{2.2}$$

Optimization problems can be solved by step-by-step algorithms, known as solvers in this context. These solvers iterate and attempt to converge towards an optimal solution. There exist several categories to optimization problems based on properties of both the objective function and its constraints. Some examples are if the objective function is linear or non-linear, or that binary variables are included in the problem specification. There are many different solvers available for all the different optimization problem variants [17]. This allows us to define our own optimization problems without worrying about these properties, as long as we choose the right solver to use. Therefore, the differences of these variations are not described further. Instead focus is directed towards some areas within optimization theory which will be utilized, namely the Lagrangian relaxation and the Bender's dual decomposition method.

2.3.1 Lagrangian relaxation

The Lagrangian relaxation is a technique introduced to replace hard constraints in an optimization problem with a penalty term in the objective function, known as a soft constraint. These constraints become soft because being part of the objective function ensures that the constraints will indirectly be considered in the optimization problem, but there is no hard limit posed on them. This infers that a solution will still be found in infeasible situations. It is then necessary to check if solutions are feasible once optimization has been performed on a problem with soft constraints. The process of converting a hard constraint to a soft one requires moving all terms of the relevant constraint to one side of the equation. These terms are then added to the objective function, but scaled by a Lagrange multiplier, λ . An example optimization problem is shown in Equation 2.3 and its corresponding relaxed Lagrangian formulation follows in Equation 2.4. It is also worth noting, that partial Lagrangian relaxations can be performed, the only implication is that not all constraints have been relaxed [18, 19].

$$\begin{aligned} \min_{x,y,z} \quad & f(x,y,z) \\ \text{s.t.} \quad & x + y = 25 \end{aligned} \tag{2.3}$$

$$\min_{x,y,z} \quad f(x,y,z) + \lambda(x + y - 25) \tag{2.4}$$

2.3.2 Benders dual decomposition method

The Benders dual decomposition has been intensely researched the last decade as it has shown promise in several fields. Some examples are from power generation [20], supply chain management [21] and network design [22]. A full overview of the method is presented by Rahmani et al. [23]. At the core of this method lies the concept of divide and conquer. Given an optimization problem, the complicating constraints are removed by generating a relaxed Lagrangian, as described above. The constraint is then still a part of the optimization, but only as a soft constraint. This allows the main part of the method. With the complicating constraint added to the objective function, this function is now split into two, hence the name dual decomposition. An example of a non-decomposed problem is shown in Equation 2.5. After decomposing the relaxed Lagrangian the resulting equation is shown in Equation 2.6. In the equations, the variable z is part of the complicating coupling constraint.

$$\min_{x,y,z} \quad f(x,z) + g(y,z) \tag{2.5}$$

$$\min_{x,z1} \quad f(x,z1) + \lambda z1 \quad + \quad \min_{y,z2} \quad g(y,z2) - \lambda z2 \tag{2.6}$$

The next step in the method is to solve each of the two minimizations in Equation 2.6 and update the λ variable based on these optimizations. This is the gradient ascent step where this process is then repeated until convergence to a lambda is reached. At this point convergence to a z that minimizes both decomposed objective functions has occurred. Finally, this z value can then be input as a static parameter in the original problem, Equation 2.5, and an optimal solution is found.

State of the Art

In this chapter, we introduce state of the art research that has been the main source of inspiration for the new hybrid model this thesis introduces.

3.1 Dual decomposition for load balancing optimization

In a recent paper by Hidalgo-Rodríguez et al.[2] several optimization models for power balancing between a main power grid and connected home-microgrids were explored. These microgrids were independent of the others and have individual costs and objectives. This approach is deemed interesting for this thesis as a simplification can be made to this model, with micro-grids being replaced by agents. From this point the simplified system is looking very similar to the situation of charging optimization for EVs in a grid. Therefore, one of the models that was introduced in the paper will be a starting point for our own hybrid model approach.

The model we take interest in is called the "Hierarchical-distributed operation strategy" [2]. This model defines an objective function which encompasses all the individual objective functions of the home-microgrids as well as the objective function for the grid itself. A simple way to visualize this is to think of the decentral objectives added to the central objective. A constraint is then defined for the system where the amount of power supplied by the grid must be equal to the total power required by the home-microgrids. This constraint connects the central and decentral aspects of the model, and thus becomes a complication coupling constraint for the optimization process. It is because of this complication that the concepts of the Lagrangian relaxation and the Benders dual decomposition come into play. These methods are described thoroughly in section 2.3. By incorporating these methods, the main objective function can now be split into two parts. One will contain the decentral objectives and the term of the relaxed constraint regarding the power demand. The other contains the central objective and the term of the relaxed constraint regarding power supply. The terms from the relaxed constraint include the Lagrange multiplier λ .

To solve this new structure to the problem, an algorithm is introduced. The steps of this algorithm require that the decentral part is solved first followed by the central part. When both have been computed, the Lagrangian multiplier is updated. This process is then repeated until convergence occurs. Initially the Lagrangian multipliers are set to zero. The update of the multiplier uses the difference of the decentral and central results, scaled by a factor α .

In the paper by Hidalgo-Rodríguez et al. [2], this approach results in a load distribution and cost that is similar to a central approach, but not as sporadic as a decentral approach. These results are shown in Figure 3.1.

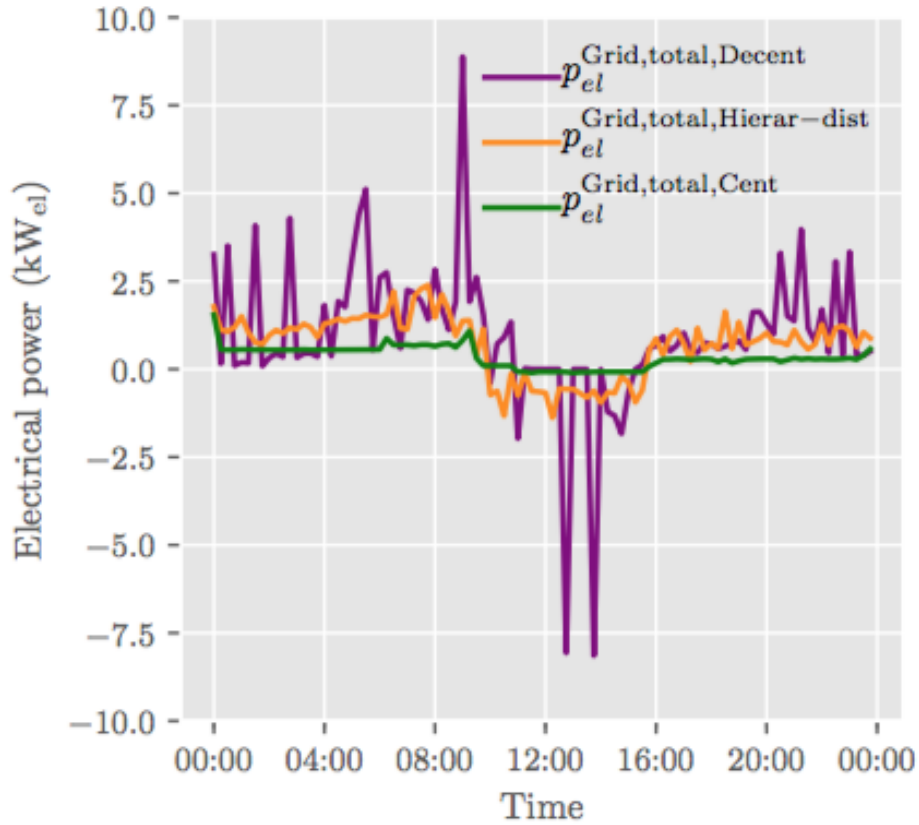


Figure 3.1: Resulting power profiles for the experiments performed in the original paper [2]

3.2 Linearization of non-linear power flow equations

An additional complication in the optimization of charging and load balancing in power grids is that conventional load flow equations can't be used for realistic modelling [24]. This is due to the fact that power grids are subject to multiple non-linear loads. Therefore, it is necessary to use harmonic load flow equations for modelling [24]. These are non-linear in nature, thus complicating the solver requirement for the optimization and increasing necessary computation to perform optimization.

The paper by Pirouzi et al. [24] focuses on replacing these non-linear harmonic load flow equations with a linear simplification. This is done by using Taylor series [25] and linearization techniques for the AC power flow formulation. The equations shown in 3.1 and 3.2 are valid for both the non-linear and linear versions. Meanwhile, equations 3.3 and 3.4 show the non-linear approach, while equations 3.5 and 3.6 show the linear approach. All the equations are found in the original paper by Pirouzi et al. [24]. For all the models introduced in the next chapter, the linear equations have been used. Even though the linear variations are simplifications, and thus deliver an approximate result compared to the full, non-linear equations, these differences are minor enough so they are barely noticeable. Because of this, the positives of using a linear set of equations outweigh the minor negative that the simplification imposes.

$$Pg_{b,t} - \sum_j^{j \neq b} Al_{b,j} Pl_{b,j,t} = Pd_{b,t} \quad \forall b, t \quad (3.1)$$

$$Qg_{b,t} - \sum_j^{j \neq b} AL_{b,j} \times Ql_{b,j,t} = Qd_{b,t} \quad \forall b, t \quad (3.2)$$

$$Pl_{b,j,t} = g_{b,j}V_{b,t}^2 - V_{b,t}V_{j,t} \times [g_{b,j}\cos(\theta_{b,t} - \theta_{j,t}) + b_{b,j}\sin(\theta_{b,t} - \theta_{j,t})] \quad \forall b, j, t \quad (3.3)$$

$$Ql_{b,j,t} = -b_{b,j}V_{b,t}^2 + V_{b,t}V_{j,t} \times [b_{b,j}\cos(\theta_{b,t} - \theta_{j,t}) - g_{b,j}\sin(\theta_{b,t} - \theta_{j,t})] \quad \forall b, j, t \quad (3.4)$$

$$Pl_{b,j,t} = g_{b,j}V_{min} \sum_l^L (\Delta V_{b,t,l} - \Delta V_{j,t,l}) - b_{b,j}V_{min}^2 (\theta_{b,t} - \theta_{j,t}) \quad \forall b, j, t \quad (3.5)$$

$$Ql_{b,j,t} = -b_{b,j}V_{min} \sum_l^L (\Delta V_{b,t,l} - \Delta V_{j,t,l}) - g_{b,j}V_{min}^2 (\theta_{b,t} - \theta_{j,t}) \quad \forall b, j, t \quad (3.6)$$

Method

In this chapter, we begin by introducing and describing the models created for the purpose of this research. Then follows a detailed look at the modelling and optimization software that was used. Next the different scenarios defined for running the different models are described. Finally, the foundation for travel pattern extensions are detailed.

4.1 Proposed charging pattern optimization models

The first model introduced is intended as a reference model, then, based on recent relevant research, described in chapter 3, several improved models have been formulated. The first two, the central and the decentral model, are the base models which are found at opposite ends of the possibilities of charging pattern optimizations. The last two models are hybrid models, meaning they are in between the central and the decentral approach and hence combine elements from both. All of these models are of interest in the charging pattern optimization field, but the hybrid variations are the least explored and most innovative ones.

4.1.1 Reference model, dumb charging

To allow a meaningful analysis of the results from the models introduced in this chapter, it is necessary to have a base reference model to which the results can be compared. For this purpose, a dumb charging model is used. Dumb charging is the name given to charging patterns where charging occurs the moment a PEV parks and connects to the grid. The word dumb is used to show how it opposes models in a smart grid as there is no consideration to if and when charging should occur. Dumb charging is most prevalent in current grids; PEVs are charged when the owners park at work during the day or when they come home in the evening on a general basis. This then, amplifies peak load hours and it is exactly this situation smart grids wish to improve.

One aspect of the implementation of dumb charging in our model has to do with how congestion situations are handled. In reality, if several agents try to charge more than the grid allows at any given time, they will all receive less power. Instead, in our simulation, power is diverted to those most in need, essentially a comparison of each PEV's SoC is performed, and a lower SoC is prioritized for charging. Even though this behaviour is not perfectly realistic, the total load on the grid remains the same. Therefore, we assume this difference not to impact our result analysis.

4.1.2 Central model

A centralized approach to charging pattern optimization entails that there is a central controlling unit (CCU), with complete knowledge of the state of the power grid and its components [26]. Relevant components here are the PEVs of the system and the base load of the system. The base load is the load the grid faces on a regular basis without considering the PEVs. The main benefit of such a control scheme is that the total knowledge allows for optimal optimization of both agent charging pattern and power flow in the grid. As there are no unknowns, there is no obstacle in finding an optimal charging pattern for any given time. The downsides to such an approach are, unfortunately,

plentiful. Firstly, it is rather uncommon, if not impossible, for a such a system to have total complete knowledge. It would require not only an active line of communication between all entities and the CCU at all times, which itself poses several challenges, but it would also need the capability to predict all future events. Secondly, utilizing a CCU puts enormous responsibility on it. All charging optimization profiles must be calculated and distributed in real time. Given a large grid and a large amount of PEVs, this quickly becomes computationally infeasible. Additionally, the collection of all known information regarding each entity in the system raises concerns with privacy of data as discussed in section 2.1. Finally, a single CCU is also a single point of failure. Such a CCU would be critical to keep online at any given time and any downtime could therefore quickly lead to chaos.

The main objective of our proposed centralized charging scheme is to reduce the total cost of power production from a generation perspective. In addition to this objective, some physical limits must be taken into account both for the PEVs and for the power grid. For the agents, the main aspect would be to maintain the minimum state of charge (SoC) of each of them, which is either the model specification minimum SoC or the minimum amount of energy required for the agent to reach its next target. Additionally, a requirement is introduced for all the PEVs to end an optimization time frame with an SoC that is at least equal to the SoC at the start of the time frame. The reason for this requirement is that the CCU only optimizes in regard to power generation cost as seen in Equation 4.1. Hence, without this requirement, logically all PEVs would be charged as little as possible to reduce the amount of power generated. The limitations connected to the power grid refer to the power flow equations described in section 3.2

The objective function for the central model is defined as:

$$z_c = \sum_{t=1}^T \sum_{g=1}^G nG(g, t) \times Pg(g, t) \quad (4.1)$$

Here, nG and Pg respectively denote the marginal cost of the generators and the amount of power they generate at each time interval. The double summation in this equation sums over all generators in the system and each time step of the time frame.

The optimization problem, which intends to minimize this objective function is then defined by:

$$\min_{Pg(g,t), pCh(a,t)} z_c \quad (4.2)$$

subject to:

$$Pg_{b,t} - \sum_{j \neq b} Al_{b,j} Pl_{b,j,t} = Pd_{b,t} \quad \forall b, t \quad (4.3)$$

$$Qg_{b,t} - \sum_{j \neq b} AL_{b,j} \times Ql_{b,j,t} = Qd_{b,t} \quad \forall b, t \quad (4.4)$$

$$Pl_{b,j,t} = g_{b,j} V_{min} \sum_l^L (\Delta V_{b,t,l} - \Delta V_{j,t,l}) - b_{b,j} V_{min}^2 (\theta_{b,t} - \theta_{j,t}) \quad \forall b, j, t \quad (4.5)$$

$$Ql_{b,j,t} = -b_{b,j} V_{min} \sum_l^L (\Delta V_{b,t,l} - \Delta V_{j,t,l}) - g_{b,j} V_{min}^2 (\theta_{b,t} - \theta_{j,t}) \quad \forall b, j, t \quad (4.6)$$

$$Pd_{b,t} = Pbl_{b,t} + Pald_{b,t} \quad \forall b, t \quad (4.7)$$

$$Pald_{b,t} = \sum_a^A \max_charging_rate_a \times charging_ratio_{a,b,t} \quad \forall b, t \quad (4.8)$$

$$Min_SOC_a \leq SOC_{a,t} \leq 1.0 \quad \forall a, t \quad (4.9)$$

$$SOC_{a,t-start} \leq SOC_{a,t-final} \quad \forall a \quad (4.10)$$

4.1.3 Decentral model

A decentral model for charging pattern optimization is a bottom-up approach while the central model is a top to bottom approach. Instead of assigning a single control unit all of the responsibility, all agents, in our case PEVs, are tasked with making their own decisions. The agents have little to no information about the rest of the system and they are only interested in satisfying their own needs. There are several benefits to a decentral model, most of which are resolutions of the negative aspects of the central model [27]. This model architecture removes the need for constant communication between the agents of a system and a controller. Thus, there is no longer a large responsibility in one place as each agent make their own decisions. At the same time, the system no longer has a single point of failure. If a single agent fails, the rest of the system will remain unaffected. The fact that each agent optimizes their own charging pattern drastically reduces computational complexity as less variables need to be considered. Even though there are many positives to a decentral model, there are also some less fortunate implications. As all agents only consider themselves, there is no consideration for how the effect that the combined charging pattern of all agents will affect the power grid. One has to rely on the objective function of the agents to correlate with a charging behaviour that reduces load during peak times, and instead increases the load at low demand times. Lacking a guarantee to satisfy power grid constraints is therefore the major downside to a decentral approach.

