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Profitability of a Hydropower Producer Operating in Two Different Markets

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Specialization Project
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Problem Description

With an international focus on a larger share of renewable energy sources, solar and wind have seen an increase in installed capacity and production over the last few years. However, solar and wind are dependent on weather and other uncontrollable factors, making the actual production in constant variation throughout the day and year. These non-regulative energy sources will make the need for flexible and potent balancing services more apparent and critical in the future, mostly at short notice.

For hydropower producers, their future role and participation in power production can be changed completely. At the power exchange Nord Pool, most of the power produced from hydro today is traded at the Day-Ahead market. Hydropower with a reservoir has the advantage of being able to store potential power for future use. This ability makes hydropower plants capable of operating in the Day-Ahead market, as well as having reserve capacity for balancing services in the Balancing market. Since a hydropower producer wants to keep the revenues high for future investments or profit, one must balance the participation in each market to maximize profit and to avoid spillage/shortage of water. How much power should be used to produce electricity and how much should be kept in reserve for balancing services will be critical for this optimization.

These are some of the essentials behind this specialization project, in which a prototype optimization model for production planning run in AMPL will be used to study the potential for operating in both the Day-Ahead market and the Balancing market. The flexibility and performance of the model will be studied under different scenarios to ensure close to perfect operation at all times, and model extensions will be implemented to ensure realistic behavior.

Preface

This specialization project has been written at the Department of Electrical Power Engineering at the Norwegian University of Science and Technology (NTNU). The project has been finalized during Fall semester in 2017, in cooperating with both SINTEF Energy Research and Lyse AS. The work done is part of optimizing a prototype Long Term Scheduling model that implements spinning reserve capacity as profitable production. The model has been developed by SINTEF Energy Research, while the input data used is given by Lyse AS.

Several persons have supported and guided me during this project. I would especially like to thank my supervisor Magnus Korpås for his guidance and help throughout the semester. His contagious positive attitude has been a driving factor in accomplishing the work in this project. I would also like to thank Arild Helseth at SINTEF, who has been an immense help in learning me how to use AMPL, and patiently shown how the given model works. His availability for discussions and feedback on possible challenges, has made me search for the optimal solution to truly understand the concepts.
I would also send my thanks to Lyse AS for providing me with data regarding their hydropower station Maudal, allowing me to test the model with realistic data.

Lastly, a shout-out to my other fellow master students is in order. Without their encouraging attitude and motivation, this project would never be what it is.

Kasper Emil Thorvaldsen
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Summary

In this project, the main focus has been to study the potential profit increase when introducing hydropower producers to the balancing market alongside the energy market. The goal was to find how the producer should allocate resources to optimize participation in both markets, and how this affects scheduling of the hydropower station.

SINTEF Energy Research has developed a prototype model that extends the possible markets that hydropower producers can participate in, to include secondary reserves in the balancing market. This model, that has been used for this project, is a long term scheduling model operating at a multi-stage, multi-scenario stochastic level. Lyse AS has provided with data from the hydropower station that will be analyzed, including inflow and price data. Since the model is currently a prototype under development, a major part of this report has analyzed the scheduling behavior for reserve capacity, to find and improve the scheduling decision.

To find the potential increase in profit for a producer when participating in both markets, potential profit was analyzed both with and without this extension. The results showed that the hydropower producer could expect an increase in total profit by selling reserve capacity. For the given run, the increase was 8.59 % compared to only selling power, despite having the same inflow and price pattern.

The model was also tested at low and high reservoir levels to study the optimal scheduling decisions for reserve capacity sales. The results showed that the model needs to include a guarantee of having excess water stored in the reservoir for sudden up-regulation of power due to reserve capacity sales to prevent the producer from taking unnecessary risks. This implementation made the model find additional value in storing water to allow reserve capacity sales even at low reservoir levels and high energy prices, which also gives the producer a buffer against sudden incidents.

Sammendrag

Hovedfokuset for dette prosjektet har vært å studere profittfortjenesten for en vannkraftsprodusent ved å aktivt delta i både energi- og balansemarkedet. Målet var å finne ut hvordan produsenten burde fordele ressursutnyttelsen for best deltakelse, og hvordan dette påvirker produksjonsplanleggingen for kraftverket.

SINTEF Energi har utviklet en prototypemodell som utvider de potensielle markedene en vannkraftsprodusent kan delta i til å inkludere sekundærreserver i balansemarkedet.
Modellen er en langtidsmodell som opererer på et flerstegs, multi-scenario stokastisk nivå. Lyse AS har gitt både stasjonsdata, pris- og tilsigsdata for å kunne kjøre denne modellen. Denne modellen er i skrivende stund under utvikling, noe som har medført at store deler av denne rapporten også vil analysere modellens beslutninger av reservekapasitetssalg for å finne forbedringer.

Det ble gjennomført simuleringer av modellen både med og uten reservekapasitetssalg, for å finne den potensielle økningen av profitt for produsenten. Resultatene viste at det ligger en potensiell profittøkning ved deltakelse i begge markedene. For de aktuelle simuleringene lå økningen på 8,59 % sammenlignet med kun å delta i energimarkedet. Dette til tross for at det ble brukt samme utgangspunkt og lik tilsigsflyt i systemet.

Modellen som ble brukt, ble testet under lave og høye reservoarnivåer for å kunne studere den optimale beslutningen som ble tatt angående reservekapasitetssalg. Resultatene viste at modellen trenger å inkludere en vannreservegaranti for å kompensere for plutselig oppregulering av produksjon. Dette vil medføre mindre risiko for produsenten. Implementeringen av en slik garanti medførte at modellen fant en verdi i å spare litt vann i reservoaret for muligheten til å selge reservekapasitet, til tross for høye energipriser. Dette vil gi produsenten en buffer som kan benyttes ved plutselige uforutsette hendelser.

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1 Introduction

Hydropower is an important renewable source that is well implemented in Norway, contributing to over 95 % of total power production. With the focus on reducing non-renewable energy sources in the European power grid to reduce climate change impact, hydropower will play a key role during this transition. Renewable energy sources like wind and solar have experienced an increase in installation in Europe over the last years, leading to more renewable production in the market. However, these sources are dependent on the weather, causing varying immediate generation, resulting in challenges to keep power balance in the grid. Norwegian hydropower can contribute to mitigate this imbalance with flexible production. This will give Norwegian hydropower producers a new power market they can increase their contribution in, creating new business opportunities.

The main goal of this work is to check the increased profitability for a hydropower producer by contributing to both the energy and balancing market. This is done by using a prototype long term scheduling model. The model has been implemented in A Mathematical Programming Language, also known as AMPL, by SINTEF Energy Research. It is a multi-stage, multi-scenario stochastic model that describes a single hydropower producer operating in both the energy and balancing market.

Since the model is considered a prototype, the behavior and scheduling under several conditions must be checked to find abnormalities or invalid operations when compared to the realistic scheduling. Therefore, the model will be analyzed during several conditions to look for unexpected decisions.

There are different methods for performing the optimization model analysis that can be utilized, each with their strengths and weaknesses. Two of these will be shortly explained and used when testing the model, finding the differences and consequences behind each method.

This specialization project is divided into three main parts: Literature explanation of key areas, model description and performed runs of the model. Section 2 will focus on the theoretical background needed for understanding this project. Section 3 will explain the optimization model and what factors it considers. Section 5 sets up the case that will be studied using this model, with Section 7 and 8 contain the results from the performed analyzes.

2 Theory, Background and Methodologies

2.1 European Power Mix Changing

Due to ambitious goals regarding reduction of greenhouse gases (GHG) in the European Union (EU), the European power grid is expected to go through restructuring in the future. This is mainly due to solar and wind power replacing coal and gas power production. However, these changes will result in less flexibility as more unregulated power production increases [21].
The following sections will take a closer look at the generation mix in the EU, the changing market coupling, and some possible solutions to increase flexibility in the grid.

2.1.1 European Generation Mix

With the new climate and energy policies, the EU is striving towards a more renewable production portfolio. At present, the EU bases their environmental work on two main policies: The 2020 climate and energy package and the 2030 climate and energy framework, in which the main goals are presented in Table 1 [5].

Table 1: Overview of the EUs climate and energy goals for year 2020 and 2030.

Goals	2020	2030
Cut in GHG (from 1990 levels)	20 %	40 %
Share of renewable production	20 %	27 %
Improved energy efficiency	20 %	27 %

In 2015, the generation mix for Europe had a share of 17 % renewable energy (excl. hydro), as shown in Figure 1. As of right now, the goal of 20 % renewable before 2020 looks reachable due to the shared focus.

Figure 1 also shows an increase in annually renewable production from 2011 to 2015, while fossil fuels experience a reduction. To achieve the 2020 and 2030 goals, this trend must continue to increase over time. Contributing to this, the main investments will be in solar and wind [21]. With the increase in unregulated power sources, the power grid will be increasingly in need of power sources to maintain power balance in the future.

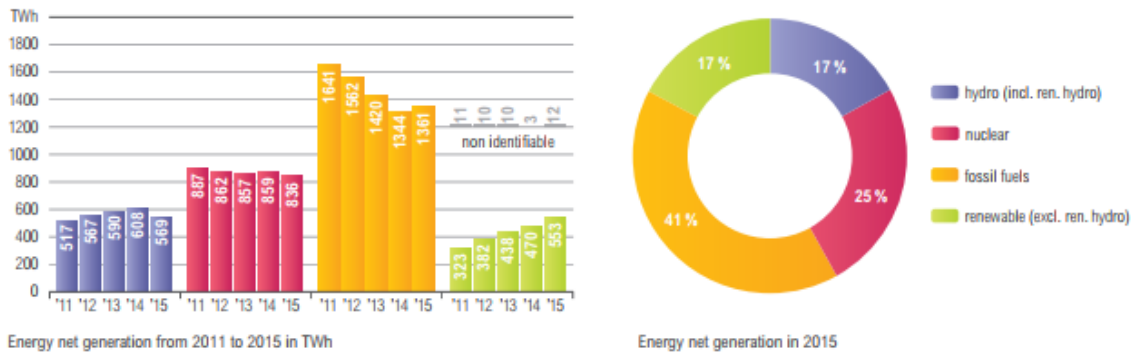

Figure 1: ENTSO-E overall net generation per year between 2011 and 2015, and percentage production for 2015 [8].

2.1.2 Market Coupling

Europe will need to improve the electrical interconnections across borders to accomplish the climate and energy goals, which will lead to more renewable energy in the market, as well as improved security of electrical supply [6].

The Nordic countries have steadily increased their transmission capacity to the rest of Europe over the last years, and proposed further installations in the future. In 2016, the capacity was at roughly 7 000 MW, with a proposed total capacity at around 11 000 MW in 2025 [20]. Table 2 shows the proposed new capacities from the Nordic countries to some European countries.

Table 2: Proposed new cables between the Nordic countries and the rest of Europe.

Name	Country 1	Country 2	Capacity [MW]
NordLink	Norway	Germany	1 400
NSL	Norway	UK	1 400
Cobra	Denmark	Netherlands	700
Viking Link	Denmark	UK	1 000
Krigers Flak	Denmark	Germany	600
HansaPowerBridge	Sweden	Germany	600

This extension of transmission capacity is important to maintain stability in the European power grid, and will become more critical as the unregulated renewable energy sources become more dominant for production.

2.1.3 Better Generation Flexibility

With an increasing installation of unregulated energy sources and a more interconnected power grid, the grid will experience changes in behavior in the future. The power balance in the system will be subject to more fluctuation that must be kept under control. Therefore, the role of flexible power production will be important due to the ability to up- and down-regulate at a quick pace, a role that coal and gas plants so far has been important contributors to.

Hydropower is able to perform this type of flexible power production, due to storage possibilities in reservoirs and high regulation in power production. With an increase in interconnections between European countries, Sweden and Norway can contribute to keeping balance in the European grid, due to their high amounts of hydropower.

2.2 The Nordic System

The Nordic countries are countries located in Northern Europe, consisting of Iceland, Finland, Denmark, Norway, and Sweden. With the exception of Iceland, the Nordic countries have connections to the Baltic countries and to continental Europe, interacting with both export and import of electricity. The Nordic countries primarily have hydro and thermal power production, with the addition of nuclear and wind power. However, each country has their own generation mix, as can be seen in Figure 2. The generation mix varies, from almost purely hydro in Norway, to an almost even distribution of different sources in Finland.

With these different energy sources, the countries have the possibilities to trade between them to fully utilize each source. For instance, hydropower in Norway allows for export during wet years, and import during dry years. Denmark can export wind power during high wind conditions, and import when the wind is low.

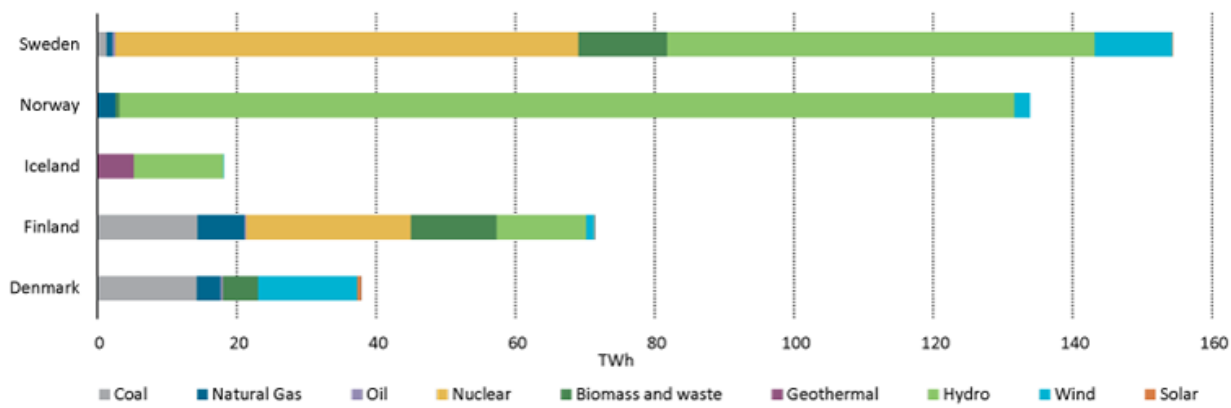

Figure 2: Power production in the Nordic countries for 2013 [12].

In Norway, about 85 TWh of power can be stored in reservoirs, giving a high flexibility of storing water for later usage [19]. Other characteristics such as quick production regulation and cost-efficient flexibility make hydropower a major contributor in the balancing market, as it will be able to improve stability and security of supply. With the European trend, Norwegian hydropower will play an important role in the future European power system, based on the increased amount of unregulated power production.

2.3 The Nord Pool Market

For Norwegian power producers, all sales and bids of power are done through the European power market Nord Pool. Nord Pool consists of multiple markets that producers can participate in, based on the wanted role for production. The financial market of Nord Pool will not be included in this project, only the physical market will be described. This is due to the focus of this project.

The physical markets in Nord Pool include the Day-Ahead (DA) and Intra-Day (ID) market, while the Transmission Operators (TSO) of each country run the Balancing (BM) and Capacity (RKOM) market. An illustration of their time window is shown in Figure 3. The following subsections will describe their role and purpose in the power market, and the opportunities each producer has. The information is obtained from Nord Pool [4] and the Norwegian TSO Statnett [3].

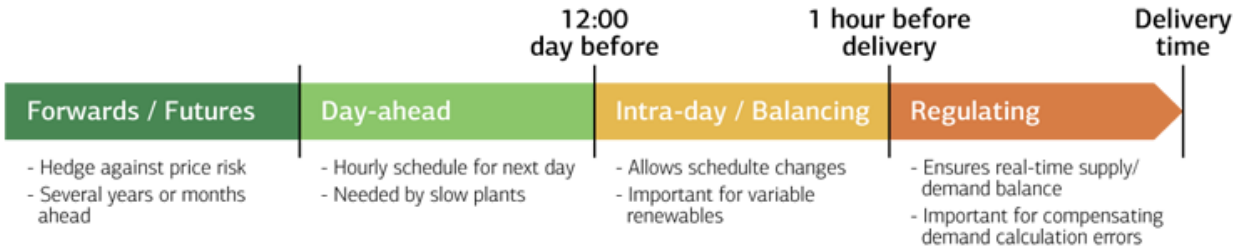

Figure 3: Overview of the different markets in Nord Pool [12].

