

Assessment of a semi-probabilistic safety concept for reinforced concrete columns using non-linear finite element analyses

Morten Lynnebakken Håkon Hammer Normann

Civil and Environmental Engineering Submission date: June 2018 Supervisor: Jochen Kohler, KT Co-supervisor: Max Hendriks, KT Morten Engen, Multiconsult

Norwegian University of Science and Technology Department of Structural Engineering



MASTER THESIS 2018

SUBJECT AREA:	DATE:	NO. OF PAGES:
Structural Design	08.06.2018	108

TITLE:

Assessment of a semi-probabilistic safety concept for reinforced concrete columns using non-linear finite element analyses

Vurdering av et semi-probabilistisk sikkerhetskonsept for armerte betongsøyler ved bruk av ikke-lineære elementanalyser

BY:

Morten Lynnebakken Håkon Hammer Normann



SUMMARY:

This thesis considers the reliability of methods for slender concrete column design, including Non-Linear Finite Element Analysis (NLFEA). The applicability of the Eurocode in slender column design is investigated, and the current safety format is assessed.

The Partial Safety Factor (PSF) method is a semi-probabilistic method where partial safety factors have been calibrated based on a linear Limit State Function (LSF). In slender structures, significant second-order effects cause geometric non-linearity. The combination of geometric non-linearity and the non-linear behavior of concrete, assessed in an NLFEA software, violate the assumption of a linear limit state. Nevertheless, the Eurocode suggests applying the PSF method to problems solved with NLFEA.

The PSF method is compared with two alternative safety formats, namely the Global Resistance Factor Method (GRFM) and the method of Estimate of Coefficient Of Variation (ECOV). Since the PSF method currently is embedded in the Eurocode, a new approach for applying PSFs to slender column design is sought. A new set of PSFs is inquired through reliability analyses combined with both hand-calculation methods and NLFEA.

Five stochastic variables are used in the analyses, including: The concrete compressive strength, the reinforcement yield strength, the concrete stiffness, the eccentricity and the load. Inverse reliability analyses are conducted to find the optimal combination of PSFs for the different slenderness ratios.

The minimum eccentricity in the Eurocode is considered too conservative and a new approach to treat eccentricity is suggested. GRFM is a more conservative alternative to the PSF method, while ECOV might be non-conservative if the material parameters are included by the values given in Eurocode 2-1-1. It is, therefore, proposed to apply the in-situ adjusted concrete strength with the ECOV method. The results from the inverse analyses indicate that a new slenderness limit should be developed to distinguish between compression and yield failure. Two separate sets of PSFs are proposed, for columns below and above the slenderness limit.

RESPONSIBLE TEACHER: Jochen Köhler

SUPERVISOR(S): Jochen Köhler, Max Hendriks, Morten Engen

CARRIED OUT AT: Department of Structural Engineering

MASTER THESIS 2018

for

Morten Lynnebakken and Håkon Hammer Normann

Assessment of a semi-probabilistic safety concept for reinforced concrete columns using non-linear finite element analyses

Vurdering av et semi-probabilistisk sikkerhetskonsept for armerte betongsøyler ved bruk av ikke-lineære elementanalyser

In the spring of 2017, Hanna Eklund, Astrid Skorve and Arne Strand finished their MSc thesis at NTNU, entitled "Reliability Assessments of Concrete Structures using Non-Linear Finite Element Analyses". In this thesis, methods have been explored on how to perform safety assessments of slender reinforced concrete columns, using non-linear finite element analyses. The report includes an assessment of the properties of the design point ("alpha values") for these columns and the dependency of these properties on the geometrical proportions of the system - all exemplified based on a "round robin column example".

The current MSc project is a continuation of this research and will focus more on the reliability issues. The column example is revisited first. Next, the reliability assessment is completed. This will involve a much more rigorous development and assessment of the response surface, based on the results of the FEA. The study should also involve a comparison of the response surface to analytical limit states, derived from simplified methods. Finally, it should include an assessment of model and modelling uncertainty.

The output of the thesis can deliver insights on the placement of advanced non-linear mechanical analysis in the safety concept of design codes of the Eurocodes. It is anticipated that the results of the thesis will form the basis for an international journal publication.

Supervisor(s): Jochen Köhler, Max Hendriks & Morten Engen

NTNU, 11.01.2018

Preface

This thesis is the conclusive part of the Master of Science in Civil and Environmental Engineering at Norwegian University of Science and Technology (NTNU) in Trondheim. The research was conducted for the Department of Structural Engineering from January to June 2018.

The field of structural reliability was relatively new to the authors, and it provided many interesting challenges. A significant amount of time was spent in developing and investigating the DIANA model and the Matlab scripts that were applied in the reliability analyses. It was educative to assess the potential and limitations of the reliability analyses using Matlab and an NLFEA software.

We would like to thank our supervisors Professor Jochen Köhler, Professor Max Hendriks and PhD Morten Engen at Multiconsult. The help and guidance we have received is highly appreciated. The conversations and discussions have been crucial for our ability to finish this thesis.

The thesis is a continuation of the work of Hanna Eklund, Astrid Skorve and Arne Strand. Their effort of creating an extensive framework for the calculation procedures in Matlab and DIANA, has been of great value.

ii

Abstract

This thesis assesses the reliability of methods for slender concrete column design, including Non-Linear Finite Element Analysis (NLFEA). The applicability of the Eurocode in slender column design is investigated, and the current safety format is assessed.

The Partial Safety Factor (PSF) method is a semi-probabilistic method where partial safety factors have been calibrated based on a linear Limit State Function (LSF). In slender structures, significant second-order effects cause geometric non-linearity. The combination of geometric non-linearity and the non-linear behavior of concrete, assessed in an NLFEA software, violate the assumption of a linear limit state. Nevertheless, the Eurocode suggests applying the PSF method to problems solved with NLFEA.

The PSF method is compared with two alternative safety formats, namely the Global Resistance Factor Method (GRFM) and the method of Estimate of Coefficient Of Variation (ECOV). Since the PSF method currently is embedded in the Eurocode, a new approach for applying PSFs to slender column design is sought. A new set of PSFs is inquired through reliability analyses combined with both hand-calculation methods and NLFEA.

Five stochastic variables are used in the analyses, including: The concrete compressive strength, the reinforcement yield strength, the concrete stiffness, the eccentricity and the load. Inverse reliability analyses are conducted to find the optimal combination of PSFs for the different slenderness ratios.

The minimum eccentricity in the Eurocode is considered too conservative and a new approach to treat eccentricity is suggested. GRFM is a more conservative alternative to the PSF method, while ECOV might be non-conservative if the material parameters are included by the values given in Eurocode 2-1-1. It is, therefore, proposed to apply the in-situ adjusted concrete strength with the ECOV method. The results from the inverse analyses indicate that a new slenderness limit should be developed to distinguish between compression and yield failure. Two separate sets of PSFs are proposed, for columns below and above the slenderness limit.

Sammendrag

I denne oppgaven er det gjort en pålitelighetsvurdering av beregningsmetoder for slanke betongsøyler, inkludert ikke-lineær elementanalyse. Eurokodens anvendelighet for bruk i beregninger av slanke betongsøyler er undersøkt, og det nåværende sikkerhetsformatet er vurdert.

Metoden med partielle sikkerhetsfaktorer (PSF) er en semi-probabilistisk metode der partielle sikkerhetsfaktorer har blitt kalibrert ved hjelp av en lineær grensetilstandsfunksjon, eller *limit state function* (LSF). I slanke konstruksjoner vil betydelige andre ordens effekter føre til geometrisk ikke-linearitet. Kombinasjonen mellom geometrisk ikke-linearitet og de ikke-lineære materialegenskapene til betong er ikke forenelig med en lineær LSF. Likevel anbefaler Eurokoden å benytte PSF-formatet i ikke-lineære elementanalyser.

PSF-formatet er sammenlignet med to alternative sikkerhetsformater som er mer forenlige med en ikke-lineær LSF, kalt GRFM og ECOV. Siden PSF-formatet er dypt innebygd i Eurokoden, er det tilstrebet å finne et nytt sett med PSF som er optimalisert for bruk til beregninger av slanke betongsøyler. Dette er utført gjennom pålitelighetsanalyser i kombinasjon med både håndberegningsmetoder og ikke-lineær elementanalyse.

Fem stokastiske variabler er inkludert i analysene; betongens trykkfasthet, armeringens flytespenning, betongens E-modul, lasteksentrisiteten og lasten. Inverse analyser ble benyttet for å finne den optimale kombinasjonen av partielle sikkerhetsfaktorer for søyler med ulik slankhet.

Minimumseksentrisiteten i Eurokoden er for konservativ for søyler som går til brudd grunnet trykk i betongen, og en ny tilnærming til minimumseksentrisiteten er foreslått. GRFM er et konservativt alternativ til PSF-formatet. ECOV kan være ikke-konservativ dersom materialegenskapene fra Eurokode 2-1-1 er benyttet. Det er derfor anbefalt å bruke in-situ justerte verdier for betongstyrken. Resultatene fra de inverse analysene indikerte at et nytt slankhetskriterium burde bli utviklet for å skille mellom søyler utsatt for trykkbrudd og knekking. Det er foreslått et unikt sett med PSF for søyler over og under slankhetskriteriet.

Contents

	Pref	face		i
	Abs	tract .		iii
	Sam	imendra	ag	v
Li	st of	Figur	es	xi
Li	st of	Table	S	xiii
Li	st of	' Symb	ols	xv
A	bbre	viation	IS	xix
1	Intr	roduct	ion	1
2	Col	umn S	pecifications	5
	2.1	Geom	etry and Load	5
	2.2	Mater	ials	6
		2.2.1	Eurocode 2 - material parameters	7
		2.2.2	NLFEA - material parameters	7
		2.2.3	Stochastic variables	8
		2.2.4	Creep	11
3	Cap	oacity	Calculation Methods	13
	3.1	Desig	a capacities	13
		3.1.1	MN-diagram	14
		3.1.2	Safety format	15
	3.2	Nonlin	near Finite Element Modeling	18
		3.2.1	Constitutive model	20

		3.2.2	Geometric model	21
		3.2.3	Finite element discretization	21
		3.2.4	Boundary conditions and load application	22
		3.2.5	Analysis	22
4	Stru	uctural	Reliability Methods	25
	4.1	Limit	State	25
	4.2	Reliab	ility methods	28
		4.2.1	Monte Carlo	28
		4.2.2	First Order Reliability Method	28
		4.2.3	Response Surface Method	29
		4.2.4	RSM-FORM	30
	4.3	Reliab	ility assessment with NSM and NCM	31
		4.3.1	Monte Carlo	31
		4.3.2	FORM	32
		4.3.3	RSM-FORM	34
		4.3.4	System reliability	35
	4.4	Reliab	ility assessment with NLFEA	35
	4.5	Assess	ment of calculation methods	36
	4.6	Inverse	e analyses and PSF	38
5	Res	ults &	Discussion	39
	5.1	Design	a capacities	39
		5.1.1	Impact of creep	41
	5.2	Safety	format study	43
	5.3	Reliab	ility analyses	46
		5.3.1	Simplified calculation methods	46
		5.3.2	NLFEA	54
	5.4	Assess	ment of model deviation	55
	5.5	Inverse	e analyses	57
	5.6	Valida	tion of PSF	68
6	Cor	nclusio	n	71

7	Further work	73
Bi	bliography	75
\mathbf{A}	Verification of NLFEA	77

List of Figures

2.1	Column geometry	6
2.2	Bi-linear stress-strain relation adapted from EC2 [1]	8
2.3	The three parts of the eccentricity variable adapted from the JCSS Probabilist	ic
	Model Code [9]	10
3.1	Rectangular compressive stress distribution [1].	14
3.2	Concrete stress-strain models [12]	20
3.3	Geometric and finite element discretization models for NLFEA. \ldots .	21
4.1	Probability density function of resistance, load and safety margin [16]	26
4.2	Stress and strain in the cross-section for compression failure [8]. \ldots	33
4.3	Stress and strain in the cross-section at the two end points of the line	
	representing yield failure [8]	33
4.4	Compression failure LS, yield failure LS and an arbitrary work-diagram	
	with NSM	34
5.1	MN-diagram for the two analytic methods in EC2	40
5.2	Design loads for NSM, NCM and NLFEA	41
5.3	Ratio between NSM and NCM	42
5.4	Comparison of safety formats applied to NLFEA	44
5.5	Design capacities for all methods, relative to NLFEA PSF	45
5.6	Monte Carlo for design loads calculated with NCM - 3000 mm column. $% \mathcal{M} = \mathcal{M} = \mathcal{M} = \mathcal{M} + \mathcal{M} + \mathcal{M}$.	47
5.7	Monte Carlo for design loads calculated with NCM - 5000 mm column. $% \mathcal{M} = \mathcal$	48
5.8	Concrete strength - Load for 4000 mm column.	48
5.9	Yield strength - Load for 4000 mm column.	49
5.10	Concrete stiffness - Load for 3000 mm column.	49

5.11	Ratios between NLFEA and simplified methods for 20 random realizations.	56
5.12	Inverse FORM with NSM for the 2000 mm column	58
5.13	Inverse FORM with NSM for the 3000 mm column	58
5.14	Inverse RSM-FORM with NLFEA for 2000-4000 mm columns	62
5.15	Results from inverse RSM-FORM with NLFEA	62
5.16	Regression analyses based on e_d for NLFEA	63
5.17	Regression analyses based on $e_{\rm d}$ for NSM, NCM and NLFEA	63
5.18	Ratio between design capacities with new PSFs and best estimate	69
A.1	The graphs illustrate the three critical load steps shown in Figure A.2-A.7.	78
A.2	Stress and strain distribution along the critical cross section for the column	
	of length 1000 mm	79
A.3	Stress and strain distribution along the critical cross section for the column	
	of length 2000 mm	80
A.4	Stress and strain distribution along the critical cross section for the column	
	of length 3000 mm	81
A.5	Stress and strain distribution along the critical cross section for the column	
	of length 4000 mm	82
A.6	Stress and strain distribution along the critical cross section for the column	
	of length 5000 mm	82
A.7	Stress and strain distribution along the critical cross section for the column	
	of length 6000 mm	83

List of Tables

2.1	Length-slenderness relation.	5
2.2	EC2 Parameters.	7
2.3	Mean, standard deviation and coefficient of variation for the material	
	parameters	9
3.1	Coefficients of variation for steel and concrete as recommended in Eurocode	
	2 Commentary [6]	16
3.2	Adopted Solution Strategy	19
4.1	Overview of the capacity calculation methods and the performed reliability	
	analyses	25
4.2	Strategy to calculate partial safety factors	38
5.1	Design capacities for NSM, NCM and NLFEA in kN	39
5.2	Design capacities for GRFM and ECOV relative to the PSF method. $\ .$.	44
5.3	Results from Monte Carlo with NSM and NCM	47
5.4	$\beta\text{-values}$ with FORM and NSM	49
5.5	Results from RSM-FORM with NSM	50
5.6	Results from RSM-FORM with NCM.	51
5.7	Results from RSM-FORM with NLFEA.	54
5.8	Ratio between capacities calculated with NLFEA and NSM/NCM	56
5.9	Results from the inverse FORM analysis with NSM and $\beta_{target} = 3.8.$	57
5.10	Results from the inverse RSM-FORM analysis with NSM and $\beta_{\text{target}} = 3.8$.	59
5.11	Results from the inverse RSM-FORM analysis with NCM and $\beta_{\text{target}} = 3.8$.	60
5.12	Results from the inverse RSM-FORM analysis with NLFEA and $\beta_{\text{target}} = 3.8$.	61
5.13	Eccentricity as functions of column length.	64

5.14	Proposed sets of PSFs	68
5.15	Design capacities in kN with the PSFs proposed in Table 5.14	69

List of Symbols

θ_i	Model uncertainty parameter
Φ	Standard normal distribution
Φ_n	n-dimensional standard normal distribution
Ω_{F}	Failure domain of series system
$\alpha_{ m cc}$	Coefficient for long-term load
$lpha_{ m fc}$	Sensitivity of concrete strength
α_i	Sensitivity factors
β	Reliability index
$\beta_{\rm sys}$	System reliability index
β_{target}	Target reliability index
$\gamma_{\rm CE}$	Partial factor for Young's modulus of concrete
$\gamma_{ m G}$	Partial factor for permanent actions
$\gamma_{\rm O}$	Overall safety factor
$\gamma_{\rm O}$,	Overall safety factor with model uncertainty
$\gamma_{ m R}$	Global safety factor
$\gamma_{ m Rd}$	Model uncertainty for resistance
$\gamma_{ m c}$	Partial factor for concrete
$\gamma_{ m s}$	Partial factor for steel
$\varepsilon_{\mathrm{cu3}}$	Ultimate concrete strain
$\varepsilon_{ m s}$	Reinforcement strain
ϵ_{FORM}	Convergence criterion in first order reliability method
η	Effective concrete strength

