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Hydro power scheduling in multi-owner river systems

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Problem Description

1. Consolidation of model from the master's project (autumn 2008) .
1.1 Check and follow-up of various comments
1.2 Test the spillage model and confirm that it works correctly.
If the model is sound:
1.3 Include additional constraints like minimum discharge and minimum bypass requirements
1.4 Test for the multi scenario case. Test for the general case of 60-70 different scenarios and verify the results to ensure that the method is robust.

2. If the model is not sound, develop an alternative approach based on iteration between the owners. In this case, apply points 1.2-1.3 for the new model.

3. Build the model for a general hydro power system: extend the model for one reservoir and one power plant to a general case with multiple reservoirs and power plants and varying owner shares in reservoirs and power plants. The model should finally be verified and tested for the multi scenario case.

Assignment given: 17. September 2009
Supervisor: Gerard Doorman, ELKRAFT

Preface

This master's thesis is a part of the Master program in Technology at the Norwegian University of Science and Technology NTNU in Trondheim, Norway as well as a requirement for the Engineering diploma at Grenoble Institute of Technology G-INP, France. It has been written under the supervision of Professor Gerard L. Doorman from the department of Electric Power Engineering at NTNU.

In this project, the building of a mathematical model for seasonal hydro power scheduling in multi-owner river systems is considered. The initial approach consists in solving the problem, maximizing the profits from electricity generation through the scheduling period, as one single optimization problem including all the river system owners. As this modeling was rejected after further testing, a new one was proposed based on an iterative approach. The technique used is fully described in this report and documented with test results for several hydro power system topographies. It has been proven to work well, so far, and should therefore be submitted to extensive testing for large-scale water courses.

I would like to give special thanks to my head supervisor Gerard L. Doorman and to Birger Mo for giving me the opportunity to work on this interesting subject, and for their support and advice throughout the project. Also thanks to Raphael Marguet who supported me during the entire master. I improved my knowledge about energy planning and mathematical programming on many levels during this thesis and these fields are particularly relevant to me in preparation for my future career.

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Abstract

This study was undertaken to build a mathematical model for the calculation of individual water values for reservoirs within a multi-owner river system. After the rejection of a first modeling based on the maximization of the owners' profits solving one single optimization problem, the profit maximization was achieved in separate optimizations with exchange of information about reservoir levels and discharges between them. The solving was proceeded iteratively until a stable solution was reached for a system made up of one reservoir and its power plant. Seventy inflow scenarios were considered and the testing was made under various levels and system shares. Since this approach turned out to work properly, further testing was carried out for two and four cascaded reservoirs forming a part of Sira-Kvina water course. The approach was shown to be robust for seventy five inflow scenarios and important variations over the reservoir levels giving owners' water values that reflected correctly their hydrological situation. The number of iteration required depended on the information exchanged between the procedures as well as the testing conditions but was often equal to three iterations.

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Nomenclature

The parameters start with a capital letter while the variables are written with small-letters.

General variables and parameters

xt global reservoir level in Mm3
ut global discharge in Mm3/week
vt global spillage in Mm3/week
st global bypass in Mm3/week

Tt weekly inflow in Mm3

K number of weeks in the testing period

$X0$ global (physical) initial reservoir level in Mm3
XtK global final reservoir level in Mm3
Xt_{min} global minimum reservoir level in Mm3
Xt_{max} global maximum reservoir level in Mm3

Ut_{min} global minimum discharge in Mm3/week
Ut_{max} global maximum discharge in Mm3/week

St_{min} global minimum bypass in Mm3/week
St_{max} global maximum bypass in Mm3/week

e energy equivalent of the plant in kWh/m3

P weekly prices in NOK/MWh (NOK= Norwegian crowns)
P_{spill} weekly spillage penalty in NOK/MWh to postpon spillage as late as possible

Variables and parameters for each owner

As Owner S's share of the system
Aa Owner A's share of the system

xs Owner S's reservoir level in Mm3
us Owner S's discharge in Mm3/week
vs Owner S's spillage in Mm3/week
ss Owner S's bypass in Mm3/week
WvS Owner S's water value in NOK/Mm3

xa Owner A's reservoir level in Mm3
ua Owner A's discharge in Mm3/week
va Owner A's spillage in Mm3/week
sa Owner A's bypass in Mm3/week
WvA Owner A's water value in NOK/Mm3

$x2s$ Owner S's reservoir level in Mm3 run 2
$u2s$ Owner S's discharge in Mm3/week run 2
$v2s$ Owner S's spillage in Mm3/week run 2
$s2s$ Owner S's bypass in Mm3/week run 2

$x2a$ Owner A's reservoir level in Mm3 run 2
$u2a$ Owner A's discharge in Mm3/week run 2
$v2a$ Owner A's spillage in Mm3/week run 2
$s2a$ Owner A's bypass in Mm3/week run 2

$d12r1$ discharge capacity left by S and available for A after run 1, s->a run 1
$x12r1$ reservoir capacity left by S and available for A after run 1, s->a run 1

$d21r1$ discharge capacity left by A and available for S after run 1, a->s run 1
$x21r1$ reservoir capacity left by A and available for S after run 1, a->s run 1

$d12r2$ discharge capacity left by S and available for A after run 2, s->a run 2
$x12r2$ reservoir capacity left by S and available for A after run 2, s->a run 2

$d21r2$ discharge capacity left by A and available for S after run 2, a->s run 2
$x21r2$ reservoir capacity left by A and available for S after run 2, a->s run 2

Introduction

The Norwegian electricity is mainly produced by hydroelectric power plants. The hydro producers try to maximize their profits covering the electricity consumption and taking into account all constraints in their hydro system. However, to bid on the markets and establish their operational plans, they need to figure out how the consumption, the hydrological situation, the prices will look like in the future. Is it better to produce now or wait for a few weeks? The so called hydro power scheduling process provides them decision support to answer this question.

To include long term electricity contracts and the multi-year storage capacity of some reservoirs, the hydro producers might need to estimate what will happen up to 5 years ahead. For expansion planning purposes, this horizon is even longer and is generally higher than 20 years. Thus, the scheduling process is split up into three stages with different time horizons. The long term scheduling allows the producers to get price forecasts several years ahead and is achieved by a stochastic model to take the uncertainties into account. With a much higher degree of details, the short term model, which is deterministic, is used to find out the optimal asset combination day J for day J+1 with an horizon of two weeks. The way the system is described in the two models is rather different and another model, the seasonal model, is therefore required to make the link between them.

The seasonal model is at the same time coupled with the short and long term models and has a time horizon from 3 to 18 months. As the model used is deterministic despite the period considered, it is run for up to 70 scenarios, each of them with a known price and inflow. The price forecasts and the reservoir information come from the long run and are used as parameters in the multi scenario deterministic optimization to calculate the value of the water within the reservoirs for a later use in the short term optimization task.

The concept of water value is not a priori obvious since the water in itself is for free. It just says how much the profits from generation would have increased by having the equivalent of one more MWh of water available for production. In other words, it gives the expected marginal value of the water stored in the reservoirs with respect to market prices, inflow and load. This value is low when the reservoirs are almost full since there is a risk of overflow while it is really high when the reservoirs are empty due to the risk

of curtailment.
The water values are calculated at an aggregate level in the long term optimization mostly because of the computation limitations of the stochastic dynamic programming method used. Nevertheless, it is necessary to get water values for individual reservoirs for short term planning since these values are compared to the expected market prices to decide if water should be drawn or not from one reservoir or another and finally take the corresponding purchase/sale decisions. The aim of the seasonal optimization is thus the individual water value calculation.
The hydro producers run the seasonal model once a week with inflow, reservoir level and prices updates to get water values as realistic as possible. The short term optimizers then use the first week water value for next week physical operation of the system.

Mostly due to the costs at stake to finance large hydro projects and the deregulation process that started early in the 90s in Norway, several companies share the hydro power facilities. The state owned company, Statkraft, owns around 30% of the system with various degrees of ownership depending on the water course. The hydro power scheduling task is much more complicated in multi-owner river systems. Indeed, the decisions taken by one owner to operate the system: store or draw more water for instance, reduce the degrees of freedom of the other ones: there is less capacity available for them. Therefore, it has a potential impact on their water value.

The proposed work for the master's project and master's thesis at the Norwegian University of Science and Technology, is to develop a seasonal scheduling model for such river systems - most of all for water value calculations. At first, it was based on a original approach written by Birger Mo from SINTEF Energy Research.

The first part of this report will present this approach and discuss its limitations. Then, a new approach will be proposed, explained and finally tested for complex river systems.

Chapter 1

Initial problem formulation

The master's project, a six month mandatory project at NTNU, was based on the formulation of the multiple owner problem proposed by Birger Moe. During autumn 2008, the mathematical model of the problem was written in AMPL, a programming language for large-scale optimizations. Some improvements were made and the model was tested for a few combination of parameters and inflow scenarios, see [1] for specific information. Further testing of this model was complete in autumn 2009 for the master's thesis to figure out if the approach was robust or not.

1.1 Model presentation

The initial model is as simple as possible: one single power plant and its reservoir are modeled. This hydro electric system belongs to several companies: company S, which is the considered one, and the other ones, grouped together in so called company A. Each company owns a share of the system, Ai, that defines the amount of inflow it receives. A diagram of the system is shown in Figure 1.1.

The approach used consists in maximizing the two companies' profits, all system constraints taken into account, by solving one single optimization problem. The objective formulation is therefore the maximization of the total revenues: the sum of the profits for the two owners through the K-week-period. The problem constraints are the physical limitations of the system as well as the restrictions for its use by the two owners. The optimization problem is summed-up in equation 1.1 while the detailed model as well as the list of its variables and parameters is given in Appendix A. The scheduling period is split up into weekly time steps and the relation between production and discharge is assumed to be linear.

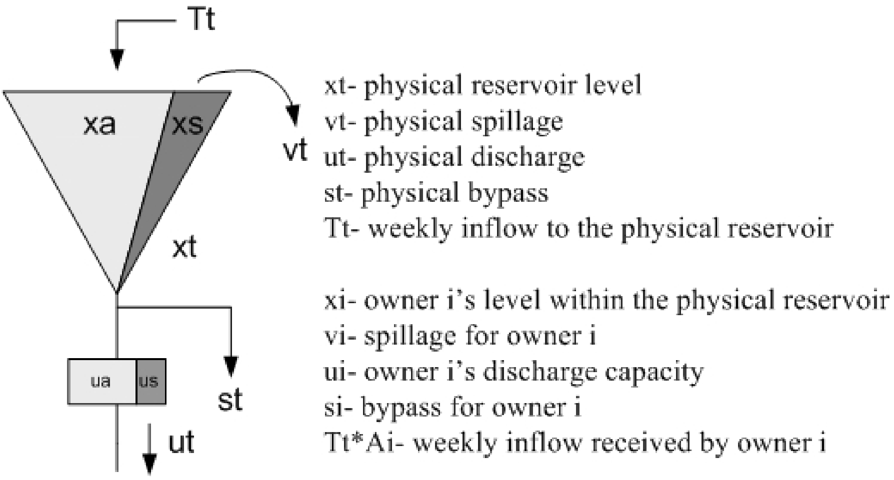

Figure 1.1: Sketch of the system

$$\begin{cases} Max \sum_{k=1}^{K}(P[k] \times (us[k] + ua[k]) \times e) \\ \\ \text{Subject to:} \\ \text{Reservoir balance equations} \\ \text{Reservoir, discharge, bypass limits and minimum requirements} \\ \text{Initial and final states of the system} \\ \text{Fair spillage distribution between the owners} \end{cases} \tag{1.1}$$

In statement 1.1, e is the energy equivalent and says how much energy is stored in each m3 of water in the reservoir [2, p.64]. us, the discharge in Mm3/week for owner S, and ua are multiplied with e and the weekly electricity price, P in NOK/MWh to get the incomes in NOK.

The reservoir balance equation is particularly important. It gives the reservoir level in week k according to the reservoir level in week k-1 and the utilization of the water.
The balance equation for the physical system is:
$xt[k] = xt[k-1] - ut[k] - vt[k] - st[k] + Tt[k]$.
Although the mathematical demonstration will not be achieved in this report, it should be noted that the water values are the dual values of the balance equation and are given in NOK/Mm3. The reservoir equation is also written for each owner's reservoir since the purpose of the modeling is to get their individual water values.

The spillage is to be defined according to the individual reservoir levels to be fair. If one owner has more water than what his share says in the reservoir, then his water must be spilled first. As these if-then-else statements introduce non linearities in the modeling, the conditions are expressed with integer variables. An easier way to describe the spillage distribution was found out last autumn but its implementation in integers was not correct.

1.2 Spillage conditions

The new formulation of the spillage conditions is reminded below. To reduce the lines of code, x[i,k] corresponds to the reservoir level of owner i in week k.

```
case 1:
if x[1,k]> A[1]*xt[k] then v[2,k]<=0 (v[i,k] is defined >= 0 in the model)
case 2:
if x[1,k]< A[1]*xt[k] then v[1,k]<=0
case 3:
if x[1,k]=A[1]*xt[k] then A[1]*v[2,k]=A[2]*v[1,k]
```

The programming with integer variables is based on the method described in [3]. The mistakes of last autumn are corrected in Listing 1.1. $sigxc[i,k]$ stands for case 1 and 2 integer variables whereas sigxc3 is used for case 3 that requires intermediate integer variables to code the equality signs. As one way implications are written, $sigxc[1,k] = 1$ does not necessarily means $x[i,k] > A[i] \times xt[k]$, another constraint is added to ensure that only one sigma is true at a time.

The spillage conditions only say who should get the overflow with respect to the reservoir levels but not when it should take place. If overflow is inevitable, it can happen every moment during the period but one wants it only when the reservoir is higher than Xt_{max}. So, it should occur as late as possible in the period, when there is no other options for using the water, and this can be done easily by introducing a spillage penalty. A decreasing cost is assigned to the spillage (for a 18-week-period, it varies from 1.8 NOK/MWh in week 1 to 0.1 NOK/MWh in week 18) and the term $-P_{spi} \times v[i,k]$ is added in the objective function. The later the spillage, the smaller the profit reduction.

The complete AMPL code, written with the help of [4], is given in Appendix B. The model and a data file containing the reservoir characteristics just have to be called to run the tests. Previous ones were completed for two inflow scenarios (1 and 8) and two periods of the year during the master's project work. Testing is to be carried out for more combination of scenarios and parameters using the correct spillage conditions to prove the validity of the modeling.

1.3 Further testing

During the filling season, the snow starts melting and a huge inflow amount comes into the reservoirs. This water has to be stored for later use in the winter, where the energy

```
# case1&2

# if x[i,k]>A[i]*xt[k] then sigxc[i,k]=1
subject to level_case12 {i in 1..OWNER, k in 1..K}:
(x[i,k]-A[i]*Xt_max[k]-(Xmax+Tmax)*sigxc[i,k]) <= 0;

# if sigxc[1,k]=1 then v[2,k]<=0
subject to spillage_case1 {k in 1..K}:
(v[2,k]+Tmax*(sigxc[1,k])) <= Tmax;

# if sigxc[2,k]=1 then v[1,k]<=0
subject to spillage_case2 {k in 1..K}:
(v[1,k]+Tmax*(sigxc[2,k])) <= Tmax;

# case3

# if x[1,k] <= A[1]*xt[k] then sigl31[k]=1
subject to level_case31 {k in 1..K}:
(x[2,k]-A[2]*Xt_max[k]-(-(Xmax+Tmax)-EPS)*sigl31[k]) >= EPS;

# if x[1,k] >= A[1]*xt[k] then sigl32[k]=1
subject to level_case32 {k in 1..K}:
(x[2,k]-A[2]*Xt_max[k]-(Xmax+Tmax+EPS)*sigl32[k]) <= (-EPS);

# if sigl31[k]=1 and sigl32[k]=1 then sigxc3[k]=1
subject to level_case33 {k in 1..K}:
(sigl31[k]+sigl32[k]-sigxc3[k]) <= 1;

# if sigxc3[k]=1 then A[1]*v[2,k]-A[2]*v[1,k] <= 0
subject to spillage_case31 {k in 1..K}:
(A[1]*v[2,k]-A[2]*v[1,k]+Tmax*sigxc3[k]) <= Tmax;

# if sigxc3[k]=1 then A[1]*v[2,k]-A[2]*v[1,k] >= 0
subject to spillage_case32 {k in 1..K}:
(A[1]*v[2,k]-A[2]*v[1,k]-Tmax*sigxc3[k]) >= (-Tmax);

# only one case is true

subject to sigmas {k in 1..K}:
sigxc[1,k]+sigxc[2,k]+sigxc3[k] <=1;
```

Listing 1.1: Spillage conditions

consumption increases due to low temperatures and there is almost no inflow getting in the reservoir since the precipitations come as snow. The challenge is therefore to keep enough water for the winter period while avoiding overflowing during the summer. The scheduling during this season is consequently really interesting and the tests presented in this report will mainly be focused on it.

The considered period is from week 20 to week 38, the electricity prices of year 2007 and 2008 are taken from NordPool website for the testing, [5].

The reservoir and plant specifications are the followings:
Mean annual inflow: 336 Mm3
Energy conversion factor: 1 kWh/m3
Reservoir: 100 Mm3
Installed capacity: 100 MW

A large number of inflow records are available for this reservoir. An example of results is given in Table 1.1 for inflow scenario 15.

The testing parameters are:
Ownerships: S 50% A 50%
Corresponding discharge upper limit: S 8.4 A 8.4 Mm3/week
Initial individual levels: S 3 A 27 Mm3
Final global level: 85 Mm3
Both owners' discharge capacities are limited by their share of the system.

The water values seem to take the value of some weekly prices. Let's try to understand exactly what it means.

1.4 Gaining insight into water values

The water value is the expected marginal value of the water stored in the reservoirs. If an hydro producer has one more Mm3 of water in his reservoir, it says the maximum amount of money he can get for it.

An extra mega cubic meter of water can be produced during a week with an high expected price if there is some production capacity left within this week. Basically, the program looks for weeks where the best prices are forecast one after another until it finds some discharge capacity available to draw one more Mm3. Looking at Table 1.1, one can see that us, u for owner 1, reaches its maximum between week 21 and 29. It is only possible to discharge an extra Mm3 of water in week 30 for a price of 107.8 NOK/MWh. As the energy equivalent of the considered plant is 1kWh/m3, the water value is 107.8 NOK/Mm3 and $107.8 \times e = 107.8$ NOK/MWh for all the weeks before week 31. After week 30 and up to week 36, the reservoir is filled to overflowing, an extra Mm3 of water coming to the reservoir within these weeks has to be produced directly or turns out to be spilled.

OWNER	Week	x	u	Wv	v	s	Profits/week	P
1	20	3						
1	21	0.4	8.4	107.8	0	0	1446.9	172.3
1	22	2.3	8.4	107.8	0	0	1494.3	177.9
1	23	10.8	8.4	107.8	0	0	1413.5	168.3
1	24	12.8	8.4	107.8	0	0	1634.9	194.6
1	25	18.6	8.4	107.8	0	0	1718.4	204.6
1	26	27.3	8.4	107.8	0	0	1755.3	209.0
1	27	29.8	8.4	107.8	0	0	1512.9	180.1
1	28	31.7	8.4	107.8	0	0	1393.9	165.9
1	29	32.6	8.4	107.8	0	0	913.3	108.7
1	30	35.5	2.5	107.8	0	0	269.6	107.8
1	31	38.4	0	101.8	0	0	0.0	101.8
1	32	36.8	5.1	127.1	0	0	648.3	127.1
1	33	40.1	0	126.8	0	0	0.0	126.8
1	34	42.5	0	129.3	0	0	0.0	129.3
1	35	44.9	0	173.2	0	0	0.0	173.2
1	36	46	0	195.8	0	0	0.0	195.8
1	37	46.5	0.4	207	0	0	82.8	207.0
1	38	42.5	8.4	207	0	0	1772.1	211.0
2	20	27						
2	21	24.4	8.4	107.8	0	0	1446.9	172.3
2	22	26.3	8.4	107.8	0	0	1494.3	177.9
2	23	34.8	8.4	107.8	0	0	1413.5	168.3
2	24	36.8	8.4	107.8	0	0	1634.9	194.6
2	25	42.6	8.4	107.8	0	0	1718.4	204.6
2	26	51.3	8.4	107.8	0	0	1755.3	209.0
2	27	53.8	8.4	107.8	0	0	1512.9	180.1
2	28	55.7	8.4	107.8	0	0	1393.9	165.9
2	29	56.6	8.4	107.8	0	0	913.3	108.7
2	30	58.7	3.2	107.8	0	0	345.1	107.8
2	31	61.6	0	101.8	0	0	0.0	101.8
2	32	56.7	8.4	127.1	0	0	1067.7	127.1
2	33	59.9	0	126.8	0	0	0.0	126.8
2	34	57.5	4.9	129.3	0	0	633.6	129.3
2	35	55.1	4.7	173.2	0	0	813.9	173.2
2	36	54	2.3	195.8	0	0	450.4	195.8
2	37	46.5	8.4	207	0	0	1738.9	207.0
2	38	42.5	8.4	207	0	0	1772.1	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) Profit/week- (NOK)

Table 1.1: Unbalanced profits for the two owners, scenario 15

Moreover, if a producer knows that he could get one more Mm3 of water week k, he can decide to take out one more Mm3 of water in week j earlier than k, if he still has some water in his reservoir in week j. In other words, knowing he will receive some extra water later in the period, the producer discharge one more Mm3 now. This is due to the fact that the model is deterministic, the future is known with certainty. The water value is therefore 207 NOK/MWh in week 37 and 38 in Table 1.1. If more inflow is expected in the reservoir week 38, more water should be drawn during week 37.

The understanding of the water value is easier for one single plant with an energy equivalent of 1kWh/m3 but can become really tricky for more complicated river systems. It will be further discussed in Chapter 3.

After looking carefully at Table 1.1 and randomly running tests for scenarios with this formulation, a tendency is observed. The owner who starts with the lowest reservoir level does not have a good production repartition through the period. It does not draw his water for the best prices. One sees that the reservoir of owner S, 1 in the table, never reaches 50 Mm3. He does not use his storage space to produce when the best prices are expected. Instead of producing in week 36 and 37 for 195.8 and 207 NOK/MWh, he discharges in week 28, 29, 30 and 32 for less attractive prices. Unlike him, owner A always produces at the best moment.

The strategy expressed by the model is to store water to produce the last weeks avoiding spillage but it does not take into account who is keeping the water or producing. Then, A has water for weeks 35, 36 and 37 while S does not since he has to meet the final reservoir requirement in week 38. He uses all his water to produce the first weeks to prevent owner A's water to be spilled. The solution is consequently optimal from a system point of view - no spillage and huge global profits - but individually there is an important prejudice for owner S. The next step is to reduce the inequity and urges S to produce at the right moment.

1.5 Reducing unfairness between owners

The only way to include such a concern is to add some constraints in the model and/or change its objective. The first attempt was made on reservoir levels.

1.5.1 Constraint on reservoirs

In the previous section, owner S's reservoir handling was not acceptable since he did not keep water to produce later in the period. To correct this, a penalty, Pcor, is included in the objective for every Mm3 of water below $Xt_{max} \times As$ so that xs is as close as possible of its maximum value. The AMPL modifications are shown in Listing 1.2.

```
# New objective
maximize total_profit:
sum {k in 1..K} ((u[1,k]+u[2,k])*P[k]*E-Pspill[k]*(v[2,k]+v[1,k])*E-diff[k
    ]*Pcor[k]);

# Where diff is defined as:
subject to optimal_prod {k in 1..K}:
diff[k]=Xt_max[k]*A[1]-x[1,k];
```

Listing 1.2: Keep company S's reservoir as high as possible

Several values for Pcor were tried but it should be maintained as low as possible since it is included in the objective. In table 1.2, the value of Pcor is 0.1 NOK/MWh.

The production for S is higher during week 37, the discharge goes from 0.4 Mm3/week to 8 Mm3/week. The penalty turns into a profit when xs is above $Xt_{max} \times As$ and it explains why S's level is so high in weeks 35 and 36. The water values should be 107.8 NOK/MWh from week 21 to 29, instead they take different values every week that come from the penalty introduced.
The value of the water for week 21 is 109.4 NOK/MWh which does not corresponds at all to $107.8 + (50 - 0.4) \times 0.1 = 112.8$ as one could think. So, the penalty has an important impact on the water values but it is hardly predictable.

By entering a higher value of Pcor, say 0.3, 2.3 Mm3 are discharged in week 36 but the water values do not reflect the best opportunity to draw an extra kWh anymore. For instance, owner A's water values are 106.6 NOK/MWh for weeks 21 to 30.

This solution is not working at all; the value chosen for Pcor strongly affects the water values in a quite unpredictable way. Moreover, the production is not so well corrected and what has been gained for S is lost for A. With Pcor=0.1, the former produces 8 Mm3 in week 37 while the latter only draws 0.9 Mm3 of water. The production should have been split up between the two owners to be fair but that is not seen by the model since the objective is to maximize the sum of the profits.
Other ideas can still be explored to try to increase owner S's production towards the end of the period.

1.5.2 Other ideas and conclusions

An intuitive idea to force owner S's production is to increase the price he sees. If the price for S is set up at price for A plus 10, there is absolutely no changes in the distribution of the production. The water values only raise by 10 NOK/MWh.

The price growth should follow the price fluctuations to be coherent. Let's say that the prices for S are increased by 10% which leads to the corresponding change in the water values for S. The solution achieved looks very similar to the one in Table 1.2. The

OWNER	Week	x	u	v	Wv	s	Profits/week	Prices
1	20	3						
1	21	0.4	8.4	0	109.4	0	1446.92	172.3
1	22	2.3	8.4	0	109.3	0	1494.3	177.9
1	23	10.8	8.4	0	109.2	0	1413.5	168.3
1	24	12.8	8.4	0	109.1	0	1634.88	194.6
1	25	18.6	8.4	0	109	0	1718.43	204.6
1	26	27.3	8.4	0	108.9	0	1755.32	209.0
1	27	29.8	8.4	0	108.8	0	1512.85	180.1
1	28	31.7	8.4	0	108.7	0	1393.86	165.9
1	29	32.6	8.4	0	108.6	0	913.27	108.7
1	30	37.9	0	0	108.5	0	0	107.8
1	31	40.9	0	0	102.4	0	0	101.8
1	32	44.4	0	0	127.3	0	0	127.1
1	33	47.6	0	0	127.2	0	0	126.8
1	34	50	0	0	129.6	0	0	129.3
1	35	52.4	0	0	173.4	0	0	173.2
1	36	53.6	0	0	195.9	0	0	195.8
1	37	46.5	8	0	207	0	1656.08	207.0
1	38	42.5	8.4	0	206.9	0	1772.07	211.0
2	20	27						
2	21	24.4	8.4	0	107.8	0	1446.92	172.3
2	22	26.3	8.4	0	107.8	0	1494.3	177.9
2	23	34.8	8.4	0	107.8	0	1413.5	168.3
2	24	36.8	8.4	0	107.8	0	1634.88	194.6
2	25	42.6	8.4	0	107.8	0	1718.43	204.6
2	26	51.3	8.4	0	107.8	0	1755.32	209.0
2	27	53.8	8.4	0	107.8	0	1512.85	180.1
2	28	55.7	8.4	0	107.8	0	1393.86	165.9
2	29	56.6	8.4	0	107.8	0	913.27	108.7
2	30	56.2	5.7	0	107.8	0	614.68	107.8
2	31	59.1	0	0	101.8	0	0	101.8
2	32	54.3	8.4	0	126.8	0	1067.71	127.1
2	33	52.4	5.1	0	126.8	0	646.6	126.8
2	34	50	4.9	0	129.3	0	633.58	129.3
2	35	47.6	4.7	0	173.2	0	813.92	173.2
2	36	46.4	2.3	0	195.8	0	450.41	195.8
2	37	46.5	0.9	0	207	0	186.31	207.0
2	38	42.5	8.4	0	207	0	1772.07	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) Profit/week- (NOK)

Table 1.2: Results urging xs to be as high as possible, scenario 15

production is 8.4 Mm3 in week 37 for S but goes down to 0.4 for A. Here again the results are not satisfying.

To reduce the difference between the profits per week, see Table 1.1 where S does not make any from week 33 to 36, it is also possible to set conditions that says owner S's profits cannot be less or more than x% of the profits for A within a y-week-interval. How to choose the value for x and y without affecting the results and in a way that it is still valid for other scenarios?

By adding constraints or changing the objective, the water values are strongly affected. Moreover, the approach should not only be specific to one scenario but has also to be applied generally for many scenarios and periods of the year.
After a discussion with Bjørn Nygreen, an optimization professor at NTNU, the model was abandoned. He concluded that the modifications to be made are not obvious and that the validity of the water values is not guaranteed when the integer conditions for the spillage distribution are engaged.
As the problem of multi-owner river systems tends to be more and more common due to the deregulation of the electricity sector, the development of a scheduling approach is quite important. Therefore, the work goes on to figure out another modeling that gives better individual results for the owners.

Chapter 2

Iterative approach building for one reservoir

As the previous model did not achieve the excepted results, a new formulation of the problem is developed for the same simple system including one reservoir and its power plant downstream.

2.1 Model building for one reservoir

2.1.1 Model description

In the first modeling, the system was described as one reservoir shared by two owners. The basis for this new approach is to split up this physical reservoir in two, one virtual reservoir for the first owner, S, and one for the other owner, A. The production and reservoir capacity of the owners are bounded by their share of the system. See Figure 2.1 for a sketch of owner S's system.