2.3.1 The Day-Ahead Market

The largest market for power trading is the Day-Ahead market, in which both sellers and buyers make contracts for delivery of power the next day. Nord Pool gathers bids from both sellers and buyers, specifying the quantity of power to sell/buy, and at what price. At noon the day before, the market will close, and the gathered bids will be sorted and separated into supply and demand bid curves. These curves will be determined on an hourly basis, and the final market price and quantity of power will be where the two curves cross, shown in Figure 4. The market price will be the same for all participants.
The ideal setup would be to have the same bidding zone for all participants, but due to transmission constraints, this is hard to achieve. As a result, multiple bidding areas are introduced with their own area price based on their own supply, demand, and transmission capacity to other areas. This is why different regions in Norway can experience different prices at the same time.

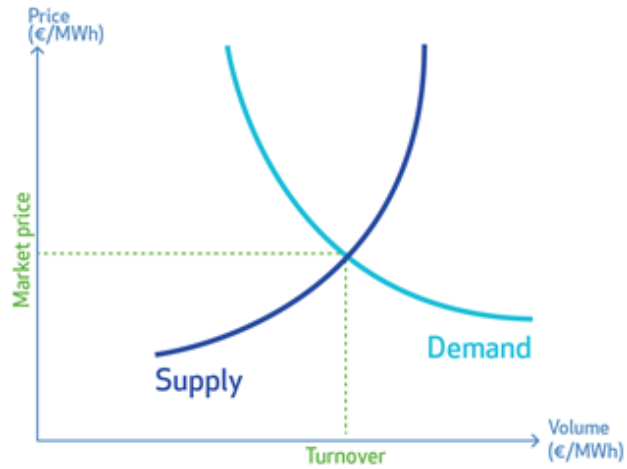

Figure 4: Illustration showing how the market price is determined [4].

2.3.2 The Intra-Day market

The Intra-Day market supplements the Day-Ahead market to prevent imbalances in the market. Since the DA market terminates one day before the chosen hour of operation, there can be events in-between that create problems for balance between demand and supply, such as an unexpected shutdown, or unplanned high production yield. The ID market however, allows buyers and sellers to trade quantities from 14:00 CET till 1 hour before delivery, and thereby contributes to keep the market balanced.

The ID market is expected to have an increased share of trade in the future, due to the higher share of unregulated renewable sources like solar and wind. The uncertainty in these energy sources is reduced as the actual production time approaches, decreasing the imbalance possibility they can create.

2.3.3 The Balancing Market

When there is only one hour left, and the ID closes, the TSOs are responsible for any imbalances that occur from here. They must maintain power balance in the grid at all times, and must thus acquire production sources that can be utilized to help achieve balance. The possible imbalances can be caused by end-user variations, outage of lines or industry, to name a few. The BM is where producers can participate by selling power regulation. There are three regulation reserves; primary, secondary and tertiary reserves.

The primary goal is to keep the system frequency stable in the Nordic system, which is 50 Hz. Figure 5 illustrates how each reserve contributes and at what level. When the frequency devia-

tion occurs, the primary reserves (FCR) will kick in, an automatic regulation that will dampen the deviation till the secondary reserves (FRR) automatically activate after a few minutes.
FRR is activated by a regulation signal from the TSO to the producers control system. This signal will affect the power output from the producer, causing either a down- or up-regulation of power to restore the frequency, releasing FCR. The type of regulation is based on if there is a power surplus or deficit, respectively. Thus, the producer is obliged to have spinning reserves that can be altered both up and down.
If the frequency cannot be restored, then the TSO will manually activate the tertiary reserves (manual FRR) to secure optimal frequency and release primary/secondary reserves.

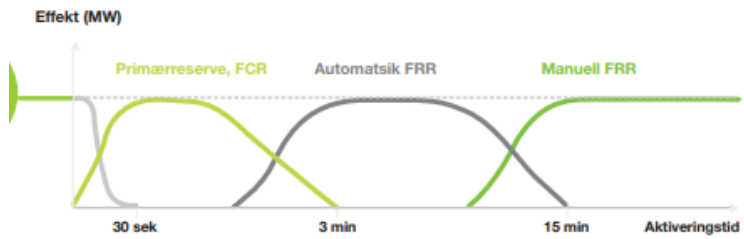

Figure 5: Graphical presentation of when each reserve kicks in [21].

2.3.4 The Norwegian Capacity Market

To ensure stability in the system even during unexpected events, there is a capacity market within the Norwegian Balancing market (other Nordic countries have their own solutions). This market provides up-regulation of power, and is known as the "Regulerkraftopsjonsmarkedet" (RKOM). The principle of this market is that the TSOs pay producers to have excess capacity available, and to contribute with this capacity to the balancing market if prompted. There are two different options in the market, weekly or seasonal trading. The seasonal trading requires producers to have their capacity on standby for the entire season for potential up-regulation. The required demand is considered before every season. The weekly trading is based on current power system and on future forecasts. Each day is split into two periods for weekly trading: RKOM-day [00:00 – 05:00], and RKOM-night [05:00-00:00].

Selling reserve capacity does not imply that the capacity will be utilized. However, the hydro producer is committed to produce when prompted, and must therefore ensure that both the capacity on the generator and stored water in the reservoir can meet the sudden demand at all times. If the producer cannot meet this demand, the producer will be subject to a penalty.

2.4 Hydro Power Scheduling

This section and the following subsections will focus on how hydropower scheduling is laid out to provide information about the optimization model used in this report. The information presented is based on old lecture and litterature given by the course "ELK-15 - Hydro Power Scheduling" held at NTNU by Gerard Doorman during fall 2017 [7].

The main objective of a hydropower producer is to maximize profit from production. The market the producer operates in, is considered a deregulated market, where the price is considered unknown. Scheduling this has proved to be difficult, due to the complicated physical setup and system size, the long time horizon and the uncertainty in inflow, price, and demand. To find ways to meet these challenges, the scheduling process has been divided into different phases, as shown in Figure 6. The optimization model used in this report is considered a Long Term Scheduling model (LTS), and will be the main focus here.

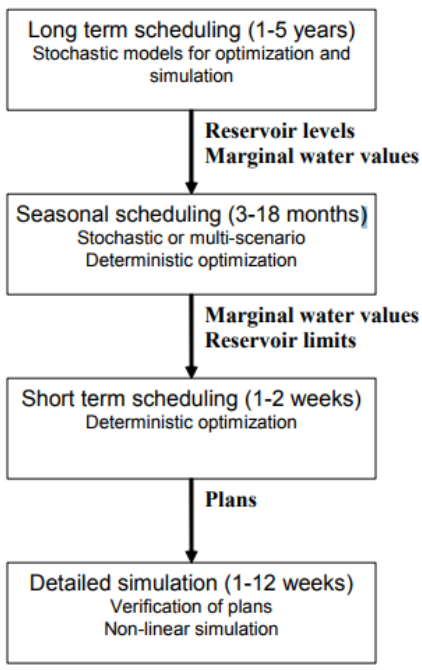

Figure 6: Figure showing the hierarchy of each scheduling type.

2.4.1 Long Term Scheduling

The main goal of LTS is to find the optimal use of the available resources and capacities within a time horizon of normally 1-5 years. A well-known example is the EFI's Multi-Area Power-market Simulator (EMPS) model, used by many producers. Parameters like inflow, price, demand etc.

are represented as discretized stochastic parameters, which include uncertainty into the model. Including stochastic parameters will make the model consider scenarios and events that have a low probability, but where the consequences are considerable. The model allows for global and local analysis, in which parameters like price can be represented as either internal or exogenous variables, respectively.

The LTS is split into two phases; the strategy phase and the simulation phase. In the strategy phase, water values are calculated for the system. For models with multiple areas, these are usually calculated separately. The water values are calculated for every stochastically possible scenario. The simulation phase will simulate the system scheduling with a weekly time resolution for a given number of years. Data from the water balance between reservoirs are used as restrictions in the seasonal scheduling.

The model described in this report consists of a single hydropower station, where the price variables are considered known and unaffected by production volume. Therefore, there exists only one area, and interconnections with other areas/stations can be ignored. The water values calculated in the strategy phase will thus be the optimal values for the hydropower station only. This will be described in detail later.

2.4.2 Seasonal Scheduling

Seasonal scheduling has the advantage of giving a more detailed analysis of the seasonal period than LTS, due to a shorter time horizon and because it is coupled to LTS at a preferred stage in the model. Normally, the coupling to LTS is chosen in periods with low uncertainty in reservoir levels, typically during October (end of filling season) or April (start of filling season). This scheduling will provide information about reservoir levels and their limits for the needed periods, which can be used in the short term scheduling.

2.4.3 Short Term Scheduling

The short term scheduling will find the actual production based on available resources on an hourly basis. Here, the inflow and price variables are assumed known, making it a deterministic model. The data from the seasonal scheduling will provide the limits for available resources, production etc., which will be considered for the actual production.

2.4.4 Water Value Calculation

A variable often mentioned when it comes to hydropower planning is the water value. Shortly, this is described as the expected marginal value of water, and is used as a decision variable for hydro production when compared to cost of operation. Despite water being a resource with no real cost (as water is considered free), the water value is used to set a cost on water to find out if one should produce now or store it for later. Water value is a function based on time and reservoir level. Given Figure 7, J_t is the total operation dependent cost for period t to T. This cost can be represented by Equation 1, where $S(x_t^{res})$ is the value of the reservoir level at the end of week T, and L_t is the operation dependent cost for every week t to T when moving from time step t to $t+1$.

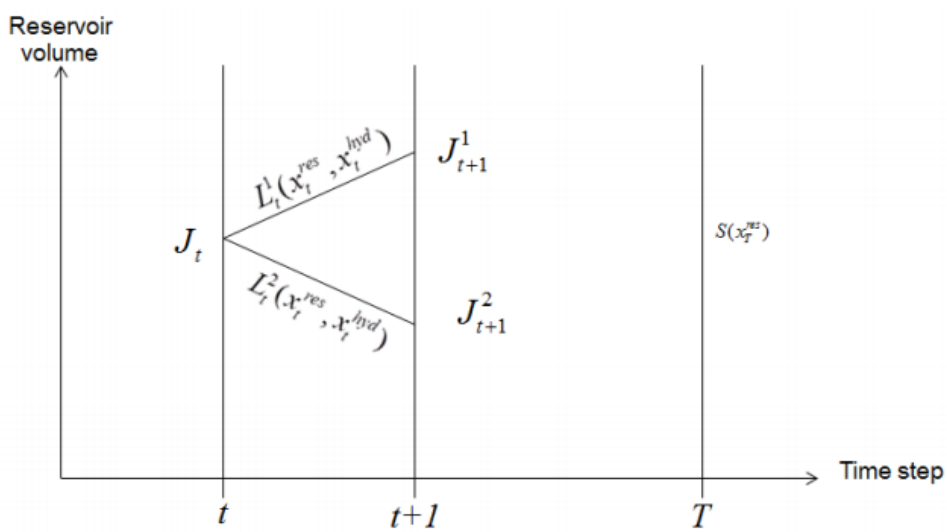

Figure 7: Illustration of planning period for water value calculation [7].

$$J_t(x_t^{res}) = min(\sum_{\tau=t}^{T} L_\tau(x_\tau^{res}, x_\tau^{hyd}) + S(x_T^{res})) \tag{1}$$

The dependent variable in these equations are the x_t^{hyd} variables, determining the released water from the reservoir for hydro production. L_t will vary based on this variable. The main goal here is to find the optimal value x_t^{hyd} that gives lowest cost J_t. Equation 1 can be altered to be dependent on the total operation cost for the next week, see Equation 2.

$$J_t(x_t^{res}) = min(L_t(x_t^{res}, x_t^{hyd}) + J_{t+1}(x_{t+1}^{res})) \tag{2}$$

Derivating from week t to $t+1$ based on released water for production will give Equation 3.

$$\frac{dJ}{dx_t^{hyd}} = \frac{dL}{dx_t^{res}} + \frac{dJ}{dx_{t+1}^{res}} \cdot \frac{dx_{t+1}^{res}}{dx_t^{hyd}} = \frac{dL}{dx_t^{hyd}} - \frac{dJ}{dx_{t+1}^{res}} = 0 \tag{3}$$

This equation gives that the marginal cost of operation connected to price, sales etc. is equal to the future total operation dependent cost depending on reservoir level (which is the marginal water value) for $t+1$. Thus, the water value should be equal to the marginal cost of operation for optimal production, and will be influenced by the production price at the given week.

For instances in which the goal is to maximize profit instead of minimizing cost, J is often referred to as α, known as the expected future profit.

When there are multiple stochastic scenarios for a given week, the water value must be based on all possible scenarios. The water value will thus be weighted on all possible considerations, and will present the optimal value when including uncertainty. The weighted water value is calculated using Equation 4, wherep_i is the probability of scenario i to occur, and $WV(t,i)$ is the water value for week t scenario i. Figure 8 shows how the water value for week t considers multiple scenarios from $t+1$.

$$WV_t^w = \sum_{i=1}^{I} p_i \cdot WV(t,i) \tag{4}$$

Calculating water values is done using backward dynamic programming, by assuming that the water value for week $t+1$ is known. Backward dynamic programming is a method used for calculating the optimal path when the end result is known, to find the optimal initial values. For water value calculation, this is the preferred approach, as the current water value is dependent on the future water values and their probabilities.

Starting at the end of the year, the water values are calculated backwards till week 1, and these values are compared to the initial values. If there is a deviation, the initial water values are adjusted and the process is repeated. This iterative process is done till the values converge.

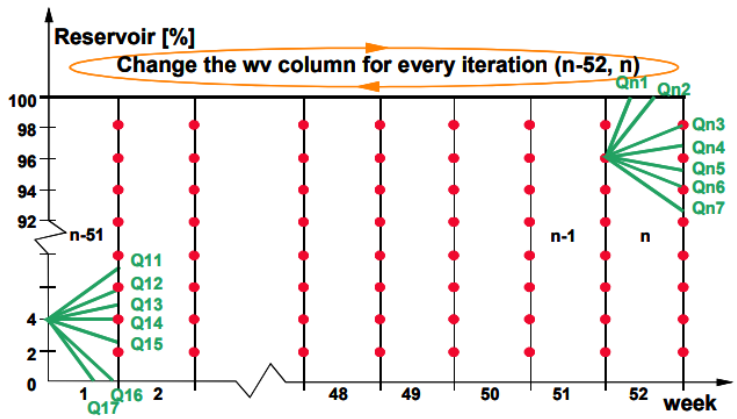

Figure 8: Illustration of the iterative process of calculating water values [7].

2.4.5 Modelling The Reservoir

When modelling using LTS, the water values must be calculated for a given reservoir level. Since the reservoir can take any value between a minimum and maximum size, the number of states that should be calculated is unlimited. To solve this, the reservoir is modelled to be a user-defined amount of segments, cutting the reservoir into equally distanced segments holding a unique reservoir level each. This will limit the reservoir to be modelled into $n \in NR$ points, in which NR is the total amount of reservoir segments. The water value will then be calculated for each reservoir segment.

2.5 The Turbine Types

When designing a hydropower station or replacing old equipment, one must decide which types of turbines to be installed. This decision is often combined with the height difference of the stored water and the turbine, often called the relative head. If the head is low, then less kinetic energy in the water will build up for production, while the opposite is true when the head is high. In Norway, three turbine types are normally installed: The Pelton, Francis and Kaplan turbines. These turbines will be discussed in the following subsections. Relevant data regarding the three types are presented in Figure 9 [14].

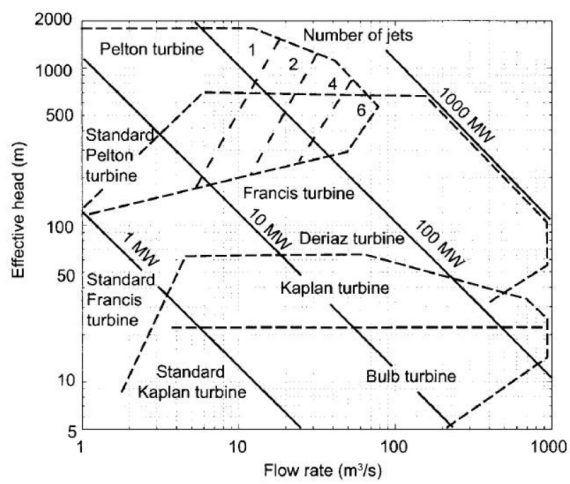

Figure 9: Figure showing the water flow and head range for Pelton, Francis and Kaplan turbines [14].

2.5.1 The Pelton Turbine

A Pelton turbine is an “impulse turbine”, and is most suited for stations with high head and low water flow. This results in low water flow for operating at minimum conditions.

2.5.2 The Francis Turbine

A Francis turbine is a “reaction turbine”, and is normally used for heads under 500 meters. It needs higher water flow for minimum production operation than the Pelton turbine.

2.5.3 The Kaplan Turbine

A Kaplan turbine is also a “reaction turbine”, and is typically used for stations with high water flow and low head. Therefore, it is normally used in run-of-river hydro stations.