λ	Slenderness ratio
$\mu_{ heta}$	Mean model uncertainty
$\mu_{ m E}$	Mean Young's modulus of concrete
$\mu_{ m M}$	Mean safety margin
$\mu_{ m N}$	Mean of the applied load
$\mu_{ m R}$	Mean of the resistance
$\mu_{ m S}$	Mean of the load action
$\mu_{\rm X}$	Mean of normal distributed parameter
$\mu_{ m fc}$	Mean concrete compression strength
μ_{fy}	Mean steel tensile strength
$\mu_{ m ln}$	Lognormal mean
$\mu_{\mathbf{x}}$	Mean of random variable X
ϕ	Out of plumpness
ϕ_{ef}	Effective creep coefficient
$\phi_{(\infty, \mathrm{t0})}$	Final creep coefficient
$\sigma_{ heta}$	Standard deviation of model uncertainty
$\sigma_{ m G}$	Standard deviation of permanent actions
$\sigma_{ m M}$	Standard deviation of safety margin
$\sigma_{ m R}$	Standard deviation of resistance
$\sigma_{ m S}$	Standard deviation of load actions
$\sigma_{\rm X}$	Standard deviation of normal distributed parameter
$\sigma_{ m ln}$	Lognormal standard deviation
$\sigma_{\rm x}$	Standard deviation for random variable X
$A_{\rm c}$	Concrete compression area
$A_{\rm s}$	Reinforcement area
$A_{\rm s,max}$	Maximum reinforcement area
$A_{\rm s,min}$	Minimum reinforcement area
a_i	Constants of the first order reliability method
b	Concrete column cross section width
d	Effective cross section height

$E_{\rm c}$	Modulus of elasticity of concrete
$E_{\rm cd}$	Design modulus of elasticity of concrete
$E_{\rm cm}$	Mean modulus of elasticity of concrete
$E_{\rm cm,is}$	In-situ adjusted mean modulus of elasticity of concrete
$E_{\rm s}$	Modulus of elasticity of steel
e	Total eccentricity
e_0	Average eccentricity
e_2	Second order deflection
$F_{\rm c}$	Compressive force resultant from concrete
$F_{\rm s}$	Tensile force resultant from reinforcement
f	Arbitrary factor
f_0	Initial curvature
$f_{ m c}$	Concrete compressive strength
$ ilde{f}_{ m c}$	Concrete compressive strength for GRFM
$f_{\rm cd}$	Design concrete compressive strength
$f_{\rm ck}$	Characteristic concrete compressive strength
$f_{\rm ck,is}$	In-situ adjusted characteristic concrete compressive strength
$f_{\rm cm}$	Mean concrete compressive strength
$f_{\rm cm,is}$	In-situ adjusted mean concrete compressive strength
$f_{\rm ct}$	Concrete tensile strength
$f_{\rm c}$	Concrete compressive strength
$f_{\rm y}$	Steel yield strength
\tilde{f}_{y}	Steel yield strength for GRFM
$f_{\rm ym,is}$	In-situ adjusted steel yield strength
G	Limit state function
$G_{\rm F}$	Fracture energy
\bar{g}	Polynomial function for RS
h	Concrete column cross section height
$h_{\rm eq}$	Equivalent length
h_{\max}	Maximum element size

$k_{0.05}$	Inverse of the normal distribution for the 5%-fractile value
L	Concrete column length
M	Safety margin
M_2	Nominal second order moment
$M_{0\rm Ed}$	First order bending moment at ultimate limit state
$M_{0\mathrm{Eqp}}$	First order bending moment at serviceability limit state
$N_{\rm Ed}$	Design load
$P_{\rm f}$	Probability of failure
R	Resistance
$R_{\mathrm{best},i}$	Resistance from NLFEA with random realizations of input variables
$R_{\rm d}$	Design resistance
$R_{\rm k}$	Characteristic resistance
$R_{\rm m}$	Mean resistance
$R_{\mathrm{simp},i}$	Resistance from simplified method with random realizations of input variables
S	Load action
U	Random variable in standard normal space
$V_{\rm G}$	Coefficient of variation for geometric uncertainties
$V_{\rm N}$	Coefficient of variation for load
$V_{\rm R}$	Coefficient of variation for resistance
V_{f}	Coefficient of variation for material uncertainties
$V_{\rm fc}$	Coefficient of variation for concrete compression strength
$V_{\rm fy}$	Coefficient of variation for steel tensile strength
$V_{\rm m}$	Coefficient of variation for model uncertainties
$V_{\rm x}$	Coefficient of variation for random variable X
X	Random variable
x	Realization of random variable X

Abbreviations

COV	Coefficient of variation		
CQ16 M	Eight-node quadrilateral isoparametric plane stress element		
EC2	Eurocode 2-1-1		
ECOV	Estimate of coefficient of variation		
FORM	First order reliability method		
GRFM	Global resistance factor method		
JCSS	Joint Committee on Structural Safety		
\mathbf{LS}	Limit state		
\mathbf{LSF}	Limit state function		
\mathbf{MC}	Monte Carlo		
NCM	Nominal curvature method		
NLFEA	Non-linear finite element analysis		
NSM	Nominal stiffness method		
PSF	Partial safety factor		
\mathbf{RS}	Response surface		
RSM	Response surface method		

1. Introduction

In the design of concrete structures, the Partial Safety Factor (PSF) method is usually applied to ensure that the required safety level is obtained. The PSF method is a semi-probabilistic method where partial safety factors have been calibrated based on a linear Limit State Function (LSF). The partial factors are often related to the load action, the geometry, the calculation model and the material parameters governing the resistance of the structure. Certain assumptions concerning the sensitivity and the probabilistic distribution of the variables form the basis for the development of PSFs. Eurocode 2-1-1 [1] has defined a set of PSFs that is assumed applicable for the design of concrete structures. In slender structures, significant second-order effects are causing geometric non-linearity. Furthermore, the non-linear material properties of concrete and the potential of different global failure mechanisms might call for a Non-Linear Finite Element Analysis (NLFEA). The assumption that a linear limit state can be used to define the PSFs is violated when the problem is highly non-linear and assessed in an NLFEA software. Nevertheless, the Eurocode suggests applying the PSF method to non-linear problems solved by NLFEA. Recent studies examine alternative safety formats, more applicable for non-linear LSFs and NLFEA [2, 3, 4].

In this thesis, the procedure for slender concrete column design proposed by the Eurocode is investigated. The investigation includes testing of the different calculation methods and the PSFs by conducting reliability analyses. The simplified hand-calculation methods suggested by the Eurocode are combined with multiple reliability methods to investigate the calculation methods' sensitivity to the uncertain variables and to verify that the reliability methods work correctly. To obtain more accurate results, NLFEA is combined with a suitable reliability method. There is no explicitly defined LSF for NLFEA, and the LSF is, therefore, approximated by a response surface deducted from a regression procedure on several sample points. By combining the response surface with the First Order Reliability Method (FORM), an applicable model is obtained, which balances accuracy and time consumption.

The thesis evaluates alternative safety formats and their applicability for use with NLFEA. Furthermore, since the PSF method currently is embedded in the Eurocode, a new approach for applying PSFs to slender concrete column design is proposed. The aim is to find a more effective way to design slender concrete columns and simultaneously satisfy the reliability requirements of the Eurocode and avoid unnecessarily costly designs. New PSFs are determined from inverse reliability analyses where the reliability index for a 50-year reference period of a residence/office building is inquired [5]. It is desirable to find a standard set of PSFs that applies to all the three methods the Eurocode suggests for use in slender concrete column design. Therefore, inverse analyses are carried out for all the calculation methods to ensure that the optimal set is acquired. An essential aspect of the study is to investigate how the slenderness ratio of the column impacts the failure mode and the ideal combination of PSFs.

The content of the thesis is structured as follows:

Section 2 - Column Specifications

The predetermined geometry of the column is presented first. Secondly, both deterministic and probabilistic material parameters are given. Then the probabilistic nature of the load and the eccentricity is specified. Finally, it is given a description of how creep is treated in the analyses.

Section 3 - Capacity calculation methods

This section presents the calculation methods used to determine the design capacities of the columns. The theoretical background for the implementation of the simplified methods is presented first. Then the different safety formats are described, before a thorough description of the NLFEA model is given.

Section 4 - Structural reliability methods

This section covers the theory and methods regarding the reliability analyses. The Monte Carlo method, the First Order Reliability Method and the Response Surface Method are introduced. The way these methods were implemented with the different capacity calculation methods is described. Finally, the inverse analyses and the approach to estimate PSFs is explained.

Section 5 - Results & Discussion

All the results are presented and discussed in this section. First, the design capacities calculated in accordance with the Eurocode and current PSFs are examined. This includes an investigation of the impact of creep and a comparison of the different safety formats. Then the reliability analyses, with the design capacities used as input, are presented. Finally, the inverse analyses are considered along with studies on the new approaches for treating eccentricity and PSFs.

Section 6 - Conclusions

Conclusions that are based on the most important findings in the discussion.

Section 7 - Further Work

Suggestions for further work are given to evoke a more extensive study.

2. Column Specifications

2.1 Geometry and Load

The geometry and materials for the column were selected in accordance with Eurocode 2-1-1 (EC2) and aimed to describe a general and realistic column. The column cross-section was set to be quadratic with height (h) and width (b) equal to 200 mm. The column length (L) was implemented as a variable ranging from 1000 mm to 6000 mm, to cover columns with varying slenderness. The relations between lengths and slenderness ratios (λ) calculated according to EC2 5.8.3.2 are shown in Table 2.1. The slenderness of all the columns exceeded the slenderness limit given by EC2 5.8.3.1, implying that second-order effects had to be considered.

Table 2.1: Length-slenderness relation.

Length [mm]	1000	2000	3000	4000	5000	6000
λ	17.3	34.6	52.0	69.3	86.6	103.9

To obtain an under-reinforced cross-section and to be within the longitudinal reinforcement limits of $A_{s,min}$ and $A_{s,max}$, four bars of $\phi 14$ were regarded as suitable reinforcement. Minimum shear reinforcement, calculated in accordance with EC2 9.5.3, was implemented. A concrete cover of 25 mm was also applied.



Figure 2.1: Column geometry.

No planned eccentricity was applied to the load because the load case was intended to be general. Therefore, it was sufficient to add a minimum design eccentricity, e = 20 mm, as suggested by EC2 6.1(4). Additional eccentricities described in EC2 Section 5.2 were not necessary because these were smaller than the minimum eccentricity and would only be necessary in the case of a planned eccentricity.

2.2 Materials

The materials that were considered in this thesis included ordinary C45/55 concrete and reinforcement steel with steel grade S500. For design capacity calculations, deterministic values were applied for all the material parameters, in accordance with Table 3.1 in EC2. For the structural reliability analyses, some of the most important material parameters were included as stochastic variables.

2.2.1 Eurocode 2 - material parameters

The most important material parameters in the design capacity calculations included: The concrete stiffness (E_c) , the concrete compressive strength (f_c) and the reinforcement yield strength (f_y) . These values are listed in Table 2.2. The concrete tensile strength (f_{ct}) was neglected in the simplified capacity calculation methods suggested by EC2 and was, therefore, not used.

Table 2.2: EC2 Parameters.

CONCRETE				
Characteristic compressive strength	$f_{ m ck}$	$45.00 \mathrm{MPa}$		
Design compressive strength	$f_{\rm cd} = \frac{f_{\rm ck}}{\gamma_{\rm c}}$	30.00 MPa		
Mean Young's modulus concrete	$E_{\rm cm}$	36.00 GPa		
Design Young's modulus concrete	$E_{\rm cd} = \frac{E_{\rm cm}}{\gamma_{\rm CE}}$	30.00 GPa		
REINFORCEMENT				
Characteristic yield strength	$f_{ m yk}$	$500.00 \mathrm{MPa}$		
Design yield strength	$f_{\rm yd} = \frac{f_{\rm yk}}{\gamma_{\rm s}}$	434.78 MPa		

The characteristic values given in EC2 Table 3.1 were scaled by partial safety factors in accordance with EC2 2.4.2.4, to achieve the design values f_{cd} and f_{yd} . The values that were used for the PSFs in this thesis were $\gamma_c = 1.5$ and $\gamma_s = 1.15$ for concrete and steel, respectively. The mean stiffness was divided by $\gamma_{CE} = 1.2$, as prescribed in EC2 5.8.6(3), to obtain the design concrete stiffness (E_{cd}). For the simplified design capacity analyses, constant stress in the compression zone and the bi-linear stress-strain relation shown in Figure 2.2 was applied. No post-yielding hardening was considered for the reinforcement.

2.2.2 NLFEA - material parameters

The NLFEA model intends to represent the column capacity as realistic as possible. Therefore, a material set that represented the most realistic values for the material parameters was implemented in the NLFEA model. The mean value of the concrete strength and concrete stiffness from an assumed lognormal distribution was reduced to account for the difference between laboratory and in-situ strength. Based on Eurocode



Figure 2.2: Bi-linear stress-strain relation adapted from EC2 [1].

2 Commentary [6], the mean cylinder strength $(f_{\rm cm})$ was scaled by a factor of 1.15 to obtain the mean in-situ strength $(f_{\rm cm,is})$, denoted $\mu_{\rm fc}$ in Table 2.3. The reinforcement steel was not scaled, because the difference between laboratory and in-situ is small. The remaining material parameters are shown in Table 3.2 and were calculated using formulas from Table 3.1 in EC2, where $f_{\rm cm}$ is substituted by $f_{\rm cm,is}$. $k_{0.05}$ is the inverse of the normal distribution for the 5%-fractile, and $V_{\rm fc}$ and $V_{\rm fy}$ are the coefficients of variation (COV) for concrete strength and yield strength, respectively.

2.2.3 Stochastic variables

Material parameters

For the probabilistic study, the material parameters that were assumed to carry most uncertainty were implemented as stochastic variables. These variables were considered to cover all material uncertainty and included:

- Concrete compressive strength, $f_{\rm c}$
- Concrete modulus of elasticity, $E_{\rm c}$
- Reinforcement yield strength, f_y

The above-mentioned variables were assumed to be lognormally distributed. The lognormal distribution was chosen because the normal distribution can cause inconvenient results, e.g., negative realizations when the coefficient of variation is large. For the lognormal distribution, however, the probability of negative values will always be zero [7]. Lognormally

distributed parameters are entirely defined by the mean value and the standard deviation. The mean, standard deviation and coefficient of variation of the stochastic material parameters are listed in Table 2.3. The remaining material parameters were assumed to be deterministic. The mean values of the concrete compressive strength ($\mu_{\rm fc}$) and the concrete stiffness ($\mu_{\rm E}$) were set to the in-situ adjusted mean values shown in Table 3.2. The coefficient of variation for both concrete and steel strength were chosen based on recommendations in the EC2 commentary [6]. Those COVs were used to derive the partial safety factors mentioned in Section 2.2.1 that are commonly used in EC2. The coefficient of variation for the concrete stiffness ($V_{\rm E}$) was calculated by Eklund, Skorve & Strand [8] based on recommendations in the Joint Committee on Structural Safety (JCSS) Model Code [9].

Table 2.3: Mean, standard deviation and coefficient of variation for the material parameters.

X	$oldsymbol{\mu}_{ m x}$	$\pmb{\sigma}_{\mathrm{x}}$	$V_{ m x}$	Probability Density Function (PDF)
$oldsymbol{f}_{ m c}$	$\frac{f_{\rm ck}}{1.15}e^{1.645V_{\rm fc}}$	$\mu_{\rm fc} V_{\rm fc}$	0.150	Lognormal
$oldsymbol{f}_{\mathrm{y}}$	$f_{\rm yk}e^{1.645V_{\rm fy}}$	$\mu_{\rm fy} V_{\rm fy}$	0.040	Lognormal
$oldsymbol{E}_{ m c}$	$22(\frac{f_{\rm cm,is}}{10})^{0.3}$	$\mu_{\rm E} V_{\rm E}$	0.158	Lognormal

Eccentricity and Load

The eccentricity was assumed to be an essential variable, which covered all uncertainty related to geometry in the model. The JCSS Probabilistic Model Code recommends using a normally distributed eccentricity [9].



Figure 2.3: The three parts of the eccentricity variable adapted from the JCSS Probabilistic Model Code [9].