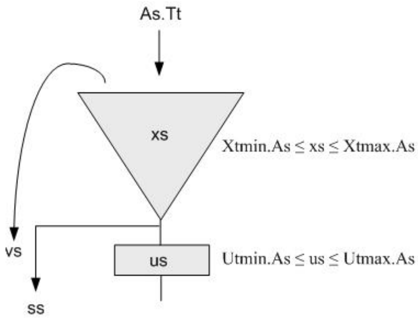

Figure 2.1: Owner S's system

The optimization is then run for each owner individually. The objective and constraints for owner S optimization are given in Listing 2.1. The code for A is exactly the same but the variables are called xa, sa, va and ua and the constraint names start with an A.

There is no need for spillage conditions. The amount of overflow should be the water above $Xt_{max}[k] \times Ai$ for each owner.

```
# OBJECTIVE
maximize S_profit:
sum {k in 1..K} (P[k]*us[k]-Pspill[k]*vs[k])*E;

# PHYSICAL CONSTRAINTS
# reservoir balance equation
subject to S_balance {k in 1..K}:
xs[k]= xs[k-1]-us[k]-vs[k]-ss[k]+Tt[k]*As;
# production constraint
subject to S_production1 {k in 1..K}:
Ut_min[k]*As <= us[k] <= Ut_max[k]*As;
# bypass constraint
subject to S_bypass {k in 1..K}:
St_min[k] <= ss[k] <= St_max[k];
# reservoir constraint
subject to S_level {k in 1..K}:
Xt_min[k]*As <=xs[k] <= As*Xt_max[k];

# CONSTRAINTS DUE TO INITIAL AND FINAL CONDITIONS
subject to S_initial_condition:
xs[0]=Xs0;
subject to S_final_condition:
xs[K]= XtK*As;
```

Listing 2.1: Optimization for owner S

Because of this modeling, the two owners do not interact with each other and the use of the system is not realistic. If, for instance, owner A has more water than $Aa \times Xt_{max}[k]$, his water will automatically be spilled. However, if owner S does not reach his level limit then this overflow could have been partially avoided by the utilization of his reservoir. A demonstration of this phenomenon is available in Appendix C. Therefore, there should be an exchange of information between these two separate optimizations.

2.1.2 Exchange of data between optimizations

As mentioned in the previous section, the first information that needs to be sent to owner A after owner S's optimization is the capacity left in owner S's virtual reservoir. This capacity is called $x12$ since the flow goes from owner 1, S, to owner 2, A. Depending on the alternative chosen for the production capacity - the owners are allowed to produce up to their shares, owner S can use A's capacity if he does not want to use it or they can both use what is left by the other owner- the remaining discharge capacity after one owner optimization is also defined as $d12$ or $d21$. Figure 2.2 sums up these explanations.

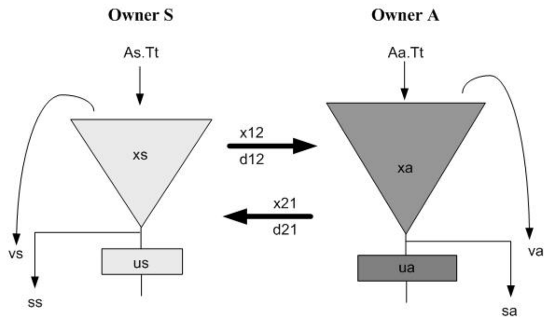

Figure 2.2: Flow of information between owners

The model for each owner needs to be updated with these elements. The capacities unused by the owners, xij and dij, are calculated during the problem solving and should be defined as variables in the model file. An example of how the new constraints look like if owner S can use owner A production capacity is given here:

```
1 First optimization for owner S :

Constraints:
Xt_min[k]*As <= xs[k] <= Xt_max[k]*As
Ut_min[k]*As <= us[k] <= Ut_max[k]*As

Capacity left for A:
x12r1[k]=Xt_max[k]*As-xs[k]

2 Optimization for owner A:

Constraints:
Xt_min[k]*Aa <= xa[k] <= Xt_max[k]*Aa+ x12r1[k]
Ut_min[k]*Aa <= ua[k] <= Ut_max[k]*Aa

Capacity left for S:
d21r1[k]=Ut_max[k]*Aa-ua[k]
x21r1[k]=Xt_max[k]*Aa-xa[k]

3 Second optimization for owner S (run 2):

Constraints:
Xt_min[k]*As <= xs[k] <= Xt_max[k]*As+ x21r1[k]
```

```
Ut_min[k]*As <= us[k] <= Ut_max[k]*As+ d21r1[k]

Capacity left for A:
x12r2[k]=Xt_max[k]*As-xs[k] etc...
```

The two optimizations are obviously performed until the capacities xij and dij remain constants otherwise the solution is not stable. This is the next step of the programming, building a model where the two optimization problems are solved until the exchanged capacities converge.

2.2 Implementation of the iterative approach

2.2.1 Model file

To know if the capacities converge, two optimizations for each owner are a minimum. Indeed, $x12r1$ that stands for owner S's first optimization is compared to $x12r2$ - the capacity left after owner S's second optimization. If all the transferred values are the same, a stable solution is reached. If not, a new iteration is done i.e a new optimization is run for S and then for A.

As four optimization problems are modeled the first idea was to create four different model files with corresponding data files. While computing, the program has to call all these files one after another and some parameters are also defined several times, which is clearly a waste of time. The best solution consists in using one model file that contains the four problems: Owner S run 1 (S1), Owner A run 1 (A1), Owner S run 2 (S2) and Owner A run 2 (A2). The variable and constraints names are changed for the second run. $x2s$ is for instance the reservoir level for owner S run 2 and $A2-balance$ the balance equation for owner A run 2.

The model and data files are written but a list of commands is created to run the optimizations in the correct order and to transfer information between optimizations.

2.2.2 Script of commands

The commands are written in a file .run. In AMPL environment, $includefile.run$; automatically executes the list of commands. To alternate easily between our four problems, each of them is defined with its variables, objective and constraints . When a problem is called it becomes the current one and all its variables are unfixed while the other ones are fixed to their current value. See [4, p.304-318] for more details.

First, we solve (S1), then (A1), (S2) and (A2). The equality between the variables from run 1 and 2 are checked and another iteration is performed until this condition is satisfied. This last part is done within a *repeat until loop.* As it is not possible to use directly the variables of an optimization dij to the next one, the capacities are entered as parameters $fdij$ in the next optimization. The number of iterations is counted with an index j.

The building principle of the run file is shown in listing 2.2.

To eliminate useless iterations, the results are rounded to one decimal. Otherwise, a difference of 0.01 in the capacities leads to another iteration for giving almost the same results. The last problem to solve is being able to run this procedure for many scenarios in an efficient way.

2.2.3 Iterating over scenarios

The inflow data are generally shown in an excel file with up to 70 years of records. For each year (scenario), the weekly inflow is given. It is possible to write a VBA macro that generates a new excel file for each scenario with the inflow for the weeks we want to run the model for. A UserForm, see Figure 2.3 asks the user the data he would like to have for his tests.

According to the user entries, clicking the OK button creates the corresponding excel files. The UserForm macro is in Appendix D.1.

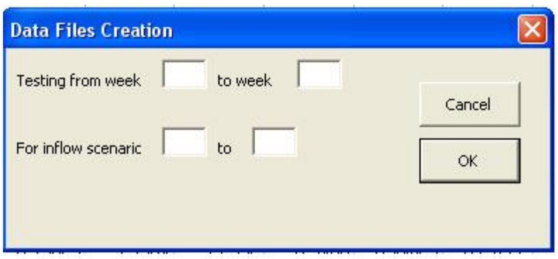

Figure 2.3: Excel UserForm

The run file is updated, see Listing 2.3, to iterate over the scenarios and a table with the results is created.

The model, the script of commands to be executed and an easy solution to run the approach for several scenarios are ready. A short testing is now carried out to have a look at the results.

2.3 First testing of the approach

The reservoir is the same as the one used in Chapter 1 and is tested for the same period from week 20 to 38 in the filling season. These characteristics are briefly summarized below:

Mean annual inflow: 336 Mm3

```
Read the model and the data files;

Define the four problems with their variables, objective and constraints;

Solve problem for owner S1;
Variables transferred as parameters for optimization A1;

Solve problem for owner A1;
Variables transferred as parameters for optimization S2;
j=1;

Solve problem for owner S2;
Variables transferred as parameters for optimization A2;

Solve problem for owner A2;
Variables transferred as parameters for optimization S1;
j=2;

Repeat loop {
  If (cond= capacities are not the same between S1 S2 and A1 A2) then
  { Solve problem for owner S1;
    Variables transferred as parameters for optimization A1;

    Solve problem for owner A1;
    Variables transferred as parameters for optimization S2;
    j=j+1;
  }
  Else break loop;

  If (cond= capacities are not the same between S1 S2 and A1 A2) then
  { Solve problem for owner S2;
    Variables transferred as parameters for optimization A2;

    Solve problem for owner A2;
    Variables transferred as parameters for optimization S1;
    j=j+1;
  }
  Else break loop;

  If (cond= capacities are not the same between S1 S2 and A1 A2) then t=1
      else t=0;
} until (t=0 or j=10)
```

Listing 2.2: Iterative approach

```
Read the model and the data files;
Define the four problems with their variables objective and constraints;

Define the scenarios the model is run for;
set SCENARIOS=19..20;

Declare the excel table containing the inflow data;
table Inflow {i in SCENARIOS} IN "ODBC" ("M:\dokument\scenario" & i & ".xls
    "): [TIME],Tt;

Declare the table that will receives the results;
table res {i in SCENARIOS} IN "ODBC" ("M:\dokument\res_sce" & i & ".xls"):
    [TIME], xs OUT,us OUT,S_balance OUT,vs OUT,d12r1 OUT,x12r1 OUT,xa OUT,
    ua OUT,A_balance OUT,va OUT,d21r1 OUT,x21r1 OUT;

Iterate over scenarios;
for {i in SCENARIOS}
{ reset data Tt;
  read table Inflow[i];
  solve problem with the iterative approach;
  write table res[i];
}
```

Listing 2.3: Iterating over scenarios

Energy conversion factor: 1 kWh/m3
Reservoir: 100 Mm3
Installed capacity: 100 MW
The testing parameters are:
As:=0.5;
Aa:=0.5;
Xa0:=27;
Xs0:=3;
XtK:=85;

A has much more water than S in his reservoir to force the reservoir capacity exchange. In this test both owners are limited to the discharge capacity given by their share of the system since the focus is on the reservoirs.

Table 2.1 shows the results of this test for inflow scenario 19. Owner A uses owner S's virtual reservoir since he needs to store more water than he is allowed by his share. But looking carefully at the table, something seems to be wrong.

Owner S has a water value, Wv S, negative due to the spillage penalty introduced to postpone it as late as possible, see Chapter 1. It suggests overflow during week 30 even though S does not reach his maximum reservoir level. His capacity is in fact used by

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1
20	3						27					
21	6.6	8.4	-1	0	0	43.4	30.6	8.4	-1.1	0	0	19.4
22	6.5	8.4	-1	0	0	43.5	30.5	8.4	-1.1	0	0	19.5
23	13.5	8.4	-1	0	0	36.5	37.5	8.4	-1.1	0	0	12.5
24	18.8	8.4	-1	0	0	31.2	42.8	8.4	-1.1	0	0	7.2
25	23.8	8.4	-1	0	0	26.2	47.8	8.4	-1.1	0	0	2.2
26	30.7	8.4	-1	0	0	19.3	54.7	8.4	-1.1	0	0	-4.7
27	35.8	8.4	-1	0	0	14.2	59.8	8.4	-1.1	0	0	-9.8
28	40.6	8.4	-1	0	0	9.4	59.4	8.4	-1.1	5.2	0	-9.4
29	41.6	8.4	-1	0	0	8.4	58.4	8.4	-1	2.1	0	-8.4
30	41.9	8.4	-0.9	0	0	8.1	58.1	8.4	-0.9	0.6	0	-8.1
31	48.8	1.8	101.8	0	6.6	1.2	51.2	8.4	-0.8	7.2	0	-1.2
32	50	8.4	126.8	0	0	0	50	8.4	-0.7	2.4	0	0
33	50	6.6	126.8	0	1.8	0	50	6.6	126.8	0	1.8	0
34	50	3.5	129.3	0	4.9	0	50	3.5	129.3	0	4.9	0
35	50	4.1	173.2	0	4.3	0	50	4.1	173.2	0	4.3	0
36	50	5.6	195.8	0	2.8	0	50	5.6	195.8	0	2.8	0
37	48.2	6.4	207	0	2	1.8	48.2	6.4	207	0	2	1.8
38	42.5	8.4	207	0	0	7.5	42.5	8.4	207	0	0	7.5

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week) xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3)

Table 2.1: First model testing for scenario 19

owner A to avoid to spill water when the physical reservoir is not full.

Writing $Xt_{min}[k] \times As \leq xs[k] \leq Xt_{max}[k] \times As + x21r1[k]$ means that S reservoir capacity is increased if $x21r1[k] > 0$ whereas it is decreased when $x21r1[k] < 0$. The last case occurs here. The model considers that S has reached his new reservoir limit since it is reduced by owner A's utilization of the reservoir. His only possible use of an extra Mm3 is spillage. The modeling of the capacities exchanged between the owners is not satisfying and should include a sort of priority for the use of the capacity left. Even if an owner does not want to fully use it, he has the priority to do so.

2.4 Correction to prioritize the owner that has the unused capacity

2.4.1 Attempt on the model file

As mentioned earlier, the issue raises when $xij[k] < 0$ - i.e one owner uses more than his limits. In such a case, the capacity for the other one should not be corrected. Intuitively, there should be two reservoir constraints instead of one to distinguish between $xij[k] < 0$ and $xij[k] > 0$. The idea is shown below for owner A.

```
Constraints:
if x12r1[k]>0 then Xt_min[k]*Aa <= xa[k] <= Xt_max[k]*Aa+ x12r1[k]
if x12r1[k]<=0 then Xt_min[k]*Aa <= xa[k] <= Xt_max[k]*Aa
Ut_min[k]*Aa <= ua[k] <= Ut_max[k]*Aa

Capacity left for S:
d21r1[k]=Ut_max[k]*Aa-ua[k]
x21r1[k]=Xt_max[k]*Aa-xa[k]
```

This solution is clearly not linear due to the use of if-then-else statements. Such constraints cannot be handled by a classical solver and are consequently rejected. Another way to introduce the differentiation is found: after an optimization, for instance solving problem S1, the capacities are passed from variables in S1 to parameters in A1 in the run file.

2.4.2 Modification of the command script

The variables names associated to the discharge and reservoir available for the use of the next owner use are dij and xij. They are passed as parameters in the following optimizations with the name $fdij$ and $fxij$. The idea is to assign the value 0 to parameters $fdij$ and $fxij$ when $dij < 0$ and $xij < 0$. In the same manner, $fdij$ equals dij and $fxij$ is set to xij when $dij > 0$ and $xij > 0$. The AMPL code is provided in Listing 2.4.

This solution finally works well and the transfer of capacities between the two owners is done as expected. The final model file as well as the final script of commands are respectively in Appendix D.2 and D.3.

2.4.3 Results after approach correction

Earlier in this chapter, an error was highlighted by scenario 19. Owner S's capacity was reduced according to Owner A's use of it which leads to false water values for owner S. Let's figure out if this mistake has been corrected by our modification attempts. The testing conditions are the same. Table 2.2 shows the results.

The solution converges after the minimum number of iterations, 2. Despite the use of his reservoir by owner A, owner S's water values for week 21 to 32 are the price in week 31. Indeed, S cannot produce an extra kWh between week 20 and 30 since his discharge is maximum. Owner A uses exactly what is left by S weeks 28-31 but S is free to use it if he wants to. When both owners reach the same level week 32, the strategy for the use of the water blends.

```
solve owner_s;
display xs,us,S_balance,vs,d12r1,x12r1;

# Remaining capacities after s optimization1 are transferred as
    parameters for owner a optimization1

for {k in 1..K}{

  if d12r1[k]>0 then let fd12r1[k]:= d12r1[k];
  else let fd12r1[k]:= 0;
    }
for {k in 1..K} {

  if x12r1[k]>0 then let fx12r1[k]:= x12r1[k];
  else let fx12r1[k]:=0;
    }

solve owner_a;
display xa,ua,A_balance,va,d21r1,x21r1;

# Remaining capacities after a optimization1 are transferred as
    parameters for owner s optimization2

for {k in 1..K}{

  if d21r1[k]>0 then let fd21r1[k]:= d21r1[k];
  else let fd21r1[k]:= 0;
    }
for {k in 1..K} {

  if x21r1[k]>0 then let fx21r1[k]:= x21r1[k];
  else let fx21r1[k]:=0;
    }
```

Listing 2.4: Going from variables in optimization 1 to parameters in optimization 2

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	P
20	3						27						
21	6.6	8.4	101.8	0	0	43.4	30.6	8.4	-1.1	0	0	19.4	172.3
22	6.5	8.4	101.8	0	0	43.5	30.5	8.4	-1.1	0	0	19.5	177.9
23	13.5	8.4	101.8	0	0	36.5	37.5	8.4	-1.1	0	0	12.5	168.3
24	18.8	8.4	101.8	0	0	31.2	42.8	8.4	-1.1	0	0	7.2	194.6
25	23.8	8.4	101.8	0	0	26.2	47.8	8.4	-1.1	0	0	2.2	204.6
26	30.7	8.4	101.8	0	0	19.3	54.7	8.4	-1.1	0	0	-4.7	209.0
27	35.8	8.4	101.8	0	0	14.2	59.8	8.4	-1.1	0	0	-9.8	180.1
28	40.6	8.4	101.8	0	0	9.4	59.4	8.4	-1.1	5.2	0	-9.4	165.9
29	41.6	8.4	101.8	0	0	8.4	58.4	8.4	-1	2.1	0	-8.4	108.7
30	42	8.4	101.8	0	0	8	58	8.4	-0.9	0.7	0	-8	107.8
31	48.8	1.9	101.8	0	6.5	1.2	51.2	8.4	-0.8	7.1	0	-1.2	101.8
32	50	8.4	101.8	0	0	0	50	8.4	-0.7	2.4	0	0	127.1
33	50	6.6	126.8	0	1.8	0	50	6.6	126.8	0	1.8	0	126.8
34	50	3.5	129.3	0	4.9	0	50	3.5	129.3	0	4.9	0	129.3
35	50	4.1	173.2	0	4.3	0	50	4.1	173.2	0	4.3	0	173.2
36	50	5.6	195.8	0	2.8	0	50	5.6	195.8	0	2.8	0	195.8
37	48.2	6.4	207	0	2	1.8	48.2	6.4	207	0	2	1.8	207.0
38	42.5	8.4	207	0	0	7.5	42.5	8.4	207	0	0	7.5	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week) xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

Table 2.2: New testing for scenario 19 after correction

From weeks 33 to 37, the water value in week i equals the price in week i. The owners are keeping as much water as possible for week 37 and 38 so their levels are 50 Mm3. The amount above it is drawn every week to avoid overflowing. The value of a marginal Mm3 is then the price in week i since its only utilization is production the week in question.

Owner A gets overflow from week 28 to 32 so his water values are negative thanks to the spillage penalty. It could have been partially avoided for week 31 by the use of S's remaining production capacity. However, it is not interesting for S to produce since he keeps his water when higher prices are foreseen. Finally, when each owner is limited to his maximum production, the solution is optimal from an individual point of view but not for the system as a whole since some spillage could have been avoided. Allowing A to use S capacity would reduce the spillage for week 31 by 6.5 Mm3 without changing any other. If the system rules do not allow the user to use directly the capacity left by the other one, they could have concluded a deal based on A's willingness to pay to borrow S's capacity.

Scenario 19 was particularly relevant for the building of the approach thus its results were presented in the report. More commented results can be found in Appendix E. The tests were not only carried out for more inflow scenarios but also for another period of the year and various ownerships and initial reservoir levels. They show that the approach works well giving different situations and that the results converge quickly -in maximum 3 iterations - which is very important.

2.5 Sensitivity Analyses

To study the impact of a small change on a parameter is a quick alternative to find out mistakes providing that one has an intuitive idea of the results. Decreasing one owner starting level should for example increases his water value. The run file is updated to carry out such type of analysis.

2.5.1 Add in commands

A sensibility analysis can be done for several parameters, the technique is given for the analysis of the impact of different starting levels but can be brought easily into general use.

The first week water value is the one to be used for short term planning. The seasonal model is run every week to take into account new data and the water values are updated. The short term optimizers use the first week water value, from the seasonal optimization, for the generation planning of the week. As a consequence, focus is on the impact of the starting levels on the first week water values.
The code to perform the analysis is given in Listing 2.5.

```
for {l in LEVELS}
{
  reset data Xs0,Xa0; # erase previous starting levels
  let Xs0:=l; # Xs0 varies over the set LEVELS from 0 to 30
  let Xa0:=X0-Xs0; # Xa0 is changed according to Xs0, the total reservoir
      X0 remains constant

  problem solving until convergence;

  let wvS[l]:= S_balance[1]; # the dual value of S_balance equation, week 1
       is assigned to wvS[l]
  let wvA[l]:= A_balance[1]; # the dual value of A_balance equation, week 1
       is assigned to wvA[l]

}
display wvS,WvA;
```

Listing 2.5: Iterating over starting levels

The next step is to test it for detecting possible programming or modeling errors.

2.5.2 Varying initial owner levels for inflow scenarios 8 and 58

For a better understanding of the water values after the sensitivity analysis, a simple test is run. The system is set symmetrical for the two owners, they start with the same reservoir level and each of them owns half of the system.

Two inflow scenarios are chosen, number 8 where a small amount of spillage is expected due to high inflow and number 58 that should result in high water values thanks to its distribution and low inflow amount. The results are shown respectively in Table 2.3 and 2.4 for owner S but are similar for A.

For scenario 8, spillage occurs in weeks 30 and 31. The inflow during the period is really high so the owners produce at maximum capacity every week except in week 33 and 34 where lower prices are forecast.
On the contrary, there is so little inflow for scenario 58 that the owners can really use their water to produce at the best prices. It gives a high water value for weeks 21 to 34 corresponding to the price in week 28.

As the understanding of the results is at stake, the sensitivity analysis is run for the simplest case - the owners' discharges are given by their ownerships.

Both owner productions are limited by their share of the system which is 50%. Owner S starts with 0 Mm3 of water in his reservoir and it increases up to 30 by steps of 2 Mm3. Correspondingly, owner A's level goes from 30 to 0 Mm3. The variation of the water values as a function of the starting level is plotted for both scenarios. The water

Week	xs	us	Wv S	vs	d12r1	x12r1	P
20	15						
21	12.5	8.4	-0.9	0	0	37.5	172.3
22	13.2	8.4	-0.9	0	0	36.8	177.9
23	21.1	8.4	-0.9	0	0	28.9	168.3
24	29.9	8.4	-0.9	0	0	20.1	194.6
25	34.3	8.4	-0.9	0	0	15.7	204.6
26	36.1	8.4	-0.9	0	0	13.9	209.0
27	37.9	8.4	-0.9	0	0	12.1	180.1
28	44.7	8.4	-0.9	0	0	5.3	165.9
29	49.2	8.4	-0.9	0	0	0.8	108.7
30	50	8.4	-0.9	5.3	0	0	107.8
31	50	8.4	-0.8	2.9	0	0	101.8
32	46.9	8.4	126.8	0	0	3.1	127.1
33	50	1.6	126.8	0	6.8	0	126.8
34	49.4	4.6	129.3	0	3.8	0.6	129.3
35	49.7	8.4	129.3	0	0	0.3	173.2
36	46.9	8.4	129.3	0	0	3.1	195.8
37	43.1	8.4	129.3	0	0	6.9	207.0
38	42.5	8.4	129.3	0	0	7.5	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week) xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

Table 2.3: Results for scenario 8 with a symmetrical system for the two owners

Week	xs	us	Wv S	vs	d12r1	x12r1	P
20	15						
21	11.5	8.4	165.9	0	0	38.5	172.3
22	24.8	8.4	165.9	0	0	25.2	177.9
23	26.7	8.4	165.9	0	0	23.3	168.3
24	28.5	8.4	165.9	0	0	21.5	194.6
25	27.4	8.4	165.9	0	0	22.6	204.6
26	24.9	8.4	165.9	0	0	25.1	209.0
27	22.7	8.4	165.9	0	0	27.3	180.1
28	22.7	5.1	165.9	0	3.3	27.3	165.9
29	26.5	0	165.9	0	8.4	23.5	108.7
30	31.8	0	165.9	0	8.4	18.2	107.8
31	39.9	0	165.9	0	8.4	10.1	101.8
32	42.3	0	165.9	0	8.4	7.7	127.1
33	45.9	0	165.9	0	8.4	4.1	126.8
34	50	0	165.9	0	8.4	0	129.3
35	50	2.1	173.2	0	6.3	0	173.2
36	50	1.6	195.8	0	6.8	0	195.8
37	47.5	3.6	207	0	4.8	2.5	207.0
38	42.5	8.4	207	0	0	7.5	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week) xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

Table 2.4: Results for scenario 58 with a symmetrical system for the two owners

value curves should be symmetrical for both owners since their initial levels vary in the opposite direction.

In Figures 2.4 and 2.5, one can see that the water values go down when the initial reservoir level increases. The more water, the lower expected price for an extra Mm3. The maximum production is reached for a larger number of weeks with best prices so, there is less opportunity to produce during them. It is important to note that the water values are given for some reservoir points and that a linear interpolation cannot be used to find out the water values in between. The line is just drawn for a better understanding of the shape of the curve.

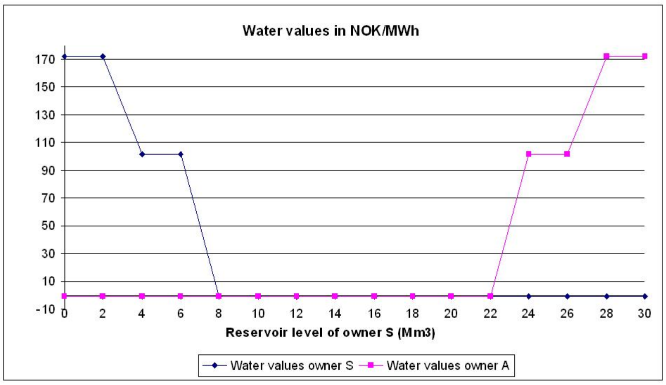

Figure 2.4: Water values as a function of owner S's initial reservoir for scenario 8

Spillage occurs for scenario 8 when owner S has a level around 8 Mm3. The water value is really high when the reservoir is at 0 or 2 corresponding to the high price in week 21. For 4 and 6 Mm3 the water value is the price for week 31: 101.8 NOK/MWh which means that the entire discharge capacity is used from week 21 to 30.

The situation is quite different for scenario 58. S's water values are high and fall down from 172.3 NOK/MWh (price in week 21) up to 127.1 NOK/MWh (price in week 32). From points 10 to 18 Mm3, the water values level off at 165.9 - it shows that the production capacity was not used at all for week 28 for an initial level of 8 Mm3. Then, the reservoir level goes up and as soon as the increase is higher than 8.4 Mm3 for point 20 for instance, the water value changes to the best price left, 129.3 NOK/MWh.

The testing is extended to ownerships. It is quite interesting to see what happens when the ownership is raised since it not only increases the production and reservoir capacity but also the inflow amount coming to one owner's virtual reservoir.

2.5.3 Varying ownerships for inflow scenario 58

When changing a little a parameter, it is important that the other ones remain constant. The initial levels for both owners are fixed and set up equal to 15 Mm3.

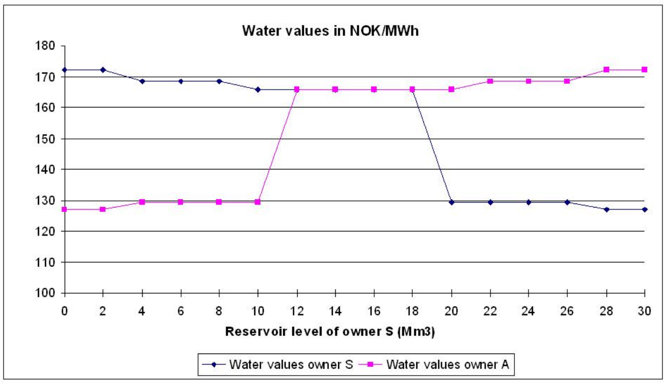

Figure 2.5: Water values as a function of owner S's initial reservoir for scenario 58

Owner S's ownership is increased from 0.1 to 0.9 by steps of 0.1 while A's decreases from 0.9 to 0.1. The results for scenario 8 are not very relevant since water is spilled whatever the ownership. On the opposite the results from scenario 58 are quite interesting and shown in figure 2.6.

The larger the share of the system, the higher the water values. Having an ownership above 50% allows a better strategy for the use of the water for this scenario - the increase in capacities compensates the inflow increase. When owner S's share equals 0.1 spillage occurs while owner A gets high water value since he has enough production capacity to produce whenever he likes.

To conclude, the approach works well under the testing conditions and the transfer of capacities between the two owners is done as expected. The final water value calculations can be carried out.