2.6 Modelling Uncertainty and Approximations

As mentioned in section 2.4, there are uncertainties and other approximations related to modelling hydropower systems. This section will cover the uncertainties regarding price and inflow, and specify some approximations done regarding PQ curves for hydro systems in LTS.

2.6.1 Inflow

To generate reliable and consistent analyses for long term scheduling, it is critical to use precise assumptions of inflow in the system. Without detailed inflow behavior, the system will not be able to make accurate decisions regarding production compared to realistic behavior. And since long term scheduling is an analysis over several years, the inflow will have an impact on reservoir levels and thus the yearly scheduling decision. The water values will also be affected by this, due to reservoir dependency and future inflow.

For long term scheduling, a good baseline is to use inflow data generated by historical data. This will give the behavior historical reliability and the inflow a realistic pattern. It will also include extreme conditions that have happened earlier. In Norway, NVE stores inflow data for several rivers in Norway that can be extracted [15].

To limit the number of inflow scenarios that the model will use, the inflow should be cut down to a user-defined number of discretized stochastic variables that represent the possible inflow values for a given week, based on the realistic inflow data.

2.6.2 Price

Power price has an uncertainty because the price is not known at the time of bidding. This occurs at both the DA and BM. Assuming that the system in the model is small, the prices can be seen as inelastic and the market unprovoked by production variation from the system. This combined with historical price data can be used in a LTS model to simulate realistic behavior, and thus be implemented in the model where the price is considered a known variable, and the producer allocates in a reactive manner.

As with inflow, price data should be converted into several discretized stochastic variables based on historical behavior for usage in LTS.

2.6.3 Inflow and Price Probabilities

When operating with stochastic models, there are multiple price and inflow nodes for each week in a hydro system that has a probability of occurring. However, price and inflow is normally

dependent on behavior in the past when deciding the future stage, making modelling and representation this long and complicated. This is especially true for backward dynamic programming, in which the previous stages have not been determined yet for the current stage.

To solve this dilemma, price and inflow can be modelled in a Markov Chain [11]. Graphically shown in Figure 10, the main function Markov Chain is that future probability decisions are made regardless of past decisions. In other words, the only factor contributing to the decision making is the probability of future states based on the current state. Modelling price and inflow like this will solve the problem for using backward dynamic programming.

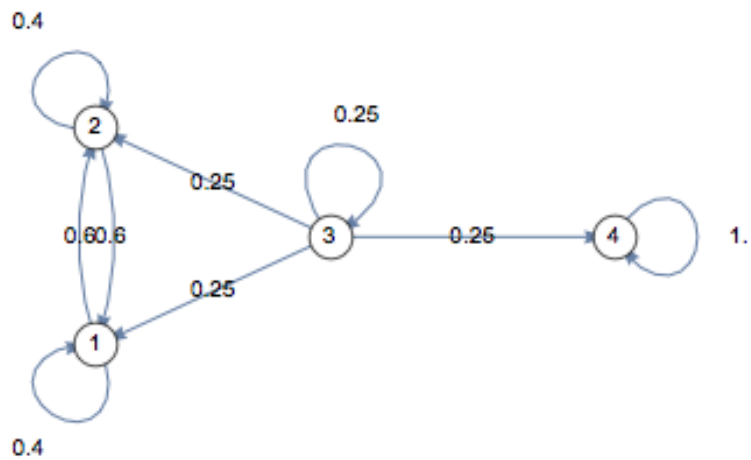

Figure 10: Illustration showing the setup for a Markov Chain [11].

2.6.4 Modelling PQ and efficiency curves

When operating a hydro turbine, the characteristic is that power output increases when water flow increases, due to the increased energy input. However, the pattern between output and input is not linear, as shown in Figure 11. This non-convex non-linear pattern up to local optimum that PQ curves and efficiency curves create, are difficult to model.

Linearizing the non-convex part of the curve can be performed in multiple ways. One possible way is the dotted line in Figure 11. This will result in less computation time for finding the optimal production profile in the line. However, as shown in the figure, the dotted line has a deviation compared to the original curve. This is the consequence of simplifying.

Another solution is to model the non-convex part using a piecewise-linear function, in which several points in the curve are set, and linear lines are drawn between them. These lines are then

modelled as SOS-2 variables, requiring that a maximum of 2 neighboring points in the curve can be non-zero, and the sum of them equal to 1. This will make the optimal production in the linear function between two points in the curve. This modelling is more time consuming when determining optimal solutions, but the deviation in production is lower than for linearizing, depending on the number of points [13].

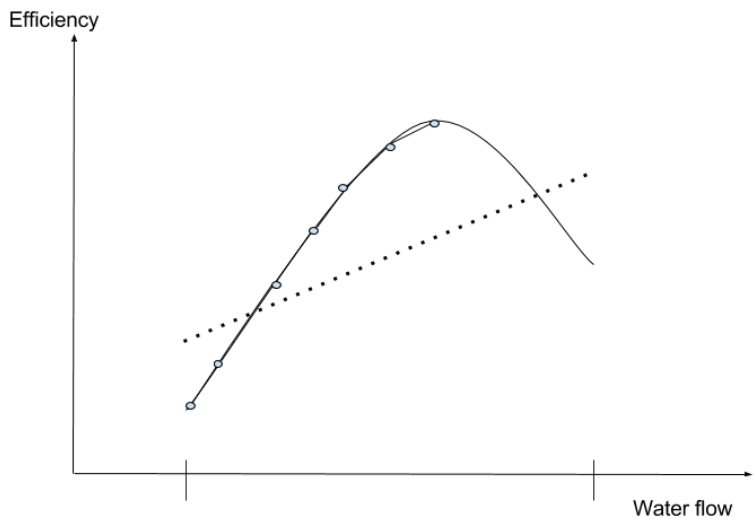

Figure 11: Illustration of a turbine efficiency curve, including linear and piecewise-linear implementation.

2.6.5 Linear Programming and Mixed-Integer Programming

When calculating optimization models for hydro systems, there are multiple variables that are either considered integer or binary variables. These constraints limit solution area and force the model to include more restrictions to find the optimal solution. This is especially true for efficiency curves mentioned in the previous subsection. Optimizing using this method is called Mixed-Integer Programming (MIP), in which variables can have binary or integer values. The consequence of these limitations is that the computation time will increase.

It is however possible to lower computation time by relaxing the model, converting it to Linear Programming (LP). By relaxing, the variables that originally had binary or integer constraints have these restrictions removed, so that they can consist of all real values between their given boundaries. For hydro systems, constraints like spinning state, a binary variable that decides whether the generator is on (=1) or off (=0), can have a value of 0.2 if relaxed, which gives no realistic information about the spinning state. This relaxation can also affect the minimum power output restrictions, creating operation in originally “illegal” areas.

2.7 Software Tools

In this report, the long term scheduling in a hydropower system is implemented in A Modelling Language for Mathematical Programming, also known as AMPL. This commercial software can solve complex optimization problems, including problems like linear, non-linear, and mixed-integer programming. The syntax of AMPL is also similar to the modelling language for optimization problems. There are different software packages for the solver, and CPLEX was chosen for this optimization problem. CPLEX is known as the most widely used large-scale solver, due to the robustness and efficiency it demonstrates [1] [18].

For plotting and presenting the obtainable results from the model, Python was chosen to be the software for this task. The possibility to visualize and work with large input data files makes it an excellent choice. Python is powerful, has many packages for different usage, and is open-source [16].

3 Model Description

To be able to analyze the performance of the hydro producers when operating in both the energy and reserve markets, modelling is the preferred approach to give realistic results. A multi-stage, multi-scenario, long-term stochastic model has been given for this occasion. The model and problem given have been implemented and solved in AMPL. The model has been used in a SINTEF project called "Assessing Hydropower Operational Profitability Considering Energy and Reserve Markets", written by Arild Helseth and Marte Fodstad. The following citations include the original explanation of the model [10] [9]. However, the explanation in this section is based on the author's understanding of the model, unless cited.

The model is classified as a prototype, and is therefore under development during the creation of this report. Alterations and extensions to the model to further optimize it has been encouraged, and the implementations will be specified in Section 4.
The whole model is split into different phases. It consists of a model setup in which the optimization problem is defined, and has a two-step solution procedure to analyze the given input scenarios. First it will execute the strategy phase to calculate water values for each possible deterministic scenario, and then simulate sequential performance in the simulation phase. Figure 12 shows the general setup of the model, in which each key part is specified. The details of the optimization problem will be described in this section, as well as the two phases.

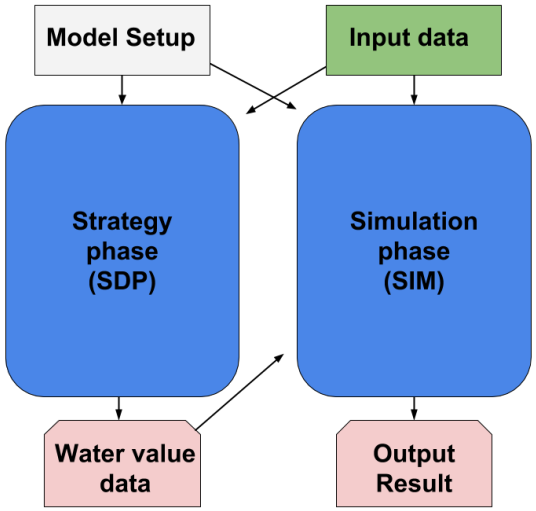

Figure 12: Illustration of the two-step solution procedure and the optimization model as a whole.

3.1 Optimization Problem

"Model setup" from Figure 12 consists of the optimization problem used in the overall model. The optimization problem is representing the hydropower systems scheduling decision, and will analyze the given input parameters to optimize scheduling for a given scenario. It is used in both the strategy and simulation phase, and is therefore key for the whole model. Following the next subsections, the optimization problem will be explained more in detail, giving an overall understanding of how it functions and what it considers.

3.1.1 Overview

The model represents a hydropower station with a single reservoir connected to multiple generators used for both energy production and spinning reserve capacity sales. The regulated water I_{annual} running in the river is stored in the reservoir with a capacity of V_{max}. Water released Q_{rel} from the reservoir can be used for production $Q_{discharge}$, bypassed Q_{bypass} or spilled Q_{spill}. $Q_{discharge}$ will flow into the station and produce power from the generators $g \in G$, and will flow out underneath where all the water ends up.

In this power station there are two inflows; regulated and unregulated $I_{unr,annual}$. The unregulated consists of other water flowing around the station that is not used for production. It is modelled to end up underneath the station. The model is also capable of considering minimum water flow in the river system, depicted by Q_{min}. Figure 13 represents a general graphical representation of the system.

3.1.2 Objective Function

The main objective of the optimization problem is to maximize total profit for the producer. The optimization problem can be found as a whole in Section A. The revenues originate from the sales of energy (Equation 10) and reserve capacity (Equation 11), the value of the remaining unused water in the reservoir and the future profit (Equation 15). The costs come from start-up of the generators (Equation 12), penalty for spilling water (Equation 13), and penalty for utilizing artificial water in the system (Equation 14).

3.1.3 Model Constraints

The optimization problem has implemented several constraints to make the model behave like a realistic hydropower system. These are crucial for correct scheduling decisions, and it is important that they are performed correctly, when implementing new strategies. The constraints in this model are listed in Section A.3, including explanations for each.

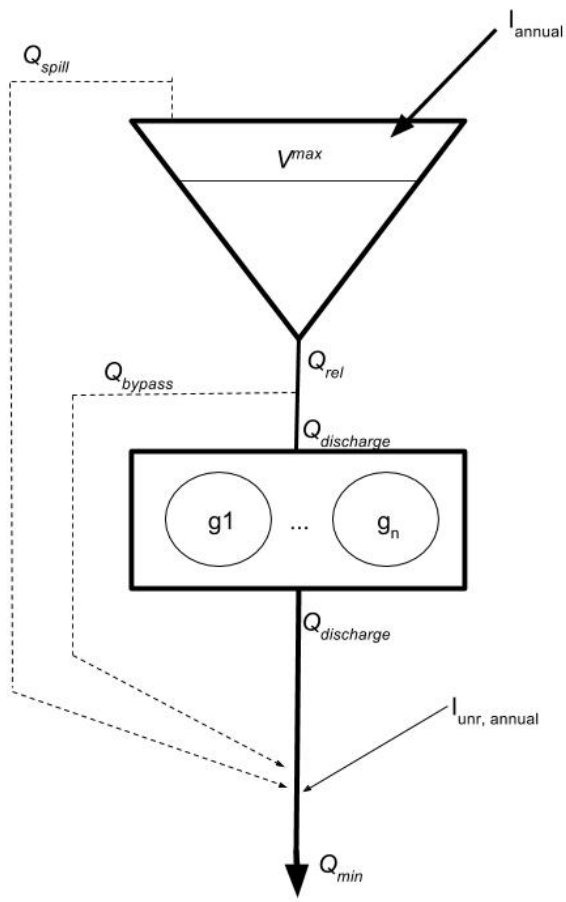

Figure 13: Illustrations of the hydro system in the optimization problem.

3.1.4 Penalty functions

As stated in the objective function, there are penalty functions implemented in the model that decrease the potential profit. These penalty functions have been implemented to restrict the model into a realistic scheduling behavior, and to help manage feasibility when approaching extreme conditions.

Equation 12 shows a cost for starting the generators when they are originally non-spinning. A generator rapidly switching between a spinning and non-spinning state will wear out the equipment, and is unwanted behavior. Therefore, adding a cost for turning it on will make it more beneficial to keep it running at minimum production than to shut it down for a short period of time.

Equation 13 represents the cost of spilling water. This is to keep the model from reaching max-

imum reservoir capacity and prevent spilling water rather than use it for production. Also, the environmental damage that can occur is included in this penalty.

Equation 14 is a cost for adding artificial water to the system. To maintain feasibility at all times, especially when there are constraints that require a minimum flow in the river, one must add another water source that can be utilized when necessary. This choice should be costly to prevent usage unless absolutely necessary.

3.1.5 Converting Water to Energy

When the water is flowing into the station, the generators convert the kinetic power in the water into energy sold in the DA market. The mathematical equation for this is found in Equation 5.

$$p[k] = 9.81 \cdot 10^{-3} \cdot \eta_{gen}(h(q), q^D[k], k) \cdot q^D[k] \cdot h(q) \tag{5}$$

Where q^D is the discharge water flowing into the turbine, η_{gen} is the total efficiency for the generator and turbine and h(q) is the head function, which will impact the total efficiency of the system based on the current reservoir level. The index k is the current time step in range $k \in K$. The efficiency η_{gen} is the total efficiency, including turbine efficiency, generator efficiency, possible transmission losses and other losses. This equation is considered to be complex and non-linear, due to the complexity of the efficiency curves from the generators, as mentioned in Section 2.6.4.

3.1.6 Reserve Capacity Implementation

As shown in the objective function, reserve capacity is included in the model as profitable production. From constraints found in Equation 23 and Equation 24, reserve capacity is modelled to be the minimum deviation between current production and max/min production capacity. This is shown in Figure 14, where the sold reserve capacity must be the lesser of down- or up-regulation of power. This is due to the capacity being sold in the secondary reserve market, where the producer is obliged to contribute with both an increase and reduction of power to the grid. If the generator is turned off, the producer will not be able to sell any reserve capacity. This can create instances where the generator is kept spinning to contribute fully to the reserve market instead of the energy market.

This model operates with multiple generators, and the restrictions show that the model only considers the total generation potential when deciding the reserve capacity sales. Therefore,

some generators can operate at a max production while others operate at the minimum while selling capacity. This up- and down-regulation of power would then go to the generator that has the potential to perform that operation.

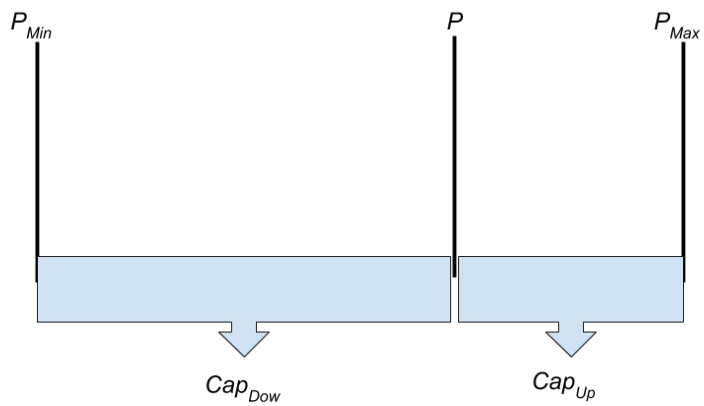

Figure 14: Illustration of reserve capacity potential for both up- and down-regulation based on current production P. The sold capacity here will be equal to the up-regulation potential.