JCSS divides the eccentricity into three different parts; the average eccentricity (e_0) , the initial curvature (f_0) and the out-of-plumbness (ϕ) , as shown in Figure 2.3. All of them are considered normally distributed with mean value zero. e_0 and f_0 have a standard deviation of $\frac{L}{1000}$ while ϕ has a standard deviation of 0.0015 rad, which results in an eccentricity of $0.0015\frac{L}{2}$ at the middle of the column length. According to 5.2(7) in EC2, the eccentricity that occurs due to an inclination of the column can be calculated as the inclination angle times $\frac{L}{2}$ [1]. Considering that all the parameters are normally distributed and independent, they can for simplicity be merged into one eccentricity variable (e). The mean and standard deviation are given as:

$$\mu_{\rm e} = \mu_{e_0} + \mu_{f_0} + \mu_{\phi} = 0$$

$$\sigma_{\rm e} = \sqrt{\sigma_{e_0}^2 + \sigma_{f_0}^2 + \sigma_{\phi}^2} = \sqrt{\left(\frac{L}{1000}\right)^2 + \left(\frac{L}{1000}\right)^2 + \left(\frac{0.0015L}{2}\right)^2} \approx 0.001601L$$

The eccentricity is a variable that depends highly on the structure surrounding the column and how the loads are transferred into the column. It can be argued that the average eccentricity is more likely to contain uncertainties than the initial curvature for concrete columns. Because most of the references used in the JCSS Probabilistic Model Code are based on steel columns, the standard deviations may not give the most accurate
description for uncertainties related to concrete columns. However, for the generality of the problem, the standard deviations advised by JCSS were used for the probabilistic analyses.

The load is a critical variable because it contains high uncertainties. The load was assumed to be normally distributed. In the JCSS, self-weight and live loads are treated separately and are dependent on the applied materials and the kind of structure that is examined. This thesis treats a general column problem, and the JCSS could, therefore, not be used directly to find the coefficient of variation. It was assumed that a coefficient of variation (V_N) of 0.2 was appropriate for the load, based on the different values proposed by JCSS. The mean load (μ_N) was given by N_{Ed}/γ_G , where N_{Ed} is the design capacity and γ_G denotes the partial factor for self-weight. In Eurocode EN 1990 [5], the partial factor γ_G is equal to 1.35 for situations where permanent loads are dominating. The standard deviation was calculated by $\sigma_N = \mu_N V_N$.

2.2.4 Creep

EC2 states in Section 5.8.4 and 5.8.6 that creep should be considered when second-order calculation methods are carried out. The effective creep ratio was calculated in accordance with EC2:

$$\phi_{\rm ef} = \phi_{(\infty, t0)} \cdot \frac{M_{0\rm Eqp}}{M_{0\rm Ed}}$$

The final creep coefficient $(\phi_{(\infty,t0)})$ was determined from Figure 3.1 in EC2. It was assumed relative humidity of 50% and loading of the concrete after 28 days. The first-order bending moment at serviceability limit state (M_{0Eqp}) was set equal to the first-order bending moment at ultimate limit state (M_{0Ed}) because the column considered in this thesis was loaded until failure. ϕ_{ef} was found to be 1.8. This was a simplified creep calculation but was assumed a valid approximation considering that this was a representation of a general case without knowledge of the loading history and the time dependency of creep.

EC2 also suggests using a nonlinear creep calculation when the stresses in the concrete are high. However, for a design case, it is unrealistic that the ultimate limit state design load would be present over a long time period, and very high stresses would, therefore, not affect creep significantly. Consequently, it was assumed that the final creep coefficient could be calculated without the inclusion of nonlinear creep. For the simplified capacity calculation methods introduced in Section 3, the effective creep ratio was included directly in the methods.

Due to limited knowledge of the loading history of the column, creep was not included directly in the NLFEA analysis. Because the load case was general and the loading history unknown, creep was not included through time-steps. A simplified method suggested in EC2 proposes to include creep by multiplying the strains with $(1+\phi_{ef})$ and thus treat creep as linearly proportional to the stress level. The most convenient way to incorporate this in the NLFEA model was to scale the concrete stiffness by a factor of $\frac{1}{1+\phi_{ef}}$. The stresses in the parabolic stress-strain relation in DIANA are proportional to the elastic modulus until peak stress is reached. After softening of the concrete occurs, the stresses are no longer dependent on the elastic modulus, but the strains will be well approximated also in this region. It was, therefore, considered a good approximation to reduce the elastic modulus to include creep in the NLFEA model.

Creep was considered an important variable, which can impact the design of slender concrete columns significantly. However, given the general case that was examined in this thesis, creep was not included as a stochastic variable in the reliability analyses. Creep was merely included through deterministic factors as mentioned above.

3. Capacity Calculation Methods

3.1 Design capacities

Eurocode 2-1-1 proposes three methods for capacity calculation of axially loaded concrete columns subjected to second-order effects. The following methods are included:

- The general method (EC2-1-1, 5.8.6)
- The nominal stiffness method (EC2-1-1, 5.8.7)
- The nominal curvature method (EC2-1-1, 5.8.8)

The Nominal Stiffness Method (NSM) and the Nominal Curvature Method (NCM) are simplified methods that estimate the second-order effects without the need of NLFEA. Both methods aim to find the corresponding maximum bending moment to the applied axial force when second-order effects are considered. NSM reduces the bending stiffness of the column to account for cracking, nonlinear material properties and creep. The first-order moment is then increased by a factor, which depends on the moment distribution in the column and the Euler buckling load calculated with reduced stiffness. NCM estimates the maximum second-order deflection (e_2) from the column's curvature and calculates the second-order moment as $M_2 = N_{\rm Ed}e_2$. The relation between the axial load and the moment is linear until the maximum moment capacity is reached. Beyond the balance point, an increasing axial load results in a decreasing deflection because the column response becomes stiffer. At some point, the second-order eccentricity is so small that the moment decreases for higher axial loads.

The general method is the most sophisticated method suggested in the Eurocode, because it applies NLFEA to handle both geometric and material non-linearity. The general rules for nonlinear analyses, proposed in EC2-1-1 5.7 and 5.8.6, applies for the analyses. DIANA FEA was the software used for the NLFEA simulations. Assumptions made for the analyses include:

- Perfect bond between concrete and reinforcement
- Parabolic stress-strain relationship for concrete in compression
- Post-yielding hardening for steel

The NLFEA model is further described in Section 3.2.

3.1.1 MN-diagram

The axial capacities estimated with NSM and NCM were found by identifying the points where the work diagrams of NSM and NCM intersected with the MN-diagram. Matlab scripts were developed based on the work of Eklund, Skorve & Strand [8] to create MN-diagrams for the chosen geometry and material parameters. The MN-diagrams were developed by demanding strain compatibility and force equilibrium over the cross-section. The calculations were based on linear strain and rectangular compressive stress distribution. The tensile strength of the concrete was neglected for the simplified calculation methods. The concrete and the reinforcement were assumed perfectly bonded and, thus, obtained the same strain at the same part of the cross-section.



Figure 3.1: Rectangular compressive stress distribution [1].

Figure 3.1 shows the assumptions made for the stresses and strains in the MN-diagrams, for an arbitrary cross-section. $A_{\rm c}$ denotes the concrete area assumed in compression, $A_{\rm s}$ denotes the reinforcement area in tension, d is the effective height of the cross-section, x is the distance from the top to the neutral axis, ε_{cu3} is the ultimate concrete strain, ε_s is the reinforcement strain, λx defines the effective height of the compression zone, η defines the effective concrete strength, F_c is the compressive resultant force and F_s is the resultant force from the tensile reinforcement.

3.1.2 Safety format

The safety format used in Eurocode 2-1-1 is based on partial safety factors. Partial safety factors scale all parameters that are considered to contain uncertainties. Material parameters are reduced to increase the level of safety for the structure. PSFs are found by assuming a limit state function and defining a level of reliability that is acceptable for the structure. The partial safety factors are based on empirical assumptions of the sensitivity and variability of the different parameters. Partial safety factors can be applied to both resistance and load. The variation of resistance is defined by

$$V_{\rm R} = \sqrt{{V_{\rm m}}^2 + {V_{\rm G}}^2 + {V_{\rm f}}^2}$$

where $V_{\rm m}$, $V_{\rm G}$ and $V_{\rm f}$ denotes the coefficient of variation of model, geometric and material uncertainties respectively. The values for $V_{\rm m}$, $V_{\rm G}$ and $V_{\rm f}$ are given in Table 3.1, as recommended in Eurocode 2 Commentary [6]. Equation 3.1 and 3.2 determined the partial safety factors for steel and concrete recommended by the Eurocode.

$$\gamma_{\rm s} = e^{(3.04V_{\rm R} - 1.64V_{\rm f})} \tag{3.1}$$

$$\gamma_{\rm c} = 1.15e^{(3.04V_{\rm R} - 1.64V_{\rm f})} \tag{3.2}$$

	Assumed coefficient of variation			
Type of uncertainty	Steel	Concrete		
Model	$V_{\rm m} = 2.5\%$	$V_{\rm m} = 5.0\%$		
Geometry	$V_{\rm G} = 5.0\%$	$V_{\rm G} = 5.0\%$		
Material	$V_{\rm f} = 4.0\%$	$V_{\rm f} = 15.0\%$		

Table 3.1: Coefficients of variation for steel and concrete as recommended in Eurocode 2 Commentary [6].

Eurocode 2-2 [10] introduces a different safety format called the Global Resistance Factor Method (GRFM). This method uses an overall safety factor, which is applied on the final capacity rather than on the uncertain parameters. The concrete compressive strength and the reinforcement yield strength are modified as seen in Equation 3.4 and 3.3.

$$\tilde{f}_{\rm c} = 1.1 \frac{\gamma_{\rm s}}{\gamma_{\rm c}} \alpha_{\rm cc} f_{\rm ck} = 1.27 f_{\rm cd} \tag{3.3}$$

$$\tilde{f}_{\rm y} = 1.1 f_{\rm yk} = 1.27 f_{\rm yd}$$
 (3.4)

GRFM is developed based on the two abovementioned variables, and it is not clear how the safety format should be used when more variables are considered. In this thesis, $E_{\rm c}$ and $f_{\rm ct}$ are calculated with the formulas given in Table 3.2, where $f_{\rm cm,is}$ and $f_{\rm ym,is}$ are substituted with $\tilde{f}_{\rm c}$ and $\tilde{f}_{\rm y}$, respectively. The design resistance is obtained by dividing the resistance by an overall safety factor, $\gamma_{\rm O} = 1.20$. The model uncertainty for resistance $(\gamma_{\rm Rd})$ should be included and can be set to 1.06 when it is not explicitly considered in the analysis, which changes the overall safety factor to $\gamma_{\rm O'} = \gamma_{\rm O}\gamma_{\rm Rd} = 1.27$ [10].

The fib Model Code [11] introduces a method with similarities to GRFM, called Estimate of Coefficient Of Variation (ECOV). The method is based on the concept that the resistance is lognormally distributed, and that the coefficient of variation for the resistance can be calculated. Two sample resistances are needed, the mean resistance (R_m) and the characteristic resistance (R_k) . The resistances are obtained by running analyses with mean and characteristic material parameters, respectively. Both mean and characteristic concrete strength were in this thesis in-situ adjusted as shown in Equation 3.5 and 3.6. This was done because in-situ adjusted values for concrete are considered more realistic. The concrete stiffness was computed by the formula for mean $E_{\rm c}$ in Table 2.3. $f_{\rm cm,is}$ was replaced by $f_{\rm ck,is}$ for the characteristic analysis.

$$f_{\rm cm,is} = \frac{f_{\rm ck}}{1.15} e^{k_{0.05} V_{\rm fc}}$$
(3.5)

$$f_{\rm ck,is} = \frac{f_{\rm ck}}{1.15} \tag{3.6}$$

The coefficient of variation for resistance $(V_{\rm R})$ is approximated, as shown in Equation 3.7.

$$V_{\rm R} = \frac{1}{1.65} \ln \left(\frac{R_{\rm m}}{R_{\rm k}} \right) \tag{3.7}$$

The global safety factor ($\gamma_{\rm R}$) can then be calculated by Equation 3.8. It is suggested to use $\alpha_{\rm R} = 0.8$ and $\beta = 3.8$, which results in Equation 3.9 [11]. Finally, the design resistance is calculated, as shown in Equation 3.10. The model uncertainty ($\gamma_{\rm Rd}$) should be set to 1.06 for well validated models [11].

$$\gamma_{\rm R} = \frac{R_{\rm m}}{R_{\rm d}} = e^{(\alpha_{\rm R}\beta V_{\rm R})} \tag{3.8}$$

$$\gamma_{\rm R} = e^{(3.04V_{\rm R})} \tag{3.9}$$

$$R_{\rm d} = \frac{R_{\rm m}}{\gamma_{\rm Rd}\gamma_{\rm R}} \tag{3.10}$$

A study that compared the different safety formats was conducted, and the results are presented in Section 5.2.

3.2 Nonlinear Finite Element Modeling

The non-linear finite element model developed for this thesis was mainly based on the work of Eklund, Skorve & Strand [8] and recommendations in the Guidelines for NLFEA of concrete structures [12], hereafter referred to as the Dutch Guidelines.

The nonlinear finite element model of the column was created in DIANA FEA 10.1. The purpose of the development of the model was to achieve a more accurate and realistic column capacity, which satisfies the criteria for the General Method in EC2.

The solution strategy is shown in Table 3.2.

CONCRETE				
Finite element				
Element type	Plane stress element CQ16M			
Interpolation scheme	Quadratic			
Integration scheme	Full (2x2 Gauss integration)			
Constitutive modelling				
Model	Total strain based			
Wodel	rotating crack model			
Tensile behaviour	Exponential softening			
Compressive behaviour	Parabolic softening			
Reduction of compressive	Vocchio & Collins 1003			
strength due to lateral cracking	Veccino & Comms 1995			
Lower bound reduction curve	0.4			
Stress confinement model	Selby & Vecchio			
Poisson's ratio reduction model	Damage based			
Material parameters (in-situ adjusted)				
Mean compressive strength	$f_{\rm cm,is} = \frac{f_{\rm ck}}{1.15} e^{k_{0.05} V_{\rm fc}}$	50.08 MPa		
Mean tensile strength	$f_{\rm ctm,is} = 0.3 (\frac{f_{\rm ck}}{1.15})^{\frac{2}{3}}$	3.46 MPa		
Fracture energy	$G_{\rm F,is} = 0.073 f_{\rm cm,is}^{0.18}$	$0.148 \ \mathrm{Nmm}/\mathrm{mm}^2$		
Compressive fracture energy	$G_{\rm C,is} = 250 G_{\rm F,is}$	$37.00 \ \mathrm{Nmm}/\mathrm{mm}^2$		
Modulus of elasticity	$E_{\rm cm,is} = 22 (\frac{f_{\rm cm,is}}{10})^{0.3}$	35.67 GPa		
Poisson's ratio	ν	0.15		
REINFORCEMENT STEEL				
Finite element				
Embedded reinforcement	Yes			
Interpolation scheme	Quadratic			
Constitutive modelling				
Hardening	Strain hardening, isotropic			
Material parameters				
Modulus of elasticity	Es	200 GPa		
Reinforcement steel diameter	6mm	$14\mathrm{mm}$		
${f Yield \ stress},\ f_{ m ym,is}=f_{ m yk}e^{k_{0.05}V_{ m fc}}$	534.00 MPa	$534.00 \mathrm{MPa}$		
Ultimate stress, $f_{\rm uk} = 1.08 f_{\rm ym,is}$	576.72 MPa	576.72 MPa		
LOADING, ITERATION AND CONVERGENCE CRITERION				
Loading	Displacement controlled			
Load steps	$0.1 \mathrm{mm}$			
Equilibrium iteration	Regular Newton-Raphson			
Maximum number of iterations	50			
Force norm	0.01			
Energy norm	0.0001			
No convergence	Continue			

Table 3.2: Adopted Solution Strategy.

3.2.1 Constitutive model

As recommended in the Dutch Guidelines [12], a total strain based crack model was chosen as the constitutive model for concrete. A rotating crack model was applied, which is well suited for reinforced concrete structures [13]. In the DIANA model, the tensile strength of the concrete was included and described by an exponential softening behavior as shown in Figure 3.2 (a). When the tensile capacity was reached, the concrete cracked but retained an exponentially decreasing strength. For the compressive behavior, the parabolic model in Figure 3.2 (b) was preferred as recommended by the Dutch Guidelines. The compressive model in EC2, shown in Figure 3.2 (c), was also evaluated. The EC2 model was based on an ultimate strain limit instead of being dependent on the fracture energy. Abrupt changes in stresses between elements could then cause large deformations locally, and the mesh dependency became an issue. The parabolic model was not sensitive to the mesh and was, therefore, considered the best choice.



(a) Tension: Exponential
 (b) Compression: Parabolic
 (c) Compression: EC2
 Figure 3.2: Concrete stress-strain models [12].

The model defined by Vecchio and Collins [14] was applied to account for losses in compressive strength due to lateral cracking. As recommended in fib Model Code 2010, a lower bound reduction curve of 0.4 was applied to avoid exaggerated reductions that could cause an unrealistic response. The Poisson effect vanishes when concrete cracks and was, therefore, reduced by a damage based reduction model for concrete in tension.

