WAVE POWER STATISTICS FOR SEA STATES BASED ON WIND STATISTICS

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Abstract

The paper addresses how the wave power for sea states can be assessed based on available wind statistics for an ocean area. Results using a simple analytical tool are exemplified for a Pierson-Moskowitz wave spectrum together with long-term wind statistics from one location in the Northern North Sea and from four locations in the North Atlantic. The present method should be useful for making preliminary assessment of wave power for sea states based on, for example, global wind statistics.

Keywords: Wave power, Wind statistics, Pierson-Moskowitz spectrum, Weibull distribution, Mean wind speed.

1. Introduction

Ocean wave energy represents a promising alternative energy resource. The engineering community has a challenge in designing appropriate devices to convert energy from waves. The response of such devices is frequency dependent, i.e. the design of the device will determine the natural frequency and the frequency range that might give a significant response. Generally, a device will be exposed to sea states representing a wide range of wave conditions. Sea states are described by the significant wave height H_s and the spectral peak period T_p (or the mean zero-crossing wave period T_z). Furthermore, the individual waves within a sea state are described by the wave height H and the wave period T.

An optimum design of a wave power device requires that it can be controlled to give maximum power at a low cost in a sea state, at the same time as the control can protect it when exposed to extreme conditions. Further details regarding such issues are given in e.g. Venugopal and Smith (2007); Valerio et al. (2007). Moreover, a comprehensive review of wave energy extraction is given in Falnes (2007).

Consequently, to know the statistical properties of the waves, both for those characterizing the sea state $(H_s, T_p \text{ or } T_z)$ and the individual random waves within a sea state (H, T), are crucial for designing an optimum wave power device. Knowledge of the joint statistics of wave power both with sea state parameters and individual wave parameters are of interest. Examples of previous works on wave power statistics for sea states are Ertekin and Xu (1994), also including a review of the literature up to that date; Pontes (1998); Beels et al. (2007); Kerbiriou et al. (2007); Myrhaug et al. (2011), while wave power statistics for individual waves has been addressed by Smith et al. (2006); Venugapol and Smith (2007); Myrhaug et al. (2009); Leira and Myrhaug (2015).

Furthermore, Izadparast and Niedzwecki (2011) used joint statistics of H and T and exemplified how to evaluate the long-term wave power potential. Sheng and Lewis (2012) as well as Pastor and Liu (2014) used numerical validation procedures to assess wave energy extraction. It should also be noted that Ertekin and Xu (1994) were the first to estimate the wave power from observations of wind statistics.

The main motivation for this work is to provide a simple analytical method which can be applied to make first assessments of the wave power potential for an ocean area based on available mean wind speed statistics, e.g. from global wind statistics. Such a simple tool might serve as a first inexpensive wave power estimation before eventually applying more work-intensive tools based on available long-term wave statistics from measurements or hindcast analysis, or by applying wave simulation models. The present method demonstrates how to estimate the wave power potential for sea states based on wind statistics for the mean wind speed at the 10 m elevation above the sea surface. Results are exemplified by using the Pierson-Moskowitz model wave energy spectrum together with long-term mean wind speed statistics from one location in the Northern North Sea and from four locations in the North Atlantic. Thus, it is demonstrated how the present analytical tool can be used to make a preliminary assessment of the wave power for sea states based on e.g. global mean wind speed statistics.

2. Background

Following e.g. Falnes (2007) the wave power within a sea state of random waves, defined as the transport of wave energy per unit crest length of a progressive wave front, is given by

$$J = \frac{1}{2}\rho g^2 \mathrm{m}_{-1} \tag{1}$$

Here ρ is the density of the fluid, g is the acceleration due to gravity, $m_{-1} = \int_0^\infty \omega^{-1} S(\omega) d\omega$, ω is the wave frequency in *rad/s*, and $S(\omega)$ is the single-sided wave energy spectrum.

3. Example of results for a Pierson-Moskowitz spectrum and long-term wind statistics

3.1 Pierson-Moskowitz spectrum

The Pierson-Moskowitz (PM) type wave energy spectrum has the form (Tucker and Pitt 2001)

$$S(\omega) = \frac{A}{\omega^5} \exp\left[-\frac{B}{\omega^4} \right] \tag{2}$$

with the spectral moments $m_n = \int_0^\infty \omega^n S(\omega) d\omega;\ n = -1, 0, 1, 2, ---$, as

$$m_n = \frac{1}{4} A B^{\frac{n}{4}-1} \Gamma(1 - \frac{n}{4}) \text{ for } n < 4 \tag{3}$$

where Γ is the gamma function. For the original form of the PM-spectrum, $A = \alpha g^2$ with $\alpha = 0.0081$, $B = 1.25\omega_p^4$, $\omega_p = 2\pi / T_p$ is the spectral peak frequency in *rad/s*, and T_p is the spectral peak period.