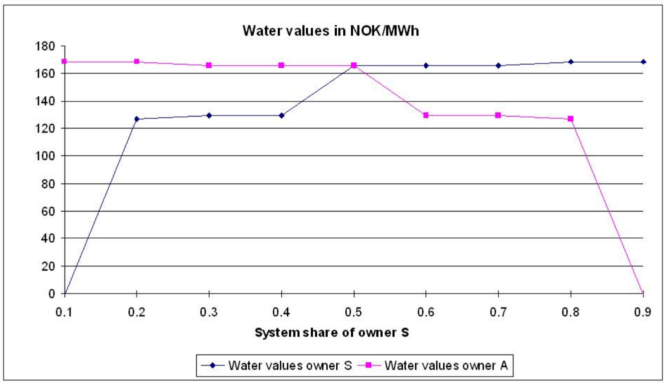

Figure 2.6: Water values as a function of owner S's ownership for scenario 58

2.6 Water value calculations and final testing

Although the seasonal optimization is run for long time horizons, the model used is deterministic and therefore does not take into account uncertainties. To get as realistic results as possible, the problem is solved for up to 70 scenarios given by hydro records. The final water values are the average of the water values for these 70 scenarios.
After creating the 70 inflow files with the excel macro, the approach is tested for all the scenarios and the water values are calculated after iterating over them.

The previous sensitivity analysis are achieved to know how the final water values look like. Nevertheless, their interpretation is slightly harder than in the previous section as it is an average over 70 scenarios. Owner S is allowed to use owner A's discharge capacity for this testing.

2.6.1 Final water values as a function of initial levels

Filling season

The testing period is the same as before (weeks 20-38) as well as the prices and spillage penalties. The physical reservoir level starts at 30 and reaches 85 Mm3 in week 38. Both owners have 50% of the system and the reservoir of owner S varies from 2 to 28 Mm3 by steps of 2 Mm3. On the contrary, the level for A decreases from 28 up to 2 Mm3.

The curves on Figure 2.7 are not symmetrical since owner S can use A's remaining discharge while A cannot. This does not make any difference when the reservoir of the owner is below 15 Mm3 - the water values decrease as the reservoir level increases.
If the owner's reservoir is above 15 Mm3 the water value strongly depends on the possibility of using the other owner's discharge capacity. If not, the water value falls down to 86 NOK/MWh which is much lower than the lowest price of the period: 101.8 NOK/MWh. It suggests that overflow happens for some scenarios. Owner S uses the other owner capacity and therefore prevents his water value from decreasing deeply. His water value levels off at around 105 NOK/MWh while his reservoir level goes from 16 to 28 Mm3.

Depletion season

During the winter period, the hydrology situation and the consumption are slightly different. There is almost no inflow catchment in the reservoir since a large part of the inflow comes as snow. The energy consumption is moreover much more important due to low temperatures. The prices are consequently much higher.

The testing period goes from week 46 year i to week 16 year i+1. The energy prices are updated with data from Nordpool's website for year 2007 and 2008. The physical

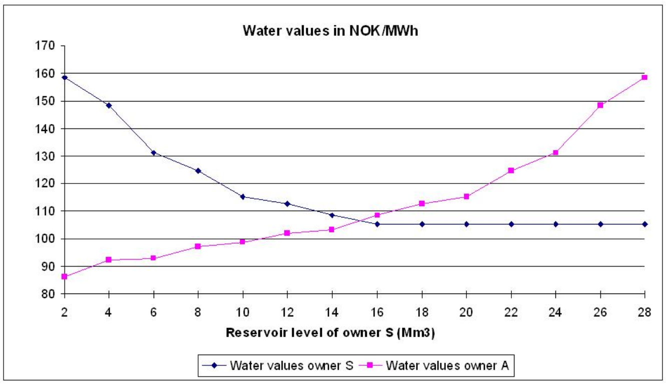

Figure 2.7: Water values as a function of owner S's initial reservoir for the filling season

reservoir level starts at 80 and ends up at 20 Mm3 week 16. Both owners have 50% of the system and the reservoir of owner S varies from 10 to 70 Mm3 by step of 10 Mm3. As a result, A's initial reservoir drops from 70 to 10 Mm3.

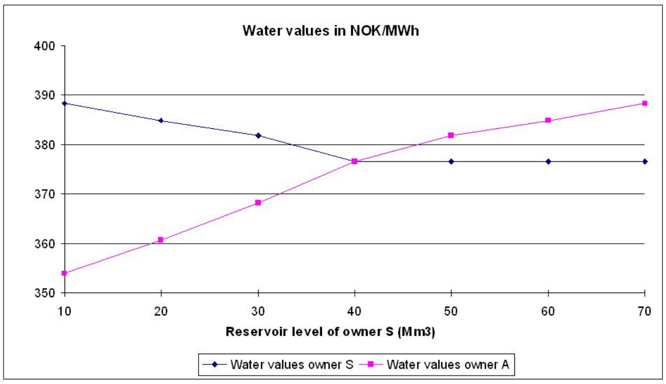

Figure 2.8: Water values as a function of owner S's initial reservoir for the depletion season

The tendency is the same as the one observed in the filling season with high water values coming from huge weekly prices, see Figure 2.8.

2.6.2 Final water values as a function of ownerships

Filling season

The starting reservoir levels are now fixed at 15 Mm3 for each owner and owner S's ownership goes from 0.1 to 0.9 by steps of 0.1. The one for A goes the other way around.

The shape of the curve in Figure 2.9 for A looks like the one pointed out for both owners in the testing for scenario 58. If the owner is bounded by his production capacity, the water values decrease with the ownership. With a low ownership, the owner looses freedom to schedule his production since he gets a little discharge capacity and few space in the reservoir. He does not benefit so much from A's remaining reservoir capacity since this owner gets a lot of inflow and stores it to produce at the highest prices. When the share decreases from 0.4 to 0.1, the number of scenarios where spillage takes place increases and the value of the water drops.

When one owner is able to use the capacity left by the other one, here it is the case for owner S, the water values stabilize for the shares below 50%. Owner S takes advantage of A's remaining capacity.

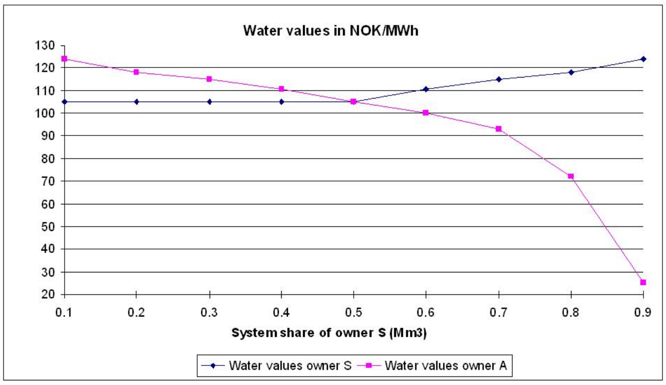

Figure 2.9: Final water values as a function of owner S's share for the filling season

Depletion season

The testing is finally done within the depletion season. Initial levels are set to 40 Mm3 for each owner.

The previous explanations are still valid for Figure 2.10. Nevertheless, the graph (that has been cut for scaling reasons) shows a strange decrease when owner A reaches a share of 10%. The water value is -0.1 which means that spillage occurs within the season. It is rather unlikely for this spillage to be realistic since the testing is done for the winter period and for a physical final reservoir much lower than the initial one. If A owns 10% of the system and cannot use owner S's production capacity, his maximum discharge is 23 weeks times 1.7 Mm3/week. He can only empty out his reservoir of 39.1 Mm3. He starts with 40 Mm3 in his reservoir and must reach 2 Mm3 while he is receiving 10% of the inflow during the period. The only way the model finds to meet this requirement is to spill the water above 2 Mm3 in week 16. The spillage is completly unrealistic but the model only says that overflow reduces the profits and not that it should only happen when the physical reservoir is higher than Xt_{max}. Instead of leading to an unfeasible problem, the problem is solved but the solution is unrealistic from a physical point of view.

Even if this could seem weird, the approach works fine. The parameters used were not very relevant - how can owner A ever reach an initial level of 40 Mm3 if he only gets 10% of the inflow and most of all why A does not store more water for the best prices period of the year- and the testing figured it out.

In this case, water is spilled since the reservoir level decreases during the period and the production capacity is not enough to meet the final reservoir requirement. On the contrary, if the level globally increases and there is too little inflow to reach the final level, the problem returns infeasible. This phenomenon occurs for some set-ups in scenario 29. It could not be corrected by allowing the reservoir to be at a lower final level paying a penalty or by "buying" virtual inflow to cover the missing amount of water at a very high cost. Finally, the tests of the approach, carried out for one reservoir, were conclusive.

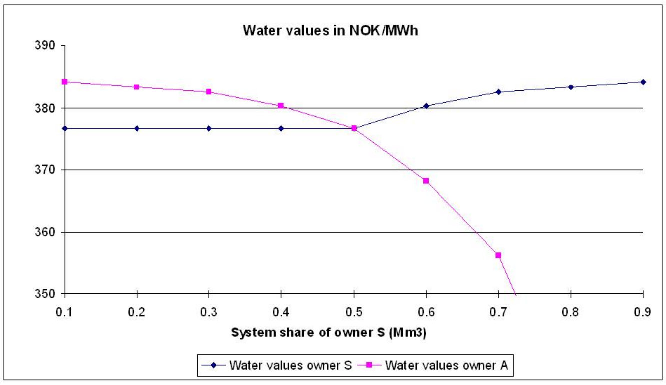

Figure 2.10: Final water values as a function of owner S's share for the depletion season

The approach has been proven to give correct results for various changing in parameters and for 70 inflow scenarios. All the results that have been shown so far were achieved in maximum three iterations. Further testing should be done to see if the results still converge for larger systems and to point out modeling mistakes that have not been discovered yet, if any.

Chapter 3

Testing for two cascaded reservoirs

In Norway, the electricity is mainly produced by hydro power plants within complex river systems. Several reservoirs and plants can follow one another or be in parallel in a watercourse. The difference between the reservoir sizes, the installed capacities, the collected inflows as well as the coupling between the reservoirs - topography of the watercourse, have to be taken into account to find out the best handling of the system.

The tests of the iterative approach were conclusive for one reservoir and were achieved with a limited number of iterations. The analysis is now carried out step by step for more and more complex hydro systems to find out if the approach is still robust or not for such configurations.

3.1 Description of the system

First, the system only consists of two reservoirs. The two reservoirs are a part of Sira-Kvina, a Norwegian river system situated in south-western Norway between Stavanger and Kristiansund, see Figure 3.1. This system is owned by four different compagnies: Lyse Produksjon AS (41,1 %), Statkraft (32,1 %), Skagerak Kraft AS (14,6 %) and Agder Energi Produksjon AS (12,2 %). As Statkraft owns a large part of the two latter compagnies, one can consider for the testing that its share of the system is 59%.
The two reservoirs are isolated from the rest of Sira-Kvina water course.

The first reservoir is a multi-year storage reservoir of 684.1 Mm3 called Roskrepp. Its power plant is quite modest with an installed capacity of 50 MW. The other reservoir downstream, Kvinen, is much smaller with a storage volume of 104.1 Mm3 but has an installed capacity of 80 MW. A sketch of the system as well as useful information are provided in Figure 3.2. The discharged, bypassed, or spilled water from Roskrepp goes directly into Kvinen reservoir without any delay. A kWh can be drawn for Roskrepp and Kvinen within the same week.

Figure 3.1: Sira-Kvina location

The energy equivalent in kWh/m3 is given by 3.1,[2, p.65]:

$$e = \frac{P}{Q} \times \frac{1000}{3600} \tag{3.1}$$

where:
P is the installed capacity in MW
Q is the discharge capacity in m3/s

The discharges need to be in Mm3/week to be coherent with the reservoir balance equation. The values in Figure 3.2 in m3/s are just multiplied by $10^{-6} \times 3600 \times 24 \times 7$ which respectively gives a discharge of 41.7 and 46.6 Mm3/week for Roskrepp and Kvinen.

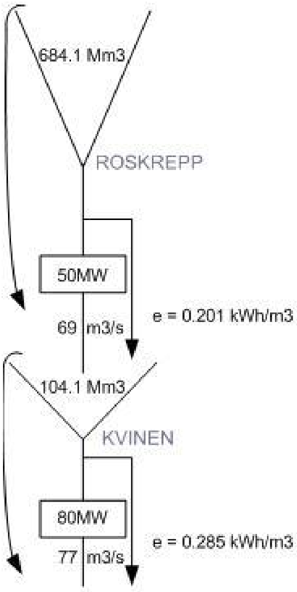

Figure 3.2: Roskrepp and Kvinen reservoirs

As these reservoirs are very different in terms of size and inflow collection, an interesting indicator to compare them is the degree of regulation α given by equation 3.2, [2, p.193].

$$\alpha(i) = \frac{R_{max}(i)}{Q_{reg-calc}(i)} \tag{3.2}$$

with

$$Q_{reg-calc}(i) = Q_{res}(i)+Q_{plant}(upstream)+Q_{bypass}(upstream)+Q_{pump}(i)+Q_{transit}(upstream) \tag{3.3}$$

and

$$Q_{transit}(i) = \frac{\alpha_{lim} - \frac{R_{max}(i)}{Q_{reg-calc}(i)}}{\alpha_{lim}} \times Q_{reg-calc}(i) \tag{3.4}$$

where:
$Q_{res}(i)$ = inflow from local catchment to reservoir i
$Q_{plant}(i)$ = non storable inflow to module i
$Q_{bypass}(i)$ = bypass from module i due to minimum bypass constraints
$Q_{pump}(i)$ = water pumped to module i
α_{lim} = usually taken as 0.45

In cascaded river systems, the calculation of $\alpha(i)$ is slightly complex since its strongly depends on what happens upstream. This is taken into account through $Q_{transit}$ calculation.
The calculation for Roskrepp is really simple since there is no power plant upstream.

$$\alpha(ROS) = \frac{R_{max}(ROS)}{Q_{reg-calc}(ROS)} = \frac{684.1}{554.5} = 1.23$$

For Kvinen, the non storable inflow coming from Roskrepp should be taken into account, but as Roskrepp reservoir is huge, it allows to store much water and $Q_{transit}(ROS)$ can be set to 0.

$$\alpha(KVI) = \frac{104.1}{231.5} = 0.45$$

Kvinen is full in less than 6 months of average inflow, while more than one year of mean inflow is needed to fill in Roskrepp. The strategy for the utilization of the water is consequently quite different for these two reservoirs. Furthermore, the water in the upper reservoir can be discharged twice, in both plants, and this has an impact on its value.

3.2 Water value calculations in cascaded hydro systems

After running the approach, the water values calculated are given in NOK/Mm3 for each reservoir. As mentioned before, if there are several reservoir within the hydro system, a cubic meter of water in the upper reservoirs can be drawn in all the plants situated downstream. As a result, such cubic meter has a higher value. To compare them with weekly prices, it is interesting to have water values in NOK/MWh. The conversion in NOK/MWh is harder than in the previous chapters and requires the introduction of two concepts: water value referred to sea level, known as global water value, and the local one.

The former takes into account the fact that the water can be used several times whereas the latter just gives the water value with respect to what happens locally at the reservoir: inflow, discharge capacity, reservoir level and so on.

If one looks at two cascaded reservoirs: 1 is the upper one and 2 the lower one. The value of the water in reservoir i in NOK/Mm3 is called Wvi and the energy equivalent of plant i ei.

The global and local water values for reservoir 2 in NOK/MWh are equal and shown in equation 3.5.

$$Wv2_{global}(NOK/MWh) = Wv2_{local}(NOK/MWh) = \frac{Wv2(NOK/Mm3)}{e2} \tag{3.5}$$

For reservoir 1, at the top, the water values in NOK/MWh are given by Equation 3.6:

$$\begin{cases} Wv1_{global}(NOK/MWh) = \dfrac{Wv1(NOK/Mm3)}{e1+e2} \\ \\ Wv1_{local}(NOK/MWh) = \dfrac{Wv1(NOK/Mm3) - Wv2(NOK/Mm3)}{e1} \end{cases} \tag{3.6}$$

Knowing this, the testing is carried out to achieve the water values for the owners of the two reservoirs.

3.3 Testing conditions

The testing is performed for the filling season between week 20 and 38. Reasonable parameters need to be estimated for the initial and final level of the reservoirs.

3.3.1 Initial reservoir levels

At the beginning of the filling season, the reservoirs are really low since a high inflow is expected for the next weeks. Starting the period with 30% of energy in the system seems to be realistic enough to carry out the analysis.

A good way to prevent water to be spilled during the filling season is to maintain equal relative dampings, D, for all reservoirs. Therefore, to determine an initial level for the reservoirs, $R_{target}(i)$, the following system of equation is solved, [2, p.122]:

$$\begin{cases} D = \frac{R_{max}(i) - R_{target}(i)}{R_{max}(i)} \times \alpha(i) \;\; equal\ for\ every\ reservoir\ i \\ \\ \sum R_{target}(i) \times e_{total}(i) = R_{esum} \end{cases} \tag{3.7}$$

where
R_{max} = maximum reservoir level for reservoir i
R_{target} = desired reservoir level for reservoir i
$\alpha(i)$ = degree of regulation of the reservoir i
$e_{total}(i)$ = total energy equivalent of the reservoir referred sea level for reservoir i
R_{esum} = desired sum energy in the total system at the aggregate level

$e_{total}(KVI)$ = is 0.285 kWh/m3 as mentioned earlier.
The water discharged from Roskrepp can be used in Kvinen afterwards so, its total energy equivalent is $e_{total}(ROS) = (0.201 + 0.285) = 0.486$ kWh/m3.
R_{esum} is $30\% \times (R_{max}(ROS) \times e_{total}(ROS) + R_{max}(KVI) \times e_{total}(KVI)) = 108.6$GWh.

From the system 3.7, the formulas for $R_{target}(ROS)$ and $R_{target}(KVI)$ are given by equations 3.8:

$$\begin{cases} R_{target}(KVI) = \dfrac{R_{esum} - R_{target}(ROS) \times e_{total}(ROS)}{e_{total}(KVI)} \\ \\ R_{target}(ROS) = \dfrac{\alpha(ROS) - \alpha(KVI) + \dfrac{\alpha(KVI) \times R_{esum}}{R_{max}(KVI) \times e_{total}(KVI)}}{\dfrac{\alpha(ROS)}{R_{max}(ROS)} + \dfrac{\alpha(KVI) \times e_{total}(ROS)}{R_{max}(KVI) \times e_{total}(KVI)}} \end{cases} \tag{3.8}$$

With $R_{esum} = 108.6$ GWh, $R_{target}(ROS) = 264.8$ Mm3 while $R_{target}(KVI)$ should be taken as -70.3 Mm3. This negative value is due to the large difference between the reservoir sizes and their degrees of regulation, the higher damping that can be achieved for Kvinen is 0.45. It means that the reservoir should be empty at the beginning of the period.

3.3.2 Final reservoir levels

Until now, the same final reservoir level requirements were used for all the inflow scenarios. This is not very relevant, the final level should be fixed according to the amount of expected inflow. If, this level is set too high compared to the inflow, there is almost no production. On the contrary, if the level is set too low and there is a huge amount of inflow that goes to the reservoir, unrealistic spillage occurs to get rid of the water above the desired final level.

The two extreme scenarios for the considered reservoirs took place in 1990 and 1996, with respectively a mean annual inflow of 863 Mm3 and 322 Mm3 in Roskrepp reservoir. The total desired energy in the system at the end of the planning period will be set at 90% for the first scenario, a very wet year, to have room left for the inflow that could come later and to cover a part of the energy consumption of the period. For the second one, a very dry year, the total energy is taken as 40% to get at least some energy production. Detailed simulations were made for these scenarios, as the results were not particularly exciting, they are moved in Appendix F. For the inflow scenarios in between, a linear interpolation will be used to determine the final energy levels in the system. Even though different price scenarios should be used with respect to each scenario, a dry year results for instance in high electricity prices, the prices from Nordpool for year 2007 and 2008 will be kept for an easier explanation of the results.

3.4 Testing for scenario 41

Scenario 41 corresponds to year 1970 and is a scenario with normal inflow - 465.3 Mm3 of water coming into Roskrepp reservoir. The desired final amount of energy in the system is set arbitrarily to 63%, for this test. The corresponding reservoir levels are calculated according to equation 3.7, and both owners have a final reservoir level proportional to their share of the sytem.

On the contrary, the initial stored energy in the system is 30%, but the owner starting levels are unbalanced compared to their ownerships. Statkraft's, company S, share of the system is 59% but it owns 70% of the water within Roskrepp reservoir at the beginning of the period though. The parameters and the limits for each owner are given in Table 3.1. With respect to production, both owners can use the discharge capacity given by their share.

The results are given in Tables 3.2 and 3.3. They were computed after two iterations of the approach. The prices times the energy equivalent referred to sea level - 0.486 kWh/m3 (0.201+0.285) for Roskrepp and 0.285 kWh/m3 for Kvinen, are calculated in the last column of the result tables for the interpretation of the results.

		Roskrepp			
	Ownership	Xt_{max}	X_{ini}	X_{final}	Ut_{max}
Statkraft S	59 %	403.6	185.4	272.9	24.6
Lyse A	41 %	280.5	79.4	189.6	17.1
Total	100 %	684.1	264.8	462.5	41.7
		Kvinen			
	Ownership	Xt_{max}	X_{ini}	X_{final}	Ut_{max}
Statkraft S	59 %	61.4	0	7.0	27.5
Lyse A	41 %	42.7	0	4.9	19.1
Total	100 %	104.1	0	11.9	46.6

x- reservoir level (Mm3/week) u- discharge (Mm3/week)

Table 3.1: Testing parameters for two cascaded reservoirs, scenario 41

3.4.1 First interpretation of the results

Owner S produces much more energy than owner A through the period, since it has an higher initial level, which explains why its water values are lower. His water value for Kvinen is 55.5 NOK/Mm3 since his best opportunity to produce an extra MWh is in week 24 for 194.6 NOK/MWh while it is 59 NOK/Mm3 for owner A that can draw it in week 37.

Globally, more water is stored in Roskrepp due to its better degree of regulation. The production of owner A is almost nonexistent from this reservoir since he would not have enough water to cover the final reservoir requirement otherwise. Looking at the last column of the Table 3.3, one sees that the highest value of owner A's water is not achieved when water is taken out from the two reservoirs within the same week. Even though, one extra kWh could have been produced in week 38 from Roskrepp, the model takes into account that it cannot be used straight afterwards by Kvinen power station since it is already producing at maximum capacity. So, the water does not take the value 102.5 NOK/Mm3 for the period and should be a bit below it.
The model is deterministic, as explained in Chapter 1. If owner A knows for sure he is going to receive some water from Roskrepp in week 38, he would use some water he has in his reservoir to produce an extra kWh in an earlier week. In this case, he has unused production capacity in week 37. This gives a water value of $0.201 \times 211 + 0.285 \times 207 =$ 101.4 NOK/Mm3.

Knowing, he will receive more water in week 25 from Roskrepp, owner S's best shot is to draw an extra kWh from Kvinen reservoir, which is not empty, in week 24. As a result, his water value is $0.201 \times 204.6 + 0.285 \times 194.6 = 96.6$ NOK/Mm3.

Kvinen												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	0					0						
21	6.9	0	55.5	27.5	54.5	4.8	0	59	19.1	37.9	172.3	49.1
22	16	0	55.5	27.5	45.4	11.1	0	59	19.1	31.6	177.9	50.7
23	24.7	0	55.5	27.5	36.7	17.2	0	59	19.1	25.5	168.3	48.0
24	24.4	7	55.5	20.5	37.1	21.8	0	59	19.1	20.9	194.6	55.5
25	9.1	27.5	55.5	0	52.4	25.1	0	59	19.1	17.6	204.6	58.3
26	10.6	27.5	55.5	0	50.8	9.1	19.1	59	0	33.6	209.0	59.6
27	15.4	0	55.5	27.5	46.1	12.4	0	59	19.1	30.3	180.1	51.3
28	21.1	0	55.5	27.5	40.3	16.4	0	59	19.1	26.3	165.9	47.3
29	23.6	0	55.5	27.5	37.8	18.1	0	59	19.1	24.6	108.7	31.0
30	26.2	0	55.5	27.5	35.2	19.9	0	59	19.1	22.8	107.8	30.7
31	27.6	0	55.5	27.5	33.8	20.9	0	59	19.1	21.8	101.8	29.0
32	28.4	0	55.5	27.5	33	21.4	0	59	19.1	21.2	127.1	36.2
33	29.2	0	55.5	27.5	32.2	22	0	59	19.1	20.7	126.8	36.1
34	30.5	0	55.5	27.5	30.9	22.9	0	59	19.1	19.8	129.3	36.9
35	31.9	0	55.5	27.5	29.5	23.9	0	59	19.1	18.8	173.2	49.4
36	7.3	27.5	55.5	0	54.1	25.9	0	59	19.1	16.8	195.8	55.8
37	8.2	27.5	55.5	0	53.3	15.7	12.7	59	6.4	26.9	207.0	59.0
38	7.0	27.5	55.5	0	54.4	4.9	19.1	59	0	37.8	211.0	60.1

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week) xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P- NOK/Mm3

Table 3.2: Detailed results for Kvinen, scenario 41

Roskrepp												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	185.4					79.4						
21	202.1	0	96.6	24.6	201.5	91	0	101.4	17.1	189.5	172.3	83.7
22	223.8	0	96.6	24.6	179.8	106.1	0	101.4	17.1	174.4	177.9	86.5
23	244.8	0	96.6	24.6	158.9	120.6	0	101.4	17.1	159.8	168.3	81.8
24	260.8	0	96.6	24.6	142.9	131.8	0	101.4	17.1	148.7	194.6	94.6
25	264.7	7.4	96.6	17.2	138.9	139.7	0	101.4	17.1	140.8	204.6	99.4
26	250.8	24.6	96.6	0	152.8	147.1	0	101.4	17.1	133.4	209.0	101.6
27	262.2	0	96.6	24.6	141.4	155	0	101.4	17.1	125.5	180.1	87.5
28	276	0	96.6	24.6	127.6	164.6	0	101.4	17.1	115.9	165.9	80.6
29	282	0	96.6	24.6	121.6	168.8	0	101.4	17.1	111.7	108.7	52.8
30	288.2	0	96.6	24.6	115.4	173.1	0	101.4	17.1	107.3	107.8	52.4
31	291.6	0	96.6	24.6	112	175.5	0	101.4	17.1	105	101.8	49.5
32	293.5	0	96.6	24.6	110.1	176.8	0	101.4	17.1	103.7	127.1	61.8
33	295.5	0	96.6	24.6	108.1	178.2	0	101.4	17.1	102.3	126.8	61.6
34	298.5	0	96.6	24.6	105.1	180.3	0	101.4	17.1	100.2	129.3	62.8
35	301.9	0	96.6	24.6	101.7	182.6	0	101.4	17.1	97.9	173.2	84.2
36	308.9	0	96.6	24.6	94.7	187.5	0	101.4	17.1	93	195.8	95.2
37	293.3	24.6	96.6	0	110.3	193.7	0	101.4	17.1	86.7	207.0	100.6
38	272.9	24.6	96.6	0	130.7	189.6	7	101.4	10.1	90.9	211.0	102.5

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week) xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P- NOK/Mm3

Table 3.3: Detailed results for Roskrepp, scenario 41

3.4.2 Water value meaning in terms of energy

Using formulas 3.5 and 3.6 in section 3.2, the water values in NOK/MWh are calculated for each owner of the reservoirs.

For the upstream reservoir, Roskrepp, the water values for owner S are:

$$\begin{aligned} WvS_{ROS,global} &= \frac{WvS_{ROS}(NOK/Mm3)}{e(ROS)+e(KVI)} \\ &= \frac{96.6}{0.486} = 198.8\ NOK/MWh \end{aligned}$$

$$\begin{aligned} WvS_{ROS,local} &= \frac{WvS_{ROS}(NOK/Mm3) - WvS_{KVI}(NOK/Mm3)}{e(ROS)} \\ &= \frac{96.6-55.5}{0.201} = 204.5\ NOK/MWh \end{aligned}$$

In the same way, the local water value for owner A is 210.9 NOK/MWh and the global one 208.7 NOK/MWh.

The local water values are above the global ones which reflects the high degree of regulation of Roskrepp reservoir. Indeed, it points out that given the prices, the water value is 208.7 NOK/MWh. However at this particular reservoir it is more valuable or "safe" to store water since the reservoir is huge compared to the inflow it receives. If the reservoir is badly regulated, high prices - meaning that the water is an expensive commodity in the system - can still give a high global value, but the local value remains low due to the risk of spillage at this reservoir.

The water values for Kvinen are respectively 55.5/0.285 = 194.7 NOK/MWh and 59/0.285 = 207 NOK/MWh for S and A.

In this example, owner S was not allowed to use the production capacity left by A but still they could have concluded a deal.

3.4.3 Modification of the system rules

If the handling rules of the system say that S is allowed to use A's remaining production capacity, the situation is different, see the results in Table 3.4. They were achieved after 3 iterations of the approach.

The production, the reservoir level and the water values for A remain unchanged and are not shown here. Owner S's water values are higher than before. $d12r1$ is negative for some weeks which points out that owner S benefits from the discharge capacity unused by A to produce at better prices. More water is discharged from Roskrepp in week 26 and 38. Even if the price is higher in week 37 than in week 25, water is drawn from Roskrepp in week 25 for a production downstream at Kvinen's plant. Owner S also uses what is left by A at the lower plant in week 25 and 37.

Between week 21 and 25, the water value should be 55 NOK/Mm3 at Kvinen since it is possible to produce an extra kWh in week 24. However the reservoir is empty in week 25 to produce at full capacity, so the water value tends to be a bit higher than 55 NOK/Mm3 and reaches 56.7 NOK/Mm3. The water value is then 55.8 NOK/Mm3 up to week 37 and suggests that if an extra kWh should be produced it should happen in week 37. The water value decreases down to 55.4 NOK/Mm3 in week 38. An extra kWh arriving in the reservoir during week 38 cannot be produced within the week or a week before since the reservoir is empty in week 37. The water takes a value closed to the last known one, 55.8 NOK/Mm3, but is lower since no more water can be taken out in week 37.