3.2.2 Geometric model

Eklund, Skorve & Strand [8] achieved similar results when applying a 2D plane stress model and a 3D solid model. Because the 2D model was considerably less computationally expensive, this model was preferred for the NLFEA analyses. The column consisted of two parts, one that described the concrete column and another that described the loading platen. All sections were assigned a thickness of 200 mm into the plane. The reinforcement was included as lines without any thickness but had cross-sectional areas corresponding to two reinforcement bars assigned to each of them. Shear reinforcement was also included with 200mm spacing to make the column more robust against lateral cracking and to describe a realistic column. The reinforcement properties are shown in Table 3.2. Figure 3.3 illustrates the implementation of the columns in DIANA.



Figure 3.3: Geometric and finite element discretization models for NLFEA.

3.2.3 Finite element discretization

In accordance with the Dutch Guidelines [12], elements with quadratic interpolation of the displacement field were applied. Quadratic regular plane stress elements, CQ16M, with 2x2 Gauss integration were used. The reinforcement was embedded in the model, meaning that it did not have degrees of freedom of its own but contributed with additional stiffness to the mother element. For embedded reinforcement, the reinforcement and the concrete are perfectly bonded, which implies that the reinforcement strain is calculated in the mother element. The element size was chosen based on the two criteria in the Dutch Guidelines, given by Equation 3.11 and 3.12.

$$h_{\rm max} \le \frac{h_{\rm eq}}{2} < \frac{E_{\rm c}G_{\rm F}}{2f_{\rm ct}^2}$$
 (3.11)

$$h_{\max} < \min\left(\frac{L}{50}, \frac{h}{5}\right) \tag{3.12}$$

Equation 3.11 is meant to avoid a snap-back behavior in the stress-strain relationship, while Equation 3.12 prevents too coarse meshing that may lead to jumps in the stress field. An element size of 25 mm was chosen to obtain a smooth mesh with evenly sized elements in both vertical and horizontal direction. The latter criterion recommends a maximum element size of 20mm for the column with a length of 1000 mm. However, for consistency and based on a comparison with results where smaller element sizes were applied, the chosen element size was considered sufficient.

3.2.4 Boundary conditions and load application

A symmetry line at the mid-length of the column was applied to reduce the computational time. The applicability of using a symmetry line was verified by Eklund, Skorve & Strand [8]. A 50 mm steel platen was placed on top of the column to avoid high concentrations of stress around the point of load application. The platen had Young's Modulus of 200 GPa and Poisson's ratio of 0.3. The load was modeled as a prescribed displacement and was placed at the top of the loading platen. The point of load application was supported in the horizontal direction while the entire cross-section at the mid-length was supported only in the vertical direction.

3.2.5 Analysis

Non-linear elasticity, plasticity and total strain based cracking were used in the non-linear structural analysis. Creep was not directly included, as mentioned in Section 2.2.4.

The geometric non-linearity was implemented by the Total Lagrange description, where stresses and strains are defined with respect to the undeformed geometry [13]. The default parallel direct sparse solver was used to solve the systems of equations.

The analysis was run with 60 load steps, which were kept constant at 2% of the prescribed total vertical displacement of 5 mm. The regular Newton-Raphson iteration scheme was preferred to solve the equilibrium equations. The method is considered effective in most cases and may reach a quadratic convergence rate [15]. The model was tested for smaller load steps to ensure that the load step size was adequate. The change in the results was negligible, and the chosen load step size was applied to avoid unnecessary computational time. In accordance with the Dutch Guidelines [12], a force norm check in combination with an energy norm check was applied with convergence criteria of 0.01 and 0.0001, respectively. A maximum of 50 iterations was considered sufficient to ensure convergence where it was possible, yet low enough to avoid excessive time consumption. If convergence was not obtained, the analysis continued to the next load step. The occurrence of non-converging steps demanded a careful check of the results. However, non-converging steps only appeared after failure had occurred. The continue option was used to avoid that Matlab scripts, where many analyses were run consecutively, stopped due to non-convergence after the occurrence of failure.

Output values chosen for further investigation included the horizontal displacement at the mid-length, the stresses and strains in the longitudinal reinforcements and the concrete elements along the cross-section at mid-length, as well as the moment and the vertical reaction forces. The vertical force in the column was calculated by summation of the reaction forces at mid-length. The moment was found from the composed line through the vertical center line of the column.

4. Structural Reliability Methods

Several reliability analyses were conducted to determine the reliability of the design methods and to compare the different reliability methods. The different reliability methods performed with the three design methods are listed in Table 4.1. Firstly, reliability analyses were conducted with the design capacities found with NSM, NCM and NLFEA used as input. The reliability indexes found for the different capacity calculation methods were then compared to the target reliability index of 3.8. RSM-FORM was applied to perform inverse analyses and calculate partial safety factors. The inverse analyses were conducted by finding the design point where the reliability index was exactly 3.8, which is the value of a 50-year reference period for a residence/office building [5].

Table 4.1: Overview of the capacity calculation methods and the performed reliability analyses.

	NSM	NCM	NLFEA
MC	Yes	Yes	No
FORM	Yes	No	No
RSM-FORM	Yes	Yes	Yes

4.1 Limit State

The Limit State Function (LSF) represents the state of a structure when it no longer satisfies the relevant design criteria. The ultimate limit state is given by G = R - S, where R and S represent the resistance and the load, respectively. The load and the resistance are associated with uncertainties and are represented by random variables. Therefore, they are assigned suitable probability density functions. When R and S have been assigned probabilistic properties, the LSF can be used in the calculation of the probability of failure ($P_{\rm f}$). The structure will fail when the load is greater than the resistance. Thus, the probability of failure is given as $Pr(G \leq 0)$.



Figure 4.1: Probability density function of resistance, load and safety margin [16].

A method developed by Basler [17], presented in the notation of Cornell [18], can be used to calculate the exact probability of failure. The method is based on the LSF, rewritten as the safety margin M = R - S. The safety margin is shown in Figure 4.1, where the load and resistance are assumed normally distributed. From statistics, it follows that the sum of two normally distributed random variables becomes a normally distributed random variable. Hence M is normally distributed with mean and coefficient of variation expressed by Equation 4.1 and 4.2.

$$\mu_{\rm M} = \mu_{\rm R} - \mu_{\rm S} \tag{4.1}$$

$$\sigma_{\rm M} = \sqrt{\sigma_{\rm R}^2 + \sigma_{\rm S}^2} \tag{4.2}$$

The reliability index (β) is calculated by Equation 4.3. The physical interpretation of β is the number of times the standard deviation ($\sigma_{\rm M}$) can be placed between 0 and the mean value ($\mu_{\rm M}$) [16]. The probability of failure can be found from standard normal distribution tables using Equation 4.4.

$$\beta = \frac{\mu_{\rm M}}{\sigma_{\rm M}} \tag{4.3}$$

$$P_{\rm f} = \Phi(-\beta) \tag{4.4}$$

Ditlevsen [19] discovered that the results from the procedure of Basler/Cornell were dependent on how the safety margin was formulated and called it the invariance problem. To avoid this issue, Hasofer and Lind [20] suggest converting the random variables and the limit state to the standard normal space. In this space, all the random variables have a mean and a standard deviation of 0 and 1, respectively. The eccentricity and the load were assumed to be normally distributed and had to be converted into the standard normal space to be used in the method proposed by Hasofer and Lind. The equations below show the transformation between the normal space (Equation 4.5) and the standard normal space (Equation 4.6). u is a realization of the random variable U in the standard normal space. μ_x , σ_x , and V_x denote the mean, standard deviation and coefficient of variation for a random variable X.

$$X(u) = u\sigma_{\rm x} + \mu_{\rm x} \tag{4.5}$$

$$U(x) = \frac{x - \mu_{\rm x}}{\sigma_{\rm x}} \tag{4.6}$$

The stochastic variables related to the material parameters $(f_c, f_y \text{ and } E_c)$ were assumed to be lognormally distributed. Equation 4.7 and 4.8 show the transformations between the lognormal space and the standard normal space, respectively.

$$X(u) = \exp(u\sigma_{\rm ln} + \mu_{\rm ln}) \tag{4.7}$$

$$U(x) = \frac{\ln(x) - \mu_{\ln}}{\sigma_{\ln}} \tag{4.8}$$

where the lognormal mean and lognormal standard deviation are given as

$$\mu_{\rm ln} = \ln\left(\mu_{\rm x}^2 \sqrt{\frac{1}{\sigma_{\rm x}^2 + \mu_{\rm x}^2}}\right), \quad \sigma_{\rm ln} = \sqrt{\ln(V_{\rm x}^2 + 1)}$$

4.2 Reliability methods

4.2.1 Monte Carlo

The Monte Carlo (MC) method is the reliability method that obtains the probability of failure with the potentially highest accuracy. The method is based on a series of simulations where the stochastic variables are given random realizations at each simulation run, dependent on their respective distribution functions. For every simulation, the column resistance is calculated and compared to a realization of the load. The column fails if the load exceeds the resistance. The probability of failure can then be estimated as the number of failures divided by the number of simulations. The reliability index can be calculated by use of the inverse normal cumulative distribution $\beta = \Phi^{-1}(P_{\rm f})$. In structural reliability problems, a low probability of failure is desired. Failure will then occur when the stochastic variables get values from the tail region of the distributions. A large number of simulations is required to obtain reliable results for the probability of failure. This calls for a limit state function that is very fast to compute, which excludes the use of Monte Carlo in combination with NLFEA.

4.2.2 First Order Reliability Method

The First Order Reliability Method (FORM) is a method that approximates the probability of failure with a considerably lower computational cost than the MC method. FORM can also indicate the sensitivity of the random variables on the limit state function, which is a helpful tool for calculation of PSFs. In FORM, the limit state function is linearized around the design point (x_i) by use of the linear terms in a Taylor series expansion. The limit state function will then be approximated on the form

$$G = a_0 + \sum_{i=1}^n a_i X_i, \tag{4.9}$$

where n denotes the number of random variables and a_i is a set of constants.

Schneider [16] proposes the following iteration procedure:

- 1. Approximate the limit state function by Equation 4.9
- 2. Determine $a_i = \frac{\partial G}{\partial X_i} | *$ and $a_0 = G(x_i^*) \sum_{i=1}^n a_i x_i^*$
- 3. Estimate the design point x_i^* , e.g. by starting with mean values
- 4. Calculate the mean $(\mu_{\rm G})$ and standard deviation $(\sigma_{\rm G})$ of G
- 5. Calculate the reliability index (β), the sensitivity factors (α_i), and the next design point (x_i^*)

$$\beta = \frac{\mu_{\rm G}}{\sigma_{\rm G}} \qquad \alpha_i = \frac{a_i(x_i^*)}{\sigma_{\rm G}} \qquad x_i^* = -\alpha_i \beta$$

6. Check for convergence of the design point,

$$\epsilon_{\text{FORM}} = \sqrt{\sum_{i=1}^{n} (x_{i,j} - x_{i,j-1})^2}$$

where j denotes the iteration number

7. If convergence is acquired, compute $P_{\rm f}$, else return to 3. with the new design point x_i^*

4.2.3 Response Surface Method

The Response Surface Method (RSM) is an effective method when a closed-form limit state is unavailable. When an NLFEA model is used for capacity calculations, a closed-form limit state does not exist and needs to be approximated. RSM creates a polynomial function to approximate the limit state (LS) dependent on the stochastic variables considered. When used in combination with an NLFEA model, several simulations with different sample points must be conducted. Each sample point includes realizations of the stochastic variables. The resistances obtained from the analyses form the basis for the regression procedure conducted by RSM. Computing the resistance for all possible combinations of realizations of the stochastic variables will result in infinite time consumption. Therefore, a simplified computational procedure must be conducted. Bucher and Bourgund [21], propose a polynomial function

$$\bar{g}(x) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2$$
(4.10)

where only 2n + 1 simulations are required to create a unique response surface (RS). n denotes the number of stochastic variables, x_i are the stochastic variables and a, b_i and c_i are parameters that need to be determined. An alternative method to increase the accuracy of the response surface is to extend the approximated LS, $\bar{g}(x)$, by including cross terms presented in Equation 4.11 [21]. This increase of accuracy requires 0.5(n+1)(n+2)simulations to be conducted.

$$\bar{g}(x) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 + \sum_{i \neq j} \sum_{i \neq j} d_{ij} x_i x_j$$
(4.11)

Bucher and Bourgund propose to set x_i to the mean values \bar{x}_i of the stochastic variables and include variations as $x_i = \bar{x}_i \pm f_i \sigma_i$, where f_i is an arbitrary factor. For the polynomial without cross terms given by Equation 4.10, the stochastic variables were all set to the mean value for the first experiment and then changed one by one with $\pm f_i \sigma_i$ for the following experiments.

In the following, the stochastic variables and the undetermined coefficients are represented by the vectors $\mathbf{A} = [1, x_1, x_2, x_1^2, x_2^2]$ and $\mathbf{b}^T = [a, b_1, b_2, c_1, c_2]$, for an example with 2 stochastic variables. The undetermined coefficients can be found by $\mathbf{b} = \mathbf{A}^{-1}\mathbf{g}$, when the number of sample points equals the number of parameters in the RS. \mathbf{g} is a vector containing the capacities computed with NLFEA at the different sample points.

4.2.4 RSM-FORM

RSM was applied in combination with FORM iterations, to obtain a better approximation for the response surface. The FORM iterations were included to optimize the design points and move them closer to the actual limit state, which reduces the error. The iterative procedure was based on the work of Eklund, Skorve & Strand [8].

- 1. A RS is created based on the initial sample points for the stochastic variables
- 2. FORM is used to find updated design points and β with the RS as input
- 3. Convergence check: $\epsilon_{\text{RSM}} = \sqrt{\beta_{i+1}^2 \beta_i^2}$
- 4. If convergence is obtained, the iterations are stopped

Else, the updated design points are used as initial sample points in the next iteration

The convergence criterion for RSM was set to 0.01. The probability of failure was calculated with FORM when the RSM iterations had converged.

4.3 Reliability assessment with NSM and NCM

Reliability analyses conducted with the simplified calculation methods were mostly carried out to verify the reliability analyses conducted in conjunction with NLFEA. The analyses were also performed to investigate which parameters that were dominant in NCM/NSM, which was important when the partial factors should be determined. Furthermore, the analyses were meant to investigate the accuracy of the simplified methods.

4.3.1 Monte Carlo

Monte Carlo simulations were used for validation of FORM and RSM-FORM, to ensure that they worked correctly. As previously mentioned, the MC method is in most cases the most accurate method for calculating the probability of failure. In this case, however, the MC simulations are based on the NSM/NCM methods, which are not considered accurate representations of the reality.

MC was used to calculate the probability of failure when design loads computed with NSM and NCM were used as input. For each MC simulation, an MN-diagram was created and combined with a work-diagram calculated by either NSM or NCM. The intersection gave the resistance of the column, which then was compared to a realization of the load and checked for failure. The MC simulations could provide information about how safe the partial safety factor method proposed by EC2 was for the simplified methods. However, the results would only be completely reliable under the assumption that the method for calculating the resistance (NSM and NCM) described the physical situation correctly. Considering that the two methods used different assumptions and simplifications to compute the resistance, the results had to be compared with other reliability methods to be verified. The most accurate results that could be drawn from the MC simulations were related to how the variables impacted the safety margin for different calculation methods and slenderness ratios and what the limit state function should look like with respect to the random variables.

NCM showed some weaknesses for certain combinations of the random variables. In NCM, the second-order eccentricity is dependent on the axial force and decreases when the axial force increases. Therefore, the work-diagram of NCM shows a descending moment after a specific axial force is reached. In some cases, the work-diagram and the MN-diagram have a similar shape in the descending branch, and could, therefore, intersect multiple times. The first intersection point was used to determine the capacity in those cases.

4.3.2 FORM

FORM was applied to differentiate the two considered failure modes of the different columns. To distinguish between compressive failure of concrete and yield failure of reinforcement, two different limit states were used. These limit states were originally developed by Eklund, Skorve & Strand [8] and are rendered briefly here.

The two limit states that were introduced should represent compressive failure of the concrete and buckling failure initiated by yielding of the tensile reinforcement. The MN-diagram was, therefore, divided into two separate parts describing each failure mode. The limit states were then represented by the intersection points between the work diagram and the two MN-diagrams. The two MN-diagrams describing the failure modes were found with Matlab's solve function and symbolic variables. Compression failure was described under the assumption of constant ultimate strain, ε_{cu3} , at the top of the cross-section. It was also assumed no yielding of the reinforcement, even if the yield strain was exceeded.



Figure 4.2: Stress and strain in the cross-section for compression failure [8].

Figure 4.2 shows the stress and strain state that represent compression failure.



Figure 4.3: Stress and strain in the cross-section at the two end points of the line representing yield failure [8].

Yield failure was as a simplification represented by the line ranging from the point of full tension in the whole cross-section to the point where ε_{cu3} was reached in the concrete and simultaneously double yield strain was reached in the bottom reinforcement. The stress and strain state in the two points can be seen in Figure 4.3.