The decentral model we define includes the price of power in the objective function of the agents. The intention with this is to incentivize charging at times with lower demand. When agents see a higher price for power, which should be the case at higher demand times, they will want to move their charging to a cheaper, and accordingly lower demand time. This will not necessarily keep all agents from charging at peak times as they do have some basic requirements that they must fulfill. They have minimum SoC which they will not allow themselves to fall under and, as with the central model, each agent wishes to have at least as much power at the end as at the start of the optimization time span. This is based on the same reasoning as detailed in subsection 4.1.2. Basically, since we want to minimize cost, and charging always has a cost, the absolute minimum is achieved when not charging.

As a side note, many decentral models are stochastic in nature and determine a charging pattern randomly [8, 27, 28, 29]. The concept is that at any time step, charging is determined by a probability. This can be combined with an approach like ours with varying the probability based on the time step, in the same way the price changes. However, our model does not include this aspect because we chose to focus our efforts on generating deterministic results.

The objective function for the decentral model is defined as:

$$z_{dec} = \sum_{a=1}^A z_{a_{dec}}(a) \quad (4.11)$$

where $z_{a_{dec}}$ is the objective function for each agent defined by:

$$z_{a_{dec}}(a) = \sum_{t=1}^T nP(a, t) \times pCh(a, t) \quad (4.12)$$

Here, nP is the price of power at the bus the agent is connected to at the given time step, and pCh is the amount of power taken from the network at the given time step. It might seem counter intuitive at first that the main objective function in Equation 4.11 is the sum of all agents, when the concept of the decentralized model is that they are all independent. Mainly, this formulation is a simplification for the simulation. In practice, each agent would run their own optimization, with the objective function defined in Equation 4.12. For our simulation though, we optimize the single function defined in Equation 4.11 to reduce computation time and complexity. The results are guaranteed to be identical because the decisions made by all agents are independent of each other. Altering the charging pattern for one, will therefore not affect any other. Hence, minimizing the sum of all agents' objective function is equivalent to minimizing each agents' objective function separately.

The optimization problem to minimize the objective function of all agents is defined by:

$$\min_{pCh(a,t)} z_{dec} \quad (4.13)$$

subject to equations 4.8, 4.9 and 4.10 defined in subsection 4.1.2.

4.1.4 Hybrid models

As described, both the central and decentral models have benefits and disadvantages. A hybrid model intends to surpass the disadvantages and combine the benefits of the two models. For this thesis a hybrid model has been defined, based on state-of-the-art research described in chapter 3. The principle of this model is to iterate towards an optimal charging pattern for both the agents and the system. The iteration process consists of a few steps: First, a decentral optimization problem is solved. Then, the charging pattern of this decentral optimization is communicated to a central control unit (CCU). The CCU now solves a central optimization problem, but with the agent charging pattern fixed as the one received by the decentral optimization. The implication of this is that the central optimization does not optimize the charging pattern of the agents anymore, instead it only optimizes the power flow of the grid. Next, the CCU detects any congestion in the grid and communicates this through a parameter to the agents, which will then affect the next decentral optimization. These steps are then repeated until convergence to an optimal solution is found or until a maximum number of iterations is reached.

Hierarchical-distributed operation model

The hierarchical-distributed operation model is based on the work of Hidalgo-Rodríguez et al. [2], described in section 3.1. The general idea is to take use of the concept of dual decomposition to optimize the objective. This entails relaxing the coupling constraint by forming a partial Lagrangian which is detailed in subsection 2.3.1. Simply stated, this process removes the constraint by adding it to objective function with a weighting term. This weighting term is the part of the new objective function, and by altering this each iteration, we are then able to converge towards a solution. The equation which updates the weighting parameter called λ is defined as:

$$\lambda^{l+1} = \lambda^l + \alpha \times (z_{dd_dec} - z_{dd_c}) \quad (4.14)$$

Here, $\alpha = \frac{0.5}{T}$, l is the iteration step and the variables z_{dd_dec} and z_{dd_c} are the dual decomposition equations of the central and decentral objective functions. They are defined as follows:

$$z_{dd_dec} = z_{dec} - \lambda^l \times \sum_{t=1}^T Pald(t) \quad (4.15)$$

$$z_{dd_c} = z_c + \lambda^l \times \sum_{t=1}^T (Pg(t) - Pbd(t) - PL(t)) \quad (4.16)$$

The decentral equation 4.15 has all the constraints of the decentral model, described in subsection 4.1.3. The central equation 4.16 has the same constraints as the central model described in subsection 4.1.2, but with one exception. The coupling constraint, being Equation 4.3 has been relaxed, and is therefore excluded.

4.2 Optimization environment - GAMS

With all the models defined as optimization problems, we will take use of a mathematical optimization suite called GAMS to both models and run them. On the software's homepage there is a good introduction to what capabilities the suite offers:

The General Algebraic Modeling System (GAMS) is a high-level modeling system for mathematical programming and optimization. It consists of a language compiler and a stable of integrated high-performance solvers. GAMS is tailored for complex, large scale modeling applications, and allows you to build large maintainable models that can be adapted quickly to new situations. GAMS is specifically designed for modeling linear, nonlinear and mixed integer optimization problems[30].

Implementing our models in GAMS introduces several benefits. The most notable of these are: The large quantity of available commercial solvers, its ease-of-use and the ability to separate model and data freely. Having access to several solvers reduces the risk of being limited by solver capabilities. If a specific situation is encountered that one solver is unable to deal with, then it is likely that one of the other available solvers is able to deal with it. The fact that it is both easy to learn and to use is a general positive, but it is also important regarding reusability of the system. This is

line index	connection 1	connection 2	R	X	S max
1	1	2	0.000574	0.000293	4.60
2	2	3	0.003070	0.001564	4.10
3	1	4	0.002279	0.001161	2.90
4	2	4	0.002373	0.001209	2.90

Table 4.1: Example format of the line table used to define the power grid in GAMS

discussed further in section 4.2.3. The independence of model and data was not a feature that we got to take advantage of in the scope of this project though, but it is highly relevant to allow for scalability and extended use. One addition which could make use of this feature is the travel pattern generation. Even though its implementation ended outside the final scope of the project, its use and functionality were defined and are therefore described in section 4.4.1.

4.2.1 Power grid implementation

When modelling the power grid aspect of our models in GAMS, the main challenge was to create the network structure in a flexible and extendable manner. A power grid can be represented by a graph where the generator and the buses can be represented by vertices and power lines by edges. Our solution to create a flexible and easily adaptable system was to require the graph of the power grid to be represented by a table describing the power lines. In this table we require technical data regarding the lines, and additionally, each line must store which buses and/or generators they are connected to. This results in a system where changing the power grid does not require anything but altering this table. This does not only allow quick and simple tweaks to the grid for testing purposes, but it also simplifies the process of applying this system on any other grid. An example on how such a table is formatted is seen in table 4.1 and the resulting graph based on this table is shown in figure 4.1. The physical limits of the network are then defined by the constraint equations for the central model, found in subsection 4.1.2.

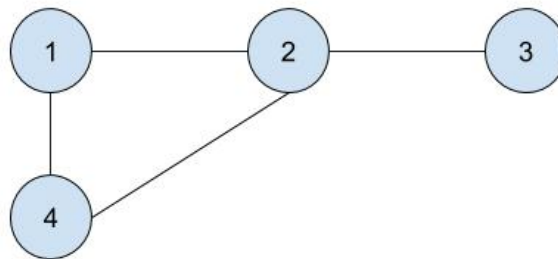


Figure 4.1: Illustration of the power grid defined by Table 4.1

4.2.2 ABM implementation

Generally, ABMs implement their agents as custom objects with their own data structure. GAMS, unfortunately, does not support creating objects with such a purpose. To incorporate an ABM in GAMS then, a set of indices was created. This set only contains the identifiers of the agents, while data regarding the agents is represented in separate GAMS parameters. These parameters are essentially arrays of the data where we use the index of each agent to retrieve the data belonging to that agent. The equations defined in GAMS to represent the ABM are those found in the decentral model, described in section 4.1.3. Additionally, PEVs can vary in their technical specification, e.g. a Tesla has a larger battery capacity than a Nisan leaf [31]. These specifications must be defined through GAMS parameters as well. For our optimizations only equal PEVs were used since the projects focus was more directed towards the load on the system, rather than specific agent behaviour. Nonetheless, adding different agents would only require additional parameters describing these differences.

4.2.3 Functional reusability of the system

One of the main goals of this project was to offer a scalable, expandable system, that can be taken advantage of by anyone for a multitude of situations. Clean code usually comes to mind when reusability of code is mentioned. While important for readability and understandability, this is not the most critical in regard to the functional reusability of the system. With functional reusability, the focus is directed towards creating a flexible and easily adaptable system that anyone can take use of. Implementing the system in GAMS is thus already a major step in this direction due to its design.

The GAMS website [30] states: The GAMS language is formally similar to common programming languages. It is therefore familiar to anyone with programming experience. But since the model is formulated in a way that is similar to its mathematical description, it can be understood and maintained not only by programmers, but also by the actual domain experts. GAMS focuses on the modeler and allows him to do all relevant things himself.

The implementation of the power grid structure is, as mentioned, a table of the power lines of the grid. This facilitates the task of implementing a new grid, of any structure, in the system. Additionally, GAMS has support for separating model and data, therefore one could easily store the grid structure as an independent file, e.g. an excel file. This separation of data can equally be useful for incorporating the travel patterns of the agents. This is currently also defined as a table in the GAMS model, where the values describe where the agents are connected at any given time. Table 5.1 is an example of such a table.

When it comes to reusability regarding the ABM, its current implementation consists of a general ABM base. All agents are equal, and they have few features that are agent dependent. As detailed previously, such features can be easily added with GAMS parameters. Additionally, if one wishes to alter the decision factors for when agents wish to charge, this only requires altering the objective equation or adding/removing constraint equations.

Finally, another practical feature of GAMS, which improves the functional reusability of the system, is the ability to separate defined equations in a GAMS file into their own models. This allows us to define all relevant variables and equations for all our models in the same file, but in a structured way. From this point, we can simply select the model of interest and then only solve that one. Because of this, modifying or extending existing models is very simple. Removing, adding or altering elements can be done on a per element basis, without affecting the rest of the model.

4.3 Scenario definitions

With the models defined and their implementation detailed, the next step is to introduce the different scenarios utilized to run the optimizations and gather performance data. Each scenario is structured in such a manner to test the models for specific circumstances.

The scenarios we ran were designed to test specific limit aspects of the system. Three run configurations were been defined:

- No congestion
Run where everything runs smoothly and capacity limits are not hit.

- Line congestion
Run where the agents need to charge more than the line capacity.

- Generation congestion
Run where the agents need to charge more than the generator can supply.

The grid used in the scenarios is the simplest available, consisting of only one generator and one bus where agents can charge. this structure is shown in Figure 4.2. The intention is to introduce our models in a very basic system, so behaviour can be explained by the model, and not by technicalities in a more advanced grid. This also means that our generator, and its marginal cost, are a simplified approach to represent respectively importing power from the grid and the market power price.



Figure 4.2: Base grid structure

4.4 Defined concepts outside project scope

These concepts were initially defined to be part of the project, but eventually fell out of scope for the main assignment. Even though they were not implemented, they are highly relevant to this research area.

4.4.1 Travel pattern generation

One aspect of the problem formulation which has not been in focus is how travel patterns are decided for optimizations. We have simply defined a table with connections for a simple system. An interesting approach was created by a master student at NTNU [28]. Here, a full travel pattern is generated for a given time period. The driving patterns are based on the regular citizen, where they need to go to work between 8am and 4pm. An aspect of randomness is added with possible drives performed in the evening. Depending on the importance of realistic travel patterns for the optimization, here there is no limit to possible extensions. Other aspects we considered as options were taking the weather into account, considering which day of the week it is and the current season.

4.4.2 Travel pattern optimization

Another aspect worth exploring is the ability to affect travel patterns based on the charging pattern optimization. The current system only decides when to charge in the span of the time connected, but not where to charge. It would be interesting to pass charging patterns resulting from our optimizations to a travel pattern generator. Based on certain conditions, this could then affect which charging buses agents decide to connect to. One could make an iterative loop here to converge towards an optimal travel pattern. Such a system would add another option for solving congested buses. Instead of only altering the cost of power, which can result in no load reduction, and more expensive power for agents in cases where they have to charge, the agents could now select another charging location instead. Of course, real world aspects regarding needs of the driver should be considered for this area of expansion.

Results

In this chapter, the results from the optimization models, defined in chapter 4, will be listed. A simple description will follow each entry, while an analysis and discussion of these results can be found in chapter 6.

5.1 Central model

For the central model, the results for a power grid without congestion are the first showed. Then results for a generation congested grid will follow. Lastly, results for a line congested grid are presented. The models all operate over five time steps. There are five agents, where all are connected at the first two time steps and at the last. It is assumed that the agents are driving and discharging when not connected. The connections are showed in Table 5.1. The marginal cost of the generator for each time step, found in the objective function of the central model, is shown in Table 5.2. The initial SoC of each agent is shown in Table 5.3. Note that agents 3, 4 and 5 start with an equal SoC. This causes some of the graphs depicting the SoC to draw these three agents as three overlapping lines if their charging pattern is equal.