The model has not implemented instances where the reserve capacity sold needs to be utilized, and will thus not affect reservoir content or water discharge. The reserve capacity is thus only modelled to be an extra profit when not utilizing full energy production. This is something that could be implemented at a later stage.

3.1.7 Minimum flow in river

As is true for many hydropower producers, a restriction regarding water flow in the river is present in the model. This is found in Equation 27, where the unregulated inflow, discharged, bypassed and spilled water are factors that are necessary to meet this requirement. This restriction must be true at all times, creating instances where the producer must decide to produce at lower cost or bypass water to prevent start-up costs.

3.1.8 Relative head

When discharging water from the reservoir, the reservoir level also plays a role in the efficiency of production. This is mainly due to the added pressure on the water flowing into the head of the station. Thus, depending on the water stored, the kinetic energy in the water can be increased for higher efficiency. This is modelled as an approximation with a reference reservoir level.

3.2 Strategy Phase

For the long term scheduling model, a strategy phase is needed to pre-emptivly find the optimal planning for each possible scenario before simulating. In other words, the marginal value of production, the water values, are calculated for each possible outcome to have the proper planning strategy. To find these water values, the strategy phase is executed using stochastic backward dynamic programming. As mentioned in Section 2.4.4, this is the preferred approach as the future stage affects the current stage decision making.
The next subsections will further describe how the strategy phase is laid out, and how the optimal water values are found.

3.2.1 Overview

Figure 15 shows a schematic of how the simulation phase is laid out. The following points will explain the role and function of each part.

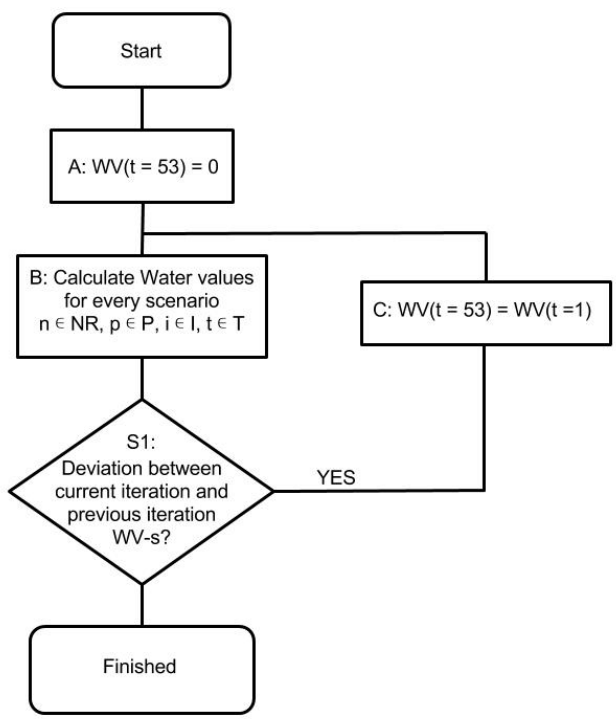

Figure 15: Flow schematic of the strategy phase.

Box A: Setting Starting Values
Since the model is using backward dynamic programming to compute the water values, the initial values must be set before calculating. Thus, the initial water values and expected future profit values for week 53 are set to 0 here, or any given number. The model operates with 52

weeks per year, however, as the water values are based on the values for the end of the week, which is equivalent to the start of the next week, a 53rd week is needed to symbolize the start of next year/end of current year. Therefore, the 53rd values are set to 0.

Box B: Calculating Water Values

Here, the water values for all scenarios are found. The number of scenarios calculated are based on the given size of inflow ($i \in I$), price ($p \in P$), reservoir segments ($n \in NR$) and weeks ($t \in T$). The water values are needed for each possible combination, resulting in a total of $T \cdot NR \cdot P \cdot I$ calculations.

Calculating the water values are done as follows:

For a given week t and reservoir segment n, the optimal production profile is calculated using the optimization problem for every $p \in P$, $i \in I$, storing the objective function α for each calculation.

Since both the price and inflow have a stochastic setup, the weighted objective function for the given week and reservoir segment must be calculated based on all probable scenarios. This is done as in Equation 6, where pr_{price} is the probability of moving from price node p to price node $p^{'}$, pr_{inflow} is the probability of moving from inflow node i to inflow node $i^{'}$, and α is the objective function for the given price and inflow node. This is done to include the uncertainty connected to multiple possible outcomes, which will affect the profit.

$$\alpha(p,i,t) = \sum_{p^{'} \in P} \sum_{i^{'} \in I} pr_{price}(p,p^{'},t) \cdot pr_{inflow}(i,i^{'},t) \cdot \alpha(p^{'},i^{'},t) \tag{6}$$

If the current reservoir segment is the first, when the reservoir is empty, then the weighted objective function is stored as the offset in value function $DVAL$, also referred to as the expected future profit. Figure 16 shows how the expected future profit varies with the reservoir level, including a positive value for an empty reservoir. This value must be included in the optimization problem for optimal calculation as it stores the potential profits [9].

For reservoir levels not equal to zero, the water values are calculated based on the found weighted objective functions. The equation for this is shown in Equation 7:

$$WV(n-1,p,i,t) = \frac{\alpha(p,i,n) - \alpha(p,i,n-1)}{ResEnd(n) - ResEnd(n-1)} \tag{7}$$

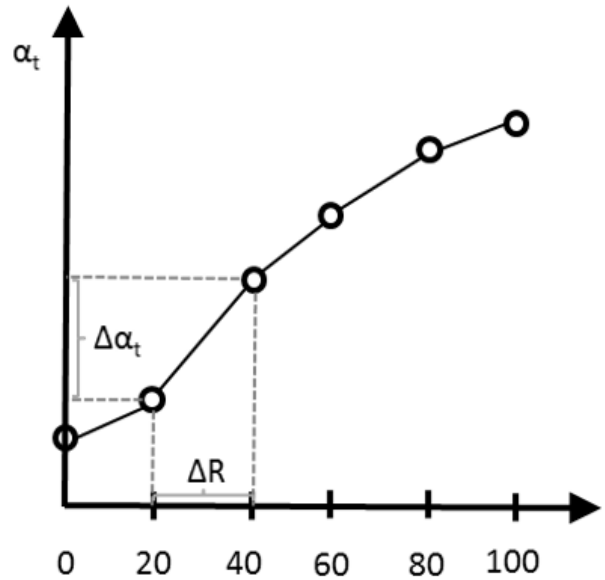

Figure 16: Plot of expected future profit for different reservoir levels. Illustration from [9].

Where α is the weighted objective function for the given price, inflow and week, and $ResEnd$ is the reservoir level for the chosen reservoir segment. This procedure is the concept in Figure 16.

This water value calculation is repeated for all $n \in NR$, $t \in T$.

Box S1: Deviation between iterations

Consistent water values are calculated through an iterative process, making sure the values at the end are close or equal to the starting values. Therefore, all the water values will be checked for consistency in this box. When having completed an iteration, the water values will be checked against the water values from the previous iteration. If this is the first iteration, then there is no other file to compare to, and the deviation will be large, causing the statement to be "YES". If, after some iterations, the deviation is under a user-defined tolerance, then the statement will be "NO", causing the water values generated to be accepted.

Box C: Adjusting the initial values

If the deviations from box S1 were too large, the process must be repeated in a new iteration. The water values and expected future profit from week 1 of the previous iteration will be the new initial condition for week 53 of the next iteration. By adjusting the initial values like this, the process will slowly reach optimum in which the starting values are close or equal to the ending values, causing convergence in the system.

3.2.2 End result of strategy phase

The final output will be a water value table listing the water values for every scenario. These values will in detail help the optimization problem to find the optimal production profile for the given values by giving an accurate value of the stored water in the reservoir. However, the water values are calculated with a stochastic approach, weighting each alternative, and will thus not give the perfect solution for each scenario. Further on, these values can be used to simulate behavior in the model for several periods like in the simulation phase, which will be further described in the next subsection.

3.3 Simulation Phase

After having calculated water values for every scenario possible for the given model, the next step is to find out how the system will function under the different inflow and price alternatives. As mentioned, the water values will not give the perfect solution for each scenario, but if used as a decision criterion, they will give the best solution when uncertainty is included. Therefore, one will want to simulate for multiple periods, finding the average behavior and trends. Therefore, the simulation phase is executed to calculate proper behavior, which will include production profile, reservoir levels and overall knowledge of properties for the given system.
The next sections will describe how the simulation phase is carried out, and what it can be used for.

3.3.1 Overview

Figure 17 shows a schematic of how the simulation phase is laid out. The following points will explain the role and function of each part.

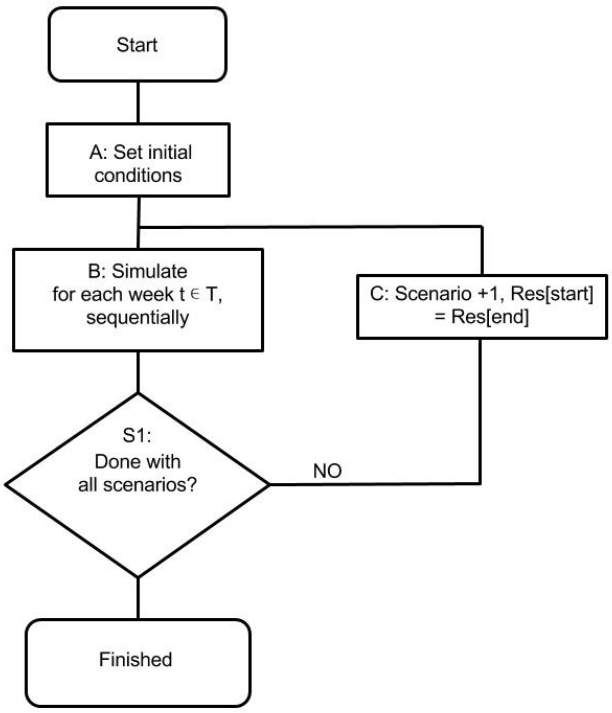

Figure 17: Flow schematic of the simulation phase.

Box A: Set initial conditions
Here, the initial conditions for the model are defined. These initial values consist of starting reservoir level, initial inflow and price nodes, generator spinning state and number of periods to be calculated. Each period will be for one year.

Box B: Simulate the given period
For every week $t \in T$, a given inflow and price node will be randomly chosen, and the optimization problem will run on the given conditions to find optimal production profile. The weeks will be run sequentially, meaning that parameters as reservoir levels and spinning state of the generators will be kept from the end of the previous week and be the starting values of the current week. The simulations start at week 1 till week 52. The price and inflow nodes for each week are set randomly, only affected by the probabilities between each transition.

The model enables the user to give a specified price/inflow pattern for the periods. The randomized prices and inflows will be determined by the pattern, which allows the user to test multiple analyzes with the same scenarios.

Box S1: Done with all periods?
If the wished number of periods are met, then the simulation phase is done, and the generated data will be stored for further analysis. If the condition is not met, then it will move to box C.

Box C: Update initial values
As with Box C in Figure 15, the initial values for the period must be set when iterating again. This time, the reservoir level at the end of the period is set to be the starting level, and the spinning state of the generators are also transferred to the start. This will keep the simulation sequential.

3.3.2 The End Result

After the simulation phase has finished, data regarding performance of the system for all periods will be generated and stored for analysis. With these data, one can check the reservoir level, generator production, energy/reserve profits etc. for each time step.

4 Model Modifications

During the project period, new features and improvements to the model have been implemented to improve performance. This is partly based on preliminary results from this project work. All modifications mentioned here are made by the author. These implementations will be mentioned in this section, including the motivation behind.

4.1 Optimization Problem

The optimization problem has implemented reserve capacity as a resource that can be utilized. As this is still considered a prototype implementation, there are still adjustments to the optimal scheduling that must be made.

4.1.1 Implementation of reserve capacity sales restriction

When selling reserve capacity, the hydropower producer is obliged to produce extra power up to the sold capacity if prompted by the TSO. Failing to do so will cause problems in the system, and a penalty will be given to the producer for failing his obligations. Therefore, selling reserve capacity should be done only when there is a guarantee that the producer has enough water stored for this increased power output. A new method for including a reserve capacity guarantee was developed, implemented and tested by the author.

This decision resulted in the test introduced in Section 5.4.3, and based on the results, a new constraint was added to the optimization problem. This constraint is shown in Equation 8.

$$T[k] \cdot cap[b] \leq \frac{\eta[g1,1] \cdot res[k]}{MM3H}, \quad k \in K, b \in B, ord(b) = BK[k] \tag{8}$$

This constraint will for all time steps $k \in K$ check that the minimum stored power in the reservoir at the end of the time step is more or equal to the total sold reserve capacity for the time step. In other words, the sold capacity is limited to the available resources left in the reservoir at the end of the time step.

This implementation will make the problem consider the available resources left in the reservoir, and limit the reserve capacity to always have enough stored water.By doing this, the optimization problem will make sure that the producer never sells more reserve capacity than he can deliver, and the TSO will be ensured the power requested. Performance of this added constraint

is the motivation behind the test introduced in Section 5.4.4.

4.2 Strategy Phase

The strategy phase does a large amount of calculations to find the optimal water values for every scenario of the system. With the iterative approach of finding these values, the computation time for this phase will be lengthy. Therefore, some implementations to store data found underway have been done.

4.2.1 Store water value tables

For each iteration, new water values are found, stored, and compared to the previous iteration values. If the convergence criteria is not met, the model will begin a new iteration and the old values will be overwritten by the new values. This prevents possible analyzes of how the water value differs between each iteration.

Therefore, the strategy phase had implemented a user-defined option that will, if enabled, store the water values from each iteration in separate output files.

4.2.2 Store deviation and time usage

Due to computation time differences when using LP and MIP, it is beneficial to include this when comparing the different strategies. This also holds true for the water value deviation between each iteration.

A user-defined option has been implemented for both of the situations mentioned. If enabled, the model will store the time usage and deviation for each iteration in an output file. This will help understand how the water values converge, and the time used.

4.3 Simulation Phase

4.3.1 Randomizing sequences

The simulation phase decides the price and inflow of a given week based on a random variable generator in AMPL. This ensures that the declared values are random. However, unless specified, the random number generator will use the same values when AMPL is reset [2]. To implement a "true" random variable, code "option randseed”;" was included, that will generate a random number generator based on the current time of execution. This way, the values will always be completely random and repetition will not occur.

4.3.2 Issues with sampled data

The model had originally included the option to use a user-defined path of the price and inflow nodes for each week instead of creating random variables each run. This enables the user to simulate the same "random" variables for each run, which is crucial to get comparable results.

However, as was discovered in Section 6.4.2 when testing the simulation phase, implementing the random number generator uncovered that the water values used for each calculation were not the correct values when using a specified inflow/price path. The water values were determined by the random decision for price/inflow, and not based on the actual used prices and inflows. This resulted in the fact that wrong water values were determined, which led to unnatural and wrong decisions.

This was fixed in the code, and the water values will now be set based on the user-specified decision of inflow and price transitions.

4.3.3 Store time usage

As with the strategy phase, information regarding time usage data is relevant when comparing LP and MIP. Therefore, time usage was recorded and stored as a separate output file for each period analyzed in the simulation phase.

5 Case Study

The motivation behind this case study is that Lyse AS considers upgrading Maudal power station by replacing the old generators with new ones while maintaining the same installed capacity. They want different combinations of generators to be analyzed to see their performance and revenue potential, both in the energy and reserve markets. For the reserve market, they want to participate in the secondary reserves (FRR) balancing market. For this project, two different hydropower generator combinations have been given, however only one of them will be used in this report.

The work will be done using the prototype optimization model, and both the model and results from it will be tested and analyzed. Implementation of reserve capacity will be analyzed, and different optimization methods will be mentioned.

5.1 Maudal Hydropower Station

The specific station used in this case study is Maudal hydropower station, located in Rogaland, Norway. In the Nord Pool overview, the station is located in the NO2 price node region. The station has today 4 generators installed with a max capacity of 25 MW, resulting in a yearly production of approximately 97 GWh [17]. The power station is connected to the reservoir "Store Myrvatn", having a capacity of 63 Mm^3 and water level regulation between 614 and 592 meters above sea level. Figure 18 shows an overview of the power station that is used in the model, including the annual inflow that has been averaged for this scenario.

