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Identifying the Effects of Eliminating Replenishment for Nonexistent Backorders, on the Performance of Perishable Inventories

A Simulation Study

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Preface

This study investigates the effects of modifying ordering policies, on the performance of perishable inventories; where the modifications entail elimination of the underlying assumption of backorders. The rationale behind these modifications is the absence or uncommonness of backorders in the retail-consumer link for food products. Due to having their roots in classical inventory theory, the policies do not take this into account.

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S.B.

Summary

Inventory control is an indispensable activity in operations management. Variability and uncertainty of demand increase the complexity of inventory control activities. The complexity further amplifies when the inventories to be controlled are of perishable products, such as fresh fruits and vegetables, dairy products, meat, blood, chemicals, etc. Such products have limited usable lifetimes and are discarded if not used within this period. For food products, the discarded items amount to food wastes, which are associated with negative social, environmental and economic impacts.

In case of unavailability of products that are of general use rather than special occasion use, such as milk, customers find alternative products or buy the products elsewhere; but are seldom found postponing the purchase of such products, and buying them later at the same store. As a result, in cases of unavailability of such products, sales are lost and the phenomenon of 'backordering' diminishes.

Majority of inventory replenishment models in classical inventory theory, are based on the assumption of backordering. If replenishment is done assuming that customers will return to fulfil their demand, while actual customer behaviour contradicts the expected behaviour; the assumption becomes the cause of over-ordering, as inventory is held in anticipation of demand that has already been lost.

This issue was observed in policies which are intended to cater to replenishment of perishables in grocery retail. These policies exhibit an underlying assumption of backordering, due to having their roots in classical inventory theory. When the inventoried product under consideration is perishable, over-ordering and holding excess inventory, can become a cause of waste. However, reducing order sizes can also be expected to fulfil lower proportions of demand, thus, lowering profits and availability. As a result, the objective of this research was to identify the simultaneous effects of eliminating the backordering assumption, on various inventory performance measures and answering the question:

How are the performance indicators: waste, fill rate, inventory level and number of deliveries; for perishable inventories with no backorders, affected when lost-sales are taken into account while ordering?

To answer the question, a simulation study was conducted on a model that was developed to represent the characteristics of a milk inventory in grocery retail store; where modified and unmodified forms of three ordering policies were compared. The unmodified forms reflected a backordering assumption. The modified forms excluded replenishment for demand that would be lost by the time the order arrives.

The study showed that the proposed modifications reduce waste for stores that experience waste under the unmodified policy. However, the percentage reduction varies with the store characteristics of weekly demand and review intervals. Stores with high weekly demands and low review intervals were found relatively immune to wastes under the unmodified policy, and have little value for such modifications. Among the stores with low weekly demands and high review intervals, the policy modification was observed to have varying level of impact on waste and availability. To summarise the measure of impact, the ratio of change in overstocking and change in understocking was used; where overstocking was represented by the percentage of items that were wasted out of those purchased; and understocking was represented by the percentage of demand that was fulfilled. This measure was referred to as the value of policy change, and was plotted for various store characteristics; for product characteristics that represented milk; and additionally for three other set of characteristics to assess the sensitivity towards these characteristics.

The policy modification can be concluded to have varying levels of value for different stores that face wastes under the unmodified policy forms, and the number of these stores varies with product characteristics. The value of policy modification is higher, for products with shelf lives lower than milk, if customer responses to stock-outs of these products reflect no or minimal backordering.

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List of abbreviations

CODP Customer Order Decoupling Point

POS Point Of Sales

OOS Out Of Stock

WIP Work In Progress

EOQ Economic Order Quantity

ELS Economic Lot Size

SKU Stock Keeping Unit

SDL Substitute, Delay, Leave

FIFO First In First Out

LIFO Last In First Out

CDF Cumulative Distribution Function

MSE Mean Squared Error

SSE Sum of Squared Errors

Chapter 1

Introduction

1.1 Background

Inventory is defined as a "stored accumulation of material resources in a transformation system" (Slack et al., 2010); and an inventory may consist of different kinds of material resources depending on the context of the transformation system, such as raw materials and purchased parts, work-in-process (WIP), finished goods, maintenance and repairs, etc. Inventory control refers to the activity of managing the quantities of items in an inventory, and is usually done with two primary concerns, namely, fulfilling customer demand and minimising costs of ordering and carrying inventories, such that profit can be maximised (Stevenson, 2012).

Within a supply chain, a customer order decoupling point (CODP) marks a boundary of which, upstream planning processes are forecast driven, while downstream planning processes are customer order driven (Olhager, 2010). Inventory levels at grocery retail stores are usually planned on the basis of forecasted customer demand, since the CODP within grocery retail supply chains is typically located at the stores (Hübner et al., 2013). However, planning based on forecasted demand rather than customer orders, involves uncertainty regarding the future demand, as the actual demand may exceed or fall short of the forecasted demand (Stevenson, 2012).

The event of actual demand exceeding the forecasted demand, can be the cause of the phenomenon of 'out-of-stock' (OOS), or stock-out. It refers to the unavailability of an item to fulfil a customer demand (ECR, 2003). This excess demand may remain unfulfilled temporarily (delayed sale) or permanently (lost sale) or may be substituted (Zinn and Liu, 2001). The unfulfilled demand is associated with stock-out costs in the form of loss of current sales, and possible reduction in likelihood of future sales (Anderson et al., 2006). On the other hand, substitution may be a cause of lower profit margins (Smith and Agrawal, 2000) and possibly, loss of customer goodwill due to unavailability of their preferred product, which may also affect sales of other products (Corsten and Gruen, 2003).

Yet another challenge associated with stock-outs is the tactical disadvantage posed due to censoring of demand. It refers to the phenomenon of absence of data about the customers who were unable to purchase their preferred item due to a stock-out (Sachs, 2015b). As a result, the point of sales (POS) data represent censored demand, and future forecasts based on this POS data are inaccurate estimates of the demand. Additionally, the censored demand, which makes up the unfulfilled and substituted components of demand, does not allow assessment of the achieved fill rate. Silver et al. (1998) define fill rate as the component of customer demand that is met without backorders (delayed sales), and is of considerable importance to practitioners. However, stock-outs eliminate the possibility of measuring fill rates, leaving cycle service level as the practically measurable service measure. The cycle service level refers to the probability of not having a stock-out in a replenishment cycle or alternatively, number of cycles over a period of several cycles, when a stock-out does not occur (Silver et al., 1998). Measuring the cycle service level is a useful alternative but provides an inaccurate measure of customer service, since it does not reflect the number of customers whose demand is not fulfilled between a stock-out and arrival of the next replenishment.

In the event of actual demand falling short of the forecasted demand, the problem is relatively straightforward, i.e., excess purchasing and holding costs for inventory are incurred (Silver et al., 1998). However, when the inventory to be controlled consists of perishable items or items with short shelf life (discussed in section 3.1), excess inventories also contribute to the risk of these items reaching the end of their shelf life before they are sold.

Inventory control for perishable items is of high importance to grocery retail industry (for perishables such as bread, meat, fruits, vegetables, dairy and poultry) as well as health care sector (for blood, platelet and pharmaceuticals) (Gürler and Özkaya, 2008). The complexity and significance (both, theoretical and practical) of managing perishable inventories is reflected in the magnitude of research and quantity of published literature in this field, which have been periodically reviewed by Bakker et al. (2012); Bhalla (2017); Goyal and Giri (2001); Janssen et al. (2016); Raafat (1991).

Literature proposing models for inventory control (of perishables and non-perishables) has increasingly inclined towards a service level approach, rather than an optimisation approach which aims at maximising profit; mainly due to the complexities of estimating shortage costs (Janssen et al., 1998). Shortage cost refers to the monetary value of the sales delayed, lost or substituted, or possible loss of customer goodwill when a stock-out occurs (discussed in subsection 3.3.1). Thus, as an alternative, planned or target levels of customer service are used to set inventory control parameters, as argued by Janssen et al. (1998) for non-perishable inventory models, and by Minner and Transchel (2010) for perishables. Minner and Transchel (2010) also point out that the optimisation approach, when used for perishables, has been predominantly pursued for identifying optimal policies. They emphasise that these policies are highly complex due to their dependencies on the ages of on-hand and in-transit inventories, as also demonstrated by Nahmias (2011).

Before further discussion, it is important to highlight the distinction between an optimal policy and an optimal decision, which has been based on combined insights from literature on inventory management (Dvoretzky et al., 1952; Silver et al., 1998; Stevenson, 2012) and decision theory (Berger, 2013; DeGroot, 2005). Consider a situation where multiple decision alternatives are available, and each alternative has a different expected utility or payoff, which has been computed using the information available about possible future consequences of the decision alternative. An optimal decision is defined as one where the decision alternative with maximum expected utility or payoff is chosen (Berger, 2013; DeGroot, 2005; Stevenson, 2012). Thus, in the inventory context, this would refer to choosing the order quantity that is either expected to earn maximum profit or is expected to achieve the highest service level among various decision alternatives. In the example, profit and service level represent two different decision criterion on which the payoff of an alternative can be measured (Stevenson, 2012). However, the common practice is to aim for achieving a target service level rather than maximising it (Minner and Transchel, 2010), also referred to as satisficing (Odhnoff, 1965). Broekmeulen and Van Donselaar (2009); Kiil et al. (2017) also use the target service level as the service objective due to its practical relevance.

An optimal policy can be defined as a long term decision which, once made, specifies a set of parameters, which can be used repeatedly with no or very trivial efforts (such as arithmetic calculations) and can be expected to have a higher expected payoff as compared to any other alternatives, in majority of the situations. An optimal inventory policy is defined by Dvoretzky et al. (1952) as one that strikes "a balance between overstocking and understocking". Thus, an optimal inventory policy can be expected to successfully trade-off overstocking with achieving a targeted service level. Common forms of inventory policies are (s, Q), (s, S), (R, S), and (R, s, S), where s, Q, S, and R represent different policy parameters (discussed in section 3.2.1). Identifying or defining constant policy parameters is useful for relatively stable demand rates, but as also pointed out by Silver et al. (1998), "when the demand rate varies with time, we can no longer assume that the best strategy is always to use the same replenishment quantity".

Time-varying nature is a characteristic of demand in grocery retail (Broekmeulen and Van Donselaar, 2009). Therefore, rather than defining policies, decisions on order quantities are made with smaller planning horizons, using "demand information over a finite period, extending from present, when determining the appropriate value of the current replenishment quantity" (Silver et al., 1998). Methods such as pre-specified analytical results, algorithms or heuristics are utilised in such cases, to determine order quantities every time an order is placed (Silver et al., 1998). For perishable items, computing optimal policies is highly complex even without the time-varying characteristic of demand. This is elegantly illustrated by Nahmias (2011), formulating a multi-period dynamic model and explaining the increasing complexity in solving it, with the increase in the maximum shelf life of product under consideration. Nahmias (2011) concludes that computing optimal policies is only feasible for product lifetimes as short as two or three periods. As the maximum life increases, the number of possible states of inventory age increase drastically, and any inventory expiration because of the immediate decision moves farther away in time. However, these challenges diminish when the horizon is limited to the time period until the next known decision instance, as is the approach in the heuristics proposed by Broekmeulen and Van Donselaar (2009); Ferguson and Ketzenberg (2006); Kiil et al. (2017).

Policies and heuristic approaches that utilise stock levels and age information of perishable items, have been found to perform better than policies and heuristics that use stock levels without considering the age of items. This can be observed in the findings from numerical studies conducted by Tekin et al. (2001) and simulation experiments conducted by Broekmeulen and Van Donselaar (2009); Ferguson and Ketzenberg (2006); Kiil et al. (2017).

Thus, the value of heuristic approaches for determining order quantities is twofold for inventory control of perishables in grocery retail; as they can address the time-varying nature of demand; and also utilise age information with lower levels of computational complexities than those faced while determining optimal policies. Ferguson and Ketzenberg (2006) also point out that since optimisation procedures become increasingly impractical for higher shelf life items, well performing heuristics are considered useful, and their utility is assessed on "a balance between simplicity and performance".

1.2 Objectives and research question

As mentioned in the previous section (1.1), satisfying the target service level has been the objective in majority of perishable inventory models in literature, due to its utility for managerial practitioners. Minner and Transchel (2010) point out that this is used as an alternative to profit maximisation, which is usually not feasible due to the difficulties faced in estimating costs of shortages which occur during a stock-out. One of the reasons for the complexity in estimating these costs, is the variety of possible customer responses to a stock-out (Aastrup and Kotzab, 2010). As a result, while modelling an inventory, it is difficult to identify what proportion of demand that arises during a stock-out, is lost or backordered.

This uncertainty is reduced in inventory models by assuming excess demand to be lost or backordered entirely, or assuming proportions of the two. Among recent literature, the assumption of excess demand being lost is made by Avinadav et al. (2017); Buisman et al. (2017); Chua et al. (2017); Kara and Dogan (2018); Li et al. (2016, 2017); Mahmoodi et al. (2016); Sazvar et al. (2016), in the literature reviewed by Bhalla (2017); and by Chao et al. (2015); Chen et al. (2014); Jammernegg and Kischka (2013); Kouki et al. (2015); Olsson (2014); Pal et al. (2015); Ramadhan and Simatupang (2012); Sachs (2015a); Sainathan (2013); Shukla and Jharkharia (2014); Wee and Widyadana (2013), in the literature reviewed by Janssen et al. (2016).

Broekmeulen and Van Donselaar (2009); Kiil et al. (2017) also conduct the simulation studies to test their heuristics under the assumption of unsatisfied demand being lost. In both, model contexts or in a real situation where the assumption holds true, there are essentially two instances when demand may not be fulfilled. First, between the instances of placing an order and receiving it, i.e., during the lead time of an outstanding order. Second, if a stock-out occurs at an instance when no order is outstanding. An outstanding order refers to an order that has been placed but not been received yet (Tekin et al., 2001). Also, henceforth, the phenomenon of unsatisfied demand due to shortage or stock-out, is referred to as lost-sales, following the common terminology in literature (Bijvank and Vis, 2011).

Broekmeulen and Van Donselaar (2009); Kiil et al. (2017)'s simulation models conceivably account for the lost-sales phenomenon occurring at these instances, by only fulfilling demand that is less than or equal to the on-hand stock. However, the first type of lost demand instance must also be taken into account while placing orders, i.e., the ordering procedure must exclude the expected demand in these periods, because the corresponding replenishment arrives after the demand has occurred already. This is discussed further in detail in section 3.3. It can be observed that the policies proposed and deployed to calculate ordering quantities in their simulation studies do not take this into account. The likely cause for this is the assumption of backordering in classical inventory models, which, as explained in section 3.3, is also the underlying assumption in the EWA and EWA_{SS} heuristic based (R, s, nQ) policies proposed by Broekmeulen and Van Donselaar (2009) and Kiil et al. (2017) respectively, as well as in the stock-based (R, s, nQ) policy which Broekmeulen and Van Donselaar (2009) use as a benchmark, to demonstrate the improvements achieved by using the EWA policy. Bijvank and Vis (2011) point out that when lost-sales systems are approximated with a backorder model, cost deviations can be substantial.

Due to underlying backordering assumption, it is suspected that utilising these policies may over-order, which, as mentioned in section 1.1, contributes to increase in waste for perishables. After modifying these policies for the lost-sales assumption, the ordered quantities are expected to reduce. Such corrections are of theoretical importance, as majority of attention in inventory theory has been on inventory models with the backorder assumption (Bijvank and Vis, 2011). However, if the reduced orders result in substantial reduction in item availability, with only marginal or no reduction in waste, the corrections can be expected to be of low value for managers and practitioners. Thus, it is important that the effect of these modifications on inventory performance is studied considering both, understocking and overstocking aspects.

With the same concern, i.e., considering understocking as well as overstocking due to a policy, Kiil et al. (2017) use the performance indicators of fill rate and waste. As Kiil et al. (2017) explain, low fill rates are representative of lack of availability, while high wastes represent an oversupply. Further, to consider other performance aspects such as transport and handling cost, and capital invested in procuring the inventory, Kiil et al. (2017) use the performance indicators of number of deliveries and average inventory levels. These performance indicators are frequently used and recommended for perishable inventories in literature, as pointed out by Kiil et al. (2017), citing Broekmeulen and Van Donselaar (2009); Hübner et al. (2013); Kaipia et al. (2013); Van Der Vorst (2006).

Thus, the objective of this research is to study the effects of modifying these ordering policies, on the performance of an inventory system where these policies are deployed for calculating order quantities. However, a necessary precursor to this study is identification of aspects of the policies that need to be modified, while clearly demonstrating these modifications, to ensure conceptual clarity and possibility for future critique. Thus, the preparatory objective is to identify and implement relevant modifications to the stock-based (R, s, nQ) policy, and age-based EWA and EWA_{SS} (R, s, nQ) policies, to account for lost-sales. By fulfilling the research objectives, the studies within this research are aimed at answering the following research question:

Research question: *How are the performance indicators: waste, fill rate, inventory level and number of deliveries; for perishable inventories with no backorders, affected when lost-sales are taken into account while ordering?*

The choice of using these policies for investigation, is supported by their utility to the current and future grocery retail practices. As Kiil et al. (2017) point out, "the EWA policy introduced by Broekmeulen and Van Donselaar (2009) is a direct extension of the policy found in traditional automatic replenishment systems, and is intended to be used for automatic replenishment of perishables", where the ‘policy found in traditional automatic replenishment systems’ refers to a (R, s, nQ) policy (Potter and Disney, 2010). Thus, any conceptual improvements are not only of theoretical, but also practical value.

Since inventories of perishable items are found in varying contexts, as mentioned earlier, it is important to scope such a study, in order to maintain a realistic time-frame. The approach taken for scoping this study, is to identify a context where the results of such a study can be of high practical value, while also ensuring a pragmatic scope of work. The process followed and arguments made to support this, are described in the subsequent section.

1.3 Scope

Frequent stock-outs in retail stores have motivated vast volumes of research on the topic (reviewed by Aastrup and Kotzab (2010)), attempting to understand causes and extent of stock-outs and customer responses to these occurrences (Corsten and Gruen, 2003, 2005; Gruen et al., 2002; Verhoef and Sloot, 2006). Citing results from Gruen et al. (2002) and Verhoef and Sloot (2006), Bijvank and Vis (2011) point out that excess demand can be considered lost in majority of real world retail settings. While results from Gruen et al. (2002) show that only 15% of the customers delay their purchase in the event of a stock-out, Verhoef and Sloot (2006) found the percentage to be 23%. Nonetheless, the proportion of delayed demand, which serves as back-orders, is much lower as compared to all other customer reactions, which amount to lost-sales. Subsection 3.3.1 discusses further in detail how various documented customer responses contribute to lost-sales and backorders.

Aastrup and Kotzab (2010) categorise the variables that have been studied in literature to assess their influence on customer responses to stock-outs. One of these categories of variables are product related variables, implying that product characteristics influence the customer responses to stock-outs. One such product related finding presented by Emmelhainz et al. (1991) was that products that are of regular usage rather than special occasion usage, are more likely to be substituted. The finding is corroborated by the results presented by McKinnon et al. (2007), where dairy products were found to have the lowest proportion of consumers who would choose to delay a purchase, as compared to the two other product categories of frozen products, and health and beauty products. Additionally, their finding for dairy products that approximately 10% of the customers delay their purchase, is even lower than the averages presented by Gruen et al. (2002); Verhoef and Sloot (2006) for grocery retail. Although the arguments might not apply for customers who refrain from use of dairy products, but the ones who do, use them often.

Since the objective within the research is to investigate the consequences of including the lost-sales assumption into the ordering decision for perishable products, a model of a dairy product inventory in a retail store was considered a logical choice of premise for investigating potential changes in inventory performance. Other arguments that provide additional logical support to this choice, follow.

As Engelseth (2012) describes, the structure of milk supply chain for the largest dairy producer in Norway is such, that milk is delivered directly to the stores rather than through a wholesaler or distributor. Additionally, the production and packaging process that transforms raw milk collected from farms, into milk cartons, is partially planned based on orders from the retail stores (Engelseth, 2012). 'Partially' emphasises on the corrective or adaptive feature of the milk production planning process, where production is planned in the morning based on forecasts, but "around noon, the total demand of the day becomes apparent and production volume is adjusted so that the production of milk meets daily order requirements" (Engelseth, 2012).

While this structure of supply chain is specific to liquid dairy products such as milk and cream, other food products (including solid dairy products) are supplied from producers through wholesalers to the retailers (Stensgård and Hanssen, 2016). As a result, an additional level of divergence is introduced in the supply chain structure. Citing Ganeshan (1999); Hwarng et al. (2005), Dominguez et al. (2014) explain that a divergent "structure is characterised by a tree-like structure, where every stock point in the system receives supply from exactly one higher echelon stock point, but can supply to one or more lower echelon stock points". Thus, while the divergence occurs at one stage in a milk supply chain, i.e., at the production facility; two stages of divergence occur in supply chains for other food products, i.e., producer and wholesaler.

With an intermediate wholesaler or warehouse storage between the producer and retailer, producers plan their production based on forecasts which are made on historical aggregated orders from various wholesalers; while the wholesalers place these orders based on forecasts which are made on the historical aggregated orders from various retailers (Chocholáč and Prša, 2016). As Chocholáč and Prša (2016) demonstrate, such supply chains are highly prone to the bullwhip effect. The bullwhip effect is defined by Lee et al. (1997) as the distortion of demand information as it moves upstream in a supply chain, and Chocholáč and Prša (2016) refer to it as the "phenomenon where order variability increases as the orders move upstream in the supply chain". Dominguez et al. (2014) demonstrated, that as the number of stages in the supply chain increases, the bullwhip effect in the supply chain can be expected to amplify. This is also demonstrated for a food supply chain by Chocholáč and Prša (2016). Due to the higher complexity of divergent food supply chains, the model of such a network would be more complex and would have to involve the occurrence of several phenomena such as change in waste, change in fill rates, bullwhip, etc. at several stages, simultaneously. This would, in turn, also increase the scope of investigation. A simulation model of such a divergent supply chain is used as premise by Kiil et al. (2017), for evaluating the performance of EWA_{SS} and EWA policies.

However, using a milk supply chain model appropriately limits the scope of investigation within this research. Due to lower number of stages in the Norwegian milk supply chain network, and direct adjustment of production plans according to the orders from retail stores, the information distortion can be expected to be lower as compared to other food supply chains.

Food waste data from Norway, presented by Stensgård and Hanssen (2016), shows a drop of 28% in waste of liquid dairy products at the producers from the year 2010 to 2015. However, an increase of 28.5% in the waste of liquid dairy products at retailers from the year 2013 to 2015, is also reported in the same study. As pointed out by Fearne et al. (2003), stores attempt to ensure high service levels due to the competitive nature of grocery retail industry, and often over-order, which increases the chances of waste. These wastes, besides the obvious negative financial impacts, also have serious social and environmental impacts (Parfitt et al., 2010). Thus, any efforts to reduce over-ordering by improving the ordering procedures used at the stores can be considered to be of value to the entire production network.

The arguments presented above qualify a milk inventory at a retail grocery store as an appropriate premise to investigate potential improvements by modifying ordering procedures, as it can directly influence waste. Thus, this study tests the effects of the proposed conceptual improvements on a simulation model of a milk inventory at a grocery retail store. However, according to information on ordering procedures known from Engelseth (2012); Herstad (2016), the current ordering procedures for milk do not make use of the age of items, because this information is not available in the barcodes, as highlighted by Damgaard et al. (2012) for most perishable grocery items. This information is a prerequisite for the use of EWA and EWA_{SS} policies.

With age information having been established to improve inventory control of perishables (Broekmeulen and Van Donselaar, 2009; Ferguson and Ketzenberg, 2006; Kiil et al., 2017; Tekin et al., 2001), and under the growing emphasis on automated replenishment among retailers (Broekmeulen and Van Donselaar, 2009; Kiil et al., 2017; Van Donselaar et al., 2006), it is important that theoretical knowledge that is expected to support these endeavours, is tested for robustness. Thus, considering the proposed modifications as conceptually significant, investigation of the effects of modifying the age-based EWA and EWA_{SS} policies is carried out under the assumption that age information for milk is available.

1.4 Research outline

Table 1.1: Thesis structure

Chapter 1 Introduction	The introduction briefly presents the background and motivation, the research objectives, research question, scope and structure of this report.
Chapter 2 Research methodology	The methodology describes the process of literature study which led to developing the research objective; explains the rationale for the choice of research method to fulfil the objective; and describes the research method.
Chapter 3 Theory and literature study	Theory and literature study provides theoretical background for the research context; consolidates arguments to support the research objective; presents propositions that the research aims to evaluate; and presents secondary information to support modelling and simulation.
Chapter 4 Modelling and simulation	Modelling and simulation describes the modelled system; describes the modelling approach; describes model notations, model elements, model verification, validation; and presents results from simulation experiments.
Chapter 5 Discussion	The discussion uses findings to answer the research question; discusses implications and interpretations of the results; discusses limitations and challenges faced; and proposes future research agenda.
Chapter 6 Conclusion	The conclusion summarises the rationale and findings of the study; highlights contributions to knowledge and practice; discusses limitations regarding methodology; highlights future research necessity and proposes prospective approaches.

Chapter 2

Research methodology

"The generally thought of most significant characteristic of good research is that, methodologically, it is well done" (Karlsson, 2010). Thus, the purpose of this chapter is to discuss the methodological aspects of this research. However, to support the methodological approach and choice of research design, this chapter also describes the process which led to the formulation of the research objective and research question as presented in the previous chapter. Thus, the chapter is organised as follows: section 2.1 describes the exploratory literature study that resulted in the curiosity which is expressed in the research question and objective; section 2.2 presents arguments to support the choice of research method and discusses corresponding expansion of the literature study to support application of the method; and section 2.3 discusses the application of the method and selected simulation approach, to the problem context.

2.1 Literature study

Among the various purposes of literature review in theses, Ridley (2012) points out that such a review provides historical background for the research, gives an overview of the current context while referring to contemporary issues and questions in the field and discusses relevant theory and terminologies. Additionally, it describes how the research addresses a gap in work in the field and provides supporting evidence for the issue that is to be addressed. While these purposes provide an extensive checklist for students to ensure that the literature review text in a thesis, addresses these aspects; it is also important to highlight the importance of literature study in identifying the problem investigated.

This research was preceded by a project where the very goal was to get familiarised with the field of perishable inventory models. The project included the application of a research design that can be categorised as a review, where a systematic literature review was conducted, to identify research gaps and problem contexts with scope of investigation (Bhalla, 2017).

Following the review, an exploratory literature study for this research was conducted, to get deeper insights into the problem context. The literature searches for this exploratory study were conducted using the following keywords individually and in combination, using Boolean operator 'OR' to expand the scope of results:

1. perishable(s);
2. inventory model(s);
3. stock out(s) OR stock-out(s) OR stockout(s);
4. out-of-stock OR OOS;
5. lost-sale(s) OR lost sale(s);
6. backorder OR backorders OR backordering; and
7. waste.

The searches were conducted on online databases Google Scholar, Scopus, and Web of Science. Assessing the content of the journals and articles found, led to highlighting the unavailability or nonexistence of research that could answer the research question in the context of interest, i.e., perishables. This led to the formulation of the objective of this research, and the rationale behind the formulation is demonstrated in Chapter 3. The following section explains the choice of simulation as the method in this research.

2.2 Choice of method

Discussing research philosophy, Croom (2010) explains that research philosophy is concerned with adopting an approach "that will provide insight into the phenomenon or process of interest" to a study, where philosophy refers to the study of truth. Croom (2010) presents Meredith et al. (1989)'s generic framework for classifying research methods, pointing out that the framework relates philosophical paradigms to the choice of appropriate research methods, by linking the knowledge generation approach to the sources of information used in research. While Meredith et al. (1989)'s framework places rational and existential approaches at the two extremes of knowledge generation approaches, the information sources and kind of information vary between the extremes of natural to artificial.

Among the various methods described, Meredith et al. (1989) refer to simulation as "a special type of analytical modeling", implying that it is not a physical model. They further state that simulation methods can be utilised to evaluate variations in policies, by varying equations in the model. As pointed out in the previous chapter, the process of interest to this research, is the ordering of perishable items. The insights that this study aims to gain is regarding the effect of a decision variable, i.e., order quantities calculated by different policies; on dependent variables, i.e., performance indicators. Meredith et al. (1989) classify the knowledge generation approach of such a research method as logical positivist, explaining that such a "perspective assumes that the phenomenon under study can be isolated from the context in which it occurs". Citing Croom (2010), Kiil et al. (2017) explain that "simulation models are typically found in the literature to evaluate different inventory scenarios as they provide a risk-free environment". Thus, following the similar logical positivist approach to knowledge generation, simulation was considered an appropriate method for this study. However, Meredith et al. (1989) also point out the risk in the use of such methods, which concerns the source and kind of information used in such research.

Pointing out that simulation is a commonly used method in operations management research, Meredith et al. (1989) explain that simulation models include a conceptual model of processes, using equations, and "an element of reality through the values set for the parameters in the equations". They further emphasise that the parameter values are, on occasions, hypothesised in the model rather than being taken from real world data; thus, reducing the fit to the actual phenomenon and risking irrelevance or reducing external validity of the model. While a simulation model is already an "artificial reconstruction of object reality" (Meredith et al., 1989), in order to ensure the validity of the reconstruction, it is important that the simulation study is supported by empirical or real world data.

To address this concern while maintaining a pragmatic scope for the investigation, the simulation study was supported by expansion of the literature study. This expansion served the following two purposes:

1. to provide empirical basis to the simulation assumptions and parameters, rather than the use of entirely hypothesised parameters;
2. to identify a real world context where the research outputs could be of value in the future.

Thus, following McKinnon et al. (2007)'s empirical finding about milk being prone to lost-sales, the literature study progressively expanded with the keywords:

1. milk OR dairy;
2. supply chain;
3. food waste;
4. grocery retail;
5. divergent supply chains;
6. bullwhip effect.

As a result, in addition to providing theoretical basis, the literature study also provided access to secondary information and data, i.e., data from sources such as scientific journals and articles, that was collected empirically by the authors. This information provided basis for assumptions made in the simulation study regarding various model parameters, the process of which is described in Chapter 4.

Croom (2010) points out "whether one is undertaking data collection in the 'real world', or conducting experiments with 'artificial' data but using proprietary software, one will often encounter issues relating to difficulties and opportunities over access to the data or the means of data analysis preferred in the study" (Gummesson, 2000). While the access to proprietary software is not a challenge that this research faced, the concern of gaining access to 'real data' was overcome by utilising secondary sources. The subsequent section describes the research method of simulation, and presents arguments to support the choice of the simulation approach.

2.3 Simulation

An idealised model is an abstraction of a real situation that provides a simplified representation of reality, meaning that the model may not reflect the complete reality, but only those aspects of reality which are considered of importance to the causal relationship being studied (Will M. Bertrand and Fransoo, 2002). In operations management practice, quantitative models are frequently used to improve understanding of systems and thus, aid decision making processes; while in operations management research, they allow to identify changes that can allow for improvements (Stevenson, 2012; Will M. Bertrand and Fransoo, 2002). Modelling refers to the process of abstracting the system, where system refers to the process or situation of interest (Law and Kelton, 2007). Models may vary in their level of abstraction, and can be classified as physical, schematic and mathematical or quantitative models, with physical models being the least abstract and quantitative models being the most abstract (Stevenson, 2012).

Will M. Bertrand and Fransoo (2002) define quantitative models in operations management context as models that are "based on a set of variables that vary over a specific domain, while quantitative and causal relationships have been defined between these variables". Simulation concerns with imitating the operation of the modelled system using the mathematical model (Jerry, 2005). Robinson (2004) defines simulation as "experimentation with a simplified imitation (on a computer) of an operations system as it progresses through time, for the purpose of better understanding and/or improving that system".

Davis et al. (2007) present a seven step roadmap for using simulation methods to develop theory, which are also discussed by Happach and Tilebein (2015). These seven steps for such experimental research design can be listed, and their rationale described, as:

1. Begin with a research question, which focuses efforts on a relevant theoretical issue for which simulation is effective.
2. Identify simple theory, which gives shape to theoretical logic, propositions, constructs and assumptions.
3. Choose a simulation approach, that is appropriate for the research at hand.
4. Create the computational representation, which sets stage for theoretical contributions.
5. Verify computational representation, which confirms robustness and accuracy of computational representation and confirms internal theoretical validity.
6. Experiment to build novel theory, building new theory through exploration, elaboration and extension of simple theory.
7. Validate with empirical data, to strengthen external validity.

This research follows the roadmap laid out by Davis et al. (2007) for using simulation as a research method, however, the external validity of the research contribution is only limited due to lack of access to empirical data. The first two steps of beginning with a research question and identifying simple theory which this research aims to develop were briefly addressed in the previous chapter (1) and are discussed further in Chapter 3. Steps four through seven are further described in Chapter 4. This discussion focuses on the third step of choosing the appropriate simulation approach.

Davis et al. (2007) discuss five different simulation approaches, while Happach and Tilebein (2015) discuss two major simulation approaches, with system dynamics being the one approach common to these discussions. Davis et al. (2007) explain that usually, a system dynamics approach "models a system as a series of simple processes with circular causality". Happach and Tilebein (2015) explain this as "a variable A influences a second variable B and at the same time, variable B influences variable A"; further pointing out that these approaches are useful when the goal is to understand the dynamic behaviour caused by several interrelated feedback loops. The second simulation approach discussed by Happach and Tilebein (2015) is agent based simulation, where "agents influence one another by interactions which are based on some predefined simple rules and preferences" (Harrison et al., 2007; Tilebein and Stolarski, 2009). Happach and Tilebein (2015) further explain that "by these interactions, the simulation method focuses on the individual parts of a system and derives the overall system's state from the sum of all interactions as an emergent phenomenon". As can be inferred, to analyse the performance of an inventory as a result of several inventory transactions over a period of time, one requires that the overall system state becomes apparent after evolving due to the several transactions. Inventory transactions refer here, to increase in inventory due to orders and decrease due to demand or waste. Thus, rather than system dynamics, an agent based simulation approach was found relevant for this research.

The classification of simulation and modelling approach used in this research, was further facilitated by the typology of modelling approaches presented by Pidd (2006), who classifies modelling approaches based on the elements included in the simulation model. These are explained further, along with the modelling process in Chapter 4.

Chapter 3

Theory and literature study

This chapter provides the necessary theoretical background for understanding the significance and characteristics of the problem situation; and presents the propositions that this research aims to test. Firstly, characteristics of perishable products and associated terminology are discussed in section 3.1, and characteristics of the product category of interest in this research are clarified. This is followed by discussions on some basic inventory management concepts (subsection 3.2.1), effect of demand uncertainty on inventory decisions considered in this research (subsection 3.2.2) and inventory management of perishable products (subsection 3.2.3) in section 3.2. The policy modifications which this research revolves around and relevant theoretical background are discussed in section 3.3. Section 3.4 presents information from empirical studies, which supports the modelling process for testing the effect of the proposed modifications, in the subsequent chapter. Finally, the chapter is summarised in section 3.5.

3.1 Perishables and deterioration

This section discusses terminology regarding perishables used within this research, and clarifies the characteristics of products relevant to this study. Items are considered 'perishable' because of their property to 'deteriorate', meaning that they lose value with time, which has to be taken into account while modelling inventories of such items (Goyal and Giri, 2001). Raafat (1991) defines deterioration as the "process that prevents an item from being used for its intended original use", further classifying the process into:

1. situations where all items in an inventory have utility during the planning horizon but simultaneously become obsolete at the end of the planning horizon, such as fashion merchandise, and
2. situation in which items continuously deteriorate throughout the planning horizon, which is further sub-classified into:
   - items with fixed shelf life, and
   - items with continuous decay and random lifetime.

Goyal and Giri (2001) use a similar approach for classifying inventoried goods, according to the phenomenon that characterises them, in the categories of obsolescence, deterioration, and no obsolescence or deterioration.

The definition of obsolescence given by Goyal and Giri (2001), i.e. "items lose their value through time because of rapid changes in technology or style, or introduction of a new product by a competitor"; is a more precise formulation of the phenomenon. It concerns more with the underlying reason for the occurrence of the phenomenon, which is what differentiates it from deterioration. On the other hand, the definition by Raafat (1991) concerns with 'when' the transition in utility from non-zero to zero occurs, which provides little clarity on how it is differentiated from deterioration.

Van Donselaar et al. (2006) include items undergoing obsolescence as well as deterioration into the umbrella of 'perishables', however, while also precisely formulating the characteristics of these items. They consider an item to be perishable if "the high rate of deterioration at ambient storage conditions requires specific storage conditions at the store and/or at the consumer to slow the deterioration rate" or "the obsolescence date of the product is such that reordering for the products with the same date is impractical".

A useful distinction between perishability and obsolescence is established by Silver et al. (1998). They state that perishability concerns with physical deterioration, and demand for further units continues when some of the items have perished. On the other hand, when obsolescence occurs, there is negligible demand for further units.

Goyal and Giri (2001) sub-classify items undergoing deterioration on similar criteria as Raafat (1991), into:

1. having a known usable life-time, known as perishable products such as fresh food and vegetables, human blood, etc., and
2. having unknown shelf life, known as decaying products, such as gasoline and alcohol (due to volatility), radioactive substances (due to radioactive decay), etc.

This research primarily revolves around the inventories of a category of items that undergo 'deterioration' (as defined by Goyal and Giri (2001); Raafat (1991)) and have a known usable life-time. It can be observed that the terminology within the field has evolved over the years, which points to a scope to formalise definitions and criteria for categorising items as perishable, decaying, or prone to obsolescence.

Based on insights from different definitions, the 'storage condition' criteria for perishables used by Van Donselaar et al. (2006) can be extended in following ways for grocery items:

- items that require specific storage conditions throughout their usable life, undergo rapid deterioration if exposed to ambient conditions for prolonged periods, but are highly prone to be unusable beyond a certain point in time, even if the storage conditions are maintained, such as liquid dairy products (which is the focus within this research),
- items whose usable life can be extended with certain storage conditions, but can also stay usable for prolonged periods in ambient conditions, and for which the end of usable life is not known with certainty, such as fruits and vegetables, and
- items with extremely low shelf lives which deteriorate rapidly irrespective of the storage conditions, such as bread.

Besides, the general practice in scientific literature concerning perishable inventories, is to clearly specify the characteristics of the product(s) and context being considered, which can be observed in literature reviewed by Bhalla (2017); Broekmeulen and Van Donselaar (2009); Janssen et al. (2016). For example, Van Donselaar et al. (2006) specify that only perishable items in a retail grocery store, with shelf life less than or equal to 30 days are considered in their study.

A similar clarification of the characteristics of the situation under consideration is carried out in section 3.4, which allows to narrow down the focus and scope of the study. As can be observed in results presented by Broekmeulen and Van Donselaar (2009); Ferguson and Ketzenberg (2006); Kiil et al. (2017), when combined with other situation characteristics such as review interval, lead time, etc., the product lifetime has considerable influence on the performance of inventory control policies. Thus, the magnitude of improvement from the same policy modification can be expected to yield substantially different results for different products (such as dairy products, bread, fruits, etc) and thus, must be taken into account before the results are generalised.

3.2 Inventory management

Silver et al. (1998) state that "inventory management encompasses decisions regarding purchasing, distribution, and logistics, and specifically addresses when and how much to order". They further consider decisions in organisations to be a hierarchy that extends from long range strategic planning, through medium range tactical planning to short range operational control. A similar hierarchy can be conceptualised for inventory management as follows (Brown, 1982; Silver et al., 1998):

- strategic planning decisions such as financial importance of an item (ABC classification), and choosing type(s) of inventory control system to be used for an item based on its financial importance.
- tactical planning decisions such as specifying policy parameters, service objectives, forecasting techniques, etc.
- operational control activities such as data collection, calculations, and decisions such as whether a replenishment order should be placed or not, what should be the size of the order, etc.

Subsection 3.2.1 discusses concepts that are utilised in, or are a result of, inventory management decisions at different hierarchical levels; and explains the rationale behind the use of heuristic based approaches for inventory control. This is followed by a discussion on uncertainty in decisions and the nature of demand uncertainty considered within this research, in subsection 3.2.2. Subsection 3.2.3 discusses inventory management in the context of perishable items.

3.2.1 Preliminaries

The primary concern of inventory management is to achieve high levels of customer service while incurring minimum possible costs in achieving the level of service, so that profits can be maximised (Stevenson, 2012). Silver et al. (1998) list some commonly used measures of customer service for inventory management as the following:

1. Cycle service level, which is defined as the fraction of inventory cycles in which a stock-out occurs, or the probability of no stock-out in a replenishment cycle.
2. Fill rate, which is defined as the fraction of customer demand that is routinely met, where non-routine refers to backordered demand.
3. Ready rate, which is the fraction of time during which the net stock (on-hand stock minus backorders) is positive, implying that it is the probability of not facing any backorders.
4. Time between stock-outs, or alternatively, stock-out occasions per year.

Further, the cost factors that are commonly considered in assessing inventory performance, as described by Silver et al. (1998), are as follows:

1. Unit variable cost, which represents the monetary value of a unit of inventory, and may have a different valuation for different actors in a supply chain. For example, while a retailer's valuation of an inventory unit would simply be the price paid to the supplier (which is set by the supplier) to acquire it; the supplier's valuation would require that the unit cost reflects all the costs that have been incurred to make the unit available in the saleable form.
2. Holding cost or carrying cost, which includes the opportunity cost of capital invested, expenses incurred for storing inventory in the form of cost of utilities, handling, warehousing, etc. and taxes; and is usually expressed as a fraction of the monetary value of the inventory. The opportunity costs of invested capital refers to the return on investment that could be earned on the most lucrative investment option, instead of using it for carrying inventory, and is usually, the largest component of holding costs.
3. Ordering cost, which is a fixed cost that does not vary with the ordered quantity, unlike purchase costs, and is incurred in the fixed quantity every time an order is placed. These costs play an important role in determining the ordering frequency.

4. Cost of insufficiency can be interpreted for a retailer as the costs that are explicitly or implicitly incurred in the event of a stock-out such as emergency shipments to avoid demand loss, substitution of demand for a less profitable alternative, loss of goodwill due to poor service, etc., which can be usually estimated empirically, and are highly dependent on customer responses to unavailability. The variability of customer responses to stock-outs, is discussed in subsection 3.3.1.

Another important cost factor for perishable items is the cost incurred when they reach the end of their saleable life without being sold. Silver et al. (1998) include the cost of deterioration, obsolescence and pilferage in the description of carrying cost. However, similar to the approach taken by Nahmias (2011) in demonstrating the preliminaries of modelling perishable inventory systems, it is common to consider a separate cost of waste or outdating cost per unit, to explicitly model and analyse the limited life characteristic of perishables, as also done by Broekmeulen and Van Donselaar (2009). However, to avoid any biases in estimating the above listed costs, a useful approach is to analyse the phenomenon which lead to these costs, such as purchased quantities, number of orders placed, average inventory level, etc., as done by Kiil et al. (2017).
The conceptual components that characterise an inventory control system can be listed as (Silver et al., 1998):

1. the frequency of determining inventory levels, i.e., continuous review or periodic review;
2. the presence or absence of condition to determine if an inventory replenishment order should or should not be placed, i.e., reorder point, or no reorder point; and
3. size of the replenishment order, i.e., fixed order quantity, fixed order up to level, or variable order quantities.

N.B.: Silver et al. (1998) point out that in practical situations with periodic review, it might be the case that the inventory levels are continuously available from transaction reporting, such as POS data records. However, the situation is characterised as periodic review due to constraints on the ordering frequency.
When demand rates are constant and deterministic, the basic economic order quantity (EOQ) model, which is derived by balancing ordering and carrying costs, is one of the simplest theoretical approaches to defining an ordering policy (Silver et al., 1998). The variation of inventory

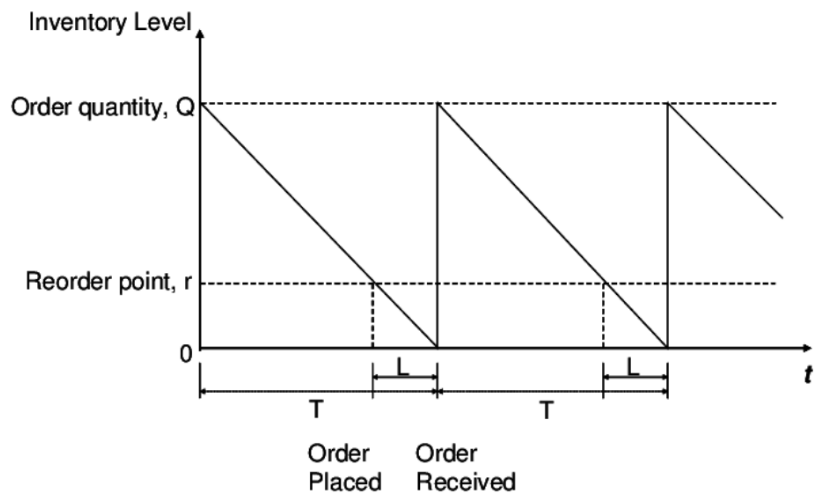

Figure 3.1: Behavior of inventory level with time in EOQ model (Herrera et al., 2009)

levels with time, for an inventory which is controlled using parameters Q and T calculated using the EOQ model, is shown in Figure 3.1. Although the basic EOQ model does not provide a reorder point, it can be easily calculated using the demand rate (Q/T) and lead time (L).
When demand is probabilistic (explained further in subsection 3.2.2), i.e. not deterministic, but average demand still remains approximately constant with time, different types of inventory control systems may be relevant according to characteristics of the situation (Silver et al., 1998). Depending on the characteristics of an inventory control system, inventory policies used for probabilistic demand can be classified into different 'forms', most common ones of which being (Silver et al., 1998):

- Order-Point, Order-Quantity (s, Q) policy (continuous review): an order of fixed quantity Q is placed whenever inventory position reaches s or lower (lower, referring to the case where a single demand transaction drops the inventory position from greater than s to smaller than s).
- Order-Point, Order-Up-To-Level (s, S) policy (continuous review): an order is placed to increase inventory position to S, whenever inventory position reaches s or lower.
- Periodic-Review, Order-Up-To-Level (R, S) policy: an order is placed to increase the inventory position to S after every R time units.
- (R, s, S) policy: this is a combination of the (s, S) and (R, S) policies. Inventory position is reviewed after every R time units and if the inventory position is found to be below

s, a sufficiently large replenishment order is placed to raise the inventory position to S, however, no order is placed if inventory position is above s.

Choosing the form of policy that is used for an item, and selecting the values for policy parameters R, s, Q, S, are a combination of strategic and tactical decisions, the process of which is discussed in detail by Silver et al. (1998). The preferred approach is to select the optimal policy parameters, i.e., parameters that form an optimal policy, which is defined as one that "strikes a balance between overstocking and understocking" (Dvoretzky et al., 1952).

If measured in terms of cost, excess costs of purchasing, holding, ordering, as well as outdating costs for perishables, can be considered contributing costs to the cost of overstocking. The cost of understocking is comprised of the cost of insufficiency. Optimal policy parameters are determined by trading-off the estimated values of these costs, such that profit can be maximised in every inventory cycle (Silver et al., 1998), where inventory cycle refers to the duration between two ordering instances. An alternative approach is to identify policy parameters to satisfy or achieve the threshold value of a service measure, i.e., fill rate (Janssen et al., 1998), or fill rate and cycle service level (Minner and Transchel, 2010). This may not provide the optimal policy, but is of practical importance due to the complexity of cost estimation and the resultant utility of the service measure approach in practical settings (Minner and Transchel, 2010).

The types of inventory control systems discussed above, address situations with deterministic and probabilistic demands, but with approximately constant average demand. The common characteristic among these situations, is that one inventory cycle can be considered representative of other inventory cycles, and once a set of policy parameters have been identified as the best performing for one cycle, they can be expected to produce similar results for subsequent cycles as well. In the absence of this characteristic, simple average costs over an inventory cycle cannot be considered to represent every inventory cycle (Silver et al., 1998). Silver et al. (1998) emphasise that when demand rates vary with time, it can no longer be assumed that using the same replenishment quantities or order up to levels is the best strategy. They explain that for such situations, "demand information over a finite period, extending from the present" has to be used to determine appropriate replenishment quantities. This period is known as the planning horizon, and should be kept as short as possible, because the farther in future it extends, the lower is the likelihood of accuracy of demand information (Silver et al., 1998).

For situations where demand is deterministic but varies with time (referred to as time-varying demand), Silver et al. (1998) list three alternative approaches for determining immediate order quantities:

- using the EOQ model, when demand variability is low;
- using exact best solution to a mathematical model of the situation, such as the Wagner-Whitin algorithm (Wagner and Whitin, 1958); or
- approximate or heuristic methods that capture the essence of complexity due to the time-varying demand, while also being simple enough to understand for practitioners and not requiring lengthy computations.

However, Silver et al. (1998) point out that when demand is time-varying and probabilistic, exact analysis is "far too complicated for routine use in practice", thus, again emphasising on the use of heuristic approaches.