The results we take note of are the charging patterns of the agents, the amount of power generated by the generator, the active load of the agents and the SoC of the agents.

	time step 1	time step 2	time step 3	time step 4	time step 5
Agent 1	1	1	0	0	1
Agent 2	1	1	0	0	1
Agent 3	1	1	0	0	1
Agent 4	1	1	0	0	1
Agent 5	1	1	0	0	1

Table 5.1: Connection pattern of agents. 1 indicates a connection and 0 indicates no connection

Time	time step 1	time step 2	time step 3	time step 4	time step 5
nG	4	7	4	7	4

Table 5.2: Marginal cost of the generator

5.1.1 Grid without congestion

The first run of the system is performed without any power limits, this emulates the fact that no congestion should happen. We can then later generate congestion by taking use of these non-congested results as detailed in subsection 5.1.2

and subsection 5.1.3. The figures 5.1-5.4 show the results of this run.

	initial SoC
Agent 1	0.25
Agent 2	0.9
Agent 3	0.5
Agent 4	0.5
Agent 5	0.5

Table 5.3: Initial SoC of all agents

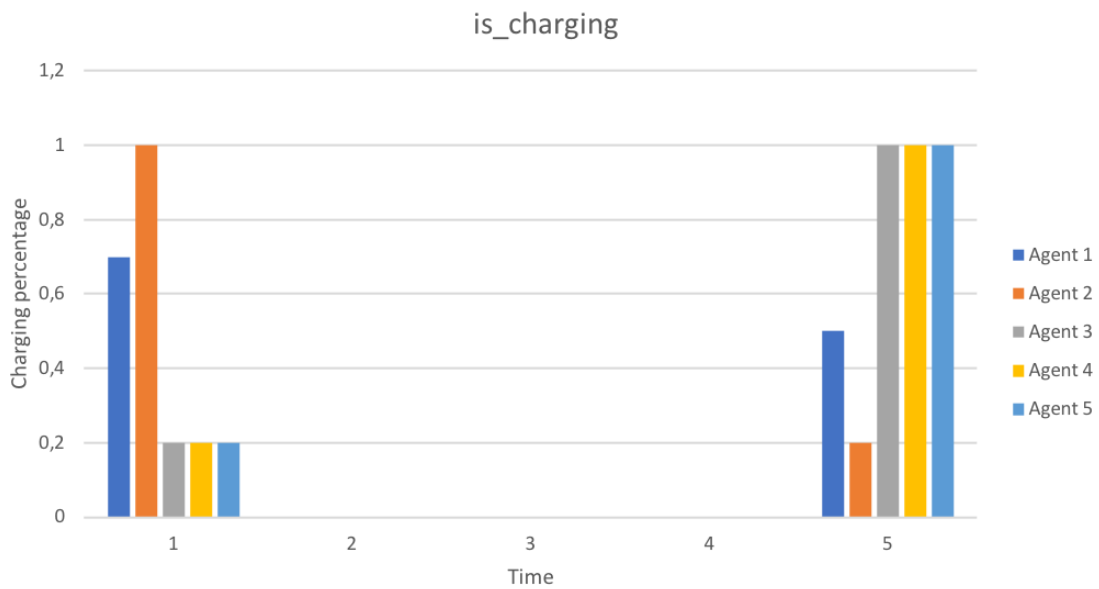


Figure 5.1: Charging pattern of all agents in a central model without congestion

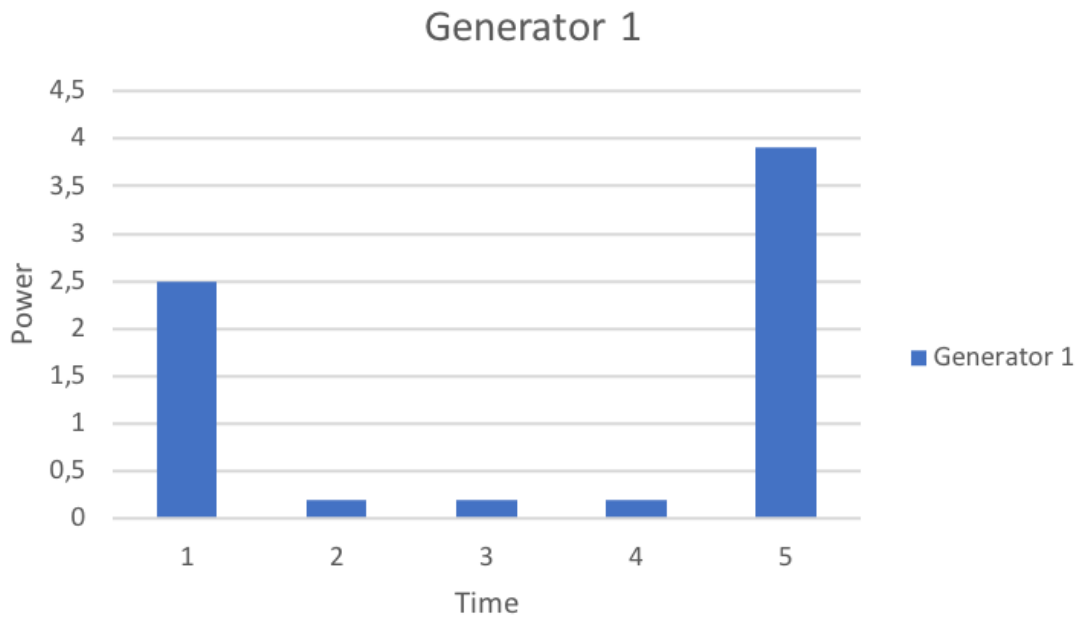


Figure 5.2: Power generated by the generators in a central model without congestion

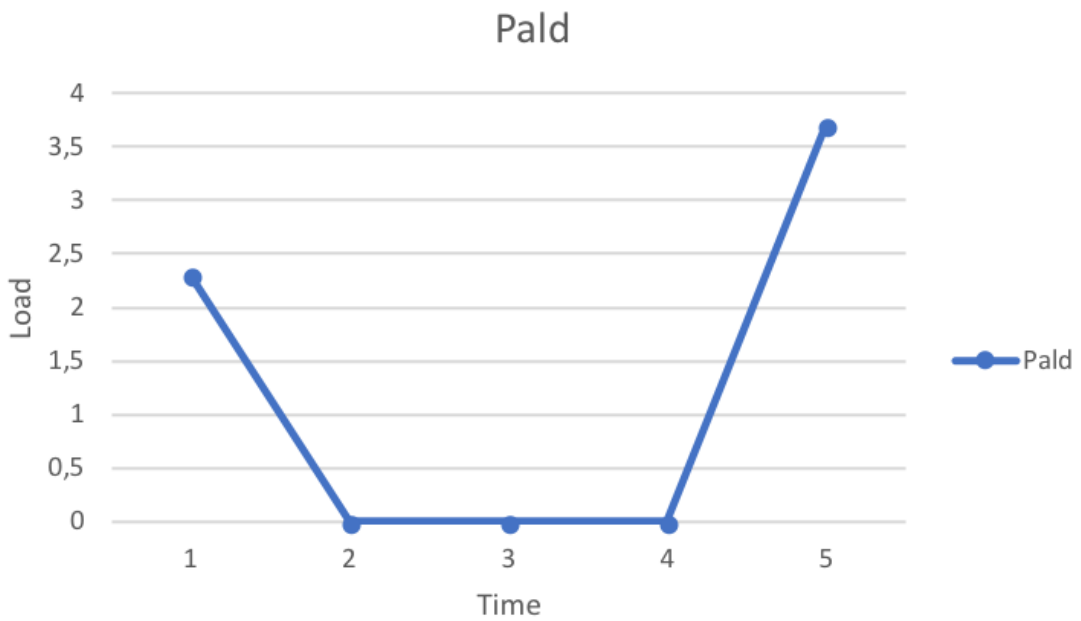


Figure 5.3: Active load of all agents in a central model without congestion

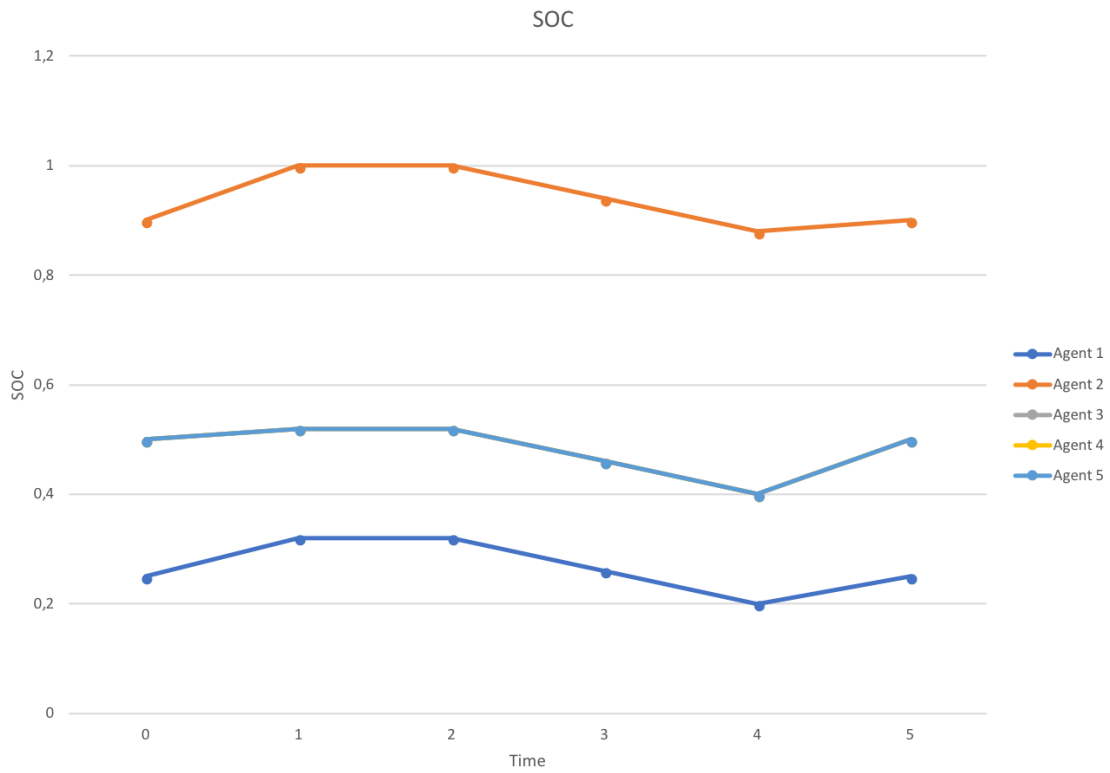


Figure 5.4: SoC level for all agents in a central model without congestion

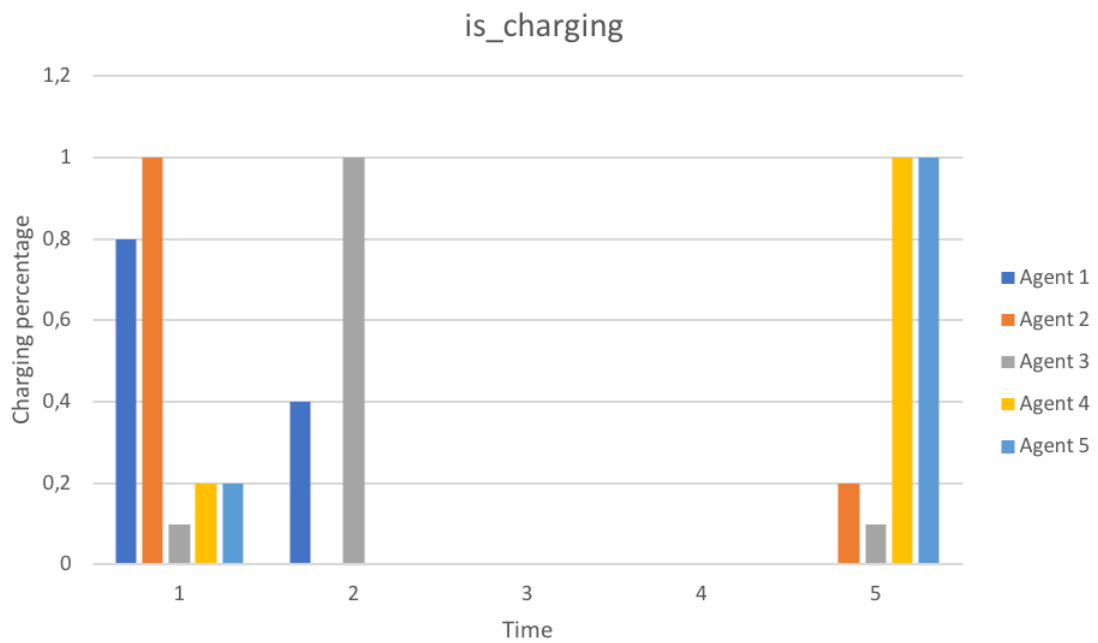


Figure 5.5: Charging pattern of all agents in a central model with generation congestion

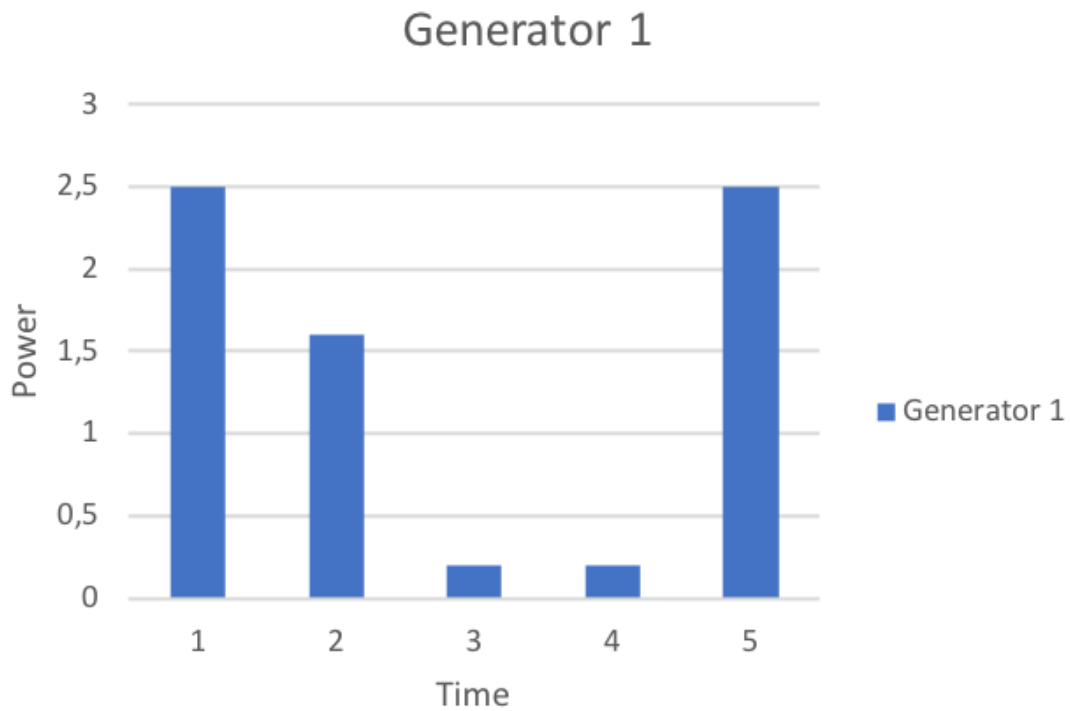


Figure 5.6: Power generated by the generators in a central model with generation congestion

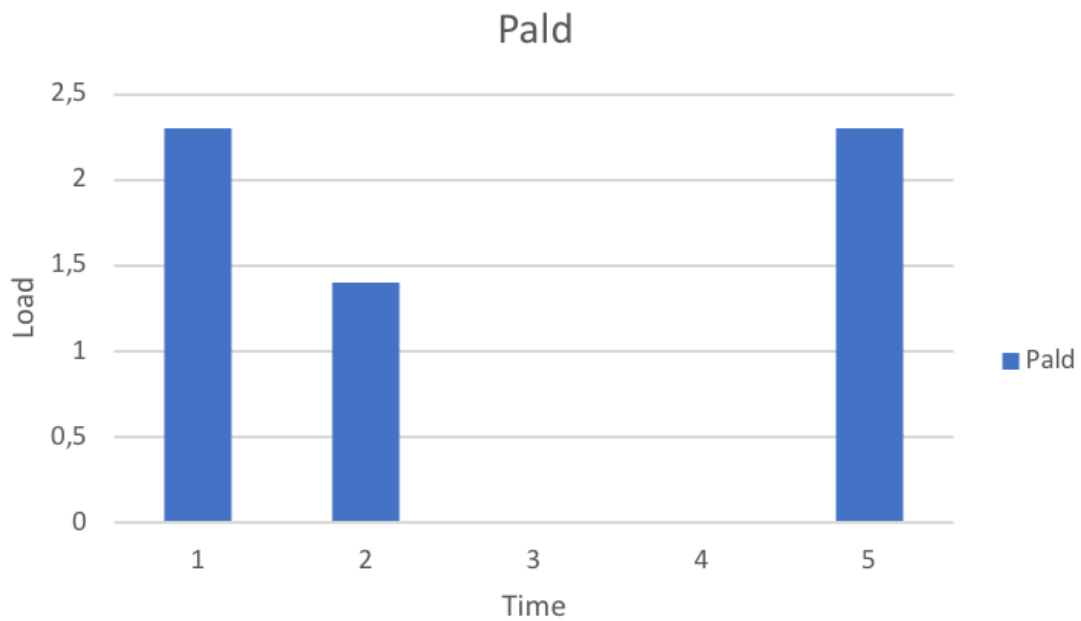


Figure 5.7: Active load of all agents in a central model with generation congestion