Figure 4.4: Compression failure LS, yield failure LS and an arbitrary work-diagram with NSM.

Figure 4.4 shows compression failure and yield failure for an arbitrary column, combined with an arbitrary work-diagram calculated with NSM. The two intersection points indicate the compression and yield limit state for the column. The limit states are given by $G1 = M_{\text{compression}} - M_{\text{work}}$ and $G2 = M_{\text{yield}} - M_{\text{work}}$, where $M_{\text{compression}}$ and M_{yield} represent the cross-section moments given for the strain states shown in Figure 4.2 and 4.3, respectively.

NSM was the only method used in conjunction with FORM. NCM has a more complex work-diagram than NSM and could cause the limit state equations in Matlab to find multiple solutions.

4.3.3 RSM-FORM

RSM-FORM analyses were conducted for both NSM and NCM to verify the Matlab scripts developed for RSM-FORM with NLFEA. The verification was carried out by comparing the results from RSM-FORM to the output from FORM and MC. The response surface was generated by nine sample points surrounding an initially assumed design point. The arbitrary factor f was set to 3.0. The results from the FORM analysis, combined with engineering judgment, were used to estimate an appropriate starting point for RSM-FORM.

4.3.4 System reliability

System reliability should be taken into consideration whenever the structure has multiple failure mechanisms. In this case, the system can be viewed as a chain that will fail if one of the links fails, i.e. one of the limit states are below zero. This is called a series system. The failure domain for a series system ($\Omega_{\rm F}$), is given by Equation 4.12, while the probability of failure ($P_{\rm fsys}$) is approximated by Equation 4.13 [22]. Φ_n is the *n*-dimensional standard normal distribution, where *n* is the number of limit states. **B** is a vector containing β -values for the two limit states in the FORM analysis, and **R** is the correlation coefficient matrix. **R** is approximated with Equation 4.14, where *i* denotes the limit state and **A**_i contains the sensitivity factors from FORM. $\beta_{\rm sys}$ is given by Equation 4.15 and is used to evaluate the significance of system reliability.

$$\Omega_{\rm F} = \bigcup_{i=1}^{n} \{g_i(\mathbf{x}) \le 0\}$$

$$(4.12)$$

$$P_{\rm fsys} = 1 - \Phi_n(\mathbf{B}, \mathbf{R}) \tag{4.13}$$

$$\rho_{ij} = \mathbf{A}_i \mathbf{A}_j^T, \quad i = 1, 2, ..., n \quad j = 1, 2, ..., n$$
(4.14)

$$\beta_{\rm sys} = \Phi^{-1}(P_{\rm fsys}) \tag{4.15}$$

4.4 Reliability assessment with NLFEA

RSM in combination with FORM was used in the reliability assessment with NLFEA. The method was chosen over the most accurate method with Monte Carlo simulations due to computational time limitations. RSM-FORM was also necessary since FORM alone demands a closed form limit state, which was impossible to obtain.

The development of the finite element model used in DIANA is described in Section 3.2 and summarized in Table 3.2. The table also contains the material parameters used in the model. The verification of the model is described in further detail in Appendix A. Choosing appropriate starting values for the design point was essential to decrease the computational time, and it also improved the response surface. Running RSM-FORM analyses with NSM and NCM gave good estimations for the first design points. For each RSM iteration, NLFEA was run in each of the sample points. Nine sample points were created for each RSM iteration to satisfy the requirement of 2n + 1 analyses, which is needed to create a RS by a second-order polynomial. For the resistance calculations, the number of variables (n) was four because the load variable was not necessary for resistance calculation.

Unlike NSM and NCM, where the capacity was found at the intersection between the MN-diagram and the work diagram, the capacity was determined by the maximum reaction force before failure. If the column was loaded further than this point it would either collapse in compression or the second-order eccentricity would grow rapidly and eventually cause buckling.

4.5 Assessment of calculation methods

The quantification of model uncertainty is a complex matter in structural reliability. A wide set of physical experiments should be conducted, to thoroughly test the uncertainty of a model. Even if physical experiments are carried out, there is no way of knowing exactly the value of the material parameters of the test specimen. Therefore, general and accurate quantification of model uncertainty is not established.

In this thesis, physical experiments were not carried out, and the model uncertainty could, therefore, not be quantified. However, the ratio between the capacities found with the different calculation methods could be quantified in a probabilistic manner, similar to the model uncertainty. The NLFEA model with the PSF method was considered the most accurate model available. Therefore, the design capacities found with this model were assumed to represent the physical experiments, while NSM and NCM represented the models to be tested. The probabilistic assessment of the calculation methods was carried out to quantify the uncertainty of the simplified methods compared to the best estimate model.

The resistances achieved with the different models were assumed lognormally distributed

because the material parameters had a log-normal distribution. The resistances from the i^{th} simulation run of NLFEA and NSM/NCM are denoted $R_{\text{best},i}$ and $R_{\text{simp},i}$, respectively. For each simulation, the capacity ratio (θ_i) is estimated from the ratio between $R_{\text{best},i}$ and $R_{\text{simp},i}$, by Equation 4.16.

$$\theta_i = \frac{R_{\text{best},i}}{R_{\text{simp},i}} \tag{4.16}$$

The quantity $X = \ln(\theta)$ is normally distributed with mean (μ_X) and standard deviation (σ_X) from Equation 4.17 and 4.18, respectively.

$$\mu_{\rm X} = \frac{1}{n} \sum_{i=1}^{n} \ln(\theta_i) \tag{4.17}$$

$$\sigma_{\rm X} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\ln(\theta_i) - \mu_{\rm X})^2}$$
(4.18)

n is the number of samples that were run for each of the calculation methods. The mean value and the standard deviation of the lognormally distributed parameter θ could then be found by Equation 4.19 and 4.20, respectively [23].

$$\mu_{\theta} = \exp\left(\mu_{\rm X} + \frac{\sigma_{\rm X}^2}{2}\right) \tag{4.19}$$

$$\sigma_{\theta} = \sqrt{\exp\left(2\mu_{\mathrm{X}} + \sigma_{\mathrm{X}}^{2}\right)\left[\exp(\sigma_{\mathrm{X}}^{2}) - 1\right]}$$
(4.20)

n simulations were run in Monte Carlo for the two simplified methods. The same random realizations of the stochastic variables were then applied to the NLFEA model. Due to computational limitations when conducting NLFEA, only a limited number of samples could be run for each column length. The results were still considered valuable to quantify the uncertainty of the results obtained by the simplified methods compared to the results from NLFEA.

4.6 Inverse analyses and PSF

Inverse analyses were conducted to evaluate the sensitivity of the stochastic variables and to propose partial factors for use in slender column design. Inverse analyses were carried out by changing the input variables to obtain a target reliability index, β_{target} . In accordance with EN-1990 [5], β_{target} was set to 3.8, which is the target reliability index for a reference period of 50 years. The mean values for the material parameters and the eccentricity were predetermined, and the only variable that could be adjusted was the load. For the reliability analyses conducted with the simplified calculation methods, the mean value of the load was adjusted until β_{target} was achieved. An accuracy of $\beta_{\text{target}} \pm 0.01$ was set as the criterion for the inverse analyses. Only FORM and RSM-FORM were used in the inverse analyses because MC was too time-consuming for iteration procedures.

When β_{target} was obtained, the corresponding design points could be used to calculate appropriate PSFs. The design values for f_{cd} , f_{yd} , E_{cd} and N_{Ed} were used to find the respective partial factors. The partial factors were calculated the same way the design values are calculated in EC2. The calculation approach used to determine the different PSFs is listed in Table 4.2.

	$oldsymbol{f}_{ ext{c}}$	$oldsymbol{f}_{ ext{y}}$	$oldsymbol{E}_{ m c}$	$oldsymbol{N}$
Abbreviation	$\gamma_{ m c}$	$\gamma_{ m s}$	$\gamma_{ m CE}$	$\gamma_{ m G}$
Reference value	Characteristic	Characteristic	Mean	Nominal
Calculation	$f_{ m ck}/f_{ m cd}$	$f_{ m yk}/f_{ m yd}$	$E_{\rm cm,is}/E_{\rm cd}$	$N_{\rm Ed}/N_{ m Nom}$
PSF in EC2	1.50	1.15	1.20	1.35

Table 4.2: Strategy to calculate partial safety factors.

The in-situ adjusted mean value of concrete stiffness, $E_{\rm cm,is}$, was used as the reference because this value was used in the reliability analyses. The load was assumed to originate from self-weight and the nominal load value was used as the reference. The nominal load was assumed to correspond to the mean capacity calculated in the inverse analyses. The eccentricity was the last stochastic variable, but the eccentricity is not treated with a PSF in EC2. In this thesis, the minimum eccentricity of 20mm was used for design, and this value could be examined by comparing it to the design point of eccentricity.

5. Results & Discussion

5.1 Design capacities

The design capacities for the different column lengths are presented in this section. The design capacities are calculated with NSM, NCM and NLFEA. The material parameters are reduced by partial safety factors, following the rules for design in Eurocode 2-1-1, for all the three methods. The partial factors are given in Section 2.2.1 and the minimum eccentricity of 20 mm is applied. The design capacities are listed in Table 5.1.

Length [mm]	NSM	NCM	NLFEA
1000	1112	1139	1069
2000	942	1034	930
3000	667	796	758
4000	423	550	596
5000	268	412	443
6000	184	252	330

Table 5.1: Design capacities for NSM, NCM and NLFEA in kN.

The two simplified methods in EC2 are compared for columns with different slenderness. Figure 5.1 shows the MN-diagrams and the work-diagrams for the nominal stiffness method and the nominal curvature method. The design capacities are found at the intersection points between the MN-diagram and the work diagrams for the different column lengths.



Figure 5.1: MN-diagram for the two analytic methods in EC2.

Discussion

It was expected that the NLFEA analyses should yield the least conservative results because the method is the most sophisticated of the three methods conducted in this thesis. However, the results in Table 5.1 show that NLFEA was least conservative only for the three slenderest columns. NCM, in particular, was non-conservative compared to NLFEA when columns with low slenderness ratio were considered. The second-order eccentricity in NCM is proportional to the square of the effective length, which seems to be a conservative assumption only for columns where second-order effects are dominating.

NSM was more conservative than NCM for all column lengths. The underlying reasons for the differences in the results between the two simplified methods can to some extent be identified by examining Figure 5.1. The work-diagram of NCM is linearly increasing until the axial load at the balance point of the MN-diagram is reached. For axial loads above the balance point, the predicted second-order eccentricity decreases because the response of the column becomes stiffer when it is loaded with high axial loads. Figure 5.1 (b) shows that this effect arises before the intersection with the MN-diagram, for the four least slender columns. Eventually, this leads to a decreasing second-order moment when the axial loads are high.

The relative difference between the design capacities found with NSM and NCM was, however, most significant for the slenderest columns. NSM is governed by a Euler buckling load calculated with reduced stiffness. Figure 5.1 (a) shows that the work diagram has a horizontal asymptote at this buckling load. The work-diagram for the slenderest columns approach this asymptote before the intersection with the MN-diagram. Therefore, the shape of the MN-diagram is of lesser importance for the slenderest columns. The governing reason for the lower Euler buckling load in NSM is that creep is treated differently in the two methods.

5.1.1 Impact of creep

How creep impacted the design capacities is examined in this section. The design loads found with the different calculation methods are shown in Figure 5.2. Figure 5.2 (b) shows the design loads when creep is neglected in the calculations.



Figure 5.2: Design loads for NSM, NCM and NLFEA.

The impact of creep is further investigated for the different calculation methods. Figure 5.3 shows the ratio between the design capacities calculated with NSM and NCM for different slenderness ratios when creep was included and neglected.



Figure 5.3: Ratio between NSM and NCM.

Discussion

It is evident that creep has a significant impact on the design capacity when second-order effects are considered. Figure 5.3 shows how the two simplified methods in EC2 yields different results when creep is included and when it is neglected. For columns with slenderness ratio roughly above 70, NCM was the most conservative method when creep was neglected or when the impact of creep was small, while NSM became the most conservative method when creep was substantial. It is, therefore, important which method that is used in slender column design. For the columns with low slenderness ratio, NSM was only slightly more conservative than NCM.

The reason for the differences originates from the criteria governing whether creep should be included in the methods or not. In NSM, creep is included through a reduction of the stiffness, like the way creep was applied to NLFEA in this thesis. The stiffness is scaled by a constant, $\frac{1}{1+\phi_{\text{ef}}}$, and is included regardless of the column length. For NCM, however, creep is included through a factor that also depends on the slenderness. Thus, creep is not included when the slenderness ratio is higher than the limit given by Equation 5.1. For the column geometry described in this thesis, creep was not included in NCM for columns of length 5000 mm or longer.

$$\lambda > \frac{105}{2} + \frac{3}{4} f_{\rm ck} \tag{5.1}$$

Figure 5.2 shows how the design capacities for all the three different methods were affected by creep. For the NLFEA model, creep was most significant for the slenderest columns. Since creep was included through a reduction of the concrete stiffness, as explained in Section 2.2.4, it seems natural that the slenderest columns were most affected. That is because a reduction in the concrete stiffness resulted in a higher strain of the tensile reinforcement, and provoked yield failure at an earlier stage. The least slender columns did not experience the same effect because the impact of the second-order moment was small and, consequently, the stress in the tensile reinforcement was insignificant. The relative difference between the design capacities found with NSM and NLFEA was not drastically changed when creep was included, because creep was treated similarly in the two methods. If NLFEA is used as a reference, NCM became far less conservative when creep was included for columns with high slenderness ratio.

5.2 Safety format study

The safety format that was used for most of this thesis was the partial safety factor method, recommended by EC2. In this section, the results from a safety format study are presented. The three safety formats introduced in Section 3.1.2 were used to find design capacities with NLFEA.



Figure 5.4: Comparison of safety formats applied to NLFEA.

Figure 5.4 shows the design capacities calculated with NLFEA for the three safety formats that were tested. In Table 5.2, the relative difference of the design capacities are presented. The partial safety factor method is used as the reference, and the design capacities computed with GRFM and ECOV are divided by the design capacities calculated with the PSF method to find the relative differences. These results are also illustrated in Figure 5.5, where the relative capacities for NSM and NCM are included.

Table 5.2: Design capacities for GRFM and ECOV relative to the PSF method.

Column length [mm]	1000	2000	3000	4000	5000	6000
PSF	1	1	1	1	1	1
GRFM	0.99	0.97	0.95	0.92	0.88	0.88
ECOV	0.99	1.00	1.00	0.99	0.97	0.96



Figure 5.5: Design capacities for all methods, relative to NLFEA PSF.

Discussion

The partial safety factor method was for most of the columns the least conservative safety format examined in this thesis. Figure 5.4 and Table 5.2 show that the two other methods, particularly GRFM, became more conservative for higher slenderness ratios. Recall that the eccentricity was treated equally for all the methods, considering that the eccentricity was not treated by a partial safety factor but rather by a minimum value of 20 mm. Thus, the most apparent difference between the methods was the treatment of the stochastic material parameters. It is emphasized that international standards do not entirely cover the way GRFM and ECOV should be implemented, and the obtained results are based on certain assumptions.

GRFM has the drawback that it is only thoroughly formulated for two variables, namely, f_c and f_y . It was, therefore, necessary to make appropriate assumptions to include variables such as E_c and f_{ct} . It was considered a valid approximation to compute these variables based on the two specified variables. For the method to be considered as an equivalent alternative to the PSF method, a more detailed description for more complex models should be developed. GRFM was more conservative than the PSF method for all column lengths, and can, therefore, be considered safe. The difference between GRFM and the PSF method increased for higher slenderness ratios because the partial factors related to material parameters have less impact on the capacity when the second-order effects are dominating. The partial factor of $\gamma_c = 1.5$, for instance, is

far more dominant for shorter columns subjected to compression failure than columns subjected to yield failure. In GRFM the total capacity is divided by the same overall safety factor independent on the slenderness. Consequently, GRFM becomes considerably more conservative than the PSF method for highly slender columns.

The primary challenge when applying ECOV as safety format is to define the mean and characteristic material parameters. In this thesis, the method was implemented with in-situ adjusted concrete strength, which was considered to give the most realistic mean value. This resulted in very similar results between ECOV and the PSF method, with ECOV mainly on the conservative side. However, it must be emphasized that this was the case because of the assumption of in-situ reduced concrete strength, and reduction of the other material parameters accordingly. It is, therefore, suggested to exercise caution when ECOV is used, given that it has the potential of being less conservative than the PSF method.

5.3 Reliability analyses

5.3.1 Simplified calculation methods

The results from the reliability analyses used in conjunction with analytic capacity calculation methods are presented in this section. The analyses include Monte Carlo simulations with NSM and NCM, FORM with NSM and RSM-FORM with NSM and NCM.