Even though the hydro producer forecasts that it is still possible to draw some water from Roskrepp in week 38, he cannot take out more water from Kvinen in week 37 since it is empty. The best utilization of the water would be to be produced in week 26 from Roskrepp and then from Kvinen in week 36. This give a water value of $209 \times 0.201 + 195.8 \times 0.285 = 97.8$ NOK/Mm3.

3.5 First week water values for 75 scenarios

To be sure that the approach works fine under different conditions, it is tested for a large number of scenarios. 75 years of inflow records are available for Sira-Kvina water course and the first week water values, used for the physical operation of the system, are calculated.

The initial energy level in the system is 30% (264.8 Mm3 in Roskrepp, 0 Mm3 in Kvinen) while the final one is defined with respect to the inflow ammount received at the reservoirs. A linear interpolation between the driest year (final energy 40%) and the wetest one (final energy 90%) is used to find out the final energy level for each inflow scenario. It gives for instance 55% for scenario 41. Owner S owns 60% of the water in the physical reservoir at the begining of the period, i.e 158.9 Mm3 in Roskrepp, and benefits from the unused production capacity of owner A.

The water values for the 75 inflow scenarios are shown in Figures 3.3 and 3.4 and are achieved in maximum 3 iterations of the approach (3 optimizations for each owner).

The water values are quite stable and level off between 87 and 102 NOK/Mm3 for Roskrepp and 49 and 60 NOK/Mm3 for Kvinen. It means that whatever the scenario considered, about the same amount of energy is produced, there is no scenario that leads to a huge discharge and thus low water values. It shows that the constraint set for the last week reservoir level has about the same weight for all scenarios.

	Roskrepp					Kvinen				
Week	xs	us	Wv S	d12r1	x12r1	xs	us	Wv S	d12r1	x12r1
20	185.4					0				
21	202.1	0	97.8	24.6	201.5	6.9	0	56.7	27.5	54.5
22	223.8	0	97.8	24.6	179.8	16	0	56.7	27.5	45.4
23	244.8	0	97.8	24.6	158.9	24.7	0	56.7	27.5	36.7
24	260.8	0	97.8	24.6	142.9	31.4	0	56.7	27.5	30
25	261.7	10.5	97.8	14.1	141.9	0	46.6	56.7	-19.1	61.4
26	234.4	38	97.8	-13.4	169.2	15	27.5	55.8	0	46.4
27	245.7	0	97.8	24.6	157.9	19.7	0	55.8	27.5	41.7
28	259.6	0	97.8	24.6	144.1	25.5	0	55.8	27.5	36
29	265.5	0	97.8	24.6	138.1	27.9	0	55.8	27.5	33.5
30	271.8	0	97.8	24.6	131.8	30.6	0	55.8	27.5	30.9
31	275.2	0	97.8	24.6	128.5	31.9	0	55.8	27.5	29.5
32	277.1	0	97.8	24.6	126.5	32.8	0	55.8	27.5	28.7
33	279.1	0	97.8	24.6	124.6	33.6	0	55.8	27.5	27.8
34	282.1	0	97.8	24.6	121.5	34.8	0	55.8	27.5	26.6
35	285.4	0	97.8	24.6	118.2	36.2	0	55.8	27.5	25.2
36	292.5	0	97.8	24.6	111.1	30.2	9	55.8	18.5	31.2
37	301.4	0	97.8	24.6	102.2	0	33.9	55.8	-6.4	61.4
38	272.9	32.8	97.8	-8.2	130.7	7.0	27.5	55.4	0	54.4

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week) xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P- NOK/Mm3

Table 3.4: Owner S's results when he can use A's capacity, scenario 41

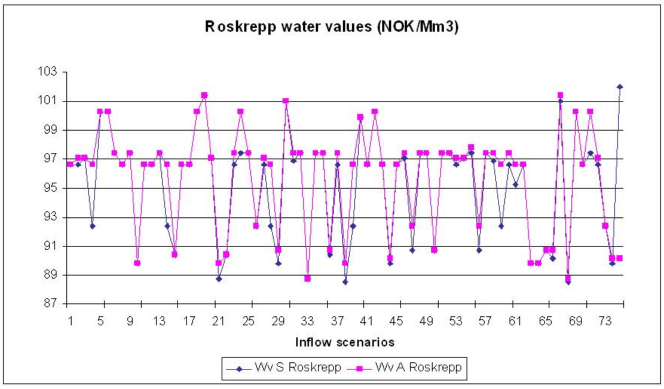

Figure 3.3: Water values as a function of scenarios for Roskrepp reservoir

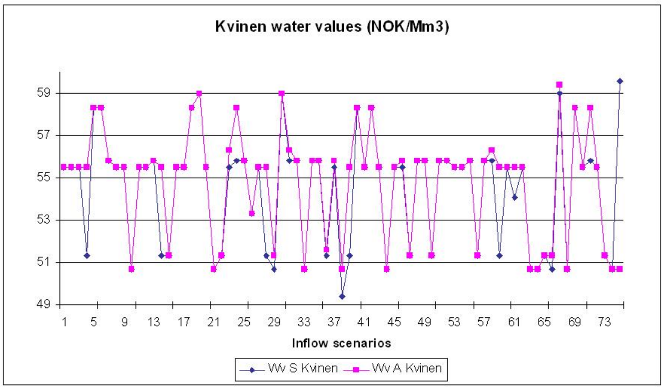

Figure 3.4: Water values as a function of scenarios for Kvinen reservoir

The difference in the water values for owner S and A is not so huge. As owner S has a higher initial level, he has to draw more water from the reservoirs and produces for lower and lower prices. Consequently, his water values should always be below the one for owner A. It is not the case for scenario 75.

In fact, the problem turned out to be unfeasible for scenario 75. Due to the unbalanced reservoirs at the beginning of the period and the inflow amount, owner A cannot reach a level that correspond to 64% of energy times his share at the end of the period. This level is pulled down to 60%, which leads to a solution with a very low energy production for A. The water values for both owners are then equal to 101.4 for the upper reservoir and 59.4 NOK/Mm3 for the lower one. Now, one knows how the water values look like, it is possible to run sensitivity analysis.

3.6 Sensitivity analyses

Looking how small changes in the initial reservoir levels influence the water values is a relevant test to ensure that the approach is robust under different conditions. For each starting level, the 75 scenarios are run and the average over the 75 water values is kept as the final water value for use in the short term optimization. As only one parameter should vary at a time for an understanding of its impact on the results, the first sensitivity analysis is done for changes of the initial level of Roskrepp reservoir.

3.6.1 Variations of individual initial levels at Roskrepp reservoir

To keep the validity of the final reservoir levels, the global initial level of Roskrepp is kept at 264.8 Mm3. Moreover, the initial amount of water owned by the two companies cannot be too much unbalanced to keep the problem solvable. To allow more flexibility on the initial levels for each owner, the previous final reservoir levels are lowered by 15%. The testing conditions do not become unrealistic for that and it strenghthens the validity of the approach in itself since it is tested for many initial levels.

60% of the water within Roskrepp reservoir belongs to owner S at the begining of the period, that is 158.9 Mm3 while owner A owns only 40%, i.e 105.9 Mm3. The level of owner A is increased by step of 5 Mm3 up to 150.9 Mm3 while the one for S is correspondingly reduced from 158.9 down to 113.9 Mm3 to keep a constant physical reservoir level of 264.8 Mm3. Figure 3.5 and 3.6, show the variations of the water values as a function of these changes.

For each starting reservoir, the average number of iteration per scenario is calculated. It was 2 for all the initial reservoir points except for the first one, 105.9 Mm3. In this case, the average number of iteration per scenario was 2.4. The transferred capacities between the two owners needed to balance each other out since S takes advantage of A unused production capacity in this case.

As the level of owner A into Roskrepp reservoir raises by 42%, his water values decrease both at Roskrepp and Kvinen by respectively 7% and 5%. The water values for S vary in the opposite direction and go up by 5% at Roskrepp and 5% at Kvinen as his reservoir level falls down by 28%. Any change on the upper reservoir level has a direct impact on the lower one since the water discharged from Roskrepp goes into Kvinen. The shape of the curves for the two reservoirs looks pretty much the same since finally a change in the upstream reservoir level involves about the same change in the lower one. The final reservoir requirement is still the same, so having more water results in having more production and not in more storage.

Let's now introduce some variations in Kvinen initial level.

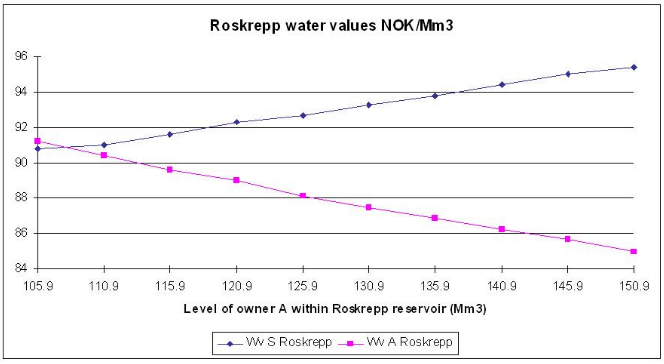

Figure 3.5: Water values for Roskrepp reservoir as a function of owner A's level within Roskrepp reservoir

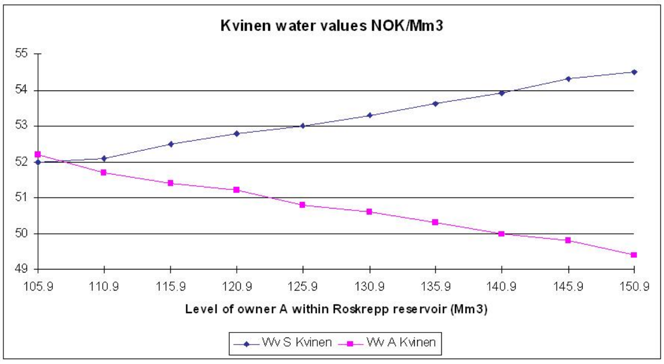

Figure 3.6: Water values for Kvinen reservoir as a function of owner A's level within Roskrepp reservoir

3.6.2 Variations of Kvinen initial level

The initial reservoir level of Kvinen that corresponds to 30% of energy in the hydro system is 0. It is not possible to reduce one owner's initial level while raising the other one. Consequently, the level of owner A within Kvinen reservoir grows from 0 to 100 Mm3 and the one for S is maintained at 0 Mm3 for the analysis. The variation of the water values are presented in Figures 3.7 and 3.8.

The water values for owner S are constant since there is no change in his reservoirs. A 29% drop in the water values for owner A at Kvinen reservoir is registered while his reservoir level increases from 0 to 100 Mm3. The water value for a reservoir of 100 Mm3 reaches 37 NOK/Mm3 which suggests that the plant downstream produces at maximum capacity almost every weeks, even the one with the lowest prices, for a large number of scenarios. Since the value in NOK/Mm3 is still above 29 NOK/Mm3 (the lowest price times e), one can conclude that there is no spillage for owner A even for the highest starting levels. He indeed benefits from owner S unused reservoir capacity.

For Roskrepp, the water value goes down by 11%. It is normal that the cubic meter of water turns to be less valuable at Roskreep since it has to be discharged afterwards from Kvinen and its value decreases there. Because it can still be discharged for higher prices from the upper reservoir, its devaluation is less important, however.

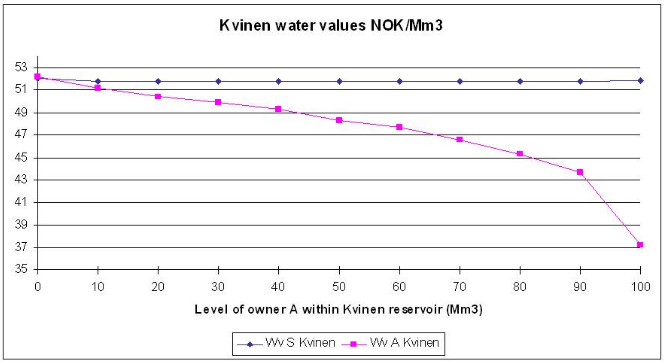

Figure 3.7: Water values for Roskrepp reservoir as a function of owner A's level within Kvinen reservoir

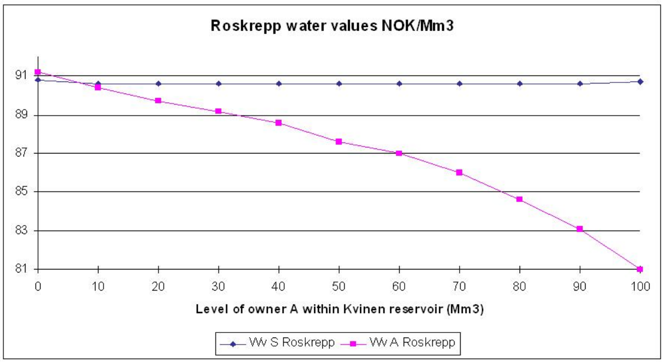

Figure 3.8: Water values for Kvinen reservoir as a function of owner A's level within Kvinen reservoir

The two sensitivity analysis shows that an increase in the reservoir levels results in a decrease in the water values and vice versa which is what was expected.

Regarding the number of iteration required to get a stable solution, it varies from one scenario to another with respect to owner A's reservoir level. It is achieved in 3 iterations or less for all the 75 scenarios and the different reservoir points except for the last one, 100 Mm3. For scenario 38, a stable solution is reached after 8 iterations for this starting level while the approach fluctuates between solutions for scenario 74 and needs 15 iterations to level off.

3.6.3 Convergence discussion for scenario 74

To figure out why the approach does not converge as fast as expected for this scenario and for an initial level of 100 Mm3 in Kvinen, the calculations are stopped and the results are saved after 3, 8 and 15 iterations of the approach. The solutions the model varies in between are really close in terms of profits and presented in Table 3.5.

When the convergence of the results is achieved for 15 iterations, the profits for owner S are a little higher than the intermediate ones while the incomes for owner A decrease.

The strategy for the utilization of the water is slightly different for the intermediate results but the amount of Mm3 discharged from the reservoirs through the period stays constant. For Roskrepp, the amount of water drawn in week 22 by owner S goes down as the number of iteration increases so, the one taken out in week 24 raises. As a consequence, the production for Kvinen is reduced in week 22 and more and more water is drawn in week 35. Furthermore, as owner S uses the capacity left by A in week 24, the handling of A's Roskrepp reservoir is modified in week 24 and 27 which lowered his profits by 0.1%.

The water value for owner S turns out to be higher after 15 iterations of the approach. For 3 and 8 iterations, the water value means that the best use of one Mm3 of water is to be drawn from Roskrepp and Kvinen in week 22 which gives it a value of $(0.201 + 0.285) \times 177.9 = 86.5$ NOK/Mm3. The solution after 15 iterations suggests that more water should be discharged in week 24. It should be noticed that the start up costs of the plant, still generally low for an hydro power plant, are not taken into account and therefore the discharge of small amounts of water is not discouraged for instance in week 22.

A solution is hard to achieve for these testing conditions since the model fluctuates between two solutions that almost lead to the same profits: more production for owner S in week 22 from both Roskrepp and Kvinen or more discharge in week 24 from Roskrepp for a later use downstream in week 35. The later option reduces owner A's incomes since owner S profits from his unused discharge capacity.

Although the strategy expressed for the reservoir handling differs from one iteration to another, the solution are globally similar. The issue is most of all raised because owner S can use the production capacity left by A at Roskrepp power plant in week 24. If, he cannot, a stable solution is reached in 2 iterations of the approach where 24.6 Mm3 of water are discharged by S in week 24 plus 6.6 in week 22 and 0.7 in week 27. Owner S water value consequently goes down to 85.6 NOK/Mm3 at the upper plant. The profits for S are finally 19334.1 NOK while the one for A are 17545.1 NOK. It is interesting to note that in this case, allowing S to take advantage of A unused production capacity turns into a small loss for owner A.

This testing points out that the approach gives correct results for two reservoirs. However, it may need an important number of iteration to meet a stable solution when an owner can use the production capacity left by the other one. As the computation time is a crucial factor for the validation of the approach, a larger hydro system should be modeled to figure out how the number of iteration increases as a function of the system size.

	Roskrepp											
	3 iterations				8 iterations				15 iterations			
Week	us	Wv S	ua	Wv A	us	Wv S	ua	Wv A	us	Wv S	ua	Wv A
20												
21	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
22	6.1	86.5	0	72.4	3.1	86.5	0	72.4	0	88.5	0	72.4
23	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
24	25.8	86.5	15.3	72.4	28.8	86.5	12.3	72.4	31.9	88.5	9.2	72.4
25	24.6	86.5	17.1	72.4	24.6	86.5	17.1	72.4	24.6	88.5	17.1	72.4
26	24.6	86.5	17.1	72.4	24.6	86.5	17.1	72.4	24.6	88.5	17.1	72.4
27	0	86.5	2.3	72.4	0	86.5	5.3	72.4	0	88.5	8.4	72.4
28	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
29	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
30	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
31	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
32	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
33	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
34	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
35	0	86.5	0	72.4	0	86.5	0	72.4	0	88.5	0	72.4
36	24.6	86.5	17.1	72.4	24.6	86.5	17.1	72.4	24.6	88.5	17.1	72.4
37	23.9	86.5	16.6	72.4	23.9	86.5	16.6	72.4	23.9	88.5	16.6	72.4
38	24	86.5	16.7	72.4	24	86.5	16.7	72.4	24	88.5	16.7	72.4
	Kvinen											
	3 iterations				8 iterations				15 iterations			
Week	us	Wv S	ua	Wv A	us	Wv S	ua	Wv A	us	Wv S	ua	Wv A
20												
21	0	50.7	19.1	33.3	0	50.7	19.1	33.3	0	50.7	19.1	33.3
22	27	50.7	19.1	33.3	24	50.7	19.1	33.3	20.9	50.7	19.1	33.3
23	0	49.4	19.1	33.3	0	49.4	19.1	33.3	0	49.4	19.1	33.3
24	27.5	49.4	19.1	33.3	27.5	49.4	19.1	33.3	27.5	49.4	19.1	33.3
25	27.5	49.4	19.1	33.3	27.5	49.4	19.1	33.3	27.5	49.4	19.1	33.3
26	27.5	49.4	19.1	33.3	27.5	49.4	19.1	33.3	27.5	49.4	19.1	33.3
27	27.5	49.4	19.1	36.2	27.5	49.4	19.1	36.2	27.5	49.4	19.1	36.2
28	0	49.4	19.1	36.2	0	49.4	19.1	36.2	0	49.4	19.1	36.2
29	0	49.4	0	36.2	0	49.4	0	36.2	0	49.4	0	36.2
30	0	49.4	0	36.2	0	49.4	0	36.2	0	49.4	0	36.2
31	0	49.4	0	36.2	0	49.4	0	36.2	0	49.4	0	36.2
32	0	49.4	9.9	36.2	0	49.4	9.9	36.2	0	49.4	9.9	36.2
33	0	49.4	0	36.2	0	49.4	0	36.2	0	49.4	0	36.2
34	0	49.4	19.1	36.2	0	49.4	19.1	36.2	0	49.4	19.1	36.2
35	14.8	49.4	19.1	36.2	17.8	49.4	19.1	36.2	20.9	49.4	19.1	36.2
36	27.5	49.4	19.1	36.2	27.5	49.4	19.1	36.2	27.5	49.4	19.1	36.2
37	27.5	44.8	19.1	30.8	27.5	44.8	19.1	30.8	27.5	46.9	19.1	30.8
38	27.5	44.1	19.1	30	27.5	44.1	19.1	30	27.5	46.1	19.1	30
Profits S	19337.2				19343.3				19349.6			
Profits A	17541.6				17532.8				17523.8			

x- reservoir level (Mm3/week) Wv- water value (NOK/Mm3) Profits- total profits for the period (NOK)

Table 3.5: Intermediate results for scenario 74

Chapter 4

Testing for four cascaded reservoirs

In the previous Chapter, two reservoirs of Sira-Kvina water course were considered. This hydro system is in reality much more huge. The focus is now on four of its cascaded reservoirs.

4.1 Description of the system

The two reservoirs situated downstream of Kvinen reservoir are added to the model. Their characteristics are given in Figure 4.1.

The last reservoir, Homstølvann, does not have any power plant, the water that arrives there is just stored or bypassed downstream. A penalty for spillage at this reservoir is still included in the optimization objective to take into account its associated scheduling challenge, stocking water while avoiding overflowing during the period. Nesjen is a rather big reservoir with 274.3 Mm3 of storage but its mean annual inflow is 582.8 Mm3. The discharges are 62.9 Mm3/week for Nesjen and 102.8 Mm3/week for Homstølvann.

The degree of regulation of the two reservoirs are calculated according to equations 3.2, 3.3 and 3.4. What happens upstream strongly influences the handling of this two reservoirs and this should be reflected in their degree of regulation.

$$\alpha(NES) = \frac{R_{max}(NES)}{Q_{reg-calc}(NES)} = \frac{274.3}{582.8 + (231.5 - 104.1) + 0} = 0.38$$

The term $(231.5 - 104.1)$ represents the non-storable inflow coming from Kvinen. Moreover, as the degree of regulation of Kvinen is 0.45, $Q_{transit}(upstream)$ is null.

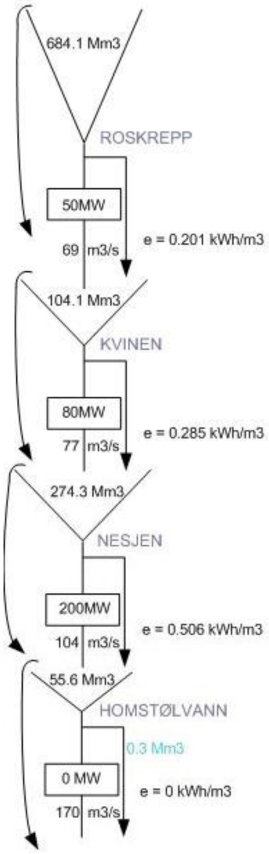

Figure 4.1: Sketch and data for a 4-reservoir-system

$$\alpha(HOM) = \frac{R_{max}(HOM)}{Q_{reg-calc}(HOM)} = \frac{55.6}{349.5 + (582.8 - 274.3) + \frac{(0.45 - 0.38) \times 710.2}{0.45}} = 0.07$$

The last term of the denominator in $\alpha(HOM)$ calculation is $Q_{transit}(upstream)$ which is not null here since the degree of regulation of the upstream reservoir, Nesjen, is 0.38. From this result, one can see that the lower reservoir is very badly regulated and thus water may be spilled from it.

4.2 Testing conditions

The utilization of the damping to find starting and final levels for cascaded reservoirs was explained in section 3.3.1. The same technique is used here and the system of equations

		Roskrepp			Kvinen			Nesjen			Homstølvann		
	O	Xt_{max}	X_{in}	Ut_{max}	Xt_{max}	X_{in}	Ut_{max}	Xt_{max}	X_{in}	Ut_{max}	Xt_{max}	X_{in}	Ut_{max}
S	59	403.6	215.6	24.6	61.4	0.0	27.5	161.8	0.0	37.1	32.8	0.0	60.7
A	41	280.5	149.9	17.1	42.7	0.0	19.1	112.5	0.0	25.8	22.8	0.0	42.1
Tot	100	684.1	365.5	41.7	104.1	0.0	46.6	274.3	0.0	62.9	55.6	0.0	102.8

O- ownership (%) S- Statkraft A- Lyse Produksjon AS
x- reservoir level (Mm3/week) u- discharge (Mm3/week)

Table 4.1: Starting parameters and limitations for four cascaded reservoirs

3.7 is solved to get starting and final reservoir levels corresponding to each desired final amount of energy in the system. The reservoir levels are now given by equations 4.1:

$$\begin{cases} R_{target}(i) = \dfrac{R_{max}(i)}{\alpha(i)} \times (\alpha(i) - \alpha(ROS) + \dfrac{R_{target}(ROS) \times \alpha(ROS)}{R_{max}(ROS)}) \\ \\ R_{target}(ROS) = \dfrac{R_{esum} - \sum_i e_{total}(i) \times \dfrac{R_{max}(i) \times (\alpha(i) - \alpha(ROS))}{\alpha(i)}}{e_{total}(ROS) + \sum_i e_{total}(i) \times \dfrac{R_{max}(i) \times \alpha(ROS)}{\alpha(i) \times R_{max}(ROS)}} \\ i = KVI,\ NES,\ HOM \end{cases} \tag{4.1}$$

while R_{esum} is calculated with $\sum_i R_{max}(i) \times e_{total}(i)$ for $i = ROS,\ KVI,\ NES$.

The desired amount of energy at the beginning of the period is set to 30% while the final one varies from 40% to 90% with respect to the inflow scenarios. The starting point of the testing and the owners' limitations are shown in Table 4.1.

The approach is run for the 75 scenarios under these conditions, first with owner S's production bounded by his share and then with the authorization to take the discharged capacity unused by owner A. The results converge in maximum three iterations in both cases. Detailed results are only presented for one scenario not to overload the report with large tables.

4.3 Detailed results for scenario 21

The initial and final reservoir levels are given in Table 4.2. Owner S is not allowed to use the discharge capacity left by A.

The two owners have the same production strategy for all the reservoirs. They produce the same weeks with an amount of water discharged proportional to their share of the system so, their water values are the same.

	Roskrepp		Kvinen		Nesjen		Homstølvann	
	X_{ini}	X_{final}	X_{ini}	X_{final}	X_{ini}	X_{final}	X_{ini}	X_{final}
S	215.6	344.6	0.0	36.9	0.0	85.2	0.0	0.0
A	149.9	239.5	0.0	25.6	0.0	59.2	0.0	0.0
Tot	365.5	584.1	0.0	62.5	0.0	144.5	0.0	0.0

x- reservoir level (Mm3/week)

Table 4.2: Levels for scenario 21

In Table 4.4 one sees that the power plant associated to Nesjen reservoir works a lot during the week with best prices. Its water value, 65.8 NOK/Mm3, corresponds to the best opportunity to draw an extra Mm3 in week 34 for 129.3 NOK/MWh. As the reservoir is empty in week 21, the water takes a slightly higher value of 67.4 NOK/Mm3.

The production at Kvinen reservoir is split up between several weeks, see Table 4.3. At the beginning of the period, water is discharged to allow it to be used at Nesjsen power plant afterward. Its value in week 21 and 22 expresses that the best shot for the owners is to draw water in week 22 from Kvinen and in week 34 from Nesjen ($177.9 \times 0.285 + 129.3 \times 0.509 = 116.5$ NOK/Mm3). As the reservoir is empty in week 22, no water can be discharged this week knowing that more will come later. The water value is therefore $173.2 \times 0.285 + 129.3 \times 0.509 = 115.2$ NOK/MWh meaning that the water should be taken out in week 35 from Kvinen but "used" in week 34 from Nesjen.

The water value for the upper reservoir means that the optimal use of an extra Mm3 of water is to be drawn in week 22 from both Roskrepp and Kvinen and finally produced in week 34 from Nesjen ($177.9 \times 0.486 + 129.3 \times 0.509 = 152.3$ NOK/Mm3). The water values really reflect the best opportunity to produce an extra MWh of energy for the three reservoirs.

The only challenge in Homstølvann handling is to prevent water to be spilled which is successfully achieved for this scenario since the water is instantaneously bypassed and not stored at all.

Let's now calculate the first week water values global and local in NOK/MWh to see if they match with the degree of regulation of the reservoirs.