The discussions above have led to highlighting the utility and importance of heuristic approaches for inventory control under probabilistic time-varying demand, even without the perishable nature of items. The added advantage of heuristics for inventory control of perishable items under time-varying probabilistic demand is discussed in subsectiion 3.2.3. Before discussing the impact of introducing the limited life characteristic of perishables on inventory control, the following subsection (3.2.2) clarifies probabilistic demand and associated uncertainty.

3.2.2 Probabilistic demand

This subsection discusses the classification of decisions based on levels of uncertainty which is utilised to explain the classification of demand within this research as probabilistic, and clarify terminology for subsequent sections and chapters.

Uncertainty can be defined as "any departure from the unachievable ideal of complete determinism" (Walker et al., 2003). Based on the level of information available when a decision is made, Whalen and Churchill (1971) classify decision situations under the categories of decisions under certainty, ignorance and risk. Alternatively, Stevenson (2012) makes the distinction into the categories of decisions under certainty, uncertainty and risk. Their typologies differ in their terminology, while the essence of the classification criteria, which is degree or level of un-

Table 3.1: Classification of decision situations on level of uncertainty

Source		Walker et al. (2003)	Whalen and Churchill (1971)	Stevenson (2012)
Decision under		determinism	certainty	certainty
		statistical uncertainty	risk	risk
		scenario uncertainty	ignorance	uncertainty

certainty, is essentially synonymous. This can be observed in the description of these categories below:

- Decision making under complete knowledge of outcomes of different decision alternatives: decision under certainty (common for both, Stevenson (2012) and Whalen and Churchill (1971)).
- Decision making when possible outcomes of a decision alternative can be assigned probabilities: termed as decision under risk by Stevenson (2012); Whalen and Churchill (1971).
- Decision making when possible outcomes of a decision alternative cannot be assigned probabilities: termed as decision under ignorance by Whalen and Churchill (1971), and decision under uncertainty by Stevenson (2012).

An extensive classification scheme for uncertainty is presented by Walker et al. (2003), where uncertainty is classified along three dimensions, namely, location, nature and level of uncertainty. The classifications by Stevenson (2012); Whalen and Churchill (1971) listed above, are along the dimension of level. Combining typologies of Stevenson (2012); Whalen and Churchill (1971); Walker et al. (2003), Table 3.1 organises the three classification schemes under one column each. Terminologies in a row are synonymous, i.e., represent same level of uncertainty; and level of uncertainty increases downward. The synonymous nature of the three typologies becomes apparent after the terminology from Walker et al. (2003), is presented below.

Walker et al. (2003) define statistical uncertainty as "any uncertainty that can be described adequately in statistical terms", or uncertainty "where the functional relationships are well described and a statistical expression of the uncertainty present can be formulated". Differentiating scenario uncertainty from statistical uncertainty, Walker et al. (2003) state that "scenario uncertainty implies that there is a range of possible outcomes, but the mechanisms leading to these outcomes are not well understood and it is, therefore, not possible to formulate the proba-

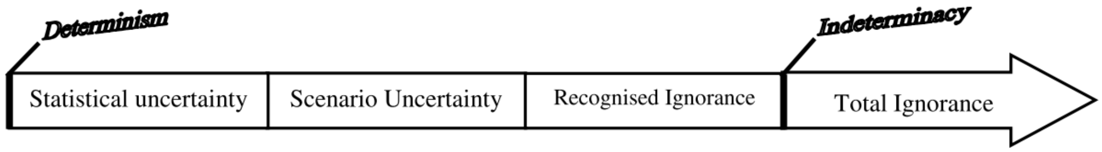

Figure 3.2: The progressive transition between determinism and total ignorance (Walker et al., 2003)

bility of any one particular outcome occurring". Considering these definitions, it appears logical to organise them as done in Table 3.1.

The advantage of the terminology used by Walker et al. (2003), is recognised when one considers the entire spectrum of levels of uncertainty presented by them (Figure 3.2). As can be observed, what Whalen and Churchill (1971) refer to as ignorance, is termed as scenario uncertainty by Walker et al. (2003), while classifying ignorance as a further higher level of uncertainty. To justify referring to this as 'ignorance', Whalen and Churchill (1971) argue that "some authors call this uncertainty rather than ignorance, but this is confusing since uncertainty also means anything other than certainty". However, this issue is resolved by the detailed classification of Walker et al. (2003).

Decisions under recognised ignorance and total ignorance (shown in Figure 3.2) are not discussed further due to the scope of investigation within this research. Improvements in decisions can almost always be expected as uncertainty reduces from ignorance to scenario uncertainty or statistical uncertainty. An example of this are the potential improvements that have been demonstrated as a result of considering age information in inventory decisions for perishables, under same levels of demand uncertainty. This can be observed in the simulation studies by Broekmeulen and Van Donselaar (2009); Kiil et al. (2017), where ordering procedures ignoring item ages, are compared with ordering procedures which assume availability of complete information about item ages, and include this information in the ordering decision. The improvements achieved due to the determinism of item ages demonstrates the value of eliminating the ignorance in the decision. However, availability of information about item ages is considered known within this research, since this information has already been established as being valuable.

Putting the discussion on levels of uncertainty, into the context of demand, inventory decisions regarding ordering quantities and policy parameters, can be categorised according to the level of demand information available. Demand forecasting is concerned with predictions for future demand by using historical demand (Silver et al., 1998). By treating historical demand as a time series, and identifying levels, trends and patterns in the data, forecasting procedures attempt to recognise the underlying stochastic process in demand (Silver et al., 1998). However, they do not eliminate uncertainty. Silver et al. (1998) clarify that after the forecasts have been used to make decisions, the only thing that is certain is that "the forecasts will be in error". To cope with this, forecasting procedures are updated to minimise these errors, however, the forecasts are always expressed with some expected error values. Thus, it can be inferred that forecasting is aimed at reducing the uncertainty regarding demand from ignorance or scenario uncertainty to statistical uncertainty. Silver et al. (1998) refer to situations where inventory decisions are based on statistically estimated forecasts, as having a probabilistic demand. As retail inventory planning is also dependent on demand forecasts (Hübner et al., 2013), demand uncertainty within the context of this research is also considered probabilistic.

An additional comment can be made here about the nature of uncertainty (Walker et al., 2003) with regard to situations with probabilistic demand. While forecasts may reduce uncertainty in decisions, as explained above, they are bound to have errors. This occurs due to the inherent variability in the demand process which forecasting aims to predict, which points to the difference between epistemic and variability uncertainties. Walker et al. (2003) explain that epistemic uncertainty arises due to imperfection in knowledge and can be reduced by empirical efforts and research activities; while variability uncertainty arises due to the inherent variability of the real world process, such as those involving human and natural systems or concerning social and economic developments. Thus, improvements in forecasting techniques, capturing censored demand data, etc. can be efforts that reduce epistemic uncertainty in the decisions. However, it may not be possible to reduce the variability uncertainty in demand, but only to predict it.

It is important to clarify here that the variability discussed here is not same as the concept of time-varying demand, which is concerned with variation in average demand rates. As discussed in subsection 3.2.1, demand can be time-varying while being deterministic. Alternatively, demand can be probabilistic without having a time-varying characteristic. However, within the context of this research, demand is considered to be time-varying and probabilistic, as is typical to grocery retail (Broekmeulen and Van Donselaar, 2009; Hübner et al., 2013).

3.2.3 Perishable inventories

The increasing level of complexity in inventory management due to introduction of time-varying and probabilistic characteristics in demand, was discussed in subsection 3.2.1. This section discusses some added complexities that occur as a result of the perishable nature of items in inventory.

Nahmias (2011) demonstrates the transformations required in the EOQ model for perishable items with deterministic and stable demand, and states that the modifications required to include perishability, are straightforward. For varying deterministic demand, Nahmias (2011) points that the exact requirements or economic lot size (ELS) policy proposed by Wagner and Whitin (1958), may not always be optimal for perishables. To provide some background, the premise for Wagner and Whitin (1958)'s model is a steel mill, whose operator wishes to determine a production schedule for number of beams to produce out of different strength varieties, where a higher strength beam can replace a lower strength beam, and different strength beams have different setup costs, and demands in future periods are considered known, and the objective is to determine the schedule that minimises costs. The key result utilised by Wagner and Whitin (1958) to construct their efficient algorithm, is that an optimal policy only orders in periods when starting inventory is zero, which Nahmias (2011) refers to as the zero inventory property. Nahmias (2011) explains that the first to consider extending the ELS problem for perishables was Smith (1975), followed by Friedman and Hoch (1978), who pointed out flaws in the algorithm presented by Smith (1975), proving that the zero inventory property does not always hold true for perishables.

For the case of stable probabilistic demand, through formulation of a dynamic programming problem, Nahmias (2011) demonstrates that optimal policies for periodic review systems can be computed only for relatively short lifetimes, such as one or two periods. As the item lifetime increases, the state vector that describes the inventory age, also grows. Since the optimal policy depends on the entire age distribution, the feasibility of computing optimal policies decreases with increasing item lifetime (Nahmias, 2011). Similarly, other complexities regarding computing optimal policies for continuous review systems are also highlighted by Nahmias (2011).

Proposed policies can be found in literature for different inventory systems, under varying assumptions. Gürler and Özkaya (2008) consider a continuous review order up to system with random product life, and zero as well as non-zero lead times. Berk and Gürler (2008) consider a continuous review and constant order quantity system with positive lead times and fixed product life. Tekin et al. (2001) compare age based and stock level based policies to conclude the superiority of the former for items with high service level requirements. Besides these, several other publications with varying set of assumptions can be found in the literature reviewed by Bakker et al. (2012); Bhalla (2017); Janssen et al. (2016).

The examples given above belong to a stream of literature that is focused on identifying modified policy forms and optimal policy parameters for perishables. Another stream of research focus can be observed to be on developing and testing heuristics, such as in the work of Broekmeulen and Van Donselaar (2009); Ferguson and Ketzenberg (2006); Kiil et al. (2017). As discussed in subsection 3.2.1 and as pointed out by Silver et al. (1998), defining constant policy parameters is of utility when demand rates are stable. However, with time-varying probabilistic demands, variable quantity policies using heuristics to compute order quantities are more practical (Silver et al., 1998).

Time-varying probabilistic nature is characteristic to demand in grocery retail (Broekmeulen and Van Donselaar, 2009). Further, the demand levels at different stores of one franchise may also vary substantially, as can be observed in the demand data utilised by Kiil et al. (2017). As a result, the value of a heuristic that can be used on operational level, is of wider utility than determining different policy parameters for every setting, on a tactical level. The importance of performance under varying conditions and extended periods is also emphasised by Broekmeulen and Van Donselaar (2009), in explaining the advantages of their heuristic over that of Ferguson and Ketzenberg (2006)'s.

The situation can be simplified for non-perishables by ignoring the time-varying nature of demand and using constant parameter policies which are not optimal, because their financial implications might not be very severe, due to marginal holding costs (Broekmeulen and Van Donselaar, 2009). However, for perishables, ignoring the time-varying nature of demand may result in wastes which have substantial financial as well as social and environmental implications (Parfitt et al., 2010).

The first stream of research focus (on finding constant order policies) is of high theoretical importance, and contributes substantially to the development of intuition in the field of perishable inventory modelling (Bakker et al., 2012). However, the second stream of research (varying orders using heuristics) simultaneously addresses various industrial needs, which is an important quality in operations management research (Karlsson, 2010). These heuristics capture the time-varying nature of demand, while also being sufficiently simple for practitioners to understand, which are highlighted as advantages of heuristic approaches by Silver et al. (1998). Additionally, they utilise the age information to improve decisions, as demonstrated by Broekmeulen and Van Donselaar (2009); Kiil et al. (2017), while maintaining their simplicity. Thus, it can be concluded that the value of heuristic approaches to determining order quantities for perishables, is significant.

This section has provided an overview of preliminary inventory management concepts; described demand uncertainty in the context of this research; and concluded by discussing some aspects of inventory management for perishables, under deterministic as well as probabilistic time-varying demand, and pointing out the importance of heuristic approaches. While it is important that they are simple enough to understand and implement for practitioners, it is also essential that these heuristics are theoretically robust, which is the premise of the policy modifications presented and tested in this research. The following section discusses evidence from empirical and theoretical research that validates the relevance of the proposed policy modifications, which is followed by description of the propositions.

3.3 Lost-sales vs. backorders

As discussed in the previous section, when demand is probabilistic, the inventory management decisions are taken under uncertainty. However, besides the uncertainty in demand, retail stores also face uncertainty regarding the responses of customers when they have stock-outs, which theoretically translate to lost-sales or backorders. The following subsection (3.3.1) elaborates on this. As a result of the varying inventory behaviours, models and policies aimed specifically at lost-sales systems have been considered in literature, and demonstrated the increase in mathematical complexity as compared to backordering systems (Nahmias, 1979; van Donselaar et al., 1996; Huh et al., 2009). However, Bijvank and Vis (2011) point out that lost-sales systems have received lower attention in literature, than is required considering their importance to grocery retail, where excess demands are often lost. Referring to the results of Zipkin (2008), Bijvank and Vis (2011) emphasise that the cost impacts of approximating lost-sales systems with backordering systems can be substantial. In their review of literature that considers lost-sales models for perishable items, they refer Broekmeulen and Van Donselaar (2009) as studying a lost-sales system under constant life time. However, as later explained in subsections 3.3.3 and 3.3.4, the ordering procedures utilised by them, are based on an underlying assumption of backordering. Thus, this section focuses on highlighting the importance of studying lost-sales systems, and demonstrating some relevant changes that are required in policies to account for lost-sales. Subsection 3.3.1 discusses how studies on customer responses to stock-outs, validate the lost-sales assumption; subsection 3.3.2 discusses the concept of inventory position and how it changes from backordering to lost-sales systems; and subsections 3.3.3 and 3.3.4 demonstrate relevant modifications to policies.

3.3.1 Customer responses to stock-outs

Aastrup and Kotzab (2010) give an overview of the literature on stock-outs, where the research on stock-outs is divided into two streams, namely demand and supply side. The former is concerned essentially with the customer responses to stock-outs. This subsection discusses some possible responses studied in literature and contextualises them with respect to this research.

Discussing progress in the literature on customer responses to stock-outs, Aastrup and Kotzab (2010) explain that most research in the field until 1991 focused on identifying and mapping different responses to stock-outs observed among customers. However, due to the significant variations observed in customer responses, literature since 1991 has mostly focused on explaining and understanding the factors that influence customer responses, which Aastrup and Kotzab (2010) classify into product-related variables, store-related variables, situation-specific variables and consumer related variables. As Aastrup and Kotzab (2010) point out, these variations were demonstrated by Grocer (1968) and the results have been replicated by ECR (2003); Gruen et al. (2002); McKinnon et al. (2007).

The three broad categories of behaviours observed in customers during stock-outs are described by Zinn and Liu (2001) with the abbreviation 'SDL', which stands for Substitute, Delay or Leave. Corsten and Gruen (2003) point out that while academic research identifies up to 15 different customer responses to stock-outs, these responses essentially fall into one of the following five categories of managerial importance:

1. buy item at another store (store switch);
2. delay purchase (buy later at the same store);
3. substitute – same brand (for a different size or type);
4. substitute – different brand (brand switch); and
5. do not purchase the item (lost sale);

and that all of these responses result in direct or indirect loss risks for retailers and manufacturers. They classify these loss risks as shopper loss risk, which refers to the risk of permanent store switch by a customer; and sales loss risk, which refers to the risk of all other customer responses except delay, i.e., buying item at another store, cancelling purchase, or substituting with a smaller or lower priced item. Putting these responses into the context of SDL typology as Zinn and Liu (2001) explain, the first and last categories of responses fall under 'leave', the second into 'delay' and the remaining two into the 'substitute' category.

Relating different customer responses to costs of understocking, Aastrup and Kotzab (2010) explain that higher the likelihood of a 'leave' response, higher is the cost of understocking; while higher likelihoods of 'delay' and 'substitute' responses translate to lower understocking costs. The variations in the likelihoods or probabilities of these responses with variation in characteristics related to products, stores, situations and customers, is what causes the complexities in estimating the costs of insufficiency or understocking (Aastrup and Kotzab, 2010). The consequence of this has been the prominence of the approach of satisficing service levels rather than optimisation of service levels, as also pointed out by Aastrup and Kotzab (2010), and explained while justifying their approaches by Janssen et al. (1998); Minner and Transchel (2010).

However, an important breakthrough of empirical research on customer responses to retail stock-outs has been establishing that 'delay' responses are consistently less likely, as compared to the 'leave' and 'substitute' responses, where 'consistently' implies that this is true for a wide variety of products and situations. This can be observed in the results of Corsten and Gruen

(2003, 2005); Emmelhainz et al. (1991); McKinnon et al. (2007); Van Woensel et al. (2007). Emmelhainz et al. (1991) conclude that the purchase of a product is less likely to be delayed and more likely to be substituted if it is of regular usage, rather than special occasion usage. The corroboration of this can be inferred from the results of McKinnon et al. (2007), who studied frozen foods, health and beauty products, and dairy products; and reported a decreasing probability of 'delay' response in these product categories respectively.

Silver et al. (1998) explain two extremities of customer responses to a stock-out when demand is probabilistic, that can be assumed in an inventory model, as:

1. Complete backordering: Unfulfilled demand due to stock-out is backordered and is fulfilled as soon as a replenishment of adequate size arrives. Examples of such situations are captive markets, exclusive dealerships, link between the wholesale and retail stages of a distribution system, etc.
2. Complete lost-sales: Unfulfilled demand due to a stock-out is lost, as the customers satisfy their demand with a substitute product or find another source for the preferred product, a common example of which, is the retail-consumer link in a distribution system.

Silver et al. (1998) also point out that in practical situations, it is likely to have a combination of these two extremes, i.e., some customers use other sources or substitute products to fulfil their demand, resulting in lost sales, while some come back at a later time to get the specific desired product, resulting in backordered demand.

Putting the SDL response categories into the context of lost-sales and backorders in inventory control, it becomes apparent that while the 'delay' response is the cause of backorders, the 'substitute' and 'leave' responses account for lost-sales. Even if a customer substitutes demand for a certain stock-keeping-unit (SKU) with another SKU, although the customer's demand is fulfilled, the original demand for the preferred SKU ceases to exist. This is also pointed out by Bijvank and Vis (2011), justifying the assumption of lost-sales for several inventory models in literature, which are placed in the retail context.

Briefly discussing the classifications of customer responses to stock-outs observed in literature, and varying likelihoods of these responses, this subsection has provided background for the lost-sales assumption in inventory models. The following subsection (3.3.2) begins to highlight the differences between inventory control under lost-sales and backordering situations.

3.3.2 Inventory position

This subsection discusses the concept of inventory position, to highlight the implications of this quantity for situations with lost-sales. For the case of probabilistic demand, Silver et al. (1998) classify inventory stocks into following four categories:

1. On-hand stock, which is defined as inventory that is physically available on the shelf to satisfy customer demand immediately.
2. Net stock = (On-hand stock) - (Backorders).
3. Inventory position (IP) = (On-hand stock (S_{OH})) + (On-order stock (S_{OO})) - (Backorders) - (Committed), where on-order stock refers to the stock that has been requisitioned but has not arrived at the inventory location under consideration. Alternatively, it is the number of items ordered as part of an outstanding order. The committed stock can be understood as confirmed component of future demand, for which stock is kept reserved; for example, an advance bulk order placed by a customer.
4. Safety stock, which refers to the component of a replenishment order that acts as a buffer against the scenario of actual demand exceeding the forecasted demand.

N.B.: The prospect of a committed stock is not pursued or addressed further in this research, considering it a rare occurrence. Thus, committed stock is assumed as zero.
For situations when backordering does not occur, one intuitively modifies the definitions of net stock and inventory position as the following:

$$\text{Net stock} = S_{OH} \tag{3.1}$$

$$IP = S_{OH} + S_{OO} \tag{3.2}$$

This definition of inventory position is also utilised by Broekmeulen and Van Donselaar (2009), describing the mathematical formulation of the policies they compare. However, additional modifications in other calculations are also required to account for lost-sales.
As briefly explained in Chapter 1, in a periodic review situation where backordering does not occur, there are essentially two instances where sales may be lost due to excess demand. They are as follows:

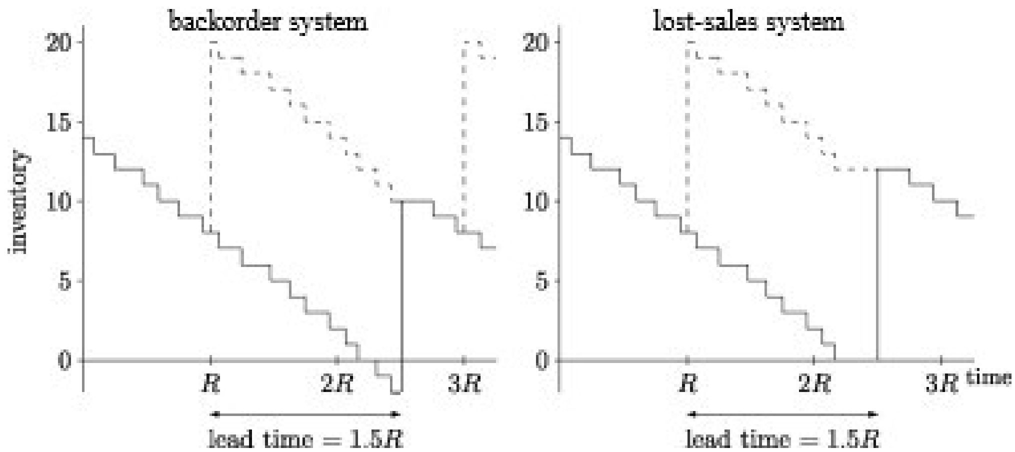

Figure 3.3: Comparison of a backorder and lost-sales model (Bijvank and Vis, 2011), where on-hand stock is represented by a solid line and inventory position by a dashed line

1. during the lead time of an outstanding order (as shown in the second depiction in Figure 3.3, which represents a lost-sales system), i.e., an order is placed at the review instance R, however the on-hand stock is exhausted before the order arrives at $R + 1.5R$. Any demand that occurs between the exhaustion of on-hand stock, and arrival of order, amounts to lost-sales.
2. when there is no order outstanding, i.e., if the on-hand stock were to be exhausted just before the instance R in Figure 3.3.

Bijvank and Vis (2011) explain "when the demand is lost instead of backordered, the inventory position does not decrease if the system is out of stock. It is no longer true that the amount of inventory after the lead time equals the inventory position after the order placement minus the demand during the lead time." This is explained through an example below.

Consider a periodic review and variable order quantity inventory system where backordering is possible. On a day t when an order is to be placed, inventory is reviewed at the start of the day, and order placed is available to fulfil demand from start of day $t+L$, where L represents the lead time. Assuming there are no orders outstanding at day t, a quantity n_t is ordered. When the ordered stock becomes becomes available to fulfil demand, inventory position (IP_{t+L}) is given by:

$$IP_{t+L} = (IP_t + n_t) - \sum_{i=t}^{t+L-1} D_i \tag{3.3}$$

where,

IP_t = inventory position just before order is placed;

$\sum_{i=t}^{t+L-1} D_i$ = demand during lead time.

However, Bijvank and Vis (2011) state that for a lost-sales inventory system,

$$IP_{t+L} \neq (IP_t + n_t) - \sum_{i=t}^{t+L-1} D_i \tag{3.4}$$

but rather, to account for lost-sales during the lead time, inventory position at $t+L$ is calculated as:

$$IP_{t+L} = (IP_t - \sum_{i=t}^{t+L-1} D_i)^+ + n_t \tag{3.5}$$

$$\text{where } (x)^+ = \begin{cases} x \text{ if } x \geq 0 \\ 0 \text{ if x<0} \end{cases}$$

(referred henceforth as positive only function)

While the calculation in equation 3.3 allows the inventory position to be negative, implying backorders; this is eliminated in eq. 3.5 by imposing a lower limit of zero on the inventory position. The difference can be understood through following expressions:

$$x - y = (x - y)^+, \text{ if } x - y \geq 0 \tag{3.6}$$

$$x - y < (x - y)^+, \text{ if } x - y < 0 \tag{3.7}$$

In a model that represents an inventory system with lost-sales, if the inventory position calculations use eq. 3.3, the model would inaccurately represent the behaviour of the inventory system as the inventory position may take negative values, thus, allowing backordering. If the model uses the inventory position variable to fulfil demand, it is important that attention is paid at incorporating the lost-sales behaviour for both types of excess demand instances. For any demand that occurs during the lead time, only the on-hand stock should be considered available to fulfil this demand, rather than the inventory position, which is the idea behind the modification in eq. 3.5.

Broekmeulen and Van Donselaar (2009) do not address this explicitly for their simulation model, however, it is assumed here that both these excess demand instances are accounted for lost sales within their simulation model.

However, in a model that represents a lost-sales system, the first excess demand instance must also be accounted for, in the ordering procedure deployed in the model. The following subsection (3.3.3) describes how the first excess demand instance is not addressed by the ordering procedure in Broekmeulen and Van Donselaar (2009)'s base policy. For clarification, 'base policy' refers to the benchmark policy, over which their improvements are demonstrated. 'Stock-based policies' are those which do not use age information of perishables, and consequently 'age-based policies' are those which also utilise age information also.

3.3.3 A stock-based (R, s, nQ) policy

This subsection discusses the base policy which is used by Broekmeulen and Van Donselaar (2009) to demonstrate the improvements achieved by using EWA policy, which uses age information. The purpose of the discussion is to highlight the modifications required in the ordering procedure, to account for lost-sales. Age information of items is not used within this policy and decisions are made only using the inventory levels. This acts as a precursor to similar modifications proposed for the EWA age-based policy in the next subsection.

The premise used by Broekmeulen and Van Donselaar (2009) is the model of a lost-sales inventory system with a variable order policy of the form (R, s, nQ), where inventory is periodically reviewed, and if the inventory position is lower than s, then n packs of size Q are ordered. Items arrive with a lead time of L days. R is the fixed review interval and Q is the fixed batch size or case pack size, as referred to by Broekmeulen and Van Donselaar (2009). The variable reorder point, s, is calculated after every R periods, based on the demand forecast for the decision horizon, i.e., $R+L$. Decision horizon is used here to refer to the farthest time instance in future which the decision is concerned with, and not the length of the time period, which is in fact only R due to lost-sales, i.e., $R+L-L$. Due to their variable nature, reorder point and order quantity are referred henceforth as s_t and n_t respectively, i.e. s and n for day t.

The events that occur in a day in their simulation, in the sequence of occurrence, are as follows: store opens and inventory decreases due to customers' demand, store closes and perished/outdated inventory is removed, remaining inventory is counted and arrived goods are stacked on shelves, and finally orders are placed. Some events, may of course, not occur on every day, for example, ordering decision will be made on alternate days, if $R = 2$. Forecast or expected demand on day t is represented by $E[D_t]$; and inventory position at the end of day t, just before an order is placed, is represented by IP_t.

Broekmeulen and Van Donselaar (2009) explain that at the end of every review period t, a decision is made which determines the state of inventory on $t+L+1$. Thus, if an order is placed on day t, it arrives at the end of day $t+L$. However, since no demand occurs on day $t+L$ after the arrival of the order, it effectively becomes available to fulfil demand on day $t+L+1$.

Their base policy is expressed mathematically as,

$$s_t = SS + \sum_{i=t+1}^{t+L+R} E[D_i] \tag{3.8}$$

if $IP_t < s_t$, then:

$$n_t = \left\lceil \frac{s_t - IP_t}{Q} \right\rceil \tag{3.9}$$

where $\lceil x \rceil$ rounds up x to the nearest integer.

To analyse the policy, two types of situations are considered, namely, when there can be no other outstanding orders when an order is placed ($R \geq L$); and when it is possible to have another outstanding order when an order is placed ($R < L$).

The case of $R \geq L$

If there are no other outstanding orders when an order is placed, the inventory position is same as the on-hand stock, i.e.,

if $R \geq L$, at any ordering instance t,

$$IP_t = S_{OH,t} \tag{3.10}$$

As pointed out previously, an order placed on day t becomes available to fulfil demand at the start of day $t+L+1$. Thus, assuming that demand during lead time is same as expected demand, and order of size n_t is placed, the expected inventory position when the current order becomes available to fulfil demand, can be calculated as:

$$E[IP_{t+L}] = \left(IP_t - \sum_{i=t+1}^{t+L} E[D_i]\right)^+ + n_t \tag{3.11}$$

where,

$\left(IP_t - \sum_{i=t+1}^{t+L} E[D_i]\right)^+$ = expected on-hand stock just before current order arrives.

Since inventory is counted at the end of everyday, IP_{t+L} is the amount that fulfils demand on day $t+L+1$.

Again, the important detail here is the positive only function which adds a lower limit of zero, to the inventory position. This further clarifies when the order quantity calculation is considered. If an order is placed at the present instance, it must be concerned with fulfilling demand from the arrival instance of this order until the arrival instance of next possible order, i.e., the instance when the lead time of the next review instance would have elapsed (since there is no possibility to acquire any inventory before that instance) (Silver et al., 1998). Thus, using the expression for expected on-hand stock from eq. 3.11, the policy can be accordingly modified as:

$$s_t = SS + \sum_{i=t+L+1}^{t+L+R} E[D_i] \tag{3.12}$$

$$\text{if } \left(IP_t - \textstyle\sum_{i=t+1}^{t+L} E[D_i]\right)^+ < s_t,$$

$$\text{then:}$$

$$n_t = \left\lceil \frac{SS + \sum_{i=t+L+1}^{t+R+L} E[D_i] - \left(IP_t - \sum_{i=t+1}^{t+L} E[D_i]\right)^+}{Q} \right\rceil \tag{3.13}$$

What differentiates the calculation of order quantity in the original form of the base policy (eq. 3.9) from the modified form (eq. 3.13) is the underlying assumption of backordering. By logically accounting for lost-sales, the order quantity is reduced. As shown below in eq. 3.14, the quantity calculated by the original policy is either equal to the modified form, or larger when it attempts to fulfil demand that will be lost by the time the order arrives.

$$\sum_{i=t+L+1}^{t+R+L} E[D_i] - IP_t + \sum_{i=t+1}^{t+L} E[D_i] \geq \sum_{i=t+L+1}^{t+R+L} E[D_i] - \left(IP_t - \sum_{i=t+1}^{t+L} E[D_i]\right)^+ \tag{3.14}$$

As can be observed, the two separate expected demand summation terms $\left(\sum E[D]\right)$ in the modified form (eq. 3.13) , are simply added in the original form (eq. 3.9), essentially imposing the backordering assumption and consequantly, ordering for demand that is lost during lead time. Thus, the modification eliminates the possibility of a negative inventory position in the order quantity calculation, as inventory position cannot take negative values under lost-sales (Bijvank and Vis, 2011).

Thus, the two policies would behave in the same way if expected demand during the lead time is less than the on-hand stock. However, the original policy would over order, when the expected demand during lead time exceeds the on-hand stock.

The case of $R < L$

When there are other outstanding orders when an order is placed, the inventory position is not equal to the on-hand stock but rather the sum of the on-hand stock and outstanding orders, i.e.,

$$\text{if } R < L \text{, at any ordering instance } t,$$

$$IP_t = S_{OH_t} + \sum_{i=t-L+1}^{t-1} n_i \tag{3.15}$$

Thus, even assuming that demand during lead time is same as expected demand, the expected inventory position when the current order becomes available to fulfil demand (start of day $t+L+1$), cannot be calculated as simply as in eq. 3.11. Instead, it requires a recursive formulation for calculating the expected inventory position day by day, finally leading to the expected inventory position at the end of day $t+L$. This is mathematically demonstrated below:

$$E[IP_{t+1}] = \left(IP_t - E[D_{t+1}]\right)^+ + n_{t-L+1} \tag{3.16}$$

$$E[IP_{t+2}] = \left(E[IP_{t+1}] - E[D_{t+2}]\right)^+ + n_{t-L+2} \tag{3.17}$$

$$E[IP_{t+3}] = \left(E[IP_{t+2}] - E[D_{t+3}]\right)^+ + n_{t-L+3} \tag{3.18}$$

$$\vdots$$

$$E[IP_{t+L}] = \left(E[IP_{t+L-1}] - E[D_{t+L}]\right)^+ + n_t \tag{3.19}$$

where,

$\left(E\left[IP_{t+L-1}\right]-E\left[D_{t+L}\right]\right)^{+}$ = expected on-hand stock just before current order arrives. Thus, the base policy can be accordingly modified as:

$$s_t = SS + \sum_{i=t+L+1}^{t+L+R} E\left[D_i\right] \tag{3.20}$$

$$\text{if } \left(E\left[IP_{t+L-1}\right]-E\left[D_{t+L}\right]\right)^{+} < s_t,$$

then:

$$n_t = \left\lceil \frac{SS+\sum_{i=t+L+1}^{t+R+L} E\left[D_i\right] - \left(E\left[IP_{t+L-1}\right]-E\left[D_{t+L}\right]\right)^{+}}{Q} \right\rceil \tag{3.21}$$

Among the various combinations of parameters considered by Broekmeulen and Van Donselaar (2009), one parameter combination where $R < L$ is ($R = 1, L = 2$). Kiil et al. (2017)'s simulation also includes one such combination, where stores can order everyday, and $L \approx 1.5$ (38 hours). Thus, further analysis of $R < L$ can be useful for such cases. However, since majority of combinations considered by Broekmeulen and Van Donselaar (2009); Kiil et al. (2017) have $R \geq L$, only brief notes on $R < L$ cases are made while discussing modifications to EWA and EWA_{SS} policies, without the corresponding recursive formulations for expected inventory positions. Cases where $R < L$ are also not tested for potential improvements in the following chapter, as it is assumed that $L = 1$.

3.3.4 The EWA and EWA_{SS} policies

This subsection extends the modifications proposed in the previous subsection, to the age-based EWA and EWA_{SS} policies for ordering of perishables which were proposed by Broekmeulen and Van Donselaar (2009) and Kiil et al. (2017), respectively.

The premise of the EWA policy was described in the previous subsection (3.3.3). The EWA policy essentially modifies the base policy by introducing an estimate of number of items, that are expected to outdate in the decision horizon, which is computed using the heuristic Broekmeulen and Van Donselaar (2009) propose. The proposed heuristic is used to compute the estimated amount of waste due to outdating by assuming that demand during the days within the decision horizon, will be equal to the forecast or expected demand; customers pick items in an assumed sequence, i.e., First-In-First-Out (FIFO)/Last-In-First-Out (LIFO) picking by customers; and assuming item ages to be known. The recursive equations that they propose are not described here, but in the next chapter (subsection 4.4.3). Broekmeulen and Van Donselaar (2009) can also be referred for this.

The estimated waste computed using the heuristic at day t, is represented as $\sum_{i=t+1}^{t+L+R-1} \hat{O}_i$, where $\hat{O}_i$ represents estimated units outdating at the end of day i; $i = t+1$ represents the next day; and $t+L+R-1$ represents the penultimate day of the decision horizon. Again, the decision horizon refers to the farthest time instance, and not the length of the time period considered. Broekmeulen and Van Donselaar (2009) describe the calculations of s_t and n_t as:

$$s_t = SS + \sum_{i=t+1}^{t+L+R} E[D_i] \tag{3.22}$$

$$\text{if } IP_t - \textstyle\sum_{i=t+1}^{t+L+R-1} \hat{O}_i < s_t, \text{ then:}$$
$$n_t = \left\lceil \frac{s_t - IP_t + \sum_{i=t+1}^{t+L+R-1} \hat{O}_i}{Q} \right\rceil \tag{3.23}$$

Similar to proposed modifications for the base (R, s, nQ) policy in cases where $R \geq L$, the ordering procedure for the EWA policy in cases where $R \geq L$, can be modified as:

$$s_t = SS + \sum_{i=t+L+1}^{t+L+R} E[D_i] \tag{3.24}$$

$$\text{if } \left(IP_t - \textstyle\sum_{i=t+1}^{t+L} E[D_i]\right)^+ - \textstyle\sum_{i=t+1}^{t+L+R-1} \hat{O}_i < s_t, \text{ then:}$$
$$n_t = \left\lceil \frac{s_t - \left(IP_t - \sum_{i=t+1}^{t+L} E[D_i]\right)^+ + \sum_{i=t+1}^{t+L+R-1} \hat{O}_i}{Q} \right\rceil \tag{3.25}$$

The difference is again, instead of current inventory position, the expected inventory position expression for $t+L$ is used, to limit it to positive values. This accordingly influences the reorder point calculation, and the 'if' condition for ordering. However, for the case of $R < L$, one would require including the $\hat{O}_i$ term in the recursive formulation similar to equations 3.16 to 3.19.

Broekmeulen and Van Donselaar (2009) present improvements of the EWA age-based (R, s, nQ) policy over the (R, s, nQ) stock-based policy as percentage improvements. Thus, it is possible that comparing the modified forms of these policies shows marginal changes from the results as reported by Broekmeulen and Van Donselaar (2009). Nonetheless, the proposed modifications

are a conceptual improvement for any further theoretical or practical utilisation of the policy.
A previously proposed modified form of the EWA policy, is the EWA_{SS} policy proposed by Kiil et al. (2017). The two propositions presented by Kiil et al. (2017) to modify the EWA policy are as follows:

1. Instead of using a constant safety stock (SS), as done by Broekmeulen and Van Donselaar (2009) while proposing the EWA heuristic, Kiil et al. (2017) propose calculating the safety stock (SS_t) at every ordering instance as ($\sigma_{R+L}k$), i.e., the product of:
   - standard deviation of forecast error during review interval and future replenishment lead time (σ_{R+L}), and
   - safety factor (k) which is determined using the target service level.
2. Instead of calculating the buffer quantity as the sum of the estimated waste ($\sum_{i=t+1}^{t+L+R-1} \hat{O}_i$) and the constant safety stock (SS), as done by Broekmeulen and Van Donselaar (2009); Kiil et al. (2017) propose calculating the buffer as the largest of the two, the dynamic safety stock (SS_t) and the estimated waste $\left(\sum_{i=t+1}^{t+L+R-1} \hat{O}_i\right)$.

The EWA_{SS} policy is also compared to the EWA policy by Kiil et al. (2017), under the assumption that "demand which cannot be satisfied is lost". Kiil et al. (2017) simulate an inventory system where lead time is 38 hours, i.e., approximately 1.5 days. The sequence of simulated events in a day, as described by Kiil et al. (2017) in the order of occurrence are: previously ordered items arrive and are added to inventory, inventory reduces due to customer demand, orders are placed, and inventory levels are recorded. While the smallest time unit considered by Kiil et al. (2017) in the simulation model is hours; approximating the lead time into days, it becomes apparent that items ordered at the end of day t become available to fulfil demand on the start of day $t+2$. This is explained by Kiil et al. (2017) as "an order placed Monday afternoon is added to the inventory Wednesday morning". This is similar to the $L = 1$ case for Broekmeulen and Van Donselaar (2009)'s simulation model.

N.B.: To avoid any misinterpretations due to different notations or time units, the policy modifications for EWA_{SS} policy as described below, are also based on the premise of Broekmeulen and Van Donselaar (2009)'s simulation model, as explained earlier. Thus, the smallest time unit is still a day, and IP_t still represents inventory position on day t after demand for the day has occurred but order is yet to be placed. Any adjustments required in the policies to fit an hourly model paradigm, should not take much effort. Also, to clarify some differences in notations; Kiil et al. (2017) refer:

- to the modified safety stock as SS, which is referred here as SS_t, i.e., safety stock as calculated on day t;
- to batch size as B, which is referred here as Q, following Broekmeulen and Van Donselaar (2009);
- to order quantity as Q_t which is referred here as n_t, again, following Broekmeulen and Van Donselaar (2009); and
- to inventory position as I_t which is referred here as IP_t, again, following Broekmeulen and Van Donselaar (2009).

Calculations of order quantity in the EWA_{SS} policy are carried out by Kiil et al. (2017) as follows:

$$\text{if } IP_t - \textstyle\sum_{i=t+1}^{t+L+R-1} \hat{O}_i < \sum_{i=t+1}^{t+L+R} E[D_i] + SS_t, \text{ then:}$$

$$\text{if } SS_t < \textstyle\sum_{i=t+1}^{t+L+R-1} \hat{O}_i, \text{ then:}$$

$$n_t = \left\lceil \frac{\sum_{i=t+1}^{t+L+R} E[D_i] + \sum_{i=t+1}^{t+L+R-1} \hat{O}_i - IP_t}{Q} \right\rceil \tag{3.26}$$

$$\text{if } SS_t \geq \textstyle\sum_{i=t+1}^{t+L+R-1} \hat{O}_i, \text{ then:}$$

$$n_t = \left\lceil \frac{\sum_{i=t+1}^{t+L+R} E[D_i] + SS_t - IP_t}{Q} \right\rceil \tag{3.27}$$

Based on the same propositions as presented previously for the stock-based (R, s, nQ) and EWA policies, the ordering procedure in the EWA_{SS} policy, for cases where $R \geq L$, can be modified as:

$$\text{if } \left(IP_t - \textstyle\sum_{i=t+1}^{t+L} E[D_i]\right)^+ - \sum_{i=t+1}^{t+L+R-1} \hat{O}_i < \sum_{i=t+L+1}^{t+L+R} E[D_i] + SS_t, \text{ then:}$$

$$\text{if } SS_t < \textstyle\sum_{i=t+1}^{t+L+R-1} \hat{O}_i, \text{ then:}$$

$$n_t = \left\lceil \frac{\sum_{i=t+L+1}^{t+L+R} E[D_i] + \sum_{i=t+1}^{t+L+R-1} \hat{O}_i - \left(IP_t - \sum_{i=t+1}^{t+L} E[D_i]\right)^+}{Q} \right\rceil \tag{3.28}$$

$$\text{if } SS_t \geq \textstyle\sum_{i=t+1}^{t+L+R-1} \hat{O}_i, \text{ then:}$$

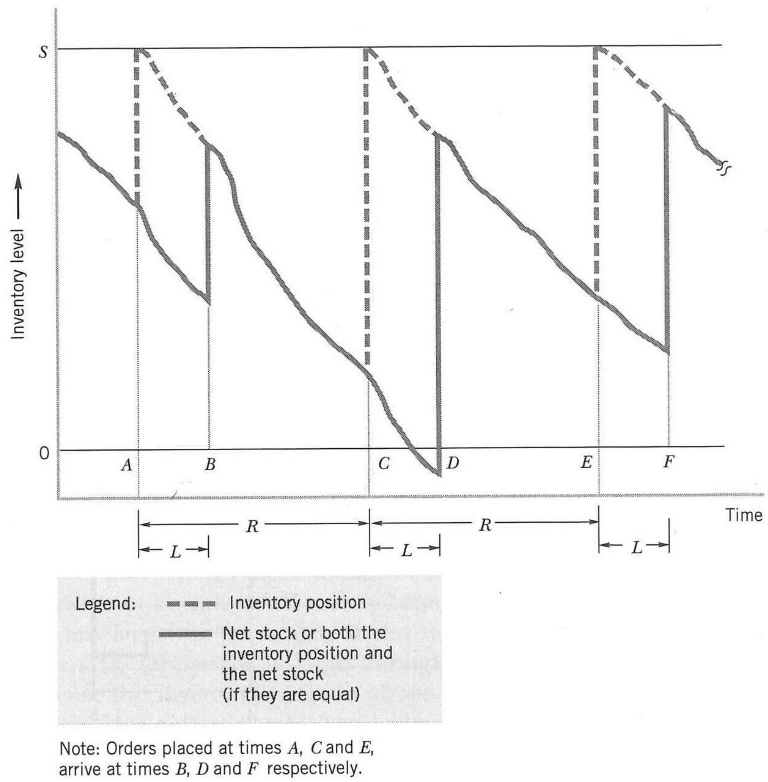

Figure 3.4: (R,S) inventory system (Silver et al., 1998)

$$n_t = \left\lceil \frac{\sum_{i=t+L+1}^{t+L+R} E[D_i] + SS_t - \left(IP_t - \sum_{i=t+1}^{t+L} E[D_i]\right)^+}{Q} \right\rceil \tag{3.29}$$

Again, the difference from the original EWA_{SS} policy can be observed in the use of expected inventory position for $t+L$. It can be observed that the structure of EWA_{SS} policy equations differs from the structure of EWA and stock-based policy equations mentioned previously. This has been done to keep the sequence of terms as similar to the original policies as possible, to facilitate comparison and to make the modifications clearly observable.

An additional modification is required on the EWA_{SS} policy for the safety stock calculation. The safety stock calculation used in the EWA_{SS} policy by Kiil et al. (2017) while citing Silver et al. (1998), is:

$$SS_t = \sigma_{R+L} k \tag{3.30}$$

however, following the same argument as made for the previous propositions, the safety stock calculation should be modified as:

$$SS_t = \sigma_{R-L_1+L_2} k \tag{3.31}$$

where L_1 refers to the lead time period of the current order, while L_2 refers to the lead time period for replenishment for the next ordering instance. This can be understood using Figure 3.4. In Figure 3.4, assuming that the letters A to F represent time duration counted from zero; when an order is placed at point C assuming lost-sales, the order quantity aims to fulfil demand between points D and F. Thus, the safety stock must be calculated for the period $((E-C)-(D-C)+(F-E))$, where the three terms represent corresponding R, L_1 and L_2 respectively. The safety stock calculation method in eq. 3.30 is again based on the underlying assumption of backordering which would be relevant for a system that behaves as shown in Figure 3.4. But when the inventory level or position cannot go below zero or demand is not backordered, eq. 3.31 corrects it for such lost-sales situations.

This section has discussed the relevance of the lost-sales assumption in inventory models for real-world retail situations; highlighted the importance of including the lost-sales assumption in calculating the inventory position; and proposed modifications to a stock-based (R, s, nQ) policy and the age-based EWA and EWA_{SS} policies for ordering of perishables, to account for lost-sales. As a result, it has contributed to fulfilling the preparatory objective of the study. The following section discusses the characteristics of a real-world inventory system, which are used as the basis for developing a simulation model, to test the effect of the proposed policy modifications, and fulfil the main objective of this research by answering the research question.

3.4 Milk inventories at grocery retail stores

As discussed in section 1.3, the structure of milk supply chain in Norway is such, that milk is delivered directly from the dairies or packaging and production facilities, to the retail stores (Stensgård and Hanssen, 2016). This is also described by Engelseth (2012) in a case study on Norway's largest dairy producer. Due to such a structure, the retail echelon in milk supply chain becomes a suitable test case for the improvements suggested in the previous section, as also explained in section 1.3. The arguments made in section 1.3, for choosing retail milk inventories as the context, are briefly summarised here.

Firstly, the responses of customers to milk stock-outs strongly support the assumption of lost-sales (Emmelhainz et al., 1991; McKinnon et al., 2007). Secondly, any improvements in the ordering policies for milk at the retailers can directly influence the production planning at the dairies, as the plans are adjusted based on orders (Engelseth, 2012), thus preventing overproduction. Additionally, due to lower number of stages of divergence in the milk supply chain as compared to other food supply chains, the bullwhip effect can be expected to be low (Dominguez et al., 2014). These factors contribute to the argument that any improvements in ordering process at retailers, can contribute to reducing waste.

As discussed in the section 3.1, perishable or deteriorating items vary in their characteristics of deterioration. Additionally, inventory control systems also vary in their characteristics, as discussed in subsection 3.2.1. Various combinations of characteristics of products and inventory systems can also be observed in literature on perishable inventory modelling (Bakker et al. (2012); Bhalla (2017); Goyal and Giri (2001); Janssen et al. (2016); Raafat (1991) may be referred for examples). Thus, for studying the behaviour of an inventory by using a model that represents it, it is important that the characteristics of the inventory are identified, so that the model can be developed.

This section discusses some characteristics of milk inventories at grocery retail stores. Literature in the domains of grocery retail, dairy supply chains, and perishable inventory modelling, is used to identify important characteristics that are relevant for modelling such an inventory. These characteristics serve as background for developing a simulation model, using which, the changes in inventory performance as a result of the modifications proposed in the previous section, can be identified.

Common distinctions found in perishable inventory models discussed in literature, are reviewed and discussed in preceding work of this research (Bhalla, 2017), and can also be observed in literature reviewed by Bakker et al. (2012); Janssen et al. (2016). These distinctions can be listed as:

- fixed lifetime vs. random lifetime,
- deterministic vs. stochastic demand,
- LIFO vs. FIFO (issuing policy, or customers' picking preference in retail context),
- lost-sales vs. backordering (customer response to stock-outs),
- periodic review vs. continuous review,
- age-based vs. stock-based policies,

- zero lead time vs. non-zero lead time, and
- service level approach vs. cost optimisation (performance objective).

The following subsections discuss the characteristics of a milk inventory, to clarify the position of this inventory on the distinction dimensions listed above. The characteristics, as discussed here, are based directly on, or on inferences drawn from, information found in empirical research in the context of dairy and grocery industries such as works of Damgaard et al. (2012); Engelseth (2012); Hübner et al. (2013); Kiil et al. (2017); and are expected to sufficiently describe the corresponding real-world situation and challenges.