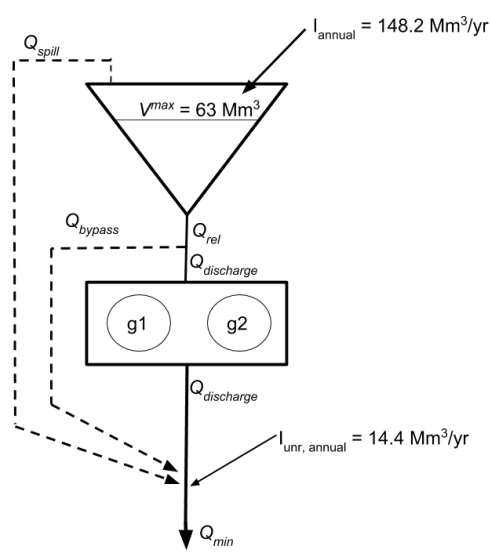

Figure 18: Illustration of Maudal hydropower station.

5.2 Input Data

For a hydropower system model, there is a wide variety of input data that must be set. For this system, the input data consist of inflow, price, and the hydro system setup. This section will specify the input data given and describe their origin, as well as the reasoning behind their setup. The information regarding the origin of these input data is retrieved from [10], and information on time series models can be found in [22].

5.2.1 Price Data

The price nodes in the model originate from historical data from 2013-2015 for NO2, the price area in Norway in which Maudal is located. These data were used in an ARIMA(2,0,3) time series model, which samples them into a wished number of price nodes. The appropriate number of price nodes ended at 15 in total per stage. These 15 nodes consist of 5 different energy and 3 different reserve capacity prices.
The reserve price nodes are also originally taken from the same historical data, but sampled through GARCH(1,1) and ARMA(1,1). Figure 19 shows parts of the data used.

```
 param EpriceState (tr):
 p1 p2 p3 p4 p5 p6 p7 p8 p9 p10 p11 p12
t1     256.2     256.2     256.2     239.3
t2     268.0     268.0     268.0     245.3
t3     273.8     273.8     273.8     247.8
t4     278.9     278.9     278.9     249.5

 param CpriceState (tr):
 p1 p2 p3 p4 p5 p6 p7 p8 p9 p10 p11 p1:
t1      42.8      34.4      27.7      42.8
t2      44.4      31.9      23.7      44.4
t3      44.9      29.7      20.9      44.9

 param TrProbPrice :=
 [*,*,t1]: p1 p2 p3 p4 p5 p6 p7 p8 p9 p
p1 3.77e-01 1.77e-01 4.11e-02 1.84e-01
p2 1.62e-01 2.42e-01 1.91e-01 7.90e-02
p3 5.54e-02 1.77e-01 3.63e-01 2.70e-02
```

Figure 19: Screenshot of the price nodes. Notice that for each price node of power, three different reserve prices are possible.

As each node will be the price of production at a given stage, varying prices within the stage were implemented to simulate hourly variation. This was done with correction factors, known as $Eprice$ and $Cprice$.

5.2.2 Inflow Data

The inflow is originally from 1000 inflow samples generated from a VAR(1) model based on historical inflow data. These samples were then used as a baseline to an appropriate number of used in this model, which was set to 5 inflow nodes per stage as shown in Figure 20. This ensures that the instances of unusually high or low inflow is taken into consideration in the model. The probabilities between the price and inflow nodes are based on the Markov Chain, explained in Section 2.6.3.

```
 param InflowState (tr):
 i1 i2 i3 i4 i5 :=
t1   12.985    5.017    1.609    0.446    0.186
t2   11.623    4.861    1.644    0.476    0.215
t3    9.788    4.650    1.659    0.520    0.223
 param TrProbInflow :=
 [*,*,t1]: i1 i2 i3 i4 i5 :=
i1  0.34000  0.50000  0.16000  0.00000  0.00000
i2  0.12667  0.34000  0.52000  0.01333  0.00000
i3  0.02337  0.12187  0.69783  0.14023  0.01669
i4  0.00000  0.00667  0.52667  0.30667  0.16000
i5  0.00000  0.00000  0.32000  0.36000  0.32000

 [*,*,t2]: i1 i2 i3 i4 i5 :=
i1  0.34000  0.50000  0.16000  0.00000  0.00000
i2  0.12667  0.34000  0.52000  0.01333  0.00000
```

Figure 20: Screenshot of inflow nodes used for the case study. Notice that the inflow values for a specific week vary greatly.

5.2.3 Time Step and Resolution

The planning period of the model is one year, with decision stages set to every week. This will result in 52 stages per year. To prevent a static production and decision pattern over the whole week, each decision stage consists of 21 time steps, enabling a more flexible variation of decisions within the week. Every 3 time steps simulates a day, with 8, 12 and 4 hours within each time steps.

5.3 Hydro System

5.3.1 Turbine and Generators

The main motivation for this analysis is that Lyse AS is considering upgrading Maudal with new turbines and generators, and wants to see the performance of different combinations. For this analysis, two combinations were given; Case A and Case D. Each combination consists of two turbines installed, with a total installed capacity of approx. 25 MW. Table 3 displays turbine data for both cases, including the number of discretized piecewise-linear points used for modelling the PQ and efficiency curves.

Table 3: Turbine data for the two cases.

Case	Gen. num	Discr. points	P_{max}	P_{min}	Turbine Type
A	1	10	21.11	6.57	Francis
A	2	12	3.71	0.49	Pelton
D	1	13	17.64	2.34	Pelton
D	2	13	7.5	0.966	Pelton

As mentioned earlier in Section 2.5, the Pelton and the Francis turbines have different characteristics for water flow and relative head. The Pelton turbines have the advantage of running at much lower minimum water flow than the Francis, enabling up-regulation of reserve capacity with low water usage. However, they have slightly lower efficiency than the Francis turbines. Therefore, the production profile between the two cases is worth studying and analyzing based on how the producer wants to participate in each market.

As previously mentioned, only Case A will be studied in this report.

5.3.2 Reservoir

The reservoir needs to be cut into segments for the computation, to be converted to state variables. The number of segments plays a huge role in the model, as a higher amount will result in more accurate values, at the cost of high computation time. Therefore, the initial number of segments was set to $N_p = 21$, recommended by the co-advisor Arild Helseth from his experience with this model. The initial starting level of the reservoir was set to 30 Mm^3 for the simulation phase.

5.3.3 Flow Requirement

Lyse AS has a restriction on keeping the water flow in the river underneath the station at minimum 0.35 $\frac{m^3}{s}$ at all times. To meet this requirement, the unregulated inflow combined with the released water from the reservoir must be optimally balanced.

5.4 Performed Model Runs

The following sections will describe some of the goals that this report is aiming to cover. Each run will specify the goal and motivation behind it, and the results will be shown in Section 7, unless stated otherwise.

5.4.1 Testing the Model

To fully understand how the model worked and how it could be utilized for the wanted analyzes, the author felt it was important to fully test the different phases of the model. Most importantly, the variable parameters that could be chosen by the user, needed to be tested. Section 6 consists of the different tests that were performed.

5.4.2 Optimal scheduling with and without reserve capacity sales

With reserve capacity sales implemented, comparing the impact this should have on the model would give a better understanding of the results. Therefore, the model was run twice, with two different cases defined.
Case EC consisted of running the model with both energy and reserve capacity sales enabled.
Case E only had energy sales enabled.
The main motivation is to check the different scheduling decisions the model makes.
The starting values of the model for the simulation phase are:

1. Reservoir level at 30 Mm^3
2. Price node 1, inflow node 2
3. Both generators turned off

5.4.3 Reserve capacity allocation under specific scenarios

The implementation of reserve capacity must be checked under various conditions to see how the optimal scheduling is decided, and to see if there are any instances of "illegal" or unwanted decisions. To check this, two tests were performed; one in which the reservoir level is low, and one where it is high. The inflow will also be altered to be lower/higher than normal, respectively. This will make the results unnatural and not give any realistic scheduling results for the given station, the only goal is to check how the model schedules.

Case LOW will check the production profile during low reservoir levels and low inflow. The initial reservoir level will be at 0.01 Mm^3, and the inflow will be halved for every node.

Case HIGH will check the production profile during high reservoir levels and high inflow. The initial reservoir level is at 62.9 Mm^3, while the inflow is untouched. However, the inflow nodes are pre-emptivly altered to give the highest inflow for the first 11 weeks.

5.4.4 Implementation of capacity restriction

As mentioned in Section 4.1.1, a new constraint was found based on the results from testing the model scheduling during low reservoir levels. This resulted in the implementation of Equation 8, a restriction that limits reserve capacity sales to actual water stored for the period. The hypothesis is that this will prevent reserve capacity sales when the reservoir is too low.

Case LIMCAP is the same test as case LOW, but with the new restriction included. The goal is to check how the production profile will change.

6 Testing the Model

To get a better grasp of how the model functions and the opportunities it can create, it felt necessary to test the model with different settings. This helped to see how all the parameters and variables were linked together, and also which parts were critical for the model to compute at best rate. Included with the model was a water value table created from the strategy phase, that was said to be made using MIP. To check the quality and reliability of this table, the first goal was to recreate this from the strategy phase.

6.1 First Test Run of the Strategy Phase

As mentioned in Section 3.2, the strategy phase of the model will calculate water values for all state variables set, and consider itself converged if the water values have a total deviation of less than 0.01. With the current setup of $N_w = 52$ weeks, $N_r = 21$ reservoir segments, $N_p = 15$ price nodes and $N_i = 5$ inflow nodes, there are 1575 discrete states. This means that 81 900 different deterministic scenarios must be calculated for each iteration to find the stochastic behavior.

The model had initially a user-defined time limit of 3 seconds for each optimization problem run, which results in up to 68.25 hours of computation time for a single iteration. This shows that for this computation, a computer with a strong processor and big memory storage is strongly recommended to lower time usage.

6.1.1 Processor strength and computation time

In the first attempt, a test was carried out on an **AMD A9-9410 Radeon R5 processor** with **2.90 GHz and 8 GB RAM** installed. The strategy phase was run using MIP, as was the supposed origin of the water value table given. This test had to be aborted midway as the computation took too long. The test was carried out before implementation of storing time usage, but approximately 30 hours had been spent on the first iteration. This computation also used all the processor capacity on the computer, making it unavailable for other work while running. This time usage, without knowing when or if the model would converge at 7 iterations as was done for the same model in the paper [10], made calculating the water value table impractical with the current computation tools.

The main problem compared to the given water value table is that the processor is less powerful. This means that a larger amount of the 81 900 scenarios reached the time limit of 3 seconds. This also results in less of the computations finding optimal results, which can make the model

unable to converge despite running for a reasonable amount of time.
The reason behind the time usage is that the model is running using MIP. As mentioned in Section 2.4, this is due to the number of mixed-integer variables and the way the PQ and efficiency curves of the generators are set up. Another reason is the profit gain from both energy and reserve capacity, making the model search for their optimal scheduling between each other, another time-consuming process.

6.1.2 Possible solutions

To decrease the computation time, there are two possible solutions and alterations to the original setup: Decreasing the stochastic variables in the input data or relaxing the model to use LP. Decreasing the stochastic variables will result in fewer computation steps for each iteration, but will come at the cost of accuracy. Since the number of weeks and the number of reservoir segments should be kept constant, the best solution would be to decrease the number of price and inflow nodes. However, as these nodes are used to describe all possible inflow and price node changes, it would be optimal to keep them fixed. A second test run of the strategy phase was attempted with both solutions implemented.

6.2 Second Test Run Strategy Phase

6.2.1 Changes done to the setup

In this run, the goal was to create a successful run of the strategy phase. To do this, the price and inflow nodes were cut down to one of each, creating a deterministic model setup. This would shorten the number of scenarios to 1575 per iteration, making the maximum computation time 1.3 hours per iteration.

To allow this, the price and inflow input files would have to be changed. This was accomplished by using handmade Python scripts , attached to this report by the author (named "Simplify_inflow_spec.py" and "Simplify_pricemod_spec.py"). The code extracted one node from each original input file, and altered the probability between each stage to 1, keeping the same format as the original. The second run was also done twice; one using MIP and one using LP, to check the performance of each method. The computer used was also changed, and the new one had an **Intel i5-3570k processor with 3.40 GHz and 16 GB ram** installed. Table 4 below shows the computation time for both runs, and for a third run using MIP with the time limit increased to 5 seconds.

Table 4: Data for each separate run of the Strategy Phase with deterministic layout.

Simulation	Time limit [s]	Average time per iteration	Iterations	Converged?	Total time usage [h]	Final deviation
Det. MIP	3	1767	10	No	4.91	847.7
Det. MIP	5	2833	10	No	7.87	535.1
Det. LP	3	38	5	Yes	0.052	0.006

6.2.2 Results from the second test run

From the first two simulations, it can be observed that the LP run converges after 5 iterations, using only a fraction of the time compared to MIP which did not converge. This shows the complexity of including MIP for solving the non-convex behavior of the PQ curves compared to linearizing the curve. Another noticeable point is the convergence behavior of MIP for a time limit of 3 seconds, shown as the blue line plotted in Figure 21. Here, the water value deviation is reduced until the 4th iteration. From here, the value is almost stable with no improvement for each additional iteration. This is probably the result of the time limit set in the computation, which makes the model unable to find optimal production profile with the available time. Therefore, the number of tolerated iterations will not necessarily guarantee convergence.

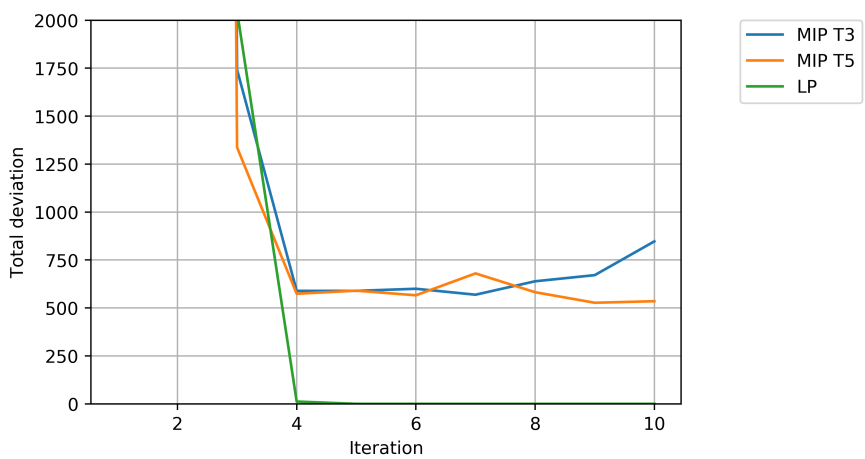

Figure 21: Plot of the deviation for each run.

Based on the results given by MIP with a time limit of 3 (referred to as T3), a new simulation was done with the time limit increased to 5 seconds per computation (referred to as T5). Comparing the two different deviation patterns, the deviation for T5 is lower than for T3 at the final iteration. They both converge to the same value in iteration 4, but have different deviation patterns from there. It can be discussed if this lack of convergence is due to computation time or if the deterministic case cannot converge with the set input data. However, as the deterministic

approach is used only to understand how the model works, these factors are not of great importance. Since T5 has been computed and has a lower deviation than T3, it will be chosen for analysis further in this section.

6.2.3 Comparing water values generated

After completing the iterations, both T5 and the deterministic run using LP (referred to as DLP) managed to create water value tables that can be used in further simulations. However, as the MIP did not converge, the water values from that calculation cannot be considered perfect. Also, since each method uses different solving methods, the tables and thus the considered optimal water values can vary. Thus, the water values have been plotted for both cases, seen in Figure 22.

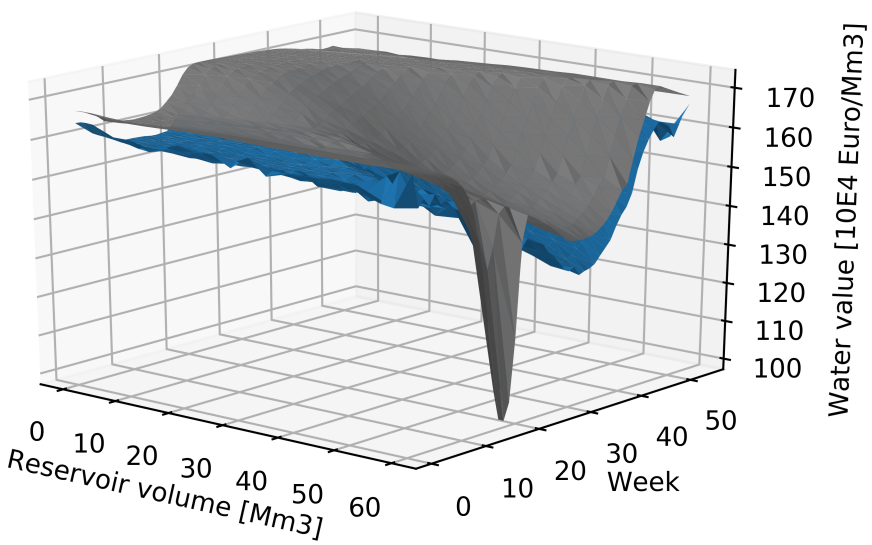

Figure 22: Plot of water values for deterministic behavior. Grey is DLP, blue is T5.