Monte Carlo was run with 10^7 simulations for both NSM and NCM. Table 5.3 shows the β -value and $P_{\rm f}$ for the different column lengths. 10 million simulations with NSM resulted in zero failures for columns of length 1000 mm and 2000 mm. The probability of failure could, therefore, not be calculated for these column lengths, but it indicated that $P_{\rm f} < 10^{-7}$.
Column Length [mm]		1000	2000	3000	4000	5000	6000
NSM	$oldsymbol{eta}$	-	-	3.46	2.24	1.95	1.78
	$oldsymbol{P}_{ m f}$	-	-	2.69E-04	1.24E-02	2.56E-02	3.73E-02
NCM	β	5.20	5.07	4.31	3.05	2.61	2.16
	$oldsymbol{P}_{ m f}$	1.00E-07	2.00E-07	8.30E-06	1.20E-03	4.50E-03	1.54E-02

Table 5.3: Results from Monte Carlo with NSM and NCM.

Figure 5.6-5.10 show all 10 million MC simulations. The black dots indicate failure. The absolute value of the eccentricity was used, and the mean value of the load was given by the design capacities in Table 5.1 divided by 1.35, which is the safety factor for governing self-weight. Figure 5.6 and 5.7 show the Monte Carlo results where the design capacities calculated with NCM were examined for columns of length 3000 mm and 5000 mm, respectively.



(a) Concrete strength - Load (b) Yield strength - Load (c) Eccentricity - Load

Figure 5.6: Monte Carlo for design loads calculated with NCM - 3000 mm column.



Figure 5.7: Monte Carlo for design loads calculated with NCM - 5000 mm column.

Figure 5.8-5.10 compare the stochastic variables related to resistance for NSM and NCM. All the material parameters are plotted against realizations of the load.



Figure 5.8: Concrete strength - Load for 4000 mm column.



Figure 5.9: Yield strength - Load for 4000 mm column.



Figure 5.10: Concrete stiffness - Load for 3000 mm column.

Table 5.4 shows the β -values calculated with FORM for the two closed-form failure modes using NSM. β_{sys} denotes the system reliability index, which is described in Section 4.3.4.

Column Length [mm]	1000	2000	3000	4000	5000	6000
$oldsymbol{eta}_{ ext{comp}}$	5.43	5.96	3.66	2.53	2.26	2.08
$oldsymbol{eta}_{ ext{yield}}$	6.97	7.30	3.68	2.40	2.10	1.93
$oldsymbol{eta}_{ m sys}$	5.43	5.96	3.63	2.39	2.09	1.92

Table 5.4: β -values with FORM and NSM.

The results from the analytic RSM-FORM analyses are summarized in Table 5.5 and 5.6 for NSM and NCM, respectively. The mean loads are the design capacities presented in Section 5.1 divided by the partial safety factor for governing self-weight.

Column Length [mm]	1000	2000	3000	4000	5000	6000
NSM mean load [kN]	824	698	494	313	199	136
$oldsymbol{eta}$	5.30	5.69	3.54	2.41	2.14	1.98
$lpha_{ m fc}$	0.700	0.574	-0.224	-0.335	-0.284	-0.241
$oldsymbol{lpha}_{\mathrm{fy}}$	0.052	0.041	0.004	0.007	0.007	0.006
$lpha_{ m Ec}$	0.002	0.082	0.304	0.262	0.222	0.193
$oldsymbol{lpha}_{ m e}$	-0.157	-0.445	-0.560	-0.434	-0.414	-0.385
$lpha_{ m N}$	-0.695	-0.681	-0.737	-0.794	-0.836	-0.870
$f_{ m cd}~[{ m MPa}]$	28.5	30.4	55.8	55.9	54.2	53.2
$f_{ m yd}~[{ m MPa}]$	528	529	533	533	533	533
$E_{ m cd}~[{ m GPa}]$	35.18	32.73	29.75	31.90	32.69	33.18
$e_{ m d} [m mm]$	1.3	8.1	9.6	6.7	7.1	7.3
$N_{ m Ed}~[{ m kN}]$	1431	1269	752	433	270	183

Table 5.5: Results from RSM-FORM with NSM.

Column Length [mm]	1000	2000	3000	4000	5000	6000
NCM mean load [kN]	844	766	590	407	305	187
$oldsymbol{eta}$	5.16	5.30	4.25	3.18	2.80	2.37
$lpha_{ m fc}$	0.700	0.648	0.414	0.423	0.296	0.201
$lpha_{ m fy}$	0.051	0.020	-0.229	-0.208	-0.195	-0.201
$lpha_{ m Ec}$	0.000	-0.002	-0.026	0.001	-0.002	0.000
$lpha_{ m e}$	-0.145	-0.396	-0.694	-0.525	-0.578	-0.516
$lpha_{ m N}$	-0.698	-0.650	-0.542	-0.709	-0.734	-0.815
$f_{ m cd}~[{ m MPa}]$	28.9	29.7	38.1	40.5	43.8	46.1
$f_{ m yd}~[{ m MPa}]$	528	531	555	548	545	542
$E_{ m cd}~[{ m GPa}]$	35.23	35.30	35.85	35.25	35.27	35.23
$e_{ m d}~[m mm]$	1.2	6.7	14.2	10.7	13.1	12.0
$N_{ m Ed}~[{ m kN}]$	1451	1294	862	591	430	259

Table 5.6: Results from RSM-FORM with NCM.

Discussion

The β -values from the Monte Carlo simulations, shown in Table 5.3, were used to verify that the other reliability methods were sufficiently accurate and worked as expected. The β -values were highly dependent on the slenderness of the columns considered. The reason for the increased probability of failure with higher slenderness ratio can be explained by the different calculation methods' sensitivity of the stochastic variables. Generally, Figure 5.6-5.10 show that the axial load is the governing variable regardless of calculation method or column length. Figure 5.6 and 5.7 show that the 3000 mm column failed solely for high realizations of the eccentricity while the 5000 mm column failed for all eccentricities. This observation is supported by the results from RSM-FORM with NCM in Table 5.6, where α_e was very dominant for the 3000 mm column, in particular, while α_N was much more significant for the slenderest columns.

The α -values in Table 5.5 and 5.6 show that the impact of the material parameters was lower for the slenderest columns and, consequently, the corresponding partial factors influenced the capacity calculation to a lesser extent. The decreased influence of the partial factors is part of the explanation why β was lower for the slenderest columns. The 1000 mm and 2000 mm columns were assumed to get such high β -values partly because of the significant impact f_c , and thus γ_c , had on this failure mode. The RSM-FORM results with NSM and NCM show that α_{fc} was equally big as α_N for the two shortest columns, which indicates that the impact of f_c is substantial. Another considerable contribution to the decrease in β for higher slenderness ratios was caused by the eccentricity, which was included in the design capacity calculations with the same deterministic value independent on the column length. The effect of the eccentricity is further discussed in Section 5.5.

Figure 5.8 shows that f_c and, thus, γ_c influence the results differently in NCM than in NSM. For realizations of the concrete strength above the mean value, NCM had few failures. For NSM, however, failures occurred almost independently of $f_{\rm c}$. It was more remarkable that NSM had fewer failures for realizations of very low concrete strengths. Table 5.5 and 5.6 show that $\alpha_{\rm fc}$ was substantial for the shortest columns in both methods. For the four columns dominated by yield failure, the value of $\alpha_{\rm fc}$ was very different for the two methods. In the formulation of NSM, $f_{\rm c}$ is included in the relative axial force, which causes a reduction in the bending stiffness when $f_{\rm c}$ increases. The RSM-FORM results in Table 5.5 show this effect through a negative $\alpha_{\rm fc}$ for columns where yield failure is dominating. Negative values for $\alpha_{\rm fc}$ indicate that the partial factor for the concrete strength is counterproductive. The yield strength is not an important variable in NSM but has a slight impact in NCM. Again, the partial factor is counterproductive because a lower yield strength would decrease the probability of failure. In the formulation of NCM, the yield strain is proportional to the yield strength, which results in a larger second-order eccentricity when the yield strength is higher. $E_{\rm c}$ is not included in NCM at all, while it has an impact in NSM as shown in Figure 5.10. There was a good correspondence between the failure patterns from the MC analyses and the α -values from the RSM-FORM analyses, which indicates that the RSM-FORM procedure worked correctly.

Generally, the reliability analyses with NSM and NCM indicate that existing PSFs and e_{\min} make the simplified EC2 methods safe for columns with low slenderness ratio. For the slenderer columns, partial factors are less influential either because second-order effects are dominant or in some cases because the partial factors work against their purpose. It

should be emphasized that failure in this context refers to a load realization that exceeds the capacity yielded by the examined method and not the actual column capacity. The simplified methods are considered less accurate than NLFEA and the β -values from Table 5.5 and 5.6 are, thus, not given much emphasis.

The FORM analyses with NSM and two separate failure modes, presented in Table 5.4, support the assumption that compression failure was dominant for the two shortest columns. For the 3000 mm column, however, $\beta_{\rm comp}$ and $\beta_{\rm yield}$ were very close to each other. Since β_{comp} was slightly lower than β_{yield} , the design capacity was equivalent to an intersection with the MN-diagram just above the balance point. A column length of 3000 mm, which corresponds to $\lambda=52$, seems to indicate where the transition zone from compression to yield failure occurs. The NLFEA model, which is carefully studied in Appendix A, indicated that yield failure initiated the failure of the 3000 mm column. Since the NLFEA model was considered more accurate than NSM, the results from NLFEA were used to identify the most probable failure mode. Yield failure was dominant for the three slenderest columns. β_{comp} was close to β_{yield} also for the slenderest columns because of the buckling load asymptote in NSM, described in Section 5.1. Figure 4.4 shows that the axial capacity was similar at the two limit states for the slenderest columns. Hence, the β_{comp} -values for the three slenderest columns were considered inaccurate. The impact of including system reliability was negligible for all columns except for the 3000 mm column. System reliability for NLFEA would give a more accurate evaluation of the importance of system reliability. However, it was not examined in this thesis as it would demand models that evoked both failure modes separately for all column lengths.

A comparison of the β -values calculated for the simplified methods reveal that FORM and RSM-FORM yield higher β -values than MC. The results from MC for the two shortest columns are somewhat inaccurate considering that no failures occurred for NSM and only one or two occurred for NCM. For the four other columns, however, MC was considered the most accurate reliability method. The reason why FORM yielded higher β -values than MC, is that FORM can miss a part of the failure domain for a non-linear limit state. MC covers the whole area of the failure domain for a sufficiently high number of simulations. Figure 5.12 (b) gives a graphical representation of this situation, where the compression LS is non-linear and has a larger failure domain than the FORM LS.

5.3.2 NLFEA

The output from the RSM-FORM analyses with NLFEA is listed in Table 5.7

Column length [mm]	1000	2000	3000	4000	5000	6000
NLFEA mean load [kN]	791	689	561	441	328	244
$oldsymbol{eta}$	5.29	5.72	4.88	4.12	3.67	3.35
$lpha_{ m fc}$	0.652	0.570	0.228	0.109	0.066	0.055
$oldsymbol{lpha}_{\mathrm{fy}}$	0.077	0.092	0.108	0.066	0.007	0.002
$lpha_{ m Ec}$	-0.034	0.241	0.472	0.547	0.506	0.517
$oldsymbol{lpha}_{ m e}$	-0.001	-0.112	-0.449	-0.340	-0.418	-0.400
$lpha_{ m N}$	-0.753	-0.772	-0.716	-0.754	-0.751	-0.755
$f_{ m cd}~[{ m MPa}]$	29.6	30.4	41.9	46.3	47.8	48.2
$f_{ m yd} \; [{ m MPa}]$	525	522	522	528	533	533
$E_{ m cd}~[m GPa]$	36.24	28.37	24.53	24.71	26.29	26.84
$e_{ m d} [{ m mm}]$	0.01	2.0	10.5	9.0	12.3	12.9
$N_{ m Ed}~[m kN]$	1423	1297	954	716	509	368

Table 5.7: Results from RSM-FORM with NLFEA.

Discussion

The NLFEA model was expected to give the most accurate results and describe the failure mechanisms most precisely. Thus, the β -values in Table 5.7 are expected to be considerably more realistic than the ones calculated with the simplified methods. In NCM and NSM, it is crucial how the material parameters are included in the equations for the partial factors to have an impact. In NLFEA the most influential material parameters are directly determined from the mechanics in the different columns. The β -values in Table 5.7 are, as expected, higher for the slenderest columns than the ones calculated with NSM and NCM. However, β was below $\beta_{target}=3.8$ for the two slenderest columns. The design capacities for the two shortest columns, on the other hand, were very conservative for NLFEA as well as for NSM and NCM. These results indicate that the partial factors in the EC2 should be modified for use in slender column design. New PSFs are proposed in the

discussion in Section 5.5.

Table 5.7 shows that $\alpha_{\rm N}$ is less dependent on the column length when NLFEA is conducted than when the simplified methods are used. The load is the most important variable for all the columns. There is a clear distinction between the remaining α -values for the two shortest columns and the four slenderest columns. The two columns associated with compression failure are governed by the concrete strength, while the eccentricity and concrete stiffness dominate the capacity of the slenderest columns. The most apparent difference between the α -values from NLFEA versus NSM/NCM is the significance of $E_{\rm c}$. The concrete stiffness influence the capacity calculated with NLFEA substantially, for the slenderest columns. $E_{\rm c}$ is significant for slender columns because a lower concrete stiffness will allow for higher strains at a lower load level and increase the stresses in the tensile reinforcement. The failure mechanisms in the NLFEA models are further described in Appendix A.

5.4 Assessment of model deviation

Several analyses were run with NLFEA and the simplified methods, with the same random realizations for the stochastic variables, to estimate the consistency and deviation of the different calculation methods. The number of analyses was limited to 20 for each column length due to time constraints. Figure 5.11 (a) shows the ratio between the capacity from NLFEA and NSM, while NSM is substituted with NCM in (b). The red line inside the boxes indicates the median value while the top and bottom box edges represent the 75th and 25th percentile, respectively. The whiskers outside the boxes mark the maximum and minimum ratios that were found, except points that are considered outliers, which are marked by red crosses.



Figure 5.11: Ratios between NLFEA and simplified methods for 20 random realizations.

The mean and the standard deviation for the ratio between NLFEA and NSM/NCM were calculated with the equations in Section 4.5. This was done for all of the column lengths for both simplified methods. The outcome is presented in Table 5.8. The coefficient of variation is the standard deviation divided by the mean.

Column	length [mm]	1000	2000	3000	4000	5000	6000
	μ	0.9622	0.9322	1.4459	2.1137	2.4765	2.6372
NSM	σ	0.0091	0.0361	0.0797	0.1472	0.2219	0.2874
	COV	0.0095	0.0387	0.0551	0.0696	0.0896	0.1090
NCM	μ	0.9577	0.8324	0.8248	1.3036	1.4044	1.7727
	σ	0.0090	0.0521	0.1342	0.1542	0.1757	0.2268
	COV	0.0094	0.0626	0.1627	0.1183	0.1251	0.1280

Table 5.8: Ratio between capacities calculated with NLFEA and NSM/NCM.

Discussion

Figure 5.11 and Table 5.8 show the tendencies of how the capacities resulting from NSM/NCM compare to the NLFEA results. For columns expected to fail in compression, the results indicate that all the three methods yield similar capacities independent of the input variables. For the three slenderest columns, the tendency is clearly that the

simplified methods become more conservative for increasing slenderness ratios. The most noticeable results are related to NCM for the 3000 mm column because the capacity can be much less conservative when NCM is conducted than when NLFEA is used. When NCM is applied to columns around the transition zone between compression and yield failure, the intersection with the MN-diagram can occur in the ascending or descending branch of the work-diagram. Thus, small changes in the parameters affecting the resistance can cause significant differences in the capacity yielded by NCM. This effect is elaborated in Section 5.5. It is difficult to use the numbers in Table 5.8 directly, given the limited number of analyses and that the model uncertainty of the NLFEA model is absent. However, it provides useful information concerning how conservative results the methods are expected to give for different slenderness ratios and how consistent results they provide.

5.5 Inverse analyses

The results from the inverse analyses with $\beta = 3.8$, are presented in this section. Table 5.9 shows the results from the inverse FORM analyses. Recall that FORM only was used in conjunction with NSM and used equation solving to find the two different limit states. The design points listed in Table 5.9 refer to the limit state that reached β_{target} for the lowest load. The three shortest columns reached β_{target} first for the compression failure limit state, while the yield limit state governed the three slenderest columns.

Column Length [mm]	1000	2000	3000	4000	5000	6000
$\mu_{ m N}~[m kN]$	1085	1003	483	248	151	101
$f_{ m cd}~[{ m MPa}]$	33.8	36.8	55.9	56.7	55.5	54.8
$f_{ m yd}~[{ m MPa}]$	534	534	534	532	532	532
$E_{ m cd}~[m GPa]$	35.22	33.44	29.07	30.27	31.02	31.48
$e_{ m d} [m mm]$	0.7	5.4	8.9	12.1	14.6	16.9
$N_{ m Ed}~[m kN]$	1688	1553	770	397	246	167

Table 5.9: Results from the inverse FORM analysis with NSM and $\beta_{target} = 3.8$.