The water values in NOK/MWh for week 21 are calculated with equation 3.6 and given below:

	Wv Global	Wv Local
Roskrepp	153.1	178.1
Kvinen	146.7	172.3
Nesjen	132.4	132.4

As expected, the water values are lower for badly regulated reservoirs such as Nesjen. It points out that water should be discharged first from this reservoir and then from Kvinen. The global water values are still close to one another. The results for this scenario are

Roskrepp												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	215.6					149.9						
21	241	0	152.3	24.6	162.7	167.5	0	152.3	17.1	113	172.3	171.4
22	247.9	12.5	152.3	12.1	155.7	172.3	8.7	152.3	8.4	108.2	177.9	177.0
23	281.9	0	152.3	24.6	121.7	195.9	0	152.3	17.1	84.6	168.3	167.4
24	274.9	24.6	152.3	0	128.7	191	17.1	152.3	0	89.4	194.6	193.7
25	280.4	24.6	152.3	0	123.3	194.8	17.1	152.3	0	85.7	204.6	203.6
26	278.2	24.6	152.3	0	125.4	193.3	17.1	152.3	0	87.2	209.0	207.9
27	292.6	0	152.3	24.6	111	203.3	0	152.3	17.1	77.1	180.1	179.2
28	309	0	152.3	24.6	94.7	214.7	0	152.3	17.1	65.8	165.9	165.1
29	321.8	0	152.3	24.6	81.8	223.6	0	152.3	17.1	56.9	108.7	108.2
30	330.3	0	152.3	24.6	73.3	229.6	0	152.3	17.1	50.9	107.8	107.3
31	336	0	152.3	24.6	67.7	233.5	0	152.3	17.1	47	101.8	101.3
32	340.2	0	152.3	24.6	63.4	236.4	0	152.3	17.1	44.1	127.1	126.5
33	351.5	0	152.3	24.6	52.2	244.2	0	152.3	17.1	36.3	126.8	126.2
34	366	0	152.3	24.6	37.7	254.3	0	152.3	17.1	26.2	129.3	128.7
35	383	0	152.3	24.6	20.7	266.1	0	152.3	17.1	14.4	173.2	172.3
36	374.9	24.6	152.3	0	28.8	260.5	17.1	152.3	0	20	195.8	194.9
37	360.9	24.6	152.3	0	42.7	250.8	17.1	152.3	0	29.7	207.0	206.0
38	344.6	24.6	152.3	0	59.0	239.5	17.1	152.3	0	41.0	211.0	209.9
Kvinen												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	0					0						
21	0.1	10.5	116.5	17	61.3	0.1	7.3	116.5	11.8	42.6	172.3	136.8
22	0	20.6	116.5	6.9	61.4	0	14.4	116.5	4.7	42.7	177.9	141.2
23	14.2	0	115.2	27.5	47.3	9.8	0	115.2	19.1	32.8	168.3	133.6
24	18.6	27.5	115.2	0	42.8	12.9	19.1	115.2	0	29.7	194.6	154.5
25	28.2	27.5	115.2	0	33.2	19.6	19.1	115.2	0	23.1	204.6	162.4
26	34.7	27.5	115.2	0	26.7	24.1	19.1	115.2	0	18.6	209.0	165.9
27	13.2	27.5	115.2	0	48.2	9.2	19.1	115.2	0	33.5	180.1	143.0
28	20	0	115.2	27.5	41.4	13.9	0	115.2	19.1	28.8	165.9	131.8
29	25.3	0	115.2	27.5	36.1	17.6	0	115.2	19.1	25.1	108.7	86.3
30	28.9	0	115.2	27.5	32.5	20.1	0	115.2	19.1	22.6	107.8	85.6
31	31.2	0	115.2	27.5	30.2	21.7	0	115.2	19.1	21	101.8	80.8
32	33	0	115.2	27.5	28.4	22.9	0	115.2	19.1	19.8	127.1	100.9
33	37.7	0	115.2	27.5	23.7	26.2	0	115.2	19.1	16.5	126.8	100.7
34	43.7	0	115.2	27.5	17.7	30.4	0	115.2	19.1	12.3	129.3	102.7
35	30.8	20	115.2	7.5	30.6	21.4	13.9	115.2	5.2	21.3	173.2	137.5
36	34.8	27.5	115.2	0	26.7	24.2	19.1	115.2	0	18.5	195.8	155.5
37	36.3	27.5	115.2	0	25.1	25.2	19.1	115.2	0	17.4	207.0	164.4
38	36.9	27.5	115.2	0	24.5	25.6	19.1	115.2	0	17.1	211.0	167.5

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

Table 4.3: Results for Roskrepp and Kvinen, scenario 21

Nesjen												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	0					0						
21	0	37.1	67.4	0	161.8	0	25.8	67.4	0	112.5	172.3	87.7
22	3.8	37.1	65.8	0	158	2.8	25.8	65.8	0	109.7	177.9	90.5
23	2.5	37.1	65.8	0	159.4	1.8	25.8	65.8	0	110.7	168.3	85.7
24	11.4	37.1	65.8	0	150.5	8	25.8	65.8	0	104.5	194.6	99.1
25	33.3	37.1	65.8	0	128.5	23.2	25.8	65.8	0	89.2	204.6	104.1
26	47.3	37.1	65.8	0	114.6	32.9	25.8	65.8	0	79.5	209.0	106.4
27	52.8	37.1	65.8	0	109	36.8	25.8	65.8	0	75.7	180.1	91.7
28	32.9	37.1	65.8	0	128.9	22.9	25.8	65.8	0	89.5	165.9	84.5
29	46.4	0	65.8	37.1	115.5	32.3	0	65.8	25.8	80.2	108.7	55.3
30	55.3	0	65.8	37.1	106.5	38.5	0	65.8	25.8	73.9	107.8	54.9
31	61.3	0	65.8	37.1	100.6	42.6	0	65.8	25.8	69.8	101.8	51.8
32	65.7	0	65.8	37.1	96.1	45.7	0	65.8	25.8	66.7	127.1	64.7
33	77.5	0	65.8	37.1	84.3	54	0	65.8	25.8	58.5	126.8	64.5
34	76.1	16.7	65.8	20.4	85.8	52.9	11.7	65.8	14.1	59.6	129.3	65.8
35	76.8	37.1	65.8	0	85	53.4	25.8	65.8	0	59.1	173.2	88.1
36	84.6	37.1	65.8	0	77.3	58.8	25.8	65.8	0	53.7	195.8	99.7
37	86.1	37.1	65.8	0	75.7	59.9	25.8	65.8	0	52.6	207.0	105.4
38	85.2	37.1	65.8	0	76.6	59.2	25.8	65.8	0	53.2	211.0	107.4

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

Table 4.4: Results for Nesjen, scenario 21

satisfying, more detailed table of results are displayed in Appendix G.Let's see if the approach is still robust for different starting level of the reservoirs.

4.4 Water value variations over reservoir levels

To keep the validity of the final reservoir levels, small variations are introduced in the initial levels of the three upper reservoirs. The tests are run for 75 inflow scenarios and the water values of the first week of the period are stored for each of them.

4.4.1 Variations at Roskrepp reservoir

The initial level of owner A within the reservoir is raised from 149.9 to 189.9 Mm3 by steps of 10 Mm3 while the one for S goes from 215.6 down to 175.5 Mm3. All the other parameters are kept at their previous values and both owners' discharge limitations are given by their share of the system.

At first, the approach does not converge at all when it is run for the 75 scenarios one after another. Looking at the results into details, the problem comes from the handling of Homstølvann. As it was assumed that there was no power generation downstream, there is no reason to take out water in one week rather than in the next or previous ones. So, for each iteration, the solution gives a different distribution of the water discharged or stored and thus the capacity transferred between the two owners are not constant. To get the convergence of the results, some economical stakes should be introduced at Homstølvann but they must lead to tiny profits for the owners not to damage the water values of the upper reservoirs. The efficiency of the plant below the lake is set to 10^{-5} kWh/m3. This value is small and does not affect the results at all. The average of the water values (over the 75 inflow scenarios), for all the reservoirs, as a function of the initial level of owner A within Roskrepp reservoir are shown in Figure 4.2.

As expected the water values for owner A go down for all the reservoirs with a significant drop when A's level in Roskrepp goes from 169.9 to 179.9 Mm3. It means that these additional Mm3 of water are transformed for less attractive prices most likely in Nesjen plant. On the opposite, the water values for S increase slowly. Since S owns a larger share of the system, he has an upper discharge bound in all the plants, so the water amount in the reservoir should vary hugely to reduce all the production in one week down to 0.

The water values in NOK/MWh both local and global are given for the two upper reservoirs in Figures 4.3 and 4.4. For Nesjen (water value local=water value global), they level off at 172.3 NOK/MWh for owner S and fall down from 172.3 to 130.3 NOK/MWh for owner A. This reservoir is badly regulated and there is less degree of freedom to handle it. The decrease in the global water values for owner A both in Roskrepp and Kvinen reservoirs is mainly due to the drop of the value for Nesjen.

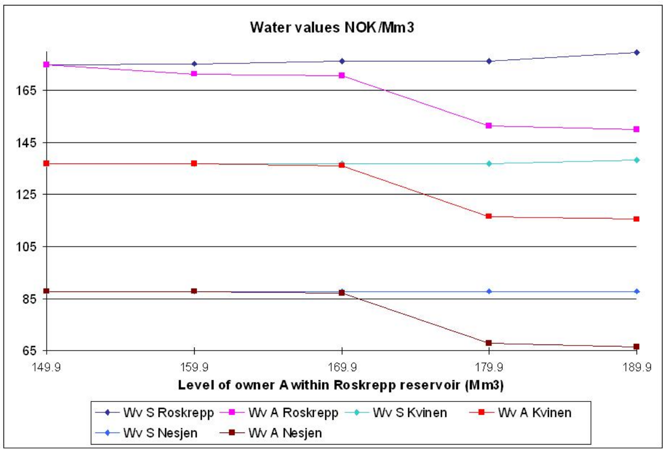

Figure 4.2: Water values of the reservoirs (NOK/Mm3) as a function of owner A's level within Roskrepp reservoir

The local water values are always higher than the global ones indicating that the upper reservoirs are globally well regulated. The depreciation for A is less important from a local point of view. The decrease at Nesjen implies a devaluation of A's water in the hydro system but as it is very safe to store water in the upper reservoirs, their local values are still high. For owner S, the curve of the local water values is very steep for the lower levels of S. Given that the level of owner S decreases, it is hardly recommended to store the water in Roskrepp since it is the most valuable there. It can be discharged in the downstream plants later in the scheduling period.

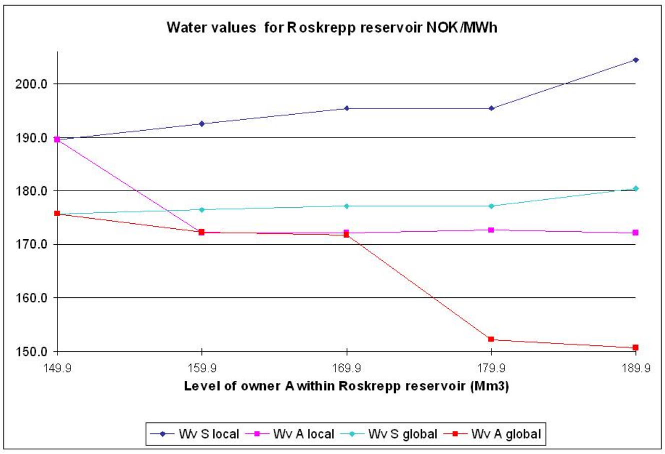

Figure 4.3: Water values of Roskrepp reservoir (NOK/MWh) as a function of owner A's level within Roskrepp reservoir

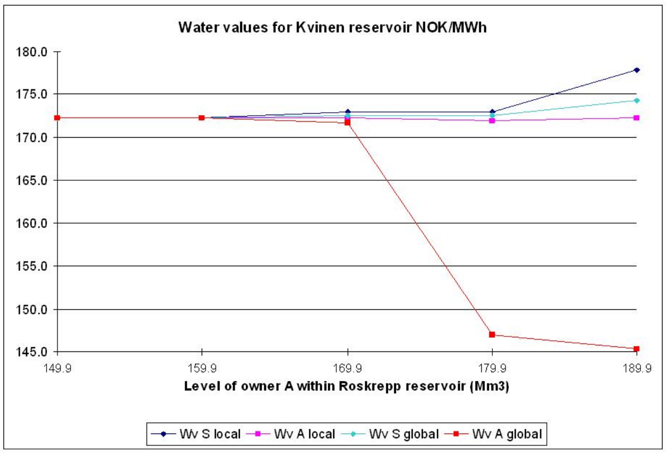

Figure 4.4: Water values of Kvinen reservoir (NOK/MWh) as a function of owner A's level within Roskrepp reservoir

4.4.2 Variations at Kvinen reservoir

The initial level of A in Kvinen reservoir grows from 0 to 100 Mm3 by steps of 10 Mm3 and the one for S is kept at 0 Mm3. The other parametersare the ones shown in 4.1.

Figure 4.5 shows the evolution of the water values in Mm3 for all the reservoirs. The water values for owner S remain constant since his reservoirs are not affected by the changes. This can be questionable because there is more water in the whole system as the level of owner A in Kvinen raises. So the value of S's water, a commodity more present in the system, should also decrease and this is not taken into account by the modeling. Different weekly prices could have been introduced with respect to the amount of water in the reservoirs.

As expected, the water values for A decrease by 14% at Roskrepp, 22% at Kvinen and 19% at Nesjen as his reservoir level in Kvinen increases. The drop is more important at Kvinen since it is there that the water level goes up.

The variation of owner A's water values in NOK/MWh are particularly interesting, see Figure 4.6. The water values referred to sea level (global) follow about the same decrease for the three reservoirs. The local water value for Kvinen is lower than the global one which highlights the fact that the water should not be stored in this reservoir since it is almost full and there is a higher risk of spillage. On the contrary, the local value for Roskrepp rises sharply to 225 NOK/MWh. Even if the water of A is less valuable in the system, it has an high value at Roskrepp since it is the best place to be kept - because of the good ratio reservoir size/inflow. Energy should be produced first from the downstream reservoirs.

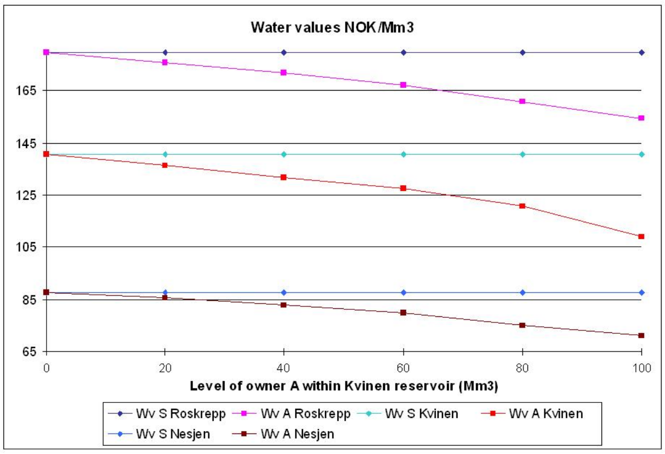

Figure 4.5: Water values of the reservoirs (NOK/Mm3) as a function of owner A's level within Kvinen reservoir

4.4.3 Variations at Nesjen reservoir

The initial level of A in Nesjen reservoir grows from 0 to 200 Mm3 by steps of 20 Mm3 and the one for S is kept at 0 Mm3. The other paramters are set according to 4.1. The water values in NOK/Mm3 and MWh as a function of owner A level within Kvinen reservoir are given in Figures 4.7 and 4.8.

The water value reduction for owner A, is pretty much the same for all the reservoir: 45 Mm3. It means that the water values from the upper reservoirs are affected indirectly by the drop of the value at Nesjen.

In terms of energy, the global water values go down for all the reservoirs since there is much more water in A's system. However, the local values for the upper plants are still important and higher than the global ones (198 for Roskrepp and 189 NOK/MWh for Kvinen compared to 135 NOK/MWh and 119 NOK/MWh referred to sea level) and stable due to the fact that there is no change in the local inflow received by theses reservoirs.

The water values for each owner, in Mm3 as well as in MWh, reflects correctly their hydrological situation which demonstrates that the approach works well. Moreover, the convergence of the results was achieved in 3 iterations of the approach or less. Allowing owner S to use A's remaining production capacity gives the same water values for the three sensitivity analyses performed and here as well the number of iteration is around 3 per scenario. The previous convergence problem is solved by the addition of the two reservoirs.

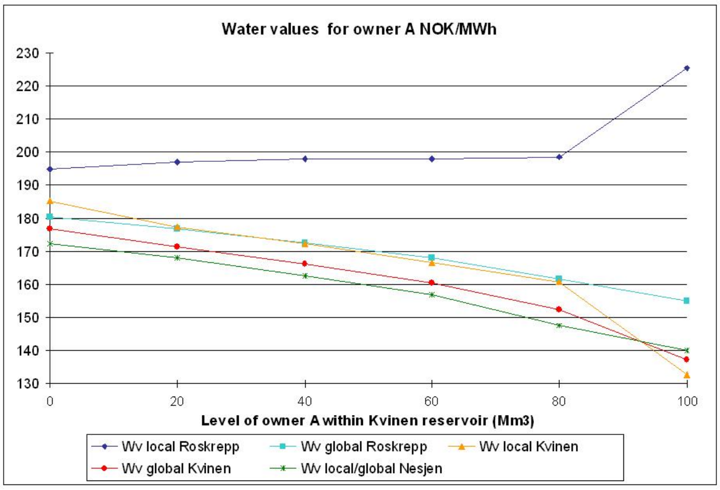

Figure 4.6: Water values of owner A (NOK/MWh) as a function of owner A's level within Kvinen reservoir

	Roskrepp				Kvinen				Nesjen			
Level	Wv S	Diff	Wv A	Diff	Wv S	Diff	Wv A	Diff	Wv S	Diff	Wv A	Diff
0	179.8	0	179.7	0	140.6	0	140.5	0	87.7	0	87.7	0
20	179.8	0	178.3	2.5	140.6	0	138.8	2.6	87.7	0	87.2	1.6
40	179.8	0	176.2	4.5	140.6	0	136.5	4.6	87.7	0	86.2	3.4
60	179.8	0	174.9	7.7	140.6	0	135	7.6	87.7	0	85.2	5.3
80	179.8	0	173.5	12.7	140.6	0	133.1	12.2	87.7	0	84	8.9
100	179.8	0	171.8	17.5	140.6	0	130.8	21.8	87.7	0	83	11.8

Wv- water value (NOK/Mm3)
Diff- water values when A can use S's discharge minus the ones when he is limited by his share (NOK/Mm3)

Table 4.5: Water values for variations over owner A's level in Kvinen reservoir

4.5 Modification of the system rules

In the sensitivity analyses, the level of owner A was always raised but his discharge upper bound was still the same. It can be interesting not only to see how much his water values would have increased having more production capacity but also how it would have affected the ones for owner S. The reservoir level of owner A within Kvinen reservoir is now increase from 0 to 100 Mm3 and owner A is allowed to use the discharge capacity left by S. The new water values in NOK/Mm3 are given in Table 4.5, they were achieved in 2 iterations of the approach for all the inflow scenarios. The column Diff is the difference between the water values achieved when A can take S's unused production capacity and the ones when he is limited by his share, in section 4.4.2.

In opposition to what happened in section 3.6.3, the results converge quickly and the

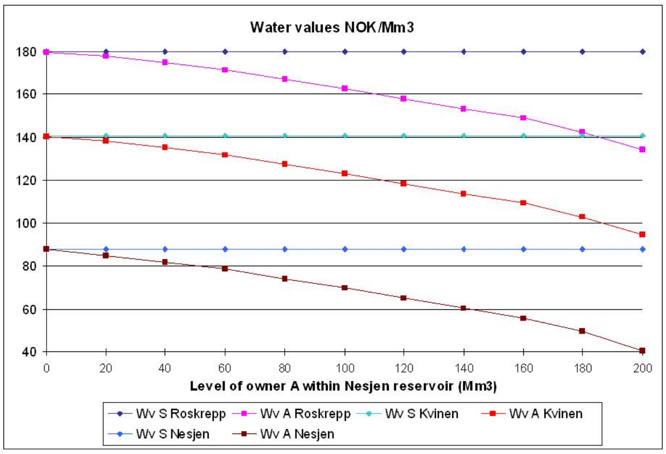

Figure 4.7: Water values of the reservoirs (NOK/Mm3) as a function of owner A's level within Nesjen reservoir

water values for owner S are not damaged at all by the utilization of his discharge capacity by A. Furthermore, the water values of owner A are higher than before especially when Kvinen level is above 40 Mm3 since A takes out water at better prices taking advantage of S's capacity.

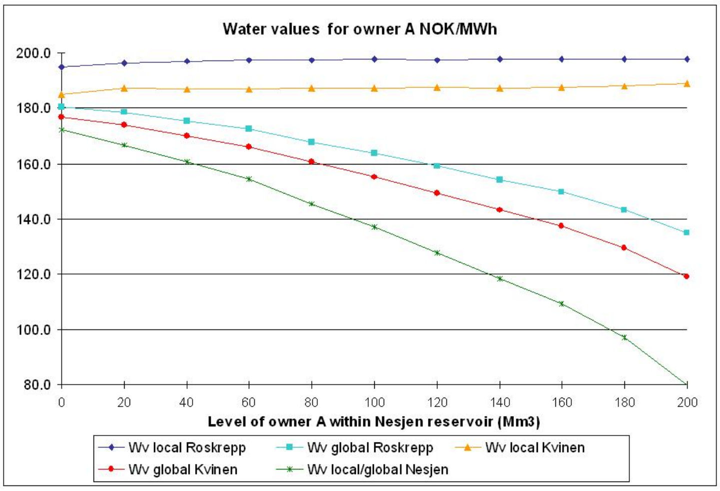

Figure 4.8: Water values of owner A (NOK/MWh) as a function of owner A's level within Nesjen reservoir

To conclude, the approach demonstrates to give coherent water values as well as a good handling of the reservoirs for a four-cascaded-reservoir-system under various testing conditions, with respect to reservoir levels and inflow scenarios, and with a rather limited number of iteration generally equal to three.

Chapter 5

Further work

This section is divided in several parts to cover different aspects that can be further investigated.

5.1 Clarifications

The alternative approach, based on individual optimizations for each owner of the water course, has proven to work well so far. A stable solution was generally reached after 3 or 4 iterations of the model depending on the case. Nevertheless, some convergence troubles were observed for the two reservoir case when one owner was allowed to use the other one's discharge capacity. In addition to this, it turned out into a small income loss for the lender owner. This issue should be looked more into details since it did not occur for the four cascaded reservoir case. An alternative is to limit both owners to their share of the system and assume that they can conclude a deal afterwards, if one owner is disposed to pay to borrow the capacity left by the other one.

5.2 Testing conditions

The testing parameters can still be improved to be as realistic as possible. Different price scenarios should be used according to the inflow scenario selected and it can be interesting to introduce different price forecasts for each owner since they do not necessarily use the same tools to estimate the expected market prices. Moreover, the final reservoir levels, calculated to maintain equal dampings for all the reservoirs, could be obtained from long term model simulations.

The most interesting period of the year was studied but the simulations have to be done as well for the depletion season and for longer time horizons than eighteen weeks because the seasonal optimization can be performed, in extreme cases, for up to 18 months.

5.3 Modeling

In the objective function to be maximized, the relation production discharge was assumed to be linear which is not the case in reality. It should be split into piecewise linear functions to take into account the changes in the plant efficiency as a function of the amount of water discharged.

Finally, the testing has also to be carried out for larger systems including pumping plants to see how many iterations of the approach are required to achieve a stable solution with a piecewise relation production-discharge. Therefore, the whole modeling of Sira-Kvina water course should be considered.

If the solution does not converge as fast as expected, master-slave optimizations can still be used to achieve a stable solution more quickly. A master algorithm takes handling decisions for the upper reservoir while a slave one tries the best effort for the lower ones. The impact of such technique on the results should be looked carefully especially when the system consists of a number of cascaded reservoirs since the results can be hardly damaged for the downstream reservoirs. For instance, if the master algorithm suggests that a huge amount of water should be drawn from the upper reservoirs it can turn into spillage for the lower ones if they are very badly regulated.

Conclusion

The purpose of this master's thesis was to build a mathematical model for seasonal hydro power scheduling in multi-owner river systems, that is, the water value calculations for individual reservoirs. The first modeling, formulated by Birger Mo and consisting in solving the problem running one single optimization with the help of integer variables, was rejected after further testing. The profits of the owners were too much unbalanced due to the objective definition of the optimization and the results were not even guaranteed when the integer constraints were engaged.

The development of the alternative approach, based on separate optimizations for each owner and an iterative procedure, was successful. The testing for seventy inflow scenarios and various parameters demonstrated that the approach was robust and converged quickly for one reservoir and its power plant. As a result, the system was extended to two and then four reservoirs forming a part of Sira-Kvina water course.

Sensitivity analyses with respect to reservoir levels were performed for seventy five scenarios for these two hydro systems. The water values achieved reflected correctly the hydrological situation of the owners as well as the handling of the reservoirs. Moreover, they were obtained after few iterations of the procedure, three in general. In the case where one owner was allowed to use the capacity left by the other ones, more iterations were required to stabilize the solution for the two-reservoir-system since the exchanged capacities needed to balance each other out.

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Appendix A

Initial problem formulation by Birger Moe

Initially we look at a very simple river system, existing of only one reservoir and a power station as shown in Figure 1.

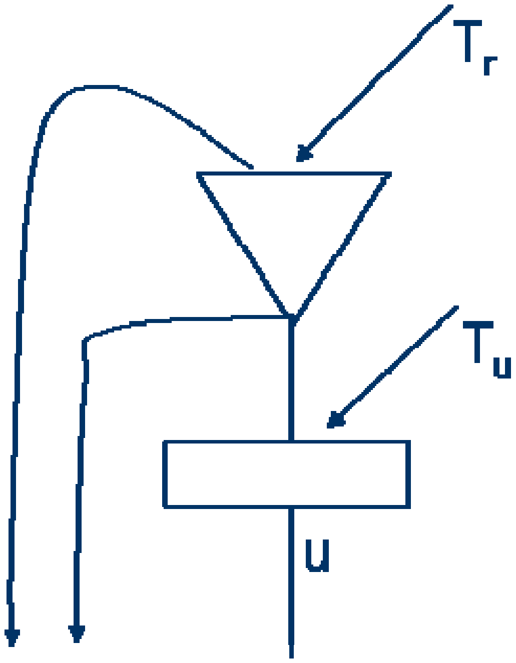

Figure 1 Simple hydro generation system

The system is described by the following variables:

$x(k)$ - Actual reservoir level (Mm^3)
$x^M(k)$ - Maximum reservoir level (Mm^3)
$x^N(k)$ - Minimum reservoir level (Mm^3)
$u(k)$ - Physical discharge at time k (Mm^3/week)
$u^M(k)$ - Maximum discharge at time k (Mm^3/week)
$u^N(k)$ - Minimum discharge at time k (Mm^3/week)
$F(u)$ - Relation between production and discharge
$V(k)$ - Spillage at time k (Mm^3/week)
$S(k)$ - Bypass at time k (Mm^3/week)
$T(k)$ - Storable inflow at time k (Mm^3/week)
k - Week number in the scheduling period
K - Last week in the scheduling period

We assume that the system is owned by two ore more companies. We focus on owner *s*, and assume that its owner share is given by the parameter *A*. The respective companies' initial share of the physical reservoir is given.

$x_s(0)$ - Company *s*'s initial reservoir (Mm^3)

$x_a(0)$ - The sum of the other owners' reservoirs (Mm^3)

The sum of $x_a(0)$ and $x_s(0)$ must be equal to the physical reservoir $x(0)$

We look at the model description for the seasonal scheduling model, where the final reservoir $x(K)$ is known. We assume that the final reservoir, given by the EMPS model for each inflow alternative, is divided between the owners according to their respective owner shares, i.e. initial deviations from the owner shares will be equalized in week K:

$$x_s(K) = x(K)*A \tag{1}$$

$$x_a(K) = x(K)*(1-A) \tag{2}$$

Later we will try to generalize this to any given distribution of the final reservoir.

Below we define the mathematical equations that follow from the physical reality of the handling and settlement of the owner shares. This description does not take into account cascades of power plants, which will be pursued subsequently.

The reservoir balance of the physical system:

$$x(k+1) = x(k)-u(k)- V(k)- S(k) +T(k) \tag{3}$$

Physical reservoir constraints:

$$x^N(k)\leq x(k) \leq x^M(k) \tag{4}$$

Physical production constraints:

$$u^N(k)\leq u(k) \leq u^M(k) \tag{5}$$

Reservoir account for owner s:

$$x_s(k+1) = x_s(k)-u_s(k)- V_s(k)- S_s(k) +T(k)*A \tag{6}$$

Reservoir account for the other owners:

$$x_a(k+1) = x_a(k)-u_a(k)- V_a(k)- S_a(k) +T(k)*(1-A) \tag{7}$$

The sum of the reservoir shares must equal the physical reservoir:

$$x_s(k+1)+ x_a(k+1) = x(k+1) \tag{8}$$

We now look at different agreement on how the owners are allowed to use the system. For the "other" owners it is assumed that they only can use production capacity given by their owner share:

$$u^N(k)*(1-A)\,)\le u_a(k) \le u^M(k)(1-A) \qquad (9)$$

For owner s we assume two different alternatives;

Alternative 1: s can use the whole physical capacity if the other owners do not wish to utilize it:

$$u^N(k) \le u_a(k)+ u_s(k) \le u^M(k) \qquad (10.a)$$

Alternative 2: s can only use production capacity given by its owner share:

$$u^N(k)*A \le u_s(k) \le u^M(k)*A \qquad (10.b)$$

From a mathematical point of view, the distribution of spillage is a complicating factor, that must be formulated with the help of integer numbers:

Case 1:
Owner s's reservoir level is higher than his owner share. If there is spillage, owner s reservoir water will be spilled first, until the actual reservoir shares correspond with the owner shares. The physical spillage if subtracted from s's reservoir balance:

If $(V(k) >0.0 \text{ and } x_s(k)>A\cdot x(k)) \text{ then } V_s(k)=V(k)$ *(11a)*

Case 2:
Owner s's reservoir level is lower than his owner share. If there is spillage, the other owners' reservoir water will be spilled first, until the actual reservoir shares correspond with the owner shares.

If $(V(k) >0.0 \text{ and } (x_s(k) <A\cdot x(k)) \text{ then } V_a(k)=V(k)$ *(11.b)*

Case 3:
Owner s's reservoir level is exactly equal to his owner share. The physical spillage is divided between the owners according to their owner shares:

If $(V(k) >0.0 \text{ and } (x_s(k)=A\cdot x^M(k)) \text{ then } V_s(k)=V(k)*A \text{ and } V_a(k)=V(k)*(1-A))$ *(11.c)*

We assume that minimum bypass requirements are divided according to owner shares:

$$S^N(k)*A \le S_s(k) \qquad (12)$$

$$S^N(k)*(1-A) \le S_a(k) \qquad (13)$$

The following object function is maximized:

$$\max\left[\sum_{k=1}^{K} p(k)AF(\frac{u_s(k)}{A})+p(k)(1-A)F(\frac{u_a(k)}{1-A})\right] \qquad (14)$$

with constraints equations (3-14). The shadow prices of equations (6) and (7) represent the water values for owner *s* and the other owners respectively at time *k*.

Added December 2009:

It should be noted that because $u(k) = u_s(k) + u_a(k)$, the objective function in (14) can be written as:

$$\sum_{k=1}^{K} p(k)F\left(A\frac{u_s(k)}{A}) + (1-A)(\frac{1-u_s(k)}{1-A})\right) = \sum_{k=1}^{K} p(k)Fu(k) \qquad (15)$$

i.e. that the objective is purely a maximization of the total revenues.