By analyzing the water values, one can see that the values from DLP are in overall higher than the values from T5. This is due to DLP linearizing the PQ and efficiency curves, which leads to an overestimating of the value of the water. This combined with the relaxation of other variables make the model increase the water value for every node.

6.2.4 Conclusion

With these tests, the strategy phase is working as intended and a stochastic layout can be tested. The problem with computation time has not been solved, other than that the use of LP should find an optimal solution within reasonable time. This hypothesis will be tested in the third test run.

6.3 Third Test Run of Strategy Phase

Based on the results from the second test run of water value calculation, the next step would be to see how accurate MIP and LP are compared to each other based on the original setup. If the results from LP are close to MIP, it would justify using LP to calculate water value tables for future tests to lower computation time and allow multiple scenario analyzes.

Therefore, a LP run of the strategy phase for the original case was done. It converged after 8 iterations with a time usage of 5 hours. The total deviation was 0.0013. Compared to the attempted MIP run, this is way faster and will make the strategy phase doable for several attempts.

6.3.1 Results from using LP

Now that the test run using LP converges and reached optimal solution within reasonable time, it can be considered a success. This means that the water value table given with the model hopefully can be compared to find out how it was made. As with the deterministic water values, LP should generate higher water values than MIP. Figure 23 shows the two water value series compared for the same inflow and price node. Studying the plots, they appear similar and have no noticeable difference like in Figure 22.

6.3.2 Findings

One can conclude that the origin of the given water values are unknown, and that a MIP run must be done to compare the two methods for the strategy phase. As this would require high computation time with the risk of not converging as with the deterministic attempt, this is something that must be accomplished at a later stage when stronger software tools have been acquired.

To perform the strategy phase for the intended analyzes, the current solution will therefore be to use LP to calculate the water values. Also, the water values from using LP in this test can be used in the simulation phase.

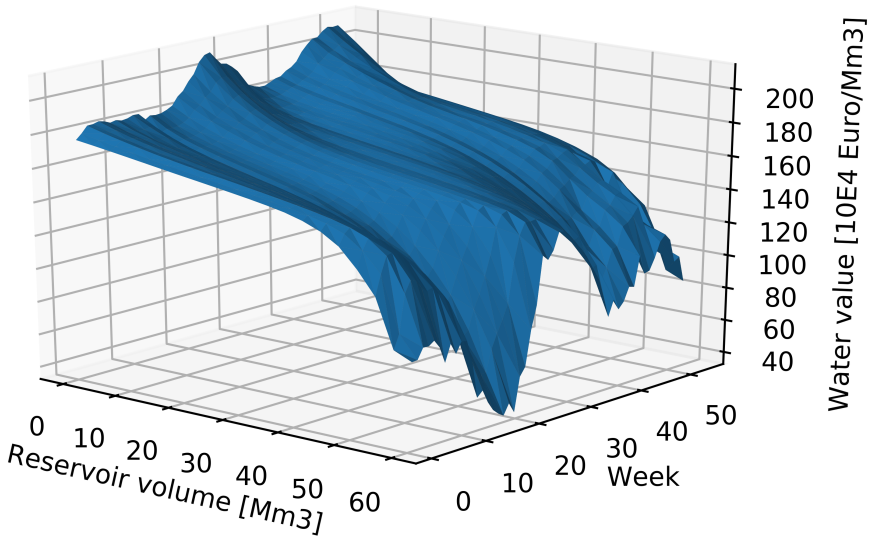

Figure 23: Plot of water values for price node 5 and inflow node 3. The two plots are from LP and the given WV table.

6.4 Test Round of the Simulation Phase

With the completion of testing the strategy phase of the model, the next logical thing was to attempt running the simulation phase, modelling and observing behavior for multiple periods. An explanation of how the simulation phase works is found in Section 3.3.

Since only weekly stages will be analyzed for each period, the number of calculations needed are much fewer than for the strategy phase. Because of this, the anticipated computation time will make it possible to include many periods for a wider analysis. Therefore, the number of periods for each analysis was set to 100, resulting in a maximum of 5200 calculations. Another point is that with the low amount of calculations, the struggle between using MIP and LP will have less consequences timewise.

The third factor to be set is the time limit. As the time limit had an impact on the results in the previous subsection, it is expected that this will occur in this part of the model as well. Therefore, the results of using different time limits with the LP calculated water values will be presented and analyzed.

6.4.1 First test of the simulation phase

The time limits of 3 and 5 seconds had been performed to see the different results in the simulation phase. Both had the same initial values and the same amount of periods, 100 periods each. MIP was also used for both, with the time usage of each run found in Table 5. This run was performed before implementing the random number generator to the simulation phase, as mentioned in Section 4.3.1.

Table 5: Time usage for simulation phase with different time limit.

Case	Time limit	Avg. Time per period [s]	Total time [h]
T3 MIP	3	78.4	2.18
T5 MIP	5	125	3.48

The table shows, that there is a significant difference in time usage for each scenario, which is to be expected. Both have a maximum time usage that can be considered acceptable when running it multiple times with different scenarios. Therefore, this does not limit the simulation phase to be run only using LP.

However, to see the important impact, the results they create must be analyzed and evaluated. The simulation phase gives data related to many elements, from production to reservoir level to spinning state and even energy and reserve revenues. The changes in reservoir level and production profile are considered key aspects to understand optimal scheduling, they will be studied further here.

Figure 24 explains the reservoir level plotted for different percentiles. The trend is similar in both simulations, having only small differences in reservoir level at some occasions in percentile 25 % and 75 %. The plot also shows that the reservoir levels are well regulated, covering almost the whole region. This shows that the number of periods are well chosen, and that the station manages to fully utilize the reservoir capacity.

Figure 25 and 26 show the duration curves for production and reserve capacity. The pattern is the same, that is, the two time limits have close to equal values. The plots show that the generators do operate at max production, but also have periods where their production is reduced. The period when generator 1 is operating at medium is almost as long as the period when the max reserve capacity is sold. This shows the co-relation between production and reserve capacity, that is, the system balances both sources to maximize profit.

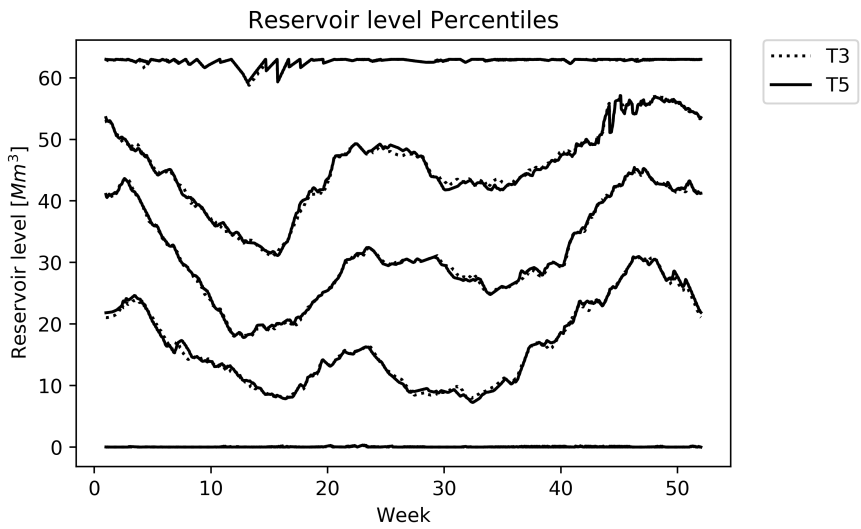

Figure 24: Plot of reservoir level for time limits 3 and 5. The percentiles are 0-25-50-75-100 %.

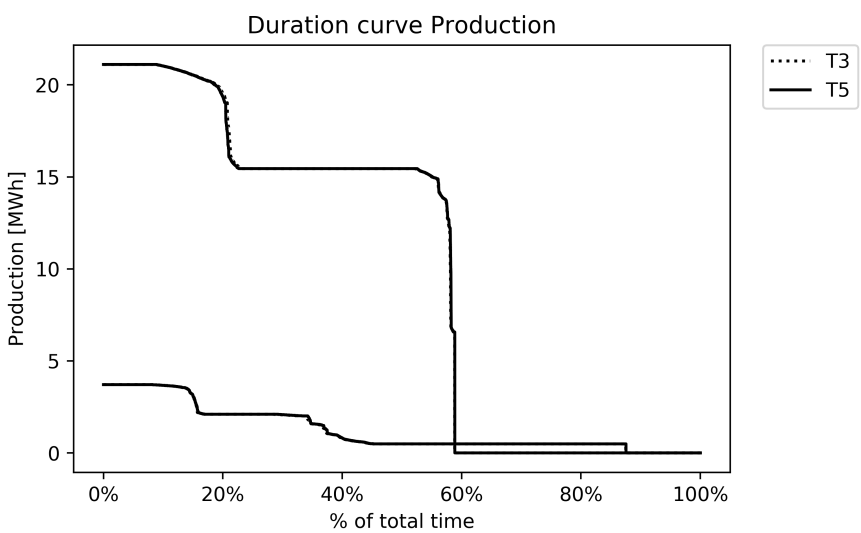

Figure 25: Duration curve for production for both generators for both T3 and T5. Generator 1 is the largest generator.

6.4.2 Backup test revealed inconsistency

At a later stage, a backup test of the same run previously mentioned was performed to double-check results. This run was done after implementing the random number generator, which should have had no impact on the decision as the model was run using a user-defined inflow and price pattern to be comparable. The plotted reservoir levels can be seen in Figure 27.
The now two plots of the same setup have different and inconsistent reservoir level behavior. This is not something that should occur as all randomized variables should have been equal.

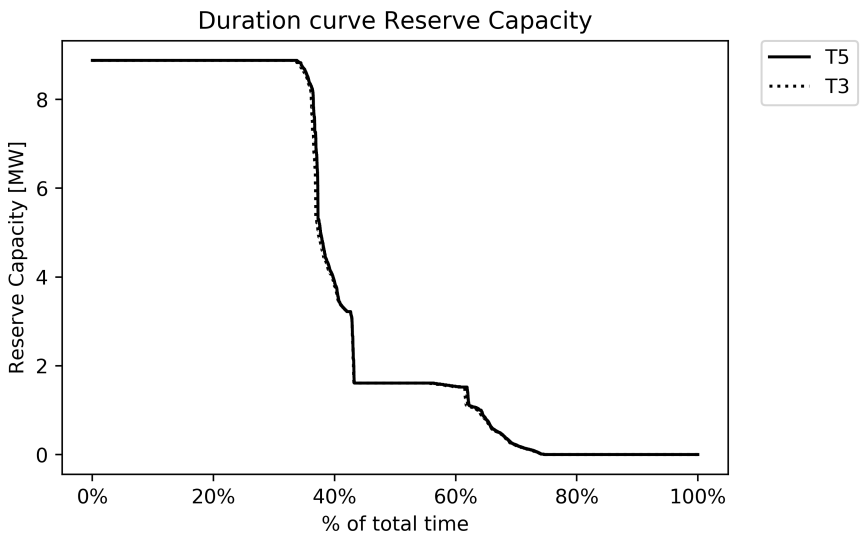

Figure 26: Duration curve for reserve capacity for both generators for both T3 and T5.

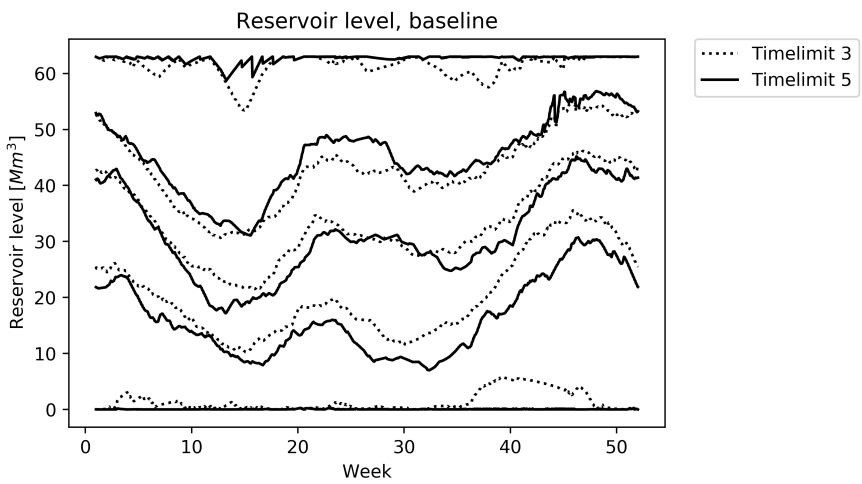

Figure 27: Plot of reservoir level for time limit 3 and 5, with random number generator implemented.

This check resulted in finding inconsistency in the code for the simulation phase, in which the water value definition for each week was based on the randomized variables, also when the pattern was fixed. The first test had the same randomized values, which caused them to have equal behavior, however the backup test did not, due to the implemented random number generator. This is why it was not found in the first attempt, but during the backup test.

This incident resulted in a modification to the simulation phase, explained in Section 4.3.2.

6.4.3 Findings

Being that the water values were based on random variables, the results given here cannot be used to realistically explain the optimal scheduling behavior for the hydropower station. The first simulations however, can be used to show the importance of time limits, and also how the system behaves. This test shows that the model operates as intended. It also helped uncover an error with the water value that was patched up.

6.5 Parameters Defined for Future Runs

Based on the tests performed, many of the parameters can be finalized for further work in this project. The finalized parameter values are specified in Table 6.

Table 6: List of finalized parameters/decisions

Strategy Phase		Simulation Phase	
Parameter	Value	Parameter	Value
Max iterations	10	Num. Periods	100
Time Limit	3	Time Limit	5
Num. Discretized states	81 900	Solving Solution	MIP and LP
Solving Solution	LP		

7 Results

This section covers the results acquired through simulations of the model. A completed original run of the model with hydropower Case A has been analyzed, with and without selling reserve capacity. Then the model was tested during low and high reservoir levels. Next a proposed restriction regarding reserve capacity sales at low reservoir levels was included in the model and simulated. All of these results will be discussed and analyzed in Section 8.
Results from the strategy and simulation phase were used in handmade Python scripts to plot the results in the desired way. These are attached with the report. "Plot 3D.py" plotted the 3D water values, while "main.py" and "functions.py" were used for the other plots.

7.1 Optimal Scheduling With and Without Reserve Capacity Sales

Table 7 shows the technical data regarding computation time for both cases. Table 8 contains the expected average total profit for each case for all 100 periods in the simulation phase. Regarding hydropower performance, Figure 28 shows the water values for both cases on a specific price and inflow node. Figure 29 shows the reservoir level behavior in percentiles, giving an indication about expected trends throughout the year. The duration curve for both production and reserve capacity can be found in Figure 30 and Figure 31, respectively.

Table 7: Data regarding computation time for both Case EC and E.

Case	Comp. time SDP [h]	Iterations	Final deviation	Comp. time SIM [h]
EC	5.03	8	0.0013	3.93
E	3.75	6	0.00167	0.29

Table 8: Expected average total profit for both energy and reserve capacity sales. The values are in percentage of the expected profit from energy sales in case E.

Case	Energy sales	Capacity sales	Total
EC	93.63 %	14.96 %	108.59 %
E	100 %	0.00 %	100 %

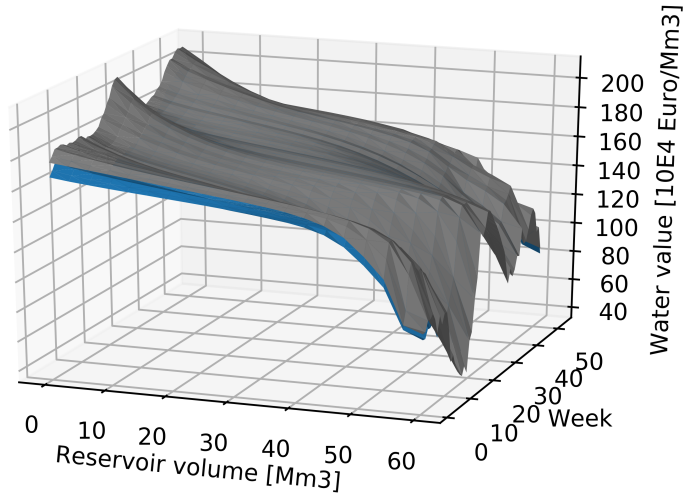

Figure 28: Water value plot for case EC (grey) and case E (blue). The values are for price node 5 and inflow node 3.