Originally the PM spectrum was given with the mean wind speed at the 19.5 m elevation above the sea surface as the parameter, while the formulation with the mean wind speed at the 10 m elevation with $U_{10} = 0.93\, U_{19.5}$ gives $T_p = 0.785\, U_{10}$ (Tucker and Pitt 2001), and from Eq. (3) that $m_{-1} = \frac{1}{4} \alpha g^2\, 1.25^{-1.25}\, \omega_p^{-5} \Gamma(1.25)$. Thus, from Eq. (1) the wave power is

$$J = \frac{1}{8}\rho g^4 \alpha \, 1.25^{-1.25} \omega_p^{-5} \, \Gamma(1.25) \tag{4}$$

which can be expressed in terms of U_{10} as (by taking $\rho = 1025\,\text{kg/m}^3$)

$$J = 0.20 U_{10}^5 \tag{5}$$

3.2 Long-term wind statistics

For an ocean area results for *J* can be obtained from available wind statistics for that area, for example, from a long-term distribution of U_{10}. Here examples are given by using five cumulative distribution functions (*cdfs*) of U_{10}; one from the Northern North Sea (NNS) and four from the North Atlantic (NA). The *cdf* from NNS is given by Johannessen et al. (2001), based on 1 hourly U_{10}-values from wind measurements over the years 1973 – 1999. The four *cdfs* from NA are given by Mao and Rychlic (2017), based on 19 years of wind speed data at four locations along ship routes in NA (see Table 1). The five *cdfs* of U_{10} are given by the two-parameter Weibull model

$$P(U_{10}) = 1 - \exp\left[-(\frac{U_{10}}{\theta})^{\beta} \right]; U_{10} \geq 0 \tag{6}$$

with the Weibull parameters θ and β as given in Table 1.

3.3 Statistical properties of wave power

The wave power statistics can be deduced by using the *cdfs* of U_{10} given in Eq. (6) and Table 1. Relevant statistical quantities are e.g. the expected (mean) value of $J, E[J]$, and the variance of $J, Var[J]$, which are proportional to $E[U_{10}^5]$ and $Var[U_{10}^5]$, respectively. Thus, the calculation of $E[U_{10}^n]$ is required, which for a two-parameter Weibull distribution are (Bury 1975)

$$E[U_{10}^{n}] = \theta^{n}\,\Gamma(1+\frac{n}{\beta}) \tag{7}$$

$$Var[U_{10}^{n}] = E[U_{10}^{2n}] - (E[U_{10}^{n}])^{2} \tag{8}$$

where Γ is the gamma function. The results for $E[J]$ and the ratio between the standard deviation of $J(=\sqrt{Var[J]})$ and $E[J]$ (st.dev./m.v.) are given in Table 1.

From Table 1 it appears that $E[J] = 47\,\mathrm{kW/m}$ for NNS, which agrees well with the result $E[J] = 43\ \mathrm{kW/m}$ in Myrhaug et al. (2011, Table 1) for NNS based on wave statistics for sea states. However, the present results for location 1 (NA), $E[J] = 66\ \mathrm{kW/m}$, and location 3 (NA), $E[J] = 67\,\mathrm{kW/m}$ appear to be lower than the results in Myrhaug et al. (2011, Table 1) with $E[J]$ in the range 83 to 100 kW/m for five NA sites. From Table 1 it is also noticed that the standard deviation is large; the ratios st.dev./m.v. are in the range 2.3 – 2.5 for NA and 4.2 for NNS. Thus, if the standard deviation is taken into account there is agreement between the present results and those referred to in Myrhaug et al. (2011, Table 1).

3.4 Discussion

Here some comments are given on the present method versus a procedure which commonly is used. Ideally, long-term wave data from measurements or from hindcast analysis should be available at the actual site, but that will rarely be the case. Thus, for a more complete and thorough assessment of the wave power potential at the site, common practice would be to start with the available data on the joint statistics of significant wave height and spectral peak period (or other characteristic wave periods), preferably within directional sectors at a nearby location. The next step would be to apply an appropriate wave simulation model to obtain the joint statistics of significant wave height and characteristic wave period within directional sectors; then finally to use this to assess the wave power potential. However, if wind statistics is the only available information at the site or at a nearby location, then this can be used as input in a wave model. Here an alternative is presented, providing a simple analytical method which can be used to make preliminary assessment of available wave power based on wind statistics and a model wave spectrum. It is important, however, to assess the accuracy of the approach versus common practice, but this is only possible to quantify by comparing with such method over a wide parameter range, which is beyond the scope of this short communication.