Appendix B

AMPL code for the first formulation

```
# VARIABLES AND PARAMETERS DECLARATION

param OWNER > 0; # 2 owner: s called owner 1 and other owners called owner
    2
param K > 0; # number of weeks

# FOR THE GLOBAL SYSTEM

param Xt_min {0..K};
param Xt_max {k in 0..K} >= Xt_min[k]; # physical reservoirs limits
param Xmax;
param imax symbolic in 0..K; # parameters for max(Xt_max[k]) research

param Ut_min {1..K} >= 0;
param Ut_max {k in 1..K} >= Ut_min[k]; # physical production limits

param Tt {1..K} >= 0; # storable inflow
param Tmax;
param itmax symbolic in 1..K; # parameters for max(Tt[k]) research

param St_min {1..K} >= 0;
param St_max {k in 1..K} >= St_min[k]; # bypass requirement

param Pspill {1..K}; # spillage "beneficit" to spill water as later as
    possible

param P {1..K} >= 0; # week price NOK/MWh

var xt {0..K} >= 0; # reservoir level

var ut {1..K} >=0; # discharge

var st {1..K} >= 0; #bypass

var vt {1..K} >= 0; # spillage
```

```
# FOR EACH OWNER

param A {1..OWNER} >= 0, <= 1; # owner share
check: sum {i in 1..OWNER} A[i]= 1; # check if sum of owner shares is equal
     to 1

var x {1..OWNER,0..K} >= 0; # reservoir level

var u {1..OWNER,1..K} >= 0; # discharge

var s {1..OWNER,1..K} >= 0; # bypass

var v {1..OWNER,1..K} >= 0; # spillage

# PARAMETERS FOR THE RELATION PRODUCTION DISCHARGE

param E; # energy equivalent

# PARAMETERS FOR INITIAL AND FINAL CONDITIONS

param X10 >= 0; # owner s initial reservoir
param X20 >= 0; # owner a initial reservoir
param XtK>=0; # final global reservoir level known
check: Xt_min[0]<= (X10+X20) <= Xt_max[0];
check: Xt_min[K]<= XtK <= Xt_max[K]; # check reservoir physical limits for
    week 0 and K

# INTEGER VARIABLES USED FOR SPILLAGE CONDITIONS ESTABLISHEMENT

param B >0;
param EPS >0; # usefull parameters for the link between real and binary
    variables

# INTEGER VARIABLES USED FOR SPILLAGE CONDITIONS

var sigxc {1..OWNER,1..K} binary; # integer variable linked to x[1,k] < or
    > A[1]xt[k]

var sigl31 {1..K} binary;

var sigl32 {1..K} binary;

var sigxc3 {1..K} binary; # integer variables linked to x[1,k]= A[1]xt[k]


# OBJECTIVE AND CONSTRAINTS

# OBJECTIVE

maximize total_profit:
sum {k in 1..K} (P[k]*(u[1,k]+u[2,k])*E-Pspill[k]*(v[1,k]+v[2,k])*E);
```

```
# PHYSICAL CONSTRAINTS

subject to global_balance {k in 1..K}:
xt[k]= xt[k-1]-ut[k]-vt[k]-st[k]+Tt[k]; # balance equation for the whole
    system

subject to reservoir_cstr {k in 1..K-1}:
Xt_min[k]<= xt[k] <= Xt_max[k]; # reservoir limits

subject to production_cstr {k in 1..K}:
Ut_min[k]<= ut[k] <= Ut_max[k]; # production constraints

subject to owner_balance {i in 1..OWNER,k in 1..K}:
x[i,k]=x[i,k-1]-u[i,k]-v[i,k]-s[i,k]+A[i]*Tt[k]; # balance for each
    reservoir

subject to level_and_reservoir_shares {k in 0..K-1}:
sum {i in 1..OWNER} x[i,k+1]=xt[k+1]; # sum of owner levels equal to global
     level

subject to discharge {i in 1..OWNER, k in 1..K}:
Ut_min[k]*A[i] <= u[i,k] <= Ut_max[k]*A[i]; # discharge constraints for
    alternative 2

subject to bypass {i in 1..OWNER, k in 1..K}:
St_min[k]*A[i] <= s[i,k] <= St_max[k]*A[i];  # bypass minimum requirement
    for each owner


# CONSTRAINTS DUE TO INITIAL AND FINAL CONDITIONS

subject to owner1_initial_condition:
x[1,0]=X10; # initial conditions for owner s given

subject to owner2_initial_condition:
x[2,0]=X20; # initial conditions for owner a given

subject to global_initial_condition:
sum {i in 1..OWNER} x[i,0]=xt[0];# sum of owner initial levels is equal to
    global level

subject to global_final_condition:
xt[K]= XtK; # global reservoir level for the week K given

subject to owner_final_condition {i in 1..OWNER}:
x[i,K]=XtK*A[i]; # final owners reservoir level must be equal to their
    owner share


# CONSTRAINTS DUE TO SPILLAGE CONDITIONS
```

```
# case1&2

subject to level_case12 {i in 1..OWNER, k in 1..K}:
(x[i,k]-A[i]*Xt_max[k]-(Xmax+Tmax)*sigxc[i,k]) <= 0; # if x[i,k]>A[i]*xt[k]
     then sigxc[i,k]=1

subject to spillage_case1 {k in 1..K}:
(v[2,k]+Tmax*(sigxc[1,k])) <= Tmax; # if sigxc[1,k]=1 then v[2,k]<=0

subject to spillage_case2 {k in 1..K}:
(v[1,k]+Tmax*(sigxc[2,k])) <= Tmax; # if sigxc[2,k]=1 then v[1,k]<=0


# case3

subject to level_case31 {k in 1..K}:
(x[2,k]-A[2]*Xt_max[k]-(-(Xmax+Tmax)-EPS)*sigl31[k]) >= EPS; # if x[1,k] <=
     A[1]*xt[k] then sigl31[k]=1

subject to level_case32 {k in 1..K}:
(x[2,k]-A[2]*Xt_max[k]-(Xmax+Tmax+EPS)*sigl32[k]) <= (-EPS); # if x[1,k] >=
     A[1]*xt[k] then sigl32[k]=1

subject to level_case33 {k in 1..K}:
(sigl31[k]+sigl32[k]-sigxc3[k]) <= 1; # if sigl31[k]=1 and sigl32[k]=1 then
     sigxc3[k]=1

subject to spillage_case31 {k in 1..K}:
(A[1]*v[2,k]-A[2]*v[1,k]+Tmax*sigxc3[k]) <= Tmax; # if sigxc3[k]=1 then A
    [1]*v[2,k]-A[2]*v[1,k] <= 0

subject to spillage_case32 {k in 1..K}:
(A[1]*v[2,k]-A[2]*v[1,k]-Tmax*sigxc3[k]) >= (-Tmax); # if sigxc3[k]=1 then
    A[1]*v[2,k]-A[2]*v[1,k] >= 0


# only one case is true

subject to sigmas {k in 1..K}:
sigxc[1,k]+sigxc[2,k]+sigxc3[k] <=1; # as the only write one way
    implications we need to ensure that only one sigma is equal to 1
```

Appendix C

Result example for a scenario without any interactions between the two owners

The reservoir characteristics as well as the prices and spillage penalties are not changed from the previous testing in Chapter 1. Mean annual inflow: 336 Mm3
Energy conversion factor: 1 kWh/m3
Reservoir: 100 Mm3
Installed capacity: 100 MW
The testing parameters are:
As:=0.5;
Aa:=0.5;
Xa0:=27;
Xs0:=3;
XtK:=85; The two owners starting levels are unbalanced for the demonstration. The scenario selected is number 19 which should result in a huge amount of spillage.

Week	xs	us	WvS	vs	xa	ua	WvA	va	P
20	3				27				
21	6.6	8.4	101.8	0	30.6	8.4	-1.3	0	172.3
22	6.5	8.4	101.8	0	30.5	8.4	-1.3	0	177.9
23	13.5	8.4	101.8	0	37.5	8.4	-1.3	0	168.3
24	18.8	8.4	101.8	0	42.8	8.4	-1.3	0	194.6
25	23.8	8.4	101.8	0	47.8	8.4	-1.3	0	204.6
26	30.7	8.4	101.8	0	50	8.4	-1.3	4.7	209.0
27	35.8	8.4	101.8	0	50	8.4	-1.2	5.1	180.1
28	40.6	8.4	101.8	0	50	8.4	-1.1	4.8	165.9
29	41.6	8.4	101.8	0	50	8.4	-1	1.1	108.7
30	42	8.4	101.8	0	50	8.4	-0.9	0.3	107.8
31	48.8	1.9	101.8	0	50	8.4	-0.8	0.3	101.8
32	50	8.4	101.8	0	50	8.4	-0.7	1.2	127.1
33	50	6.6	126.8	0	50	6.6	126.8	0	126.8
34	50	3.5	129.3	0	50	3.5	129.3	0	129.3
35	50	4.1	173.2	0	50	4.1	173.2	0	173.2
36	50	5.6	195.8	0	50	5.6	195.8	0	195.8
37	48.2	6.4	207	0	48.2	6.4	207	0	207.0
38	42.5	8.4	207	0	42.5	8.4	207	0	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week) v- spillage (Mm3/week) s- bypass (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

Owner A starts to spill his water week 16 since he reaches its reservoir maximum bound. However, owner S does not use his maximum capacity before week 32. There should be spillage only when the reservoir is full.

Appendix D

Iterative approach running files for one reservoir

D.1 Excel macro to create inflow files

```
' Clicking cancel unload the UserForm
Private Sub Cancelbutton_Click()
Unload Datafiles
End Sub

' Clicking OK starts the excel file creation
Private Sub OKButton_Click()
Dim i, j, k, nrow, ncol, val As Integer
Dim MyArea As String
Dim Zone As Range
Set Zone = Sheets("Hydro_records").Range("A6:BS58") ' where to pick up the data

i = TextBox3.Value ' first inflow scenario selected

' Iterate over each inflow scenario to create the corresponding number of files
For i = TextBox3.Value To TextBox4.Value

    Dim MyXL As Object 'Excel application object
    Dim XL_File As String
    Dim SheetName As String

    ' Directory where the files are created
    XL_File = "M:\dokument\master_thesis\loop\inflow_sce\scenario" & i & ".xls"
    SheetName = "Scenario " & i & " Inflow"

    'Create the excel application object
    Set MyXL = CreateObject("Excel.Application")

    'Create new excel workbook
    MyXL.Workbooks.Add

    ' Give a name to the new work sheet according to the scenario number
    MyXL.Worksheets(1).Name = SheetName
```

```
    ' Create the first columns
    MyXL.Worksheets(SheetName).Range("A1") = "TIME"
    MyXL.Worksheets(SheetName).Range("B1") = "Tt"

    ' Initial values of the integer variables
    j = TextBox1.Value
    k = 1
    nrow = 1
    ncol = 1

   ' Iterate over the week selected to cpoy the data in the new file
    For j = TextBox1.Value To TextBox2.Value

        'Copy inflows for the week selected
        ' Week number
        MyXL.Worksheets(SheetName).Cells(nrow + k, ncol) = k
        ' When j>52 we need to pick up the data in the next column i.e for the next year
        If j <= 52 Then
        MyXL.Worksheets(SheetName).Cells(nrow + k, ncol + 1) = Zone.Cells(j + 1, i + 1)
        Else
        MyXL.Worksheets(SheetName).Cells(nrow + k, ncol + 1) = Zone.Cells(j - 51, i + 2)
        End If
        k = k + 1

    Next

    'To use the data from excel in AMPL, the name inflow is assign to the zone
    'where the data are in the new file
    val = TextBox2.Value - TextBox1.Value + 2
    MyArea = "='Scenario " & i & " Inflow'!R1C1:R" & val & "C" & 2
    MyXL.Names.Add Name:="Inflow", RefersToR1C1:=MyArea

    'Save the new excel file
    MyXL.Worksheets(1).SaveAs (XL_File)
    MyXL.Quit

    Set MyXL = Nothing
    Next

' Close the UserForm
Unload Datafiles
End Sub
```

D.2 Model file

```
#### VARIABLES AND PARAMETERS DECLARATION

# GENERAL PARAMETERS

param K > 0; # number of weeks

param As; # owner S's share

param Aa; # owner A's share

param Xt_min {0..K};
param Xt_max {k in 0..K} >= Xt_min[k]; # physical reservoirs limits

param Ut_min {1..K} >= 0;
param Ut_max {k in 1..K} >= Ut_min[k]; # physical production limits

param Tt {1..K} >= 0; # storable inflow

param St_min {1..K} >= 0;
param St_max {k in 1..K} >= St_min[k]; # bypass requirement

param P {1..K} >= 0; # week price NOK/MWh

param Pspill {1..K} >= 0; # spillage penalty

param E; # energy equivalent

param X0 >=0; # global initial reservoir level known

param XtK>=0; # final global reservoir level given

# PARAMETERS FOR INITIAL AND FINAL CONDITIONS

param Xs0 >= 0; # initial reservoir for s

param Xa0 >= 0; # initial reservoir for a

param fd21r2 {1..K}; # discharge transfer a->s run 2

param fx21r2 {1..K}; # level transfer a->s run 2

param fd12r1 {1..K}; # discharge transfer s->a run 1

param fx12r1 {1..K}; # level transfer s->a run 1

param fd21r1 {1..K}; # discharge transfer a->s run 1

param fx21r1 {1..K}; # level transfer a->s run 1

param fd12r2 {1..K}; # discharge transfer s->a run 2
```

```
param fx12r2 {1..K}; # level transfer s->a run 2

# VARIABLES
# RUN 1
var xs {0..K} >= 0; # reservoir level s
var us {1..K} >=0; # discharge s
var ss {1..K} >= 0; # bypass s
var vs {1..K} >= 0; # spillage s

var xa {0..K} >= 0; # reservoir level
var ua {1..K} >=0; # discharge
var sa {1..K} >= 0; # bypass
var va {1..K} >= 0; # spillage

var d12r1 {1..K}; # discharge transfer s->a
var x12r1 {1..K}; # level transfer s->a

var d21r1 {1..K}; # discharge transfer a->s
var x21r1 {1..K}; # level transfer a->s
# RUN 2
var x2s {0..K} >= 0; # reservoir level s
var u2s {1..K} >=0; # discharge s
var s2s {1..K} >= 0; # bypass s
var v2s {1..K} >= 0; # spillage s

var x2a {0..K} >= 0; # reservoir level
var u2a {1..K} >=0; # discharge
var s2a {1..K} >= 0; # bypass
var v2a {1..K} >= 0; # spillage
```

```
var d12r2 {1..K}; # discharge transfer s->a

var x12r2 {1..K}; # level transfer s->a


var d21r2 {1..K}; # discharge transfer a->s

var x21r2 {1..K}; # level transfer a->s


#### PROBLEM FOR S RUN 1


# OBJECTIVE

maximize S_profit:
sum {k in 1..K} (P[k]*us[k]-Pspill[k]*vs[k])*E;

# PHYSICAL CONSTRAINTS

subject to S_balance {k in 1..K}:
xs[k]= xs[k-1]-us[k]-vs[k]-ss[k]+Tt[k]*As; # reservoir balance equation

subject to S_production {k in 1..K}:
Ut_min[k]*As<= us[k] <= Ut_max[k]*As ;
# Ut_min[k]*As<= us[k] <= (Ut_max[k]*As + fd21r2[k]); if S can use A
    remaining capacity

subject to S_bypass {k in 1..K}:
St_min[k] <= ss[k] <= St_max[k];  # bypass requirements

subject to S_level {k in 1..K}:
Xt_min[k]*As <= xs[k] <= (As*Xt_max[k]+fx21r2[k]);# reservoir limits


# Capacities left for the other owner

subject to S_discharge_transferred {k in 1..K}:
d12r1[k]=Ut_max[k]*As-us[k];

subject to S_level_transferred {k in 1..K}:
x12r1[k]=Xt_max[k]*As-xs[k];

# CONSTRAINTS DUE TO INITIAL AND FINAL CONDITIONS

subject to S_initial_condition:
xs[0]=Xs0; # reservoir initial condition

subject to S_final_condition:
xs[K]= XtK*As; # reservoir level for week K given
```

```
#### PROBLEM FOR A RUN 1


# OBJECTIVE

maximize A_profit:
sum {k in 1..K} (P[k]*ua[k]-Pspill[k]*va[k])*E;

# PHYSICAL CONSTRAINTS

subject to A_balance {k in 1..K}:
xa[k]= xa[k-1]-ua[k]-va[k]-sa[k]+Tt[k]*Aa; # reservoir balance equation

subject to A_production {k in 1..K}:
Ut_min[k]*Aa<= ua[k] <= (Ut_max[k]*Aa); # production constraints
# Ut_min[k]*Aa<= ua[k] <= (Ut_max[k]*Aa + fd12r1[k]); if A can use S
    remaining capacity

subject to A_bypass {k in 1..K}:
St_min[k] <= sa[k] <= St_max[k]; # bypass requirements

subject to A_level {k in 1..K}:
Xt_min[k]*Aa<= xa[k] <= (Xt_max[k]*Aa+fx12r1[k]); # reservoir limits

# Capacities left for the other owner

subject to A_discharge_transferred {k in 1..K}:
d21r1[k]=(Ut_max[k]*Aa-ua[k]);

subject to A_level_transferred {k in 1..K}:
x21r1[k]=(Xt_max[k]*Aa-xa[k]);


# CONSTRAINTS DUE TO INITIAL AND FINAL CONDITIONS

subject to A_initial_condition:
xa[0]=Xa0; # reservoir initial condition

subject to A_final_condition:
xa[K]= XtK*Aa; # reservoir level for week K given


#### PROBLEM FOR S RUN 2


# OBJECTIVE

maximize S2_profit:
sum {k in 1..K} (P[k]*u2s[k]-Pspill[k]*v2s[k])*E;

# PHYSICAL CONSTRAINTS

subject to S2_balance {k in 1..K}:
```

```
x2s[k]= x2s[k-1]-u2s[k]-v2s[k]-s2s[k]+Tt[k]*As; # reservoir balance
    equation

subject to S2_production {k in 1..K}:
Ut_min[k]*As<= u2s[k] <= Ut_max[k]*As;
#  Ut_min[k]*As<= us[k] <= (Ut_max[k]*As + fd21r1[k]); if S can use A
    remaining capacity

subject to S2_bypass {k in 1..K}:
St_min[k] <= s2s[k] <= St_max[k];  # bypass requirements

subject to S2_level {k in 1..K}:
Xt_min[k]*As <= x2s[k] <= (As*Xt_max[k]+fx21r1[k]);# reservoir limits


# Capacities left for the other owner

subject to S2_discharge_transferred {k in 1..K}:
d12r2[k]=Ut_max[k]*As-u2s[k];

subject to S2_level_transferred {k in 1..K}:
x12r2[k]=Xt_max[k]*As-x2s[k];

# CONSTRAINTS DUE TO INITIAL AND FINAL CONDITIONS

subject to S2_initial_condition:
x2s[0]=Xs0; # reservoir initial condition

subject to S2_final_condition:
x2s[K]= XtK*As; # reservoir level for week K given


#### PROBLEM FOR A RUN 2


# OBJECTIVE

maximize A2_profit:
sum {k in 1..K} (P[k]*u2a[k]-Pspill[k]*v2a[k])*E;

# PHYSICAL CONSTRAINTS

subject to A2_balance {k in 1..K}:
x2a[k]= x2a[k-1]-u2a[k]-v2a[k]-s2a[k]+Tt[k]*Aa; # reservoir balance
    equation

subject to A2_production {k in 1..K}:
Ut_min[k]*Aa<= u2a[k] <= (Ut_max[k]*Aa); # production constraints
# Ut_min[k]*Aa<= ua[k] <= (Ut_max[k]*Aa + fd12r2[k]); if A can use S
    remaining capacity

subject to A2_bypass {k in 1..K}:
St_min[k] <= s2a[k] <= St_max[k];  # bypass requirements
```

```
subject to A2_level {k in 1..K}:
Xt_min[k]*Aa<= x2a[k] <= (Xt_max[k]*Aa+fx12r2[k]); # reservoir limits


# Capacities left for the other owner

subject to A2_discharge_transferred {k in 1..K}:
d21r2[k]=(Ut_max[k]*Aa-u2a[k]);

subject to A2_level_transferred {k in 1..K}:
x21r2[k]=(Xt_max[k]*Aa-x2a[k]);

# CONSTRAINTS DUE TO INITIAL AND FINAL CONDITIONS

subject to A2_initial_condition:
x2a[0]=Xa0; # reservoir initial condition

subject to A2_final_condition:
x2a[K]= XtK*Aa; # reservoir level for week K given
```

Both owners are limited by their ownership for the production capacity. If one of them is allowed to use the other one production capacity, $+fdijr1$ and $+fdijr2$ have to be added in the right side of the production constraint for the concerned owner.

D.3 Script of commands

```
reset;
option solution_round 1; # 1 decimal for the results

# Model
model M:\dokument\master_thesis\loop\ite_verif.mod;

# Data
# Price and spillage penalties
table Datas IN "ODBC" "M:\dokument\master_thesis\loop\data.xls": [TIME],P,
    Pspill;
read table Datas;
# Reservoir initial and final level, ownership
data M:\dokument\master_thesis\loop\param.dat;
# Initially fd21r2 and fx21r2 are set to 0
table transfer_A2 "ODBC" "M:\dokument\master_thesis\loop\results\
    trans_A2_init.xls": [TIME], fd21r2, fx21r2;

set SCENARIOS=19..19;
param num_ite {SCENARIOS};

# Inflow data
table Inflow {i in SCENARIOS} IN "ODBC" ("M:\dokument\master_thesis\loop\
    Inflow_sce\scenario" & i & ".xls"): [TIME],Tt;

# Table for the results
table res {i in SCENARIOS} IN "ODBC" ("M:\dokument\master_thesis\loop\
    results\res_sce" & i & ".xls"): [TIME], xs OUT,us OUT,S_balance OUT,vs
    OUT,d12r1 OUT,x12r1 OUT,xa OUT,ua OUT,A_balance OUT,va OUT,d21r1 OUT,
    x21r1 OUT;

# Problem definition
problem owner_s: xs,vs,ss,us,d12r1,x12r1,S_profit,S_balance,S_production,
    S_bypass,S_level,S_discharge_transferred,S_level_transferred,
    S_initial_condition,S_final_condition;
problem owner_a: xa,va,sa,ua,d21r1,x21r1,A_profit,A_balance,A_production,
    A_bypass,A_level,A_discharge_transferred,A_level_transferred,
    A_initial_condition,A_final_condition;
problem owner_s2: x2s,v2s,s2s,u2s,d12r2,x12r2,S2_profit,S2_balance,
    S2_production,S2_bypass,S2_level,S2_discharge_transferred,
    S2_level_transferred,S2_initial_condition,S2_final_condition;
problem owner_a2: x2a,v2a,s2a,u2a,d21r2,x21r2,A2_profit,A2_balance,
    A2_production,A2_bypass,A2_level,A2_discharge_transferred,
    A2_level_transferred,A2_initial_condition,A2_final_condition;

param j symbolic default 2;
param t default 0;

for {i in SCENARIOS} {

  # The value for the flux between a and s run 2 are intialized to 0
  reset data fd21r2,fx21r2;
```

```
read table transfer_A2;
display fd21r2,fx21r2;

let j:=2;

reset data Tt;
read table Inflow[i];
display Tt;

solve owner_s;
display xs,us,S_balance,vs,d12r1,x12r1;

# Remaining capacities after s optimization1 are transferred as
    parameters for owner a optimization1

for {k in 1..K}{

  if d12r1[k]>0 then let fd12r1[k]:= d12r1[k];
  else let fd12r1[k]:= 0;
    }
for {k in 1..K} {

  if x12r1[k]>0 then let fx12r1[k]:= x12r1[k];
  else let fx12r1[k]:=0;
    }

solve owner_a;
display xa,ua,A_balance,va,d21r1,x21r1;

# Remaining capacities after a optimization1 are transferred as
    parameters for owner s optimization2

for {k in 1..K}{

  if d21r1[k]>0 then let fd21r1[k]:= d21r1[k];
  else let fd21r1[k]:= 0;
    }
for {k in 1..K} {

  if x21r1[k]>0 then let fx21r1[k]:= x21r1[k];
  else let fx21r1[k]:=0;
    }

solve owner_s2;
display x2s,u2s,S2_balance,v2s,d12r2,x12r2;

# Remaining capacities after s optimization2 are transferred as
    parameters for owner a optimization2

for {k in 1..K}{

  if d12r2[k]>0 then let fd12r2[k]:= d12r2[k];
  else let fd12r2[k]:= 0;
```

```
    }
for {k in 1..K} {

  if x12r2[k]>0 then let fx12r2[k]:= x12r2[k];
  else let fx12r2[k]:=0;
    }

solve owner_a2;
display x2a,u2a,A2_balance,v2a,d21r2,x21r2;

# Remaining capacities after a optimization2 are transferred as
    parameters for owner s optimization3

for {k in 1..K}{

  if d21r2[k]>0 then let fd21r2[k]:= d21r2[k];
  else let fd21r2[k]:= 0;
    }
for {k in 1..K} {

  if x21r2[k]>0 then let fx21r2[k]:= x21r2[k];
  else let fx21r2[k]:=0;
    }

display j;

### Calculations up to the results convergence
repeat conv_loop {

  if exists {k in 1..K} ((round(d12r1[k],1)<>round(d12r2[k],1)) or (round
      (x12r1[k],1)<>round(x12r2[k],1)) or (round(x21r1[k],1)<>round(x21r2
      [k],1)) or (round(d21r1[k],1)<>round(d21r2[k],1))) then {
  # if the flow between run 1 and 2 are not the same, another iteration
      is performed

    solve owner_s;
    display xs,us,S_balance,vs,d12r1,x12r1;


    for {k in 1..K}{

      if d12r1[k]>0 then let fd12r1[k]:= d12r1[k];
      else let fd12r1[k]:= 0;
        }
    for {k in 1..K} {

      if x12r1[k]>0 then let fx12r1[k]:= x12r1[k];
      else let fx12r1[k]:=0;
        }

    solve owner_a;
    display xa,ua,A_balance,va,d21r1,x21r1;
```

```
    for {k in 1..K}{

      if d21r1[k]>0 then let fd21r1[k]:= d21r1[k];
      else let fd21r1[k]:= 0;
        }
    for {k in 1..K} {

      if x21r1[k]>0 then let fx21r1[k]:= x21r1[k];
      else let fx21r1[k]:=0;
        }

    let j:=j+1;

    display j;

  }

  else break conv_loop;

  # The condition is checked again after these optimizations and two new
      optimizations are performed if necessary
  if exists {k in 1..K} ((round(d12r1[k],1)<>round(d12r2[k],1)) or (round
      (x12r1[k],1)<>round(x12r2[k],1)) or (round(x21r1[k],1)<>round(x21r2
      [k],1)) or (round(d21r1[k],1)<>round(d21r2[k],1))) then {

    solve owner_s2;
    display x2s,u2s,S2_balance,v2s,d12r2,x12r2;

    for {k in 1..K}{

      if d12r2[k]>0 then let fd12r2[k]:= d12r2[k];
      else let fd12r2[k]:= 0;
        }
    for {k in 1..K} {

      if x12r2[k]>0 then let fx12r2[k]:= x12r2[k];
      else let fx12r2[k]:=0;
        }

    solve owner_a2;
    display x2a,u2a,A2_balance,v2a,d21r2,x21r2;

    for {k in 1..K}{

      if d21r2[k]>0 then let fd21r2[k]:= d21r2[k];
      else let fd21r2[k]:= 0;
        }
    for {k in 1..K} {

      if x21r2[k]>0 then let fx21r2[k]:= x21r2[k];
      else let fx21r2[k]:=0;
        }
```

```
      let j:=j+1;

      display j;

    }

    else break conv_loop;


    # After these 2 iterations we check the condition again, if true we go
       into the entire loop again
    if exists {k in 1..K} ((round(d12r1[k],1)<>round(d12r2[k],1)) or (round
       (x12r1[k],1)<>round(x12r2[k],1)) or (round(x21r1[k],1)<>round(x21r2
       [k],1)) or (round(d21r1[k],1)<>round(d21r2[k],1))) then let t:=1;

    else let t:=0;

    display t;

  } until (t=0 or j=10);# the loop is stopped when the convergence is reach
      or after 10 iterations (=20 optimizations)
let num_ite[i]:=j;
write table res[i];
}

display num_ite;
```

Appendix E

Sample of detailed results for one reservoir

Reservoir characteristics: Mean annual inflow: 336 Mm3
Energy conversion factor: 1 kWh/m3
Reservoir: 100 Mm3
Installed capacity: 100 MW

Filling season testing: The testing is done during the spring period when the reservoir is filled in from week 20 to 38. The initial physical reservoir level is 30 Mm3 and the final one 85 Mm3.

Depletion season testing: The testing is done during the winter period when the reservoir is emptied out (due to high energy consumption and precipitations coming as snow) from week 45 year i to week 16 year i+1. The initial physical reservoir level is 80 Mm3 and the final one 20 Mm3.