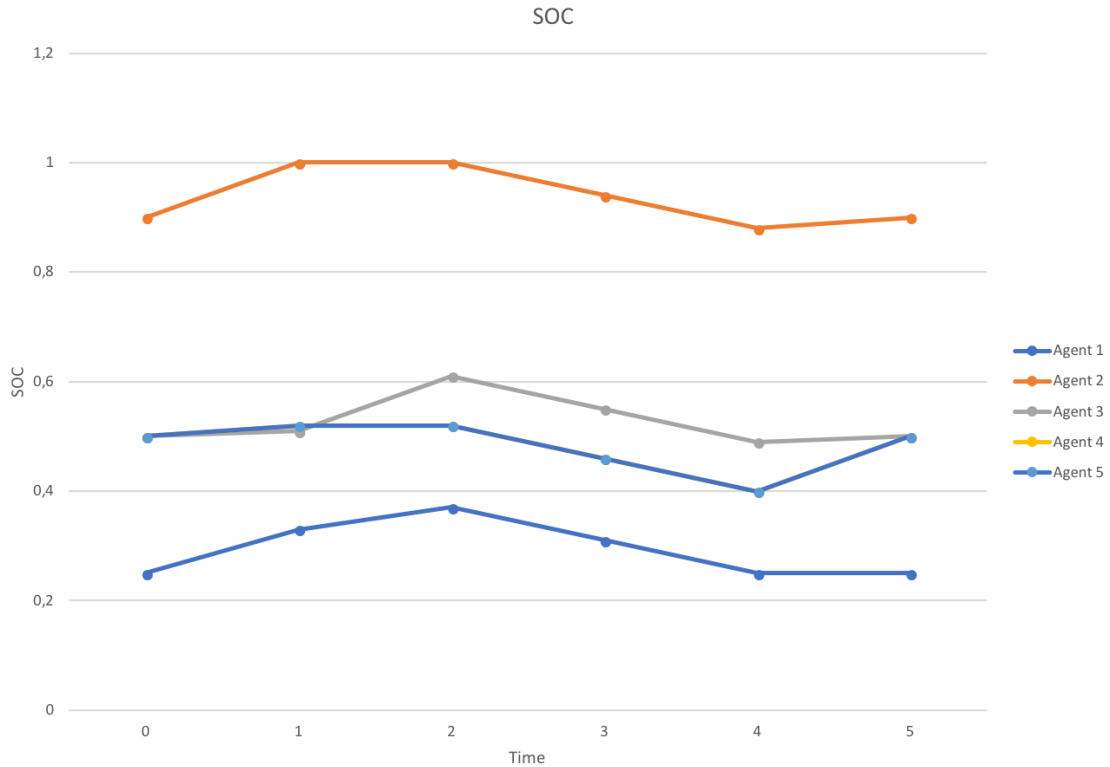


Figure 5.8: SoC level for all agents in a central model with generation congestion

5.1.2 Grid with generation congestion

To intentionally add generation congestion to the grid we set the generation limit to a lower value than what is generated in the non-congested run of the system. We see from Figure 5.2 that the power generated in time step 1 is 2.5 p.u. and almost 4 p.u. in time step 5. We then set the generation limit to 2.5 p.u., so there is generation congestion in time step 5. The figures 5.5-5.8 show how the grid adapts to this.

5.1.3 Grid with line congestion

To add line congestion, we follow the same procedure as in subsection 5.1.2, but instead of limiting the generation to 2.5, we now set the line limit between the generator and the bus to 2.5. The figures 5.9-5.12 show how the system adapts to this congestion.

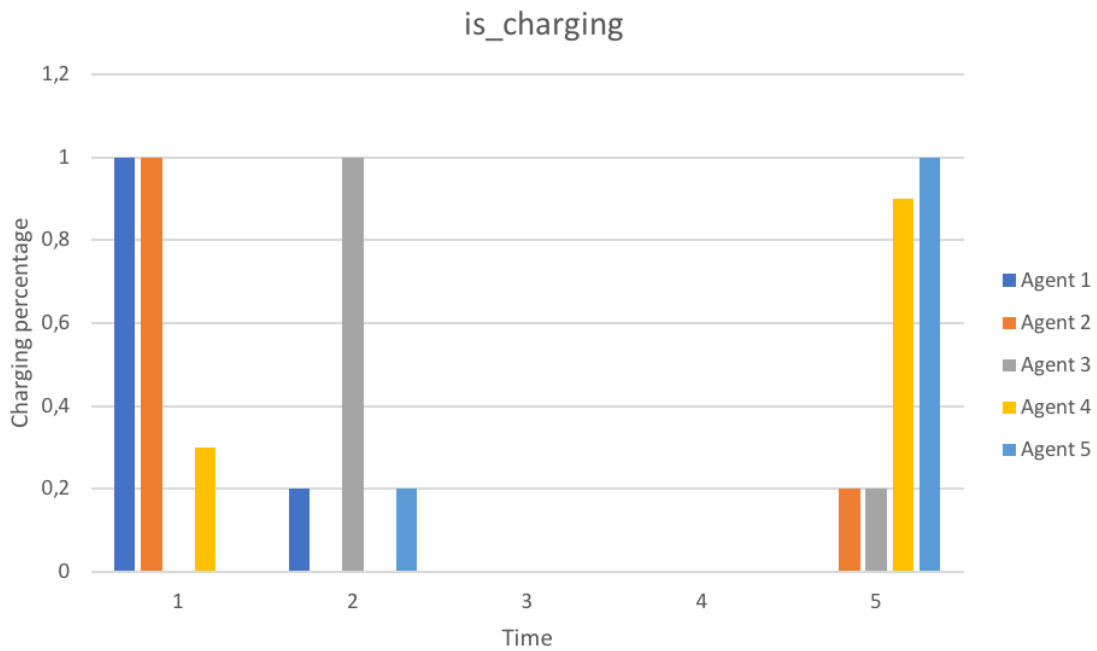


Figure 5.9: Charging pattern of all agents in a central model with line congestion

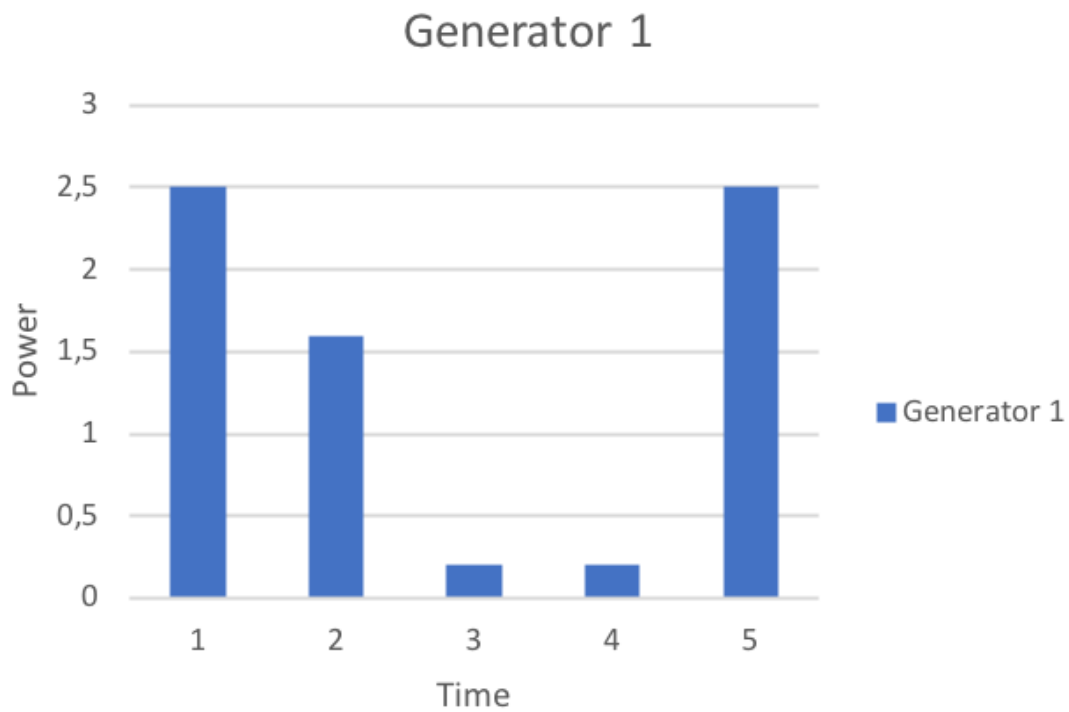


Figure 5.10: Power generated by the generators in a central model with line congestion