3.4.1 Product lifetime

While 'raw-milk' or untreated cow milk is durable under refrigeration (below 4°C) for 14 days, the 'best-before' date of treated and packaged milk is printed as 10 days after the date of production (Engelseth, 2012). In the best case, these boxes can be assumed to be available for sale in a store at late evening on the same date, or the next morning. Thus, the maximum shelf life of boxes that are available in a store can be considered as 9 days. A recent practice adopted by Norwegian dairy producers is the 'best before, but not bad after' stamp (Cornall, 2018), which has been recently modified to 'best before, often good after'. Although it is unlikely that this enables the retailers to sell products beyond this date (due to legal reasons), it could prove effective in reducing waste of these products at the consumers' end.

3.4.2 Demand

As mentioned earlier, the 'moment of truth' when actual demand for a product becomes apparent in grocery retail is when the demand occurs (Hübner et al., 2013). Thus, forecasts based on historical sales have to be used for making inventory replenishment decisions, since customer demand is not known in advance. Additionally, seasonal and weekly variations in grocery retail demand have also been documented (Van Donselaar et al., 2006). Thus, demand can be considered as time-varying and probabilistic (explained in section 3.2).

3.4.3 Customers' picking preference

Similar to the magnitude of demand, the nature of demand, i.e., the preferred choice between older and newer items, is also uncertain and can be expected to be a matter of personal choice. Inventory models consider either one of the two (Tekin et al., 2001), or both separately (Broekmeulen and Van Donselaar, 2009), or consider a shift from LIFO to FIFO when price discounts are introduced on older items (Buisman et al., 2017), or an assumed proportion of both (Kiil et al., 2017). Nahmias (1982) points out that a FIFO withdrawal is optimal, which is an intuitive result, since FIFO withdrawal decreases the possibility of outdating. Further, Janssen et al. (2016) point out that while a FIFO depletion is relevant in health care, for inventories of blood, pharmaceuticals, etc.; a depletion with LIFO or mix of both policies is more relevant for food retail. This can be understood as a consequence of higher control that an inventory manager would have over issuing from a blood inventory as compared to a grocery store, where customers pick items from the shelves and may have a choice between older and newer items. However, store managers can decrease uncertainty regarding customers' choice by limiting their access to items with different lives. Ferguson and Ketzenberg (2006) also point this out as a common characteristic of dairy products in grocery retail, where products are loaded to the display shelf from the back, and customers only get access to the older items in front. Thus, it can be considered that majority of the withdrawal is FIFO. This argument is followed up with an assumed large percentage of FIFO picking (same as Kiil et al. (2017)), while describing the characteristics of the modelled system in section 4.1.

3.4.4 Customer responses to stock-outs

Based on the extensive discussion in subsection 3.3.1 and the findings of Emmelhainz et al. (1991); McKinnon et al. (2007), it can be considered that customers who do not find their preferred milk type or package size, are expected to choose other alternatives, rather than postponing their purchase to a later time. Thus, customers whose demand for milk is not fulfilled on their arrival at the store, can be considered as lost.

3.4.5 Review frequency

As indicated by Engelseth (2012), deliveries of milk to retail stores is carried out by the milk supplier while combining orders of different products and from multiple stores. Additionally, different retail stores, depending on their size, demand, and location, may have different permitted ordering days in a week (Kiil et al., 2017). Thus, even though the inventory levels might be known continuously from digital POS data records which are generated when barcodes are scanned for billing, it can be considered that the levels are only used when an order is to be placed, thus characterising the situation as having periodic review.

3.4.6 Level of information utilised in decision

As highlighted in subsection 3.2.3, and in the discussions on policies in subsections 3.3.3 and 3.3.4, inventory decisions for perishables can be made using only the stock levels, or also using the age information of items in stock. Although age information may not necessarily be available for most perishable grocery items (Damgaard et al., 2012), it has been established that age-based policies usually perform better (Broekmeulen and Van Donselaar, 2009; Ferguson and Ketzenberg, 2006; Tekin et al., 2001). However, the comparison within this research is not of the age-based policy vs. stock-based policy nature. Thus, even though both, stock-based and age-based policies are studied; the original form of stock-based policy is compared with its proposed modified form; and original age-based policies are compared with their respective modified forms, to investigate the effect of modifications proposed in the previous section (3.3).

3.4.7 Lead time

Delivery lead times are a realistic phenomenon in grocery retail and are included in various models that are concerned with such a setting (Broekmeulen and Van Donselaar, 2009; Buisman et al., 2017; Kiil et al., 2017; Potter and Disney, 2010). Lead times are also important for the concept of interest, i.e., lost-sales. Describing the ordering process for milk at retail stores, Engelseth (2012) mentions that products are ordered in the morning and ordered items are delivered later on the same day.

3.4.8 Performance objective

As Silver et al. (1998) highlight, performance objectives essentially govern how safety stocks are calculated in a policy. As has been pointed out on several occasions in previous sections while highlighting the challenges in implementing cost optimisation approaches, approaches that aim at satisfying target service levels are predominant in practice, as can also be observed in the data from retail stores, used by Kiil et al. (2017). Additionally, modified and original safety stock calculations described in the previous section are also based on service levels. Thus, exploring similar modifications for policies where safety stock calculations are based on optimisation approaches, can be a subject of future research. However, within this study, a service level approach is presumed.

3.5 Summary

This chapter has provided relevant theoretical background for the investigation domain, described the propositions that this research aims to test, and finally identified characteristics of the system to be modelled for testing the propositions. This section summarises the discussions from the chapter.

Types of perishable items and different deterioration characteristics were discussed, clarifying the characteristics of product relevant to this research, i.e., milk. Using insights from literature on perishable inventories, it is categorised as a product with a known usable life that requires specific storage conditions throughout the usable life, and undergoes rapid deterioration if exposed to ambient conditions for prolonged periods.

Inventory management concepts have been discussed to highlight the importance of heuristic approaches for time-varying probabilistic demand, while also discussing and clarifying the level of uncertainty in probabilistic demand. This was followed by a discussion on inventory management for perishable items, highlighting the added utility of heuristic approaches for these items.

Different customer responses to stock-outs and their implications for inventory models were discussed, to highlight the difference between backorders and lost-sales. Additionally, the predominance of lost-sales for milk, was highlighted using results from empirical research. This was followed by a discussion on difference between inventory position calculations for lost-sales and backorder systems, highlighting the premise of the propositions presented and tested in this research.

Policy modifications required to account for lost-sales, in policies which are based on the underlying assumption of backorders, were demonstrated, while focussing on the case where there are not outstanding orders when an order is placed, i.e., $R \geq L$.

Finally, in order to test the effects of the propositions presented, characteristics of a milk inventory have been identified, which serve as the basis for simulation modelling in the subsequent chapter.

Chapter 4

Modelling and simulation

The previous chapter proposed modifications to order procedures of three variable order quantity policies, to account for lost-sales in the ordering procedures. However, since these modifications are essentially expected to reduce the ordered quantities, it is important that their influence on inventory performance measures such as fill rates, waste, average inventory levels and number of deliveries; is measured, to assess the value of the proposed changes. This activity is the subject of this chapter, including the development and description of the simulation model which is used to conduct this activity. The policy performance measures listed here are same as those used by, and highlighted as frequently used and recommended, by Kiil et al. (2017), citing Broekmeulen and Van Donselaar (2009); Hübner et al. (2013); Kaipia et al. (2013); Van Der Vorst (2006). Broekmeulen and Van Donselaar (2009) also use costs to demonstrate improvements, however, to avoid any biases in estimating costs, other measures that essentially lead to incurring costs, are used instead.

While a model refers to the abstraction of a real system (Will M. Bertrand and Fransoo, 2002), simulation refers to imitating the operation of such a system (Jerry, 2005). Simulation models provide a risk free environment for conducting tests where the performance of a system is to be tested under different operating conditions (Croom, 2010). For the context of this research, the system of interest is an inventory, while the different operating conditions refer to different inventory policies, and more specifically, the ordering procedures of these policies. The characteristics of the situation are governed by parameters such as the product shelf life, demand, batch size, customer picking preferences, etc. However, to assess the effect of policy modifications, other parameters are kept constant in a comparison. The importance of simulation to study comparative performance of policies is also highlighted by Pidd (2006).

This chapter describes the process of setting up a model that represents an inventory system; and the mathematical operations used to simulate processes such as demand, ordering, etc. The model development is followed by experimentation using this model. The characteristics of the system to be modelled are described in section 4.1, where the characteristics are listed and then organised in a conceptual model which represents the intended behaviour of the simulation model. This is followed by description of the characteristics of the simulation model in section 4.2, and discussion on the choice of modelling approach. Section 4.3 explains notations and assumptions for the model; and section 4.4 describes the model, explaining different model elements, associated modelling approaches, and the verification and validation processes. Section 4.5 describes the simulation experiments and presents results, while section 4.6 further refines the results to facilitate the interpretation of the results.

4.1 System characteristics and conceptual model

This section describes the characteristics of an inventory, which serve as basis for modelling and simulations. The characteristics are hypothesised based on the characteristics of a grocery retail milk inventory, as discussed in section 3.4. This is done to provide context for the assumptions made while modelling in the subsequent sections.

The system under consideration is an inventory of a milk SKU at a Norwegian retail grocery store, and the operational decision variable of interest is the ordering policy, which is used to compute an order quantity at every ordering occasion. The characteristics of the inventory are as follows:

- Demand is probabilistic, meaning that there is uncertainty regarding its actual realisation; and discrete, as number of milk boxes can only take integral values. The ordering decisions are based on a day level forecast, i.e, it provides an expected demand value for each day.
- Inventory review is periodic, and replenishment orders can only be placed at the specified periodic intervals and these are the instances when the decision on ordering quantities

has to be made. These ordering instances occur on the start of an ordering day, and the demand for that day occurs after the order has been placed.

- Replenishment lead time is assumed as deterministic and known to be 1 day. This implies that orders that are placed at the start of an ordering day, become available to fulfil demand at the start of the next day. However, the goods may arrive at the end of the same day, as Engelseth (2012) describes.
- The inventory is shared between two locations within the store boundary, namely, a display shelf, and a backroom storage. The backroom storage is utilised to ensure that older boxes are exhausted first, thus, ensuring a FIFO sequence. The appropriateness of FIFO picking for dairy products is also pointed out by Ferguson and Ketzenberg (2006). However, to account for practical errors and discrepancies, such as situations when store clerk has a specific schedule of activities, and old boxes on the display shelf have not been exhausted when he/she loads the display shelf; 90% of the demand withdrawal is considered as FIFO, while the remaining 10% of the customers use the opportunity to reach behind the older boxes and take the newer boxes instead. Basis for this assumption was the approach taken by Kiil et al. (2017), who also use these percentages to model customer picking in grocery retail.
- The inventory is perishable, meaning that it has a maximum usable shelf life. A best case scenario is considered where all milk boxes that reach the store have the maximum possible shelf life remaining. This shelf life is considered as 9 days which is one day less than the remaining shelf life (to account for lead time) when the boxes are packed, i.e., 10 days (Engelseth, 2012). The milk boxes are removed from the shelf on the night before the 'best before' date, meaning that a box will be available to fulfil demand before its printed 'best before' date, but not on that date. The boxes removed from inventory at the end of their shelf life are counted as waste. In real situations, it is possible that products close to the end of their shelf life are sold for discounted prices to avoid expiration before selling, and extract some salvage value from these items, as also described by Herstad (2016). However, discounting is not considered in the model, to limit the scope of investigation.
- Orders can only be placed in multiples of 10, due to the batch size set by the supplier (Engelseth, 2012).

- The demand that occurs during a stock-out is considered lost due to the characteristics of the product under consideration, i.e., considering milk a product which is used generally rather than on special occasions, and has a high urgency of requirement (as discussed in subsection 3.3.1).

The ordering policy used is of the form (R, s, nQ), i.e., ordering can be done after every R (constant) days, and if the inventory position is lower than s (which is calculated at every review instance), the quantity nQ is ordered, where Q is constant and n is calculated if the order is to be placed. Thus, the sequence of events in a day would be as follows:

1. If an ordering day, order quantity is calculated and ordered.
2. Demand occurs and inventory reduces.
3. Items with 'best before' date same as the next day, are removed and counted as waste.
4. Arrived products are added to the inventory for fulfilling next day's demand.

Considering the inventory described above, as a system that is to be modelled for testing the performance of this inventory while operating under different policies, the requirement can be conceptually modelled as shown in Figure 4.1. The conceptual model is developed using the conceptual modelling framework presented by Robinson (2008). As can be observed, the framework provides a systematic visualisation tool for the causal input-output relation between experimental factors and responses. Additionally, it provides a simplified yet structured overview of the model content and modelling objectives. The importance of the model content, as briefly presented in the conceptual model, becomes clearer as the conceptual model is translated to the simulation model in the subsequent sections.

4.2 Characteristics of the simulation model

According to Pidd (2006), deciding on principal elements of a simulation model is a prerequisite for producing the model, as it allows the modeller or analyst to decide on the level of accuracy and detail required from the simulation; and should be based on:

- the nature of the system being simulated, as the model needs to be a good representation of the system, and

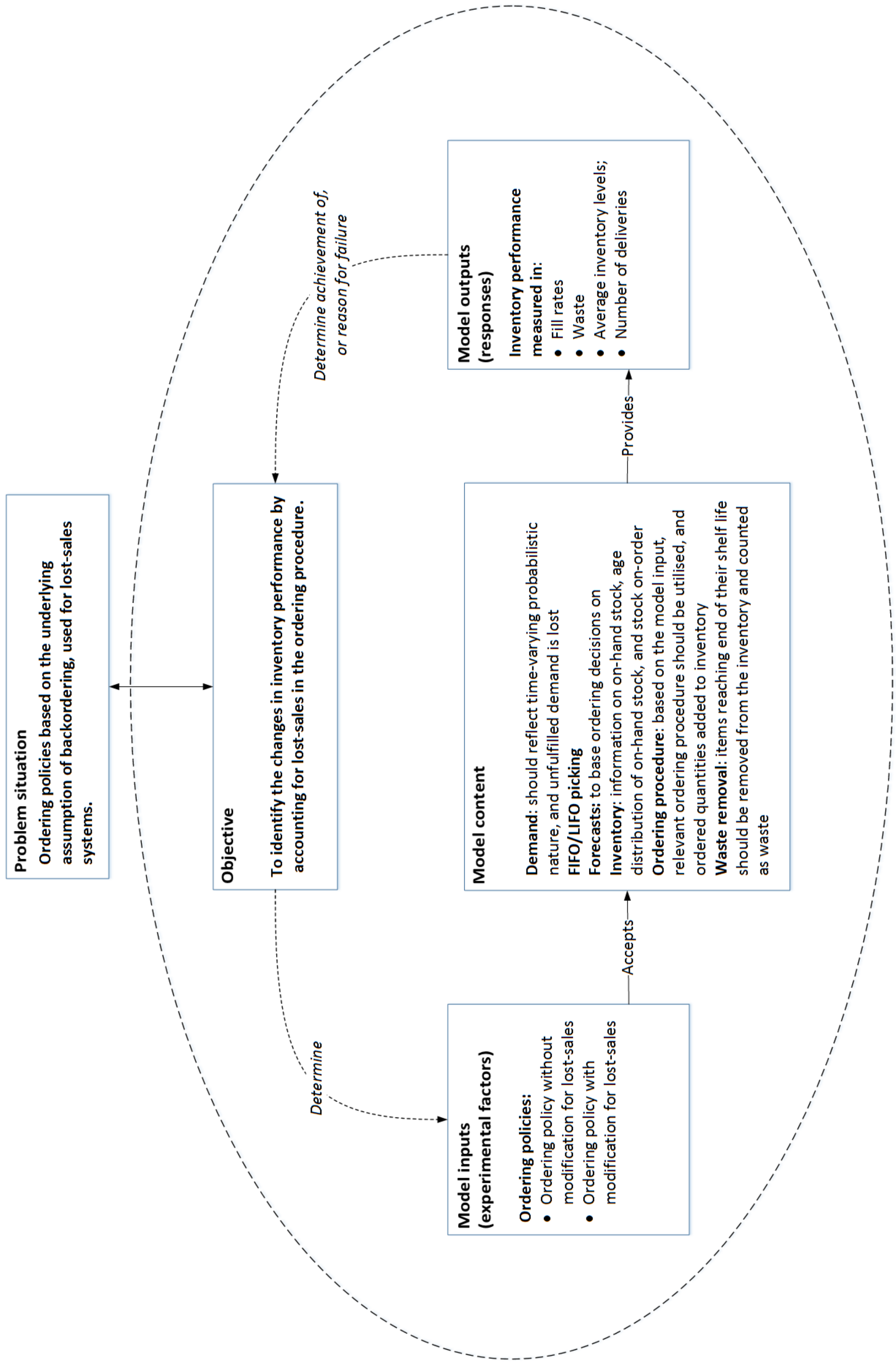

Figure 4.1: Conceptual model for simulating the milk inventory in grocery retail store, for testing different policies (developed using conceptual model development framework by Robinson (2008))

- nature of study, i.e., the objectives, and expected results.

Based on these considerations, the decisions regarding the following aspects of the model should be made (Pidd, 2006):

- time handling: time slicing or next event,
- stochastic or deterministic occurrences and duration, and
- discrete or continuous change.

This section discusses the characteristics of the simulation model used to conduct the simulation experiments, and presents arguments to justify the use of the chosen modelling approach. The model is a time-slicing stochastic discrete simulation model, based on the terminology of simulation modelling approaches explained by Pidd (2006). Each of these characteristics of the model are explained below.

'Time-slicing' refers to the time handling approach used in the model. Time handling refers to the way time-flow is handled in a simulation model (Pidd, 2006). Time-slicing implies that time increments within the model are uniform, unlike the next-event technique, where time increments may occur in varying magnitudes (Pidd, 2006). As Pidd (2006) explains, the next-event approach skips the time-slices when the system state does not change, and jumps to the next event, where the 'next event' refers to the time instance when a system state change occurs. However, for the system of interest, the time slices used are one day, and all events occurring in a day, always occur in the same sequence. Thus, there is no randomness regarding the timing of events. The advantage of the next-event technique could be utilised if shorter time slices, such as hours were used (Kiil et al., 2017). This would, for example, require several reductions in the inventory to fulfil demand. However, since none of the events in a day coincide with each other, it is sufficient to use one mathematical operation for every event transaction, i.e., reduce the inventory with the day's demand, increase inventory with ordered quantity after lead time has elapsed, etc.

'Stochastic' implies that the model includes stochastic elements, and probability distributions are used to model these elements (Pidd, 2006). As described above, the model does not have any stochastic 'duration', meaning that timings of events are deterministic. However, the magnitude of occurrence of one of the events, i.e., demand, is stochastic. As is explained later, demands are generated from a set of probability distributions, using random sampling. Due to this, any two simulation runs can be expected to provide different results. However, using several runs can provide a considerable estimate of the performance, and thus, such models should be run multiple times before any conclusions are drawn from the results they produce (Pidd, 2006).

'Discrete' implies that functional variables within the model undergo discrete changes as opposed to continuous ones (Pidd, 2006). Since the model represents a milk inventory, all increments and reductions in the inventory are integral, thus, qualifying it as discrete.

The objective of the model is to study the effect of modifying ordering procedure in a policy, to account for lost-sales. The effect of these modifications is studied using the performance indicators of waste, fill rates, average inventory levels, and number of deliveries, as done by Kiil et al. (2017). Broekmeulen and Van Donselaar (2009) use costs as the performance indicator. However, as discussed in Chapter 3, service level approaches to inventory control are used instead of cost or profit optimisation approaches because of the difficulties in estimating costs. Thus, to avoid any biases in estimating these costs, the phenomenon which lead to the costs being incurred, are used as performance indicators.

The use of simulation for this purpose is justified by the stochastic nature of demand in the model. Demands are generated by random sampling from probability distributions, and the parameters for these distributions are defined beforehand. Although once the demand has been generated, the calculations are essentially deterministic. However, as Pidd (2006) points out, simulation modelling is useful even for deterministic computations, if the extent of computations is large. Thus, if the performance of the ordering procedures is to be studied over a period of one year, the required calculations can be carried out manually, once random demand samples are generated. However, simulation simplifies this process. The random sampling technique is not programmed explicitly, and in-built functionality of the programming environment (MATLAB) is used to generate random samples from distributions. An agent based simulation of such nature could also be implemented on an Excel spreadsheet, as demonstrated for several examples by Pidd (2006). However, due to a larger in-built mathematical functionality and intuitive syntax of MATLAB, it was chosen as the platform for modelling.

4.3 Model notations and assumptions

This section describes the notations used and assumptions made, in modelling the inventory system. Some of the variables are known model parameters, but variable notations are used to denote them, in order to explain the functional equations in the following section.

- t denotes the day number, starting from 1 to the maximum simulation length.
- The maximum shelf life of the product is represented by m, and is the remaining shelf life of an item when it becomes available to fulfil demand.
- The inventory position or level at the start of day t, is denoted by IP_t. The 'start of the day' implies the time of day when orders have not been placed yet and no demand has occurred yet. Since the tested combinations of parameters is such that $R \geq L$, the inventory position would only represent the on-hand stock at this point of time, since any orders placed on previous day would have already arrived and if an order is placed on the present day, it would be placed after this instant.
- The on-hand stock comprises of items with different ages. Items with remaining shelf life of r days on day t form a batch, which is represented by B_{tr}, meaning that B_{tr} is the number of items on-hand which have remaining shelf life of r days on day t. Batches with different remaining shelf lives put together in a vector $\boldsymbol{B}_t$, represent the state of inventory, i.e., $\boldsymbol{B}_t = (B_{tm}, B_{tm-1}, B_{tm-2}, ..., B_{t1})$, where

$$IP_t = \sum_{r=1}^{m} B_{tr} \tag{4.1}$$

  Thus, while inventory position is a scalar quantity, state of inventory at any day is a vector with m elements. The boldface notation for vectors follows the convention, as done by Nahmias (2011).

- The forecasted or expected demand in period t is denoted by $E[D_t]$.
- The actual demand in period t is denoted by D_t.
- The satisfied or fulfilled demand in period t is represented by FD_t, and is calculated as:

$$FD_t = min(D_t, IP_t) \tag{4.2}$$

$$\text{where } min(x, y) = \begin{cases} x \text{ if } x \leq y \\ y \text{ if } y < x \end{cases}$$

- The unmet or lost demand is denoted by LD_t and is calculated as:

$$LD_t = max(0, D_t - IP_t) \tag{4.3}$$

$$\text{where } max(x, y) = \begin{cases} x \text{ if } x \geq y \\ y \text{ if } y > x \end{cases}$$

  Such a measurement is possible in the model since the actual demand D_t is known, however, in real situations, if $D_t > IP_t$, the excess demand $(D_t - IP_t)$ is censored, meaning that it is not known (Sachs, 2015b). Also, note that IP_t denotes the inventory position at the start of day t, thus, even if there are outstanding orders when demand occurs, IP_t would only represent the on-hand stock available to fulfil demand.

- The delivery lead time is considered deterministic and constant, and is denoted by L, while R denotes the review interval.

- σ_i denotes the standard deviation of forecast error for day i, while k represents the safety factor which is calculated as the inverse cumulative distribution function (CDF) $(F_x^{-1}(X))$ of the target service level on a standard normal distribution $(\mu = 0, \sigma = 1)$, i.e., if target service level is 98%, $k = F_x^{-1}(0.98) = 2.05$.

- As described in subsections 3.3.3 and 3.3.4, the stock-based policy and the EWA policy were compared under the assumption of a constant safety stock by Broekmeulen and Van Donselaar (2009). Since the constant safety stock is calculated by optimising costs, this approach is not pursued here. Rather, similar to the EWA_{SS} policies, the other two policies are also assumed to have dynamic safety stocks. The safety stock in the original forms of the policies is calculated as $\sigma_{R+L}k$ and the safety stock in the modified forms of the policies is calculated as $\sigma_{R-L_1+L_2}$, as explained in subsection 3.3.4 for the EWA_{SS} policy.

4.4 Model description

This section describes the development of the simulation model. First, the process of modelling demand and forecasts is explained in subsection 4.4.1, which is followed by the explanation of calculation of forecast errors in the model, in subsection 4.4.2. Subsection 4.4.3 describes the implementation of the EWA heuristic for waste estimation. The functional equations which are used in the model to simulate various processes/events are described in subsection 4.4.4. Subsection 4.4.5 describes verification and validation activities for the model.

4.4.1 Demand and forecast

The Poisson distribution is usually an appropriate representation of the distribution of time duration between consecutive customer arrivals at a retail store (Haight, 1967). However, it is also often utilised to represent the distribution of discrete demand (Alizadeh et al., 2014; Buisman et al., 2017; Duan and Liao, 2013; Mahmoodi et al., 2016; Tekin et al., 2001). Thus, following this approach, demand is modelled as a Poisson process.

To specify the λ parameter for the Poisson process, which represents the expected value of the corresponding Poisson distribution, demand data from a Norwegian retailer, used by Kiil et al. (2017), is utilised. The data (Table 4.1) represents 21 different store variations, which have different mean weekly sales, target service levels, permitted ordering days, and number of stores. The permitted ordering days listed by Kiil et al. (2017), have been used to assign approximate values of R to the different store variations.

For a store variety, the mean weekly sales is assumed to represent the mean weekly demand, and used to calculate the mean demand for seven weekdays, assuming the daily demand fractions as $[0.12, 0.13, 0.13, 0.16, 0.18, 0.18, 0.10]$ (taken from Kahn and Schmittlein (1989)). For example, if weekly mean demand is 100, the mean demands for days Monday to Sunday are given as [12 13 13 16 18 18 10]. Thus, an underlying assumption is that the stores are open on all seven weekdays.

Although Kiil et al. (2017) consider perishable products with shelf lives ranging from 4 to 20 days, it is not explicitly mentioned that the data represents characteristics for dairy products. However, it is expected that the wide spectrum of characteristics (sales, service levels, and review

Table 4.1: Characteristics of store variations modelled (Kiil et al., 2017)

S.no.	Mean weekly sales(units)	Planned service level	Review interval	Number of stores
1	5	96%	3	12
2	8	96%		23
3	13	96%		25
4	16	97%		11
5	18	96%		17
6	25	97%		11
7	26	96%		11
8	30	97.5%	2	3
9	34	97%		22
10	37	96%		10
11	46	97.5%		9
12	49	97%		17
13	62	97%		8
14	62	97.5%		12
15	74	98%	1	4
16	86	97.5%		14
17	108	98%		6
18	128	97.5%		9
19	172	98%		2
20	222	97.5%		3
21	696	97.5%		3

interval) provides considerable variations of testing environments for comparing the policies. The daily mean demand values are used as the expected value (λ parameter) for creating seven Poisson distributions, one for each weekday. Random numbers generated cyclically from these distributions are used as forecasts and actual demand for the model. Thus, forecasts and actual demands for days 1, 8, 15, etc. will be generated from the distribution which represents demand for Mondays. Similarly, forecasts and actual demands for days 2, 9, 16, etc. will be generated from the distribution for Tuesdays, and so on.

It should be pointed out that using random sampling to generate forecasts is adopted as a scope limiting measure in the modelling process. Ideally, a forecasting model should be included in the simulation, as done by Kiil et al. (2017). The effect of censored demand on forecasting accuracy is of high importance, and only the demand that is fulfilled is used for forecasting (Sachs, 2015a). Using the same forecasts for two simulations with different policies, would imply that irrespective of the demand that the policy fulfils, the forecasts are not affected by the censored demand, thus, representing reality inaccurately. To account for this, forecast error calculations are used in the model, which are described in the next subsection. Due to the forecasts being generated by random sampling, the model essentially represents a situation where forecasts for a year have been generated at the start of the year, and are not adjusted. Additionally, seasonal variations that occur during a year, also get neglected in this model, unlike Kiil et al. (2017)'s model, where actual POS data is utilised to simulate demand, and can be expected to include seasonal variations. Thus, testing the proposed policy modifications on such realistic models, should be considered in the future.

Pidd (2006) points out that "if a simulation is being used to compare various ways of operating a system, it is clearly important to ensure that each policy is examined under the same conditions". Pidd (2006) further points out that these test conditions are partially determined by the random samples used to model the stochastic processes in the model, and to ensure fair comparisons, it may be important that the same random numbers are used for every alternative tested.

As explained by Bell et al., random number generation on computer applications uses a seed which is a number that, by default, is based on the device time, and changes continuously. Thus, it is also possible to control the random number generation by specifying a seed, which would mean that the same number will be generated every time the program is run, until the seed is changed. This highlights the characteristic of such numbers being pseudo-random rather than random as also pointed out by Pidd (2006). It was also observed that often, the random number generator function returned the same numbers, if two numbers were generated from the same distribution in consecutive lines of code, meaning that the two lines of code were executed before the seed could change.

Using a specified seeds to generate the same random samples repeatedly is one of the two approaches suggested by Pidd (2006) for being able to reuse the samples. The second approach is to generate a stream of random numbers beforehand, and treat them as random. For the current context, it can be interpreted as generating the simulated demand beforehand, but not basing decisions on it, but rather on the forecasts only. This approach is implemented in the model using two separate loops to generate forecasts and actual demand, to ensure that different seeds are used for both. These loops are run from 1 to the maximum number of days for which demand is to be simulated, and random numbers are generated from the seven distributions cyclically, with one loop generating the forecasts, and other generating the actual demand.

4.4.2 Forecasting error

The importance of forecast errors within the model lies in the safety stock calculation. Although any forecasting technique is not explicitly modelled, it is expected that the generated forecasts and demand provide a sufficiently valid test environment, since they are generated from the same distributions. And regularly updating forecasts errors is expected to reflect the effect of censored demand due to lost-sales, in the ordering decision, by influencing safety stock calculation.

The usual safety stock calculation based on target service level is done as $k\sigma_{R+L}$, where k is calculated as the inverse CDF of the service level on a standard normal distribution ($\mu = 0, \sigma = 1$), and σ_{R+L} is the standard deviation of the forecast errors (Silver et al., 1998). However, as mentioned previously, in the modified policy forms, this calculation is done as $\sigma_{R-L_1+L_2}$, where L_1 represents lead time of present order and L_2 represents the lead time of the next possible future order, since demand during L_1 would be lost by the time the order arrives.

Since the forecast and demands are generated from a weekly repeating pattern of distributions, the forecast errors are also updated at the same frequency. This implies that demand on a Monday only influences the expected forecast error for the forthcoming Monday, but not other days, and so on. On the first day of every week, the forecast error for all seven days is corrected for the previous week's demand. Although the lost demand and fulfilled demand are known in the model, only the fulfilled demand value is used for correcting the forecast errors. This is done to reflect the influence of censored demand, that occurs in real situations (Sachs, 2015b).

The measure of forecast error used in the model is the Mean Squared Error (MSE), which is calculated as:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(x_t - \hat{x}_{t-1,t})^2 \tag{4.4}$$

where x_t represents the actual demand in period t, while $\hat{x}_{t-1,t}$ represents the one period ahead forecast for period t, i.e., forecast for period t made at $t-1$. The use of MSE is considered appropriate here because it provides a reasonable estimate of the standard deviation of forecast errors for calculating safety stock (Silver et al., 1998), through the relation:

$$\sigma_1 = \sqrt{MSE} \tag{4.5}$$

where σ_1 gives the standard deviation for forecast error, and the subscript one denotes that it is the standard deviation for forecast error in demand for one time period.

Thus, at any simulation time, seven values of σ_1 are maintained, one for each weekday, and are updated on the first day of every week, correcting them for the demand that has occurred in the previous week. Further, for calculating safety stock, it is required that the corresponding σ_1 values are used to calculate σ_{R+L} or $\sigma_{R-L_1+L_2}$. This is computed in the model on day t as:

$$\sigma_{R+L} = \sqrt{\sum_{i=0}^{R+L-1}\sigma^2_{1(t+i,d)}} \tag{4.6}$$

or

$$\sigma_{R-L_1+L_2} = \sqrt{\sum_{i=L}^{R+L-1}\sigma^2_{1(t+i,d)}} \tag{4.7}$$

where $d \in 1:7$ represents the day number in a week, and is calculated as:

$$d = rem(t,7), \text{ i.e., remainder from the division } t \div 7 \tag{4.8}$$

As can be seen in eq. 4.6 and 4.7, the safety stock is calculated until the day before the next order will be expected to arrive. The initial forecast error is set to zero which is not updated until the second week, and thus, no safety stock would be ordered in the first week. However, since the simulation allows for the first 94 days (12 weeks) as stabilisation duration, performance for the initial 94 days is not recorded. Additionally, the initial inventory position is also randomised with a maximum value of 50, which is then distributed among different age batches randomly. So, the initial inventory status is also not expected to influence the measured performances.

4.4.3 The EWA heuristic

The order calculation procedure for the different policies were discussed in subsections 3.3.3 and 3.3.4, without describing the EWA heuristic which is used in the age-based (EWA and EWA_{SS}) policies to calculate $\sum_{i=t+1}^{t+L+R-1} \hat{O}_i$. This computation is described here, explaining how the recursive formulation described by Broekmeulen and Van Donselaar (2009) is implemented in the simulation model to calculate $\sum_{i=t}^{t+L+R-1} \hat{O}_i$. The estimated waste of present day is also included here, since the ordering is done at the start of the day, unlike Broekmeulen and Van Donselaar (2009) and Kiil et al. (2017)'s models, where ordering is done at the end of day. Additionally, Broekmeulen and Van Donselaar (2009) list the recursive expressions for the heuristic assuming either FIFO or LIFO withdrawal. However, the assumption of 90% FIFO is also included in the EWA computations demonstrated below.

On day t, the waste estimation is carried out assuming $FIFO = 0.9$, using the following steps:

1. Current age distribution is copied to another variable which represents the estimated batch.

$$\boldsymbol{EB_t} = \boldsymbol{B_t} \tag{4.9}$$

2. From $i = 1$ to $i = R + L$ (for $i \in int$, i.e., integers), steps (a) to (g) are repeated:

(a) Amounts withdrawn due to demand in FIFO and LIFO sequences are calculated:

$$FE = \left\lceil FIFO \cdot E\left[D_{t+i-1}\right]\right\rceil \tag{4.10}$$

$$LE = E\left[D_{t+i-1}\right] - FE \tag{4.11}$$

(b) A temporary vector is created:

$$\boldsymbol{EB_{int}} = \boldsymbol{EB_{t+i-1}} \tag{4.12}$$

(c) From $r = 1$ to $r = m$ (for $r \in int$), repeat (eq. 4.13 to 4.14):

$$EB_{int(1,r)} = max\left(0, EB_{int(1,r)} - FE\right) \tag{4.13}$$

$$FE = max\left(0, FE - EB_{t+i-1,r}\right) \tag{4.14}$$

(d) From $r = m$ to $r = 1$ (for $r \in int$), repeat (eq. 4.15 to 4.16):

$$EB_{int(1,r)} = max\left(0, EB_{int(1,r)} - LE\right) \tag{4.15}$$

$$LE = max\left(0, LE - EB_{t+i-1,r}\right) \tag{4.16}$$

(e) From $r = 1$ to $r = m-1$ (for $r \in int$), repeat (equation 4.17):

$$EB_{t+i,r} = EB_{int(1,r+1)} \tag{4.17}$$

(f) Estimated waste for day $t+i-1$ is calculated:

$$\hat{O}_{t+i-1} = EB_{int(1,1)} \tag{4.18}$$

3. Finally, the estimated waste is calculated as the sum of all $\hat{O}_{t+i-1}$ terms, i.e., $\sum_{i=1}^{L+R} \hat{O}_{t+i-1}$.

As discussed later, a similar logic is applied to simulate inventory reduction due to customer demand. The following subsection describes functional equation which are used for simulating different processes.

4.4.4 Functional equations

To simulate various events or processes that happen in a day, the model requires functional variables to represent real-world entities or phenomenon, such as the inventory position, demand, age distribution, etc.; and functional equations that represent the mathematical operations applied to these variables to simulate relevant real-world processes. This subsection describes the latter.

When $t > 7$, a week of demand would have occurred, and forecast errors can be updated. However, forecast errors are to be updated only once a week. Thus, the forecast errors are updated as:

$$\text{if } t > 7, \text{ AND } rem(t,7) = 1$$

$$\text{from } i = 1 \text{ to } i = 7 \text{ (for } i \in int\text{), repeat steps 1 to 3:}$$

1. Update sum of squared errors (SSE):

$$SSE_i = SSE_i + \left(FD_{t+i-8} - E\left[D_{t+i-8}\right]\right)^2 \tag{4.19}$$

2. Update mean squared errors:

$$MSE_i = \frac{SSE_i}{\left((t-1)/7\right)} \tag{4.20}$$

3. Update standard deviation of forecast errors:

$$\sigma_i = \sqrt{MSE_i} \tag{4.21}$$

It should be clarified that the subscripts for σ ranging from 1 to 7 reflect the days in a week, and do not refer to the number of time periods the error is for, as in eq. 4.5.

Further, if it is an ordering day, relevant ordering procedure is applied to calculate the order quantity. Thus, for any experiment, the three sets of policies that are tested are (also summarised in Table 4.2):

1. Original stock-based (R, s, nQ) policy with dynamic safety stock calculation; and modified stock-based (R, s, nQ) policy with modified dynamic safety stock calculation.

Table 4.2: Policy comparisons

Base policy	**Modified policy**
SB_{base} $s_t = SS_t + \sum_{i=t}^{t+L+R-1} E[D_i]$ if $IP_t < s_t$, then: $n_t = \left\lceil \frac{s_t - IP_t}{Q} \right\rceil$	SB_{mod} $s_t = SS_{tmod} + \sum_{i=t+L}^{t+L+R-1} E[D_i]$ if $\left(IP_t - \sum_{i=t}^{t+L-1} E[D_i]\right)^+ < s_t$, then: $n_t = \left\lceil \frac{s_t - \left(IP_t - \sum_{i=t}^{t+L-1} E[D_i]\right)^+}{Q} \right\rceil$
EWA_{base} $s_t = SS_t + \sum_{i=t}^{t+L+R-1} E[D_i]$ if $IP_t - \sum_{i=t}^{t+L+R-1} \hat{O}_i < s_t$, then: $n_t = \left\lceil \frac{s_t - IP_t + \sum_{i=t}^{t+L+R-1} \hat{O}_i}{Q} \right\rceil$	EWA_{mod} $s_t = SS_{tmod} + \sum_{i=t+L}^{t+L+R-1} E[D_i]$ if $\left(IP_t - \sum_{i=t}^{t+L-1} E[D_i]\right)^+ - \sum_{i=t}^{t+L+R-1} \hat{O}_i < s_t$, then: $n_t = \left\lceil \frac{s_t - \left(IP_t - \sum_{i=t}^{t+L-1} E[D_i]\right)^+ + \sum_{i=t}^{t+L+R-1} \hat{O}_i}{Q} \right\rceil$
EWA_{SS} $s_t = SS_t + \sum_{i=t}^{t+L+R-1} E[D_i]$ if $IP_t - \sum_{i=t}^{t+L+R-1} \hat{O}_i < s_t$, then: $n_t = \left\lceil \frac{\sum_{i=t}^{t+L+R-1} E[D_i] - IP_t + max\left(SS_t, \sum_{i=t}^{t+L+R-1} \hat{O}_i\right)}{Q} \right\rceil$	EWA_{SSmod} $s_t = SS_{tmod} + \sum_{i=t+L}^{t+L+R-1} E[D_i]$ if $\left(IP_t - \sum_{i=t}^{t+L-1} E[D_i]\right)^+ - \sum_{i=t}^{t+L+R-1} \hat{O}_i < s_t$, then: $n_t = \left\lceil \frac{s_t - \left(IP_t - \sum_{i=t}^{t+L-1} E[D_i]\right)^+ + max\left(SS_{tmod}, \sum_{i=t}^{t+L+R-1} \hat{O}_i\right)}{Q} \right\rceil$
where, $SS_t = \sqrt{\sum_{i=t}^{t+R+L-1} \sigma_i^2} \cdot k$	where, $SS_{tmod} = \sqrt{\sum_{i=t+L}^{t+R+L-1} \sigma_i^2} \cdot k$

2. Original EWA (R, s, nQ) policy with dynamic safety stock calculation; and modified EWA (R, s, nQ) policy with modified dynamic safety stock calculation.
3. Original EWA_{SS} (R, s, nQ) policy with original safety stock calculation; and modified EWA_{SS} (R, s, nQ) policy with modified safety stock calculation.

Thus, the safety stock calculation procedure for all original policies is essentially same as that of the original EWA_{SS} policy. This is done to avoid any biases in calculating the constant safety stock used by Broekmeulen and Van Donselaar (2009), since they state that the safety stock is optimised to minimise simulated costs but the process is not described explicitly.

The items ordered on day t are added to $B_{t+L,m}$, which is currently zero, i.e.,

$$B_{t+L,m} = n_t Q \tag{4.22}$$

This is followed by the reduction in on-hand stock due to demand which occurs with 90% of FIFO picking. After the demand has been fulfilled, items reaching the end of their shelf lives are removed from the shelf, counting them as waste. These processes are simulated with the following steps, where the remaining items which do not expire, are moved to the next day's inventory in the final step:

1. The demand that will be lost and demand that can be fulfilled are calculated using following eq. (4.23 and 4.24):

$$LD_t = max\big(0, D_t - IP_t\big) \tag{4.23}$$

$$FD_t = D_t - LD_t \tag{4.24}$$

2. Amounts to be withdrawn in FIFO and LIFO sequences are calculated using $FIFO = 0.9$ in the following eq. (4.25 and 4.26):

$$FW = \left\lceil FIFO \cdot FD_t \right\rceil \tag{4.25}$$

$$LW = FD_t - FW \tag{4.26}$$

3. A temporary vector is created to mirror the current age distribution:

$$\boldsymbol{B_{int}} = \boldsymbol{B_t} \tag{4.27}$$

4. From $r = 1$ to $r = m$ (for $r \in int$), repeat (eq. 4.28 to 4.29):

$$B_{int(1,r)} = max\big(0, B_{int(1,r)} - FW\big) \tag{4.28}$$

$$FW = max\big(0, FW - B_{t,r}\big) \tag{4.29}$$

5. From $r = m$ to $r = 1$ (for $r \in int$), repeat (eq. 4.30 to 4.31):

$$B_{int(1,r)} = max\big(0, B_{int(1,r)} - LW\big) \tag{4.30}$$

$$LW = max\big(0, LW - B_{t,r}\big) \tag{4.31}$$

6. From $r = 1$ to $r = m - 1$ (for $r \in int$), repeat (eq. 4.32):

$$B_{t+1,r} = B_{int(1,r+1)} \tag{4.32}$$

   Eq. 4.32 moves remaining inventory to the next day.

The items to be discarded are calculated as:

$$waste_t = B_{int(1,1)} \tag{4.33}$$

And finally, the inventory position for the next day is calculated as:

$$IP_{t+1} = \sum_{i=1}^{m} \big(B_{t+1,i}\big) \tag{4.34}$$

All of the equations described in this subsection are used repeatedly, to simulate the processes day after day. The number of days simulated is 460, i.e., t is varied from 1 to 460 (for $t \in int$), where the initial 94 days are not used to record inventory performance. Thus, calculating a year starting from 95, we get (94+365), which gives 459, and is rounded off to 460.

The simulation model is run with all 21 sets of parameters from Table 4.1, i.e., 21 varieties of stores, with each of the six policies and finally the comparison is made between the original and modified forms of corresponding policies. One simulation run for one store variety is considered representative of all stores of that variety. These simulation experiments are further elaborated in section 4.5.

The policy comparisons made can be seen in Table 4.2. It can be noticed that the policies have been adjusted for the simulation model, to reflect that ordering is done at the start of the day, unlike Broekmeulen and Van Donselaar (2009) and Kiil et al. (2017)'s models. The policies are also allotted different names in Table 4.2. This is done since the original forms of the stock-based and EWA policies are used here with variable safety stocks, which are calculated as in the original EWA_{SS} policy. Thus, SB_{base} refers to the stock-based policy without the lost-sales modifications, and SB_{mod} refers to the stock-based policy after the lost-sales modifications. Similarly for the EWA policy. The 'base' subscript is not added to the EWA_{SS} policy, since it is used in its original form.

Before moving to the description of simulation experiments and results in the next section, the following subsection (4.4.5) discusses the verification and validation processes for the model.

4.4.5 Verification and validation

Citing Law and Kelton (2007), Kleijnen (1995) defines verification as "determining that a simulation computer program performs as intended, i.e., debugging the computer program", while validation "is concerned with determining whether the conceptual simulation model (as opposed to the computer program) is an accurate representation of the system under study". Kleijnen (1995) points out that validation cannot be expected to lead to a perfect model, as the perfect model is the real system itself, however, the model should be 'good enough' which depends on the objective of modelling. Kleijnen (1995) further emphasises that some applications only need relative or comparative simulation responses to different scenarios, thus, if the simulated and real responses are positively correlated, the simulation can be considered valid. The study within this research is of a comparative nature, and this form of validity 'for comparison' is demonstrated by the model, where decrease in order quantities result in decreased fill rates, which would also be expected in a real situation. Thus, any deviation from reality would affect both scenarios compared. However, other validation activities were also conducted to assess the absolute validity of the model, which are described below.

Describing their model validation process, Kiil et al. (2017) point out that since the same benchmark policy is used in their and Minner and Transchel (2010)'s simulations, the waste percentages from both simulations under the same policy are compared, to assess model validity. However, in the current context, since all the policies used in the model, except the EWA_{SS} policy, have been adjusted in terms of safety stocks, the EWA_{SS} policy in its base form was used to validate the model by comparing waste percentages. During the validation process, it was found that the model consistently gives lower wastes for the same shelf lives; and the fill rates are much less sensitive to the shelf lives, as compared to Kiil et al. (2017)'s simulation. This can be expected to be a result of the pseudo-forecasts used in the model, which reduces the variability between demand and forecasts, as they are generated from the same probability distributions; as opposed to Kiil et al. (2017)'s simulation model where real POS data are used to simulate demand, and a forecasting model is also deployed. Thus, conducting the policy comparisons while including a forecasting model in the simulation, is a prospect for future improvement in the validity of the simulation model. For conducting the validation activity described above, the following steps were used for each item shelf life from $m = 4$ (minimum considered by Kiil et al. (2017)) to $m = 9$ (shelf life for milk considered in the model):

1. Waste percentages and fill rates for the 21 store variations were calculated for one simulation run, assuming that EWA_{SS} policy is used for ordering and batch size of 10 is used, where waste percentages and fill rates are respectively calculated as:

$$W\%_{m,sv} = \frac{\sum_{t=1}^{460} waste_t}{\sum_{t=1}^{460} n_t} \cdot 100 \tag{4.35}$$

$$FR_{m,sv} = \frac{\sum_{t=1}^{460} FD_t}{\sum_{t=1}^{460} D_t} \cdot 100 \tag{4.36}$$

   where $W\%_{m,sv}$ and $FR_{m,sv}$ denote waste percentage and fill rate for shelf life m at a store variety sv, and remaining notations are same as explained before. Kiil et al. (2017) also use the same waste percentage measure, describing it as the "fraction of products wasted compared to received".

2. Using the store specific waste percentages and fill rates, the weighted averages of these

Table 4.3: Waste percentages and fill rates for model validation

Shelf life, m	Waste percentage, $\overline{W\%_m}$	Fill rate, $\overline{FR_m}$
4	23.75% (± 1.1)	89.63% (± 1.06)
5	19.20% (± 1.2)	92.60% (± 0.85)
6	13.06% (± 1.06)	93.42% (± 1.04)
7	9.56% (± 0.98)	94.25% (± 0.86)
8	7.48% (± 1.09)	95.37% (± 0.71)
9	5.5% (± 0.86)	95.44% (± 0.79)

waste percentages and fill rates were calculated for every simulation run as:

$$W\%_m = \frac{\sum_{sv=1}^{21} S_{sv} \cdot W\%_{sv}}{\sum_{sv=1}^{21} S_{sv}} \tag{4.37}$$

$$FR_m = \frac{\sum_{sv=1}^{21} S_{sv} \cdot FR_{sv}}{\sum_{sv=1}^{21} S_{sv}} \tag{4.38}$$

where S_{sv} denotes number of stores of a store variety sv (from Table 4.1).

3. After calculating $W\%_m$, and FR_m for 500 replications of the simulation, the mean of these 500 values was calculated as:

$$\overline{W\%_m} = \frac{\sum_{i=1}^{500} W\%_{m,i}}{500} \tag{4.39}$$

$$\overline{FR_m} = \frac{\sum_{i=1}^{500} FR_{m,i}}{500} \tag{4.40}$$

4. Finally, these mean waste percentages and fill rates (listed in Table 4.3) were compared with those reported by Kiil et al. (2017) in the plot for fill rates against waste percentage for different shelf lives, for the EWA_{SS} policy.