E.1 Scenario 13

Parameters for the tests are listed below:
Ownerships: S 50% A 50%
Corresponding discharge upper limit: S 8.4 A 8.4 Mm3/week
Initial individual levels: S 20 A 10 Mm3
Final global level: 85 Mm3
Both owners' productions limited by their share of the system
Filling season

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	P
20	20						10						
21	18.1	8.4	-1.1	0	0	31.9	8.1	8.4	-1	0	0	41.9	172.3
22	18.8	8.4	-1.1	0	0	31.2	8.8	8.4	-1	0	0	41.2	177.9
23	22	8.4	-1.1	0	0	28	12	8.4	-1	0	0	38	168.3
24	32.1	8.4	-1.1	0	0	17.9	22.1	8.4	-1	0	0	27.9	194.6
25	41.5	8.4	-1.1	0	0	8.5	31.5	8.4	-1	0	0	18.5	204.6
26	49.6	8.4	-1.1	0	0	0.4	39.6	8.4	-1	0	0	10.4	209.0
27	55	8.4	-1.1	0	0	-5	45	8.4	-1	0	0	5	180.1
28	51.8	8.4	-1.1	6.4	0	-1.8	48.2	8.4	-1	0	0	1.8	165.9
29	50	8.4	-1	6.6	0	0	50	8.4	-1	3	0	0	108.7
30	50	8.4	-0.9	3.7	0	0	50	8.4	-0.9	3.7	0	0	107.8
31	50	8.4	-0.8	0.9	0	0	50	8.4	-0.8	0.9	0	0	101.8
32	48.7	8.4	126.8	0	0	1.3	48.7	8.4	126.8	0	0	1.3	127.1
33	50	2.3	126.8	0	6.1	0	50	2.3	126.8	0	6.1	0	126.8
34	50	7.2	129.3	0	1.2	0	50	7.2	129.3	0	1.2	0	129.3
35	50	5.3	173.2	0	3.1	0	50	5.3	173.2	0	3.1	0	173.2
36	46.6	7.5	195.8	0	0.9	3.4	46.6	7.5	195.8	0	0.9	3.4	195.8
37	41.5	8.4	195.8	0	0	8.5	41.5	8.4	195.8	0	0	8.5	207.0
38	42.5	8.4	195.8	0	0	7.5	42.5	8.4	195.8	0	0	7.5	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

The scenario leads to spillage for both owners for weeks 28 to 31 for S and 29 to 31 for A. Owner S uses the reservoir space left by A for weeks 27 and 28. After the level equalization in week 29, both owners use the same strategy as they receive the same price signal. For weeks 33-36, the water values are the week prices for these weeks since the reservoir is full and no storage is possible. Either an extra kWh is produced or wasted.

E.2 Scenario 15

Parameters for the tests are listed below:
Ownerships: S 50% A 50%
Corresponding discharge upper limit: S 8.4 A 8.4 Mm3/week
Initial individual levels: S 3 A 27 Mm3
Final global level: 85 Mm3
Both owners' productions limited by their share of the system
Filling season

The results for this scenario are shown to be compared with the results from the initial modeling described in Chapter 1. Owner S starts the period with a much lower level than A.

Before week 31, S has a greater water value equal to week 32 price due to his low level. His reservoir is used by A for weeks 26 to 31.

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	P
20	3						27						
21	0.4	8.4	127.1	0	0	49.6	24.4	8.4	101.8	0	0	25.6	172.3
22	2.3	8.4	127.1	0	0	47.7	26.3	8.4	101.8	0	0	23.7	177.9
23	10.8	8.4	127.1	0	0	39.2	34.8	8.4	101.8	0	0	15.2	168.3
24	12.8	8.4	127.1	0	0	37.2	36.8	8.4	101.8	0	0	13.2	194.6
25	18.6	8.4	127.1	0	0	31.4	42.6	8.4	101.8	0	0	7.4	204.6
26	27.3	8.4	127.1	0	0	22.7	51.3	8.4	101.8	0	0	-1.3	209.0
27	29.8	8.4	127.1	0	0	20.2	53.8	8.4	101.8	0	0	-3.8	180.1
28	31.7	8.4	127.1	0	0	18.3	55.7	8.4	101.8	0	0	-5.7	165.9
29	41	0	127.1	0	8.4	9	56.6	8.4	101.8	0	0	-6.6	108.7
30	46.3	0	127.1	0	8.4	3.7	53.5	8.4	101.8	0	0	-3.5	107.8
31	49.3	0	127.1	0	8.4	0.7	50.7	5.8	101.8	0	2.6	-0.7	101.8
32	46.8	6	127.1	0	2.4	3.2	46.8	7.4	127.1	0	1	3.2	127.1
33	50	0	127.1	0	8.4	0	50	0	127.1	0	8.4	0	126.8
34	50	2.4	129.3	0	6	0	50	2.4	129.3	0	6	0	129.3
35	50	2.4	173.2	0	6	0	50	2.4	173.2	0	6	0	173.2
36	50	1.2	195.8	0	7.2	0	50	1.2	195.8	0	7.2	0	195.8
37	46.5	4.4	207	0	4	3.5	46.5	4.4	207	0	4	3.5	207.0
38	42.5	8.4	207	0	0	7.5	42.5	8.4	207	0	0	7.5	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

E.3 Scenario 41

Parameters for the tests are listed below:
Ownerships: S 30% A 70%
Corresponding discharge upper limit: S 5 A 11.8 Mm3/week
Initial individual levels: S 5 A 25 Mm3
Final global level: 85 Mm3
Both owners' productions limited by their share of the system
Filling season

Owner A gets overflow for this scenario despite the fact that he owns 70% of the system. He starts with a high level and his discharge capacity is not enough to cover the 70% of the total inflow he receives. The situation is better for S that has a water value corresponding to the price in week 30 since his discharge it not maximum this week. If one more kWh is available, its best opportunity to be drawn is during week 30.

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	P
20	5						25						
21	0.6	5	107.8	0	0	29.4	14.7	11.8	-1.4	0	0	55.3	172.3
22	6	5	107.8	0	0	24	27.3	11.8	-1.4	0	0	42.7	177.9
23	16.8	5	107.8	0	0	13.2	52.6	11.8	-1.4	0	0	17.4	168.3
24	24.7	5	107.8	0	0	5.3	70.9	11.8	-1.4	0	0	-0.9	194.6
25	26.8	5	107.8	0	0	3.2	73.2	11.8	-1.4	2.7	0	-3.2	204.6
26	27.9	5	107.8	0	0	2.1	72.1	11.8	-1.3	3.5	0	-2.1	209.0
27	28.6	5	107.8	0	0	1.4	71.4	11.8	-1.2	2.3	0	-1.4	180.1
28	28.5	5	107.8	0	0	1.5	71.2	11.8	107.8	0	0	-1.2	165.9
29	27.3	5	107.8	0	0	2.7	68.5	11.8	107.8	0	0	1.5	108.7
30	27.3	2.3	107.8	0	2.8	2.7	63.7	10.1	107.8	0	1.7	6.3	107.8
31	30	0	107.8	0	5	0	70	0	107.8	0	11.8	0	101.8
32	28	5	126.8	0	0	2	65.2	11.8	126.8	0	0	4.8	127.1
33	30	0.7	126.8	0	4.3	0	70	1.6	126.8	0	10.1	0	126.8
34	30	2.8	129.3	0	2.2	0	70	6.6	129.3	0	5.2	0	129.3
35	30	4.6	173.2	0	0.5	0	70	10.7	173.2	0	1.1	0	173.2
36	30	2.3	195.8	0	2.7	0	70	5.4	195.8	0	6.4	0	195.8
37	28.1	2.7	207	0	2.3	1.9	65.6	6.4	207	0	5.4	4.4	207.0
38	25.5	5	207	0	0	4.5	59.5	11.8	207	0	0	10.5	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

E.4 Scenario 25

Parameters for the tests are listed below:
Ownerships: S 40% A 60%
Corresponding discharge upper limit: S 6.7 A 10.1 Mm3/week
Initial individual levels: S 18 A 12 Mm3
Final global level: 85 Mm3
Both owners's productions limited by their share of the system
Filling season

This scenario highlights that the solution is not optimal for the physical system when both owners are limited by their discharge capacity. Indeed, owner S makes an extensive utilization of his discharge capacity from week 21 to 30 to avoid overflow. Owner A, on the contrary, has a high water value since he does not produce at maximum capacity weeks 21, 23 and 30. Let's see what happens if S can use this capacity.

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	P
20	18						12						
21	11.9	6.7	101.8	0	0	28.1	6.2	6.7	172.3	0	3.4	53.8	172.3
22	7.7	6.7	101.8	0	0	32.3	0	10.1	172.3	0	0	60	177.9
23	5.9	6.7	101.8	0	0	34.1	2.4	4.9	168.3	0	5.2	57.6	168.3
24	4.3	6.7	101.8	0	0	35.7	0	10.1	168.3	0	0	60	194.6
25	6.6	6.7	101.8	0	0	33.4	3.4	10.1	107.8	0	0	56.6	204.6
26	14.2	6.7	101.8	0	0	25.8	14.8	10.1	107.8	0	0	45.2	209.0
27	21.5	6.7	101.8	0	0	18.5	25.8	10.1	107.8	0	0	34.2	180.1
28	30.9	6.7	101.8	0	0	9.1	39.9	10.1	107.8	0	0	20.1	165.9
29	36.2	6.7	101.8	0	0	3.8	47.9	10.1	107.8	0	0	12.1	108.7
30	36.8	6.7	101.8	0	0	3.2	52.7	6.2	107.8	0	3.9	7.3	107.8
31	40	1.7	101.8	0	5	0	60	0	107.8	0	10.1	0	101.8
32	37	6.6	127.1	0	0.1	3	55.6	9.9	127.1	0	0.2	4.4	127.1
33	40	0	127.1	0	6.7	0	60	0	127.1	0	10.1	0	126.8
34	40	2.7	129.3	0	4	0	60	4.1	129.3	0	6	0	129.3
35	40	1.8	173.2	0	5	0	60	2.7	173.2	0	7.4	0	173.2
36	40	2.5	195.8	0	4.2	0	60	3.7	195.8	0	6.3	0	195.8
37	38.2	4	207	0	2.8	1.8	57.3	5.9	207	0	4.1	2.7	207.0
38	34	6.7	207	0	0	6	51	10.1	207	0	0	9	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

In the next table, the testing is done in the same conditions but S is allowed to use A remaining capacity. It occurs week 21 and 23 where S uses all that was left by A. Compare to the previous situation S has higher water values and his profits increase by 2% while the one for A stay constants.

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	P
20	18						12						
21	8.5	10.1	168.3	0	-3.4	31.5	6.2	6.7	172.3	0	3.4	53.8	172.3
22	4.3	6.7	168.3	0	0	35.7	0	10.1	172.3	0	0	60	177.9
23	1.6	7.6	168.3	0	-0.9	38.4	2.4	4.9	168.3	0	5.2	57.6	168.3
24	0	6.7	168.3	0	0	40	0	10.1	168.3	0	0	60	194.6
25	2.3	6.7	107.8	0	0	37.7	3.4	10.1	107.8	0	0	56.6	204.6
26	9.9	6.7	107.8	0	0	30.1	14.8	10.1	107.8	0	0	45.2	209.0
27	17.2	6.7	107.8	0	0	22.8	25.8	10.1	107.8	0	0	34.2	180.1
28	26.6	6.7	107.8	0	0	13.4	39.9	10.1	107.8	0	0	20.1	165.9
29	31.9	6.7	107.8	0	0	8.1	47.9	10.1	107.8	0	0	12.1	108.7
30	35.1	4.1	107.8	0	2.6	4.9	52.7	6.2	107.8	0	3.9	7.3	107.8
31	40	0	107.8	0	6.7	0	60	0	107.8	0	10.1	0	101.8
32	37	6.6	127.1	0	0.1	3	55.6	9.9	127.1	0	0.2	4.4	127.1
33	40	0	127.1	0	6.7	0	60	0	127.1	0	10.1	0	126.8
34	40	2.7	129.3	0	4	0	60	4.1	129.3	0	6	0	129.3
35	40	1.8	173.2	0	5	0	60	2.7	173.2	0	7.4	0	173.2
36	40	2.5	195.8	0	4.2	0	60	3.7	195.8	0	6.3	0	195.8
37	38.2	4	207	0	2.8	1.8	57.3	5.9	207	0	4.1	2.7	207.0
38	34	6.7	207	0	0	6	51	10.1	207	0	0	9	211.0

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

E.5 Scenario 5

Parameters for the tests are listed below:
Ownerships: S 50% A 50%
Corresponding discharge upper limit: S 8.4 A 8.4 Mm3/week
Initial individual levels: S 45 A 35 Mm3
Final global level: 20 Mm3
Owner S can use A's discharge capacity if some is left
Depletion season

During the depletion season there is little inflow coming into the reservoir. The two owners do not produce much energy during these period and choose the best moment to use the small amount of water they have. In this scenario, the water values are the same - price in week 2 - for the two owners and owner S uses A's capacity this week. He has more water at the beginning so he can produce more during the period.

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	P
45	45						35						
46	46.1	0	383.3	0	8.4	3.9	36.1	0	383.3	0	8.4	13.9	370.3
47	38.9	7.8	383.3	0	0.6	11.1	36.7	0	383.3	0	8.4	13.3	383.3
48	31.1	8.4	383.3	0	0	18.9	28.8	8.4	383.3	0	0	21.2	392.4
49	31.8	0	383.3	0	8.4	18.2	29.6	0	383.3	0	8.4	20.4	362.6
50	23.8	8.4	383.3	0	0	26.2	21.6	8.4	383.3	0	0	28.4	388.7
51	15.8	8.4	383.3	0	0	34.2	13.5	8.4	383.3	0	0	36.5	386.4
52	16.1	0	383.3	0	8.4	33.9	13.8	0	383.3	0	8.4	36.2	340.9
1	16.4	0	383.3	0	8.4	33.6	14.1	0	383.3	0	8.4	35.9	377.7
2	7.1	9.5	383.3	0	-1.1	42.9	7.1	7.3	383.3	0	1.1	42.9	383.3
3	7.6	0	383.3	0	8.4	42.4	7.6	0	383.3	0	8.4	42.4	358.6
4	8	0	383.3	0	8.4	42	8	0	383.3	0	8.4	42	351.0
5	8.3	0	383.3	0	8.4	41.7	8.3	0	383.3	0	8.4	41.7	329.3
6	8.5	0	383.3	0	8.4	41.5	8.5	0	383.3	0	8.4	41.5	320.8
7	8.7	0	383.3	0	8.4	41.3	8.7	0	383.3	0	8.4	41.3	339.4
8	8.9	0	383.3	0	8.4	41.1	8.9	0	383.3	0	8.4	41.1	294.0
9	9	0	383.3	0	8.4	41	9	0	383.3	0	8.4	41	245.0
10	9.1	0	383.3	0	8.4	40.9	9.1	0	383.3	0	8.4	40.9	251.4
11	9.2	0	383.3	0	8.4	40.8	9.2	0	383.3	0	8.4	40.8	215.9
12	9.4	0	383.3	0	8.4	40.6	9.4	0	383.3	0	8.4	40.6	219.3
13	9.6	0	383.3	0	8.4	40.4	9.6	0	383.3	0	8.4	40.4	259.4
14	9.8	0	383.3	0	8.4	40.2	9.8	0	383.3	0	8.4	40.2	260.0
15	9.9	0	383.3	0	8.4	40.1	9.9	0	383.3	0	8.4	40.1	318.3
16	10	0	383.3	0	8.4	40	10	0	383.3	0	8.4	40	350.6

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

E.6 Scenario 16

Parameters for the tests are listed below:
Ownerships: S 30% A 70%
Corresponding discharge upper limit: S 5 A 11.8 Mm3/week
Initial individual levels: S 30 A 50 Mm3
Final global level: 20 Mm3
Owner S can use A's discharge capacity if some is left
Depletion season

This scenario is very similar to scenario 5. See previous section.

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	P
45	30						50						
46	30.9	0	383.3	0	5	-0.9	52	0	383.3	0	11.8	18	370.3
47	20.7	10.8	383.3	0	-5.8	9.3	48.2	5.2	383.3	0	6.6	21.8	383.3
48	16.5	5	383.3	0	0	13.5	38.6	11.8	383.3	0	0	31.4	392.4
49	16.9	0	383.3	0	5	13.1	39.4	0	383.3	0	11.8	30.6	362.6
50	12.1	5	383.3	0	0	17.9	28.2	11.8	383.3	0	0	41.8	388.7
51	7.3	5	383.3	0	0	22.7	17	11.8	383.3	0	0	53	386.4
52	7.6	0	383.3	0	5	22.4	17.6	0	383.3	0	11.8	52.4	340.9
1	7.7	0	383.3	0	5	22.3	18	0	383.3	0	11.8	52	377.7
2	2.9	5	383.3	0	0	27.1	6.7	11.8	383.3	0	0	63.3	383.3
3	3.1	0	383.3	0	5	26.9	7.3	0	383.3	0	11.8	62.7	358.6
4	3.4	0	383.3	0	5	26.6	7.8	0	383.3	0	11.8	62.2	351
5	3.6	0	383.3	0	5	26.4	8.4	0	383.3	0	11.8	61.6	329.3
6	3.8	0	383.3	0	5	26.2	8.9	0	383.3	0	11.8	61.1	320.8
7	4	0	383.3	0	5	26	9.4	0	383.3	0	11.8	60.6	339.4
8	4.2	0	383.3	0	5	25.8	9.8	0	383.3	0	11.8	60.2	294
9	4.4	0	383.3	0	5	25.6	10.2	0	383.3	0	11.8	59.8	245
10	4.5	0	383.3	0	5	25.5	10.5	0	383.3	0	11.8	59.5	251.4
11	4.6	0	383.3	0	5	25.4	10.8	0	383.3	0	11.8	59.2	215.9
12	4.8	0	383.3	0	5	25.2	11.1	0	383.3	0	11.8	58.9	219.3
13	4.9	0	383.3	0	5	25.1	11.5	0	383.3	0	11.8	58.5	259.4
14	5.1	0	383.3	0	5	24.9	12	0	383.3	0	11.8	58	260
15	5.4	0	383.3	0	5	24.6	12.7	0	383.3	0	11.8	57.3	318.3
16	6	0	383.3	0	5	24	14	0	383.3	0	11.8	56	350.6

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

E.7 Scenario 26

Parameters for the tests are listed below:
Ownerships: S 20% A 80%
Corresponding discharge upper limit: S 3.4 A 13.4 Mm3/week
Initial individual levels: S 50 A 30 Mm3
Final global level: 20 Mm3
Owner S can use A's discharge capacity if some is left
Depletion season

The results of scenario 26 are a bit more exciting than the one for scenario 5 and 16 and deserves much more comments. Owner S starts with a really high reservoir level compared to his share of the system therefore he uses a part of A's virtual reservoir up to week 52. He also has more water to produce and he uses his discharge as well as the one from A week 47, 51, 1 and 2 to get the highest prices.
His water value for the whole period is 377.7 NOK/MWh which corresponds to price in week 1. It is possible for him to go for an extra kWh this week assuming that owner A does not want to do so.

Even though owner S uses all what is left by A week 51 (the sum of the owner discharges is 16.8 Mm3/week), A still has the priority to use his production if he had more water to produce another marginal kWh. This is reflected by his water value up to week 51. The same phenomenon takes place week 2.

It can seem strange that owner A's water value is not 386.4 NOK/MWh for week 52 and 1. Let's take an example. For owner S the water value week 12 is 377.7 NOK/MWh. It means what if owner S knows that an extra kWh can be stored in the reservoir week 12 he would have produced one extra kWh week 1. This is not possible for owner A week 52 and 1. If an extra kWh is stored these weeks, there is no possibility to produce it week 51 since the reservoir is empty.

Week	xs	us	Wv S	vs	d12r1	x12r1	xa	ua	Wv A	va	d21r1	x21r1	Prices
45	50						30						
46	50.3	0	377.7	0	3.4	-30.3	31.1	0	386.4	0	13.4	48.9	370.3
47	33.8	16.8	377.7	0	-13.4	-13.8	32.1	0	386.4	0	13.4	47.9	383.3
48	30.7	3.4	377.7	0	0	-10.7	20	13.4	386.4	0	0	60	392.4
49	31.2	0	377.7	0	3.4	-11.2	21.7	0	386.4	0	13.4	58.3	362.6
50	28.2	3.4	377.7	0	0	-8.2	9.8	13.4	386.4	0	0	70.2	388.7
51	23.1	5.5	377.7	0	-2.1	-3.1	0	11.4	386.4	0	2.1	80	386.4
52	23.8	0	377.7	0	3.4	-3.8	2.6	0	383.3	0	13.4	77.4	340.9
1	9.6	14.4	377.7	0	-11	10.4	3.4	0	383.3	0	13.4	76.6	377.7
2	0.1	10.4	377.7	0	-7	19.9	0.6	6.4	383.3	0	7	79.4	383.3
3	0.5	0	377.7	0	3.4	19.5	2.1	0	383.3	0	13.4	77.9	358.6
4	1	0	377.7	0	3.4	19	4	0	383.3	0	13.4	76	351
5	1.3	0	377.7	0	3.4	18.7	5.3	0	383.3	0	13.4	74.7	329.3
6	1.6	0	377.7	0	3.4	18.4	6.6	0	383.3	0	13.4	73.4	320.8
7	1.9	0	377.7	0	3.4	18.1	7.6	0	383.3	0	13.4	72.4	339.4
8	2.1	0	377.7	0	3.4	17.9	8.4	0	383.3	0	13.4	71.6	294
9	2.3	0	377.7	0	3.4	17.7	9	0	383.3	0	13.4	71	245
10	2.4	0	377.7	0	3.4	17.6	9.5	0	383.3	0	13.4	70.5	251.4
11	2.7	0	377.7	0	3.4	17.3	10.9	0	383.3	0	13.4	69.1	215.9
12	2.9	0	377.7	0	3.4	17.1	11.6	0	383.3	0	13.4	68.4	219.3
13	3.1	0	377.7	0	3.4	16.9	12.5	0	383.3	0	13.4	67.5	259.4
14	3.5	0	377.7	0	3.4	16.5	14.1	0	383.3	0	13.4	65.9	260
15	3.8	0	377.7	0	3.4	16.2	15.1	0	383.3	0	13.4	64.9	318.3
16	4	0	377.7	0	3.4	16	16	0	383.3	0	13.4	64	350.6

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh)

The maximum number of iterations to reach a stable solution was 3 for all these tests.

Appendix F

Extreme scenario testing for two cascaded reservoirs

The reservoirs are Roskrepp and Kvinen. The parameters and the restrictions for each owner are given in the following table:

		Roskrepp				
	Ownership	Xt_{max}	$X_{initial}$	$X_{final}(dry)$	$X_{final}(wet)$	Ut_{max}
Statkraft S	59 %	403.6	156.2	193.3	368.3	24.6
Lyse A	41 %	280.5	108.6	134.3	255.9	17.1
Total	100 %	684.1	264.8	327.6	624.2	41.7

x- reservoir level (Mm3/week) u- discharge (Mm3/week)

		Kvinen				
	Ownership	Xt_{max}	$X_{initial}$	$X_{final}(dry)$	$X_{final}(wet)$	Ut_{max}
Statkraft S	59 %	61.4	0	0	46.7	27.5
Lyse A	41 %	42.7	0	0	32.5	19.1
Total	100 %	104.1	0	0	79.2	46.6

x- reservoir level (Mm3/week) u- discharge (Mm3/week)

The two owners start and end up the period with reservoir levels equal to their share of the system times the global desired level.
$X_{final}(wet)$ is the final reservoir level considering the inflow scenario in year 1990 while $X_{final}(dry)$ is the one in year 1996.
Both owners's productions are limited by their share of the system.

F.1 Extremely wet year - 1990

Due to the high inflow amount, the final desired energy in the system is 90%. The end of period levels are given in the previous table. As the energy equivalent of both plants are not equal to 1kWh/m3, the understanding of the water values is slightly harder than before. The prices times the energy equivalent referred to sea level - 0.486 kWh/m3 for Roskrepp and 0.285 kWh/m3 for Kvinen, are calculated in the last column of the result tables.

To begin with, the results for Kvinen are explained since the water values for Roskrepp strongly depend on what happens downstream at Kvinen power plant. The strategy for the two owners is the same. They produce at maximum capacity when the best prices are expected in week 24, 25, 26, 37 and 38. Some capacity is still available in week 36 which explains the water values in week 36, 37 and 38. For the first weeks water values, the explanation is rather obscure.
The reservoir reaches his limits in week 35. It means that an extra kWh available between week 1 and 35 cannot be drawn later in the period, let's say in week 36, because it cannot be stored. There is either full production or no production at all in the weeks before. So, the model do not "see" another price to take more water out instead, it considers that an extra kWh cannot be produced for the price in week 36, but for a price a bit below it.

Kvinen												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	0					0						
21	8.4	0	54.1	27.5	53	5.9	0	54.1	19.1	36.8	172.3	49.1
22	18.3	0	54.1	27.5	43.1	12.7	0	54.1	19.1	30	177.9	50.7
23	32.2	0	54.1	27.5	29.2	22.4	0	54.1	19.1	20.3	168.3	48.0
24	17.4	27.5	54.1	0	44	12.1	19.1	54.1	0	30.6	194.6	55.5
25	11.1	27.5	54.1	0	50.3	7.7	19.1	54.1	0	35	204.6	58.3
26	16.7	27.5	54.1	0	44.7	11.6	19.1	54.1	0	31.1	209.0	59.6
27	27	0	54.1	27.5	34.4	18.8	0	54.1	19.1	23.9	180.1	51.3
28	33.9	0	54.1	27.5	27.6	23.5	0	54.1	19.1	19.2	165.9	47.3
29	39.5	0	54.1	27.5	21.9	27.5	0	54.1	19.1	15.2	108.7	31.0
30	43.6	0	54.1	27.5	17.8	30.3	0	54.1	19.1	12.4	107.8	30.7
31	47.7	0	54.1	27.5	13.7	33.1	0	54.1	19.1	9.5	101.8	29.0
32	50.4	0	54.1	27.5	11	35	0	54.1	19.1	7.7	127.1	36.2
33	54.7	0	54.1	27.5	6.7	38	0	54.1	19.1	4.7	126.8	36.1
34	57.9	0	54.1	27.5	3.6	40.2	0	54.1	19.1	2.5	129.3	36.9
35	61.4	0	54.1	27.5	0	42.7	0	54.1	19.1	0	173.2	49.4
36	45.7	21.5	55.8	6	15.7	31.8	15	55.8	4.1	10.9	195.8	55.8
37	44.6	27.5	55.8	0	16.8	31	19.1	55.8	0	11.7	207.0	59.0
38	46.7	27.5	55.8	0	14.7	32.5	19.1	55.8	0	10.2	211.0	60.1

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

Roskrepp												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	156.2					108.6						
21	176.4	0	95.2	24.6	227.2	122.7	0	95.2	17.1	157.8	172.3	83.7
22	200.2	0	95.2	24.6	203.4	139.2	0	95.2	17.1	141.3	177.9	86.5
23	233.6	0	95.2	24.6	170	162.4	0	95.2	17.1	118.1	168.3	81.8
24	264.1	0	95.2	24.6	139.5	183.6	0	95.2	17.1	96.9	194.6	94.6
25	288.2	7.8	95.2	16.8	115.4	200.3	5.4	95.2	11.7	80.1	204.6	99.4
26	284.1	24.6	95.2	0	119.6	197.5	17.1	95.2	0	83	209.0	101.6
27	308.8	0	95.2	24.6	94.8	214.7	0	95.2	17.1	65.8	180.1	87.5
28	325.3	0	95.2	24.6	78.3	226.1	0	95.2	17.1	54.4	165.9	80.6
29	338.9	0	95.2	24.6	64.7	235.6	0	95.2	17.1	44.9	108.7	52.8
30	348.7	0	95.2	24.6	55	242.3	0	95.2	17.1	38.1	107.8	52.4
31	358.5	0	95.2	24.6	45.1	249.2	0	95.2	17.1	31.3	101.8	49.5
32	365.1	0	95.2	24.6	38.6	253.7	0	95.2	17.1	26.7	127.1	61.8
33	375.4	0	95.2	24.6	28.3	260.9	0	95.2	17.1	19.6	126.8	61.6
34	383	0	95.2	24.6	20.7	266.2	0	95.2	17.1	14.3	129.3	62.8
35	391.5	0	95.2	24.6	12.1	272.1	0	95.2	17.1	8.4	173.2	84.2
36	401.2	1.3	95.2	23.3	2.5	278.8	0.9	95.2	16.2	1.7	195.8	95.2
37	380.8	24.6	95.2	0	22.8	264.6	17.1	95.2	0	15.9	207.0	100.6
38	368.3	24.6	95.2	0	35.3	255.9	17.1	95.2	0	24.6	211.0	102.5

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

Since Roskrepp is the reservoir on the top, its water values are higher since the water can also be drawn from Kvinen, the water is used twice. The production is maximum when the best prices are expected in week 26, 37 and 38. The water value is 95.2 NOK/Mm3, the best use of an extra kWh is to be stored for production in week 36 in Roskrepp and taked out from kvinen straight afterwards the same week. This is not obvious that this gives the best utilization of the water and one can calculate what would have happened if an extra kWh would have been produced in week 25 from Roskrepp. If a kWh is produced in week 25 from Kvinen, it cannot be taken out before week 27 in Kvinen since the production is maximum for the week in between and it cannot get an higher price later in the period since Kvinen is full in week 35. So, it's value would have been $204.6 \times 0.201 + 180.1 \times 0.285 = 92.4$ NOK/Mm3 which is lower than the one achieved by taking out the water in week 36.

F.2 Extremely dry year - 1996

With respect to the poor amount of water coming in the reservoir, the final energy level is set to 40%. The production is still very low. Even if the same prices were used for both scenarios, the driest year leads to the higher water values.

The water values for Kvinen match the price in week 37. An extra kWh can be stored every moment in the period since the reservoir is almost empty and be produced in that week.