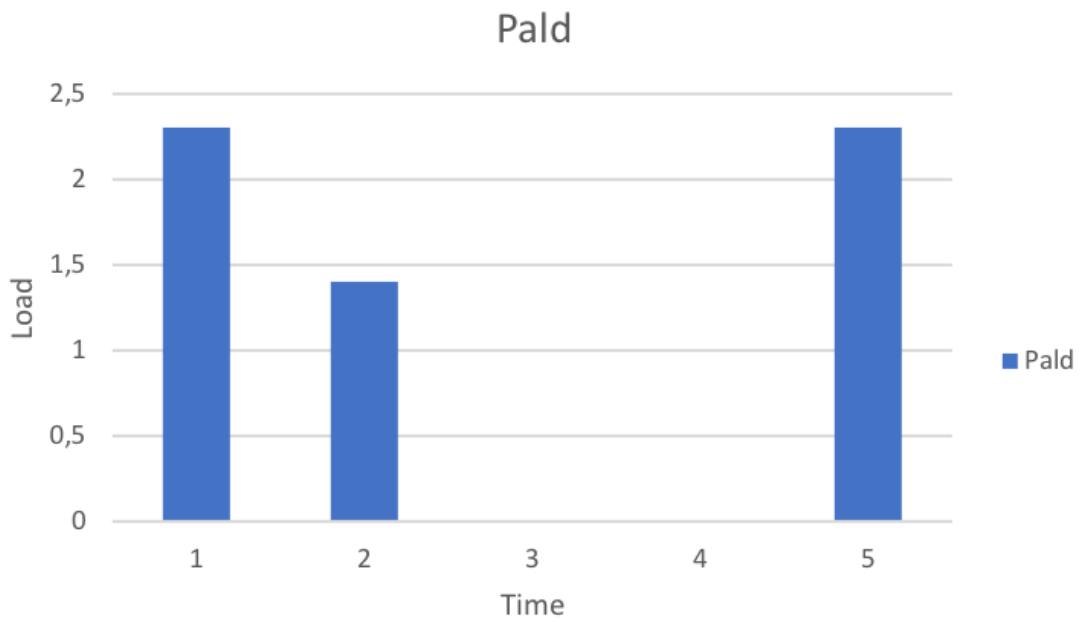


Figure 5.11: Active load of all agents in a central model with line congestion

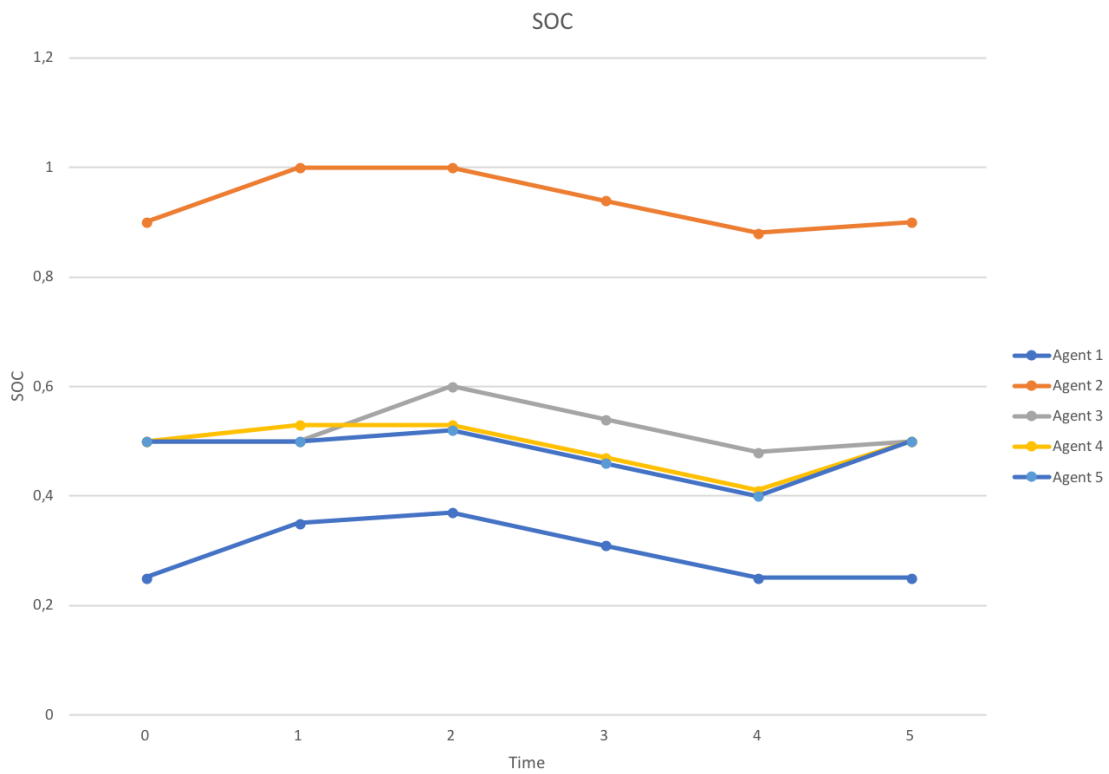


Figure 5.12: SoC level for all agents in a central model with line congestion

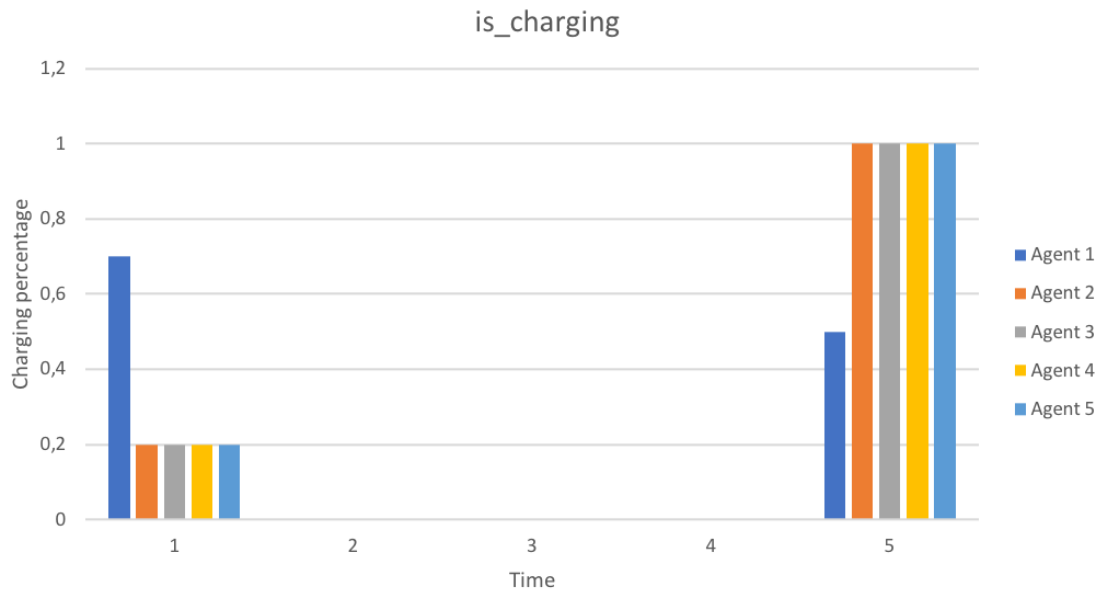


Figure 5.13: Charging pattern of all agents in a decentral model

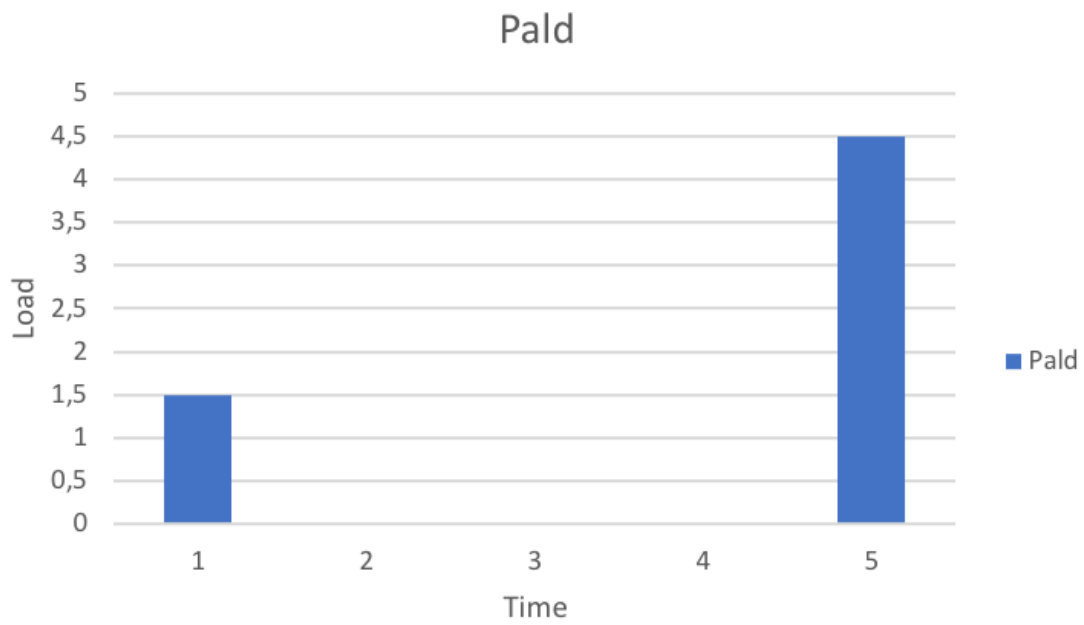


Figure 5.14: Active load of all agents in a decentral model

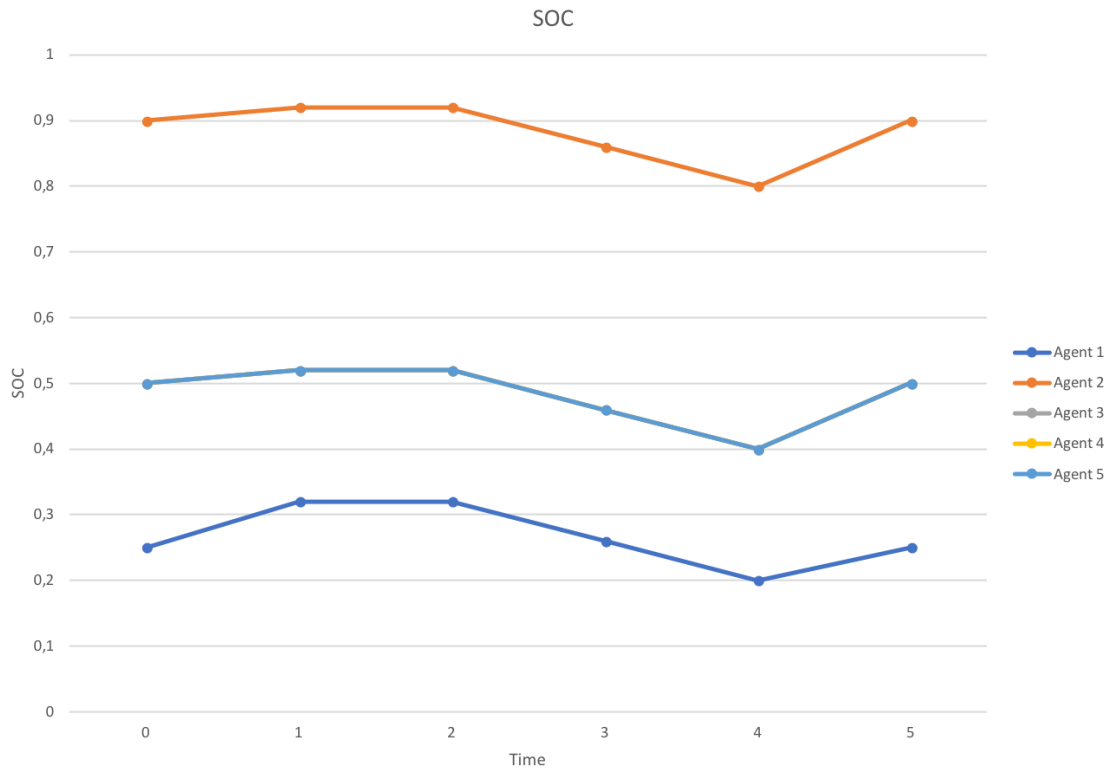


Figure 5.15: SoC level for all agents in a decentral model

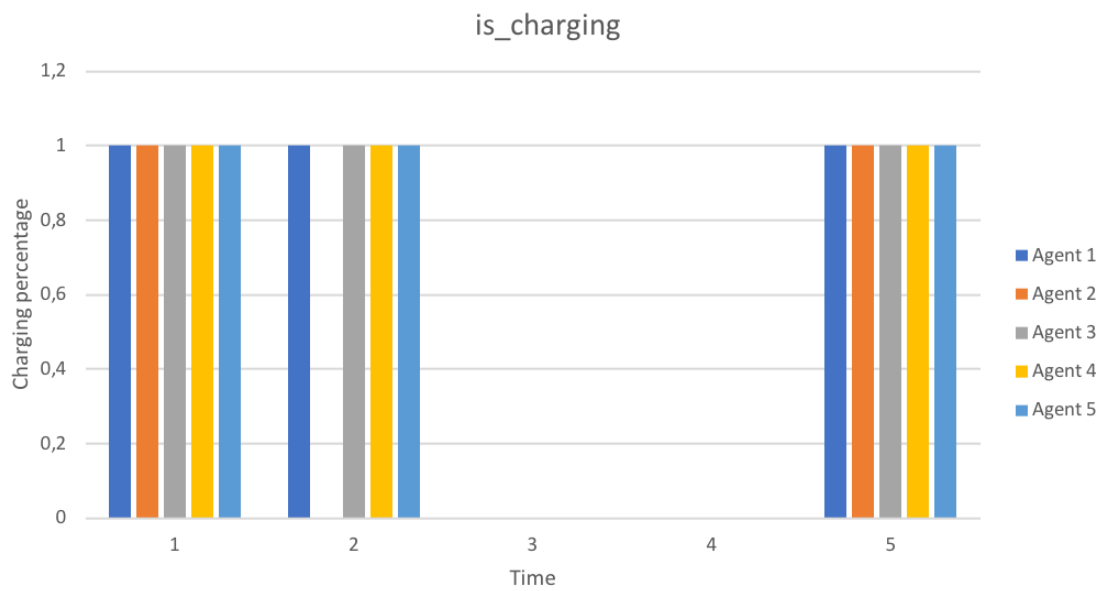


Figure 5.16: Charging pattern of all agents in a dumb charging model without congestion

5.2 Decentral model

The decentral model, by design, is not aware of any power generation limits or power line limits. This results in identical results regardless of congestion. Therefore, only one set of results is shown here. Additionally, the graph of the power generated is not included, as the decentral model does not see this value in any way. The travel pattern of the agents is the same as for the central model, and as well as the initial SoC, these values were presented in Table 5.1 and Table 5.3. Figures 5.13-5.15 show the results of this decentral model.

5.3 Dumb charging model

For the dumb charging model, the results are presented with the same structure as the central model is presented in section 5.1.

5.3.1 Grid without congestion

The results for dumb charging without grid constraints are show in Figures 5.16-5.19.

5.3.2 Grid with generation congestion

The results for dumb charging with generation constraints are show in Figures 5.20-5.23.

5.3.3 Grid with line congestion

The results for dumb charging with line grid constraints are show in Figures 5.24-5.27.

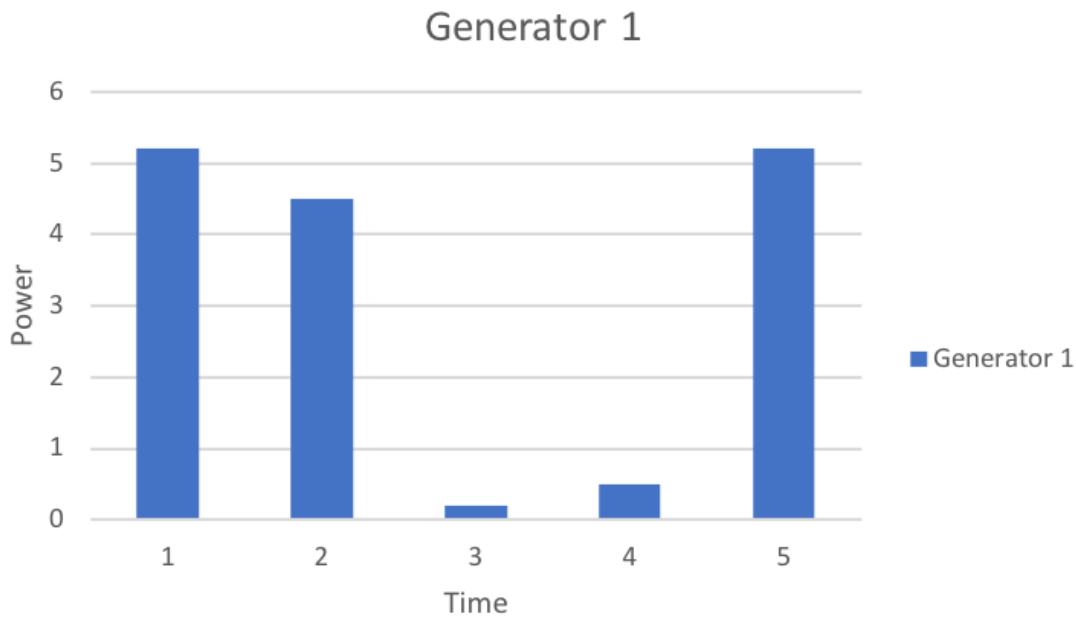


Figure 5.17: Power generated by the generators in a dumb charging model without congestion

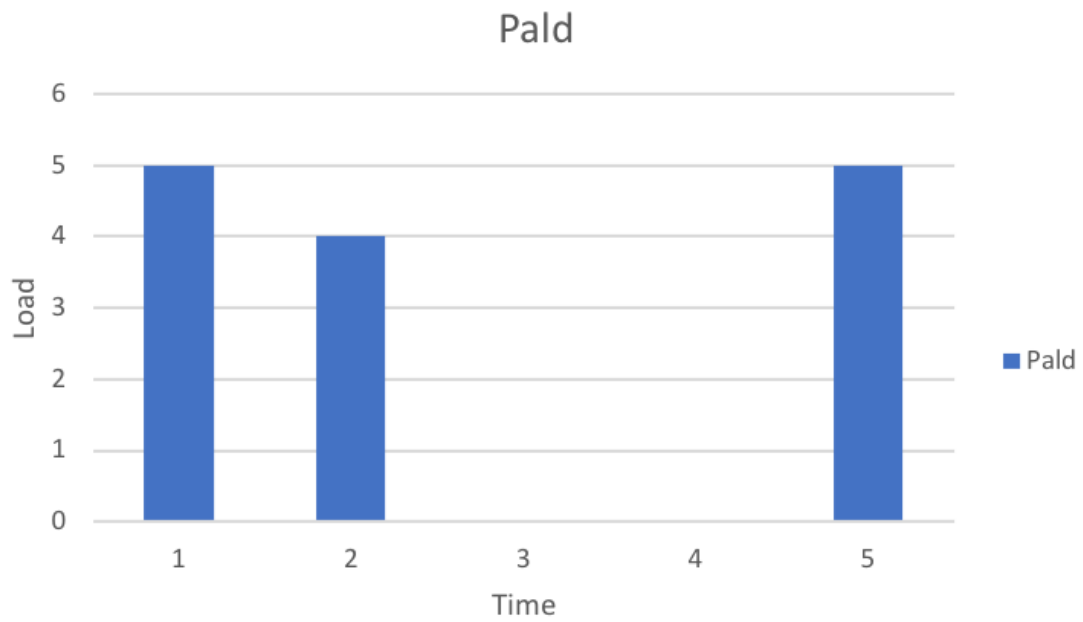


Figure 5.18: Active load of all agents in a dumb charging model without congestion

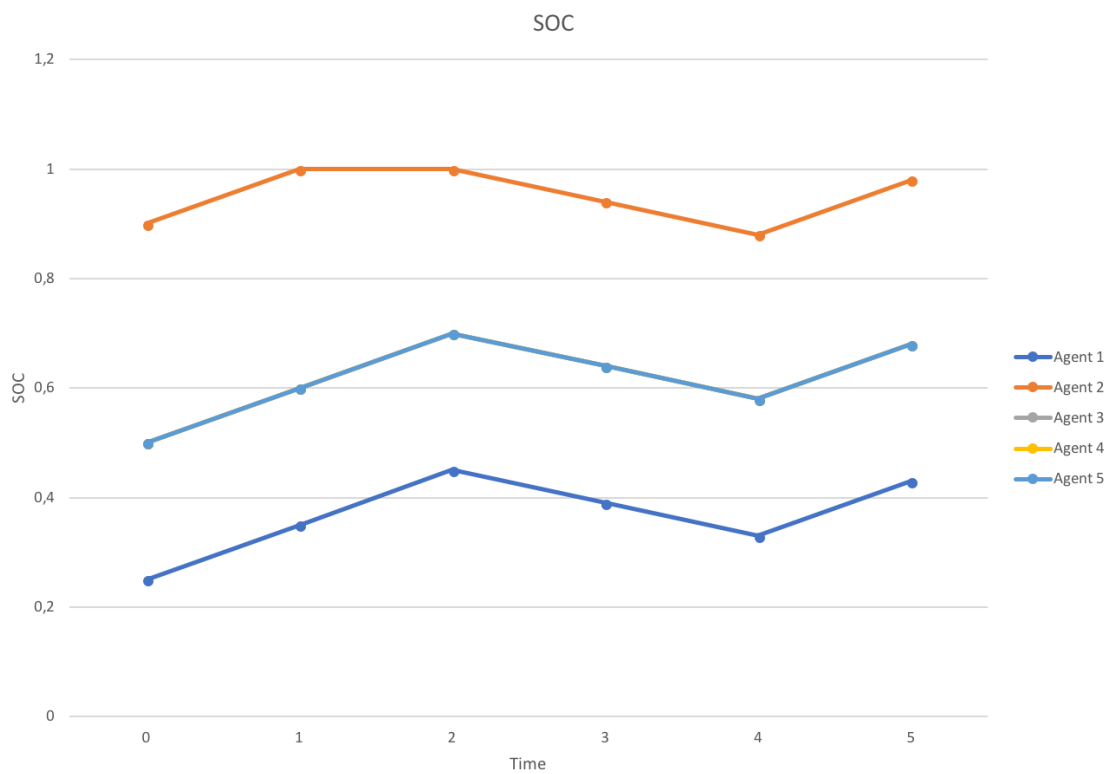


Figure 5.19: SoC level for all agents in a dumb charging model without congestion

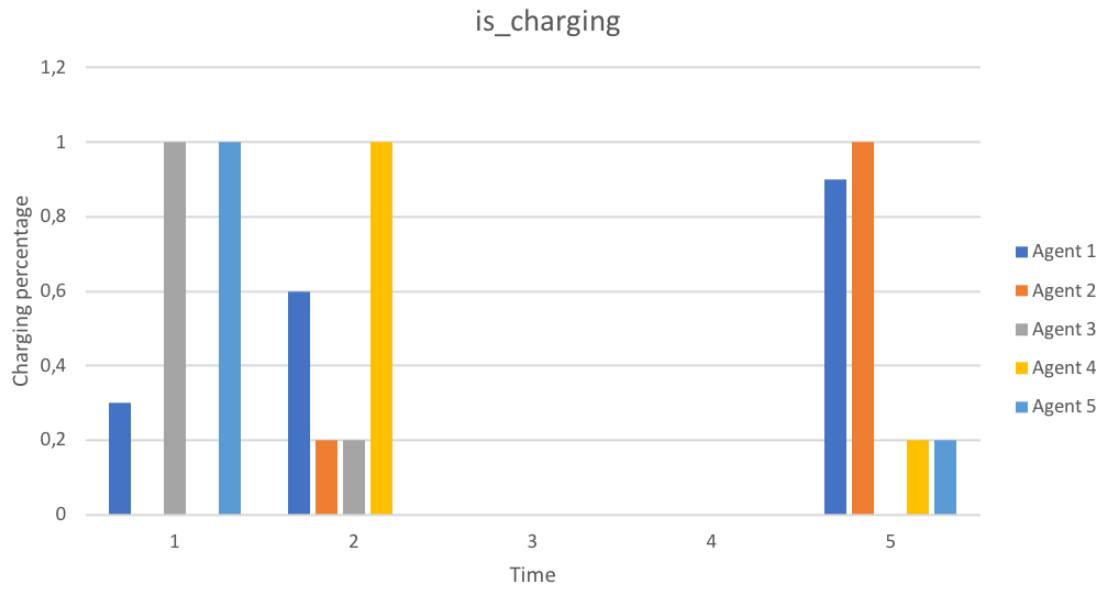


Figure 5.20: Charging pattern of all agents in a dumb charging model with generation congestion