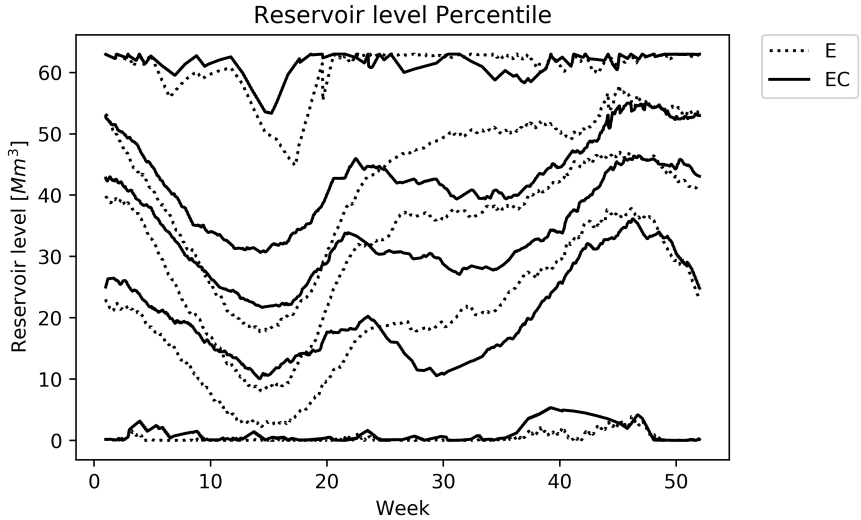

Figure 29: Plot of reservoir levels for both case EC and E. The percentiles are for 0, 25, 50, 75 and 100 %.

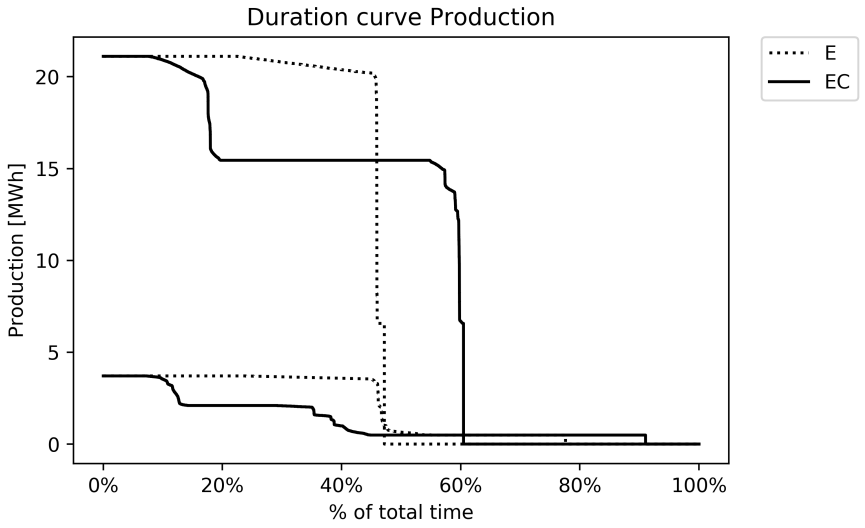

Figure 30: Duration curve for production for both Case EC and E.

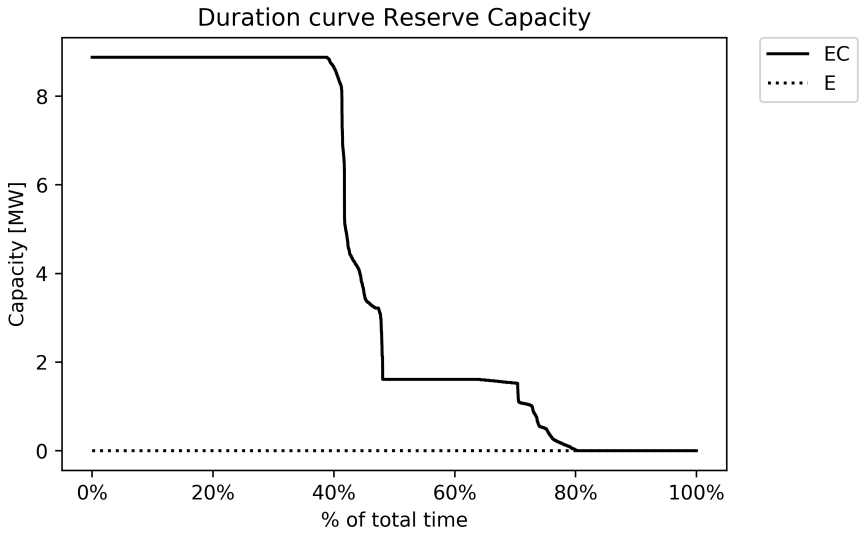

Figure 31: Duration curve for reserve capacity for both Case EC and E.

7.2 Reserve Capacity Allocation under Specific Scenarios

To check how reserve capacity sales are prioritized under extreme reservoir level conditions, the simulation phase of the model was run twice for each case, earlier referred to as case LOW and HIGH. The results are given below, and will be analyzed in Section 8.2.

7.2.1 Case LOW: Low reservoir level

With a scenario with low reservoir level and low inflow present, the model scheduling decision for a specific week is shown in Figure 32. The given week is when the starting reservoir level is low and the inflow is low, and the price for energy sales is higher than spinning reserves sales, as shown in Table 9.

Table 9: Technical information regarding the analyzed stage for case LOW.

Parameter	Value
Res_{start}	0 $[Mm^3]$
Res_{end}	0 $[Mm^3]$
Inflow	3.293 $[m^3/s]$
E_{Price}	283.9 $[mu/MWh]$
C_{Price}	18.4 $[mu/MW]$

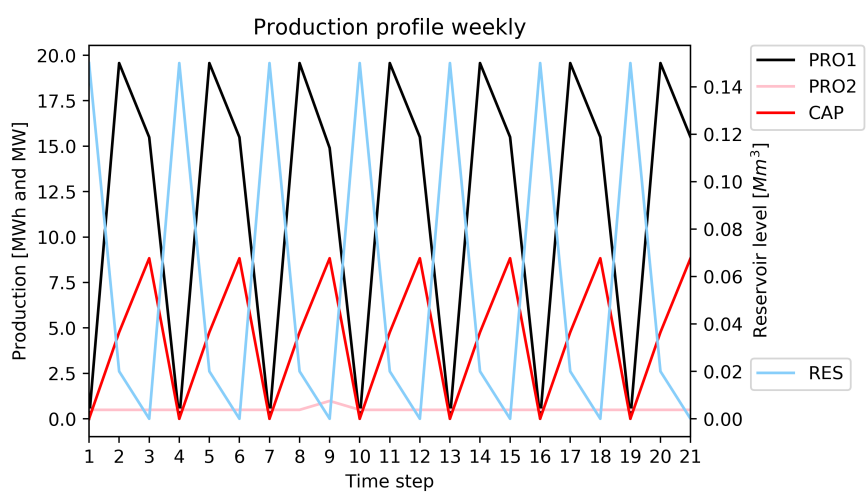

Figure 32: Production profile including reservoir level for a specific week with high reserve capacity price.

7.2.2 Case HIGH: High reservoir level

When the reservoir level is high and the inflow is high, the water value will approach 0 and water will be bypassed to prevent spillage. Figure 33 and Figure 34 show the production profile and water scheduling for a week when the price of reserve capacity sales is higher than the energy sales. Table 10 shows relevant information regarding the chosen week that is analyzed.

Table 10: Technical information regarding the analyzed stage for case HIGH.

Parameter	Value
Res_{start}	57.71 $[Mm^3]$
Res_{end}	63 $[Mm^3]$
Inflow	13.78 $[m^3/s]$
E_{Price}	88 $[mu/MWh]$
C_{Price}	211 $[mu/MW]$

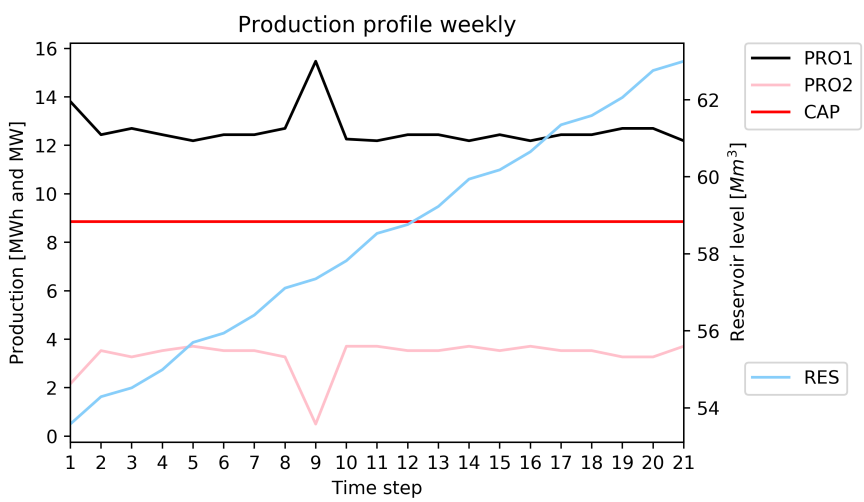

Figure 33: Production profile including reservoir level for a specific week with high reserve capacity price.

7.3 Implementation of Capacity Restriction

The results of case LIMCAP regarding production profile can be found in Figure 35 and Table 11.

Table 11: Technical information regarding the analyzed stage for case LIMCAP.

Parameter	Value
Res_{start}	0.06 $[Mm^3]$
Res_{end}	0.07 $[Mm^3]$
Inflow	3.293 $[m^3/s]$
E_{Price}	283.9 $[mu/MWh]$
C_{Price}	18.4 $[mu/MW]$

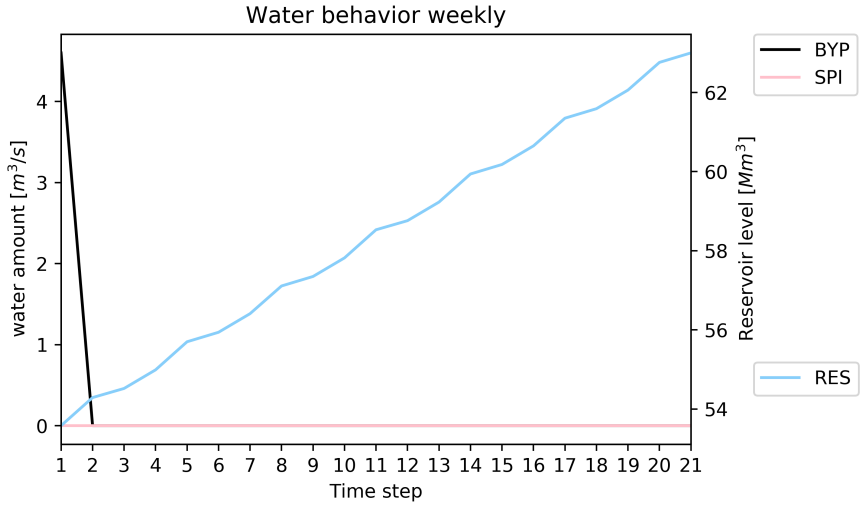

Figure 34: Water flow scheduling for same week as Figure 33.

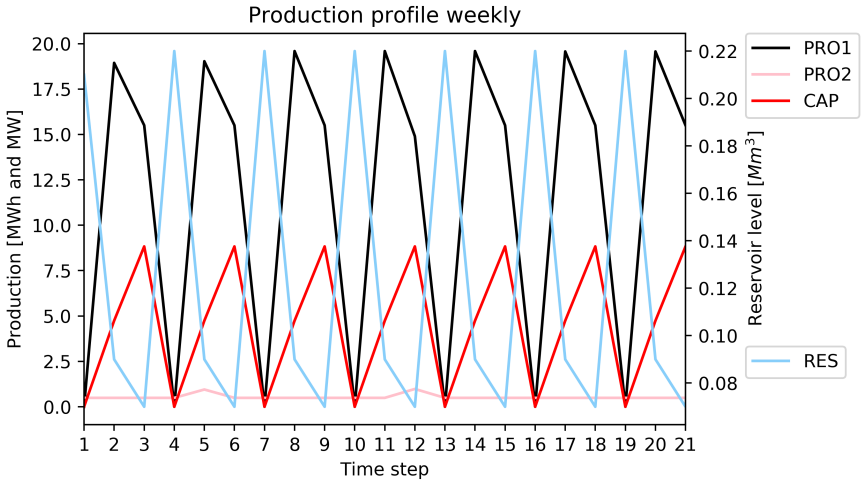

Figure 35: Production profile for a given week with new capacity restriction implemented.

8 Analysis of the Results

8.1 Optimal Scheduling With and Without Reserve Capacity Sales

Computation time comparison

When comparing the computation time between case EC and E in Table 7, the strategy phase needed more iterations to find a converged situation when including reserve capacity sales than without. Since reserve capacity would affect the scheduling of production for sales, this is an expected result as the optimal solution will be based on more options. The extra computation time is mainly due to the extra number of iterations needed.

The simulation phase had massive differences in computation time, with reserve capacity implementation resulting in nearly 4 hours of computation time compared to around 20 minutes without. Including the balancing market has a great impact on the complexity of calculating optimal solution, especially when using MIP. This shows that strong software tools are necessary for this addition to the model to be handled smoothly.

Water values

Figure 28 shows that the water values vary between the two cases. Since EC includes another income source, an increase in water values should be expected, as is the case in the plot. Despite the same price scenarios, the increased water values show that the model considers more potential in revenue for each unit of water. The difference is not that huge in terms of value, but these small deviations could result in huge differences in optimal hydropower scheduling.

Another pattern is how the water values varies depending on reservoir volume and week. The changes in value during different weeks show the different season for hydropower, with pits during depletion season to promote high production and peaks during filling season to prioritize filling the reservoir. For the reservoir volume, the sudden decrease in price when nearing maximum capacity is due to the potential of spilling water.

Expected profit from each market

Case E only has one market that revenue can be acquired from, and will thus maximize profit from that market exclusively. With reserve capacity included, the model will try to maximize the total profit of participating in both. As shown in Table 8, this causes less expected profit from the energy market for case EC, but results in a larger total profit. This shows the potential increase for the producer if planning to operate in both markets compared to only contributing to the energy market.

Reservoir level behavior

An important consideration for the hydropower producer is how the reservoir levels will be affected by selling reserve capacity. As Figure 29 shows, the result is that the reservoir level percentiles have huge differences over the whole year, especially during depletion and filling seasons. For the depletion season, case E sees a much lower reservoir level during this period than case EC. Especially the 50 % percentiles show a difference in week 15 by about 13 Mm^3, approx. 25 % of the total reservoir capacity. The opposite is true during the filling season, where case E normally has a higher reservoir level than EC.

Including the reserve capacity sales creates less variation in reservoir levels during the two seasons, due to the reduced need of stored water to create profit. This will make the producer less dependent on variations in inflow during the year and from year to year.

Production variation

As has been stated earlier, with reserve capacity sales included the production decisions will optimize both markets. This is the case for the duration curves for production and reserve capacity, shown in Figure 30 and Figure 31. For case E, the only major decision factor for production is to produce or not to produce. If there is profit in producing, then it would maximize production for most profit gain. For case EC, one can see that the duration curve has multiple steps of production compared to case E. Other than maximum production, both generators produce in an intermediate production step that is neither max nor minimum potential. For generator 1, with the largest capacity, it has the longest duration at around 15 MWh production. At this step, it will contribute to both up- and down-regulation, giving additional revenue from the balancing market. This increases the duration generator 1 is turned on with 10-15 % compared to case E.

Another interesting point is how the production profile for generator 2 varies. It has 4 different productions steps: Maximum, medium, minimum and no production. For the minimum production period, it can provide with up-regulation capacity that could be sold when generator 1 is on and have excess down-regulation. During the medium production period, it can provide with both up- and down-regulation, making capacity sales possible without the need of generator 1. This shows that they can help each other maximize the reserve capacity sales by letting one provide with up-regulation while the other provides with down-regulation. This shared flexibility will definitely help maximize contribution to the balancing market.

8.2 Reserve Capacity Allocation under Specific Scenarios

The goal of this test was to check the performance of the optimization problem under abnormal reservoir conditions, to understand how the reserve capacity sales are prioritized. As two different tests were done, they both will be described in detail here.