Kvinen												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	0					0						
21	1.5	0	59	27.5	59.9	1.1	0	59	19.1	41.6	172.3	49.1
22	5.2	0	59	27.5	56.3	3.6	0	59	19.1	39.1	177.9	50.7
23	9.5	0	59	27.5	51.9	6.6	0	59	19.1	36.1	168.3	48.0
24	14.2	0	59	27.5	47.2	9.8	0	59	19.1	32.8	194.6	55.5
25	16.5	0	59	27.5	44.9	11.5	0	59	19.1	31.2	204.6	58.3
26	7.8	27.5	59	0	53.6	5.5	19.1	59	0	37.2	209.0	59.6
27	9.4	0	59	27.5	52	6.6	0	59	19.1	36.1	180.1	51.3
28	11.2	0	59	27.5	50.2	7.8	0	59	19.1	34.8	165.9	47.3
29	12	0	59	27.5	49.4	8.4	0	59	19.1	34.3	108.7	31.0
30	12.8	0	59	27.5	48.6	9	0	59	19.1	33.7	107.8	30.7
31	14.9	0	59	27.5	46.5	10.4	0	59	19.1	32.3	101.8	29.0
32	16	0	59	27.5	45.4	11.2	0	59	19.1	31.5	127.1	36.2
33	17.1	0	59	27.5	44.3	11.9	0	59	19.1	30.8	126.8	36.1
34	18.8	0	59	27.5	42.7	13.1	0	59	19.1	29.6	129.3	36.9
35	20.8	0	59	27.5	40.6	14.5	0	59	19.1	28.2	173.2	49.4
36	21.5	0	59	27.5	39.9	15	0	59	19.1	27.7	195.8	55.8
37	2.8	20.1	59	7.4	58.6	1.9	14	59	5.1	40.8	207.0	59.0
38	0	27.5	59	0	61.4	0	19.1	59	0	42.7	211.0	60.1

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

A lowest amount of water is taken out from the upper reservoir. Since it has a better degree of regulation, it stores more water. The interpreting of the water values is not straightforward since the water seems to be drawn from the two reservoirs within two different weeks.
Let's say that one Mm3 is drawn from Roskrepp in week 26 - the week where it is still possible to take out more water for the second best price of the period. It cannot be produced directly in Kvinen since the production is maximum but it is stored until week 37. This combination would give a water value of $209 \times 0.201 + 207 \times 0.285 = 101$ NOK/Mm3.

Roskrepp												
Week	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	P*e
20	156.2					108.6						
21	159.9	0	101	24.6	243.7	111.2	0	101	17.1	169.3	172.3	83.7
22	168.6	0	101	24.6	235	117.2	0	101	17.1	163.3	177.9	86.5
23	179	0	101	24.6	224.6	124.5	0	101	17.1	156	168.3	81.8
24	190.2	0	101	24.6	213.4	132.3	0	101	17.1	148.2	194.6	94.6
25	195.8	0	101	24.6	207.8	136.1	0	101	17.1	144.4	204.6	99.4
26	181.5	17.5	101	7.1	222.1	126.1	12.2	101	4.9	154.4	209.0	101.6
27	185.3	0	101	24.6	218.3	128.8	0	101	17.1	151.7	180.1	87.5
28	189.6	0	101	24.6	214	131.7	0	101	17.1	148.7	165.9	80.6
29	191.5	0	101	24.6	212.1	133.1	0	101	17.1	147.4	108.7	52.8
30	193.6	0	101	24.6	210.1	134.5	0	101	17.1	146	107.8	52.4
31	198.5	0	101	24.6	205.1	137.9	0	101	17.1	142.6	101.8	49.5
32	201.2	0	101	24.6	202.4	139.8	0	101	17.1	140.7	127.1	61.8
33	203.7	0	101	24.6	199.9	141.6	0	101	17.1	138.9	126.8	61.6
34	207.8	0	101	24.6	195.8	144.4	0	101	17.1	136.1	129.3	62.8
35	212.7	0	101	24.6	190.9	147.8	0	101	17.1	132.6	173.2	84.2
36	214.3	0	101	24.6	189.3	148.9	0	101	17.1	131.6	195.8	95.2
37	217.6	0	101	24.6	186	151.2	0	101	17.1	129.3	207.0	100.6
38	193.3	24.6	101	0	210.3	134.3	17.1	101	0	146.2	211.0	102.5

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

The results for these two scenarios were achieved in two iterations.

Appendix G

Sample of results for four cascaded reservoirs

The results presented in this section are just here to document the testing but are not commented. The testing for scenario 44 was performed when A was limited by his share of the system and when he is allowed to take the capacity unused by S. It proves that the handling of the reservoirs is the same for S in both situations and that it results in a much better utilization of the water for owner A.

G.1 Scenario 32

The testing is done for scenario 32 and the parameters listed below:

	Roskrepp		Kvinen		Nesjen		Homstølvann	
	X_{ini}	X_{final}	X_{ini}	X_{final}	X_{ini}	X_{final}	X_{ini}	X_{final}
S	185.6	307	0.0	21.2	0.0	36.5	0.0	0.0
A	179.9	213.3	0.0	14.8	0.0	25.4	0.0	0.0
Tot	365.5	520.4	0.0	36.0	0.0	61.9	0.0	0.0

x- reservoir level (Mm3/week)

Roskrepp

TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	185.6					179.9						
21	200.8	0	191.6	24.6	202.8	190.5	0	178.6	17.1	90	172.3	171.4
22	224	0	191.6	24.6	179.6	206.6	0	178.6	17.1	73.9	177.9	177.0
23	245.1	0	191.6	24.6	158.5	221.3	0	178.6	17.1	59.2	168.3	167.4
24	257.3	0	191.6	24.6	146.4	227.2	2.5	178.6	14.5	53.3	194.6	193.7
25	273	0	191.6	24.6	130.6	221	17.1	178.6	0	59.5	204.6	203.6
26	266.2	24.6	191.6	0	137.4	216.3	17.1	178.6	0	64.2	209.0	207.9
27	274.8	0	191.6	24.6	128.8	222.2	0	178.6	17.1	58.2	180.1	179.2
28	285.8	0	191.6	24.6	117.8	229.9	0	178.6	17.1	50.6	165.9	165.1
29	293.3	0	191.6	24.6	110.3	235.1	0	178.6	17.1	45.4	108.7	108.2
30	298.9	0	191.6	24.6	104.8	239	0	178.6	17.1	41.5	107.8	107.3
31	302.6	0	191.6	24.6	101	241.6	0	178.6	17.1	38.9	101.8	101.3
32	308.2	0	191.6	24.6	95.4	245.4	0	178.6	17.1	35	127.1	126.5
33	313	0	191.6	24.6	90.7	248.8	0	178.6	17.1	31.7	126.8	126.2
34	315.9	0	191.6	24.6	87.8	250.8	0	178.6	17.1	29.7	129.3	128.7
35	321.4	0	191.6	24.6	82.3	254.6	0	178.6	17.1	25.9	173.2	172.3
36	324.1	0	191.6	24.6	79.5	239.4	17.1	178.6	0	41.1	195.8	194.9
37	325.2	4.2	191.6	20.4	78.4	226	17.1	178.6	0	54.5	207.0	206.0
38	307.0	24.6	191.6	0	96.6	213.4	17.1	178.6	0	67.1	211.0	209.9

Kvinen

TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					0						
21	6.3	0	150	27.5	55.1	4.4	0	139.5	19.1	38.3	172.3	136.8
22	16	0	150	27.5	45.4	11.1	0	139.5	19.1	31.6	177.9	141.2
23	24.8	0	150	27.5	36.6	17.2	0	139.5	19.1	25.5	168.3	133.6
24	29.8	0	150	27.5	31.6	4.2	19.1	139.5	0	38.5	194.6	154.5
25	13.9	22.5	150	5	47.5	6.7	19.1	139.5	0	36	204.6	162.4
26	18.5	27.5	150	0	43	9.9	19.1	139.5	0	32.8	209.0	165.9
27	22	0	150	27.5	39.4	3.1	9.2	139.5	9.9	39.5	180.1	143.0
28	26.6	0	150	27.5	34.8	6.3	0	139.5	19.1	36.4	165.9	131.8
29	29.7	0	150	27.5	31.7	8.5	0	139.5	19.1	34.2	108.7	86.3
30	32	0	150	27.5	29.4	10.1	0	139.5	19.1	32.6	107.8	85.6
31	33.6	0	150	27.5	27.8	11.2	0	139.5	19.1	31.5	101.8	80.8
32	35.9	0	150	27.5	25.5	12.8	0	139.5	19.1	29.9	127.1	100.9
33	37.9	0	150	27.5	23.5	14.2	0	139.5	19.1	28.5	126.8	100.7
34	39.1	0	150	27.5	22.3	15	0	139.5	19.1	27.7	129.3	102.7
35	41.4	0	150	27.5	20	16.6	0	139.5	19.1	26.1	173.2	137.5
36	42.6	0	150	27.5	18.9	15.4	19.1	139.5	0	27.3	195.8	155.5
37	21.5	27.5	150	0	40	14.9	19.1	139.5	0	27.8	207.0	164.4
38	21.2	27.5	150	0	40.2	14.8	19.1	139.5	0	27.9	211.0	167.5

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

Nesjen												
TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					0						
21	16	0	91.7	37.1	145.9	11.1	0	88.1	25.8	101.4	172.3	87.7
22	40.4	0	91.7	37.1	121.5	2.3	25.8	88.1	0	110.2	177.9	90.5
23	62.6	0	91.7	37.1	99.3	17.7	0	88.1	25.8	94.8	168.3	85.7
24	38.2	37.1	91.7	0	123.7	19.9	25.8	88.1	0	92.6	194.6	99.1
25	40.1	37.1	91.7	0	121.7	24.7	25.8	88.1	0	87.8	204.6	104.1
26	49.2	37.1	91.7	0	112.7	31	25.8	88.1	0	81.5	209.0	106.4
27	28.7	29.5	91.7	7.6	133.1	20.7	25.8	88.1	0	91.8	180.1	91.7
28	40.3	0	91.7	37.1	121.6	28.7	0	88.1	25.8	83.7	165.9	84.5
29	48.1	0	91.7	37.1	113.7	34.2	0	88.1	25.8	78.3	108.7	55.3
30	54	0	91.7	37.1	107.8	38.2	0	88.1	25.8	74.2	107.8	54.9
31	57.9	0	91.7	37.1	103.9	41	0	88.1	25.8	71.5	101.8	51.8
32	63.8	0	91.7	37.1	98.1	45.1	0	88.1	25.8	67.4	127.1	64.7
33	68.8	0	91.7	37.1	93	48.5	0	88.1	25.8	63.9	126.8	64.5
34	71.9	0	91.7	37.1	90	50.7	0	88.1	25.8	61.8	129.3	65.8
35	77.6	0	91.7	37.1	84.2	34.8	19.8	88.1	6	77.6	173.2	88.1
36	43.4	37.1	91.7	0	118.4	30.2	25.8	88.1	0	82.3	195.8	99.7
37	39.4	37.1	91.7	0	122.5	27.4	25.8	88.1	0	85.1	207.0	105.4
38	36.5	37.1	91.7	0	125.3	25.4	25.8	88.1	0	87.1	211.0	107.4
Homstølvann												
TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					0						
21	9.4	0	0	60.7	23.4	6.5	0	0	42.1	16.3	172.3	0
22	19.7	4.2	0	56.5	13.1	13.7	28.7	0	13.5	9.1	177.9	0
23	32.8	0	0	60.7	0	22.8	0	0	42.1	0	168.3	0
24	26.3	51.1	0	9.6	6.5	18.3	35.5	0	6.7	4.5	194.6	0
25	12.5	60.7	0	0	20.3	8.7	42.1	0	0	14.1	204.6	0
26	0	60.7	0	0	32.8	0	42.1	0	0	22.8	209.0	0
27	4.9	29.8	0	30.8	27.9	0	29.4	0	12.7	22.8	180.1	0
28	11.6	0	0	60.7	21.2	4.7	0	0	42.1	18.1	165.9	0
29	16.2	0	0	60.7	16.6	7.8	0	0	42.1	15	108.7	0
30	19.5	0	0	60.7	13.3	10.2	0	0	42.1	12.6	107.8	0
31	21.7	0	0	60.7	11.1	11.7	0	0	42.1	11.1	101.8	0
32	25	0	0	60.7	7.8	14	0	0	42.1	8.8	127.1	0
33	27.9	0	0	60.7	4.9	16	0	0	42.1	6.8	126.8	0
34	29.5	0	0	60.7	3.3	17.1	0	0	42.1	5.7	129.3	0
35	32.8	0	0	60.7	0	22.8	16.4	0	25.7	0	173.2	0
36	32.8	38.7	0	22	0	22.8	26.9	0	15.3	0	195.8	0
37	19.7	53.4	0	7.3	13.1	13.7	37.1	0	5	9.1	207.0	0
38	0	60.7	0	0	32.8	0	42.1	0	0	22.8	211.0	0

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

G.2 Scenario 44

The testing is done for scenario 32 and the parameters listed below:

	Roskrepp		Kvinen		Nesjen		Homstølvann	
	X_{ini}	X_{final}	X_{ini}	X_{final}	X_{ini}	X_{final}	X_{ini}	X_{final}
S	185.6	304.4	0.0	20.2	0.0	33.0	0.0	0.0
A	179.9	211.5	70.0	14.0	0.0	22.9	0.0	0.0
Tot	365.5	515.9	70.0	36.0	0.0	55.9	0.0	0.0

x- reservoir level (Mm3/week)

Roskrepp

TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	185.6					179.9						
21	204.4	0	175.2	24.6	199.2	193	0	127.3	17.1	87.5	172.3	171.4
22	247.3	0	175.2	24.6	156.3	212.6	10.2	127.3	6.9	67.9	177.9	177.0
23	274	0	175.2	24.6	129.6	231.2	0	127.3	17.1	49.3	168.3	167.4
24	297.9	0	175.2	24.6	105.7	230.7	17.1	127.3	0	49.8	194.6	193.7
25	302.4	24.6	175.2	0	101.2	233.8	17.1	127.3	0	46.7	204.6	203.6
26	290.7	24.6	175.2	0	112.9	225.7	17.1	127.3	0	54.8	209.0	207.9
27	308.5	0	175.2	24.6	95.1	220.9	17.1	127.3	0	59.5	180.1	179.2
28	320.5	0	175.2	24.6	83.1	229.3	0	127.3	17.1	51.2	165.9	165.1
29	328.2	0	175.2	24.6	75.4	234.7	0	127.3	17.1	45.8	108.7	108.2
30	333.8	0	175.2	24.6	69.8	238.5	0	127.3	17.1	41.9	107.8	107.3
31	338.4	0	175.2	24.6	65.2	241.7	0	127.3	17.1	38.7	101.8	101.3
32	347.7	0	175.2	24.6	55.9	248.2	0	127.3	17.1	32.3	127.1	126.5
33	350.4	0	175.2	24.6	53.2	250.1	0	127.3	17.1	30.4	126.8	126.2
34	352.3	0	175.2	24.6	51.3	251.4	0	127.3	17.1	29.1	129.3	128.7
35	354.7	0	175.2	24.6	48.9	253	0	127.3	17.1	27.4	173.2	172.3
36	346.6	15.2	175.2	9.4	57	240.8	17.1	127.3	0	39.6	195.8	194.9
37	326.5	24.6	175.2	0	77.1	226.9	17.1	127.3	0	53.6	207.0	206.0
38	304.4	24.6	175.2	0	99.3	211.5	17.1	127.3	0	69.0	211.0	209.9

Kvinen

TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					70						
21	0	7.8	136.8	19.7	61.4	56.3	19.1	91.6	0	-13.7	172.3	136.8
22	16.7	1.1	135.8	26.4	44.7	59.8	19.1	91.6	0	-17.1	177.9	141.2
23	27.9	0	135.8	27.5	33.5	48.4	19.1	91.6	0	-5.8	168.3	133.6
24	10.3	27.5	135.8	0	51.1	53.3	19.1	91.6	0	-10.6	194.6	154.5
25	19.5	27.5	135.8	0	41.9	59.7	19.1	91.6	0	-17.1	204.6	162.4
26	22	27.5	135.8	0	39.4	61.5	19.1	91.6	0	-18.8	209.0	165.9
27	13.2	16.2	135.8	11.2	48.2	64.6	19.1	91.6	0	-21.9	180.1	143.0
28	18.2	0	135.8	27.5	43.3	49	19.1	91.6	0	-6.3	165.9	131.8
29	21.4	0	135.8	27.5	40	51.2	0	91.6	19.1	-8.5	108.7	86.3
30	23.7	0	135.8	27.5	37.7	52.8	0	91.6	19.1	-10.1	107.8	85.6
31	25.6	0	135.8	27.5	35.8	54.2	0	91.6	19.1	-11.5	101.8	80.8
32	29.5	0	135.8	27.5	31.9	52.1	4.7	91.6	14.4	-9.5	127.1	100.9
33	30.6	0	135.8	27.5	30.8	52.9	0	91.6	19.1	-10.3	126.8	100.7
34	31.4	0	135.8	27.5	30	34.4	19.1	91.6	0	8.3	129.3	102.7
35	32.4	0	135.8	27.5	29	16	19.1	91.6	0	26.7	173.2	137.5
36	23	27.5	135.8	0	38.4	16	19.1	91.6	0	26.7	195.8	155.5
37	22	27.5	135.8	0	39.4	15.3	19.1	91.6	0	27.4	207.0	164.4
38	20.1	27.5	135.8	0	41.3	14.0	19.1	91.6	0	28.7	211.0	167.5

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

Nesjen

TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					0						
21	0	27.6	87.7	9.5	161.8	7.1	25.8	55.3	0	105.4	172.3	87.7
22	9	37.1	85.1	0	152.8	31.7	25.8	55.3	0	80.8	177.9	90.5
23	0	37.1	85.1	0	161.8	44.5	25.8	55.3	0	67.9	168.3	85.7
24	15.4	37.1	84.5	0	146.4	55.2	25.8	55.3	0	57.2	194.6	99.1
25	36.4	37.1	84.5	0	125.4	69.8	25.8	55.3	0	42.6	204.6	104.1
26	40.3	37.1	84.5	0	121.5	72.6	25.8	55.3	0	39.9	209.0	106.4
27	38.2	37.1	84.5	0	123.6	78.9	25.8	55.3	0	33.6	180.1	91.7
28	48.3	2.5	84.5	34.6	113.6	80.9	25.8	55.3	0	31.5	165.9	84.5
29	56.4	0	84.5	37.1	105.4	73.7	12.9	55.3	12.9	38.8	108.7	55.3
30	62.3	0	84.5	37.1	99.5	77.8	0	55.3	25.8	34.7	107.8	54.9
31	67.1	0	84.5	37.1	94.7	81.1	0	55.3	25.8	31.3	101.8	51.8
32	76.9	0	84.5	37.1	85	66.8	25.8	55.3	0	45.7	127.1	64.7
33	79.8	0	84.5	37.1	82.1	43	25.8	55.3	0	69.5	126.8	64.5
34	81.7	0	84.5	37.1	80.1	37.7	25.8	55.3	0	74.8	129.3	65.8
35	47.1	37.1	84.5	0	114.7	32.7	25.8	55.3	0	79.7	173.2	88.1
36	44.9	37.1	84.5	0	116.9	31.2	25.8	55.3	0	81.3	195.8	99.7
37	40	37.1	84.5	0	121.8	27.8	25.8	55.3	0	84.6	207.0	105.4
38	33.0	37.1	84.5	0	128.8	22.9	25.8	55.3	0	89.5	211.0	107.4

Homstølvann

TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					0						
21	0	39.3	0	21.3	32.8	0	33.9	0	8.2	22.8	172.3	0
22	3.3	60.7	0	0	29.5	2.3	42.1	0	0	20.5	177.9	0
23	32.8	24.3	0	36.4	0	22.8	16.9	0	25.3	0	168.3	0
24	24.1	60.7	0	0	8.7	16.8	42.1	0	0	6	194.6	0
25	18.7	60.7	0	0	14.1	13	42.1	0	0	9.8	204.6	0
26	3.1	60.7	0	0	29.7	2.2	42.1	0	0	20.6	209.0	0
27	0	51.3	0	9.4	32.8	0	35.6	0	6.5	22.8	180.1	0
28	9.9	0	0	60.7	22.9	0	30.9	0	11.2	22.8	165.9	0
29	14.6	0	0	60.7	18.2	16.2	0	0	42.1	6.6	108.7	0
30	17.9	0	0	60.7	14.9	18.5	0	0	42.1	4.3	107.8	0
31	20.6	0	0	60.7	12.2	20.4	0	0	42.1	2.4	101.8	0
32	26.3	0	0	60.7	6.5	8	42.1	0	0	14.8	127.1	0
33	27.8	0	0	60.7	5	27.8	7.1	0	35.1	-5	126.8	0
34	28.8	0	0	60.7	4	26.8	27.5	0	14.7	-4	129.3	0
35	32.8	34.5	0	26.2	0	22.8	30.7	0	11.4	0	173.2	0
36	32.8	41.4	0	19.3	0	22.8	28.7	0	13.4	0	195.8	0
37	22.2	50.4	0	10.2	10.6	15.4	35	0	7.1	7.4	207.0	0
38	0	60.7	0	0	32.8	0	42.1		0	22.8	211.0	0

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

The testing conditions are still the same but owner A is allowed to use the capacity left by S.

Roskrepp												
TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	185.6					179.9						
21	204.4	0	175.2	24.6	199.2	193	0	154.3	17.1	87.5	172.3	171.4
22	247.3	0	175.2	24.6	156.3	222.8	0	154.3	17.1	57.7	177.9	177.0
23	274	0	175.2	24.6	129.6	241.4	0	154.3	17.1	39.1	168.3	167.4
24	297.9	0	175.2	24.6	105.7	223	35	154.3	-17.9	57.5	194.6	193.7
25	302.4	24.6	175.2	0	101.2	226.1	17.1	154.3	0	54.4	204.6	203.6
26	290.7	24.6	175.2	0	112.9	218	17.1	154.3	0	62.5	209.0	207.9
27	308.5	0	175.2	24.6	95.1	230.3	0	154.3	17.1	50.1	180.1	179.2
28	320.5	0	175.2	24.6	83.1	238.7	0	154.3	17.1	41.8	165.9	165.1
29	328.2	0	175.2	24.6	75.4	244.1	0	154.3	17.1	36.4	108.7	108.2
30	333.8	0	175.2	24.6	69.8	247.9	0	154.3	17.1	32.5	107.8	107.3
31	338.4	0	175.2	24.6	65.2	251.1	0	154.3	17.1	29.3	101.8	101.3
32	347.7	0	175.2	24.6	55.9	257.6	0	154.3	17.1	22.9	127.1	126.5
33	350.4	0	175.2	24.6	53.2	259.5	0	154.3	17.1	21	126.8	126.2
34	352.3	0	175.2	24.6	51.3	260.8	0	154.3	17.1	19.7	129.3	128.7
35	354.7	0	175.2	24.6	48.9	262.4	0	154.3	17.1	18	173.2	172.3
36	346.6	15.2	175.2	9.4	57	240.8	26.5	154.3	-9.4	39.6	195.8	194.9
37	326.5	24.6	175.2	0	77.1	226.9	17.1	154.3	0	53.6	207.0	206.0
38	304.4	24.6	175.2	0	99.3	211.5	17.1	154.3	0	69.0	211.0	209.9
Kvinen												
TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					70						
21	0	7.8	136.8	19.7	61.4	53.9	21.5	115.2	-2.4	-11.2	172.3	136.8
22	16.7	1.1	135.8	26.4	44.7	20.8	45.5	115.2	-26.4	21.9	177.9	141.2
23	27.9	0	135.8	27.5	33.5	28.5	0	115.2	19.1	14.1	168.3	133.6
24	10.3	27.5	135.8	0	51.1	51.3	19.1	115.2	0	-8.6	194.6	154.5
25	19.5	27.5	135.8	0	41.9	57.7	19.1	115.2	0	-15	204.6	162.4
26	22	27.5	135.8	0	39.4	59.4	19.1	115.2	0	-16.8	209.0	165.9
27	13.2	16.2	135.8	11.2	48.2	34.3	30.3	115.2	-11.2	8.4	180.1	143.0
28	18.2	0	135.8	27.5	43.3	37.7	0	115.2	19.1	4.9	165.9	131.8
29	21.4	0	135.8	27.5	40	40	0	115.2	19.1	2.7	108.7	86.3
30	23.7	0	135.8	27.5	37.7	41.6	0	115.2	19.1	1.1	107.8	85.6
31	25.6	0	135.8	27.5	35.8	42.9	0	115.2	19.1	-0.3	101.8	80.8
32	29.5	0	135.8	27.5	31.9	45.6	0	115.2	19.1	-2.9	127.1	100.9
33	30.6	0	135.8	27.5	30.8	46.4	0	115.2	19.1	-3.7	126.8	100.7
34	31.4	0	135.8	27.5	30	46.9	0	115.2	19.1	-4.3	129.3	102.7
35	32.4	0	135.8	27.5	29	6.6	41.1	115.2	-22	36.1	173.2	137.5
36	23	27.5	135.8	0	38.4	16	19.1	115.2	0	26.7	195.8	155.5
37	22	27.5	135.8	0	39.4	15.3	19.1	115.2	0	27.4	207.0	164.4
38	20.1	27.5	135.8	0	41.3	14.0	19.1	115.2	0	28.7	211.0	167.5

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)

Nesjen

TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					0						
21	0	27.6	87.7	9.5	161.8	0	35.3	66.1	-9.5	112.5	172.3	87.7
22	9	37.1	85.1	0	152.8	51	25.8	65.8	0	61.4	177.9	90.5
23	0	37.1	85.1	0	161.8	44.7	25.8	65.8	0	67.7	168.3	85.7
24	15.4	37.1	84.5	0	146.4	55.5	25.8	65.8	0	57	194.6	99.1
25	36.4	37.1	84.5	0	125.4	70	25.8	65.8	0	42.4	204.6	104.1
26	40.3	37.1	84.5	0	121.5	72.8	25.8	65.8	0	39.7	209.0	106.4
27	38.2	37.1	84.5	0	123.6	90.3	25.8	65.8	0	22.2	180.1	91.7
28	48.3	2.5	84.5	34.6	113.6	38.7	60.4	65.8	-34.6	73.8	165.9	84.5
29	56.4	0	84.5	37.1	105.4	44.3	0	65.8	25.8	68.1	108.7	55.3
30	62.3	0	84.5	37.1	99.5	48.4	0	65.8	25.8	64.1	107.8	54.9
31	67.1	0	84.5	37.1	94.7	51.8	0	65.8	25.8	60.7	101.8	51.8
32	76.9	0	84.5	37.1	85	58.5	0	65.8	25.8	53.9	127.1	64.7
33	79.8	0	84.5	37.1	82.1	60.5	0	65.8	25.8	51.9	126.8	64.5
34	81.7	0	84.5	37.1	80.1	15.7	46.2	65.8	-20.4	96.8	129.3	65.8
35	47.1	37.1	84.5	0	114.7	32.7	25.8	65.8	0	79.7	173.2	88.1
36	44.9	37.1	84.5	0	116.9	31.2	25.8	65.8	0	81.3	195.8	99.7
37	40	37.1	84.5	0	121.8	27.8	25.8	65.8	0	84.6	207.0	105.4
38	33.0	37.1	84.5	0	128.8	22.9	25.8	65.8	0	89.5	211.0	107.4

Homstølvann

TIME	xs	us	Wv S	d12r1	x12r1	xa	ua	Wv A	d21r1	x21r1	P	e*P
20	0					0						
21	0	39.3	0	21.3	32.8	0	43.4	0	-1.3	22.8	172.3	0
22	3.3	60.7	0	0	29.5	2.3	42.1	0	0	20.5	177.9	0
23	32.8	24.3	0	36.4	0	22.8	16.9	0	25.3	0	168.3	0
24	24.1	60.7	0	0	8.7	16.8	42.1	0	0	6	194.6	0
25	18.7	60.7	0	0	14.1	13	42.1	0	0	9.8	204.6	0
26	3.1	60.7	0	0	29.7	2.2	42.1	0	0	20.6	209.0	0
27	0	51.3	0	9.4	32.8	0	35.6	0	6.5	22.8	180.1	0
28	9.9	0	0	60.7	22.9	0	65.5	0	-23.4	22.8	165.9	0
29	14.6	0	0	60.7	18.2	3.3	0	0	42.1	19.5	108.7	0
30	17.9	0	0	60.7	14.9	5.6	0	0	42.1	17.2	107.8	0
31	20.6	0	0	60.7	12.2	7.5	0	0	42.1	15.3	101.8	0
32	26.3	0	0	60.7	6.5	11.4	0	0	42.1	11.4	127.1	0
33	27.8	0	0	60.7	5	12.5	0	0	42.1	10.3	126.8	0
34	28.8	0	0	60.7	4	26.8	32.6	0	9.6	-4	129.3	0
35	32.8	34.5	0	26.2	0	22.8	30.7	0	11.4	0	173.2	0
36	32.8	41.4	0	19.3	0	22.8	28.7	0	13.4	0	195.8	0
37	22.2	50.4	0	10.2	10.6	15.4	35	0	7.1	7.4	207.0	0
38	0	60.7	0	0	32.8	0	42.1	0	0	22.8	211.0	0

x- reservoir level (Mm3/week) u- discharge (Mm3/week)
v- spillage (Mm3/week) s- bypass (Mm3/week)
dijr1- owner i's unused discharge capacity (Mm3/week)
xijr1- owner i's unused reservoir capacity (Mm3/week)
Wv- water value (NOK/Mm3) P- prices (NOK/MWh) e*P (NOK/Mm3)