4. Summary and conclusions

The paper provides a simple analytical method which can be used to estimate the wave power potential for sea states, based on wind statistics for the mean wind speed at the 10 m elevation above the sea surface. Results are exemplified by using the Pierson-Moskowitz wave spectrum together with long-term wind statistics from one location in the Northern North Sea (NNS) and from four locations in the North Atlantic (NA). The mean value and the standard deviation of the wave power are provided, showing that the standard deviation is large. It appears that the present results agree with previous results for the same ocean areas based on estimating the wave power from long-term wave statistics for sea states.

The present analytical tool should represent a useful method for making a preliminary assessment of available wave power for an ocean area based on e.g. available global wind statistics.

References

Beels C, Henriques JCC, De Rouck J, Pontes MT, De Backer G, Verhaeghe H (2007) Wave energy resource in the North Sea. In: *Proceedings of the 7th European Wave and Tidal Energy Conference.* Porto, Portugal. CD-Rom

Bury KV (1975) *Statistical Models in Applied Science.* Wiley, New York

Ertekin RC, Xu Y (1994) Preliminary assessment of the wave-energy resource using observed wave and wind data. *Energy* 19(7):729-738

Falnes J (2007) A review of wave-energy extraction. *Marine Structures* 20(4):185-201. doi:10.1016/j.marstruc.2007.09.001

Izadparast AH, Niedzwecki JM (2011) Estimating the potential of ocean wave power resources. *Ocean Engineering* 38(1): 177-185. doi:10.1016/j.oceaneng.2010.10.010

Johannessen K, Meling TS, Haver S (2001) Joint distribution for wind and waves in the Northern North Sea. In: *Proceedings of the 11th Offshore and Polar Engineering Conference.* Stavanger, Norway, Vol. III, pp. 19–28

Kerbiriou M-A, Prevosto M, Maisondien C, Clément A, Babarit A (2007) Influence of sea-states description on wave energy production assessment. In: *Proceedings of the 7th European Wave and Tidal Energy Conference*, Porto, Portugal, CD-rom

Leira BJ, Myrhaug D (2015) Long-term wave -power statistics for individual waves. ASME *Journal of Energy Resources Technology*,137(4): 041202-041202-7. doi:10.1115/1.4029869

Mao R, Rychlik I (2017) Estimation of Weibull distribution for wind speeds along ship routes. In: *Proceedings of IMechE Part M: Journal of Engineering for the Maritime Environment* 231(2): 464-480. doi:10.1177/1475090216653495

Myrhaug D, Leira BJ, Holm H (2009) Wave power statistics for individual waves. *Applied Ocean Research* 31(4): 246-250. doi:10.1016/j.apor.2009.07.001

Myrhaug D, Leira BJ, Holm H (2011) Wave power statistics for sea states. ASME *Journal of Offshore Mechanics and Arctic Engineering* 133(4): 044501-044501-5. doi:10.1115/1.4002739

Pastor J, Liu Y (2014) Power absorption modeling and optimization of a point absorbing wave energy converter using numerical method. ASME *Journal of Energy Resources Technology* 136(2): 021207-021207-8. doi:10.1115/1.4027409

Pontes MT (1998). Assessing the European wave energy resource. ASME *Journal of Mechanics and Arctic Engineering* 120(4): 226-231. doi:10.1115/1.2829544

Sheng W, Lewis A (2012) Assessment of wave energy extraction from seas: numerical validation. ASME *Journal of Energy Resources Technology* 134(4): 041701-041701-8. doi:10.1115/1.4007193

Smith GH, Venugopal V, Wolfram J (2006) Wave period group statistics for real sea waves and wave energy extraction. In: *Proceedings of IMechE Part M: Journal of Engineering for the Maritime Environment* 220(3): 99-115. doi:10.1243/14750902JEME37

Tucker MJ, Pitt EG (2001) *Waves in Ocean Engineering.* Elsevier, Amsterdam

Valerio D, Beirao P, Sa da Costa J (2007) Optimization of wave energy extraction with the Archimedes wave swing. *Ocean Engineering* 34(17-18): 2330-2340. doi:10.1016/j.oceaneng.2007.05.009

Venugopal V, Smith GH (2007) The effect of wave period filtering on wave power extraction and device tuning. *Ocean Engineering* 34(8-9): 1120-1137. doi:10.1016/j.oceaneng.2006.08.003

Table 1. Weibull parameters (see Eq. (6)) and results for the NA and NNS locations.

	θ[m/s]	β	$E[J]$ (kW/m)	$\frac{\text{st.dev.}}{\text{m.v.}}$	$E[U_{10}]$ (m/s)
1. 20°W 60°N	10.99	2.46	66	2.3	9.75
2. 10°W 40°N	7.11	2.30	9	2.5	6.30
3. 40°W 50°N	11.04	2.48	67	2.3	9.79
4. 20°W 45°N	9.32	2.47	29	2.3	8.27
North Atlantic (NA) location	8.426	1.708	47	4.2	7.52