The limits shown with the waste percentages and fill rates represent the 95% confidence interval, which implies that 95% of the calculated values lie within this interval around the mean (Rees, 1987). As can be observed in the waste percentages and fill rates listed in Table 4.3:

- the wastes are considerably lower than those reported by Kiil et al. (2017), for example: 68% waste for $m = 4$, and 13% waste for $m = 9$, without even considering smaller batch sizes, which can be expected to further reduce the wastes (Eriksson et al., 2014); and

- the fill rates are considerably higher for smaller shelf lives, and the sensitivity of fill rates to changes in shelf life is much lower than that observed in the results of Kiil et al. (2017), for example: 59% for $m = 4$, and 94% for $m = 9$.

Thus, it can be concluded that the model under-estimates waste for all shelf lives, and over-estimates fill rates for lower shelf lives. However, as mentioned previously, since the model is intended to be used for a comparative study, and any of these discrepancies will influence both, the base and modified policy forms, it can be expected to give an estimate of the effects of the suggested policy modifications. It was also observed that the variations in fill rates and waste (if any) for the same shelf life, were substantial from one store variation to another. Thus, the experiments conducted and results reported in the next section show the variation in performance measures over the entire spectrum of store variations.

Another comparison for assessing validity was made with real waste data for milk, from a grocery retailer, over a period of five months, presented by Herstad (2016). This data presented by Herstad (2016) is measured as a percentage ratio of the monetary values of waste and sales turnover or revenue. Assuming that the monetary values denoted to these are equal, the ratio translates for the current model as:

$$WR_{\%} = \frac{\sum_{t=1}^{460} waste_t}{\sum_{t=1}^{460} FD_t} \cdot 100 \tag{4.41}$$

where $WR_{\%}$ is used to denote percentage waste ratio. The same procedure was followed as the previous comparison, however, using the SB_{base} policy, to reflect the ordering situation described by Herstad (2016), where age information is neither available nor used for ordering. As compared to a value of 1.2%, as reported by Herstad (2016), this ratio was calculated in the simulation as 6.5%, which is closer to other lower volume milk varieties. Additionally, almost half of the 1.2% reported by Herstad (2016) are actually products sold for a discount, rather than reaching the end of their shelf life. Thus, for future simulations with higher validity, along with forecasting models and real demand data, pricing models and their effect on demand could also included in the simulation model to improve closeness to reality.

Verification of the model was done on various occasions, by verifying intermediate outputs and statuses of functional variables, which were checked by stopping the simulation midway. At such occasions, the values of variables such as inventory level, age distribution, fulfilled demands, etc., were checked through manual calculations. This verification technique is also explained by Kleijnen (1995). Additionally, throughout the process of developing the model, frequent short runs of the developed modules were conducted, and relevant modifications were made, if not found to perform as intended.

4.5 Simulation experiments

This section describes the experiments which are conducted on the simulation model which was described in the previous section. The objective of experimentation, as mentioned earlier, is to identify the changes in inventory performance measures when the ordering policy is modified to account for lost-sales.

The policies compared with each other were listed and named in Table 4.2. Firstly, the comparisons are made for the milk inventory premise that the model description is based on, i.e., shelf life of 9 days, batch size of 10 and 90% of FIFO picking. Following this, to test the sensitivity of the results of policy modification towards three parameters: item shelf life, batch size, % of FIFO picking; additional comparisons are made by varying only one parameter at a time. Thus, for a wider range of results, a factorial experimental design, similar to that of Broekmeulen and Van Donselaar (2009)'s could be conducted in the future.

The comparative experiments can be listed as comparing:

1. SB_{base} and SB_{mod};
2. EWA_{base} and EWA_{mod}; and
3. EWA_{SS} and EWA_{SSmod};

where each comparison is conducted under the parametric inputs:

I. $m = 9$, $Q = 10$, $FIFO = 0.9$;

II. $m = 4$, $Q = 10$, $FIFO = 0.9$;

III. $m = 9$, $Q = 5$, $FIFO = 0.9$; and

IV. $m = 9$, $Q = 10$, $FIFO = 0.1$.

Considering all comparisons with every set of parameters, results in 12 experiments. Results from the 12 experiments are presented in subsection 4.5.2. The result for every policy comparison comprises of graphical and numerical results, which are organised as follows:

- graphs are used to depict the change in performance measures as a result of policy change for milk inventories, for stores with different mean weekly demands; and
- numerical results represent percentage changes in the performance measures as a result of the policy change for all 232 stores, i.e., also considering the number of stores of each type.

After presenting these results for all policy comparisons, plots for an additional measure are presented in section 4.6. This measure is referred to as the 'value of policy change', and is calculated as the ratio of change in overstocking and change in understocking for every store variety. This measure is plotted for all parameter settings, and these plots demonstrate the objective implications of these results for every store variety, in terms of understocking and overstocking. A similar approach of comparing different performance measures is used by Kiil et al. (2017), however, plotting fill rates against waste percentage. The value of policy change is plotted here instead to show the variation in impact of the proposed policy modification for different store sizes. The next subsection (4.5.1) describes the calculation procedure for this policy change value and other percentage changes reported.

4.5.1 Calculation procedure

This subsection clarifies the calculation procedure for percentage increase or decrease in performance measures, which are reported in the next subsection. Additionally, the calculation for the value of policy change for various store varieties is also explained, for which plots are presented in the next section. The performance indicators measured in the simulation are:

1. fill rate (FR);
2. wasted units ($waste$);
3. waste percentage, i.e., ratio of wasted and purchased amounts ($W\%$);
4. average inventory level (I_μ); and
5. number of deliveries (D_n).

The changes in these performance indicators are calculated as the percentage change in the weighted sum, where the weights depend on the number of stores. Thus, for any performance indicator x, percentage effect of policy modification is calculated as:

$$\Delta_{\%x} = \frac{\sum_{sv=1}^{21} S_{sv} \cdot x_{sv,mod} - \sum_{sv=1}^{21} S_{sv} \cdot x_{sv,base}}{\sum_{sv=1}^{21} S_{sv} \cdot x_{sv,base}} \cdot 100 \tag{4.42}$$

For one replication of the simulation, the values of $x_{sv,mod}$ and $x_{sv,base}$ are calculated, followed by the calculation of $\Delta_{\%x}$. This process is repeated for multiple replications. Law and Kelton (2007) propose that if inexpensive, maximum possible replications of a simulation should be run, if it includes any stochastic components. Thus, the numerical results reported are calculated as the mean of changes observed over 1000 replications (run-time ≈ 6000 seconds/experiment), since demands are generated for every replication, which gives a 1000 different demand scenarios in a year for every store. This can also be interpreted as running the simulation for 1000 years and using yearly performance indicators to evaluate the policies. All mean values are reported with the 95% confidence intervals, which as explained earlier, implies that 95% of the calculated values lie within this interval.

For every store in one simulation run, value of policy change is calculated as the ratio of reduction in $W\%$ and corresponding decrease in FR. Thus, mathematically expressing,

$$value_{policy\ change} = \frac{W\%_{sv,mod} - W\%_{sv,base}}{FR_{sv,mod} - FR_{sv,base}} \tag{4.43}$$

Thus, for any store where no change in $W\%$ is observed, the value of policy change amounts to zero. For the value of policy change to be positive for a store, both the numerator and denominator should be negative, i.e., both, $W\%$ and FR should decrease. Further, stores which have value of policy change larger than one, benefit more from the change, as compared to stores which have fractional values. Additionally, if $W\%$ increases while decreasing the fill rate, the value is negative. The presented plots in section 4.6, are based on the average value of policy change over a 1000 simulation replications.

4.5.2 Results

This subsection presents results from the simulation experiments conducted for identifying the effect of proposed policy modifications. The results are presented for every policy comparison separately.

SB_{base} and SB_{mod}

For the comparison between stock-based policies under parameters representing a milk inventory, results are displayed in Figures 4.2 to 4.6. The average fill rate for all 232 stores decreases by 1.52% while, while reducing the wasted amounts and waste percentage by 17.53% and 8.46% respectively. However, as can be observed, the confidence intervals for these results have wide ranges, and the waste percentage even increases for a very small fraction of scenarios. Additionally, it can be observed in Figures 4.3 and 4.4, that any waste observed, and reduced as a result of policy change, is concentrated at the lower demand stores; and the cumulative waste reduction reported, amplifies due to the high number of stores with average weekly demand lower than 50. A significant reduction in average inventory levels is observed, which is not a surprising result, given the lowered order quantities. The number of deliveries increase marginally, however the intervals reflect a wide range of occurrences, where the number of deliveries also decrease for few cases. The only apparent benefit derived by stores with high weekly demands, are the reduction in average inventory levels. However, coming at the cost of fill rates, these inventory reductions are unlikely to be pursued by high demand stores.
The percentage changes in the performance measures when parametric settings are changed, are shown in Table 4.4. The graphical results for these parameter settings can be found in Appendix A (Figures A.1 to A.15). While the changes in fill rates and average inventory levels are similar for different parametric changes, the reduction in waste and waste percentage are consistently higher when the shelf life is changed to 4 days; and also when the FIFO percentage is reduced to 10%, where 'consistently' emphasises on their narrow confidence intervals. As was observed during the model validation process, and can be seen in Kiil et al. (2017)'s results, the waste percentages increase for the same policy as the product shelf life decreases. A similar effect can be expected from reducing the FIFO percentage (Nahmias, 1982). Thus, combining

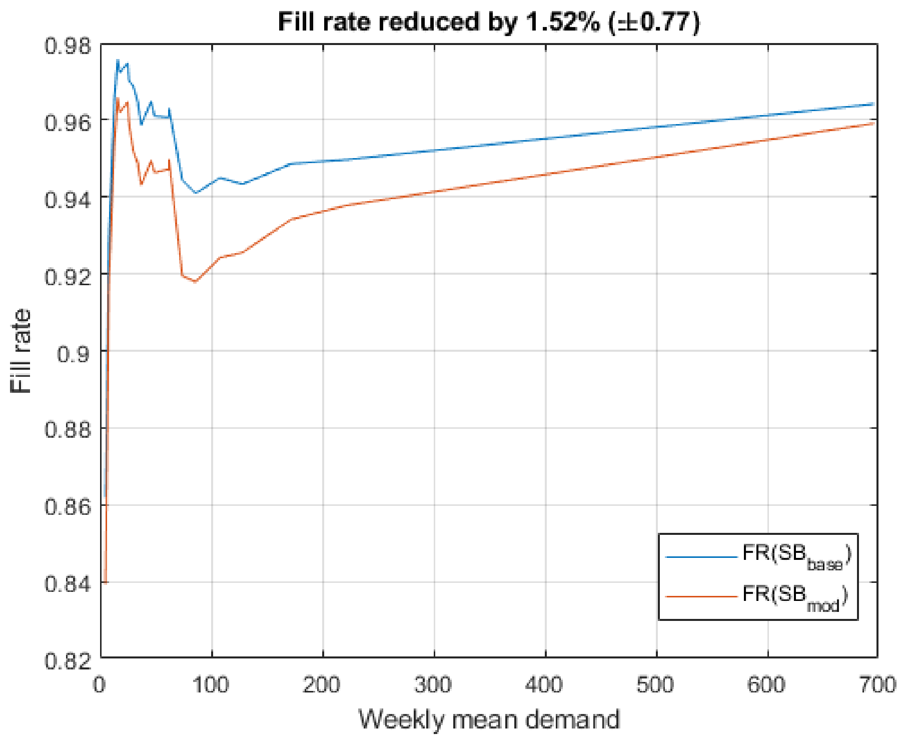

Figure 4.2: Fill rate comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

these insights with the simulation results, it can be inferred that while the values of waste and waste percentage are higher (than the first parameter setting) for lower shelf life and lower FIFO percentage under the base policy, the percentage changes in these values due to policy change are also more significant. This implies that the policy changes have higher value for lower shelf life products and products where the picking order is not as controlled as expected for milk inventories (Ferguson and Ketzenberg, 2006). The changes in number of deliveries, are again, marginal, and widely variable.

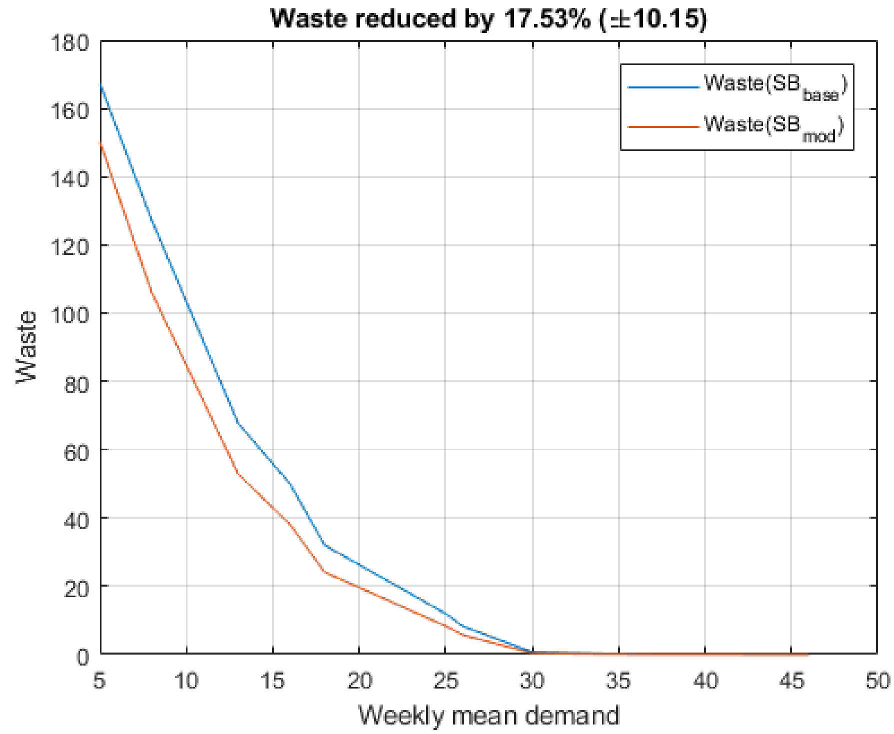

Figure 4.3: Waste comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

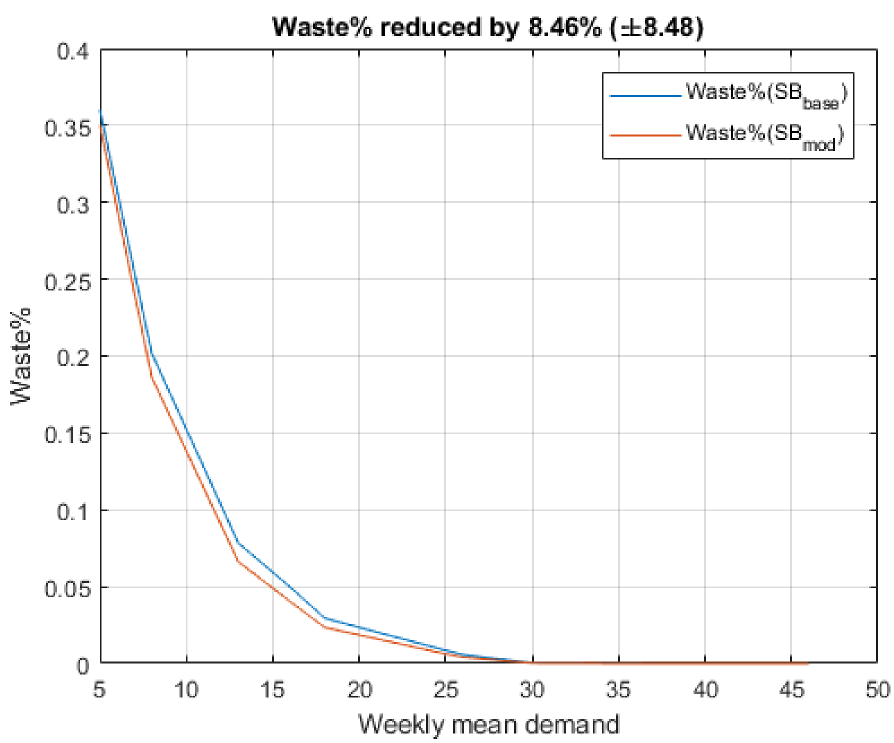

Figure 4.4: Waste% comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

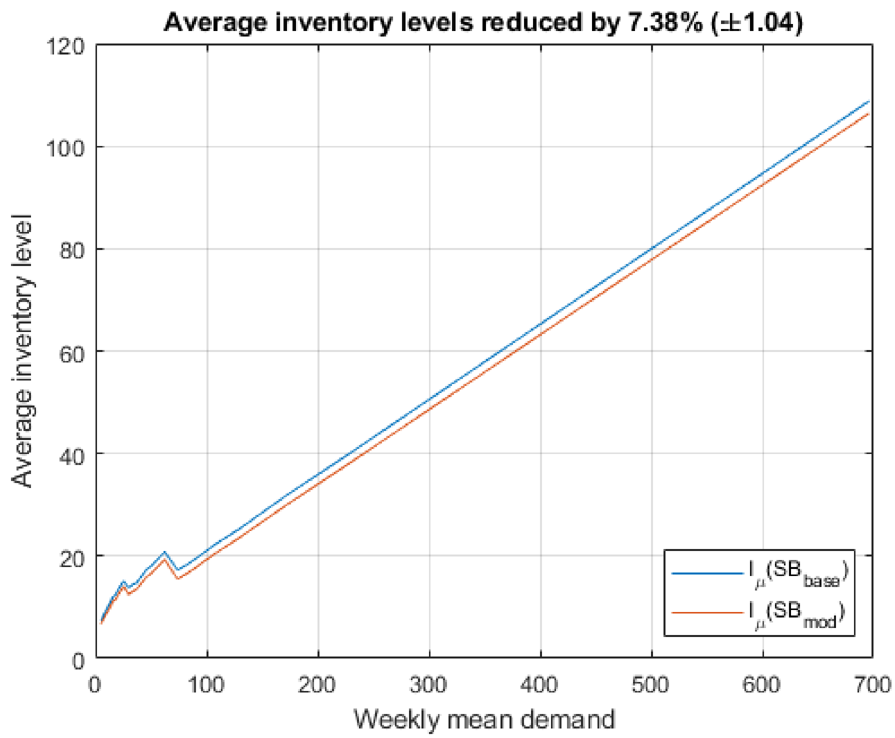

Figure 4.5: Average inventory level comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

Table 4.4: Numerical results from comparison of SB_{base} and SB_{mod}: mean values of percentage changes in performance indicators from 1000 simulation runs, calculated using eq. 4.42 for every run

Performance measure	$m = 9$, $Q = 10$, $FIFO = 0.9$	$m = 4$, $Q = 10$, $FIFO = 0.9$	$m = 9$, $Q = 5$, $FIFO = 0.9$	$m = 9$, $Q = 10$, $FIFO = 0.1$
FR	-1.52% (± 0.77)	-1.04% (± 1.07)	-1.28% (± 0.43)	-1.53% (± 0.79)
$waste$	-17.53% (± 10.15)	-19.37% (± 3.73)	-19.6% (± 9.51)	-20.2% (± 4.83)
$W\%$	-8.46% (± 8.48)	-8.66% (± 2.69)	-12.06% (± 8.56)	-10.38% (± 4.79)
I_μ	-7.38% (± 1.04)	-7.83% (± 0.92)	-7.28% (± 0.75)	-7.41% (± 0.95)
D_n	0.27% (± 1.54)	0.12% (± 1.31)	-0.07% (± 1.1)	0.09% (± 1.36)

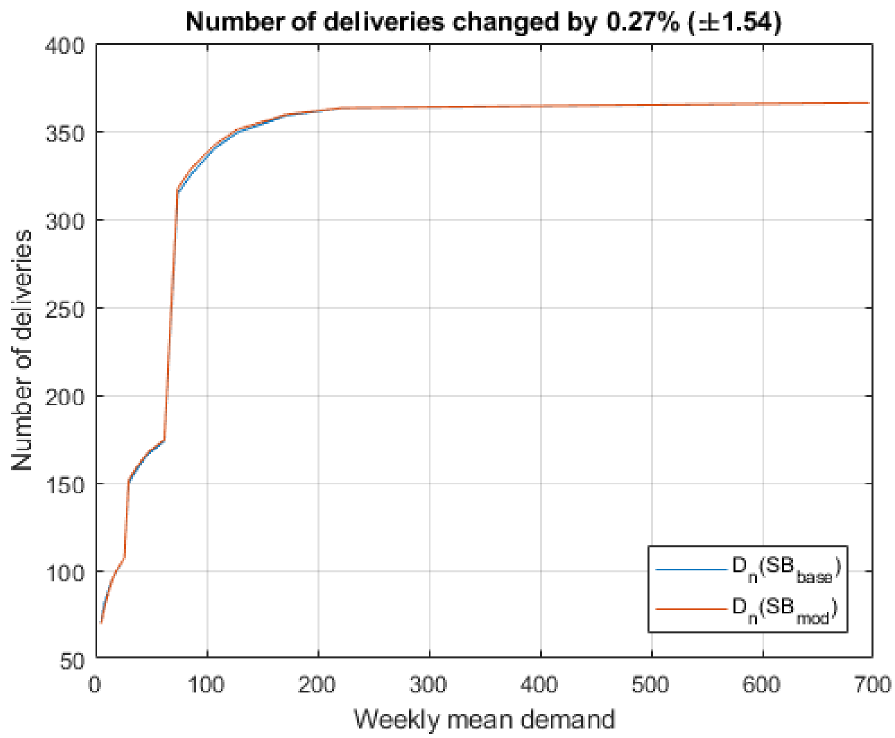

Figure 4.6: Comparison of number of deliveries for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

EWA_{base} and EWA_{mod}

For comparison between the EWA_{base} and EWA_{mod} policies under parameters representing a milk inventory, results are displayed in Figures 4.7 to 4.11. Benefiting from the EWA heuristic, the fill rates with EWA_{base} policy can be seen to be higher than the SB_{base} policy. Additionally, moving from the EWA_{base} policy to the EWA_{mod} policy shows a lower decrease in the fill rates, as compared to the stock-based policy comparison. As can be seen in Figure 4.7, the stores with the lowest fill rates when using the EWA_{base} policy, are also the ones to suffer the largest decrease in fill rates when switching to the EWA_{mod} policy. Since these stores have weekly demands between 70 and 100, the policy change is counterproductive for them, as these stores have no wastes when the EWA_{base} policy is used. Thus, moving to the EWA_{mod} policy has very limited value for these stores, i.e., only from the drop in inventory levels. Again, the stores that benefit most, are those with weekly demands lower than ≈40, as the reduction in fill rates for these stores is marginal as compared to the waste reduction they achieve by implementing the policy change.

Table 4.5: Numerical results from comparison of EWA_{base} and EWA_{mod}: mean values of percentage changes in performance indicators from 1000 simulation runs, calculated using eq. 4.42 for every run

Performance measure	$m = 9$, $Q = 10$, $FIFO = 0.9$	$m = 4$, $Q = 10$, $FIFO = 0.9$	$m = 9$, $Q = 5$, $FIFO = 0.9$	$m = 9$, $Q = 10$, $FIFO = 0.1$
FR	-1.24% (± 0.38)	-1.39% (± 0.42)	-1.21% (± 0.29)	-1.32% (± 0.41)
$waste$	-17.16% (± 11.69)	-10.78% (± 2.67)	-26.2% (± 14.52)	-20.84% (± 6.01)
$W\%$	-7.24% (± 7.16)	-5.81% (± 1.58)	-18.68% (± 9.79)	-13.21% (± 4.76)
I_{μ}	-7.77% (± 1.54)	-7.32% (± 1.1)	-7.94% (± 1.32)	-8.24% (± 1.36)
D_n	-0.27% (± 1.78)	-2.73% (± 1.28)	0.21% (± 1.12)	-0.99% (± 1.34)

The graphical results from other parameter settings can be found in Appendix A (Figures A.16 to A.30), while the numerical results are shown in Table 4.5. Considering the results from the experiment where the shelf life is lowered, the policy change shows lower reductions in absolute and percentages waste. However, these reduction values are more important for this parameter setting, as the wastes and waste percentages are much higher for the EWA_{base} policy (see Figure A.17), as compared to a shelf life of 9. Additionally, the waste reductions are concentrated about the mean, implying that the probability of achieving waste reduction as a result of policy change is higher for lower shelf life products. The change in number of deliveries is also significant and consistently negative for lower shelf lives. Additionally, this reduction occurs essentially at lower demand stores (Figure A.20), implying that the policy change is highly relevant for ordering of low shelf life products at low demand stores. The change observed in waste reduction values as a result of changing FIFO% and batch size, is similar to that in the base policy comparison; where the reductions in waste amount and waste percentages increase. It should be pointed out here, that for the case of 10% FIFO, in addition to the change in customer picking, the waste estimation is also modified. Thus, the larger waste reduction is a consequence of both, change in customer picking as well as increased order sizes. The waste estimated by the EWA heuristic would be higher for 10% FIFO than for 90% FIFO, and so, orders placed would be larger. This implies that the policy change from EWA_{base} to EWA_{mod} is much more useful where waste estimation through EWA heuristic assumes low or no FIFO.

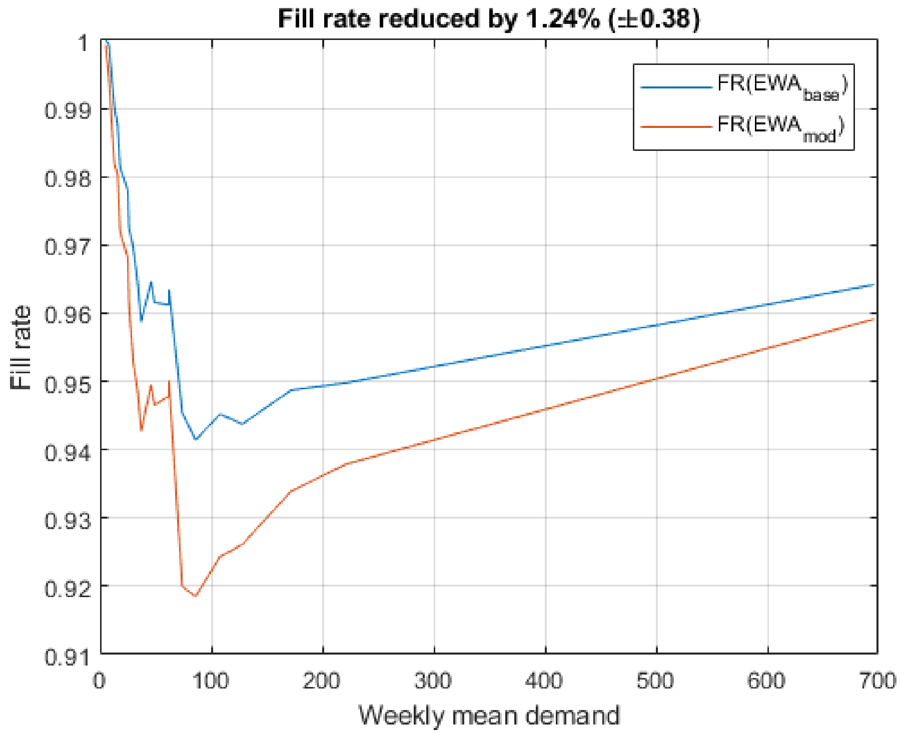

Figure 4.7: Fill rate comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

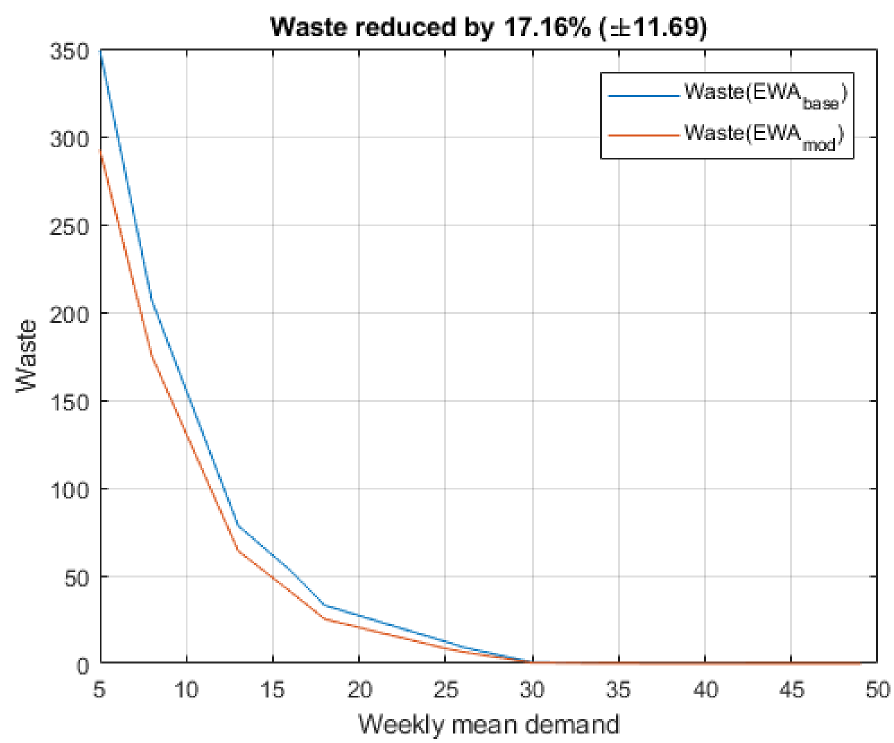

Figure 4.8: Waste comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

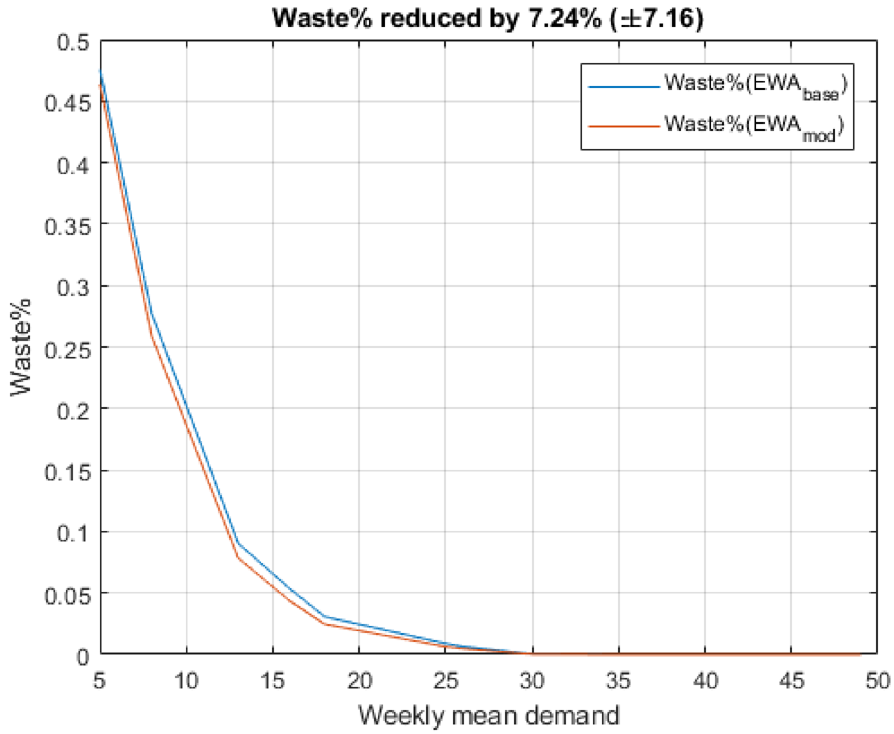

Figure 4.9: Waste% comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

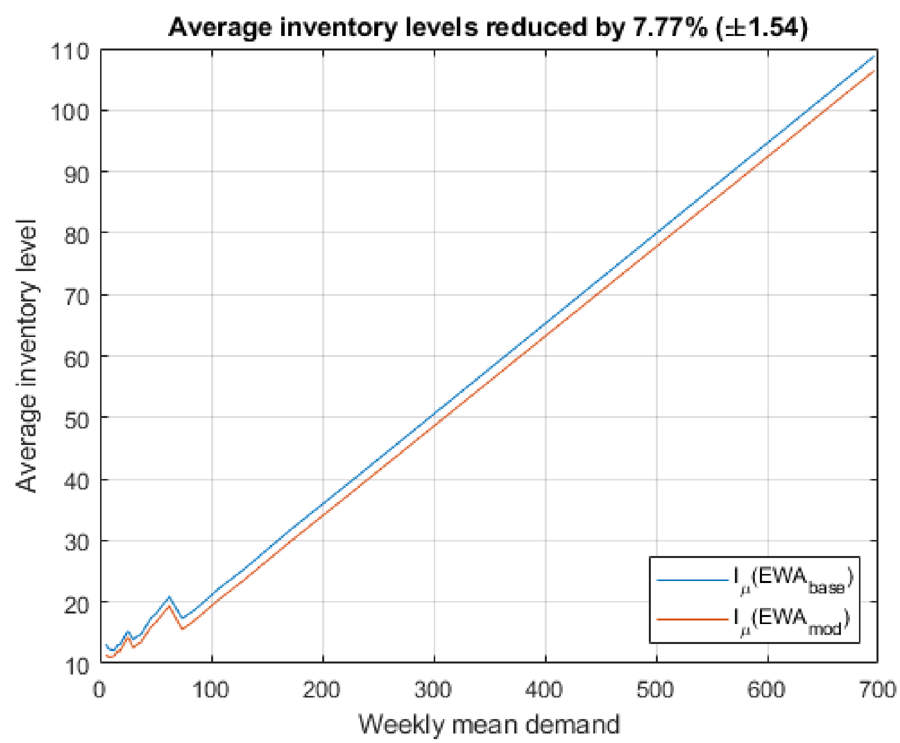

Figure 4.10: Average inventory level comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

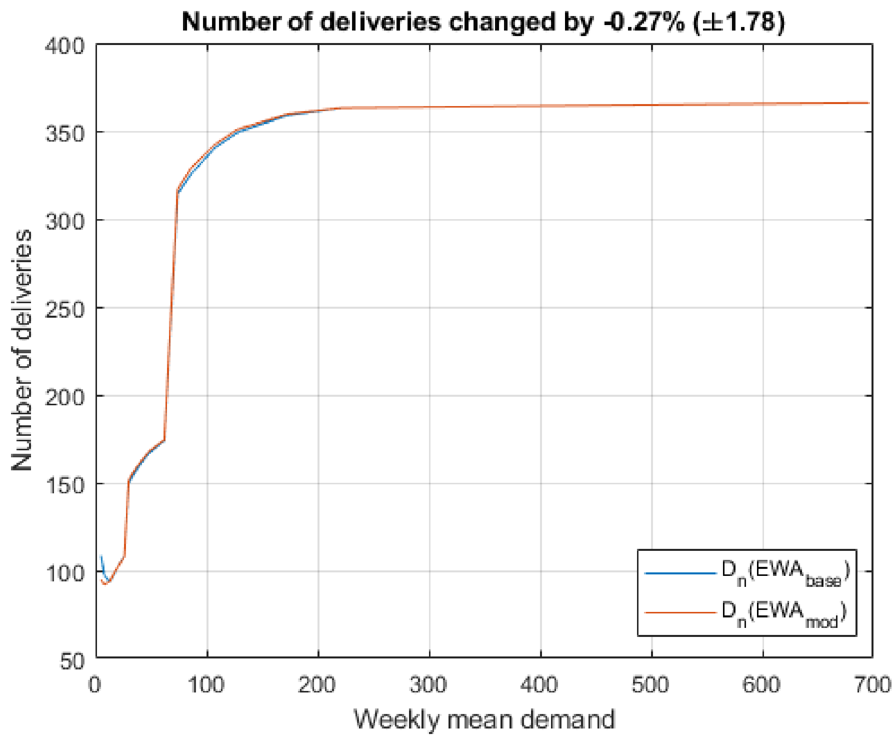

Figure 4.11: Comparison of number of deliveries for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

EWA_{SS} and EWA_{SSmod}

For comparison between the EWA_{SS} and EWA_{SSmod} policies under parameters representing a milk inventory, results are displayed in Figures 4.12 to 4.16. As can be observed, the results are similar to those observed in the previous comparisons. However, unlike the EWA_{base} policy, where the stores with lowest demands are the ones with the highest fill rates, the fill rate profile for different stores under EWA_{SS} policy is similar to that of the stock-based policy. The cause for this can be expected as the lower buffers that this policy orders, as compared to the EWA policy. Again, substantial waste reduction is observed; however, the reduction is concentrated at the lower demand stores, since the higher demand stores have no waste even with the EWA_{SS} policy. The reduction in waste amounts and waste percentages, similar to the previous two comparisons, are widely scattered around the mean. Average inventory levels reduce by similar percentage as the earlier comparisons.

The graphical results from other parameter settings can be found in Appendix A (Figures A.31 to A.45), while the numerical results are shown in Table 4.6. The reduction in waste and waste

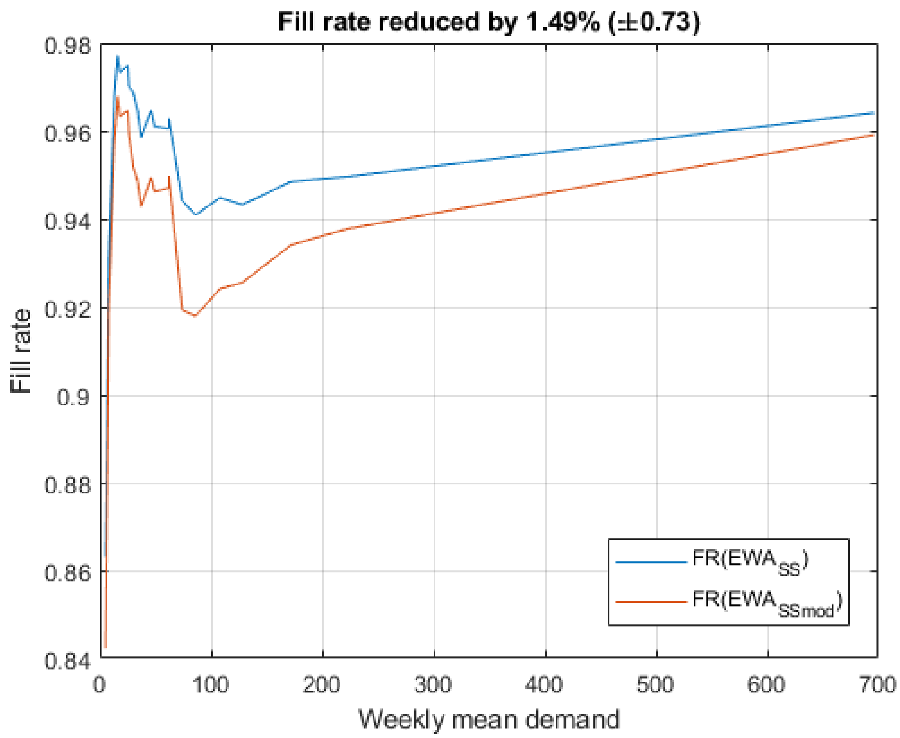

Figure 4.12: Fill rate comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

percentage values decreases for lower shelf life, while increasing for lower batch size and FIFO% changes. The policy change shows substantial value for all parameter settings in terms of waste and waste percentage reductions, while again, being more valuable for lower demand stores than higher demand stores; and more valuable for low shelf lives and low FIFO%.

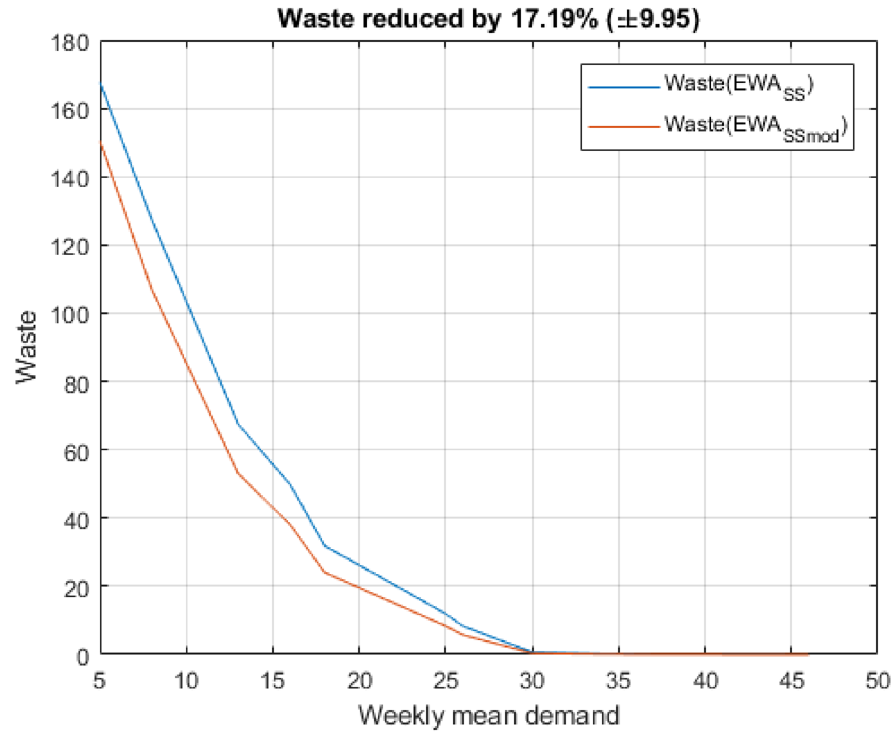

Figure 4.13: Waste comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

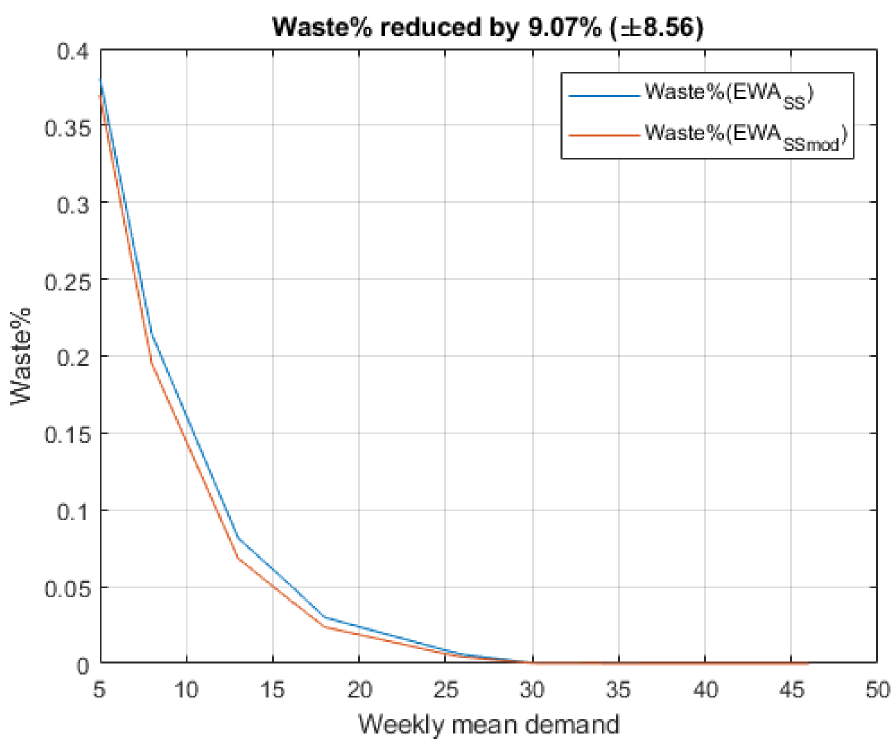

Figure 4.14: Waste% comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

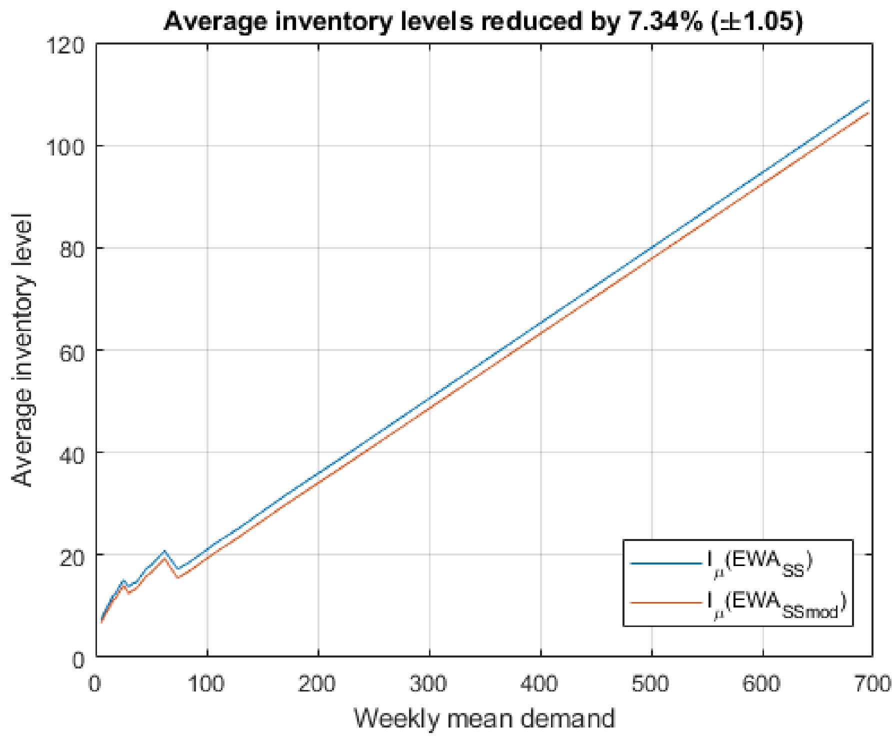

Figure 4.15: Average inventory level comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

Table 4.6: Numerical results from comparison of EWA_{SS} and EWA_{SSmod}: mean values of percentage changes in performance indicators from 1000 simulation runs, calculated using eq. 4.42 for every run

Performance measure	$m = 9$, $Q = 10$, $FIFO = 0.9$	$m = 4$, $Q = 10$, $FIFO = 0.9$	$m = 9$, $Q = 5$, $FIFO = 0.9$	$m = 9$, $Q = 10$, $FIFO = 0.1$
FR	-1.49% (± 0.73)	-1.42% (± 0.85)	-1.24% (± 0.45)	-1.47% (± 0.75)
$waste$	-17.19% (± 9.95)	-16.43% (± 3.46)	-18.85% (± 9.78)	-19.48% (± 4.96)
$W\%$	-9.07% (± 8.56)	-8.45% (± 2.53)	-13.84% (± 8.56)	-11.95% (± 4.94)
I_μ	-7.34% (± 1.05)	-7.2% (± 0.9)	-7.23% (± 0.77)	-7.27% (± 0.93)
D_n	0.42% (± 1.46)	-0.01% (± 1.07)	0.3% (± 1.09)	0.32% (± 1.22)

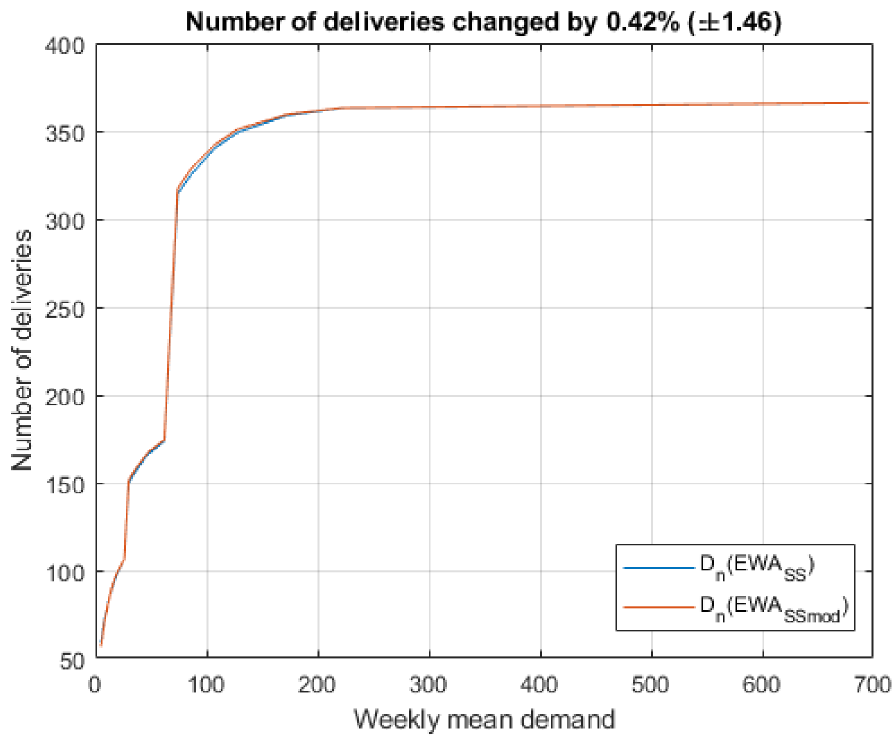

Figure 4.16: Comparison of number of deliveries for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

4.6 Value of policy change

Utilising the results presented in the previous section, this section presents plots which demonstrate the objective implications of the results for different stores. The value of policy change is plotted here against the store serial numbers (ranging 1 to 21) from Table 4.1, rather than demand, as the increase in demand from one store to another is not uniform.