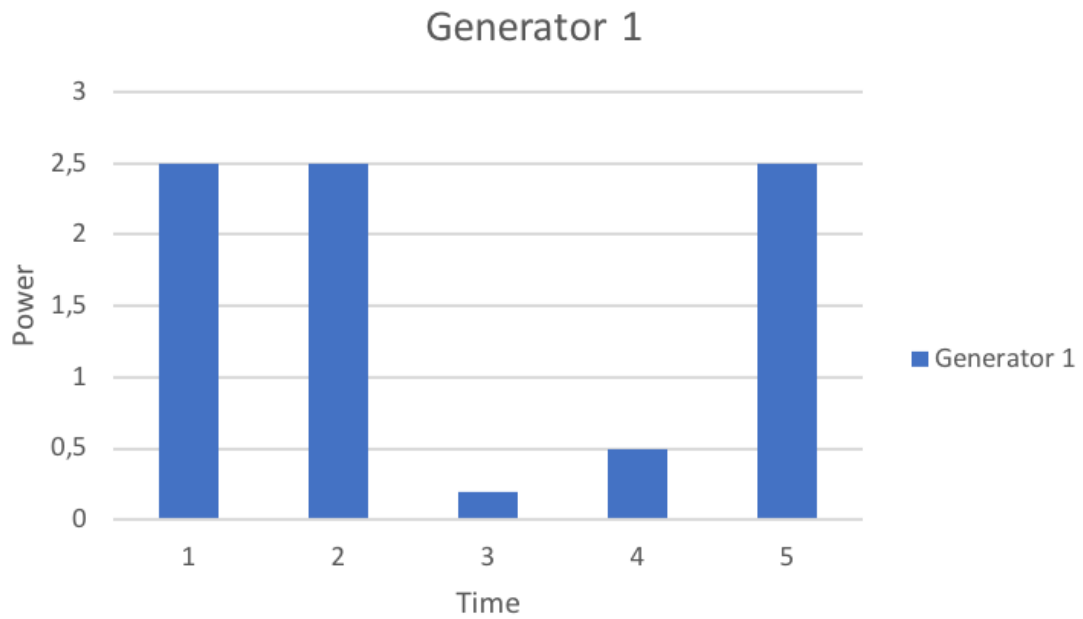


Figure 5.21: Power generated by the generators in a dumb charging model with generation congestion

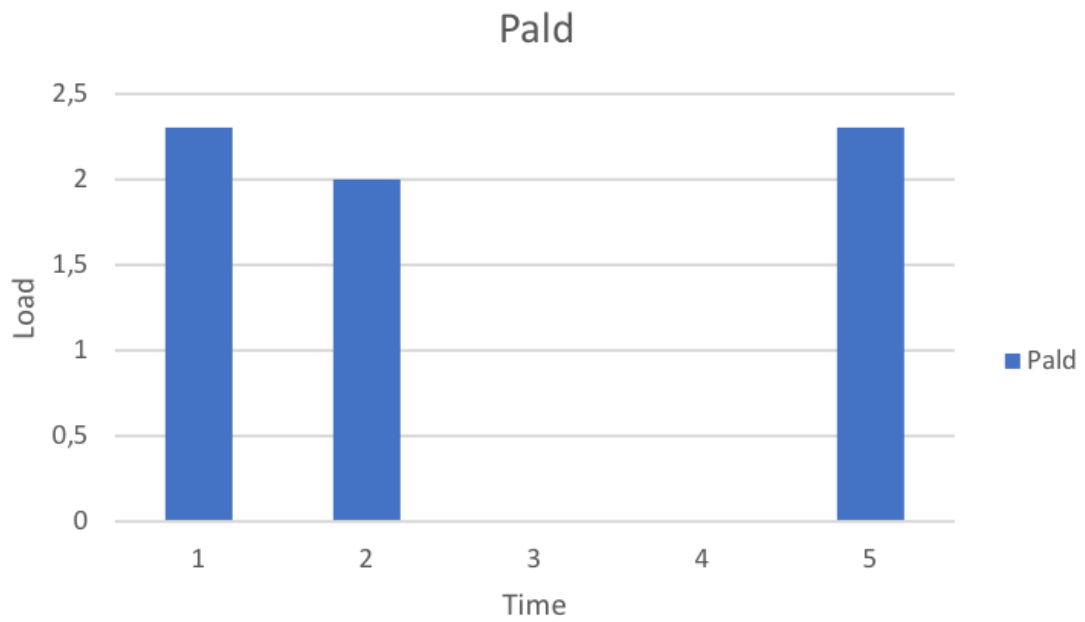


Figure 5.22: Active load of all agents in a dumb charging model with generation congestion

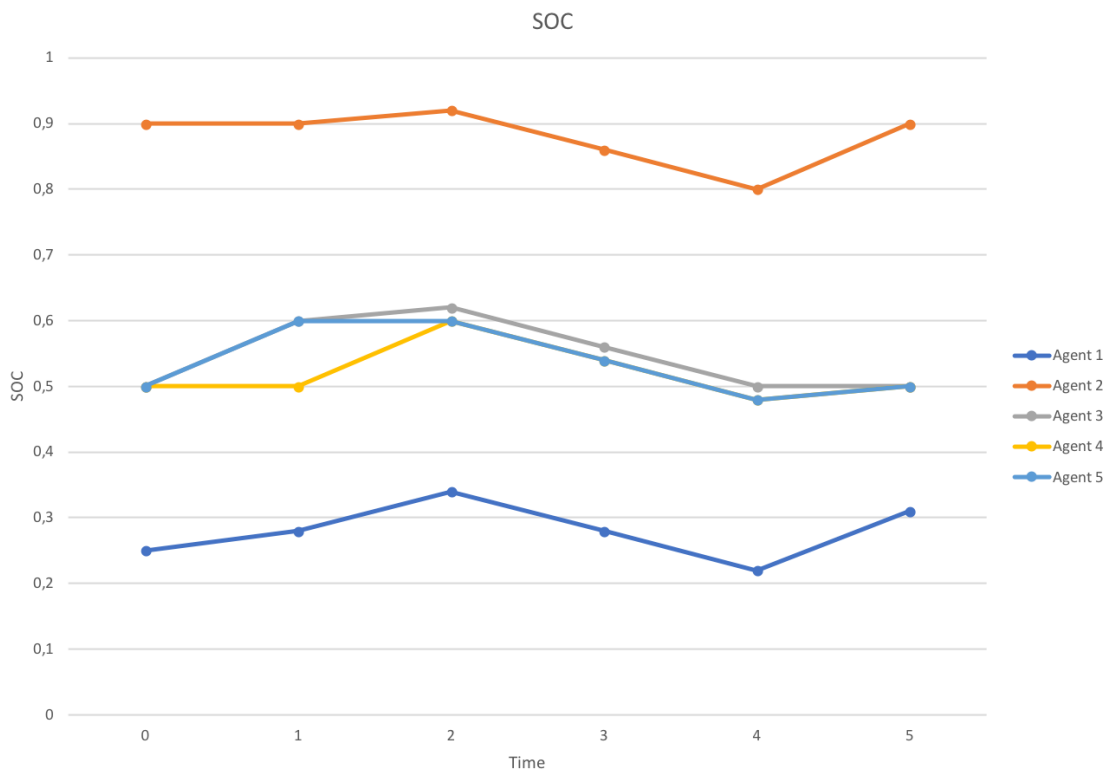


Figure 5.23: SoC level for all agents in a dumb charging model with generation congestion

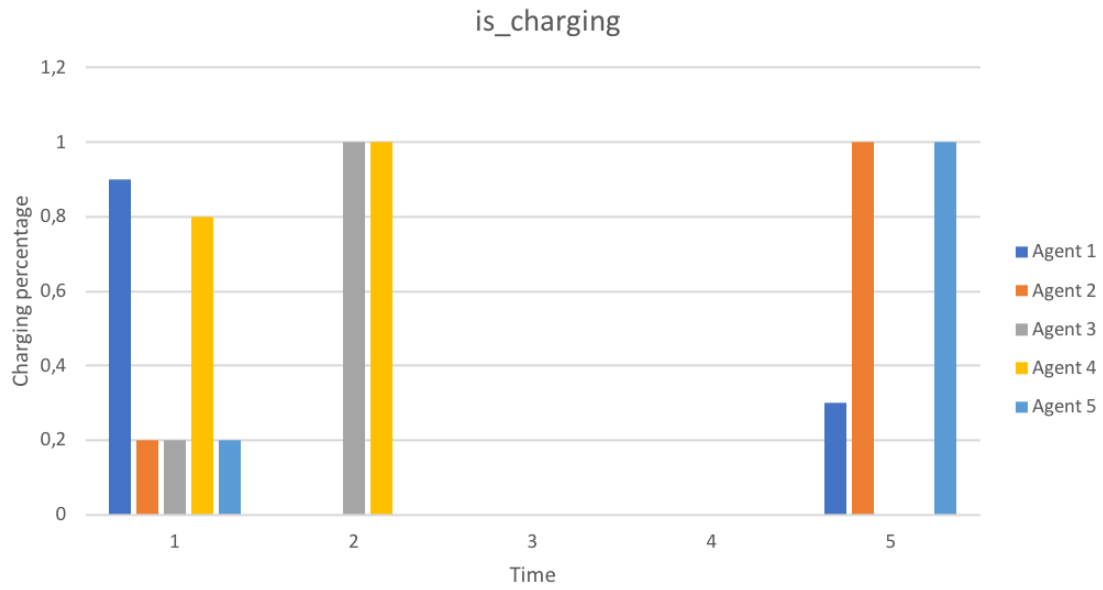


Figure 5.24: Charging pattern of all agents in a dumb charging model with line congestion

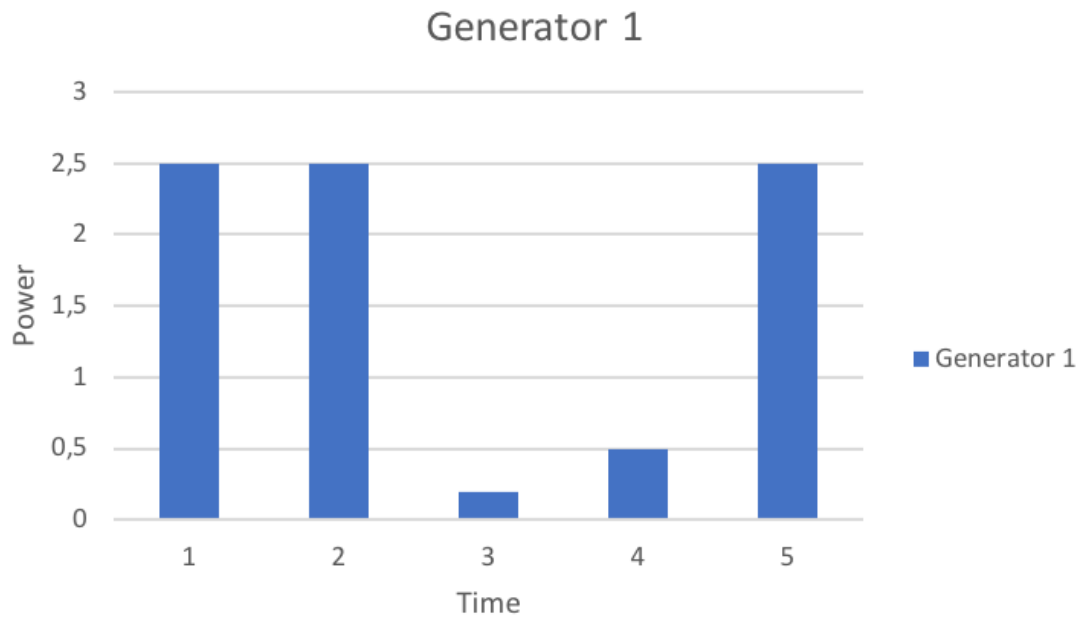


Figure 5.25: Power generated by the generators in a dumb charging model with line congestion

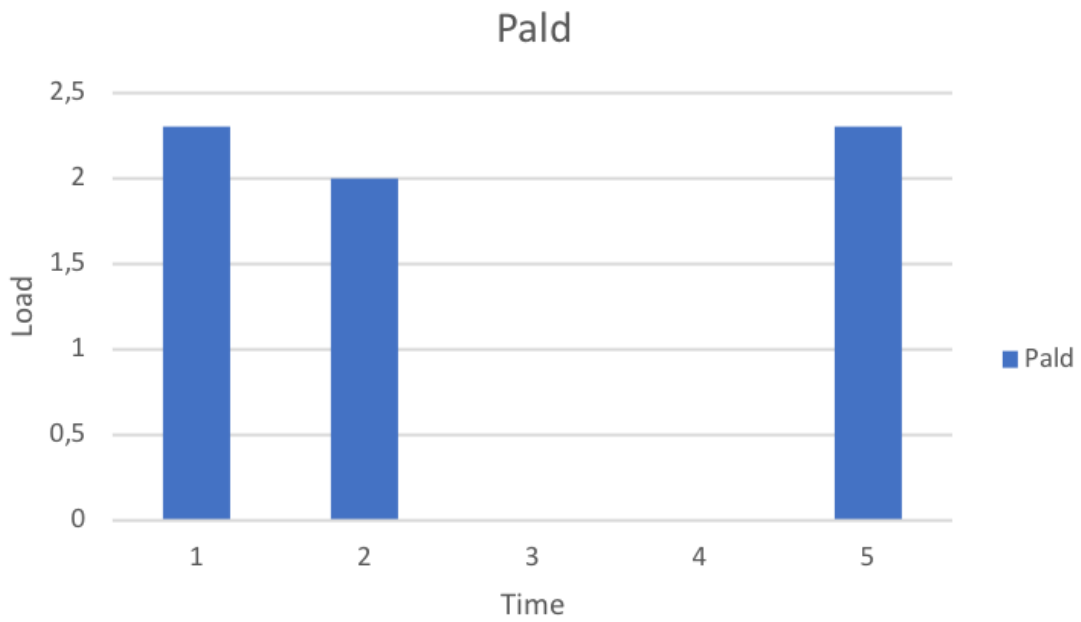


Figure 5.26: Active load of all agents in a dumb charging model with line congestion

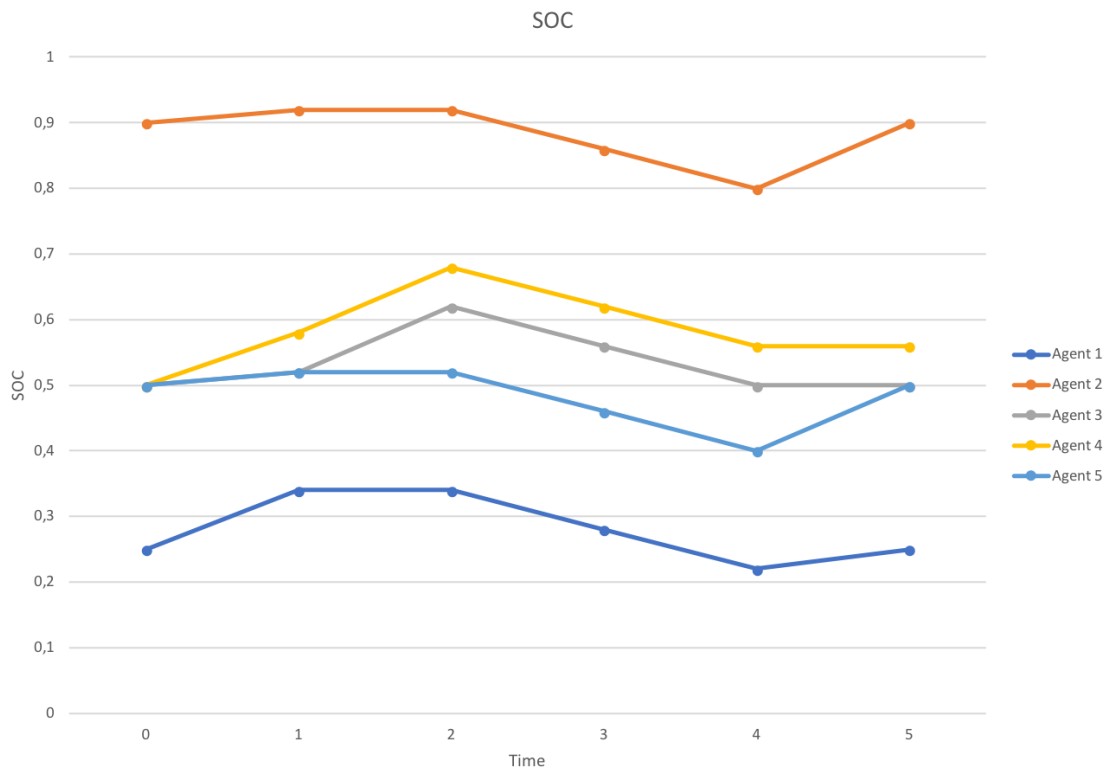


Figure 5.27: SoC level for all agents in a dumb charging model with line congestion

5.4 Hierarchical-distributed operation model

The results for this model were equal regardless of the congestion put in the system. This indicates that the model has not completed as intended. The analysis in the next chapter will discuss this aspect further. Regardless, the Figures 5.28-5.31 show these results.