Figure 5.12 and 5.13 show the two separate limit states with respect to f_c and E_c for the 2000 mm and 3000 mm column, respectively. The limit states are dependent on five variables, and the limit state functions are, therefore, five-dimensional. To be able to plot the limit states, three of the variables were kept constant at their final design points. The yield LS was simplified to be linear, as described in Section 4.3.2, which makes FORM and the closed-form limit state to overlap perfectly.



(a) Real Space

(b) Standard Normal Space

Figure 5.12: Inverse FORM with NSM for the 2000 mm column.



Figure 5.13: Inverse FORM with NSM for the 3000 mm column.

Table 5.10 and 5.11 show the results from the inverse RSM-FORM analyses with NSM and NCM, respectively. For comparison purposes, the two RSM-FORM analyses were used to calculate appropriate partial factors for NSM and NCM, which are presented in the lower section of the tables. Table 5.12 displays the output from the inverse RSM-FORM analyses with NLFEA and the corresponding partial factors calculated in compliance with Section 4.6.

Column Length [mm]	1000	2000	3000	4000	5000	6000
$\mu_{ m N}$ [kN]	1065	972	477	249	152	103
$f_{ m cd}~[{ m MPa}]$	33.8	37.4	56.1	58.0	57.3	56.0
$f_{ m yd}~[{ m MPa}]$	530	531	533	533	532	532
$E_{ m cd}~[m GPa]$	35.21	33.19	29.10	30.18	30.74	31.24
$e_{ m d} [{ m mm}]$	0.9	5.7	9.8	11.9	14.5	14.6
$N_{ m Ed}~[m kN]$	1652	1507	747	398	249	170
$lpha_{ m fc}$	0.672	0.500	-0.219	-0.277	-0.239	-0.215
$oldsymbol{lpha}_{\mathrm{fy}}$	0.043	0.032	0.010	0.011	0.017	0.016
$oldsymbol{lpha}_{ ext{Ec}}$	-0.001	0.100	0.321	0.259	0.226	0.201
$oldsymbol{lpha}_{ m e}$	-0.141	-0.465	-0.540	-0.488	-0.437	-0.401
$lpha_{ m N}$	-0.726	-0.724	-0.747	-0.786	-0.837	-0.867
$\gamma_{ m c}$	1.33	1.22	0.81	0.78	0.79	0.82
$\gamma_{ m s}$	0.94	0.94	0.94	0.94	0.94	0.94
$\gamma_{ m CE}$	1.01	1.07	1.23	1.18	1.16	1.14
$\gamma_{ m G}$	1.55	1.55	1.57	1.61	1.64	1.65

Table 5.10: Results from the inverse RSM-FORM analysis with NSM and $\beta_{\text{target}} = 3.8$.

Column Length [mm]	1000	2000	3000	4000	5000	6000
$\mu_{ m N}$ [kN]	1066	1012	660	365	256	148
$f_{ m cd}~[{ m MPa}]$	33.8	34.6	39.2	38.2	41.6	43.7
$f_{ m yd}~[{ m MPa}]$	530	532	549	551	549	547
$E_{ m cd}~[m GPa]$	35.24	35.31	35.36	35.27	35.23	35.24
$e_{ m d} [m mm]$	0.8	4.3	12.5	10.9	17.4	19.5
$N_{ m Ed}$ [kN]	1653	1541	947	569	399	238
$lpha_{ m fc}$	0.675	0.636	0.401	0.460	0.302	0.217
$oldsymbol{lpha}_{\mathrm{fy}}$	0.043	0.020	-0.192	-0.214	-0.187	-0.163
$oldsymbol{lpha}_{ m Ec}$	-0.000	-0.000	-0.006	-0.002	-0.000	-0.000
$lpha_{ m e}$	-0.133	-0.355	-0.689	-0.445	-0.585	-0.540
$lpha_{ m N}$	-0.725	-0.685	-0.572	-0.737	-0.730	-0.797
$\gamma_{ m c}$	1.33	1.30	1.15	1.17	1.08	1.03
$\gamma_{ m s}$	0.94	0.94	0.91	0.91	0.91	0.91
$\gamma_{ m CE}$	1.01	1.01	1.01	1.01	1.01	1.01
$\gamma_{ m G}$	1.55	1.52	1.44	1.55	1.56	1.61

Table 5.11: Results from the inverse RSM-FORM analysis with NCM and $\beta_{\text{target}} = 3.8$.

Column Length [mm]	1000	2000	3000	4000	5000	6000
$\mu_{\rm N}$ [kN]	1004	914	669	465	321	225
$f_{ m cd}~[{ m MPa}]$	33.9	35.4	43.6	46.6	47.7	48.0
$f_{ m yd}~[{ m MPa}]$	528	528	526	528	533	533
$E_{ m cd}~[m GPa]$	35.85	31.55	26.98	25.45	26.08	26.16
$e_{ m d} [{ m mm}]$	0.003	0.02	6.9	7.1	13.6	19.1
$N_{ m Ed}~[m kN]$	1571	1458	1063	739	501	343
$oldsymbol{lpha}_{ m fc}$	0.667	0.590	0.223	0.108	0.066	0.054
$oldsymbol{lpha}_{\mathrm{fy}}$	0.058	0.071	0.099	0.054	0.007	0.002
$oldsymbol{lpha}_{ m Ec}$	-0.029	0.185	0.446	0.546	0.503	0.498
$oldsymbol{lpha}_{ m e}$	-0.001	-0.002	-0.378	-0.291	-0.447	-0.522
$lpha_{ m N}$	-0.743	-0.783	-0.774	-0.775	-0.737	-0.691
$oldsymbol{lpha}_{ m R}$	0.670	0.622	0.508	0.559	0.507	0.501
$lpha_{ m S}$	-0.743	-0.783	-0.861	-0.828	-0.862	-0.866
$\gamma_{ m c}$	1.32	1.27	1.03	0.97	0.94	0.94
$\gamma_{ m s}$	0.95	0.95	0.95	0.95	0.94	0.94
$\gamma_{ m CE}$	0.99	1.13	1.32	1.40	1.37	1.36
$\gamma_{ m G}$	1.56	1.60	1.59	1.59	1.56	1.52

Table 5.12: Results from the inverse RSM-FORM analysis with NLFEA and $\beta_{\text{target}} = 3.8$.

The response surface and FORM limit state from the inverse RSM-FORM analyses with NLFEA are presented in Figure 5.14. The concrete stiffness in Figure 5.14 is reduced due to the effect of creep in the NLFEA analyses, as explained in Section 2.2.4. In Table 5.12, E_{cd} is scaled by a factor of 2.8 to represent the initial concrete stiffness. The columns ranging from 2000-4000 mm are denoted L2, L3 and L4 respectively.



Figure 5.14: Inverse RSM-FORM with NLFEA for 2000-4000 mm columns.

The α -values from the inverse RSM-FORM analyses with NLFEA are shown in Figure 5.15 (a). The figure shows how the impact of the stochastic variables changes with the different column lengths. In Figure 5.15 (b) the PSFs from Table 5.12 are plotted for all the considered columns.



Figure 5.15: Results from inverse RSM-FORM with NLFEA.

Eccentricity

An important aspect in the inverse analyses is the effect of the first-order eccentricity, which for existing practice have a considerable impact on the safety margin of the shortest columns considered in this thesis. A new approach to treat eccentricity is sought based on the design points in Table 5.10-5.12. It is assumed a relationship between the eccentricity and the column length. This section presents the results from the regression analyses on the design eccentricities from Table 5.10-5.12. Only the design points from inverse RSM-FORM with NLFEA were included in the regression analyses in Figure 5.16. Figure 5.17 shows the regression analyses when e_d for all the three calculation methods were included. Figure 5.16 (a) and 5.17 (a) display three different approaches to estimate the design eccentricity as a function of column length. A least square approach was used to find a linear, a second-order and an exponential function to describe the relationship between eccentricity and column length.



Figure 5.16: Regression analyses based on $e_{\rm d}$ for NLFEA.



Figure 5.17: Regression analyses based on e_d for NSM, NCM and NLFEA.

The top three functions in Table 5.13 are plotted in Figure 5.16 (a) and 5.17 (a). The bottom part displays simplified versions of the exact solutions. Figure 5.16 (b) and 5.17 (b) show the best-fit second-order function, and the modified version plotted together. Modifications were made to make the functions simpler and to give the function a shift towards the conservative side, i.e., an upward shift.

	$e_{ m d}~{ m NLFEA}$	$e_{ m d} ~{ m all} ~{ m methods}$
Linear	3.90L - 5.86	3.47L - 2.74
2nd-order	$0.46L^2 + 0.66L - 1.54$	$-0.10L^2 + 4.20L - 3.70$
Exponential	$1.04e^{(0.49L)}$	$2.53e^{(0.34L)}$
2nd-order modified	$\frac{L}{2}(L+1)$	$\frac{L}{4}(20-L)$
Exponential modified	$e^{\frac{L}{2}}$	$2.5e^{\frac{L}{3}}$

Table 5.13: Eccentricity as functions of column length.

Discussion

The inverse RSM-FORM analysis with NLFEA, presented in Table 5.12 is considered the most accurate source for determining PSFs and is used as the reference to compare the different calculation methods. Table 5.10-5.12 show that there is a clear distinction between the α -values for the two shortest columns and the four slenderest columns, for all the calculation methods. The slenderness limit in EC2 determines whether second-order effects must be considered. However, there is no slenderness limit differentiating between the expected failure modes and the corresponding, appropriate set of PSFs. It is advised that a different slenderness criterion is developed to more effectively differentiate between columns subjected to compression failure and yield failure.

The results from RSM-FORM with NLFEA cannot be compared to MC results because MC was not used in conjunction with NLFEA, due to time limitations. It is, therefore, cumbersome to state the accuracy of the RSM-FORM approximation for NLFEA. Figure 5.14 shows that FORM increases the failure domain compared to the RS, for the 3000 mm and 4000 mm columns, if only E_c and f_c are examined. The increase of the failure domain indicates that RSM-FORM might be a conservative approximation of the RS for NLFEA. It is, however, emphasized that the RS is five-dimensional and it is only assumed that the RSM-FORM approximation did not cause considerably non-conservative results. The uncertainty related to the reliability methods and the approximation of the limit state is not quantified.

Eccentricity

Table 5.10-5.12 show that the design points for the eccentricity were below 20 mm for all the columns, which confirms that the minimum value from EC2 is a conservative assumption. It should, however, be emphasized that very low values of e_d imply that using the minimum eccentricity of 20 mm will cause highly conservative results. If the design capacities from Table 5.1 are compared to the design capacities in Table 5.10-5.12, it is evident that the three shortest columns become highly conservative if the eccentricity is implemented as 20 mm. This raises the discussion of which variables that should be assigned partial factors and assure that the safety margin is sufficient. Either the minimum eccentricity or γ_G should be reduced to avoid excessively conservative design capacities for columns where compression failure is dominating. Based on the α -values from RSM-FORM with NLFEA given in Table 5.12, the eccentricity should be given less emphasis for the shortest columns, and most of the safety should be covered by γ_G and γ_c .

The design eccentricities in Table 5.10-5.12 seem to be dependent on the column length, which is reasonable considering that $\sigma_{\rm e}$ also is a function of the length. As shown in Table 5.13, a linear, a second-order and an exponential relation between the eccentricity and the column length were examined. At first, it was considered a relation based on only the design points from NLFEA. This would, however, make the simplified methods non-conservative for certain column lengths because they had considerably higher design points for eccentricity. It was considered more desirable to make the eccentricity more conservative than introducing different partial factors for the three methods. The modified, second-order function based on the design points of all the methods, is considered the best-fit relation between eccentricity and column length. Thus, the proposed relationship is given by Equation 5.2.

$$e(L) = \frac{L}{4}(20 - L) \tag{5.2}$$

Compression failure

The two columns dominated by compression failure are governed by the concrete strength in addition to the load. Even though β_{comp} was slightly lower than β_{yield} for the 3000 mm column in Table 5.4, the α -values in Table 5.10-5.12 imply that the failure mode is more similar to the slenderer columns than to the shorter ones. Figure 5.12-5.13 show how the two separate limit states with NSM change from the 2000 mm column to the 3000 mm column. For the 3000 mm column, the compression failure LS is almost identical to the yield LS, and the design points are coinciding. The response surface generated with NLFEA, plotted in Figure 5.14, shows how the RS and the design point for the 3000 mm column. It is, therefore, assumed that the slenderness limit is located between the 2000 mm($\lambda = 34$) and 3000 mm($\lambda = 52$) columns in this thesis.

The partial factors calculated for the two shortest columns are coinciding well for the three methods. The design capacities for the 1000 mm and 2000 mm columns in Table 5.10-5.12 indicate that NLFEA yields more conservative results than the hand calculation methods if the calculated partial factors from Table 5.10-5.12 are used. The partial factor for load resulting from NLFEA is, however, slightly higher than $\gamma_{\rm G}$ calculated with NSM/NCM. If the same $\gamma_{\rm G}$ is applied to all the methods, the design capacities with NSM/NCM will get closer to the NLFEA results.

If the results are compared to the PSFs recommended in EC2, it is proposed that $\gamma_{\rm G}$ should be increased and that PSFs related to material parameters could be given less emphasis. The results in Table 5.12 indicate that $\gamma_{\rm G}$ should be in the order of magnitude of 1.60 combined with $\gamma_{\rm c} = 1.30$. However, it is favorable to allocate the safety more equally between the load and the resistance side. It is, therefore, proposed to reduce the partial factor for the load to 1.50, and compensate by increasing $\gamma_{\rm c}$ to 1.40. A recommended set of PSFs for columns that are expected to fail in compression is presented below.

 $\gamma_{\rm c} = 1.40 \quad \gamma_{\rm s} = 1.00 \quad \gamma_{\rm CE} = 1.00 \quad \gamma_{\rm G} = 1.50$

Yield failure

It is considered that yield failure dominates the four slenderest columns. Table 5.10-5.12 show that the three methods give aberrant α -values for the slenderest columns. The design capacities from NLFEA are higher than $N_{\rm Ed}$ found with the simplified methods, for all the four slenderest columns. However, Table 5.11 shows how the design eccentricity was particularly high for the 3000 mm column, which can give a misleading low value for $N_{\rm Ed}$. Table 5.1 and 5.8 show that NCM is likely to be less conservative than NLFEA when the design capacity for the 3000 mm column is considered. α_e is considerably higher for the 3000 mm column is considered. α_e is considerably higher the intersection with the MN-diagram occurs in the increasing or descending branch of the work-diagram. The capacity given by NCM will be significantly higher if e_d from NLFEA is used than if the design point from inverse RSM-FORM with NCM is applied. Caution should be exercised when NCM is applied for columns around the transition zone between compression failure and yield failure. It is essential that the minimum eccentricity is sufficiently large when NCM is applied to columns with slenderness ratio in this area.

Figure 5.11 indicates that the 3000 mm column calculated with NCM is the only considered case where the simplified methods are less conservative than NLFEA. Thus, the α -values for this column are carefully considered in addition to the NLFEA results when PSFs are proposed. Table 5.10-5.12 show that α_N is higher for NSM than for the two other methods. However, as shown in Table 5.8, NSM is highly conservative for the slenderest columns. It is, therefore, less critical to account for the high α_N when determining γ_G . Based on Table 5.12, γ_G should be increased to 1.60. However, it is desirable to apply the same value for γ_G for all column lengths, given that the load is normally treated in a different part of EC2. Thus, a γ_G of 1.50 is proposed, which should be compensated for by an increase of the PSFs on the resistance side.

The material parameter that had the highest sensitivity factor for the four slenderest columns was the concrete stiffness. Table 5.12 shows that γ_{CE} also should be higher than the value recommended in EC2. Based on Table 5.12, a value of 1.40 for γ_{CE} is proposed. NCM is not affected by the PSF of concrete stiffness, while NSM will become more conservative if $\gamma_{CE} = 1.40$ is introduced. It is, however, desirable to make NLFEA

less conservative for the 3000 mm column and, thus, make the design capacities from NCM and NLFEA coincide better. If γ_{CE} is reduced to 1.35, it will make the NLFEA results less conservative. A larger eccentricity than e_d in Table 5.12 will compensate the reduction in γ_{CE} . A partial factor for concrete strength will impact the design capacity calculated by NCM more than the NLFEA result because α_{fc} is higher for NCM than for NLFEA. Thus, by including $\gamma_c > 1.0$, the NCM results would become more conservative, particularly for the 3000-4000 mm columns. Table 5.10 shows that α_{fc} is negative when NSM is used. As a consequence, a $\gamma_c > 1.0$ yields slightly less conservative results for NSM, which is favorable.