Figures 4.17 to 4.20 show the value of moving from a SB_{base} policy to SB_{mod} policy. For the first parameter setting (Figure 4.17), which represents a milk inventory, only two store varieties have value larger than one, while the remaining stores with non-zero value have fractional values. Larger number of stores benefit from policy change for lower shelf life (Figure 4.18), while two stores display negative values. The negative values occur due to increase in waste percentages at these stores as can be observed in Figure A.3. Results for lower batch size show improvements over the first parameter setting (Figure 4.19), as values for the low demand stores are larger for reduced batch size. Reduction in FIFO% shows largest improvements, as several stores benefit from policy change with values larger than one (Figure 4.20).

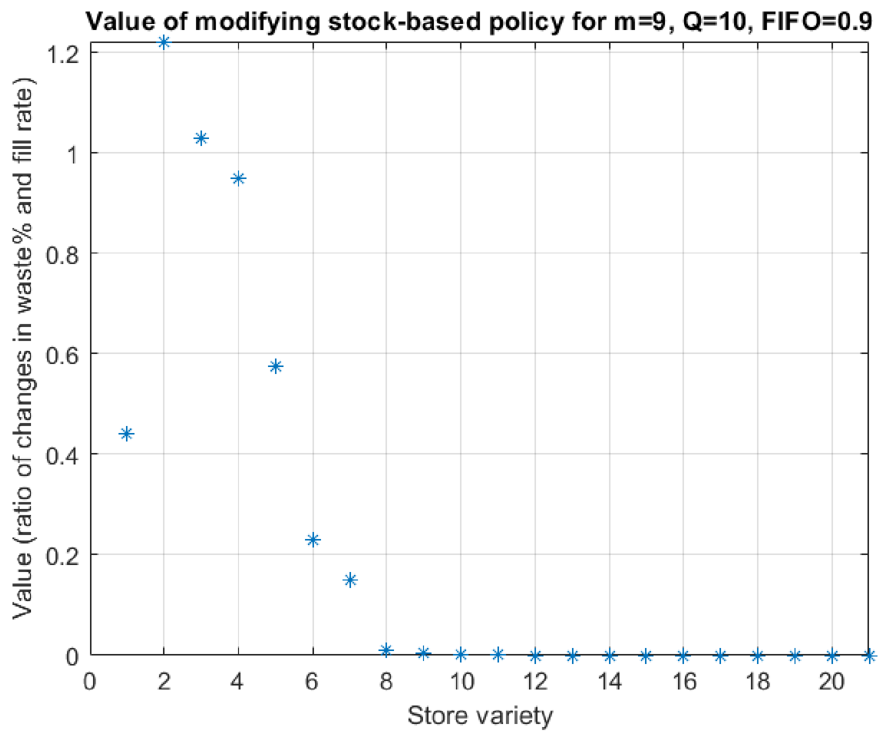

Figure 4.17: Value of changing from SB_{base} to SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

While number of stores with non-zero value for policy change is similar for the EWA policy as the stock-based policy for milk inventories, the values for these stores are larger in magnitude (Figure 4.21). The stores that do benefit, have substantial value for the policy modification. Similarly for other parameter settings (Figures 4.22 to 4.24), most of the stores that do benefit, have large reductions in waste percentage as compared to fill rate drops.

The value plots for the EWA_{SS} policy change (Figure 4.25 to 4.28), show large similarities to the value plots for the stock-based policy, which can be considered a consequence of the lower buffers that the policy orders. Thus, a useful insight from these plot comparisons is that the proposed policy changes are highly valuable for low demand stores, and the value of the modifications is larger for the EWA policy as compared to the stock-based and EWA_{SS} policies. Further, the value of modification is larger for the EWA_{SS} policy than the stock-based policy. Thus, larger the buffer that a policy orders, larger the value of proposed modifications for the policy.

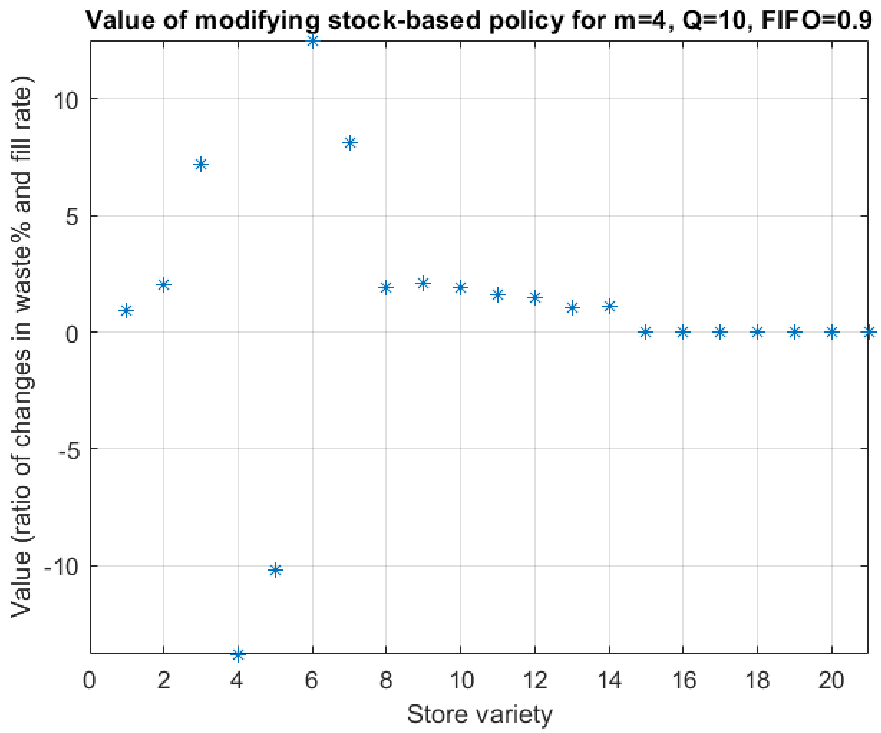

Figure 4.18: Value of changing from SB_{base} to SB_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

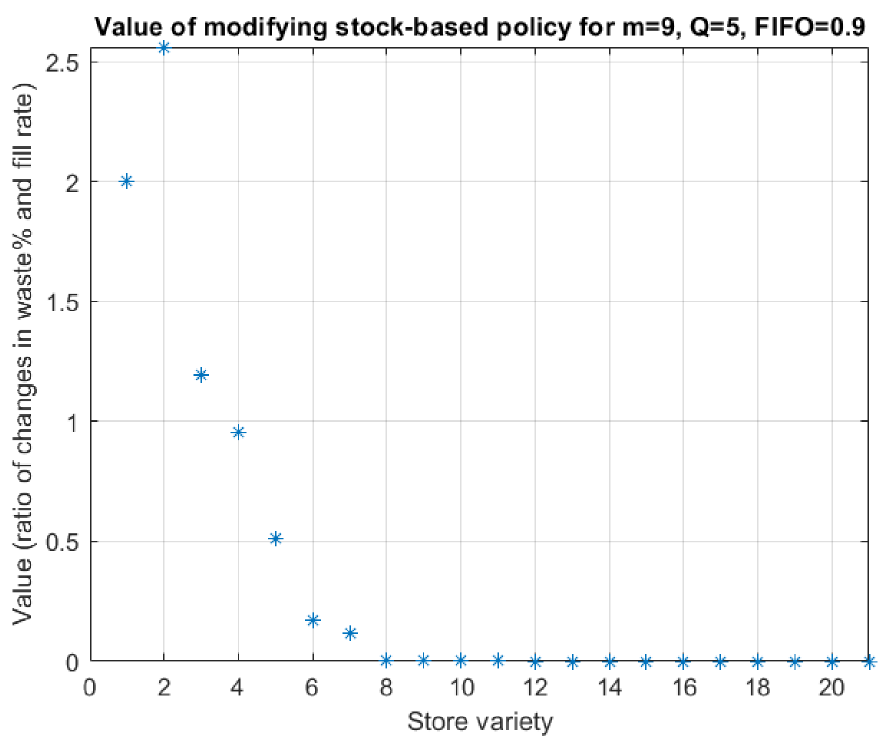

Figure 4.19: Value of changing from SB_{base} to SB_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

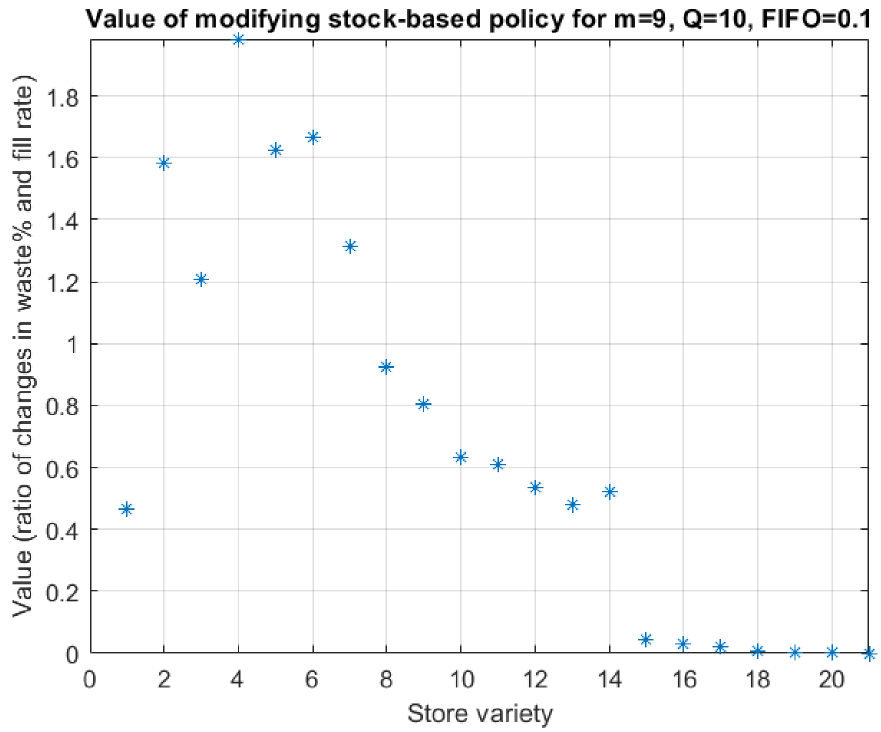

Figure 4.20: Value of changing from SB_{base} to SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

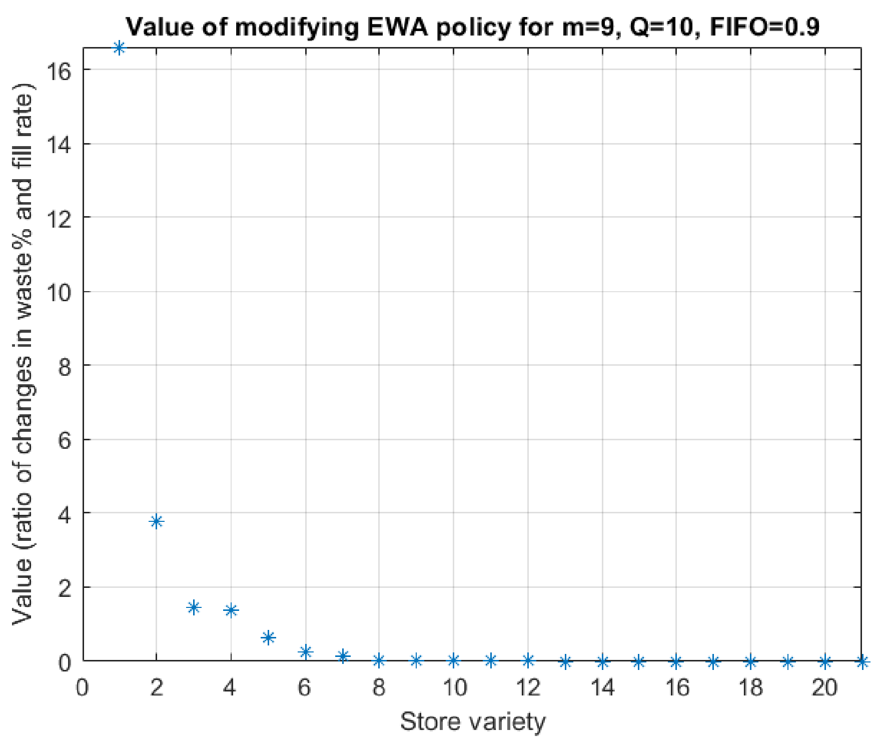

Figure 4.21: Value of changing from EWA_{base} to EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

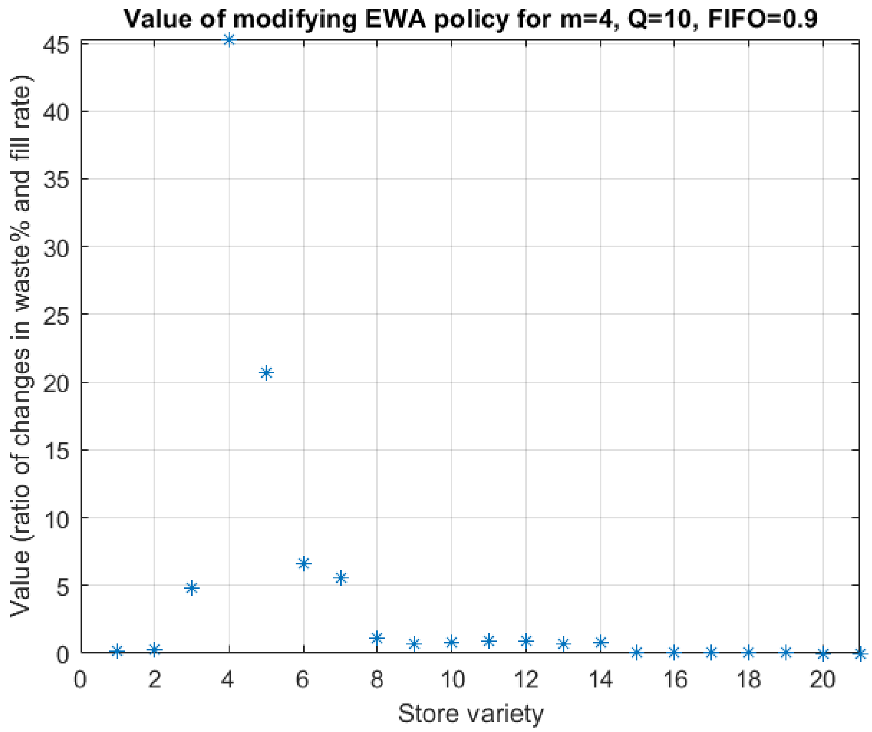

Figure 4.22: Value of changing from EWA_{base} to EWA_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

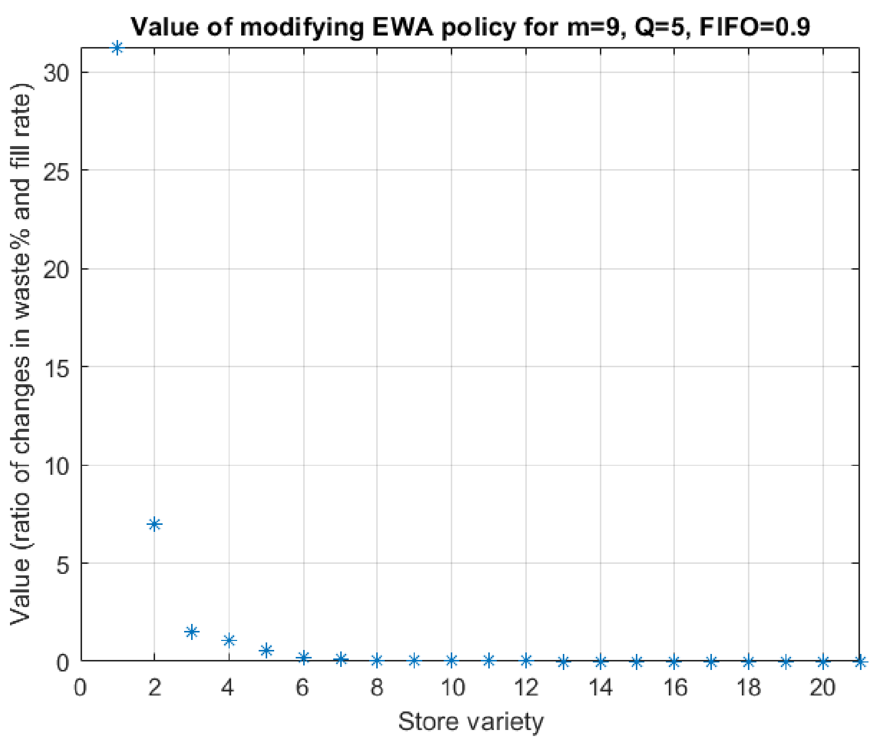

Figure 4.23: Value of changing from EWA_{base} to EWA_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

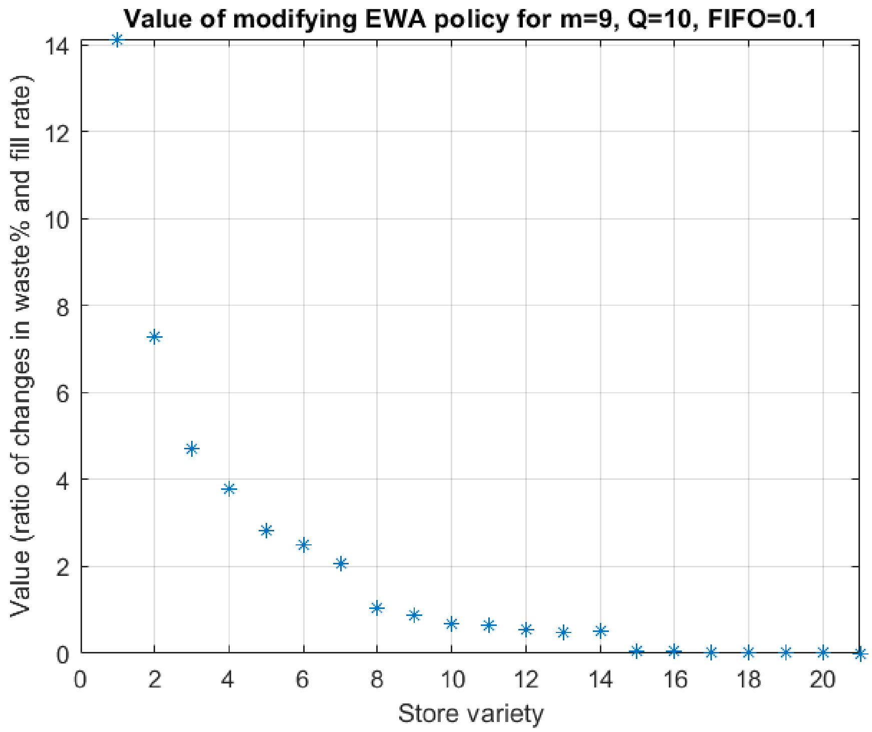

Figure 4.24: Value of changing from EWA_{base} to EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

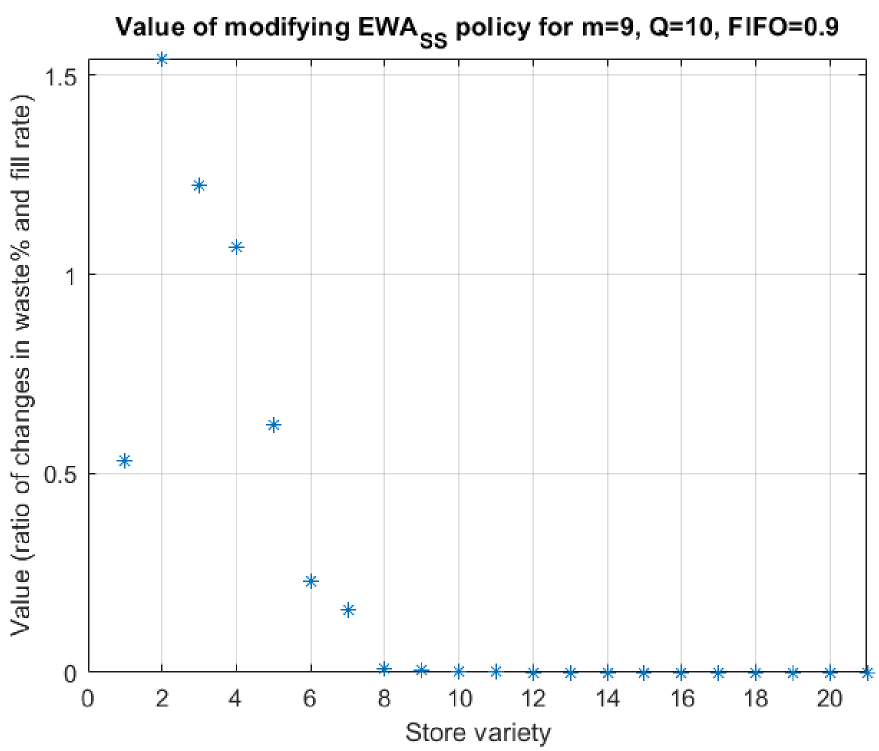

Figure 4.25: Value of changing from EWA_{SS} to EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.9$

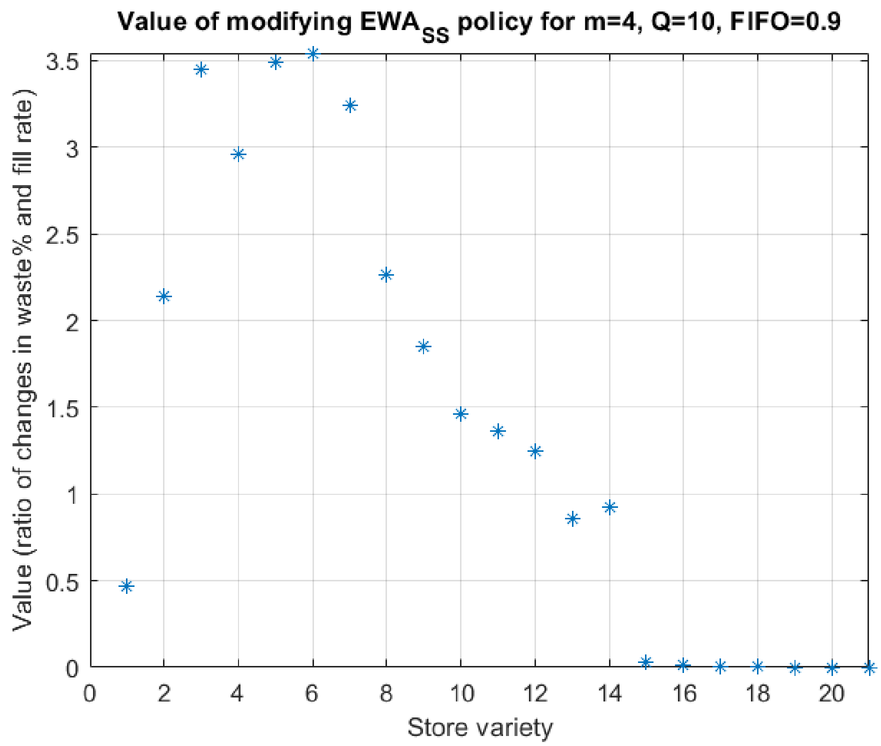

Figure 4.26: Value of changing from EWA_{SS} to EWA_{SSmod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

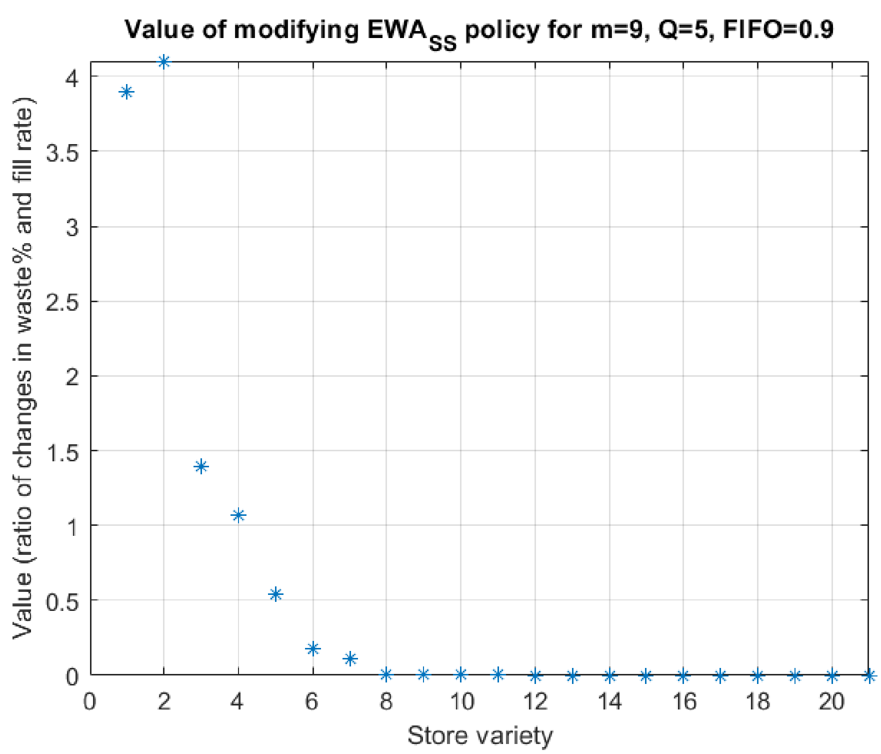

Figure 4.27: Value of changing from EWA_{SS} to EWA_{SSmod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

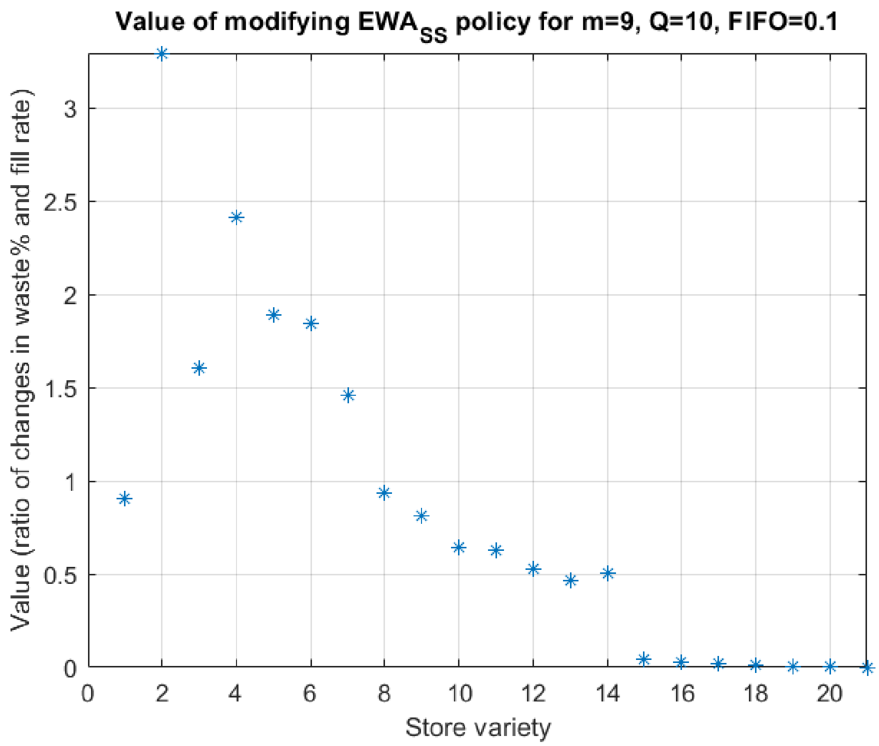

Figure 4.28: Value of changing from EWA_{SS} to EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

4.7 Summary

The characteristics of retail milk inventories, identified in the previous chapter, were utilised to develop a simulation model, and the process of development has been described in this chapter. The developed model has been utilised to conduct simulation experiments, to identify the effects of the policy modifications proposed in the previous chapter. This section summarises the chapter.

The characteristics of milk inventories were translated to the description of a system to be modelled. The system description served as basis for a conceptual model, where the modelling objective, model content, and input-output causal relationships were organised to support development of the simulation model. This was followed by description of the characteristics of the simulation model, and explanation of the rationale for the chosen simulation modelling approach.

Model notations and assumptions were stated, to aide the model description process. Modelling and simulation approaches for different model elements were described, including the mathematical operations used to implement the simulation of various activities and the EWA heuristic. Finally, the model verification and validation processes were described, followed by description of the simulation experiments and results.

The numerical and graphical results presented have provided basis for answering the research question presented in section 1.2. Additionally, value of the proposed policy changes have been plotted for different store varieties, where value is measured as the ratio of change in waste percentage and change in fill rates, as a result of the policy change. The following chapter uses these results to answer the research question and discusses the identifiable practical implications of these results.

Chapter 5

Discussion

As introduced in section 1.2, the objective of this research has been identifying and proposing modifications for a stock-based and two age-based inventory policies, such that the underlying backordering assumption in these policies, is eliminated; and identifying the effects of the proposed modifications for perishable inventories with no backorders. The modifications were proposed and explained in Chapter 3. The quantified effects of policy modifications on inventory performance indicators were identified through simulation experiments in Chapter 4. The experiments were conducted using data from a Norwegian retail chain (Kiil et al., 2017), which provided weekly demands, target service levels, approximate review intervals and number of stores, for the simulation model. The simulation model was based on a milk inventory in a retail store, where characteristics of the inventory were modelled based on information from relevant empirical literature (Engelseth, 2012; Ferguson and Ketzenberg, 2006; McKinnon et al., 2007). The use of milk as the inventoried product of interest, was explained and justified by discussing its practical significance, in section 1.3. To extend the generality of the conclusions that can be drawn from the simulation results, shelf life, batch size, and FIFO% were varied, one at a time, in different simulation experiments.

This chapter uses the quantified effects to answer the research question presented in section 1.2; discussing implications of the simulation results, that can be generalised to other retail chains and products. Additionally, limitations and shortcomings of this study are discussed.

5.1 Research question

Research question: *How are the performance indicators: waste, fill rate, inventory level and number of deliveries; for perishable inventories with no backorders, affected when lost-sales are taken into account while ordering?*

As was hypothesised in section 1.2, waste and fill rates decrease with the proposed policy changes. Among all the policy comparisons with different sets of test parameters, the average reduction in quantity of waste ranges from 10.78% to 26.2%, while the average reduction in waste percentage ranges from 5.81% to 18.68%. The average decrease in fill rate ranges from 1.04% to 1.53%. The average reduction in inventory levels ranges from 7.2% to 8.24%. The average change in number of deliveries ranges from -2.73% to 0.42%. These ranges represent the impact of policy change for 232 stores of a retail chain, however, the impact for every store variety is different. Combining insights from the graphical results (Figures 4.2 to 4.16; A.1 to A.45; 4.17 to 4.28) with characteristics of store varieties (Table 4.1), the implications for different store varieties are discussed below.

While majority of the policy comparisons under different parameter settings show substantial waste reduction against marginal reduction in fill rates, it is important to take into account that the waste eliminated comes from stores with lower demands. Additionally, the waste reduction percentages amplify due to the large number of stores in the low demand range. The policy changes might be valuable in terms of waste reduction, for a grocery retail chain that has a similar distribution of stores and demand levels. However, differentiation must be made between stores with 'low demand and high review intervals' and stores with 'high demand and low review intervals', for assessing the value of such a policy change, similar to the plots presented in the previous chapter (Figures 4.17 to 4.28). As can be seen in the demand data used for simulations of different stores (Table 4.1), stores with weekly demands below 30, have a review interval of 3; stores with weekly demand from 30 to approximately 60, have a review interval of 2; and stores with weekly demands higher than 70 have a review interval of 1. Thus, for another retail chain that has larger number of stores of the third variety, the cumulative waste reduction can be expected to be lower.

The high demand stores appear to be relatively immune to waste due to the mixed effects of high demand as well as low review intervals. As a result, the plots for value of policy modification show negligible value for these stores. The high demand store varieties do benefit from the policy modification, but only in terms of the reduction in average inventory levels, which is expected to be of little value against the reduction in fill rates, given the target service levels are as high as 98%. Additionally, due to low or no risk of waste, these stores can be expected to attempt to maximise the fulfilled demand, and any such policy modifications are expected to gain little interest from managers of such stores. However, the threshold weekly demand beyond which no wastes are observed, changes with the product shelf life, batch size, and FIFO% under consideration.

Decreasing shelf life and FIFO% increases this threshold demand value, while decreasing the batch sizes decreases the threshold. Lower the threshold, lower the number of store varieties that experience any wastes. While these parameters have been changed one at a time in the experiments, it is important to highlight that these changes can be expected to occur simultaneously in real situations. That is, if a product other than milk is considered, it may have a lower shelf life, and also a smaller batch size, with reduced control over customer picking. Since the reduction in these parameters individually, shows different effects on the threshold demand; the combined effects are difficult to predict without product specific experiments, or experiments where multiple parameters are varied simultaneously.

For a lower demand store that can successfully reduce waste by making such a policy change, the trade-off of interest for the store manager can be expected to be between the financial loss due to reduction in fill rates, and gains due to reduction in waste, which is also the basis of calculating 'value of policy change' in the previous chapter. If the former exceeds the latter, the policy change is unlikely to be accepted by the managers of such stores. On the other hand, accumulating the waste reduction from several such stores of the retail chain (same as the numerical results reported), can be of substantial value for the environmental performance of the retail chain. As Fernie and Sparks (2014) point out "as consumers are becoming more environmentally-conscious, retailers' green credentials are becoming a more important competitive differentiator. Environmental initiatives can generate higher revenues and secure greater customer loyalty." Thus, an organisation level strategic trade-off is between the reduced customer satisfaction due to decreased fill rates, and increased customer loyalty due to an initiative that reduces environmental impact, i.e., by reducing wastes.

As pointed out earlier, decreasing the shelf life and FIFO%, increases the threshold demand value, i.e., larger number of store varieties experience waste. Additionally, the percentage reduction of waste and waste percentages when changing between stock-based policies, is higher for these parameters. The useful conclusion that can be drawn from this is that the policy change has high value for products which have low shelf lives and for which, managers have low control on customer picking, if the item ages are not utilised in the ordering decisions (Figures 4.18 and 4.20). Additionally, these larger waste reductions are achieved for a lower or same fill rate reduction as the milk parameter setting (Table 4.4), which further increases the value of the policy modification for lower shelf life items and items with low customer picking control. As Damgaard et al. (2012) mention, item shelf life information is not available or used for ordering decisions for various perishables in grocery retail. As a result, such a policy change can decrease waste without making any investments in making the age information available for these perishables.

The average inventory levels can be observed to decrease consistently for all comparisons and parameter settings by approximately 7 to 8%. Thus, for the 232 stores, the excess ordering which occurs when lost-sales are not accounted for in ordering decisions, is approximately 7.5 to 8.7%, i.e., the 'reduction' as a percentage of the 'reduced' quantity ($\frac{7 \cdot 100}{93}$ to $\frac{8 \cdot 100}{92}$). On the other hand, the changes in number of deliveries vary marginally while increasing for some cases and decreasing for others. However, no useful conclusions can be drawn on these changes.

While the simulation results and this discussion have provided insights into the effects and implications of the proposed policy modifications for perishable inventories, these insights only provide a partial view of the effects and implications. This is due to various scope-limiting measures that have been taken during the course of research and due to other external limitations. These limitations and possibilities for future research, are discussed in the subsequent section.

5.2 Limitations and future research agenda

The investigation in this research has been based on the hypothesis that the use of an ordering policy which is based on an underlying backordering assumption, in a perishable inventory with lost-sales, can be a cause of over-ordering; and that eliminating this assumption can reduce waste of perishables. The investigation which was carried out through simulation experiments, confirms this hypothesis under the model assumptions, however, the aim of the investigation was to identify the simultaneous effects on various inventory performance measures. The effects have been identified for three policies, under four product related parameter settings, and the value of eliminating the excess orders has been plotted for stores (of a retail chain) with varying characteristics. This section discusses the measures taken to limit the scope of investigation within the time-frame, and exogenous challenges that were faced.

The first scoping measure taken to constrain the investigation is the choice of a product with a peculiar supply chain structure. The product choice was made with the intent of increasing the practical utility of the investigation (as described in section 1.3), and has also guided the assumptions in the development of the simulation model. While the simulation results have provided useful insights into the effects of the proposed policy modification, these effects must also be examined in the future for divergent supply chains and wider range of product characteristics. While one of the primary reasons for using the example of milk, was the large proportion of customer responses reflecting lost-sales (McKinnon et al., 2007), other products with similar customer responses but different shelf lives, and batch sizes should be considered in future studies. These effects should at least be studied for the various parametric combinations on which Broekmeulen and Van Donselaar (2009) and Kiil et al. (2017) base their simulations, so that the extent of over-ordering (if any) for all of these combinations can be identified. This can also establish further improvements in the EWA and EWA_{SS} policies for products with predominant lost-sales customer responses to stock-outs; such that marginal reduction in fill rates can provide substantial waste reductions for these products.

The second scoping measure taken is to only consider cases with no outstanding orders when an order is placed, i.e., $R \geq L$. Thus, future simulation studies where wider range of parameters are considered, can address this by including cases where ordering is done during outstanding orders, i.e., $R < L$. The recursive formulation required for such cases, to calculate the expected on-hand stock when an order is expected to arrive, was demonstrated for the stock-based policy in subsection 3.3.3. As pointed out in subsection 3.3.4, a similar recursive formulation for the EWA and EWA_{SS} policies, would require including the estimated waste into the recursive formulation. Thus, these modifications can be demonstrated, implemented in simulation models, and tested in future research.

As Bijvank and Vis (2011) point out, the literature on lost-sales inventory models is limited. As a result, scientific literature that could be used to validate and support the propositions, was limited. And the validity of the propositions has been essentially based on the reasoning demonstrated in Chapter 3. Additionally, Bijvank and Vis (2011) list Broekmeulen and Van Donselaar (2009)'s work as one of the few publications considering lost-sales models for perishables. However, as has been the premise of this research, Broekmeulen and Van Donselaar (2009) only consider lost-sales in their stated assumptions, while the ordering policies in the simulation model reveal an underlying backordering assumption. Since the impact of this underlying assumption is larger for perishables, than non-perishables, there is a definite need for further research on lost-sales inventory systems for perishable items.

As highlighted in section 4.4, demand and forecasts have been generated from the same probability distributions, which are based on specified fractions of mean weekly demand in different weekdays. In order to increase the generality and validity of the model as well as the results obtained, detailed POS data and forecasting models can be used in future simulation studies, similar to that of Kiil et al. (2017)'s simulation. This would additionally include the phenomenon of seasonal trend in retail demand and its effects on the proposed policy changes, which have been neglected in the simulation due to repeating weekly patterns.

The numerical results presented, essentially represent the distribution of store varieties for a specific retail chain. While the graphical results provide estimates of impact for stores with different characteristics, these characteristics may vary with products shelf lives as well as review intervals. For example, for another product with a shelf life of fifteen days, ordering might be possible only once a week, irrespective of weekly demand; while for yet another product with shelf life of 4 days, ordering might be possible everyday, irrespective of weekly demand. Thus, such product specific parametric combinations for products which have been found to be prone to lost-sales, can be tested in future simulation studies, and impact of the policy modifications, identified. In order to aide future simulation studies, the simulation codes for the three policy comparisons with the first set of parameters, have been appended (Appendix B). These codes can be utilised in future simulations with modifications, as suggested above and as required.

Chapter 6

Conclusion

The objective of this research has been to identify the effect of eliminating replenishment for demand that is lost during the lead time, for perishable inventories with no backorders. The rationale behind choosing perishable inventories for this investigation was the hypothesis that eliminating these orders can contribute to waste reduction in practice, if the simultaneous decrease in fill rates is relatively lower than the waste reductions achieved. Non-perishable inventories can also benefit from such policy modifications, but essentially in terms of the reduction in average inventory levels.

The importance of variable order policies for situations where demand is time-varying, was discussed to highlight the utility of such policies for grocery retail, where demand is time-varying and probabilistic (Broekmeulen and Van Donselaar, 2009). The added advantage of heuristic based variable order policies, was discussed to highlight their utility for inventory management of perishables, since such approaches can utilise the age information in ordering decisions without excessive computational complexity (Nahmias, 2011).

This was followed by discussing the importance of lost-sales inventory systems for grocery retail (Bijvank and Vis, 2011; Corsten and Gruen, 2005; Gruen et al., 2002), and why they are more important for some products than others (Emmelhainz et al., 1991; McKinnon et al., 2007), highlighting their importance for frequently used products such as milk. The differences that must be taken into account, when using the 'inventory position' in ordering decisions for backordering and lost-sales inventory systems, were discussed (Bijvank and Vis, 2011).

Relating the discussions on advantages of variable order policies, and lost-sales systems, three such policies were discussed, to highlight the underlying backordering assumption in their ordering procedure. Modifications were proposed to eliminate this assumption, such that their suitability for lost-sales situations can be increased.

The policies for which these modifications were proposed, were selected from scientific literature. The first policy was a stock-based policy that is the logic behind automated store ordering systems in grocery retail for non-perishables (Kiil et al., 2017; Potter and Disney, 2010). The second policy was a heuristic based extension of this stock-based policy, where a heuristic is deployed to estimate the waste that is expected to occur before the next earliest order arrival instance; the age-based EWA policy, proposed by Broekmeulen and Van Donselaar (2009), which uses the EWA heuristic. The third policy was an extension of the EWA policy which was proposed by Kiil et al. (2017), where they proposed changing the safety stock from constant to varying and calculated at every order instance (thus, the name EWA_{SS}); and changing the buffer stock from the sum of the safety stock and estimated waste, to the larger of the two.

After demonstrating the proposed modifications, characteristics of milk inventory in a grocery retail store were discussed. These discussions were based on information or inferences drawn from the information, that was found in publications on empirical research, which provided information about milk supply chain structure, logistical characteristics of milk supply, activities in grocery retail, etc. These inferences and information provided insights which guided the development of a simulation model, on which the effect of the proposed policy modifications could be studied. The product choice of milk was based on the rationale that milk stock-outs are highly likely to result in lost-sales (Emmelhainz et al., 1991; McKinnon et al., 2007). However, three product characteristics were varied in the experiments, one at a time, to expand the simulation results and conclusions that can be drawn from them.

Broekmeulen and Van Donselaar (2009) compare the stock-based policy and EWA policy under the assumption of a constant safety stock, and the calculation or quantity of this safety stock is not explicitly specified by them. As a result, the safety stock calculations for these policies was given the same treatment as the EWA_{SS} policy. This adjustment could be a source of discrepancies in the results derived from the simulation experiments, and the shortcoming can be overcome in the future if the constant safety stock can be identified through cost optimisation. However, using cost optimisation to determine the safety stock, while using the target service level to estimate lost-sales cost (as done by Broekmeulen and Van Donselaar (2009)), is a contradictory amalgamation of two approaches. The approach of satisficing service levels is taken due to the complexities in estimating costs (Minner and Transchel, 2010). Broekmeulen and Van Donselaar (2009) utilise the result from a single period newsvendor model to estimate cost of lost-sales for a multi-period inventory model. Since theoretical justifications or explanation for their approach could not be identified, the safety stock calculation was conducted as in the EWA_{SS} policy. It is also suspected that their approach highly overestimates the cost of lost-sales, and should be subjected to examination in future research.

After describing the modelling process and the simulation setup, experiments were conducted to answer the research question. While the model development and base experiment for every policy comparison were conducted with parameters representing a milk inventory, as mentioned earlier, three other parameter settings were also tested. Waste reduction was observed with reduced fill rates, and it was observed that majority of the waste reduction came from stores with low weekly demand and high review intervals, since the stores with high demand and low review intervals had no waste under the unmodified policy. In order to systematically visualise the impact of policy modifications for different stores, the ratio of change in overstocking and change in understocking was calculated for all stores, and this parameter was named the 'value of policy change'. The change in overstocking was calculated as the change in waste percentage, and change in understocking was calculated as the change in fill rates; where both changes represent change as a result of moving from unmodified policy to the modified policy. These value measures for different stores were plotted while discussing the implications and interpretations of the results for different stores.

These value plots showed that the value of policy change is higher for policies that order higher buffers, i.e., the EWA and EWA_{SS} policies, as compared to the stock-based policy. Additionally, it was observed the lower shelf life shows non-zero value for such a policy modification for larger number of stores; implying that for products with lower shelf life than milk, if customer responses to stock-outs reflect predominance of lost-sales rather than backorders, such modifications could be valuable. Similarly for products with lower control on customer picking. On the other hand, as the batch size decreases, the number of stores that benefit from policy change decreases while increasing the value for the benefiting stores.

The entire study has been conducted using secondary data and information, i.e., empirical data and information that were collected by other researchers for their studies. While this has provided basis for investigating the problem of interest, the results are expected to vary as model parameters and assumptions are adjusted further to reflect real world data. Not having access to POS data to simulate demand can be considered the biggest methodological shortcoming of this research. As a result, aggregated average weekly demands (Kiil et al., 2017) were used, further adding the assumption of a specific repeating weekly pattern (Kahn and Schmittlein, 1989). The research has contributed to knowledge by highlighting an aspect of ordering policies that is under-researched for all inventory systems (Bijvank and Vis, 2011), and was not found to have received much attention for perishable inventories either. While external validity of the results should be subjected to scrutiny in the future, an essential contribution has been highlighting that improvements in inventory performance, and specifically waste reduction, are feasible without necessarily investing in collection of age information. However, this finding is also subject to the assumption that literature reflects practice, i.e., the stock-based policy is used in practice as found in literature, which may not be the case. This highlights the difference between contribution to knowledge and contribution to practice. As emphasised by Karlsson (2010), both of these contributions are addressed by good operations management research. Thus, while the contribution to knowledge is sufficiently apparent for this research, the contribution to practice should be subjected to validation in the future.

Additionally, as pointed out in the previous chapter, the effects of policy modifications have been examined for cases when ordering is done under no outstanding orders. However, examining cases where ordering is done under other outstanding orders, can expand knowledge of these policy modification effects for such cases, and identify if the severity is higher or lower than the cases examined in this research.

Based on the lack of attention that lost-sales systems have received (Bijvank and Vis, 2011), and the findings of this research; it can be concluded that taking lost-sales into account while ordering for perishables has a substantial impact. While this impact may vary with situation and product related variables, the extent of variation is a subject for future studies. Thus, products for which empirical studies have established low likelihoods of backorders, should be subject of similar future studies, while conducting these studies in collaboration with practitioners to ensure higher external validity and access to data. To identify such products, insights from studies on stock-outs and customer responses to stock-outs, should serve as a good starting point for future studies.