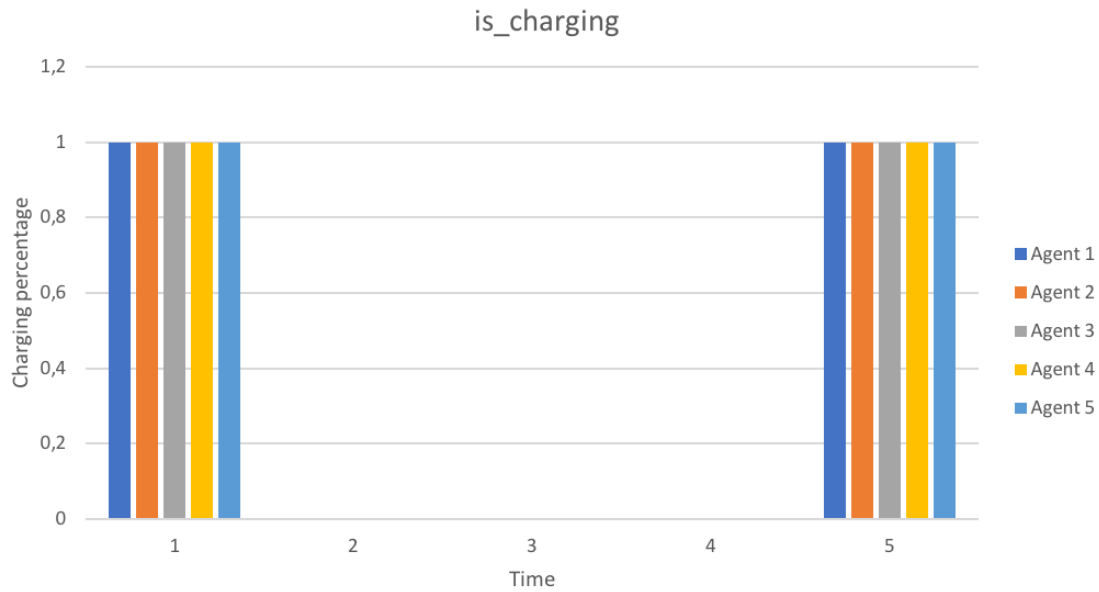


Figure 5.28: Charging pattern of all agents in a hierarchical-distributed operation model without congestion

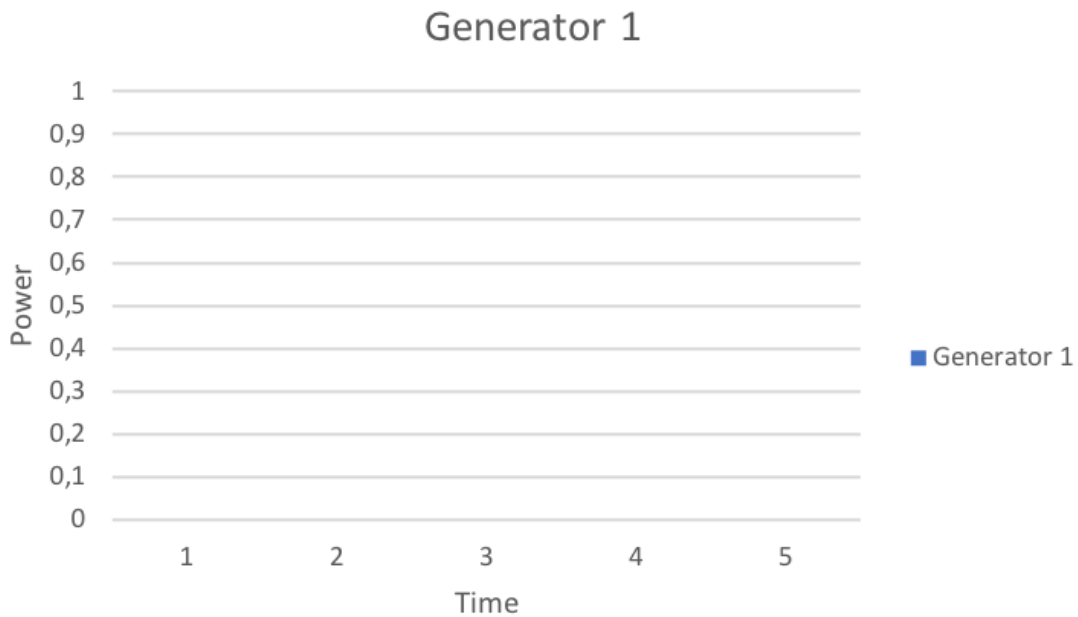


Figure 5.29: Power generated by the generators in a hierarchical-distributed operation model without congestion

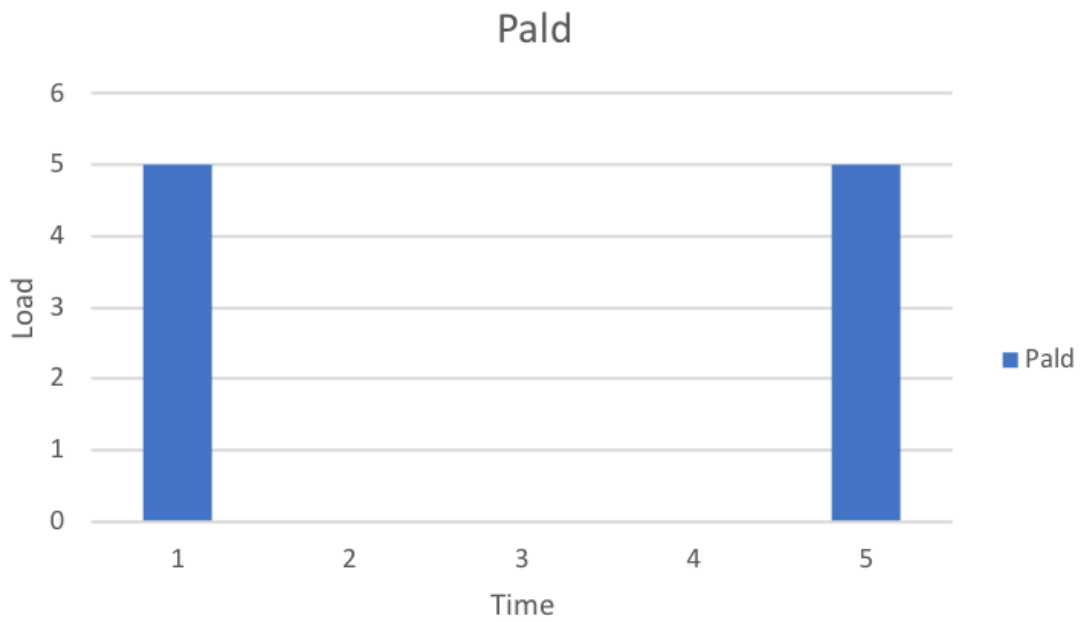


Figure 5.30: Active load of all agents in a hierarchical-distributed operation model without congestion



Figure 5.31: SoC level for all agents in a hierarchical-distributed operation model without congestion

t = 1								
iteration	1	2	3	4	5	10	50	100
λ	6	7.05	8.077	8.231	8.280	8.325	8.333	8.333

t = 5								
iteration	1	2	3	4	5	10	50	100
λ	18	6.4	6.88	7.192	7.417	8.019	8.328	8.332

Table 5.4: Values of the lambda variable vector. The values for $t = 2, 3$ & 4 are excluded as they are all 0.

Analysis and discussion

In this chapter, an analysis and a discussion will be performed based on the data presented in the previous chapter. Each model will be analyzed individually before a complete discussion follows.

6.1 Analysis

6.1.1 Dumb charging model

The first model to be described and analyzed is the dumb charging model. This is the main reference for discussing the other defined models. It is closely related to realistic charging of PEVs when there is no smart grid present. Simultaneously, this is then a good baseline for a worst case of charging patterns. The discussion for this model will be minimal because it is only intended as a reference to the other models.

No congestion

When there is no congestion in the system all agents charge their maximum amount, for all the connected time steps, as seen in Figure 5.16. The one exception to this is that Agent 2 does not charge in time step 2. This is to be expected as it has an initial SoC of 90%. After charging in the first time step, the agent acquires enough power to increase its SoC to 100% and at this point it is not physically possible to charge any more. From Figure 5.17 and Figure 5.18 it is also clear that charging happens regardless of the state of the grid, which shows how congestion and power line capacities are not being considered.

Generation congestion

In the case of congested power generation, we see that the agents charge less. They still charge as much as possible though, and as seen in Figure 5.21 the generator is at max capacity for all the time steps the agents are connected. From Figure 5.20 we are unable to see any clear pattern to which agents get to charge and when they get to charge. The behaviour seems mostly erratic. Such a behaviour puts unnecessary strain on the power grid generation wise, while simultaneously being rather unpredictable.

Line congestion

The line congestion put on the network produces very similar results as the generation congestion. The agents charge erratically and unpredictably, while keeping the grid at max capacity for each connected time step. The figures 5.24 and 5.25 show this in the same way as the generation congested network.

6.1.2 Central model

For the central model, the objective function has the sole purpose of minimizing the cost of the total power production. The marginal cost of the generator is shown in Figure 6.1. Based on this, we expect the system to perform as much as possible of the charging in the cheaper time steps 1, 3 and 5. From Table 5.1 we know that the agents are not connected for the time steps 3 and 4. Based on this information, we adjust our expectation to see most charging happen at time step 1 and 5, and only in time step 2 if it is absolutely necessary (congestion occurs). It is assumed that when agents are disconnected, they discharge. The discharge rate was set to 6% of the SoC. Additionally, the maximum charge rate per time step is set to 10% of the SoC.

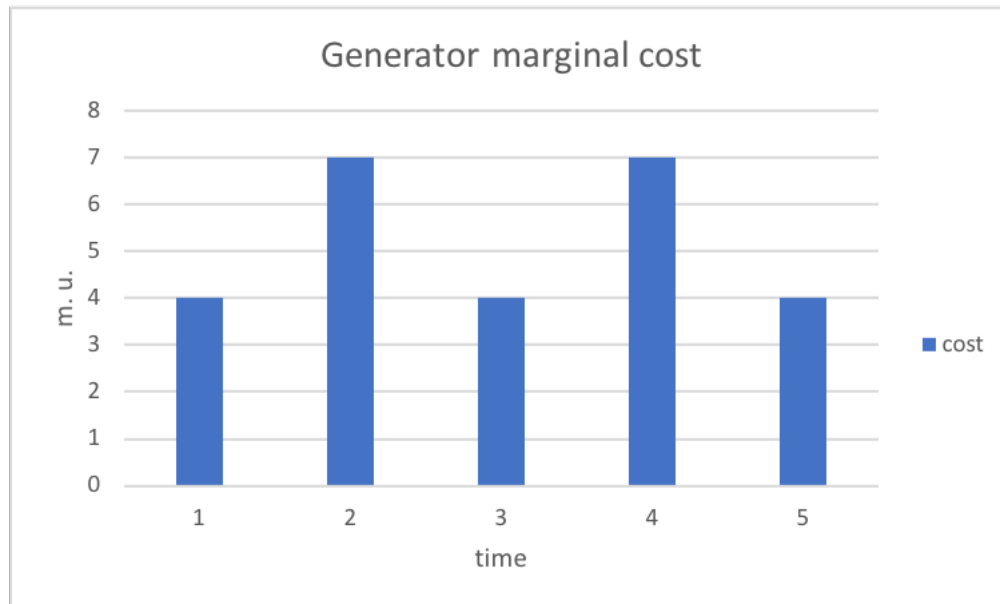


Figure 6.1: Marginal cost of the generator

No congestion

We begin by analyzing the resulting charging patterns in a grid with no congestion. For the first case where there is no congestion in the system, we see that charging occurs at time step 1 and 5. This meets our expectations. All agents charge at both time steps as they discharge more power in the two time steps they are disconnected than they can charge in a single time step. In principle, the central model does not prefer either of the time steps the agents charge as the cost of power generation is the same for these times. What is of interest though is the constraints on the PEVs. It is these that alter the agents' charging behaviours. We see the defined discharge rate in the data as all agents charge a total of 120% of the maximum charging rate, which is equivalent to the power consumed in two discharging time steps. The constraint for finishing the optimization with at least as much power as when it started lies at the core of this behaviour, without it, charging will only occur if the minimum SoC bound is not met.

The agents 2-5 all have a very similar charging pattern, in one of the time steps they charge 100% of their charging capacity and in the second they only charge 20%. Agent 2 differs from the others in reversing the order of the charging percentages, but this reversed order has no effect on the agents or the grid. This is then assumed to be caused by the stochasticity of the optimization software algorithm, and not due to a defined constraint. It is more interesting to look at how agent 1 differs. This agent charges for 70% in the first time step and for 50% in the second one. For the end result, this is still equivalent to the others, since local price for agents is ignored. There is however a constraint which causes this. Agent 1 has an initial SoC of 25%. Given that the maximum charging rate is assumed to increase the SoC by 10%, and the discharge rate 60% of this again (equivalent to a decrease in 6% of the SoC), the agent would have a remaining SoC of 13% after the discharges in time step 3 and 4. There is a defined constraint which requires the SoC to never be any lower than 20%. For this reason, agent 1 charges 70%, for a 7% increase in SoC, up to an SoC of 32%,

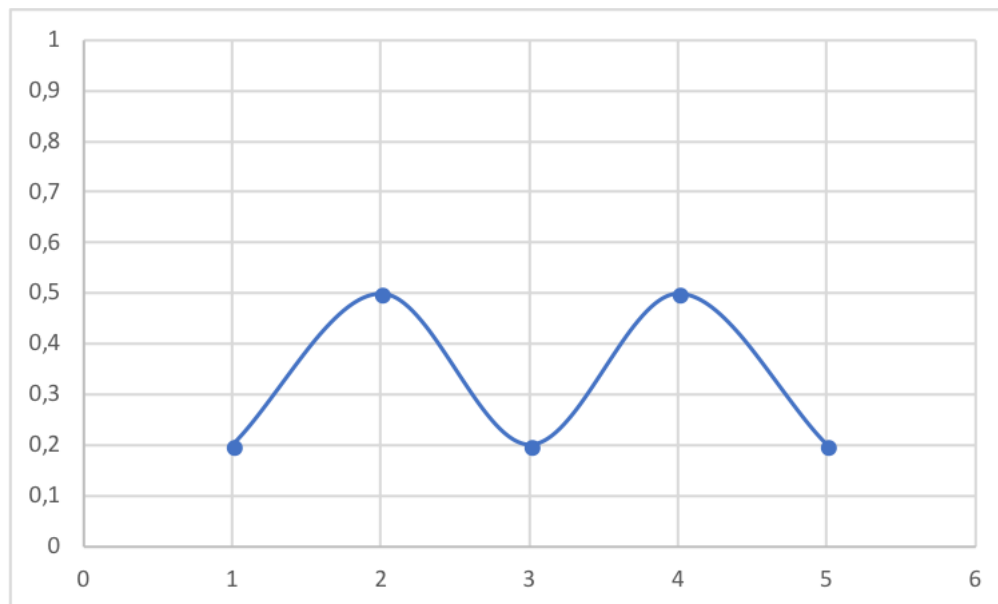


Figure 6.2: Base load of the system, disregarding any PEVs

it then discharges twice, resulting in an SoC of exactly 20%, before then charging again up to an SoC of 25%, which is equal to the initial SoC.