8.2.1 Case LOW: Low reservoir level

Performance

Studying the optimization problem performance in Figure 32, there is a constant change in production for every time step. The production profile for generator 1 is related to the reservoir level, which varies from almost empty to increasing to a cap of 0.15 Mm^3. Generator 2 is always on, but producing at minimum capacity at 0.49 MWh to provide with up-regulation. The reserved capacity sales are related to production from generator 1. When generator 1 is off, there is no down-regulation of reserve capacity available, which prevents capacity sales. When generator 1 is turned on at the following time step due to higher reservoir level, it goes to max, allowing down-regulation and thus capacity sales. During the next time step, the production is limited to remaining water, which decreases power production, increases up-regulation and thus increases total reserve capacity sales.

Here, reserve capacity is always sold when there are both up- and down-regulation possibilities. The reserve capacity appears not to be restricted by stored water in the reservoir. In reality, the hydropower producer is obliged to increase or decrease production when prompted, and should therefore have available water stored in the reservoir for this unexpected production change. Failure to meet this requirement will result in a large penalty, and is therefore unwanted. The way the model is behaving now, it does not include this into the scheduling. Having a production profile like this will induce risk to the producer, which is unwanted as the producer is considered risk-neutral.

Restriction implementation

To prevent this behavior, a restriction limiting reserve capacity depending on reservoir level must be implemented. The model will then have to choose between limiting the lower reservoir level to allow maximum reserve capacity, or lowering reserve capacity sales to reduce the reservoir level further.

Another factor is the generator efficiency. Modelled as a piecewise-linear function, the efficiency will be different depending on the power output. The efficiency for current production might not be the same efficiency if production should increase. This will be based on small deviations,

but should still be considered when formulating the constraint. To simplify the restriction and thus prevent long computation time, the efficiency should be the lowest efficiency the generators can produce at. This will make the model underestimate the potential in the stored water at worst, but only by a small margin. This will prevent the producer from taking any unnecessary risks.

With these considerations, the proposed restriction is given in Equation 9, where $T[k]$ is the number of hours in time step k, $cap[b]$ is the reserve capacity sales for this time step, η is the minimum total efficiency, $MM3H$ is the conversion factor between m^3/s and Mm^3, and $res[k]$ is the reservoir level at the end of time step k.

$$T[k] \cdot cap[b] \leq \frac{\eta[g1,1] \cdot res[k]}{MM3H}, \quad k \in K, b \in B, ord(b) = BK[k] \tag{9}$$

Expected change

This equation will limit the reserve capacity sales to be the minimum production output from the water stored at the end of the current time step. Thus, the water stored at the end must be equal or greater to the possible production increase. With this implemented, the problem should look at the reservoir level when considering reserve capacity sales, and will limit either of them to optimize production. This will only be a contributing factor at low reservoir levels, the constraint will be met most of the time.

The results from testing this restriction is found in Section 7.3.

8.2.2 Case HIGH: High reservoir level

Performance

Looking at Figure 33, reserve capacity sales are constantly at max 8.88 MW, with both generator 1 and 2 having small fluctuations in production for every time step. Reserve capacity sales are preferred over energy sales due to higher prices as shown in Table 10. The reservoir level is increasing steadily for each time step due to a higher inflow than water release, filling the reservoir to max at the end of the last time step.

In Figure 34, water flowing around the hydropower station is plotted. Here, one can see that water is bypassed during the first time step, while both spillage and bypass remain 0 for the rest of the plot. This is happening despite the reservoir not being maxed at the start of the week.

Analysis of water flow decision

The optimization problem will only focus on maximizing profit, and will choose to maximize production in the source with the highest revenue. This will happen regardless of reservoir level. As a result, reserve capacity sales will be prioritized at the cost of filling the reservoir over the limit, when the profit is greater. Instead of water flowing as spillage, the water is bypassed at the start of the week as the timing has no consequence. Since the optimization problem does not have any penalty costs for bypassing, this will be preferred over spillage. And since there is no decision factor for when to bypass, all excess water will be bypassed at the start of the week.

With this discovery, spillage will play no role in this optimization problem unless water bypass is limited. Spillage can be modelled as an alternative water way between reservoirs, but this is not the case here as only one reservoir is included. For the given case, bypass is considered an unlimited water passage. Therefore, the penalty function to make the optimization problem prevent spillage will not have an effect.

What the hydropower producer is doing in this scenario is bypassing water to prevent penalties from spilling, all while having limited production to maximize reserve capacity sales. Reserve capacity sales are hindering the producer to utilize most of the water flowing in, resulting in higher amounts of water being bypassed.

Are reserve capacity sales more important than limiting spillage?

A question that can be asked is what would a hydropower producer do in this situation? The producer is bypassing more water than necessary to sell reserve capacity, and is basically spilling free resources that could have been utilized for more power production. Previously, penalty cost related to spillage was introduced to prevent these situations and push the model to utilize the maximum potential. However this is not the case for this model with only one reservoir included.

This behavior should be discussed with a hydropower producer to find out what the optimal decision would be. Based on their feedback, constraints could be added to give unnecessary bypass a penalty cost, or prioritize production when a full reservoir is imminent.

8.3 Implementation of Capacity Restriction

Performance

Figure 35 and Table 11 show the same scenario as for case LOW. This is a good way of showing what the restriction has done with the decisions made by the optimization problem.

Looking at the reservoir levels, the reservoir is no longer being emptied during the week as was the case before. The level keeps increasing and decreasing depending on production output, but has a lower cap at 0.07 Mm^3. This reservoir level allows reserve capacity sales as there would be sufficient stored water for a sudden up-regulation of power for the given time step. This is despite high energy prices.

Change of behavior

The starting and ending reservoir level has increased compared to case LOW. Implementation of the constraint has changed the scheduling to prevent emptying the reservoir, and the optimization problem finds additional profit in keeping some extra water stored at all times. Keeping a small amount of water stored throughout the period is more profitable than using it up at the best opportunity and having too low reservoir levels for the rest of the week. The production pattern is the same for both generators as in the original scenario.

The optimization problem considers it beneficial of storing some water at low reservoir levels to allow reserve capacity sales. This constraint could therefore also result in less use of artificial water, as lack of water will occur less often.

The added restriction has prevented the risk originally present of having insufficient resources for sudden production increase. It might also reduce occasions in which an empty reservoir will lead to artificial water usage or completely stopping production. The constraint therefore adds a buffer for scenarios with low inflow. What should be studied at a later point is how the water values change with the implementation of this constraint at low reservoir levels.

9 Conclusion

In this paper it has been shown how reserve capacity sales implemented into a long term scheduling model affect the optimal scheduling, both when it comes to computation time and production allocation. Expanding the possible markets a hydropower producer can participate in is important as it gives the producer a possibility to increase profits. This model enables the producer to find the optimal participation allocation and the optimal strategy. Including reserve capacity sales resulted in a higher potential total profit compared to standard production sales for the given case. This is encouraging as more renewable energy sources installed will challenge hydropower producers in the energy market. It will also make the balancing market more beneficial for the hydropower producers.

This paper has also uncovered some possible simulation considerations that should be implemented to avoid risky participation in the balancing market. The implemented finding shows that the model encourage producers to have a small buffer of water stored in the reservoir to reduce the risk of emptying the reservoir during up-regulation in production. Another result is that the possible reserve capacity sales create situations where water rather should be bypassed than used for production. This unusual decision shows how the original focus on utilizing the most of the resources will be changed, due to other profit opportunities when it comes to water handling.

10 Future Work

The work done in this project revealed that implementing reserve capacity sales need adjustments and tuning to further optimize the scheduling. The first point of interest would be valid areas for reserve capacity sales. Further investigation of how the producer would like to juggle between energy and reserve capacity sales could lead to increased constraints for reserve capacity scheduling. The main goal of this is to find realistic decisions.

Then, one could try to implement actual up- and down-regulation of reserve capacity during scheduling. Currently reserve capacity is seen as an extra revenue source, without using any resources. This leads to scheduling that does not include the risk of actual regulation of power production. This could be implemented by introducing a regulation schedule for each time step or stage, that the production profile must meet, without extra revenue. This should lead to changes in the optimal scheduling.

Another possible implementation is introducing realistic inflow patterns. The inflow used in this study is discretized and based on historical inflow, and will therefore not perfectly simulate the inflow pattern. By acquiring the real inflow pattern and adjust the inflow sequence, this could be achieved. However, alterations to water value calculations will be needed to give realistic scheduling decisions.

Detailed analysis of using LP and MIP should be further investigated. An important factor here is to have access to strong processor power that can find the optimal MIP scheduling quickly. The possible areas to analyze are water values, production scheduling and reserve capacity sales, to name a few.

Different turbine combinations should be put to test using the optimization model. This will help uncover scheduling decisions based on different factors, which can be used to find preferred turbine combinations in case of an upgrade. It can also be used to find how the scheduling decisions change to possibly find other abnormalities.

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A Mathematical Model Description

A.1 Model functions

A.1.1 Sets

K	Time steps for sales of energy within decision stage
B	Time blocks for sales of reserve capacity
G	Generators

A.1.2 Indexes

k	Time step for sales of energy
b	Time block for sales of reserve capacity
g	Generator

A.1.3 Parameters

$HoursWeek$	Total hours for each stage
$MM3H$	Conversion value from m^3/s to Mm^3/h
$cTol$	Tolerance for consistency check
eps	Tolerance in redundant bounds
BK_k	Information regarding which index b corresponds to index k
T_k	Length of time steps $[hours]$
$Eprice$	Average energy price for current stage $[mu/MWh]$
$Cprice$	Average reserve capacity price for current stage $[mu/MW]$
$Escale_k$	Price variation for time step k
$Cscale_b$	Price variation for time step b
$Inflow$	Inflow for the current stage $[Mm^3]$
$Inflowscale$	Scaling for unregulated inflow
$FlowReq$	Minimum water flow in river $[m^3/s]$
NR	Number of discretized reservoir segments
$ResEnd(1..NR-1)$	Reservoir level for each segment $[Mm^3]$

$WV(1..NR-1)$	Water value for current decision stage[$10E3mu/Mm^3$]
$DVAL$	Offset in value function [$10E3mu$]
$ReservoirState$	Reservoir level at the beginning of the decision stage [Mm^3]
$RelHead$	The Relative head for the initial reservoir state
$ResMax$	Reservoir Capacity [Mm^3]
$ResMin$	Minimum reservoir level [Mm^3]
$RelMax$	Maximum release of water from reservoir [m^3/s]
$BypMax$	Maximum bypass [m^3/s]
$CapMax$	Maximum reserve capacity sales [MW]
$PenSpi$	Penalty for spillage [$10E3mu/m^3/s$]
$PenTank$	Penalty for use of artificial water [$10E3mu/Mm^3$]
$NPQSeg_g$	Number of piecewise-linear segments in PQ-curve
$DisMaxSeg_{g,NPQSeg_g}$	Discharge volume per PQ-curve segment [m^3/s]
$Eff_{g,NPQSeg_g}$	Generator efficiency per PQ-curve segment [$MW/m^3/s$]
$ProdMin_g$	Minimum generation [MW]
$ProdMax_g$	Maximal generation [MW]
$SCost_g$	Cost of starting generator g [$10E3mu$]

A.1.4 Variables

Continuous variables

res_k	Reservoir level at end of time step k [Mm^3]
rel_k	Water released from reservoir at time step k [m^3/s]
spi_k	Water spillage at time step k [m^3/s
byp_k	Water bypassed at time step k [m^3/s
$dis_{g,k}$	Water discharged to generator g, time step k [m^3/s]
$tank$	Artificial water added at first time step [m^3/s]
$prod_{g,k}$	Hydropower generation generator g time step k [MWh]
cap_b	Capacity sales time step b [MW]
$suc_{g,k}$	Start-up cost generator g time step k [$10E3mu$]

Binary Variables

$spg_{g,k}$	Spinning state generator g time step k [0/1]

ssi_g	Initial spinning state generator g [0/1]

A.2 Objective

$$Objective = \sum_{g \in G} \sum_{k \in K} E_{scale}[k] \cdot E_{price} \cdot prod[g,k] \cdot 10^{-3} \tag{10}$$

$$+ \sum_{k \in K} \sum_{b \in B} T[k] \cdot C_{scale}[b] \cdot C_{price} \cdot cap[b] \cdot 10^{-3} \tag{11}$$

$$- \sum_{g \in G} \sum_{k \in K} suc[g,k] \tag{12}$$

$$- \sum_{k \in K} PenSpi \cdot spi[k] \tag{13}$$

$$- PenTank \cdot MM3H \cdot T[first(K)] \cdot tank \tag{14}$$

$$+ WV[n] \cdot res[last(K)] + DVal \tag{15}$$

A.3 Constraints

A.3.1 Initial reservoir balance

Description: Reservoir level at the end of the first time step is the initial reservoir level + inflow - outflow.

$$\begin{aligned} res[first(K)] + MM3H \cdot T[first(K)] \cdot (rel[first(K)] + spi[first(K)] - tank) \\ = ReservoirState + \frac{T[first(K)]}{HoursWeek} \cdot Inflow \end{aligned} \tag{16}$$

A.3.2 Reservoir balance

Description: Reservoir level deviation between start and end of the time step must be equal to inflow and release/spillage.

$$\begin{aligned} res[k] - res[prev(k)] + MM3H \cdot T[k] \cdot (rel[k] + spi[k]) \\ = \frac{T[k] \cdot Inflow}{HoursWeek}, \quad k \in K \end{aligned} \tag{17}$$

A.3.3 Release balance

Description: Amount of water released from the reservoir must be equal to bypassed water and water discharged.

$$rel[k] = byp[k] + \sum_{g \in G} dis[g,k], \quad k \in K \tag{18}$$

A.3.4 Hydropower generation

Description: Power produced must be equal to the water released and the corresponding efficiency for each generator $g \in G$.

$$\begin{aligned} prod[g,k] = T[k] \cdot Eff[g,i] \cdot RelHead \cdot dis[g,k] \\ + T[k] \cdot (ProdMin[g] - Eff[f,1] \cdot DisMaxSeg[g,1]) \cdot spg[g,k], \quad k \in K, \quad g \in G \end{aligned} \tag{19}$$

A.3.5 Lower hydropower boundary

Description: Sets the minimal production boundary including spinning state.

$$prod[g,k] \geqslant ProdMin[g] \cdot T[k] \cdot spg[g,k], \quad g \in G, \quad k \in K \tag{20}$$

A.3.6 Upper hydropower limit

Description: Sets the upper limit for production capacity.

$$prod[g,k] \leqslant ProdMax[g] \cdot T[k] \cdot spg[g,k], \quad g \in G, \quad k \in K \tag{21}$$

A.3.7 Limit down-regulation

Description: Limits down-regulation of reserve capacity.

$$\sum_{g \in G} prod[g,k] \geq T[k] \cdot cap[b] + \sum_{g \in G} ProdMin[g] \cdot T[k] \cdot spg[g,k], \quad k \in K, \quad b \in B \backslash [ord(b) = BK[k]] \tag{22}$$

A.3.8 Limit up-regulation

Description: Limits up-regulation of reserve capacity.

$$\sum_{g \in G} prod[g,k] \leq -T[k] \cdot cap[b] + \sum_{g \in G} ProdMax[g] \cdot T[k] \cdot spg[g,k], \quad k \in K, \quad b \in B \backslash [ord(b) = BK[k]] \tag{23}$$

A.3.9 Start up of generators

Description: Checks if generator is turned from non-spinning state to spinning state.

$$suc[g,k] \geqslant SCost[g] \cdot (spg[g,k)] - spg[g, prev(k)]), \quad g \in G, \quad k \in K \backslash ord(k) > 1 \tag{24}$$

A.3.10 Initial start up of generators

Description: Start-up-cost at the first time step.

$$suc[g, first(k)] \geqslant SCost[g] \cdot (spg[g, first(k))] - ssi[g]), \quad g \in G \tag{25}$$

A.3.11 Keep initial spinning state equal to end spinning state

Description: Initial spinning state is equal to ending spinning state.

$$ssi[g] = spg[g, last(k)], \quad g \in G \tag{26}$$

A.3.12 Flow requirement

Description: Water flow out of station plus unregulated inflow must meet required water flow restriction.

$$\begin{aligned} MM3H \cdot T[k] \cdot rel[k] \geq \\ FlowReq \cdot MM3H \cdot T[k] - (\frac{T[k]}{HoursWeek} \cdot InflowScale \cdot Inflow), \quad k \in K \end{aligned} \tag{27}$$

A.3.13 Minimum Capacity requirement

Description: Reserve capacity sold is restricted to the lowest water amount stored in the reservoir. This restriction is implemented during the making of this report and is discussed in Section 8.3.

$$T[k] \cdot cap[b] \leq \frac{\eta[g1,1] \cdot res[k]}{MM3H}, \quad k \in K, b \in B, ord(b) = BK[k] \tag{28}$$