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Appendix A

Graphical simulation results

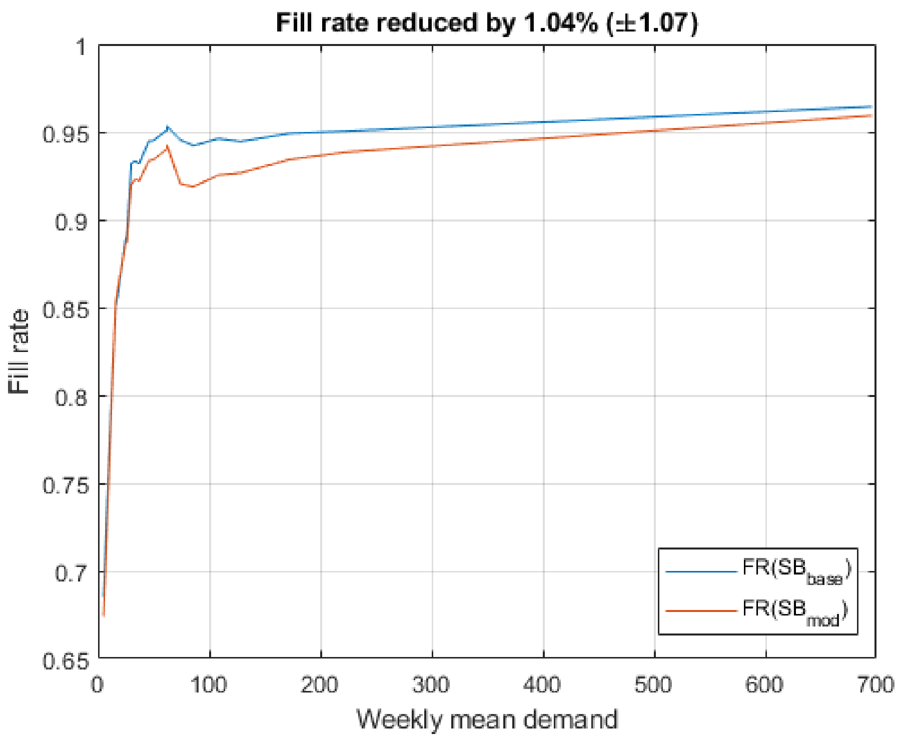

Figure A.1: Fill rate comparison for SB_{base} and SB_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

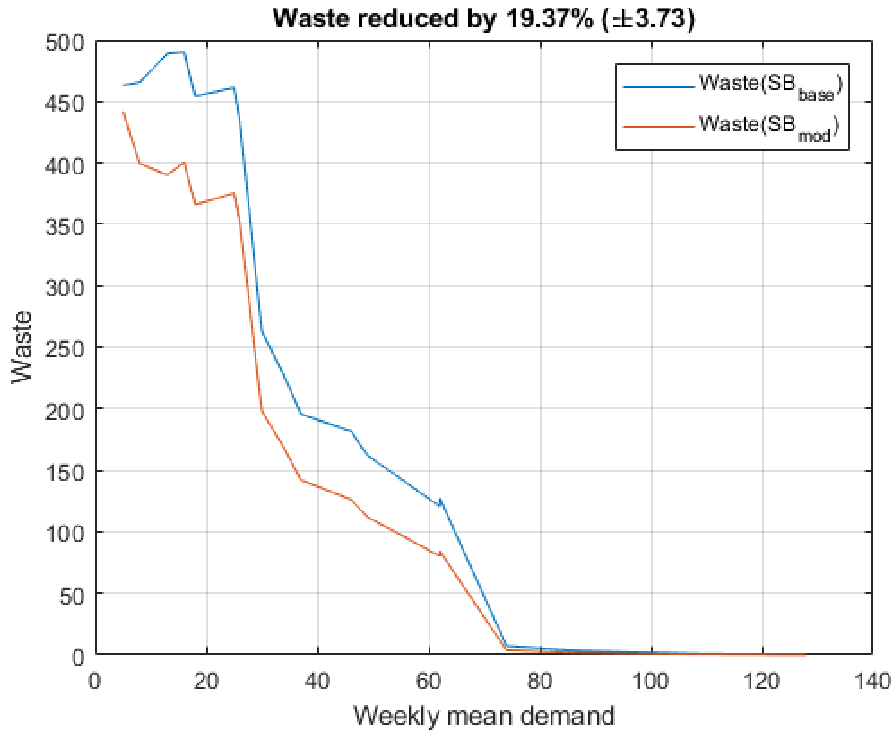

Figure A.2: Waste comparison for SB_{base} and SB_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

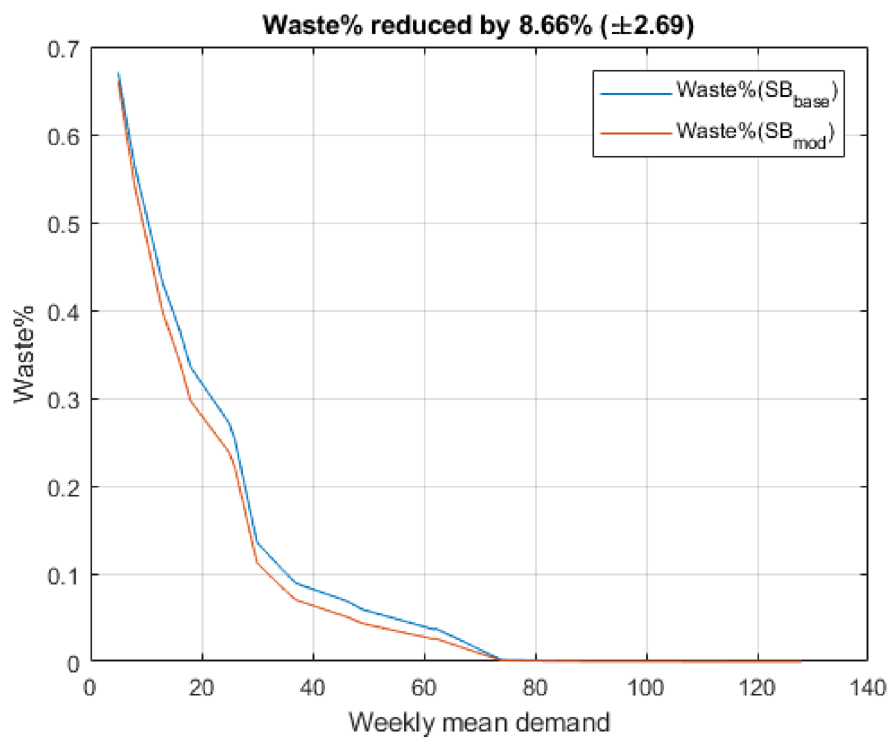

Figure A.3: Waste% comparison for SB_{base} and SB_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

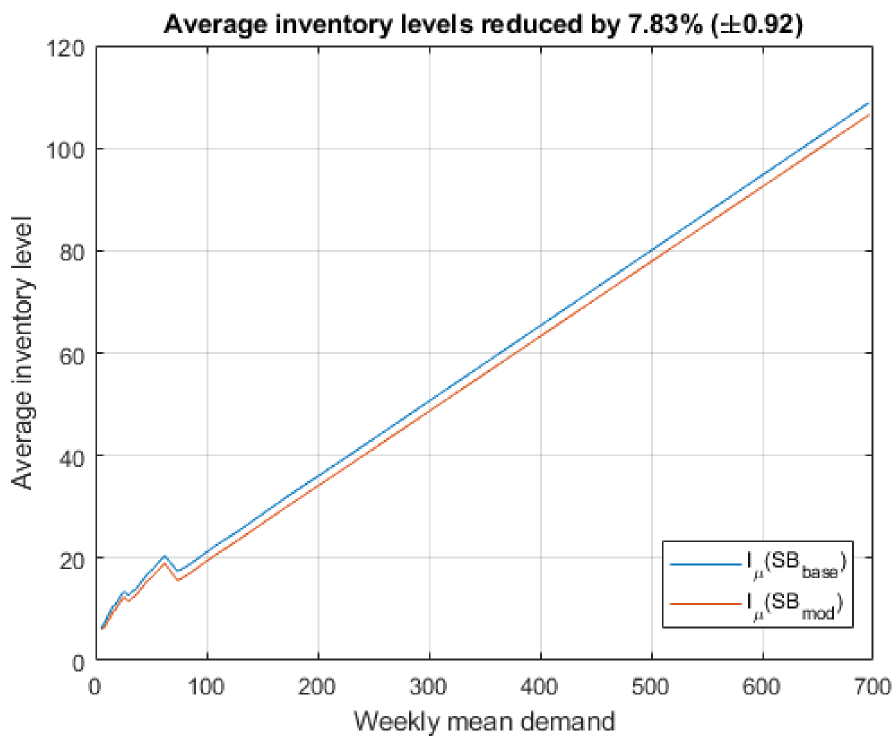

Figure A.4: Average inventory level comparison for SB_{base} and SB_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

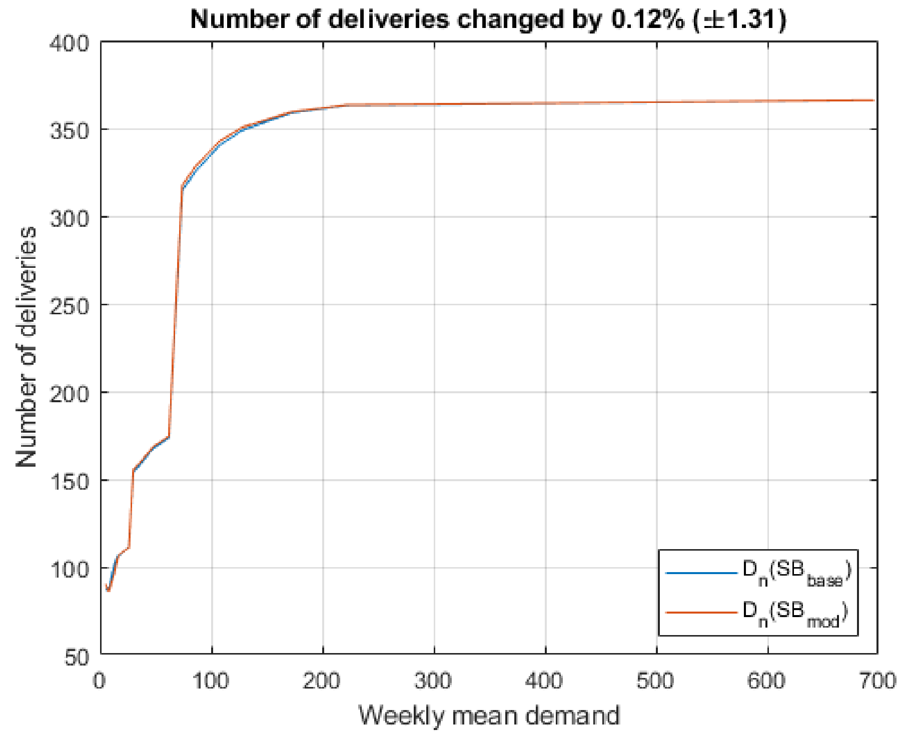

Figure A.5: Comparison of number of deliveries for SB_{base} and SB_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

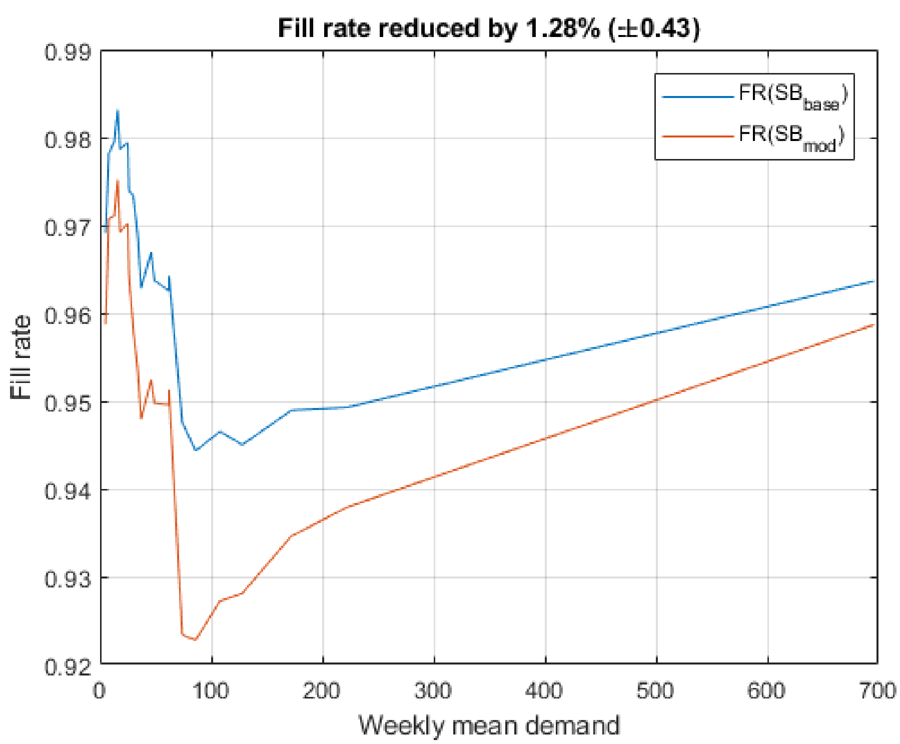

Figure A.6: Fill rate comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

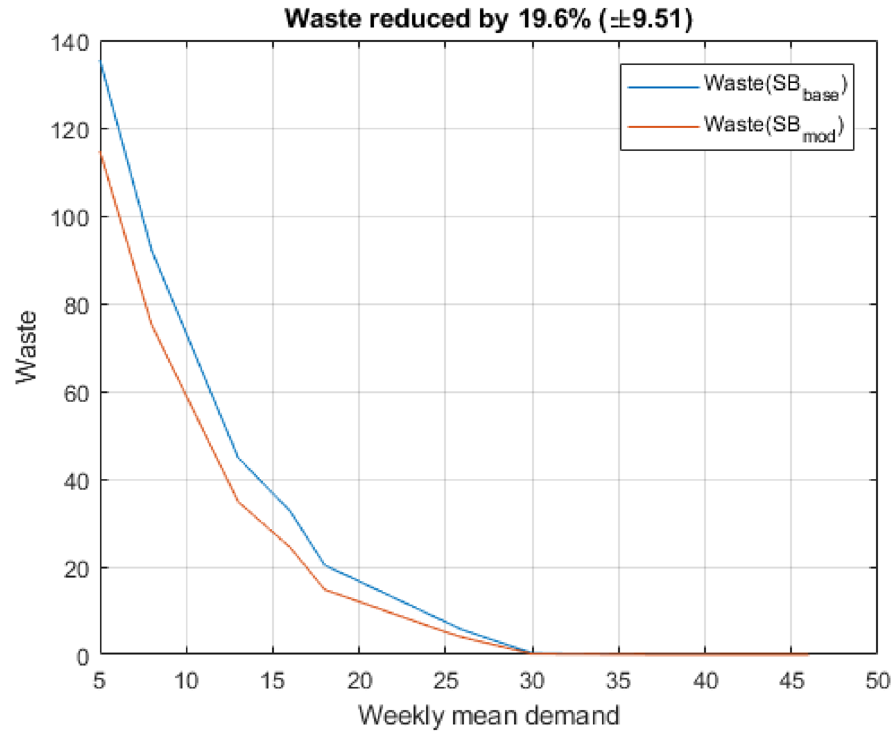

Figure A.7: Waste comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

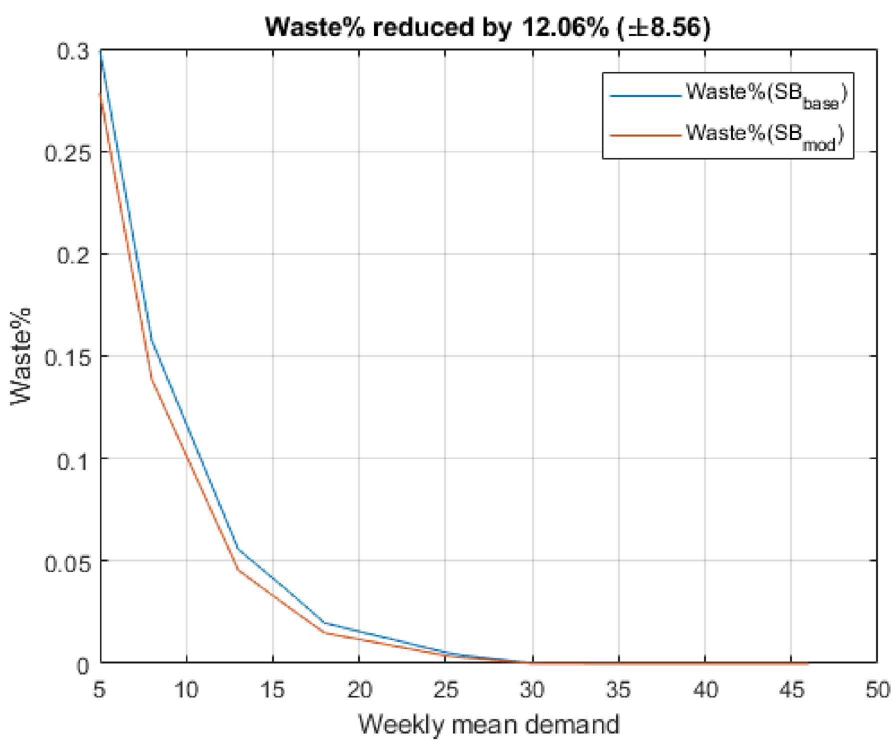

Figure A.8: Waste% comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

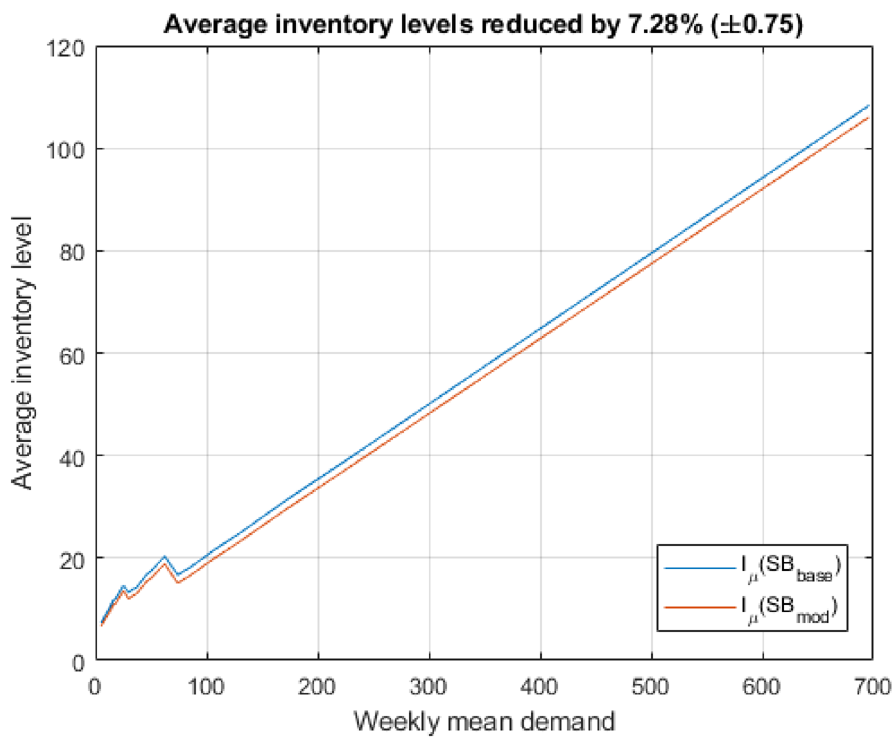

Figure A.9: Average inventory level comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

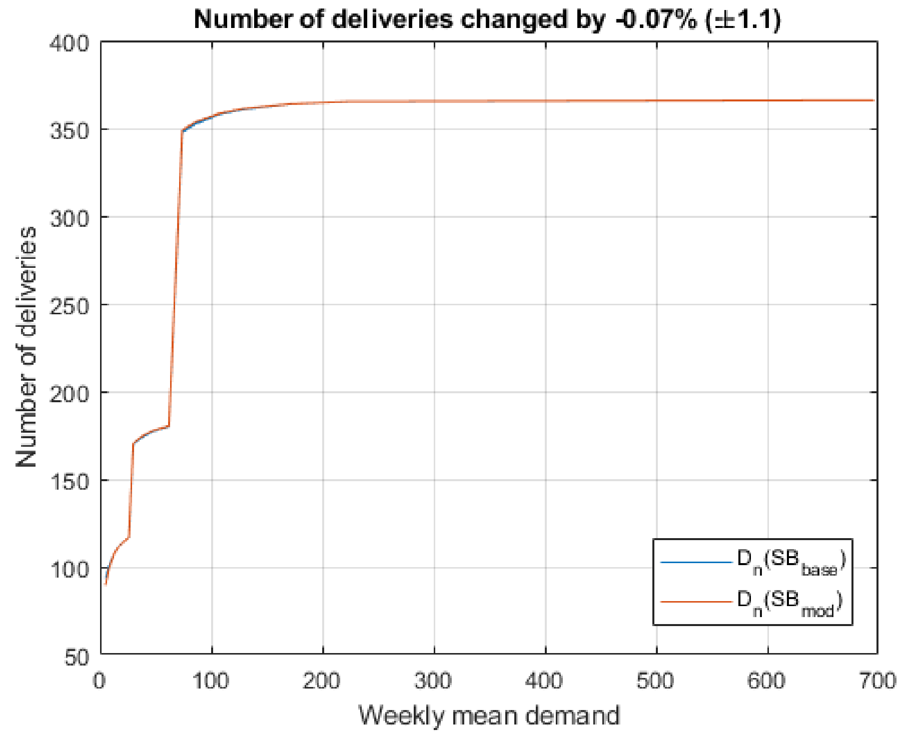

Figure A.10: Comparison of number of deliveries for SB_{base} and SB_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

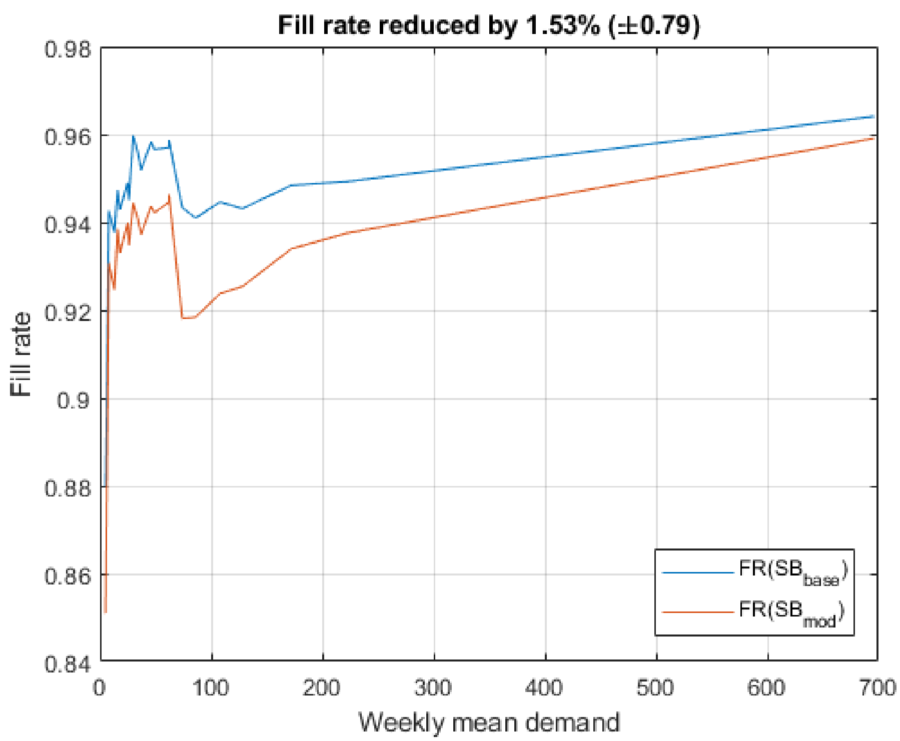

Figure A.11: Fill rate comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

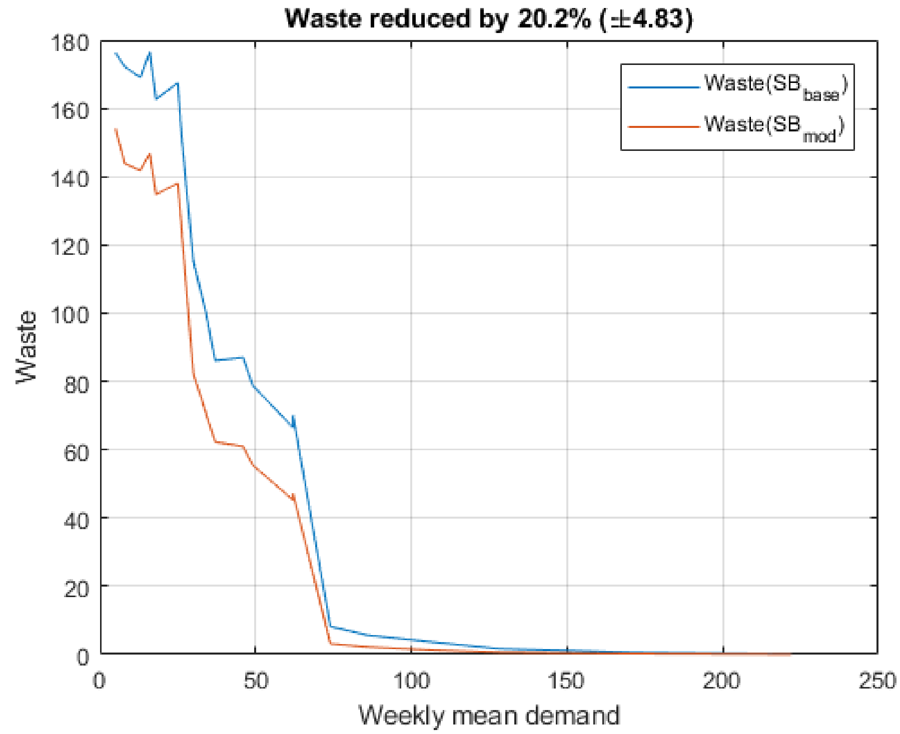

Figure A.12: Waste comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

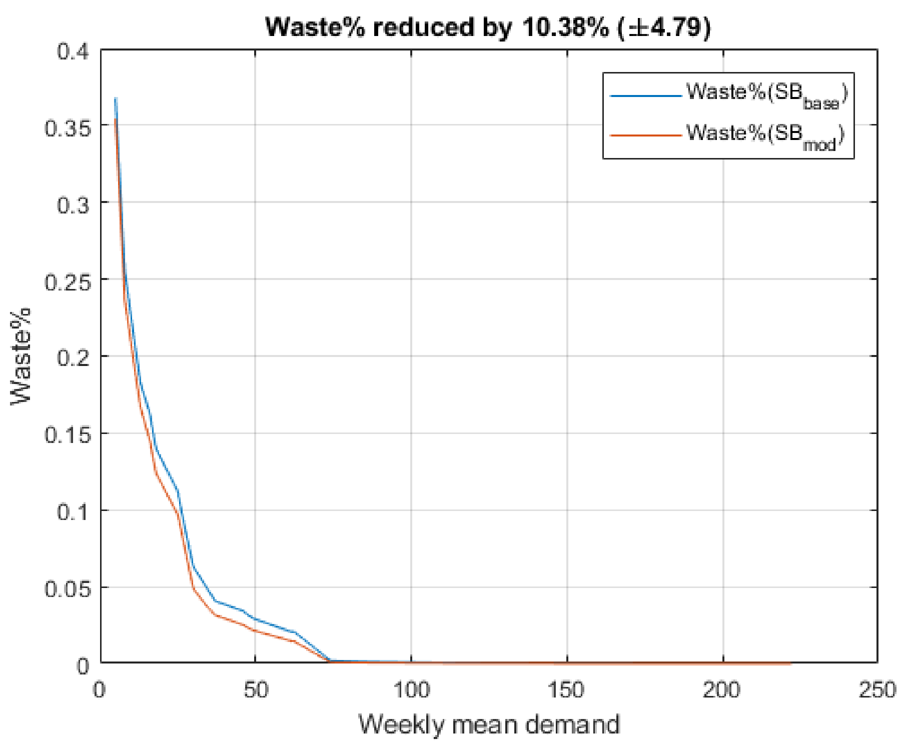

Figure A.13: Waste% comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

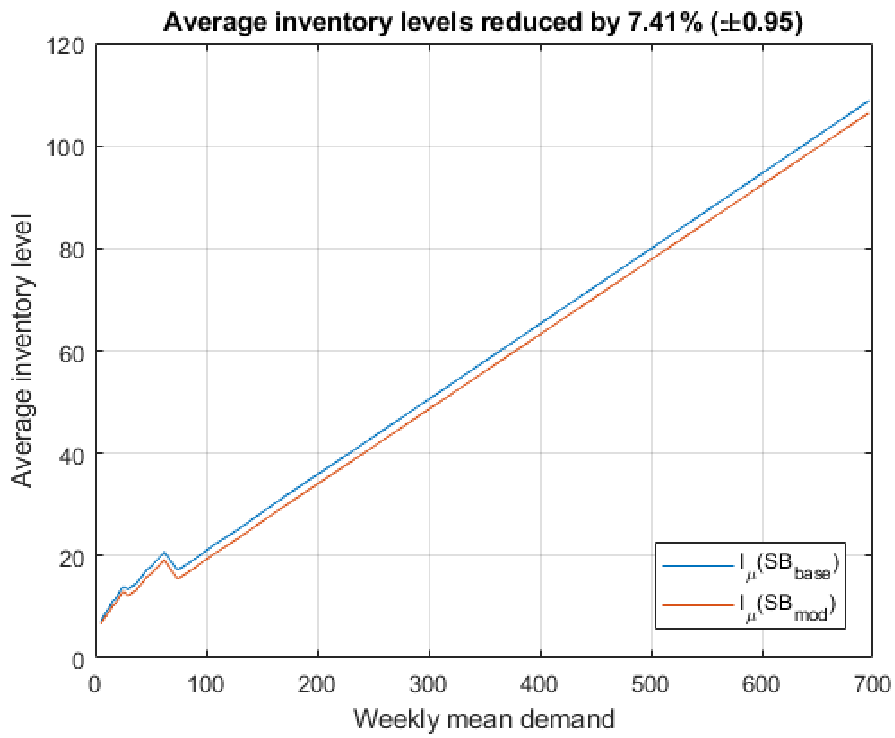

Figure A.14: Average inventory level comparison for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

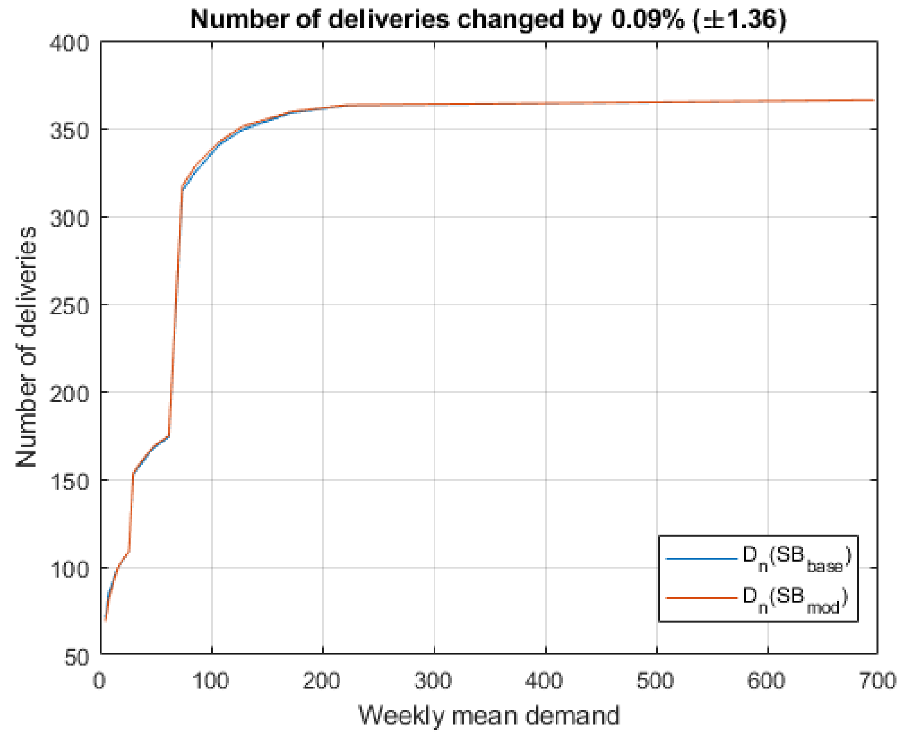

Figure A.15: Comparison of number of deliveries for SB_{base} and SB_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

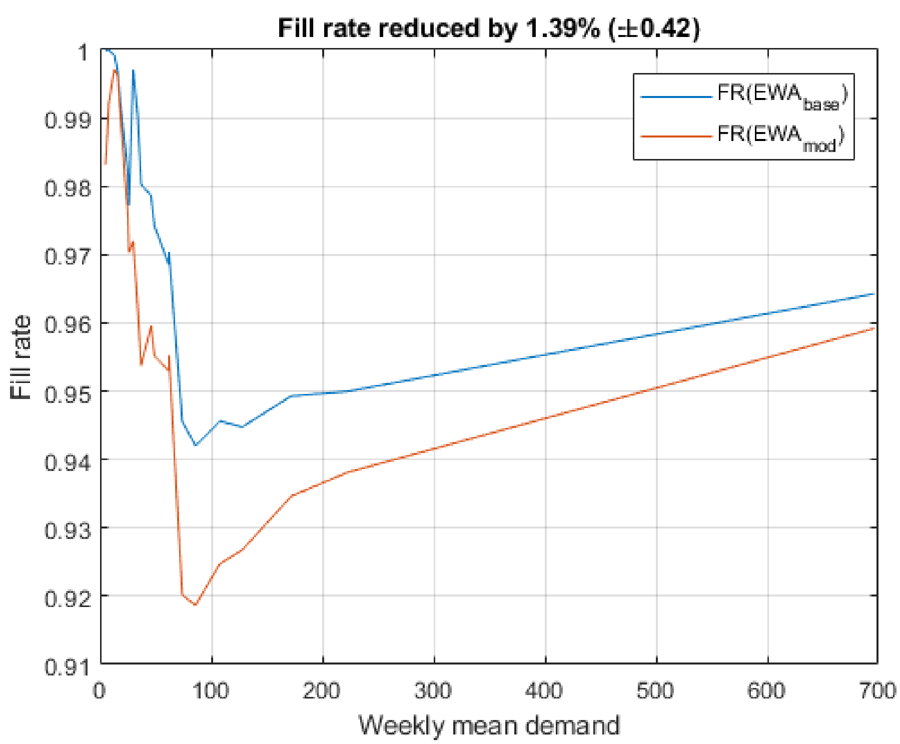

Figure A.16: Fill rate comparison for EWA_{base} and EWA_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

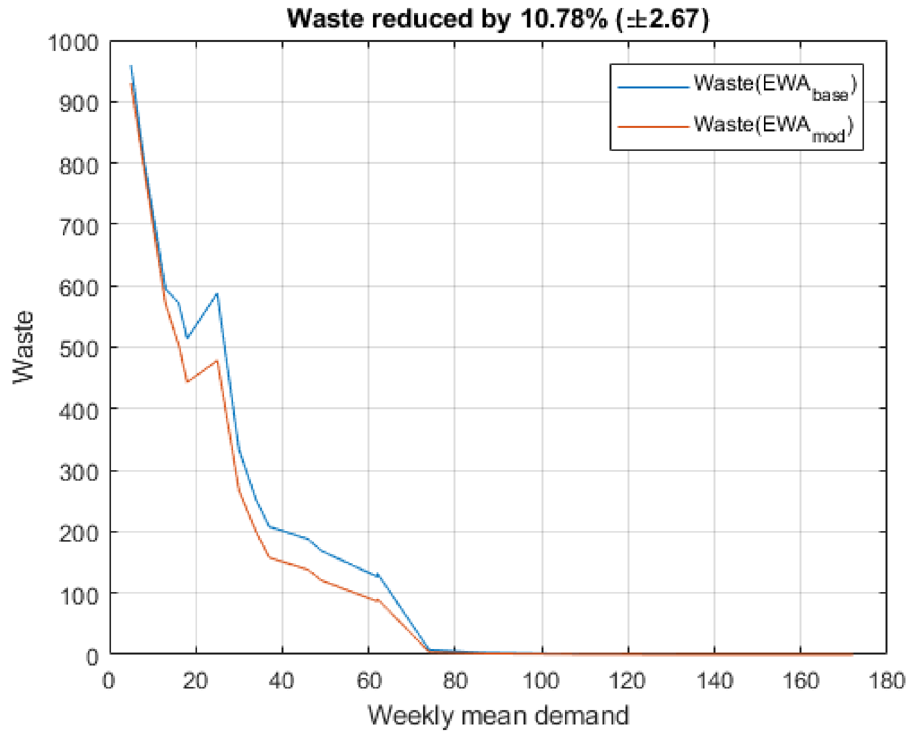

Figure A.17: Waste comparison for EWA_{base} and EWA_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

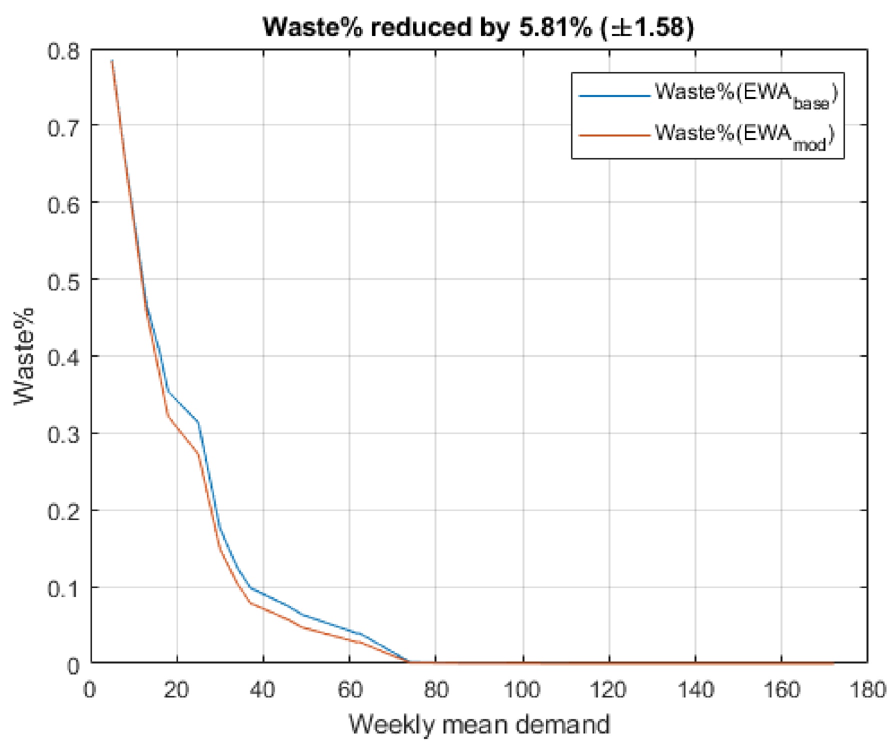

Figure A.18: Waste% comparison for EWA_{base} and EWA_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

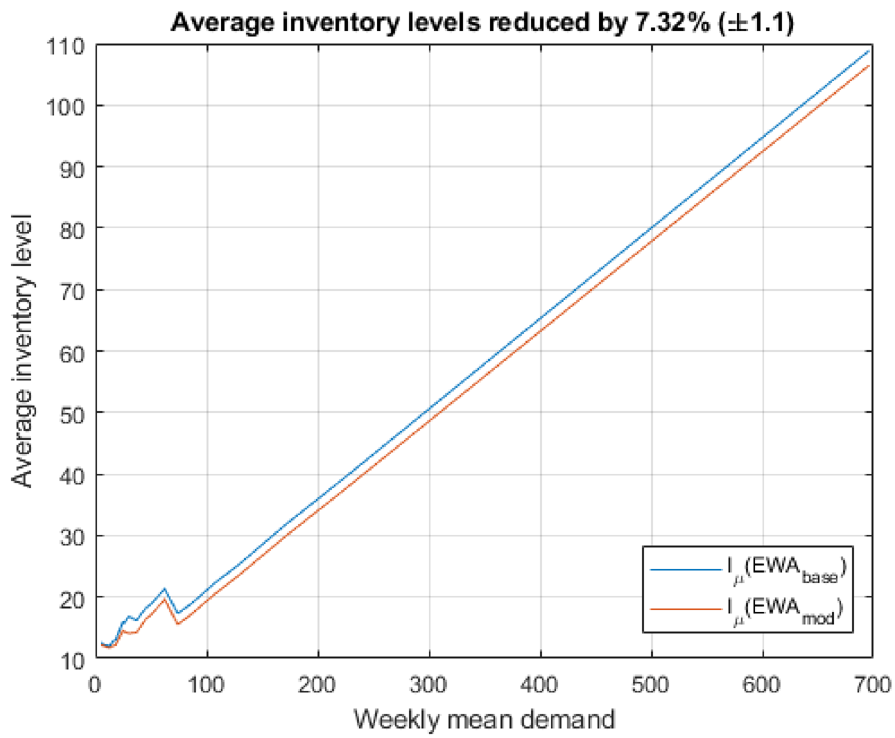

Figure A.19: Average inventory level comparison for EWA_{base} and EWA_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

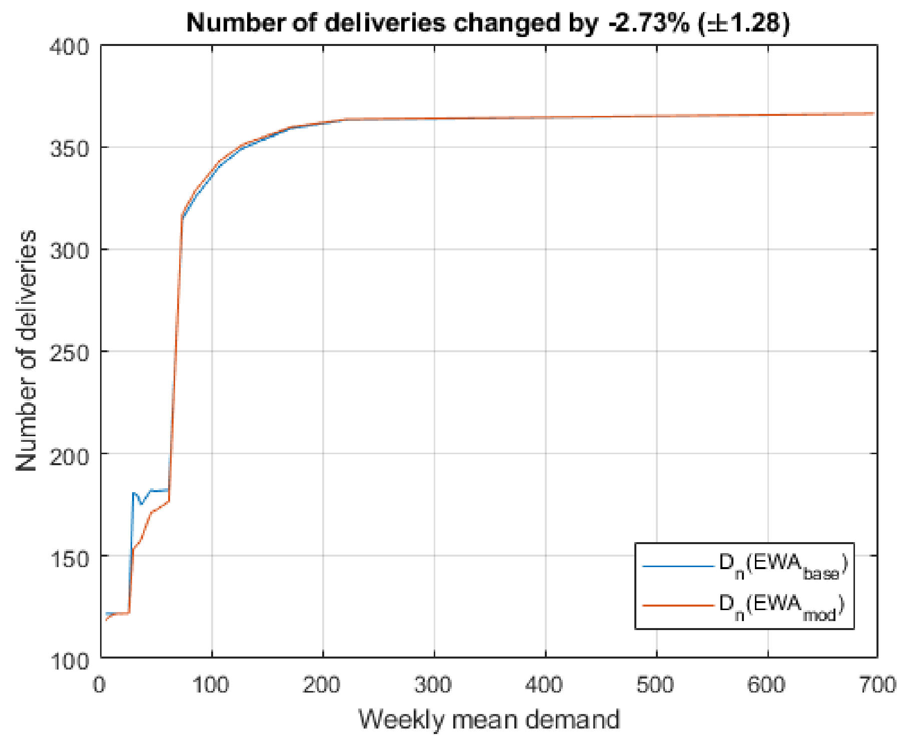

Figure A.20: Comparison of number of deliveries for EWA_{base} and EWA_{mod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

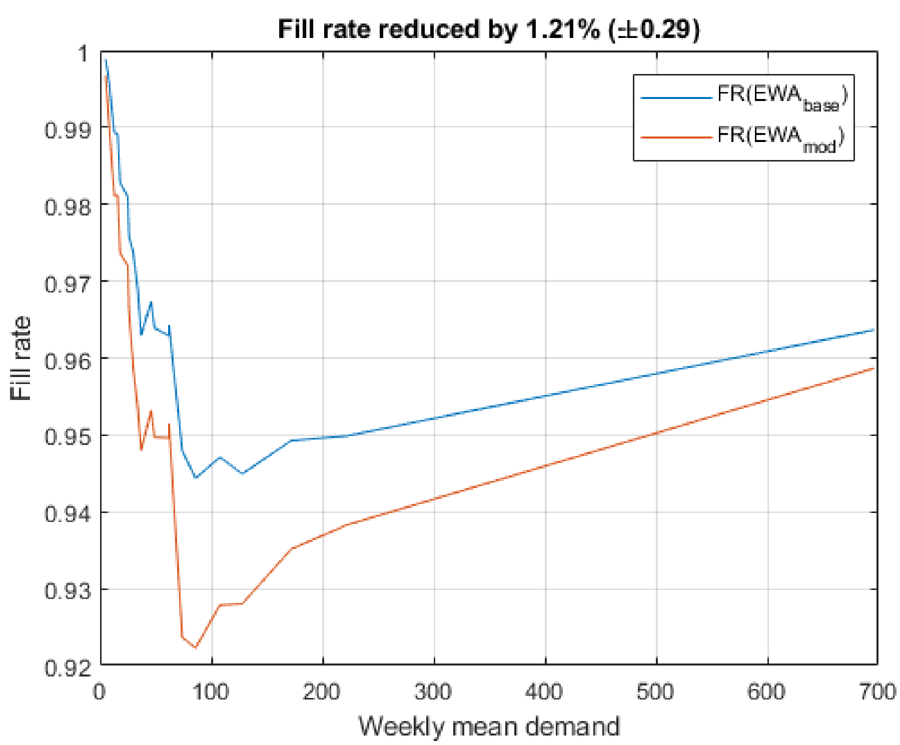

Figure A.21: Fill rate comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

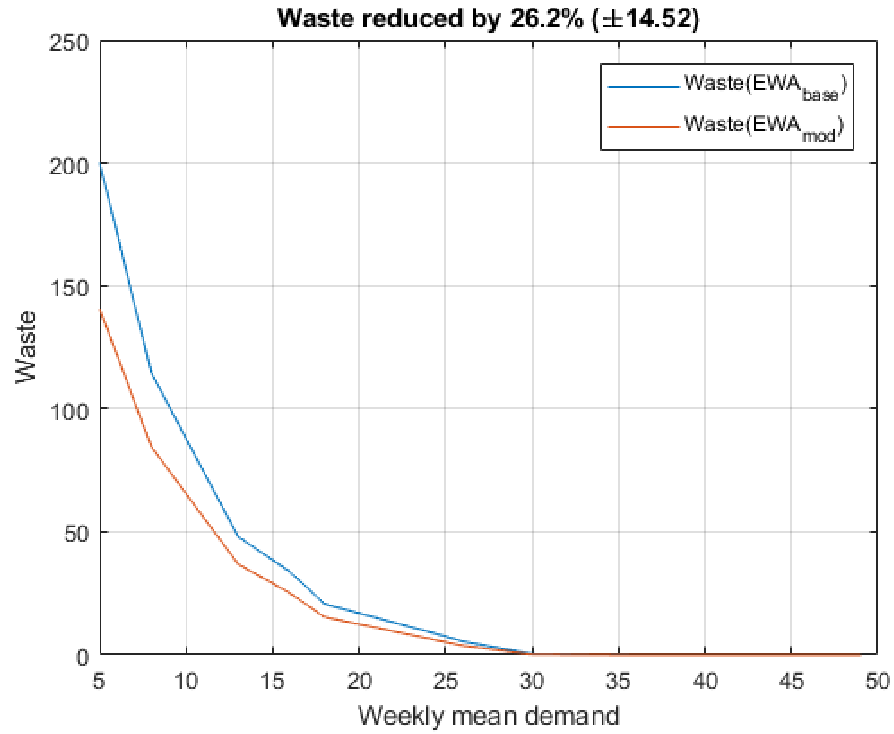

Figure A.22: Waste comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

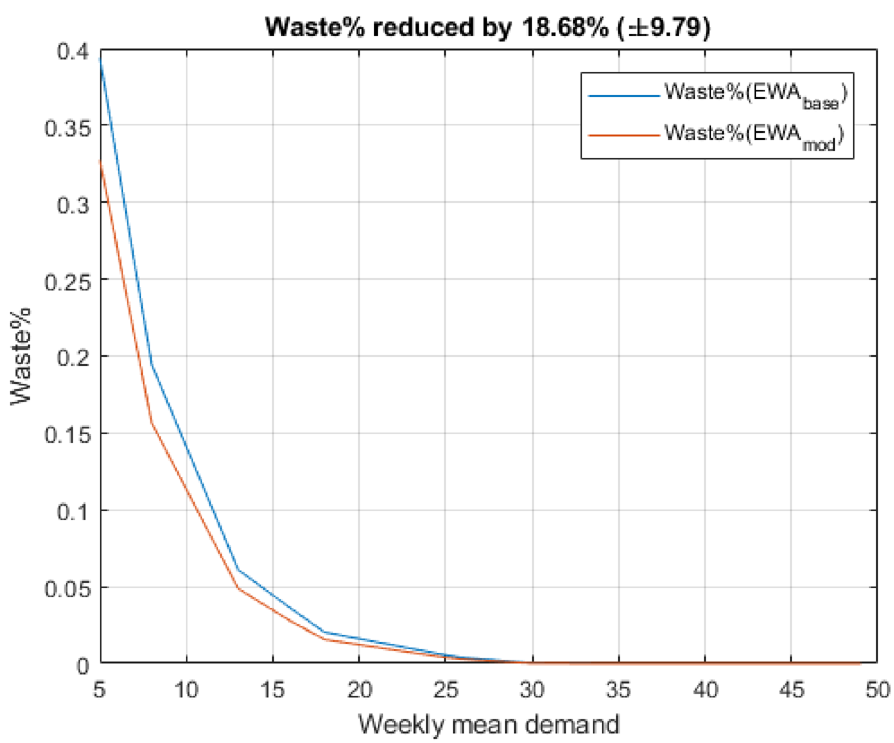

Figure A.23: Waste% comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

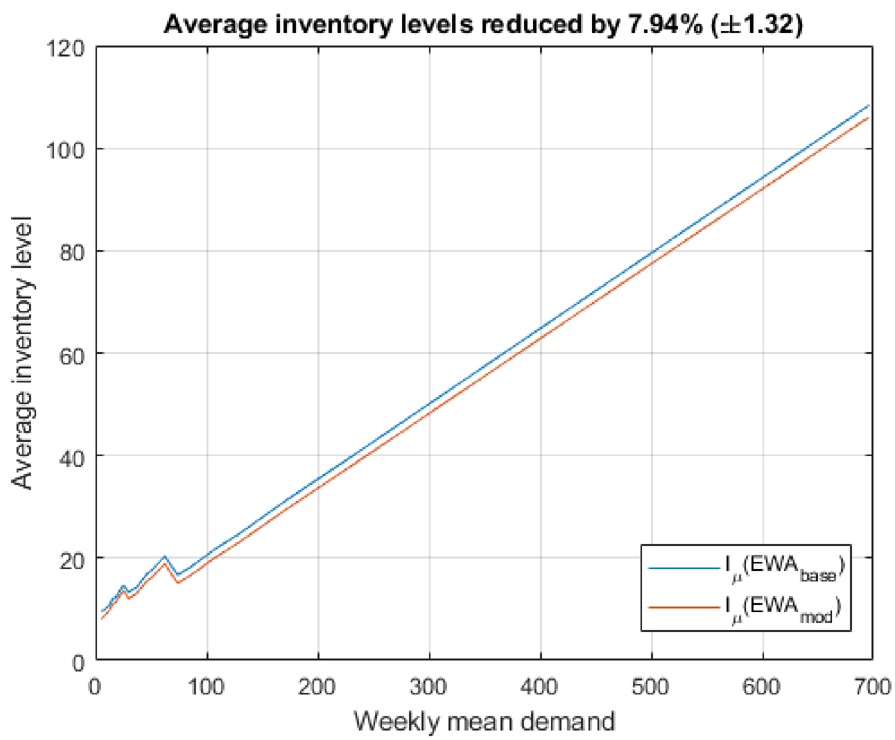

Figure A.24: Average inventory level comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