We can clearly see that this central model is able to mitigate charging in peak demand hours, and spreads the demand more evenly over the time period. We base this on the comparison of the base load seen in Figure 6.2 with the charging pattern in Figure 5.3.

Generation congestion

Next, we will examine the results from the scenario with generation congestion. In this scenario, we notice that the amount of charging in time step 1 and 5 are reduced from the non-congested scenario, and additionally, we now have charging in time step 2. This is seen in Figure 5.5. Since agent 1 has an initial SoC of 25%, it has to charge before time step 3, where it disconnects from the grid. Once disconnected, the agent will discharge 12% of its SoC. Therefore, if the agent doesn't charge, it will have an SoC below 20%, which is not allowed due to the minimum SoC of 20% constraint. This behaviour is observed when we see agent 1 perform all its charging in the first two time steps. The charging pattern of the other agents is predictable and consistent spreading out the charging in case of congestion. What isn't clear is how the system decides which agents need to charge at alternate times. From the results, it seems like this might be determined randomly by the solver. It is random which agents are selected for a given scenario and model, but multiple runs of the optimization results in the same pattern. This may indicate that the pattern is deterministically decided by the internal solver algorithm and not by a stochastic variable in the solver.

It is seen in Figure 5.8 that the SoC of all agent is exactly the same in the first and last time step. This is expected even though the constraint allows a higher SoC at the final time step. Again, the reason boils down to price. It is cheaper for the system to only charge the minimum amount necessary. From Figure 5.6 it is shown how the system generates as much as possible in the cheaper time step 1 and 5, while generating less in the more expensive time step 2.

Line congestion

When we look at the resulting output for the line congested scenario, it is remarkably similar to the generation congested scenario. The same main traits apply; agent 2 charges before it disconnects to maintain a high enough SoC, while the other agents are spread out over the time steps, with the maximum allowed charging occurs at the first and final time steps. The agent distribution in this charging pattern differs slightly from the generation congested scenario, but

this difference can be attributed to the solver algorithm and the fact that it is a different constraint that affects the optimization. It is also the case that for this scenario, that each performed optimization results in the same charging pattern. The pattern itself is random, but it is deterministic for the defined model and solver.

6.1.3 Decentral model

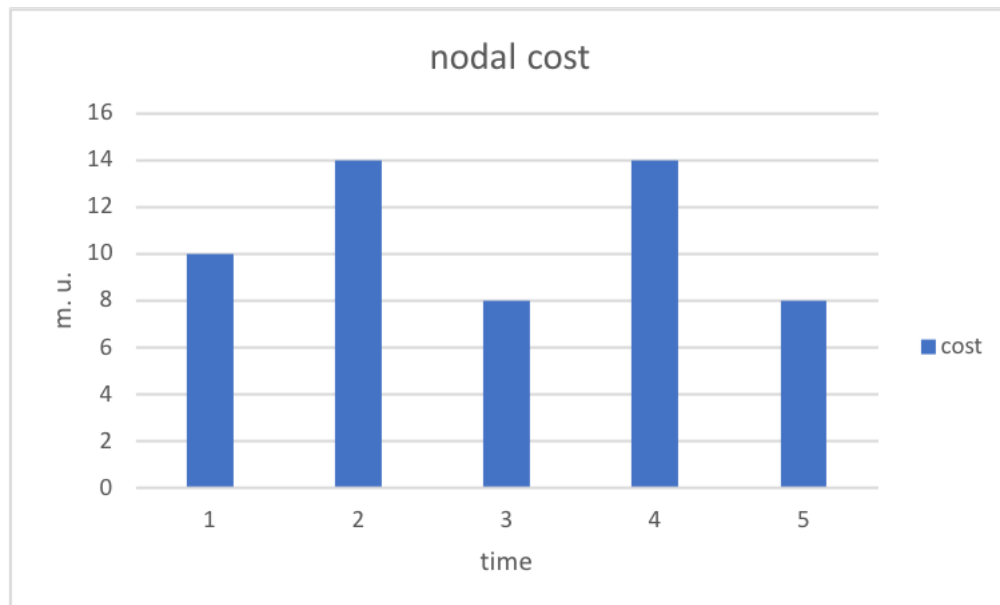


Figure 6.3: Cost of power at the bus

For the decentral model, the objective function has the sole purpose of minimizing the cost of the power charged by the agents. The cost of power at the bus is shown in Figure 6.3. As mentioned earlier, the decentral model is oblivious to the grid and its constraints, so all behaviour is related purely to the state of the agents and the price of power. Based on this, the expectations for this model are that all agents will charge just enough to end the simulation with as much charge as they had before the simulation started. This means that they will charge at the cheapest time steps, and they should not show any preference based on grid limitations.

What we see in the resulting charging pattern shown in Figure 5.13 matches our expectations. If we first look at agent 2-5 we see that they charge maximally in the final and cheapest time step, while charging a smaller amount in the first (second cheapest) time step. This minimizes their cost while retaining the amount of charge they need. Agent 1 differs in this model again because of its low initial SoC. It must charge a certain amount before it disconnects after time step 2 as it needs to remain above the minimum SoC of 20% when discharging during time step 3 and 4. It is also clear that grid limitations are not affecting the pattern as all the agents that can charge at the single cheapest time step do charge in that time step. The resulting load from the agents on the grid is mostly spread to low demand times, as seen in Figure 5.14

6.1.4 Hierarchical-distributed operation model

The results from the hierarchical-distributed operation model are unfortunately disappointing. They show that the agents all wish to charge at the first and last time step (see Figure 5.28), but there is no consideration of the congestion on the network when introduced, hence we get equal results for all scenarios. Additionally, the generator is set to not produce any power by the optimizer (see Figure 5.29), which is impossible since we have agents charging as well as a base load. When we look at the resulting lambdas generated by this model in Table 5.4 it is hard to spot any specific errors. They seem to converge, which they should in a working system, but regardless, they either don't affect the system correctly, or the system is flawed in some other way.

6.2 Discussion

This discussion will focus not only on the performance of the models, but also their applicability in the real world and as groundwork for future research.

Objectively, when only focusing on the actual results from the models, we see that the central model does the best job of spreading the load evenly, and maximally reducing the peak loads. Both in cases with and without congestion this model offers what can be considered an ideal solution for reducing peak demand and cost of generation. One could also say this was expected as it is logical. A system with full overview is able to make the best decisions. But this aspect of full overview is also what keeps this model back, and why finding an alternative model with similar results is necessary. As mentioned in section 2.1 privacy of data is an important factor in such as system. It is wished to minimize the amount of information gathered from PEVs, while still offering a smart grid with good and acceptable charging patterns which achieves good load balancing and cost minimization.

Even though such a central system is unrealistic to incorporate in a real world smart grid, it is still an interesting theoretical model to formulate and improve upon. The results of such a model are a good indication for the best case of an optimization. In combination with the dumb charging model it is then possible to perform comparisons of new models with both a lower and upper bound. One possible improvement to our introduced central model could be to incorporate the agents objective function in the main optimization. This would allow the simultaneous minimization of generation and charging costs. The same negatives regarding central models still apply though. Also, these improvements were not added to the central model for the purpose of this thesis as the price of power generation and the price of charging correlate in their variations, thus the results would be very similar, if not identical. Such an improvement would be more interesting when optimizing larger, and more complex grids.

The decentral model is the complete opposite to the central model in regard to privacy of data, as no information is gathered about the PEVs. This is great for the privacy aspect, but there is nothing holding the PEVs from all charging at the same time, and hence demanding too much of the power grid. This is a big downside, which is the main reason another option is still being researched. The most interesting aspect of the decentral model introduced in this thesis is the fact that it is based on price, and not randomness. As mentioned earlier, most existing decentral models operate on the fact that agents have a certain probability for charging at any connected time step. This reduces the determinism of the system and is in no way controllable. For our model though, even though there are no grid restrictions taken into account, the agents base their charging patterns on the price of power. This price, which can be controlled by a smart grid, then allows for some control of charging patterns without any need of knowledge about individual agents. This is clearly shown in our results when the active load of the agents occurs in low demand time steps. With a random decentral approach, the charging would most of the time be spread out evenly over all time steps, including the high demand time steps.

A decentral model is a simple and cheap model to incorporate in a smart grid, both the stochastic and non-stochastic variations. Essentially, implementing this approach only requires simple software to run on the PEV side of the grid. Due to this simplicity, this kind of model will probably be a go to solution for grids where supply is not a problem. In cases where power supply can vary and is a more critical aspect, a decentral model is not ideal as power peaks are not necessarily always mitigated, and these can cause issues.

It is clear that a system which can take power limits into account, while reducing the amount of PEV data necessary to do so, would fit very well as a best of both worlds solution. This is what the hierarchical-distributed operation model attempts to do. In essence, this model exchanges charging patterns between the agents and a central unit. The control unit then uses these charging patterns and optimizes power flow towards them. A shared parameter between the control unit and the agents is then updated in such a way that the agents might need to modify their charging patterns to better fit the state of the system. In theory, this model should have a significantly better resulting charging pattern than our decentral model, and be relatively close to the central model results. In practice, as seen in the supplied results, the model was not successful. It is worth noting that the failure of the model most likely lies in the adaption from the use case in which this approach was defined. As shown in section 3.1, the approach works well in the case implemented by Hidalgo-Rodríguez et al. [2], and since their case is theoretically similar to ours where their grid supplies home grids, while ours supplies PEVs, we are still convinced an adaption is practically possible despite our final results.

With many agents, the data flow between the agents and the central unit in a hybrid model can reach a significantly high amount, and it should therefore be taken into account in the development of a smart grid. Even though this can accumulate to a lot of data in total, the type of data is limited to charging patterns. Information regarding agent specifics are not shared. It might be possible for some of this information to be inferred by the preferred charging pattern given

the recent advancements made within machine learning. These are relevant aspects to consider when structuring smart grids, but a deeper delve in this direction is outside the scope of this project.

Future work

With this thesis, a core system for future research has been developed. As this was one of the goals of the project, there are naturally several directions that are promising for further exploration with the system. On one side, we have possible future research which can gain a head start by taking advantage of an existing base system. This bypasses the need to spend time developing a specific framework for the research task. On the other side, there are many additions to extend the system to broaden its capabilities. Some of the proposed ideas described below were initially formulated for the project, but fell out of scope once the actual goals of the thesis were narrowed down. Others are ideas that have come up during discussions related to the project. The ideas do not cover all possible directions to take in future work, but they show a general idea to which directions could be taken further.

7.1 Hybrid optimization models

What is considered the biggest contribution by our system in regard to innovative approaches to charging optimization, is the introduction of hybrid models. The first aspect of this which should be worked on next is the completion of the introduced dual decomposition method to a completely working state. As discussed earlier, the achieved results were not satisfactory, and it seems the implementation has a well-hidden problem. A working model of this would be very valuable research and therefore, attempts to locate this problem are encouraged. Additionally, introducing other variations of hybrid models would also be very valuable and interesting for research the next coming years and is equally encouraged. To improve future research which takes advantage of this system, adding additional models for experimentation is beneficial. An example starting point for a new hybrid model could be to incorporate a willingness to pay for each agent in the decentral part of the model to set a maximum price they would pay for power. Then, one could use marginal values generated by the optimization for constraints that are binding (at their limit) to update the price of power at the different buses. In this way, one could achieve greater indirect control of charging pattern through price control.

7.2 Upscale the system

Currently, the system is used on small scale models, with small power grids and few agents (PEVs). It would be interesting to analyze how the system deals with new models of different sizes. The system is fast and lightweight for these small models. All optimizations were performed on a consumer grade laptop. An implemented version of such a system in a smart grid would most likely need to run on a higher performing server. Predicting the necessary computing requirements for different scales will simplify infrastructure development for future smart grids.

7.3 Guarantee system stability

As mentioned previously in the thesis, a smart grid that controls charging would necessarily need to be up at all times, and have enough backup solutions that problems never occur. In this direction, it should be determined what measures need to be taken so as to guarantee that no issues arise, or if they do, what the consequences would be. The most straightforward analysis is in regard to having enough backup systems, but one could also implement behaviour in agents for cases where sudden and unpredicted problems occur. In general, it is worthwhile to explore what options are available to improve the stability of the system as it will have a real-world use.

7.4 Travel pattern generator

This section focuses on the idea introduced and defined in subsection 4.4.1. The current system allows for retrieving data from external sources. Because this option exists, an important addition to the system would be a travel pattern generator which could produce realistic patterns to feed into the system. Ideally this generator could be implemented in such a way that it can generate these realistic patterns for any area/power grid. In this way, the full system would be applicable in any area of interest, with minimal preparation setup for the researchers or users of the system.

7.5 Travel pattern optimizer

This section focuses on the idea introduced and defined in subsection 4.4.2. One major aspect in regard to charging which the system currently does not touch upon, is the decision of where agents should charge. The current system is only optimizing when the agents charge, based on where the agents are during the optimization period. There are multiple aspects that must be considered to be included for such decision making. There need to be rules regarding which charging location can be selected. If an agent is on its way to a specific location in an area, then it is not acceptable to select a charging too far away. Only with this in mind, there is no universal definition for what is considered too far away, so several decisions need to be made here. Such an addition to the system would add an additional layer of depth to the charging optimization, and many more possibilities for power balancing come into play. If one considers the expectation that autonomous vehicles will start to appear in the coming years. This will be increasingly relevant as agents then would be able to distribute themselves evenly, while the owners are still preoccupied at their destination.

7.6 V2G and V2H

The final possible system extension worth mentioning are the ideas presented in V2G and V2H systems. These stand for respectively vehicle to grid (V2G) and vehicle to home (V2H). The main concept is that EVs can be used as batteries in the power grid, or for the home. This idea of using existing power in the grid to help support the grid is not new [32]. Homes which generate power for themselves can sell excess power back to the grid. Additionally, Tesla is selling a product named "Powerwall" which is a large battery that can be charged by locally produced power, to supply the power back to the home when local production is low [33]. Since EVs are essentially large batteries roaming around a grid the idea of using them as a storage for the entire grid can be a game changer. Supporting V2H would essentially replicate the Powerwall developed by Tesla, while V2G would again be an additional layer of depth to how the system can optimize power flow in the grid, enabling many more options in how to charge and when.

Conclusion

In this thesis a core system for PEV charging pattern optimization has been defined and implemented. It is intended for use in development of smart grids and for further research in this field.

The system includes four different charging pattern optimization models. The central and the decentral models, as well as the reference model were all defined and implemented with positive results. The patterns follow our expected results, were the reference model, with dumb charging, increases peak demand, and is generally a bad model for a power grid. The central model results in the best charging pattern overall, but the model has several real-world limitations and which is why other alternatives are being explored. The decentral model shows good ability to distribute charging and minimize the local cost of power for each PEV, but as a decentral model it is unable to handle grid limitations.

This brings us to the hybrid model, while showing a lot of promise in theory, there were some issues with the implementation. It was not discovered what caused these issues, and the results are not what one would ideally hope for. It is clear though that the algorithm runs partially as it is supposed to, as the Lagrangian multiplier (λ) does converge to an expected value, but charging patterns are not able to take grid limitation into account as well as the generator isn't able to match the supply of power to the demand.

As a base for future work though, the system is a success. It includes different base models that perform as expected; being the reference model, central model and decentral model, and the groundwork for a hybrid model. These can all be utilized for any new scenario defined. The system is easily adaptable to use in different situations; it is able to accept external data for flexibility and adaptability, while allowing for an easy process to include new model variations, be they hybrid or any other kind. This ensures that the core system is ready for new, innovative research in this field. Additionally, the system is open for extensions and improvements, which was discussed in-depth in the previous chapter.

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