Random Loads with Data Analysis
— Home Exercises —

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Part A - Solve using Paper & Pen

A.1 Let $X(t)$ be a stationary Gaussian random process with zero mean and spectral density

$$S(\omega) = C/\omega^k, \quad \omega_1 < \omega < \omega_2, \qquad \text{with} \quad k = 5.$$

This type of spectrum is often used as models for roads.

- Compute the level upcrossing intensity, $\mu(u)$.
- How can you use $\mu(u)$ to compute an upper bound for the rainflow damage? How is this connected to the narrow band approximation? Derive the corresponding distribution of rainflow cycle amplitudes!
- How can we construct a lower bound for the rainflow damage?
- A simple approximation of the min-max distribution $f_{m,M}(u,v)$ is obtained by using

  $$M = \sigma(\alpha_2 R + \sqrt{1-\alpha_2^2}X), \qquad \text{and} \qquad m = \sigma(-\alpha_2 R + \sqrt{1-\alpha_2^2}X)$$

  where R and X are standard Rayleigh and Normal distributions, respectively. Use this to derive an approximate result for the lower bound for the rainflow damage.
- Use the extreme value approximation

  $$\mu_{\text{rfc}}(u,v) = \frac{\mu(u)\mu(v)}{\mu(u)+\mu(v)}$$

  to compute an approximation of the rainflow damage.

P.S. For the damage calculations you can assume the Basquin equation $N = \alpha S^{-\beta}$ with $\beta = 4$.

Part B - Solve using Matlab & WAFO

B.1 Use the Gaussian process in A.1 with $C = 1$, $\omega_1 = 1$, and $\omega_2 = 10$.

- Compute the limiting rainflow matrix and the damage.
- Compute the upper and lower bounds for the rainflow damage.
- Compare the exact rainflow damage with the upper and lower bound, as well as with the extreme value approximation.
- Simulate a sample path of the Gaussian process and compute the damage. How does it compare to the other computations of the damage?

B.2 Here we will analyse a measured signal. You can choose a signal of your own, or use `sea.dat` in WAFO.

- Compute the rainflow matrix and the Markov matrix from the signal.
- Now suppose that we have lost the original signal, as well as the rainflow matrix, and only have the Markov matrix. How can we then compute approximations or bounds for the rainflow damage? Use at least three different methods/approximations.