Figure A.25: Comparison of number of deliveries for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

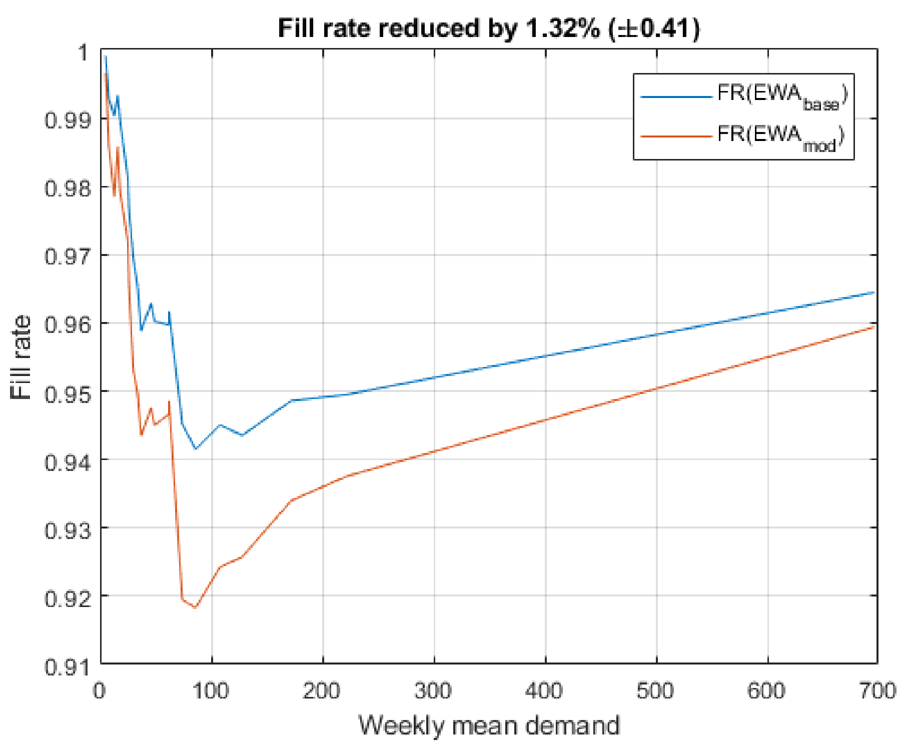

Figure A.26: Fill rate comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

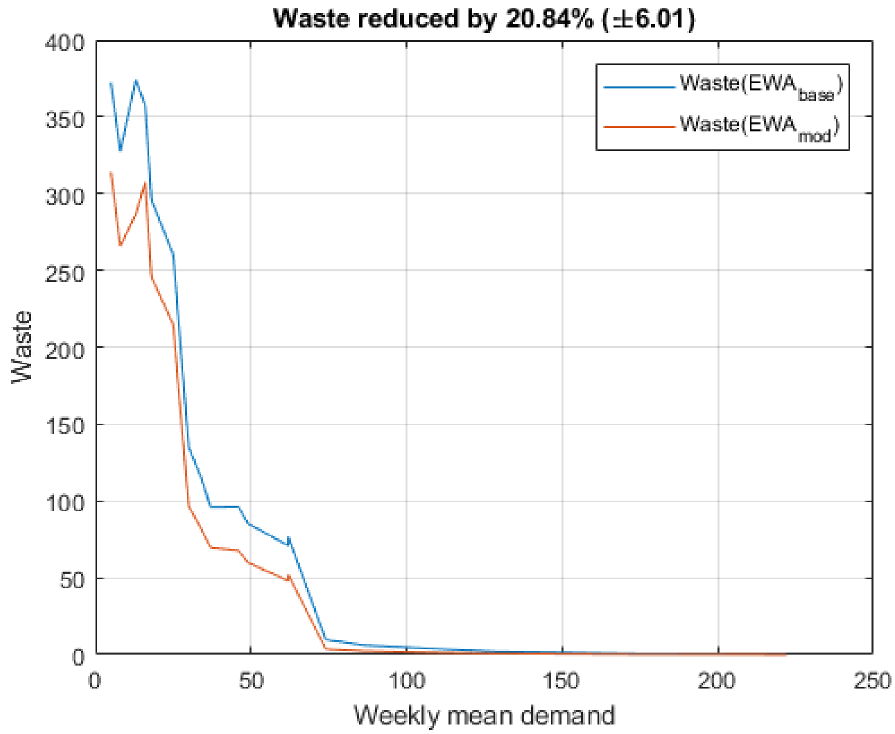

Figure A.27: Waste comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

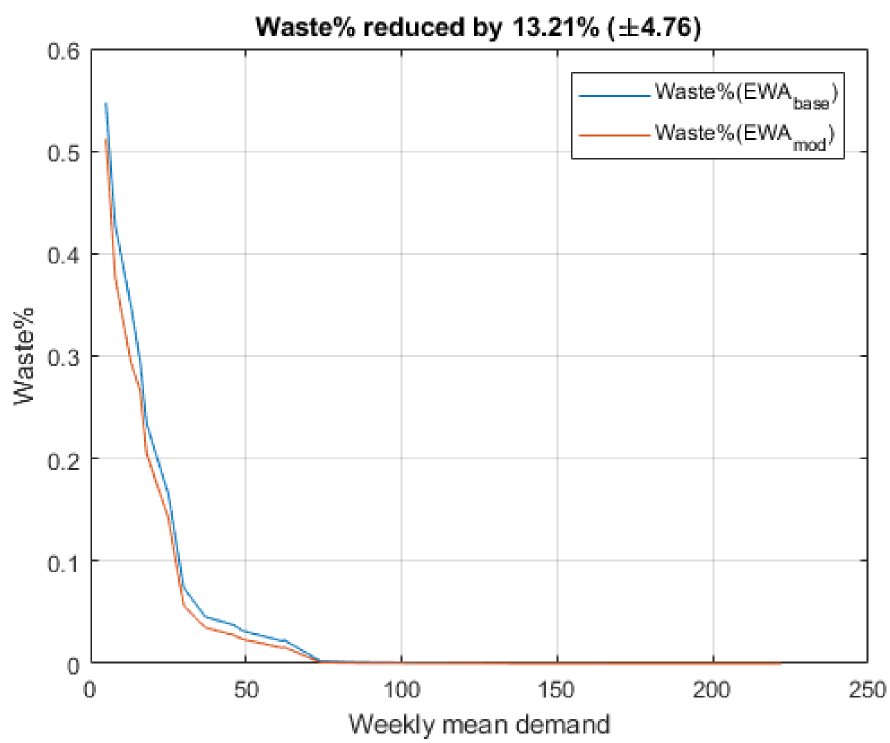

Figure A.28: Waste% comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

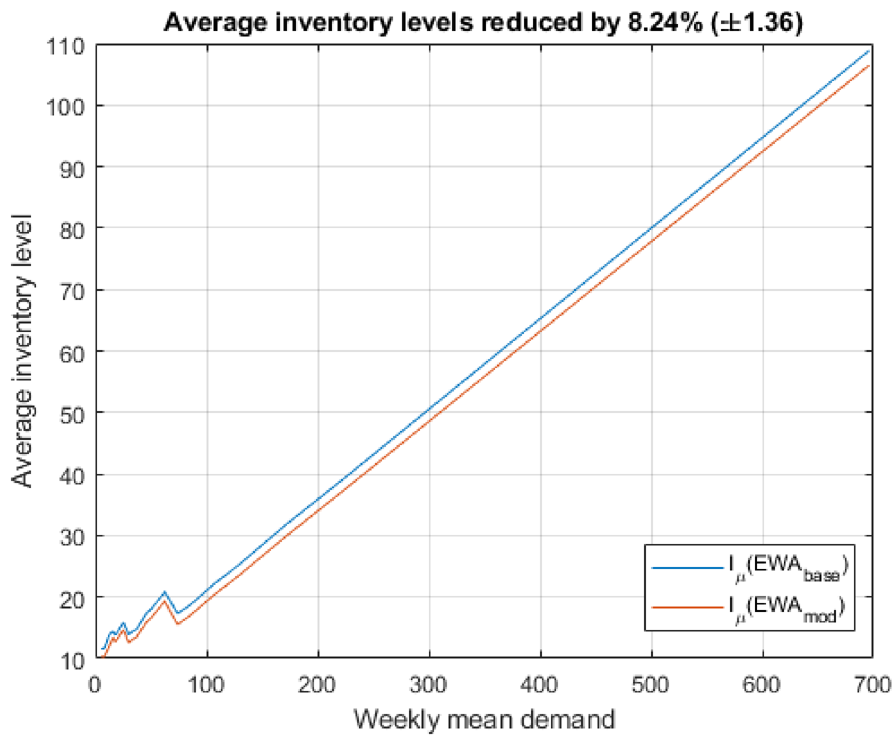

Figure A.29: Average inventory level comparison for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

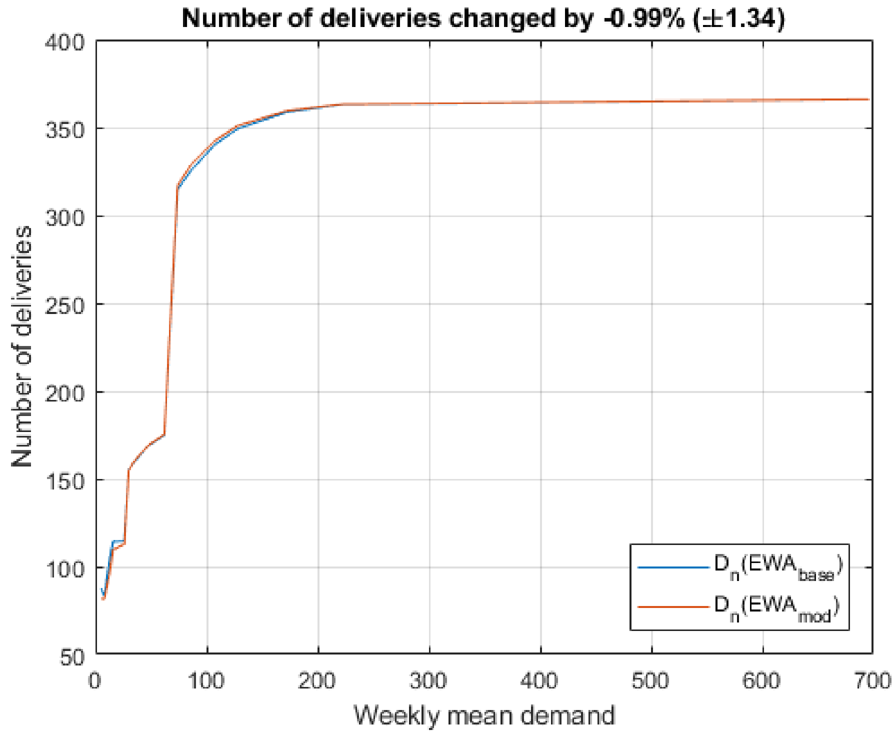

Figure A.30: Comparison of number of deliveries for EWA_{base} and EWA_{mod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

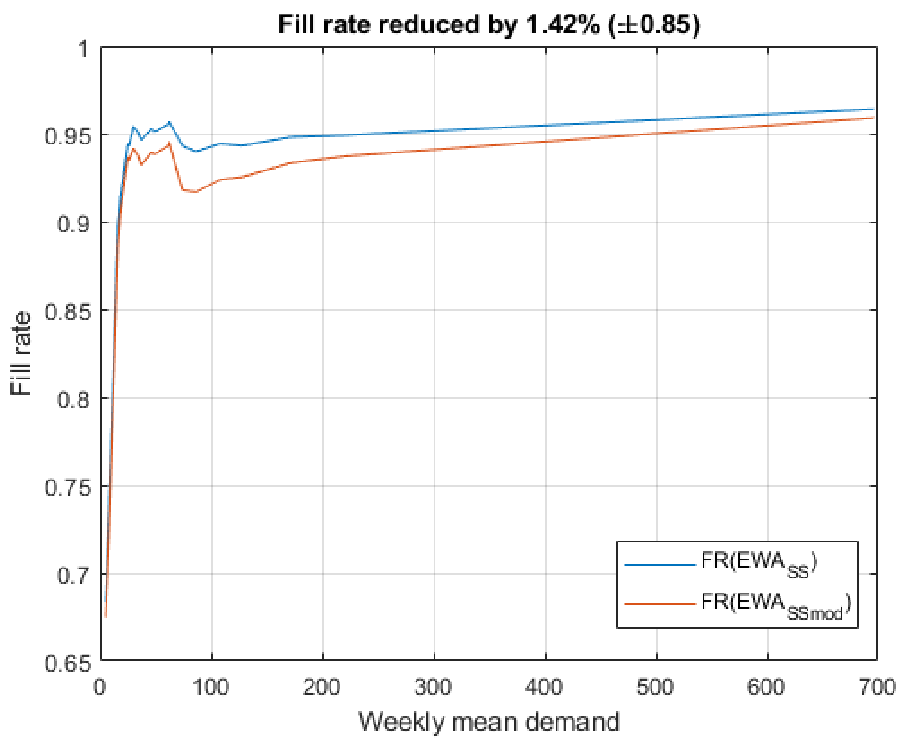

Figure A.31: Fill rate comparison for EWA_{SS} and EWA_{SSmod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

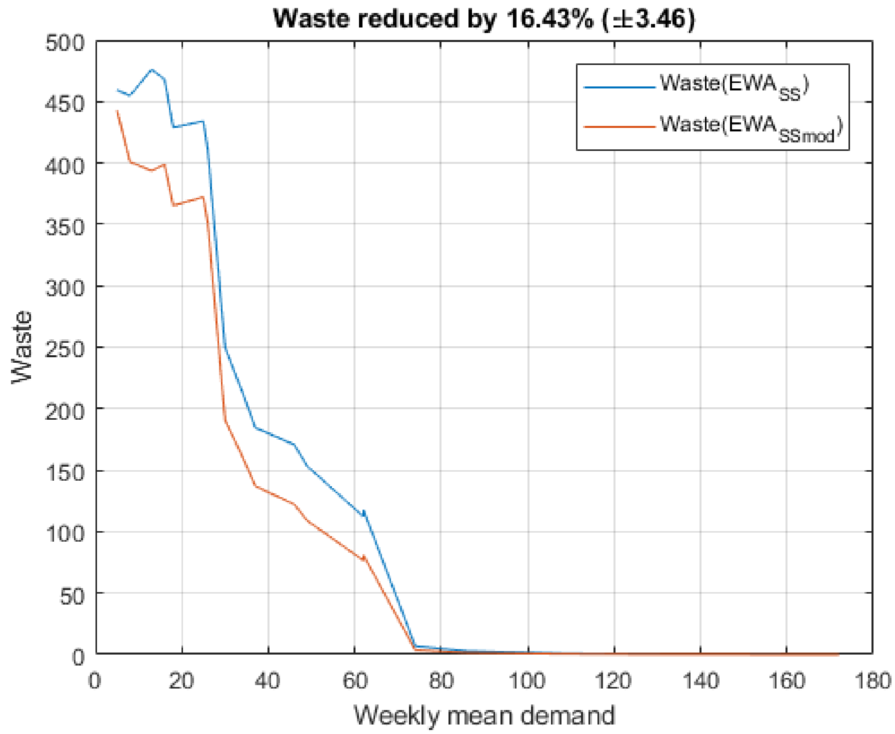

Figure A.32: Waste comparison for EWA_{SS} and EWA_{SSmod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

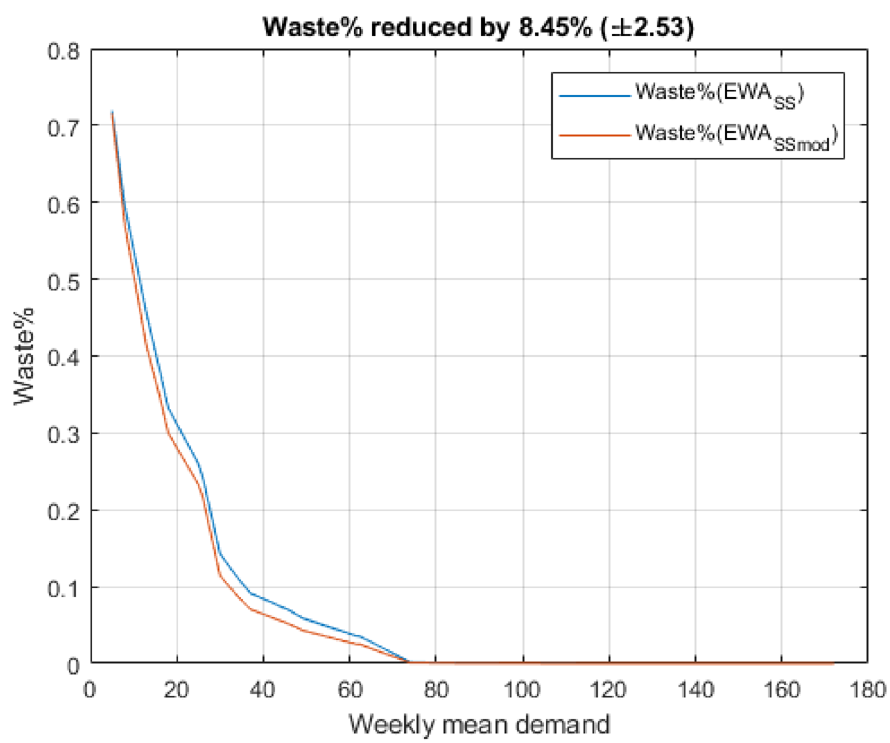

Figure A.33: Waste% comparison for EWA_{SS} and EWA_{SSmod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

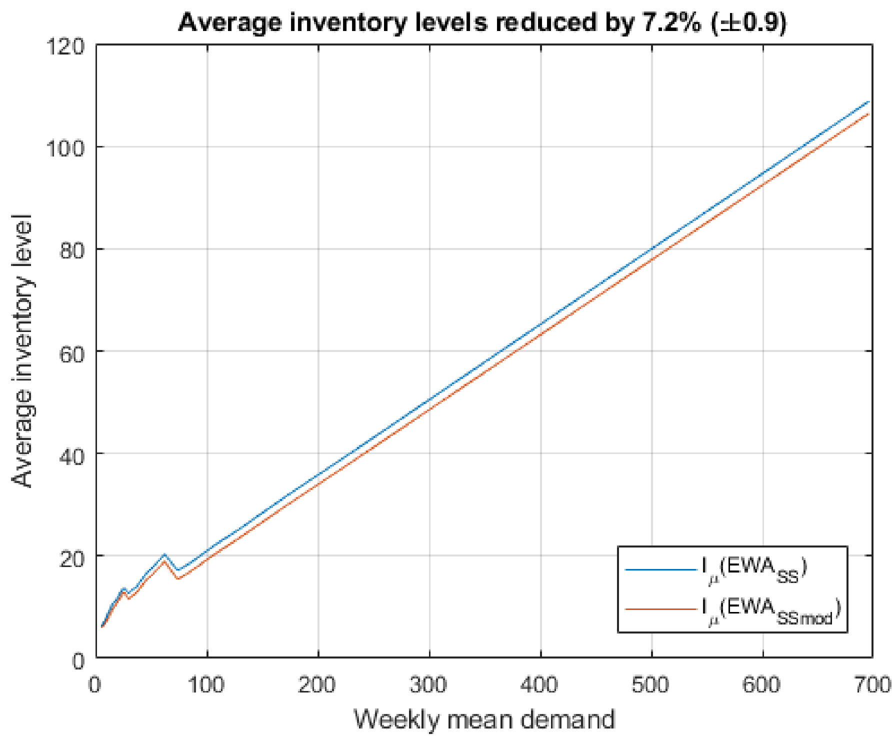

Figure A.34: Average inventory level comparison for EWA_{SS} and EWA_{SSmod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

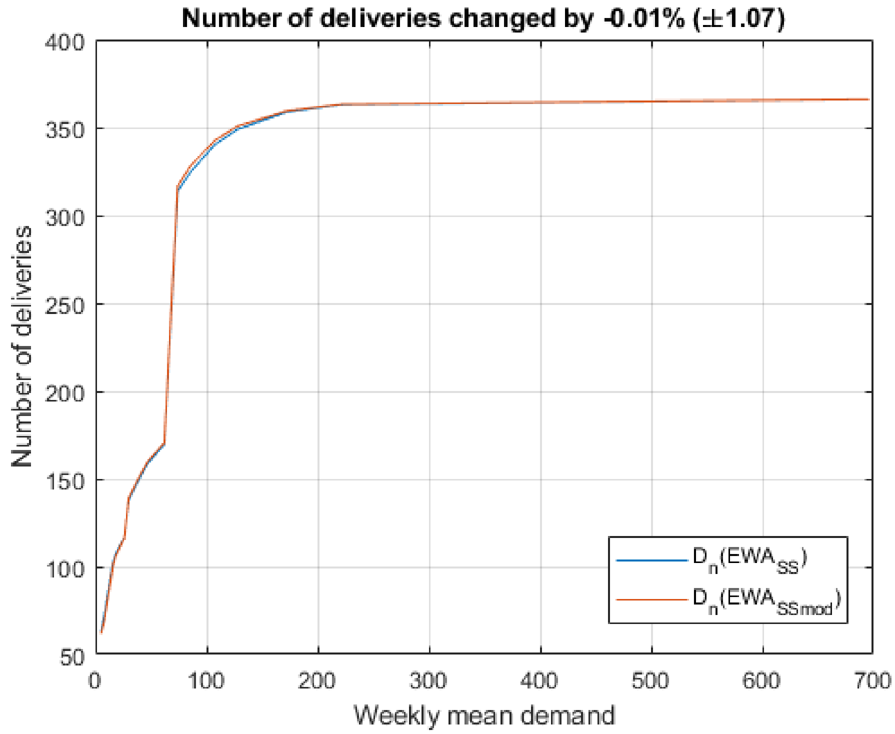

Figure A.35: Comparison of number of deliveries for EWA_{SS} and EWA_{SSmod} under $m = 4$, $Q = 10$, $FIFO = 0.9$

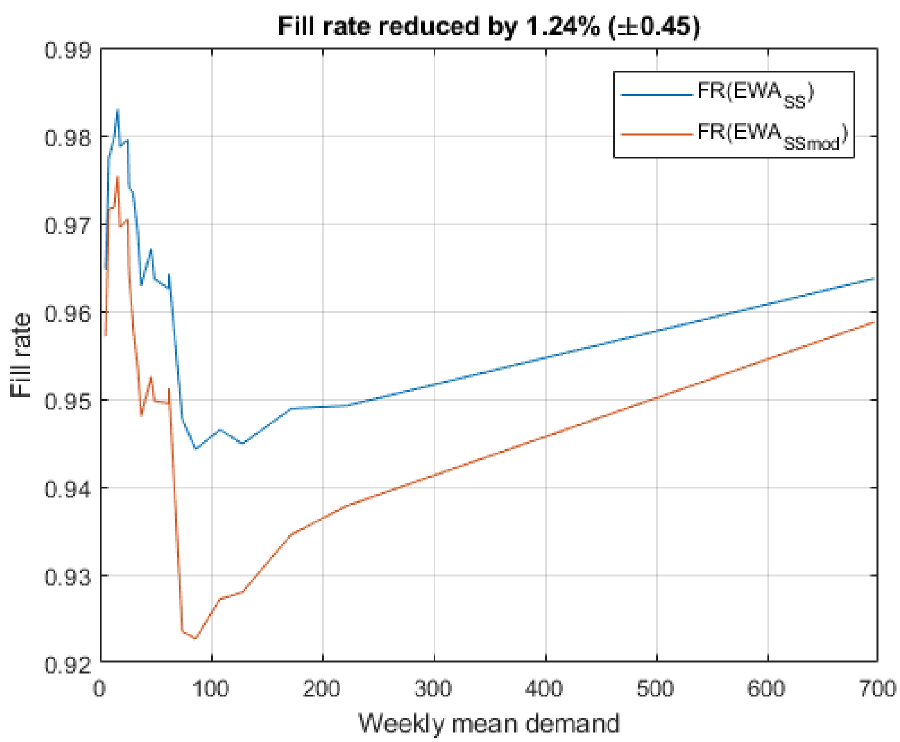

Figure A.36: Fill rate comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

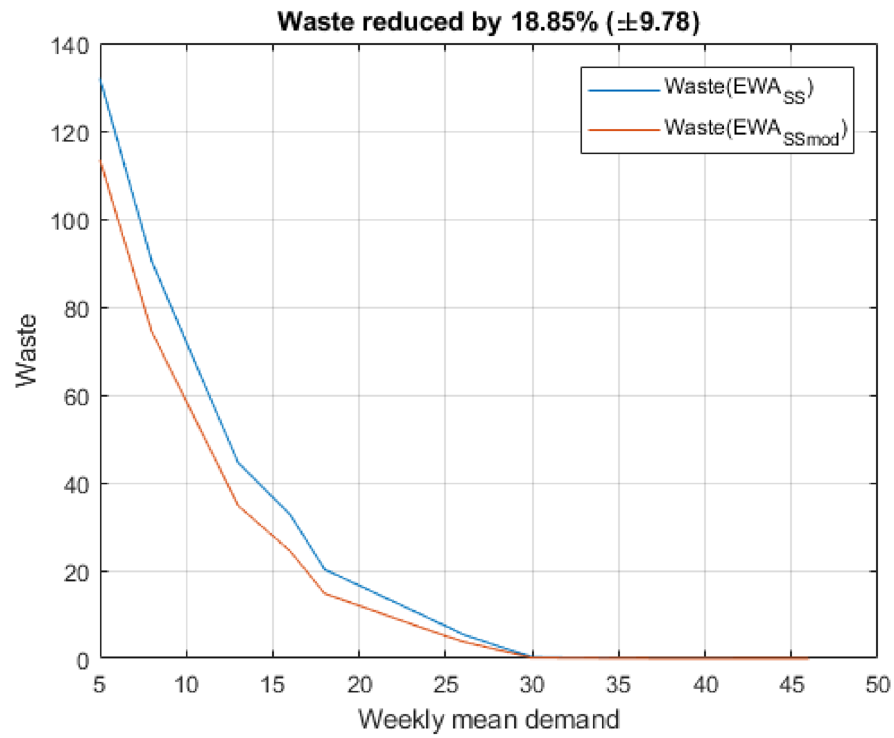

Figure A.37: Waste comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

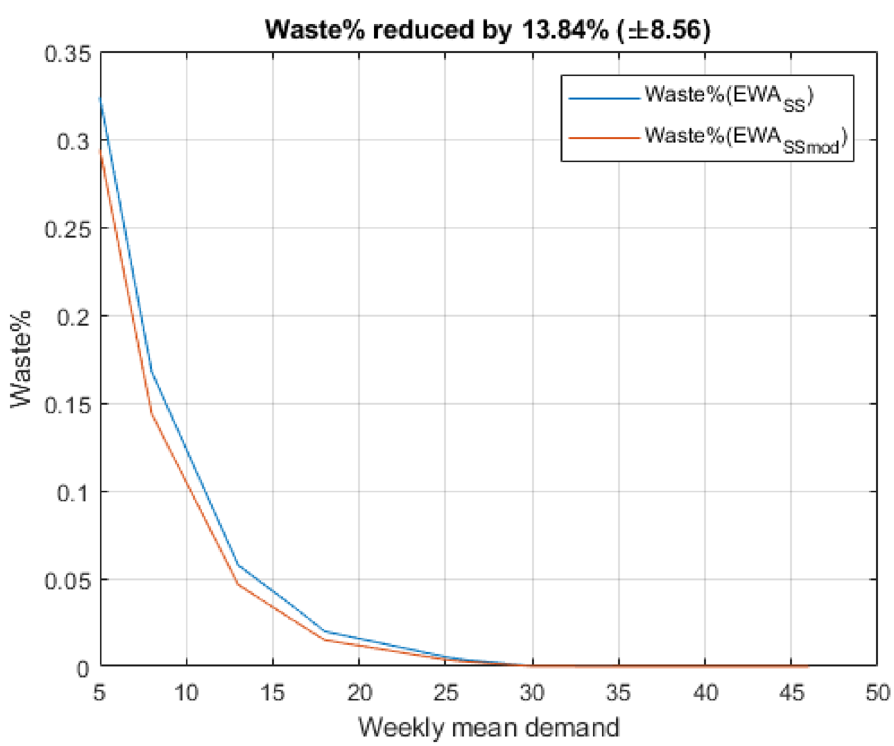

Figure A.38: Waste% comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

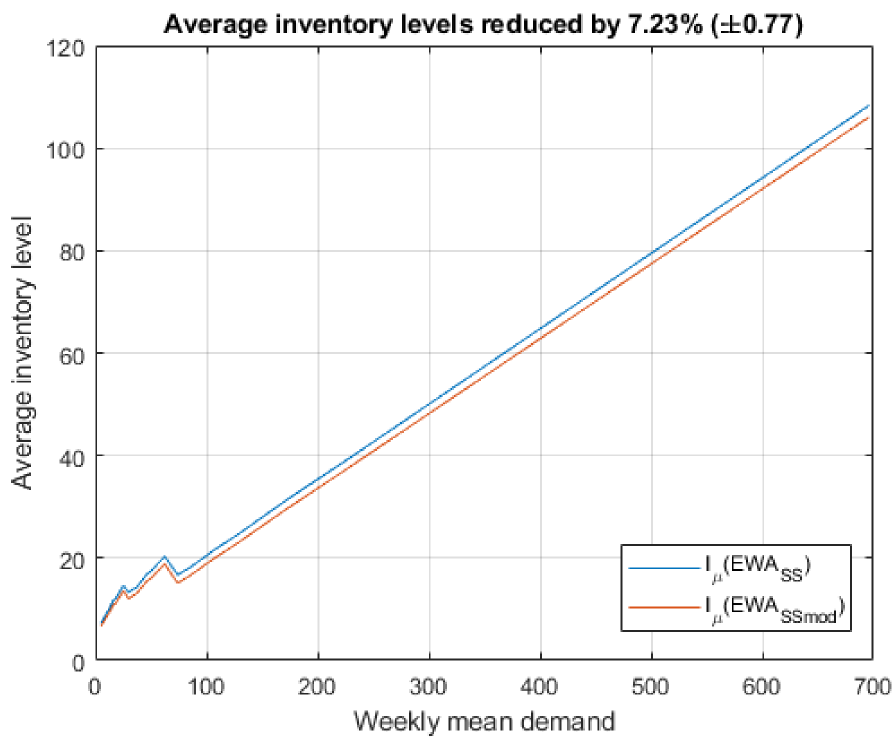

Figure A.39: Average inventory level comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

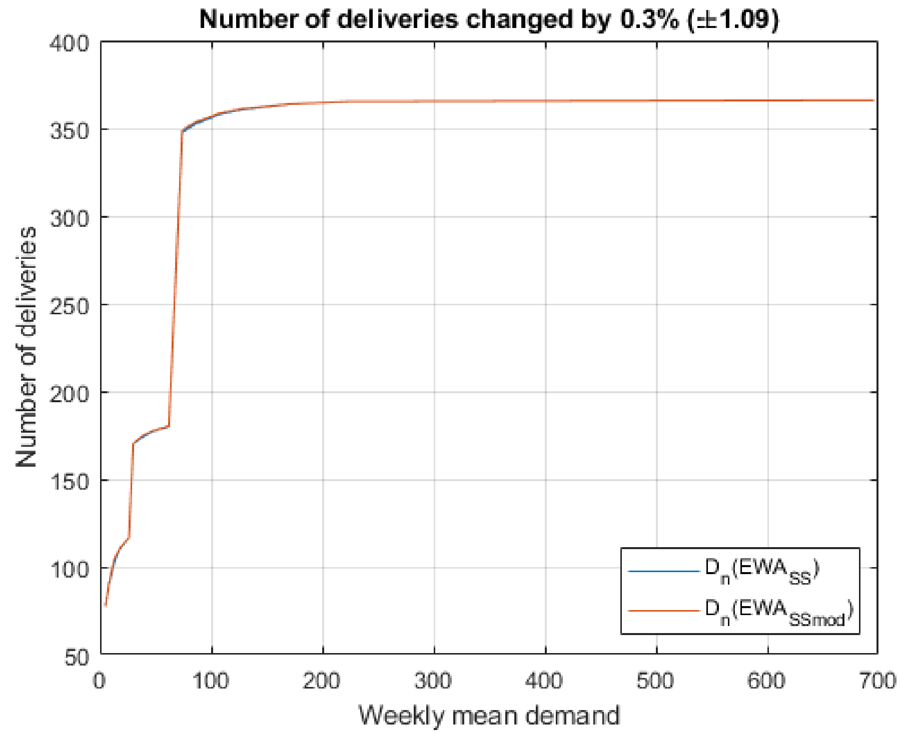

Figure A.40: Comparison of number of deliveries for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 5$, $FIFO = 0.9$

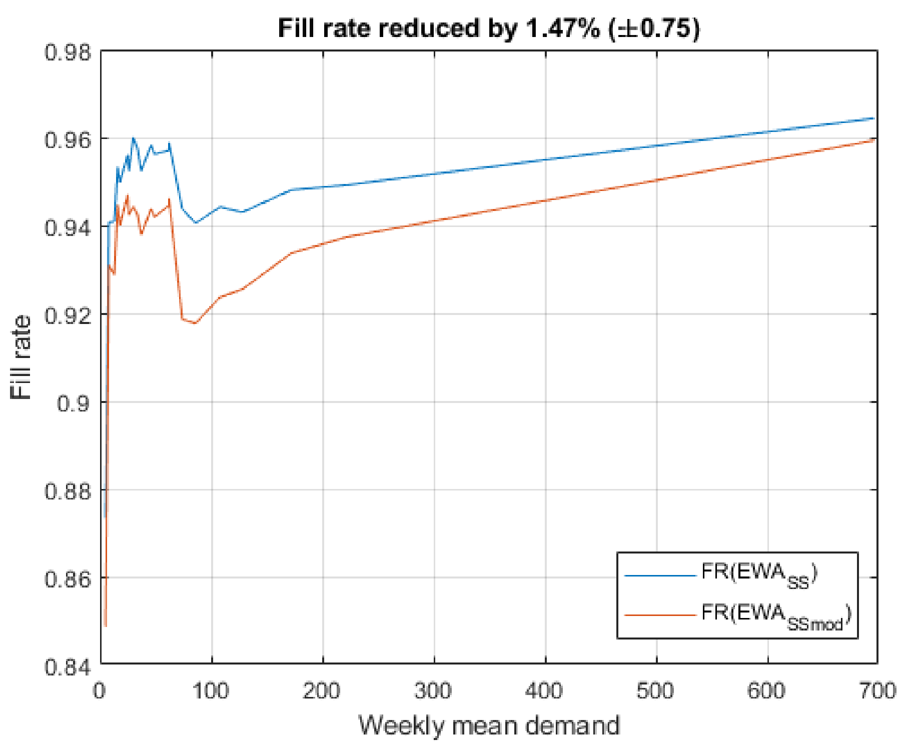

Figure A.41: Fill rate comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

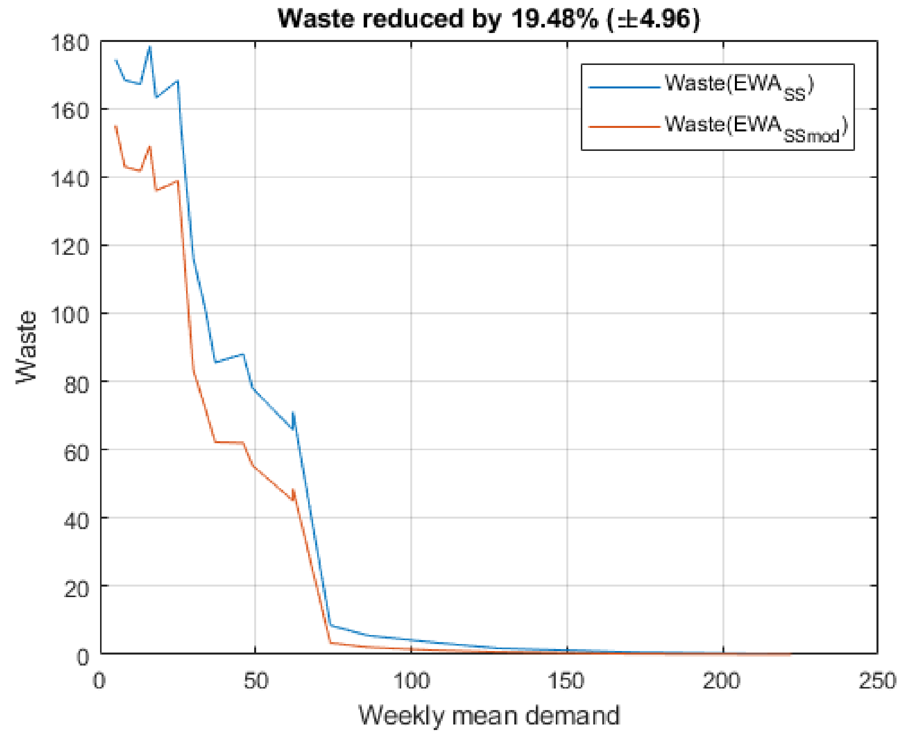

Figure A.42: Waste comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

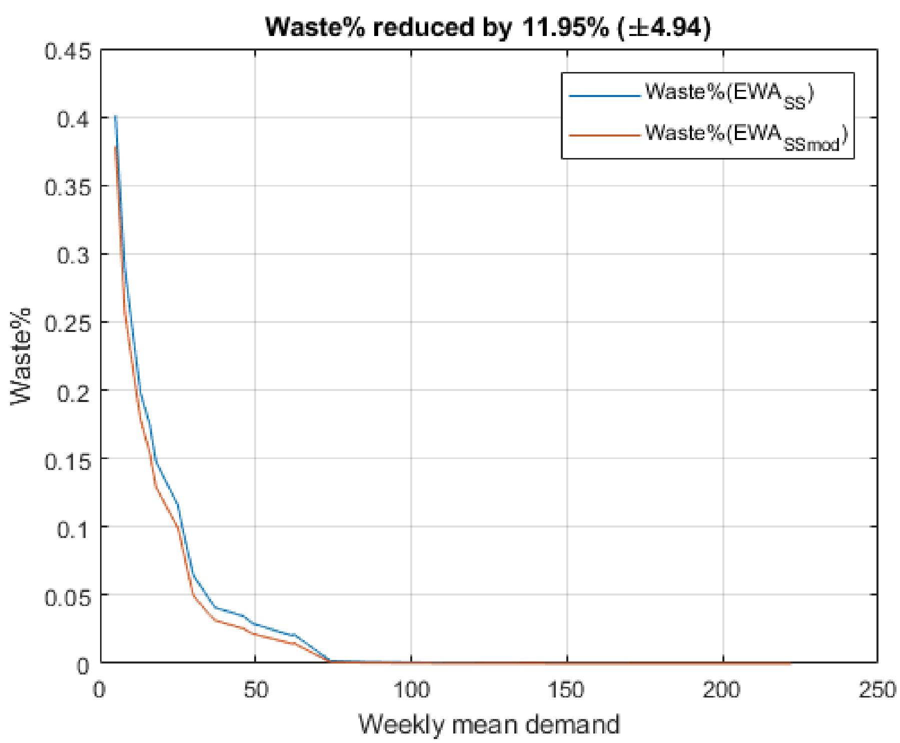

Figure A.43: Waste% comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

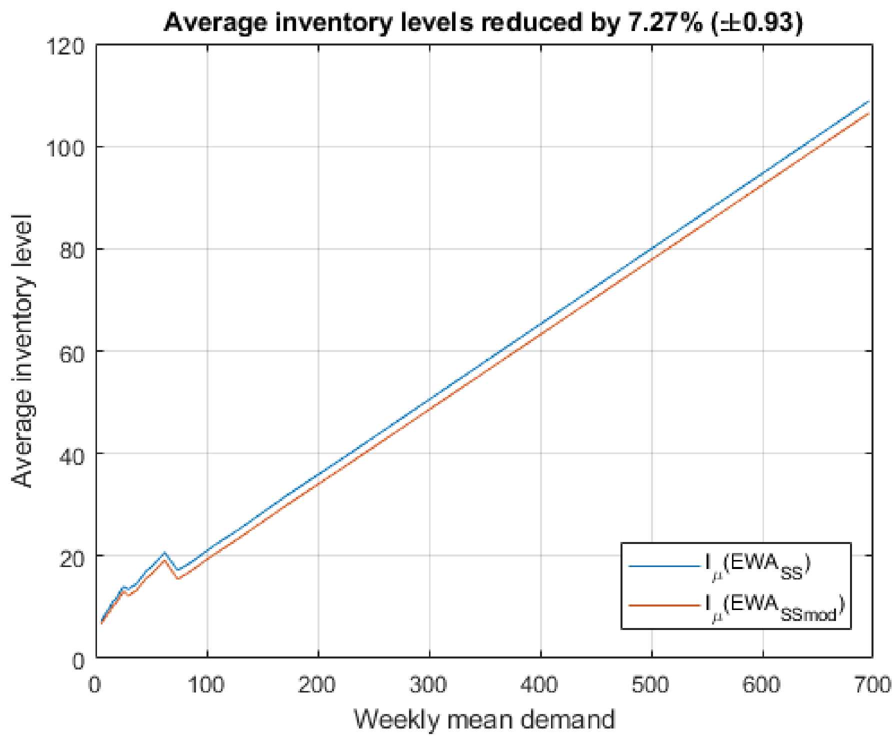

Figure A.44: Average inventory level comparison for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

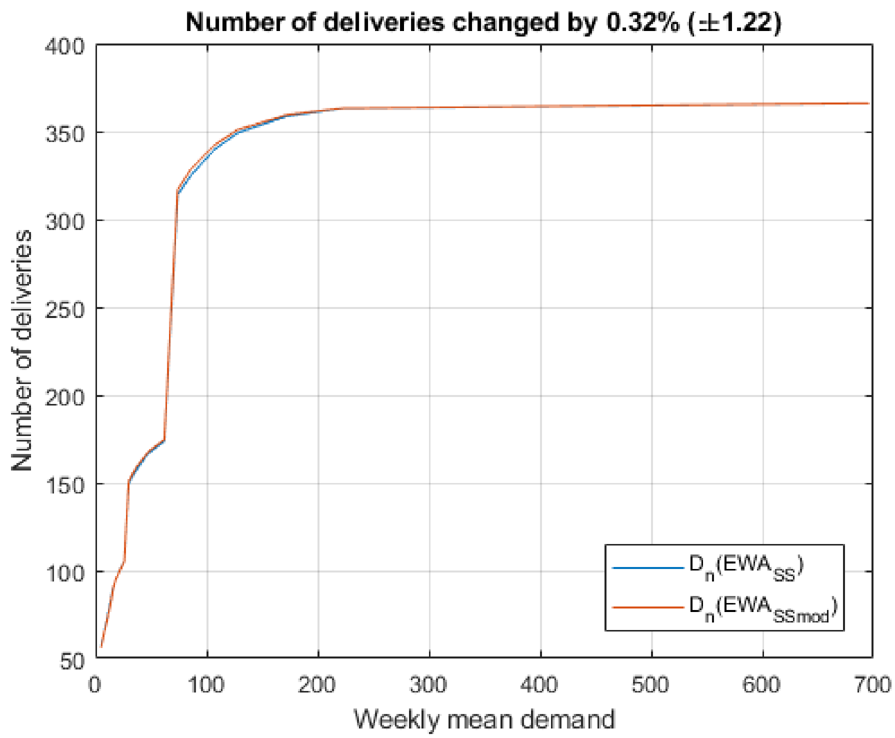

Figure A.45: Comparison of number of deliveries for EWA_{SS} and EWA_{SSmod} under $m = 9$, $Q = 10$, $FIFO = 0.1$