The yield strength was not dominant for any of the methods, and it could be set to 1.0 without affecting the NLFEA results considerably. The negative $\alpha_{\rm fy}$ in Table 5.11 implies that NCM yields less conservative results than presented in Table 5.11 if $\gamma_{\rm s}$ is higher than the proposed value of 0.91. A proposed set of PSFs for columns where yield failure is the governing failure mechanism is presented below.

 $\gamma_{\rm c} = 1.20 \quad \gamma_{\rm s} = 1.00 \quad \gamma_{\rm CE} = 1.35 \quad \gamma_{\rm G} = 1.50$

5.6 Validation of PSF

Two new sets of PSFs are presented in Table 5.14. The set on the left is produced to suit columns expected to fail in compression, while the other should be applied to columns subject to yield failure. The proposed approach to treat eccentricity is also included.

Column Length[mm]	1000	2000	3000	4000	5000	6000	
$e(L) = \frac{L}{4}(20 - L)$ [mm]	4.75	9.00	12.75	16.00	18.75	21.00	
$\gamma_{ m c}$	1.	40	1.20				
$\gamma_{ m s}$	1.	00	1.00				
$\gamma_{ m CE}$	1.00 1.35						
$\gamma_{ m G}$	1.	50	1.50				

Table 5.14: Proposed sets of PSFs.

The PSFs and the eccentricity from Table 5.14 were applied to NSM, NCM and NLFEA and the resulting design capacities are presented in Table 5.15. The top row includes the design capacities from Table 5.12.

Column Length[mm]	1000	2000	3000	4000	5000	6000
Estimated $N_{\rm Ed}, \beta_{\rm target} = 3.8$	1571	1458	1063	739	501	343
NSM	1477	1281	748	407	250	170
NCM	1484	1336	1036	586	408	240
NLFEA	1448	1227	911	650	444	313

Table 5.15: Design capacities in kN with the PSFs proposed in Table 5.14.

The ratios between the design capacities computed with the new PSFs and the best estimate $N_{\rm Ed}$ are plotted in Figure 5.18. The best estimate refers to the design capacity found with RSM-FORM and NLFEA given in Table 5.12.



Figure 5.18: Ratio between design capacities with new PSFs and best estimate.

Discussion

Table 5.15 shows that all the methods yield conservative results compared to the reference capacity. When the design capacities in Table 5.15 and Table 5.1 are compared to each other, it is clear that the results for the three shortest columns are considerably less conservative with the new PSFs and the eccentricities from Table 5.14. The 4000 mm

column got slightly more conservative results with NSM, while the results became less conservative for the two other methods. For the two slenderest columns, the results became slightly more conservative because the β -values in Table 5.7 were below 3.8. The capacity for the 5000 mm column calculated with NLFEA, in Table 5.15, is slightly higher than the corresponding capacity in Table 5.1. However, since the PSF for load is increased from 1.35 to 1.50, the result from Table 5.15 is more conservative.

Because of the general way the load was treated in this thesis, it is difficult to quantify how much the reduction of $\gamma_{\rm G}$, from the estimated value of 1.60 to 1.50, should be compensated for on the resistance side. Table 5.15 shows that the results have an additional safety margin to account for this reduction of $\gamma_{\rm G}$. It could, however, be argued that either $\gamma_{\rm G}$ or $\gamma_{\rm c}$ should be further increased to ensure that NCM is conservative for the shortest columns subjected to yield failure.

The proposed combination of PSFs and eccentricity is assumed applicable for the columns examined in this thesis. It should, however, be emphasized that the model uncertainty of the NLFEA model was not quantified. The assumption that the NLFEA model provides the true capacity introduces more uncertainty to the problem. Without physical experiments, this uncertainty can only be approximated. The comparison between the NLFEA capacities from the inverse RSM-FORM analyses and the design capacities found with the new PSFs is, therefore, not an exact validation procedure.

6. Conclusion

The results of this thesis indicate the applicability of existing procedures and provide proposals for improvements that can increase the accuracy of slender column design. The results can form a foundation for further studies on slender column design. Proposals for further work are given in Section 7.

The impact of creep should be considered in slender column design. NSM is highly conservative for columns subject to yield failure, compared to NCM and NLFEA, when creep is included in the analysis.

GRFM is a conservative alternative to the PSF method, particularly for highly slender columns. The material properties for concrete, given in EC2, can make the ECOV method non-conservative. It is proposed to apply in-situ reduced concrete strength, and reduce the concrete stiffness accordingly, to make the ECOV method a good alternative to the PSF method.

The reliability analyses indicate that existing partial factors and $e_{\min} = 20$ mm might yield excessively conservative results for columns subject to compression failure. For highly slender columns the design capacity with NLFEA might be non-conservative, and the total impact of the PSFs should be increased.

There is a clear distinction between the α -values for columns subject to compression failure and columns where yield failure is dominating. It is, therefore, proposed to develop a different slenderness criterion to effectively differentiate between the two failure modes. For the geometry that was investigated in this thesis, the transition between compression failure and yield failure occurred for slenderness ratios approximately between 35 and 50.

It is proposed a relationship between the minimum eccentricity and the column length. The proposed relation leads to a less costly design for columns that are likely to fail in compression. The nominal curvature method is particularly sensitive to the eccentricity for columns near the transition zone between the failure modes. Caution should be exercised to ensure that the minimum eccentricity is sufficiently large when NCM is applied to columns with slenderness in the mentioned area. Based on the design points for eccentricity from the inverse RSM-FORM analyses, the following second-order polynomial is proposed:

$$e(L) = \frac{L}{4}(20 - L)$$

The inverse analyses indicated that the partial factor for load should be increased from the existing value, while the PSFs related to resistance should be reduced. It is advised to apply the same value for $\gamma_{\rm G}$ independent of the slenderness limit. PSFs on the resistance side should be assigned different values depending on the expected failure mode related to the slenderness limit. Compression failure is governed by the concrete strength, while the concrete stiffness is the most crucial variable related to resistance for columns subject to yield failure. The recommended sets of PSFs, for columns with slenderness ratio over and under the slenderness limit, are presented below.

Compression failure	$\gamma_{\rm c} = 1.40$	$\gamma_{\rm s} = 1.00$	$\gamma_{\rm CE}=1.00$	$\gamma_{\rm G} = 1.50$
Yield failure	$\gamma_{\rm c} = 1.20$	$\gamma_{\rm s} = 1.00$	$\gamma_{\rm CE}=1.35$	$\gamma_{\rm G} = 1.50$

The necessity of NLFEA in slender column design seems to increase with higher slenderness ratio. For columns where compression failure is dominating, NSM and NCM give consistent and similar results to NLFEA. NLFEA is advised for highly slender columns because the hand-calculation methods are variable and excessively conservative. It is likely that the 3000 mm and 4000 mm columns in this thesis reflect the most realistic slender column designs. The simplified methods yield variable capacities for these column lengths and NLFEA is advised. If the hand-calculation methods are used, it is advised to apply both NSM and NCM to reveal huge discrepancies that can indicate non-conservative results with NCM.

7. Further work

A more extensive study on slender column design can be conducted based on the procedures in this thesis. In this thesis, only one cross-section was examined. A broader variety of columns should be examined, to establish conclusions for general concrete column design. A variety study can include different cross-sections, amounts of reinforcement, concrete strengths and boundary conditions. It would be interesting to compare rectangular and circular cross-sections where the columns have the same slenderness ratio. By following the same procedure conducted in this thesis, a variety study can potentially find a set of PSFs that is applicable for concrete columns in general.

Another important study would be to shrink the interval where the transition from compression failure to yield failure occur. It would be advantageous to quantify the slenderness ratio that best indicates the transition between the two failure modes. The results in this thesis indicate a gradual transition, which can make it necessary to adjust the PSFs at one side of the limit. The determination of the slenderness limit is a natural side study to the variation study requested above.

GRFM and ECOV are not thoroughly formulated by international model codes. GRFM should be specified for more variables than the concrete strength and the yield strength. ECOV needs specifications for how the mean and characteristic material parameters should be determined. In this thesis, ECOV was non-conservative compared to the PSF method if the mean and characteristic values from EC2 were applied. This finding should be further investigated, and a standard formulation of the method should be established. Uncertainty related to the RSM-FORM approximation of the limit state should be included to verify if the method is substantially conservative or non-conservative. A possible approach would be to run a moderate amount of Monte Carlo simulations with NLFEA and compare to the RS and the FORM approximation in the design point. MC simulations with NLFEA requires substantial computing power, and the study might be limited in accuracy due to the time cost.

Bibliography

- CEN. Eurocode 2: Design of Concrete Structures: Part 1-1: General Rules and Rules for Buildings. 2004.
- [2] Cervenka, V. "Reliability-based non-linear analysis according to fib Model Code 2010". In: Structural Concrete 14.1 (2013).
- [3] Schlune, H., Plos, M., and Gylltoft, K. "Safety formats for non-linear analysis of concrete structures". In: *Magazine of Concrete Research* 64.7 (2012).
- [4] Blomfors, M., Engen, M., and Plos, M. "Evaluation of safety formats for non-linear finite element analyses of statically indeterminate concrete structures subjected to different load paths". In: *Structural Concrete* 17.1 (2016).
- [5] CEN. Eurocode—Basis of structural design. 2002.
- [6] Jacobs, J. P. "Eurocode 2 Commentary". In: European Concrete Platform ASBL, Brussels (2008).
- [7] Torrent, R. J. "The log-normal distribution: a better fitness for the results of mechanical testing of materials". In: *Matériaux et Construction* 11.4 (1978).
- [8] Eklund, H., Skorve, A., and Strand, A. "Reliability assessments of concrete structures using non-linear finite element analyses". MA thesis. NTNU, 2017.
- [9] Structural Safety, Joint Committee on. JCSS Probabilistic Model Code. Technical University of Denmark, 2015.
- [10] CEN. Eurocode 2: Design of Concrete Structures: Part 2: Concrete Bridges-Design and Detailing Rules. 2005.
- [11] fib. fib Model Code for Concrete Structures 2010. The International Federation of Structural Concrete, 2013.

- [12] Hendriks, M.A.N., de Boer, A., and Belletti, B. "Guidelines for nonlinear finite element analysis of concrete structures". In: *Rijkswaterstaat Technisch Document* (*RTD*), *Rijkswaterstaat Centre for Infrastructure*, *RTD* 1016 (2012).
- [13] Manie, J. DIANA user's manual. Release 10.1. DIANA FEA BV, 2016.
- [14] Vecchio, F. J. and Collins, M. P. "Compression response of cracked reinforced concrete". In: *Journal of structural engineering* 119.12 (1993).
- [15] Prot, V. Lecture 2 Solution of Nonlinear Equilibrium Equations. Lecture notes -Nonlinear Finite Element Analysis, 2017. Department of Structural Engineering, NTNU.
- [16] Schneider, J. Introduction to safety and reliability of structures. Vol. 5. Iabse, 2006.
- [17] Basler, E. "Untersuchungen über den Sicherheitsbegriff von Bauwerken". PhD thesis.ETH Zurich, 1960.
- [18] Cornell, C. A. "A probability based structural code". In: ACI Journal No. 12 (1969).
- [19] Ditlevsen, O. Structural reliability and the invariance problem. University of Waterloo, Solid Mechanics Division, 1973.
- [20] Hasofer, A. M. and Lind, N. C. "Exact and invariant second-moment code format". In: Journal of the Engineering Mechanics division (1974).
- [21] Bucher, C. G. and Bourgund, U. "A fast and efficient response surface approach for structural reliability problems". In: *Structural safety* 7.1 (1990).
- [22] Köhler, J. L07 system reliability. Lecture notes Aspects of Structural Safety, 2017. Department of Structural Engineering, NTNU.
- [23] Jordaan, I. Decisions under uncertainty: probabilistic analysis for engineering decisions. Cambridge University Press, 2005.

A. Verification of NLFEA

A thorough study was conducted to validate that the NLFEA model worked correctly. The response of the column was examined and the stress and strain distribution at the critical cross section was studied. This was also meant as a study on the failure mechanism for the different columns, which was difficult to extract explicitly from the NLFEA analysis. The analyses in this section were run with the model and material parameters described in Table 3.2 and Section 3.2. The Figures A.2-A.7 show the stress and strain distribution over the cross section at mid length for the different column lengths. The values for stress and strain displayed for the concrete elements are the average nodal value in each element. The column has 8 concrete elements along the cross section and these are displayed along the x-axis in the Figures A.2-A.7, from left to right. The bars indicate the stress or strain in the reinforcement, while the graphs indicate the stress or strain in the concrete. The reinforcement at the left side of the cross section is represented by the bar plot on the left side on the Figures A.2-A.7, and vice versa on the right. The load was applied on the right side of the vertical symmetry line. Negative stress values imply compression and positive stress values represent tension. Strains and stresses in three critical load steps are shown for each column length:

- The step where the maximum reaction force was detected in the column. This force indicated the column capacity
- The step where the maximum moment occurred, which often was related to yielding of the reinforcement
- One load step after maximum moment, to identify how the redistribution of stresses change when failure has propagated

Figure A.1 (a) shows the relation between force and moment for the 4000 mm column with design values, while (b) displays the horizontal displacement in the middle of the column against the applied load. The three critical load steps listed above are marked with a cross, an asterix and a plus sign, respectively.



Figure A.1: The graphs illustrate the three critical load steps shown in Figure A.2-A.7.

The three load steps that were examined could to some extent indicate the initiation, propagation and final rupture stage of the failure mechanism.



Figure A.2: Stress and strain distribution along the critical cross section for the column of length 1000 mm

The behaviour of the shortest column is described in Figure A.2. From the stress plot, it can be seen that the compressive strength of the concrete was fully utilized in nearly half of the cross section at the load step that indicate max reaction force. The compression reinforcement was yielding while the reinforcement on the left side of the column still was in compression at this stage. When the maximum moment was reached, the furthermost right concrete elements had started softening while the elements at the middle of the cross section had reached maximum compressive stress. The tensile reinforcement was far from yielding and the compressive reinforcement was in the yield hardening stage. The next load step shows the same development with further softening of the concrete on the right side and increasing stresses in the reinforcement. The column was failing in compression and buckling did not occur at any stage for the column of length 1000 mm.



Figure A.3: Stress and strain distribution along the critical cross section for the column of length 2000 mm

The 2000 mm column, shown in Figure A.3, experienced compressive stresses in nearly all the elements along the cross section at the stage of maximum reaction force. The compressive strength of the concrete had been reached in the outermost right element. The reinforcement on the left side was still in compression while the reinforcement on the right side had reached yield stress. At the load step where maximum moment occurred, nearly half of the cross section had reached maximum compressive stress and compressive concrete softening was initiated at the right edge of the column. The tensile reinforcement had just reached yield stress and the concrete at the left side had started cracking as the stresses were minor. At the next load step, concrete softening had developed in the right half of the cross section and the reinforcement took up all the stresses at the tensile side. The failure mechanism seemed to be initiated by crushing of the concrete at the right side of the cross section, resulting in cracking of the concrete at the left side and yielding of the tensile reinforcement. Local compressive failure initiated buckling of the column.



Figure A.4: Stress and strain distribution along the critical cross section for the column of length 3000 mm

For the 3000 mm column, maximum compressive stress in the concrete was not reached at the step of maximum reaction force. However, the left side of the column had started cracking as the concrete tensile strength was reached. When the concrete at the left side lost strength, the reinforcement stresses increased and reached yield stress at the step of maximum moment. At this step, the two outermost compressive concrete elements was crushing. At the next load step, it can be seen that the concrete experienced tension failure in almost all of the cross section and the reinforcement went to failure. As opposed to the 2000 mm column, the failure in this column seemed to be initiated by tensile stresses in the left side of the column. The failure mechanism was more complicated than for the shorter ones, as this column had a combination of tension and compression failure, which eventually resulted in buckling of the column. However, yield failure was the dominating failure mechanism.



Figure A.5: Stress and strain distribution along the critical cross section for the column of length 4000 $\rm mm$



Figure A.6: Stress and strain distribution along the critical cross section for the column of length 5000 mm


Figure A.7: Stress and strain distribution along the critical cross section for the column of length 6000 mm

The three slenderest columns showed a similar behaviour and it was evident that the second-order effects were dominating. Large lateral displacement at mid length of the column caused large tensile forces at the left side of the cross section. The maximum compressive strength was not reached at the point of maximum reaction nor at maximum moment. The concrete at the left side reached maximum tensile stress when failure was initiated and when maximum moment occurred the concrete had cracked over approximately half of the cross section. Therefore, the reinforcement on the left had to take all the tensile stresses and went to failure before concrete crushing had developed. The slenderest columns were therefore clearly failing in tension.

The failure mechanisms that were found seemed to coincide with the expected response for the columns and the model seemed to work correctly. The columns with lengths 2000 mm($\lambda = 34$) and 3000 mm($\lambda = 52$) indicated the area in which the transition from compressive failure to yield failure happened.