Appendix B

Simulation code for MATLAB

Simulation code for comparison of stock-based policies:

```
clear;
tic;
%Maximum Shelf Life
m = 9;
%Maximum number of reps
maxrep=1000;
%Minimum batch size
Q(1:maxrep) = 10;
%Number of stores
Sn = [12 23 25 11 17 11 11 3 22 10 9 17 8 12 4 14 6 9 2 3 3];
%Lead time (days)
l = 1;
%Simulation duration (days)
simL = 460;
%Weekly sales pattern
wp = [0.12 0.13 0.13 0.16 0.18 0.18 0.10];
%Percentage FIFO picking
FIFO = 90;
%Review intervals
```

```
R = [3 3 3 3 3 3 3 2 2 2 2 2 2 2 1 1 1 1 1 1 1];

pd=makedist('normal');
k1=icdf(pd,0.96);
k2=icdf(pd,0.97);
k3=icdf(pd,0.975);
k4=icdf(pd,0.98);
D = [5  8  13 16 18 25 26 30 34 37 46 49 62 62 74 86 108 128 172 222 696];
k = [k1 k1 k1 k2 k1 k2 k1 k3 k2 k1 k3 k2 k2 k3 k4 k3 k4  k3  k4  k3  k3];
for i=1:21
    SL(i)=cdf(pd,k(i));
end

for rep = 1:maxrep
    for sc=1:21
        %Daily fractions of weekly mean demand (used as mean for daily demand distributions)
        f_d = D(sc)*[0.12,0.13,0.13,0.16,0.18,0.18,0.10];
        %Initialising Inventory Level Array
        IP(1:simL+7) = 0;
        %Initialising forecasted demand array
        ForD(1,1:simL+7) = 0;
        %Initialising simulated (actual) demand array
        ActD(1,1:simL+7) = 0;
        %Creating probability distributions for every weekday, to generate forecasted demand
        for wd = 1:7
            DD(wd) = makedist('poisson',f_d(wd));
        end
```

```
    %Generating forecasted demand for entire simulation period using different demand distribution for every week day
    for week = 1:(simL/7)+7
        for day = 1:7
            ForD(day+(7*(week-1))) = random(DD(day));
        end
    end
    %Generating actual demand for entire simulation period using different demand distribution for every week day
    for week = 1:(simL/7)+7
        for day = 1:7
            ActD(day+(7*(week-1))) = random(DD(day));
        end
    end
    %Batch: number of units at day "t" with shelf life "r", and Waste
    B(1:simL+7,1:m) = 0;
    W(1,1:simL+7) = 0;
    %Randomise initial inventory position
    IP(1) = randi(50);
    IIP = IP(1);
    remn = IP(1);
    for r = m:-1:1
        size = randi(20);

        if size<remn
            B(1,r) = size;
        else
            B(1,r) = remn;
        end
        remn = max(0,(remn - size));
```

```
            if remn == 0
                break;
            end
        end
        IP2 = IP;
        B2 = B;
        %Array for ordering days
        R_Day(1:simL) = 0;
        for x = 1:simL
            if rem(x,R(sc)) == 0
                R_Day(x) = 1;
            else
                R_Day(x) = 0;
            end
        end
        %Initialising various variables
        day = 1;
        waste(1:simL) = 0;
        IP(2:simL) = 0;
        B(2:simL,:) = 0;
        n(1:simL) = 0;
        SS(1:simL) = 0;
        LD(1:simL) = 0;
        FD(1:simL) = 0;
        sd_FE(sc,1:7) = 0;
        SSE(sc,1:7) = 0;
        %Simulating scenario with stock-based (base) policy
        for t = 1:simL
            %Updating forecast errors (done weekly)
            if t>7 && rem(t,7)==1
```

```
                if t>14
                    for x=1:7
                        SSE(sc,x)=SSE(sc,x)+(FD(t+x-8)-ForD(t+x-8))^2;
                        MSE(sc,x)=SSE(sc,x)/((t-1)/7);
                        sd_FE(sc,x)=sqrt(MSE(sc,x));
                    end
                else
                    for x=1:7
                        SSE(sc,x)=(FD(x)-ForD(x))^2;
                        MSE(sc,x)=SSE(sc,x);
                        sd_FE(sc,x)=sqrt(MSE(sc,x));
                    end
                end

        end
        %Computing order quantity
        if R_Day(t) == 1
            %Includes forecast for present day
            for x = 0:R(sc)
                n(t) = n(t)+ForD(t+x);
            end

            %Includes safety stock for present day
            sig_lR = 0;
            for x = 0:R(sc)
                if rem(t,7)+x > 7
                    sig_lR = sqrt((sig_lR)^2 + (sd_FE(rem(t,7)+x-7)^2));
                elseif rem(t,7)+x ~= 0
                    sig_lR = sqrt((sig_lR)^2 + (sd_FE(rem(t,7)+x)^2));
```

```
                    else
                        sig_lR = sqrt((sig_lR)^2 + (sd_FE(7)^2));
                    end
                end
                SS(t) = round(k(sc)*sig_lR);
                if IP(t) < n(t)+ SS(t)
                    n(t)=Q(rep)*round((max(0,n(t)+SS(t)-IP(t)))/Q(rep));
                    B(t+l,m) = B(t+l,m) + n(t);
                end
            end
            %Lost Demand, if any
            LD(t) = max(0,(ActD(t) - IP(t)));
            %Fulfilled demand
            FD(t) = ActD(t) - LD(t);
            %Temporary variable
            B_interim(1,:) = B(t,:);
            %Variable definitions for depleting boxes from different 'Batches'
            unsatF = round(FIFO*FD(t)/100);
            unsatL = FD(t) - unsatF;
            %Calculating forwarded batches after depletion due to fulfilled demand
            %FIFO Depletion
            for r = 1:m
                B_interim(1,r) = max(0,B_interim(1,r) - unsatF);
                unsatF = max(0,unsatF - B(t,r));
            end
            r = m;
            %LIFO Depletion
            for r = m:-1:1
```

```
                B_interim(1,r) = max(0,B_interim(1,r) - unsatL);
                unsatL = max(0,unsatL - B(t,r));
            end
            %Moving batches to next day's inventory
            for r = 1:m-1
                B(t+1,r)=B_interim(1,r+1);
            end
            %Removing items to discard and computing waste
            waste(t) = waste(t) + B_interim(1,1);
            %Inventory level for next day after reduction due to demand
                and waste
            IP(t+1) = sum(B(t+1,1:m));
        end

        %Initialising variables for modified policy run
        waste2(1:simL) = 0;
        IP2(2:simL) = 0;
        B2(2:simL,:) = 0;
        n2(1:simL) = 0;
        LD2(1:simL) = 0;
        FD2(1:simL) = 0;
        sd_FE(sc,1:7) = 0;
        SSE(sc,1:7) = 0;
        %Simulating scenario with stock-based (mod) policy
        for t = 1:simL
            if t>7 && rem(t,7)==1
                    if t>14
                        for x=1:7
                            SSE(sc,x)=SSE(sc,x)+(FD2(t+x-8)-ForD(t+x-8))
                                ^2;
```

```
                MSE(sc,x)=SSE(sc,x)/((t-1)/7);
                sd_FE(sc,x)=sqrt(MSE(sc,x));
            end
        else
            for x=1:7
                SSE(sc,x)=(FD2(x)-ForD(x))^2;
                MSE(sc,x)=SSE(sc,x);
                sd_FE(sc,x)=sqrt(MSE(sc,x));
            end
        end
end
if R_Day(t) == 1
    %Does not include forecast for present day
    for x = 1:R(sc)
        n2(t) = n2(t)+ForD(t+x);
    end
    sig_lR2 = 0;
    %Does not include safety stock for present day
    for x = 1:R(sc)
        if rem(t,7)+x > 7
            sig_lR2 = ((sig_lR2)^2 + (sd_FE(rem(t,7)+x-7)^2))^0.5;
        elseif rem(t,7)+x~=0
            sig_lR2 = ((sig_lR2)^2 + (sd_FE(rem(t,7)+x)^2))^0.5;
        else
            sig_lR2 = ((sig_lR2)^2 + (sd_FE(7)^2))^0.5;
        end
    end
    SS2(t) = round(k(sc)*sig_lR2);
```

```
            if max(0,IP2(t)-ForD(t)) < n2(t)+SS2(t)
                n2(t)=Q(rep)*round((n2(t)+SS2(t)-max(0,IP2(t)-ForD(t))
                    )/Q(rep));
                B2(t+l,m) = B2(t+l,m) + n2(t);
            end
        end
        %Lost Demand, if any
        LD2(t) = max(0,(ActD(t) - IP2(t)));
        %Fulfilled demand
        FD2(t) = ActD(t) - LD2(t);
        %Temporary variable
        B_interim(1,:) = B2(t,:);
        %Variable definitions for depleting boxes from different '
            Batches'
        unsatF = round(FIFO*FD2(t)/100);
        unsatL = FD2(t) - unsatF;
        %Calculating forwarded batches after depletion due to
            fulfilled demand
        %FIFO Depletion
        for r = 1:m
            B_interim(1,r) = max(0,B_interim(1,r) - unsatF);
            unsatF = max(0,unsatF - B2(t,r));
        end
        r = m;
        %LIFO Depletion
        for r = m:-1:1
            B_interim(1,r) = max(0,B_interim(1,r) - unsatL);
            unsatL = max(0,unsatL - B2(t,r));
        end
        %Moving batches to next day's inventory
```

```
            for r = 1:m-1
                B2(t+1,r)=B_interim(1,r+1);
            end
            %Removing items to discard and computing waste
            waste2(t) = waste2(t) + B_interim(1,1);
            %Inventory level for next day after reduction due to demand
                and waste
            IP2(t+1) = sum(B2(t+1,1:m));
            EB2 = B2;
        end

        %Calculations for performance indicators
        DelRed(sc,rep) = (sum(n2(95:simL)~=0)-sum(n(95:simL)~=0))/sum(n
            (95:simL)~=0);
        AvgInvRed(sc,rep) = (mean(IP2(95:simL))-mean(IP(95:simL)))/mean(IP
            (95:simL));
        WasteRed12(sc,rep) = (sum(waste2(95:simL))-sum(waste(95:simL)))/
            sum(waste(95:simL));
        LostSalesRed12(sc,rep) = (sum(LD2(95:simL))-sum(LD(95:simL)))/sum(
            LD(95:simL));
        nDel_base(sc,rep) = sum(n(95:simL)~=0);
        nDel_mod(sc,rep) = sum(n2(95:simL)~=0);
        FR_base(sc,rep) = sum(FD(95:simL))/sum(ActD(95:simL));
        FR_mod(sc,rep) = sum(FD2(95:simL))/sum(ActD(95:simL));
        waste_base(sc,rep) = sum(waste(95:simL));
        waste_mod(sc,rep) = sum(waste2(95:simL));
        wastepc_base(sc,rep) = sum(waste(95:simL))/sum(n(95:simL));
        wastepc_mod(sc,rep) = sum(waste2(95:simL))/sum(n2(95:simL));
        AvgInv_base(sc,rep) = mean(IP(95:simL));
        AvgInv_mod(sc,rep) = mean(IP2(95:simL));
```

```
        end
        %Percentage reduction (reduction in weighted sums)
        FRD(rep) = (sum(FR_mod(:,rep)'.*Sn)-sum(FR_base(:,rep)'.*Sn))/sum(FR_base(:,rep)'.*Sn);
        WD(rep) = (sum(waste_mod(:,rep)'.*Sn)-sum(waste_base(:,rep)'.*Sn))/sum(waste_base(:,rep)'.*Sn);
        ID(rep) = (sum(AvgInv_mod(:,rep)'.*Sn)-sum(AvgInv_base(:,rep)'.*Sn))/sum(AvgInv_base(:,rep)'.*Sn);
        WPD(rep) = (sum(wastepc_mod(:,rep)'.*Sn)-sum(wastepc_base(:,rep)'.*Sn))/sum(wastepc_base(:,rep)'.*Sn);
        DnD(rep) = (sum(nDel_mod(:,rep)'.*Sn)-sum(nDel_base(:,rep)'.*Sn))/sum(nDel_base(:,rep)'.*Sn);
end

%Plots
FRdrop=mean(FRD);
FRstd=std(FRD);
FRdist=makedist('normal',FRdrop,FRstd);
FRl=icdf(FRdist,0.025);
FRu=icdf(FRdist,0.975);
FRlim=(FRu-FRl)/2;
plot(D,mean(FR_base'),'DisplayName','FR(SB_{base})');
hold on;
plot(D,mean(FR_mod'),'DisplayName','FR(SB_{mod})');
legend('FR(SB_{base})','FR(SB_{mod})');
title(['Fill rate reduced by ',num2str(-round(100*FRdrop,2)),'% (\pm',num2str(round(100*FRlim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Fill rate');
grid on;
```

```
hold off;

avgwst_base=mean(waste_base');
avgwst_mod=mean(waste_mod');
poswas=sum(avgwst_base~=0);

Wdrop=mean(WD);
Wstd=std(WD);
Wdist=makedist('normal',Wdrop,Wstd);
Wl=icdf(Wdist,0.025);
Wu=icdf(Wdist,0.975);
Wlim=(Wu-Wl)/2;
plot(D(1:poswas),avgwst_base(1:poswas),'DisplayName','Waste(SB_{base})');
hold on;
plot(D(1:poswas),avgwst_mod(1:poswas),'DisplayName','Waste(SB_{mod})');
legend('Waste(SB_{base})','Waste(SB_{mod})');
title(['Waste reduced by ',num2str(-round(100*Wdrop,2)),'% (\pm',num2str(
    round(100*Wlim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Waste');
grid on;
hold off;

Idrop=mean(ID);
Istd=std(ID);
Idist=makedist('normal',Idrop,Istd);
Il=icdf(Idist,0.025);
Iu=icdf(Idist,0.975);
Ilim=(Iu-Il)/2;

```

```
plot(D,mean(AvgInv_base'),'DisplayName','I_{\mu}(SB_{base})');
hold on;
plot(D,mean(AvgInv_mod'),'DisplayName','I_{\mu}(SB_{mod})');
legend('I_{\mu}(SB_{base})','I_{\mu}(SB_{mod})');
title(['Average inventory levels reduced by ',num2str(-round(100*Idrop,2))
    ,'% (\pm',num2str(round(100*Ilim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Average inventory level');
grid on;
hold off;

avgwstpc_base=mean(wastepc_base');
avgwstpc_mod=mean(wastepc_mod');
poswaspc=sum(avgwstpc_base~=0);
Wpcdrop=mean(WPD);
Wpcstd=std(WPD);
Wpcdist=makedist('normal',Wpcdrop,Wpcstd);
Wpcl=icdf(Wpcdist,0.025);
Wpcu=icdf(Wpcdist,0.975);
Wpclim=(Wpcu-Wpcl)/2;
plot(D(1:poswaspc),avgwstpc_base(1:poswaspc),'DisplayName','Waste%(SB_{
    base})');
hold on;
plot(D(1:poswaspc),avgwstpc_mod(1:poswaspc),'DisplayName','Waste%(SB_{mod
    })');
legend('Waste%(SB_{base})','Waste%(SB_{mod})');
title(['Waste% reduced by ', num2str(-round(100*Wpcdrop,2)),'% (\pm',
    num2str(round(100*Wpclim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Waste%');
```

```
grid on;
hold off;

Ddrop=mean(DnD);
Ddrop=mean(DnD);
Dstd=std(DnD);
Ddist=makedist('normal',Ddrop,Dstd);
Dl=icdf(Ddist,0.025);
Du=icdf(Ddist,0.975);
Dlim=(Du-Dl)/2;

plot(D,mean(nDel_base'),'DisplayName','D_n(SB_{base})');
hold on;
plot(D,mean(nDel_mod'),'DisplayName','D_n(SB_{mod})');
legend('D_n(SB_{base})','D_n(SB_{mod})');
title(['Number of deliveries changed by ',num2str(round(100*Ddrop,2)),'%
    (\pm',num2str(round(100*Dlim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Number of deliveries');
grid on;
hold off;

WC_SB1=wastepc_mod-wastepc_base;
FRC_SB1=FR_mod-FR_base;
SB1_val=mean(WC_SB1')./mean(FRC_SB1');
plot(1:21,SB1_val,'*');
hold on;
title('Value of modifying stock-based policy for m=9, Q=10, FIFO=0.9');
ylabel('Value (ratio of changes in waste% and fill rate)');
xlabel('Store variety');
```

```
axis([0 21 -inf inf]);
grid on;
hold off;

toc;
```

Simulation code for comparison of EWA policies:

```
clear;
tic;
%Maximum Shelf Life
m = 9;
%Maximum number of reps
maxrep=1000;
%Minimum batch size
Q(1:maxrep) = 10;
%Number of stores
Sn = [12 23 25 11 17 11 11 3 22 10 9 17 8 12 4 14 6 9 2 3 3];
%Lead time (days)
l = 1;
%Simulation duration (days)
simL = 460;
%Weekly sales pattern
wp = [0.12 0.13 0.13 0.16 0.18 0.18 0.10];
%Percentage FIFO picking
FIFO = 90;
%Review intervals
R = [3 3 3 3 3 3 3 2 2 2 2 2 2 2 1 1 1 1 1 1 1];

pd=makedist('normal');
k1=icdf(pd,0.96);
k2=icdf(pd,0.97);
k3=icdf(pd,0.975);
k4=icdf(pd,0.98);
D = [5  8  13 16 18 25 26 30 34 37 46 49 62 62 74 86 108 128 172 222 696];
k = [k1 k1 k1 k2 k1 k2 k1 k3 k2 k1 k3 k2 k2 k3 k4 k3 k4  k3  k4  k3  k3];
for i=1:21
```

```
    SL(i)=cdf(pd,k(i));
end

for rep = 1:maxrep
    for sc=1:21
        %Daily fractions of weekly mean demand (used as mean for daily
           demand distributions)
        f_d = D(sc)*[0.12,0.13,0.13,0.16,0.18,0.18,0.10];
        %Initialising Inventory Level Array
        IP(1:simL+7) = 0;
        %Initialising forecasted demand array
        ForD(1,1:simL+7) = 0;
        %Initialising simulated (actual) demand array
        ActD(1,1:simL+7) = 0;
        %Creating probability distributions for every weekday, to generate
            forecasted demand
        for wd = 1:7
            DD(wd) = makedist('poisson',f_d(wd));
        end
        %Generating forecasted demand for entire simulation period using
           different demand distribution for every week day
        for week = 1:(simL/7)+7
            for day = 1:7
                ForD(day+(7*(week-1))) = random(DD(day));
            end
        end
        %Generating actual demand for entire simulation period using
           different demand distribution for every week day
        for week = 1:(simL/7)+7
            for day = 1:7
```

```
            ActD(day+(7*(week-1))) = random(DD(day));
        end
    end
    %Batch: number of units at day "t" with shelf life "r", and Waste
    B(1:simL+7,1:m) = 0;
    W(1,1:simL+7) = 0;
    %Randomise initial inventory position
    IP(1) = randi(50);
    IIP = IP(1);
    remn = IP(1);
    for r = m:-1:1
        size = randi(20);

        if size<remn
            B(1,r) = size;
        else
            B(1,r) = remn;
        end
        remn = max(0,(remn - size));
        if remn == 0
            break;
        end
    end
    IP2 = IP;
    B2 = B;
    %Array for ordering days
    R_Day(1:simL) = 0;
    for x = 1:simL
        if rem(x,R(sc)) == 0
            R_Day(x) = 1;
```

```
            else
                R_Day(x) = 0;
            end
        end
        %Initialising various variables
        day = 1;
        waste(1:simL) = 0;
        IP(2:simL) = 0;
        B(2:simL,:) = 0;
        n(1:simL) = 0;
        SS(1:simL) = 0;
        LD(1:simL) = 0;
        FD(1:simL) = 0;
        sd_FE(sc,1:7) = 0;
        SSE(sc,1:7) = 0;
        %Simulating scenario with stock-based (base) policy
        for t = 1:simL
            %Updating forecast errors (done weekly)
            if t>7 && rem(t,7)==1
                    if t>14
                        for x=1:7
                            SSE(sc,x)=SSE(sc,x)+(FD(t+x-8)-ForD(t+x-8))^2;
                            MSE(sc,x)=SSE(sc,x)/((t-1)/7);
                            sd_FE(sc,x)=sqrt(MSE(sc,x));
                        end
                    else
                        for x=1:7
                            SSE(sc,x)=(FD(x)-ForD(x))^2;
                            MSE(sc,x)=SSE(sc,x);
                            sd_FE(sc,x)=sqrt(MSE(sc,x));
```

```
                end
            end

        end
        %Computing order quantity
        if R_Day(t) == 1
            %Includes forecast for present day
            for x = 0:R(sc)
                n(t) = n(t)+ForD(t+x);
            end

            %EWA Heuristic
            for x = 1:R(sc)+1
                unsatFE = round(FIFO*ForD(t+x-1)/100);
                unsatLE = ForD(t+x-1) - unsatFE;

                EB_int(1,:) = EB(t+x-1,:);

                %Estimated FIFO dep
                for r = 1:m
                    EB_int(1,r) = max(0,EB_int(1,r) - unsatFE);
                    unsatFE = max(0,unsatFE - EB(t+x-1,r));
                end

                %Estimated LIFO dep
                for r = m:-1:1
                    EB_int(1,r) = max(0,EB_int(1,r) - unsatLE);
                    unsatLE = max(0,unsatLE - EB(t+x-1,r));
                end
                for r=1:m-1
```

```
                    EB(t+x,r) = EB_int(1,r+1);
                end
                EW(t) = EW(t)+EB_int(1,1);
            end

            %Includes safety stock for present day
            sig_lR = 0;
            for x = 0:R(sc)
                if rem(t,7)+x > 7
                    sig_lR = sqrt((sig_lR)^2 + (sd_FE(rem(t,7)+x-7)^2));
                elseif rem(t,7)+x ~= 0
                    sig_lR = sqrt((sig_lR)^2 + (sd_FE(rem(t,7)+x)^2));
                else
                    sig_lR = sqrt((sig_lR)^2 + (sd_FE(7)^2));
                end
            end
            SS(t) = round(k(sc)*sig_lR);
            if IP(t)-EW(t) < n(t)+ SS(t)
                n(t)=Q(rep)*round((n(t)+SS(t)+EW(t)-IP(t))/Q(rep));
                B(t+l,m) = B(t+l,m) + n(t);
            end
        end
        %Lost Demand, if any
        LD(t) = max(0,(ActD(t) - IP(t)));
        %Fulfilled demand
        FD(t) = ActD(t) - LD(t);
        %Temporary variable
        B_interim(1,:) = B(t,:);
        %Variable definitions for depleting boxes from different '
```

```
        Batches'
        unsatF = round(FIFO*FD(t)/100);
        unsatL = FD(t) - unsatF;
        %Calculating forwarded batches after depletion due to fulfilled demand
        %FIFO Depletion
        for r = 1:m
            B_interim(1,r) = max(0,B_interim(1,r) - unsatF);
            unsatF = max(0,unsatF - B(t,r));
        end
        r = m;
        %LIFO Depletion
        for r = m:-1:1
            B_interim(1,r) = max(0,B_interim(1,r) - unsatL);
            unsatL = max(0,unsatL - B(t,r));
        end
        %Moving batches to next day's inventory
        for r = 1:m-1
            B(t+1,r)=B_interim(1,r+1);
        end
        %Removing items to discard and computing waste
        waste(t) = waste(t) + B_interim(1,1);
        %Inventory level for next day after reduction due to demand and waste
        IP(t+1) = sum(B(t+1,1:m));
        EB=B;
    end

    %Initialising variables for modified policy run
    waste2(1:simL) = 0;
```

```
    IP2(2:simL) = 0;
    B2(2:simL,:) = 0;
    n2(1:simL) = 0;
    LD2(1:simL) = 0;
    FD2(1:simL) = 0;
    sd_FE(sc,1:7) = 0;
    SSE(sc,1:7) = 0;
    %Simulating scenario with stock-based (mod) policy
    for t = 1:simL
        if t>7 && rem(t,7)==1
            if t>14
                for x=1:7
                    SSE(sc,x)=SSE(sc,x)+(FD2(t+x-8)-ForD(t+x-8))^2;
                    MSE(sc,x)=SSE(sc,x)/((t-1)/7);
                    sd_FE(sc,x)=sqrt(MSE(sc,x));
                end
            else
                for x=1:7
                    SSE(sc,x)=(FD2(x)-ForD(x))^2;
                    MSE(sc,x)=SSE(sc,x);
                    sd_FE(sc,x)=sqrt(MSE(sc,x));
                end
            end
        end
        if R_Day(t) == 1
            %Does not include forecast for present day
            for x = 1:R(sc)
                n2(t) = n2(t)+ForD(t+x);
            end
```

```
            %EWA Heuristic
            for x = 1:R(sc)+1
                unsatFE = round(FIFO*ForD(t+x-1)/100);
                unsatLE = ForD(t+x-1) - unsatFE;

                EB2_int(1,:) = EB2(t+x-1,:);

                %Estimated FIFO dep
                for r = 1:m
                    EB2_int(1,r) = max(0,EB2_int(1,r) - unsatFE);
                    unsatFE = max(0,unsatFE - EB2(t+x-1,r));
                end

                %Estimated LIFO dep
                for r = m:-1:1
                    EB2_int(1,r) = max(0,EB2_int(1,r) - unsatLE);
                    unsatLE = max(0,unsatLE - EB2(t+x-1,r));
                end
                for r=1:m-1
                    EB2(t+x,r) = EB2_int(1,r+1);
                end
                EW(t) = EW(t)+EB2_int(1,1);
            end

            sig_lR2 = 0;
            %Does not include safety stock for present day
            for x = 1:R(sc)
                if rem(t,7)+x > 7
                    sig_lR2 = ((sig_lR2)^2 + (sd_FE(rem(t,7)+x-7)^2))
```

```
                    ^0.5;
                elseif rem(t,7)+x~=0
                    sig_lR2 = ((sig_lR2)^2 + (sd_FE(rem(t,7)+x)^2))
                        ^0.5;
                else
                    sig_lR2 = ((sig_lR2)^2 + (sd_FE(7)^2))^0.5;
                end
            end
            SS2(t) = round(k(sc)*sig_lR2);
            if max(0,IP2(t)-ForD(t))-EW(t) < n2(t)+SS2(t)
                n2(t)=Q(rep)*round(((n2(t)+SS2(t)+EW(t)-max(0,IP2(t)-
                    ForD(t))))/Q(rep));
                B2(t+l,m) = B2(t+l,m) + n2(t);
            end
        end
        %Lost Demand, if any
        LD2(t) = max(0,(ActD(t) - IP2(t)));
        %Fulfilled demand
        FD2(t) = ActD(t) - LD2(t);
        %Temporary variable
        B_interim(1,:) = B2(t,:);
        %Variable definitions for depleting boxes from different '
            Batches'
        unsatF = round(FIFO*FD2(t)/100);
        unsatL = FD2(t) - unsatF;
        %Calculating forwarded batches after depletion due to
            fulfilled demand
        %FIFO Depletion
        for r = 1:m
            B_interim(1,r) = max(0,B_interim(1,r) - unsatF);
```

```
                unsatF = max(0,unsatF - B2(t,r));
            end
            r = m;
            %LIFO Depletion
            for r = m:-1:1
                B_interim(1,r) = max(0,B_interim(1,r) - unsatL);
                unsatL = max(0,unsatL - B2(t,r));
            end
            %Moving batches to next day's inventory
            for r = 1:m-1
                B2(t+1,r)=B_interim(1,r+1);
            end
            %Removing items to discard and computing waste
            waste2(t) = waste2(t) + B_interim(1,1);
            %Inventory level for next day after reduction due to demand
                and waste
            IP2(t+1) = sum(B2(t+1,1:m));
            EB2 = B2;
        end

        %Calculations for performance indicators
        DelRed(sc,rep) = (sum(n2(95:simL)~=0)-sum(n(95:simL)~=0))/sum(n
            (95:simL)~=0);
        AvgInvRed(sc,rep) = (mean(IP2(95:simL))-mean(IP(95:simL)))/mean(IP
            (95:simL));
        WasteRed12(sc,rep) = (sum(waste2(95:simL))-sum(waste(95:simL)))/
            sum(waste(95:simL));
        LostSalesRed12(sc,rep) = (sum(LD2(95:simL))-sum(LD(95:simL)))/sum(
            LD(95:simL));
        nDel_base(sc,rep) = sum(n(95:simL)~=0);
```

```
            nDel_mod(sc,rep) = sum(n2(95:simL)~=0);
            FR_base(sc,rep) = sum(FD(95:simL))/sum(ActD(95:simL));
            FR_mod(sc,rep) = sum(FD2(95:simL))/sum(ActD(95:simL));
            waste_base(sc,rep) = sum(waste(95:simL));
            waste_mod(sc,rep) = sum(waste2(95:simL));
            wastepc_base(sc,rep) = sum(waste(95:simL))/sum(n(95:simL));
            wastepc_mod(sc,rep) = sum(waste2(95:simL))/sum(n2(95:simL));
            AvgInv_base(sc,rep) = mean(IP(95:simL));
            AvgInv_mod(sc,rep) = mean(IP2(95:simL));
        end
        %Percentage reduction (reduction in weighted sums)
        FRD(rep) = (sum(FR_mod(:,rep)'.*Sn)-sum(FR_base(:,rep)'.*Sn))/sum(
            FR_base(:,rep)'.*Sn);
        WD(rep) = (sum(waste_mod(:,rep)'.*Sn)-sum(waste_base(:,rep)'.*Sn))/sum
            (waste_base(:,rep)'.*Sn);
        ID(rep) = (sum(AvgInv_mod(:,rep)'.*Sn)-sum(AvgInv_base(:,rep)'.*Sn))/
            sum(AvgInv_base(:,rep)'.*Sn);
        WPD(rep) = (sum(wastepc_mod(:,rep)'.*Sn)-sum(wastepc_base(:,rep)'.*Sn)
            )/sum(wastepc_base(:,rep)'.*Sn);
        DnD(rep) = (sum(nDel_mod(:,rep)'.*Sn)-sum(nDel_base(:,rep)'.*Sn))/sum(
            nDel_base(:,rep)'.*Sn);
end

%Plots
FRdrop=mean(FRD);
FRstd=std(FRD);
FRdist=makedist('normal',FRdrop,FRstd);
FRl=icdf(FRdist,0.025);
FRu=icdf(FRdist,0.975);
FRlim=(FRu-FRl)/2;
```

```
plot(D,mean(FR_base'),'DisplayName','FR(EWA_{base})');
hold on;
plot(D,mean(FR_mod'),'DisplayName','FR(EWA_{mod})');
legend('FR(EWA_{base})','FR(EWA_{mod})');
title(['Fill rate reduced by ',num2str(-round(100*FRdrop,2)),'% (\pm',
    num2str(round(100*FRlim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Fill rate');
grid on;
hold off;


avgwst_base=mean(waste_base');
avgwst_mod=mean(waste_mod');
poswas=sum(avgwst_base~=0);

Wdrop=mean(WD);
Wstd=std(WD);
Wdist=makedist('normal',Wdrop,Wstd);
Wl=icdf(Wdist,0.025);
Wu=icdf(Wdist,0.975);
Wlim=(Wu-Wl)/2;
plot(D(1:poswas),avgwst_base(1:poswas),'DisplayName','Waste(EWA_{base})');
hold on;
plot(D(1:poswas),avgwst_mod(1:poswas),'DisplayName','Waste(EWA_{mod})');
legend('Waste(EWA_{base})','Waste(EWA_{mod})');
title(['Waste reduced by ',num2str(-round(100*Wdrop,2)),'% (\pm',num2str(
    round(100*Wlim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Waste');
```

```
grid on;
hold off;


Idrop=mean(ID);
Istd=std(ID);
Idist=makedist('normal',Idrop,Istd);
Il=icdf(Idist,0.025);
Iu=icdf(Idist,0.975);
Ilim=(Iu-Il)/2;

plot(D,mean(AvgInv_base'),'DisplayName','I_{\mu}(EWA_{base})');
hold on;
plot(D,mean(AvgInv_mod'),'DisplayName','I_{\mu}(EWA_{mod})');
legend('I_{\mu}(EWA_{base})','I_{\mu}(EWA_{mod})');
title(['Average inventory levels reduced by ',num2str(-round(100*Idrop,2))
    ,'% (\pm',num2str(round(100*Ilim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Average inventory level');
grid on;
hold off;


avgwstpc_base=mean(wastepc_base');
avgwstpc_mod=mean(wastepc_mod');
poswaspc=sum(avgwstpc_base~=0);
Wpcdrop=mean(WPD);
Wpcstd=std(WPD);
Wpcdist=makedist('normal',Wpcdrop,Wpcstd);
Wpcl=icdf(Wpcdist,0.025);
```

```
Wpcu=icdf(Wpcdist,0.975);
Wpclim=(Wpcu-Wpcl)/2;
plot(D(1:poswaspc),avgwstpc_base(1:poswaspc),'DisplayName','Waste%(EWA_{base})');
hold on;
plot(D(1:poswaspc),avgwstpc_mod(1:poswaspc),'DisplayName','Waste%(EWA_{mod})');
legend('Waste%(EWA_{base})','Waste%(EWA_{mod})');
title(['Waste% reduced by ', num2str(-round(100*Wpcdrop,2)),'% (\pm', num2str(round(100*Wpclim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Waste%');
grid on;
hold off;

Ddrop=mean(DnD);
Ddrop=mean(DnD);
Dstd=std(DnD);
Ddist=makedist('normal',Ddrop,Dstd);
Dl=icdf(Ddist,0.025);
Du=icdf(Ddist,0.975);
Dlim=(Du-Dl)/2;

plot(D,mean(nDel_base'),'DisplayName','D_n(EWA_{base})');
hold on;
plot(D,mean(nDel_mod'),'DisplayName','D_n(EWA_{mod})');
legend('D_n(EWA_{base})','D_n(EWA_{mod})');
title(['Number of deliveries changed by ',num2str(round(100*Ddrop,2)),'% (\pm',num2str(round(100*Dlim,2)),')']);
xlabel('Weekly mean demand');
```

```
ylabel('Number of deliveries');
grid on;
hold off;

WC_EWA1=wastepc_mod-wastepc_base;
FRC_EWA1=FR_mod-FR_base;
EWA1_val=mean(WC_EWA1')./mean(FRC_EWA1');
plot(1:21,EWA1_val,'*');
hold on;
title('Value of modifying EWA policy for m=9, Q=10, FIFO=0.9');
ylabel('Value (ratio of changes in waste% and fill rate)');
xlabel('Store variety');
axis([0 21 -inf inf]);
grid on;
hold off;

toc;
```

Simulation code for comparison of EWA_{SS} policies:

```
clear;
tic;
%Maximum Shelf Life
m = 9;
%Maximum number of reps
maxrep=1000;
%Minimum batch size
Q(1:maxrep) = 10;
%Number of stores
Sn = [12 23 25 11 17 11 11 3 22 10 9 17 8 12 4 14 6 9 2 3 3];
%Lead time (days)
l = 1;
%Simulation duration (days)
simL = 460;
%Weekly sales pattern
wp = [0.12 0.13 0.13 0.16 0.18 0.18 0.10];
%Percentage FIFO picking
FIFO = 90;
%Review intervals
R = [3 3 3 3 3 3 3 2 2 2 2 2 2 2 1 1 1 1 1 1 1];

pd=makedist('normal');
k1=icdf(pd,0.96);
k2=icdf(pd,0.97);
k3=icdf(pd,0.975);
k4=icdf(pd,0.98);
D = [5  8  13 16 18 25 26 30 34 37 46 49 62 62 74 86 108 128 172 222 696];
k = [k1 k1 k1 k2 k1 k2 k1 k3 k2 k1 k3 k2 k2 k3 k4 k3 k4  k3  k4  k3  k3];
for i=1:21
```

```
    SL(i)=cdf(pd,k(i));
end

for rep = 1:maxrep
    for sc=1:21
        %Daily fractions of weekly mean demand (used as mean for daily
           demand distributions)
        f_d = D(sc)*[0.12,0.13,0.13,0.16,0.18,0.18,0.10];
        %Initialising Inventory Level Array
        IP(1:simL+7) = 0;
        %Initialising forecasted demand array
        ForD(1,1:simL+7) = 0;
        %Initialising simulated (actual) demand array
        ActD(1,1:simL+7) = 0;
        %Creating probability distributions for every weekday, to generate
            forecasted demand
        for wd = 1:7
            DD(wd) = makedist('poisson',f_d(wd));
        end
        %Generating forecasted demand for entire simulation period using
           different demand distribution for every week day
        for week = 1:(simL/7)+7
            for day = 1:7
                ForD(day+(7*(week-1))) = random(DD(day));
            end
        end
        %Generating actual demand for entire simulation period using
           different demand distribution for every week day
        for week = 1:(simL/7)+7
            for day = 1:7
```

```
                ActD(day+(7*(week-1))) = random(DD(day));
            end
        end
        %Batch: number of units at day "t" with shelf life "r", and Waste
        B(1:simL+7,1:m) = 0;
        W(1,1:simL+7) = 0;
        %Randomise initial inventory position
        IP(1) = randi(50);
        IIP = IP(1);
        remn = IP(1);
        for r = m:-1:1
            size = randi(20);

            if size<remn
                B(1,r) = size;
            else
                B(1,r) = remn;
            end
            remn = max(0,(remn - size));
            if remn == 0
                break;
            end
        end
        IP2 = IP;
        B2 = B;
        %Array for ordering days
        R_Day(1:simL) = 0;
        for x = 1:simL
            if rem(x,R(sc)) == 0
                R_Day(x) = 1;
```

```
            else
                R_Day(x) = 0;
            end
        end
        %Initialising various variables
        day = 1;
        waste(1:simL) = 0;
        IP(2:simL) = 0;
        B(2:simL,:) = 0;
        n(1:simL) = 0;
        SS(1:simL) = 0;
        LD(1:simL) = 0;
        FD(1:simL) = 0;
        sd_FE(sc,1:7) = 0;
        SSE(sc,1:7) = 0;
        %Simulating scenario with stock-based (base) policy
        for t = 1:simL
            %Updating forecast errors (done weekly)
            if t>7 && rem(t,7)==1
                    if t>14
                        for x=1:7
                            SSE(sc,x)=SSE(sc,x)+(FD(t+x-8)-ForD(t+x-8))^2;
                            MSE(sc,x)=SSE(sc,x)/((t-1)/7);
                            sd_FE(sc,x)=sqrt(MSE(sc,x));
                        end
                    else
                        for x=1:7
                            SSE(sc,x)=(FD(x)-ForD(x))^2;
                            MSE(sc,x)=SSE(sc,x);
                            sd_FE(sc,x)=sqrt(MSE(sc,x));
```

```
                end
            end

    end
    %Computing order quantity
    if R_Day(t) == 1
        %Includes forecast for present day
        for x = 0:R(sc)
            n(t) = n(t)+ForD(t+x);
        end

        %EWA Heuristic
        for x = 1:R(sc)+1
            unsatFE = round(FIFO*ForD(t+x-1)/100);
            unsatLE = ForD(t+x-1) - unsatFE;

            EB_int(1,:) = EB(t+x-1,:);

            %Estimated FIFO dep
            for r = 1:m
                EB_int(1,r) = max(0,EB_int(1,r) - unsatFE);
                unsatFE = max(0,unsatFE - EB(t+x-1,r));
            end

            %Estimated LIFO dep
            for r = m:-1:1
                EB_int(1,r) = max(0,EB_int(1,r) - unsatLE);
                unsatLE = max(0,unsatLE - EB(t+x-1,r));
            end
            for r=1:m-1
```

```
                    EB(t+x,r) = EB_int(1,r+1);
                end
                EW(t) = EW(t)+EB_int(1,1);
            end

            %Includes safety stock for present day
            sig_lR = 0;
            for x = 0:R(sc)
                if rem(t,7)+x > 7
                    sig_lR = sqrt((sig_lR)^2 + (sd_FE(rem(t,7)+x-7)^2));
                elseif rem(t,7)+x ~= 0
                    sig_lR = sqrt((sig_lR)^2 + (sd_FE(rem(t,7)+x)^2));
                else
                    sig_lR = sqrt((sig_lR)^2 + (sd_FE(7)^2));
                end
            end
            SS(t) = round(k(sc)*sig_lR);
            if IP(t)-EW(t) < n(t)+ SS(t)
                n(t)=Q(rep)*round((n(t)+max(SS(t),EW(t))-IP(t))/Q(rep));
                B(t+l,m) = B(t+l,m) + n(t);
            end
        end
        %Lost Demand, if any
        LD(t) = max(0,(ActD(t) - IP(t)));
        %Fulfilled demand
        FD(t) = ActD(t) - LD(t);
        %Temporary variable
        B_interim(1,:) = B(t,:);
```

```
            %Variable definitions for depleting boxes from different '
                Batches'
            unsatF = round(FIFO*FD(t)/100);
            unsatL = FD(t) - unsatF;
            %Calculating forwarded batches after depletion due to
                fulfilled demand
            %FIFO Depletion
            for r = 1:m
                B_interim(1,r) = max(0,B_interim(1,r) - unsatF);
                unsatF = max(0,unsatF - B(t,r));
            end
            r = m;
            %LIFO Depletion
            for r = m:-1:1
                B_interim(1,r) = max(0,B_interim(1,r) - unsatL);
                unsatL = max(0,unsatL - B(t,r));
            end
            %Moving batches to next day's inventory
            for r = 1:m-1
                B(t+1,r)=B_interim(1,r+1);
            end
            %Removing items to discard and computing waste
            waste(t) = waste(t) + B_interim(1,1);
            %Inventory level for next day after reduction due to demand
                and waste
            IP(t+1) = sum(B(t+1,1:m));
            EB=B;
        end

        %Initialising variables for modified policy run
```

```
        waste2(1:simL) = 0;
        IP2(2:simL) = 0;
        B2(2:simL,:) = 0;
        n2(1:simL) = 0;
        LD2(1:simL) = 0;
        FD2(1:simL) = 0;
        sd_FE(sc,1:7) = 0;
        SSE(sc,1:7) = 0;
        %Simulating scenario with stock-based (mod) policy
        for t = 1:simL
            if t>7 && rem(t,7)==1
                    if t>14
                        for x=1:7
                            SSE(sc,x)=SSE(sc,x)+(FD2(t+x-8)-ForD(t+x-8))
                                ^2;
                            MSE(sc,x)=SSE(sc,x)/((t-1)/7);
                            sd_FE(sc,x)=sqrt(MSE(sc,x));
                        end
                    else
                        for x=1:7
                            SSE(sc,x)=(FD2(x)-ForD(x))^2;
                            MSE(sc,x)=SSE(sc,x);
                            sd_FE(sc,x)=sqrt(MSE(sc,x));
                        end
                    end
            end
            if R_Day(t) == 1
                %Does not include forecast for present day
                for x = 1:R(sc)
                    n2(t) = n2(t)+ForD(t+x);
```

```
end

%EWA Heuristic
for x = 1:R(sc)+1
    unsatFE = round(FIFO*ForD(t+x-1)/100);
    unsatLE = ForD(t+x-1) - unsatFE;

    EB2_int(1,:) = EB2(t+x-1,:);

    %Estimated FIFO dep
    for r = 1:m
        EB2_int(1,r) = max(0,EB2_int(1,r) - unsatFE);
        unsatFE = max(0,unsatFE - EB2(t+x-1,r));
    end

    %Estimated LIFO dep
    for r = m:-1:1
        EB2_int(1,r) = max(0,EB2_int(1,r) - unsatLE);
        unsatLE = max(0,unsatLE - EB2(t+x-1,r));
    end
    for r=1:m-1
        EB2(t+x,r) = EB2_int(1,r+1);
    end
    EW(t) = EW(t)+EB2_int(1,1);
end

sig_lR2 = 0;
%Does not include safety stock for present day
for x = 1:R(sc)
    if rem(t,7)+x > 7
```

```
                    sig_lR2 = ((sig_lR2)^2 + (sd_FE(rem(t,7)+x-7)^2))^0.5;
                elseif rem(t,7)+x~=0
                    sig_lR2 = ((sig_lR2)^2 + (sd_FE(rem(t,7)+x)^2))^0.5;
                else
                    sig_lR2 = ((sig_lR2)^2 + (sd_FE(7)^2))^0.5;
                end
            end
            SS2(t) = round(k(sc)*sig_lR2);
            if max(0,IP2(t)-ForD(t))-EW(t) < n2(t)+SS2(t)
                n2(t)=Q(rep)*round(((n2(t)+max(SS2(t),EW(t))-max(0,IP2(t)-ForD(t))))/Q(rep));
                B2(t+l,m) = B2(t+l,m) + n2(t);
            end
        end
        %Lost Demand, if any
        LD2(t) = max(0,(ActD(t) - IP2(t)));
        %Fulfilled demand
        FD2(t) = ActD(t) - LD2(t);
        %Temporary variable
        B_interim(1,:) = B2(t,:);
        %Variable definitions for depleting boxes from different 'Batches'
        unsatF = round(FIFO*FD2(t)/100);
        unsatL = FD2(t) - unsatF;
        %Calculating forwarded batches after depletion due to fulfilled demand
        %FIFO Depletion
        for r = 1:m
```

```
                B_interim(1,r) = max(0,B_interim(1,r) - unsatF);
                unsatF = max(0,unsatF - B2(t,r));
            end
            r = m;
            %LIFO Depletion
            for r = m:-1:1
                B_interim(1,r) = max(0,B_interim(1,r) - unsatL);
                unsatL = max(0,unsatL - B2(t,r));
            end
            %Moving batches to next day's inventory
            for r = 1:m-1
                B2(t+1,r)=B_interim(1,r+1);
            end
            %Removing items to discard and computing waste
            waste2(t) = waste2(t) + B_interim(1,1);
            %Inventory level for next day after reduction due to demand
                and waste
            IP2(t+1) = sum(B2(t+1,1:m));
            EB2 = B2;
        end

        %Calculations for performance indicators
        DelRed(sc,rep) = (sum(n2(95:simL)~=0)-sum(n(95:simL)~=0))/sum(n
            (95:simL)~=0);
        AvgInvRed(sc,rep) = (mean(IP2(95:simL))-mean(IP(95:simL)))/mean(IP
            (95:simL));
        WasteRed12(sc,rep) = (sum(waste2(95:simL))-sum(waste(95:simL)))/
            sum(waste(95:simL));
        LostSalesRed12(sc,rep) = (sum(LD2(95:simL))-sum(LD(95:simL)))/sum(
            LD(95:simL));
```

```
            nDel_base(sc,rep) = sum(n(95:simL)~=0);
            nDel_mod(sc,rep) = sum(n2(95:simL)~=0);
            FR_base(sc,rep) = sum(FD(95:simL))/sum(ActD(95:simL));
            FR_mod(sc,rep) = sum(FD2(95:simL))/sum(ActD(95:simL));
            waste_base(sc,rep) = sum(waste(95:simL));
            waste_mod(sc,rep) = sum(waste2(95:simL));
            wastepc_base(sc,rep) = sum(waste(95:simL))/sum(n(95:simL));
            wastepc_mod(sc,rep) = sum(waste2(95:simL))/sum(n2(95:simL));
            AvgInv_base(sc,rep) = mean(IP(95:simL));
            AvgInv_mod(sc,rep) = mean(IP2(95:simL));
        end
        %Percentage reduction (reduction in weighted sums)
        FRD(rep) = (sum(FR_mod(:,rep)'.*Sn)-sum(FR_base(:,rep)'.*Sn))/sum(
            FR_base(:,rep)'.*Sn);
        WD(rep) = (sum(waste_mod(:,rep)'.*Sn)-sum(waste_base(:,rep)'.*Sn))/sum
            (waste_base(:,rep)'.*Sn);
        ID(rep) = (sum(AvgInv_mod(:,rep)'.*Sn)-sum(AvgInv_base(:,rep)'.*Sn))/
            sum(AvgInv_base(:,rep)'.*Sn);
        WPD(rep) = (sum(wastepc_mod(:,rep)'.*Sn)-sum(wastepc_base(:,rep)'.*Sn)
            )/sum(wastepc_base(:,rep)'.*Sn);
        DnD(rep) = (sum(nDel_mod(:,rep)'.*Sn)-sum(nDel_base(:,rep)'.*Sn))/sum(
            nDel_base(:,rep)'.*Sn);
end

%Plots
FRdrop=mean(FRD);
FRstd=std(FRD);
FRdist=makedist('normal',FRdrop,FRstd);
FRl=icdf(FRdist,0.025);
FRu=icdf(FRdist,0.975);
```

```
FRlim=(FRu−FRl)/2;
plot(D,mean(FR_base'),'DisplayName','FR(EWA_{SS})');
hold on;
plot(D,mean(FR_mod'),'DisplayName','FR(EWA_{SSmod})');
legend('FR(EWA_{SS})','FR(EWA_{SSmod})');
title(['Fill rate reduced by ',num2str(−round(100∗FRdrop,2)),'% (\pm',
    num2str(round(100∗FRlim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Fill rate');
grid on;
hold off;


avgwst_base=mean(waste_base');
avgwst_mod=mean(waste_mod');
poswas=sum(avgwst_base~=0);

Wdrop=mean(WD);
Wstd=std(WD);
Wdist=makedist('normal',Wdrop,Wstd);
Wl=icdf(Wdist,0.025);
Wu=icdf(Wdist,0.975);
Wlim=(Wu−Wl)/2;
plot(D(1:poswas),avgwst_base(1:poswas),'DisplayName','Waste(EWA_{SS})');
hold on;
plot(D(1:poswas),avgwst_mod(1:poswas),'DisplayName','Waste(EWA_{SSmod})');
legend('Waste(EWA_{SS})','Waste(EWA_{SSmod})');
title(['Waste reduced by ',num2str(−round(100∗Wdrop,2)),'% (\pm',num2str(
    round(100∗Wlim,2)),')']);
xlabel('Weekly mean demand');
```

```
ylabel('Waste');
grid on;
hold off;


Idrop=mean(ID);
Istd=std(ID);
Idist=makedist('normal',Idrop,Istd);
Il=icdf(Idist,0.025);
Iu=icdf(Idist,0.975);
Ilim=(Iu-Il)/2;

plot(D,mean(AvgInv_base'),'DisplayName','I_{\mu}(EWA_{SS})');
hold on;
plot(D,mean(AvgInv_mod'),'DisplayName','I_{\mu}(EWA_{SSmod})');
legend('I_{\mu}(EWA_{SS})','I_{\mu}(EWA_{SSmod})');
title(['Average inventory levels reduced by ',num2str(-round(100*Idrop,2)),'% (\pm',num2str(round(100*Ilim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Average inventory level');
grid on;
hold off;


avgwstpc_base=mean(wastepc_base');
avgwstpc_mod=mean(wastepc_mod');
poswaspc=sum(avgwstpc_base~=0);
Wpcdrop=mean(WPD);
Wpcstd=std(WPD);
Wpcdist=makedist('normal',Wpcdrop,Wpcstd);
```

```
Wpcl=icdf(Wpcdist,0.025);
Wpcu=icdf(Wpcdist,0.975);
Wpclim=(Wpcu-Wpcl)/2;
plot(D(1:poswaspc),avgwstpc_base(1:poswaspc),'DisplayName','Waste%(EWA_{SS})');
hold on;
plot(D(1:poswaspc),avgwstpc_mod(1:poswaspc),'DisplayName','Waste%(EWA_{SSmod})');
legend('Waste%(EWA_{SS})','Waste%(EWA_{SSmod})');
title(['Waste% reduced by ', num2str(-round(100*Wpcdrop,2)),'% (\pm',num2str(round(100*Wpclim,2)),')']);
xlabel('Weekly mean demand');
ylabel('Waste%');
grid on;
hold off;

Ddrop=mean(DnD);
Ddrop=mean(DnD);
Dstd=std(DnD);
Ddist=makedist('normal',Ddrop,Dstd);
Dl=icdf(Ddist,0.025);
Du=icdf(Ddist,0.975);
Dlim=(Du-Dl)/2;

plot(D,mean(nDel_base'),'DisplayName','D_n(EWA_{SS})');
hold on;
plot(D,mean(nDel_mod'),'DisplayName','D_n(EWA_{SSmod})');
legend('D_n(EWA_{SS})','D_n(EWA_{SSmod})');
title(['Number of deliveries changed by ',num2str(round(100*Ddrop,2)),'% (\pm',num2str(round(100*Dlim,2)),')']);
```

```
xlabel('Weekly mean demand');
ylabel('Number of deliveries');
grid on;
hold off;

WC_EWA_SS1=wastepc_mod-wastepc_base;
FRC_EWA_SS1=FR_mod-FR_base;
EWA_SS1_val=mean(WC_EWA_SS1')./mean(FRC_EWA_SS1');
plot(1:21,EWA_SS1_val,'*');
hold on;
title('Value of modifying EWA_{SS} policy for m=9, Q=10, FIFO=0.9');
ylabel('Value (ratio of changes in waste% and fill rate)');
xlabel('Store variety');
axis([0 21 -inf inf]);
grid on;
hold off;

toc;
```