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# Impact of Modelling Details on the Generation Function for a Norwegian Hydropower Producer

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**Abstract.** In the following work the generation function relating water discharge to power output for a hydropower station is considered. The conventional approach to modelling the generation function has been to simplify it as a concave function. In the future it is expected that the stations will operate at more varied power outputs, motivating the need for a more detailed modelling of the generation function. Therefore, an investigation on how the nonconcave generation function may be simplified, without having to lose the function's inherent geometric shape, is performed. A greedy algorithm and a Minimum Least Square Error (MLSE) approach is used. It was found that the function can be reduced until a point where the changes become too prominent. A benchmark test against the conventional modelling approach found that it is important with a detailed modelling of the generation function when environmental constraints, such as minimum discharge, are included.

## 1. Introduction

The following work is founded on recent advancements in hydropower scheduling methods that enables a higher level of modelling details. There has been a fundamental change in the electricity sector with large shares of renewables entering the market and increased cross-border exchange. It is expected that this will lead to increased demand of flexible power plants that can provide ancillary services [1]. Given a market environment where these services can be traded across the European countries it is estimated that there is a income potential for Norwegian hydropower producers [2]. Due to the flexibility storable hydropower possesses they have an opportunity to provide many of the ancillary services at a competitive cost. However, in order for them to adapt to this changing market environment and provide these services it is evident that adequate decision support tools are needed to optimally allocate generation between different markets. Earlier studies have shown that conventional scheduling algorithms based on linear programming (LP) tend to overestimate the amount of capacity reserves that the hydropower producer can deliver [3]. There is also a tendency to increase the environmental restrictions on new concessions for hydropower systems in Norway, leading to more complex constraints in the optimization problem. Developing more advance methods should there be of priority to include these constraints and capture the added income potential by participating in multiple-markets.

## 2. Hydropower Scheduling

Due to the uncertainty of inflow, energy prices and the possibility of storing water over longer time periods, the hydropower scheduling problem is generally handled as multistage stochastic programming problem. An important element of hydropower scheduling is the water value; even though inflow to the reservoirs are free, there is an opportunity cost associated with the water stored in the reservoirs. The producer can choose to use the water for selling energy today and earn an immediate income or store it



for future use with a potential higher profit. In terms of hydropower scheduling, this opportunity cost is referred to as the water value.

Given the complex nature of the hydropower scheduling problem, it can be divided into three problems with different time horizon and modelling details [4]. The Long-Term Hydropower Scheduling (LTHS) problem aims at solving a fundamental market problem that provides energy price forecasts and aggregated water values [5] [6]. The Medium-Term Hydropower Scheduling (MTHS) problem can then use the aggregated water values as end statement and price forecasts to provide individual water values for each reservoir in a local hydropower system [7]. Further, these individual water values can be used as an end statement for the Short-Term Hydropower Scheduling (STHS) problem that has high modelling details, e.g. modelling of individual generators and water courses. The time resolution is fine, typically hourly for a period of one or two weeks. For liberalized electricity markets this problem aims at first generating bids to the power market and after the market clearing perform a re-optimization that incorporates any commitments that occurred in the clearing [8].

### 2.1. Multistage Stochastic Optimization

This paper depicts an application that will be used in further studies on the MTHS problem. A time horizon of 1-2 years is typical for this problem type, using weekly decision stages as the inflow is highly correlated on a weekly basis. The problem is cast as a Stochastic Dynamic Programming (SDP) problem, with weekly decision stages. To circumvent the “curse of dimensionality” associated with conventional (S)DP problems, a method called Stochastic Dual Dynamic Programming (SDDP) is widely used [9]. The method approximates the expected profit function (EPF), also referred to as cost-to-go function for minimization problems. One of the drawbacks with the method is that it requires the decision stages to be modelled as LP problems, ruling out the possibility of modelling nonlinearities without simplifications. A recent extension of the method has been proposed called the Stochastic Dual Dynamic integer Programming (SDDiP) method [10]. The method can solve nonlinear problems with finite convergence. However, it comes at a cost of added computation time and some adjustments on the structure of the problem, i.e. it requires all the state variables to be binary. Previous work has tested the method on a MTHS problem with promising results, compared to the SDDP method [11]. With this in mind, the following work investigates how the generation function of a hydropower plant can be modelled in the SDDiP framework and how it can be simplified to reduce the computational burden.

### 2.2. Contributions

The following paper applies a greedy algorithm and a Minimum Least Square Error (MLSE) approach to simplify the generation function of a hydropower station. The greedy algorithm is based on the idea of reducing the number of points on a bid curve in [12]. The approach is tested on two real power station equivalents and the importance of detailed modelling of the generation function is shown. The following section outlines the hydropower scheduling problem, followed by case studies and results.

## 3. Medium-Term Hydropower Scheduling

The problem aims at allocating resources simultaneously in an energy market and a capacity market. For the sake of simplification, only uncertainty of inflow is considered. The weekly decision problem can be written on dense form as

$$\max_{(x_t, y_t)} f_t(x_t, y_t) + \alpha_t(x_t) \quad (1)$$

$$\text{s.t. } Wx_t + Gy_t = h_t(\xi_t) - Hx_{t-1} \quad (2)$$

$$By_t = 0 \quad (3)$$

$$Cy_t - Dx_t \geq 0 \quad (4)$$

$$Cy_t + Dx_t \leq Cy^{\max} \quad (5)$$

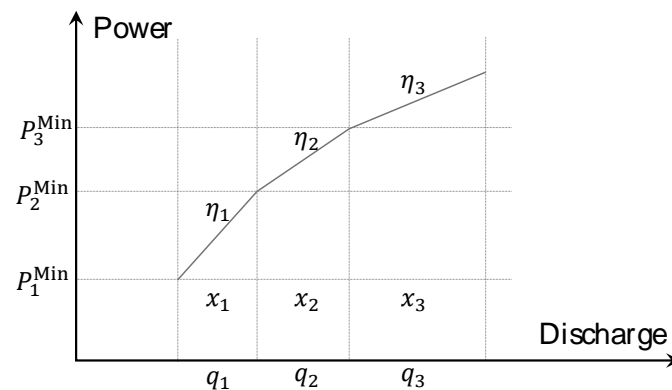
$$x_t, y_t \in Y_t \quad (6)$$

$$x_t \in \mathbb{R}^{k_1} \cdot \mathbb{Z}^{k_2}, y_t \in \mathbb{R}^{l_1} \cdot \mathbb{Z}^{l_2}, \quad (7)$$

where the objective function (1) consists of a present profit function,  $f_t(x_t, y_t)$ , and the expected future profit function,  $\alpha_t(x_t)$ .  $x_t$  and  $y_t$  are respectively the state and stage variables. State variables carry information between stages, e.g. reservoir levels, whereas the stage variables only represent variables within the stages. The inflow is given by the function  $h_t(\xi_t)$ , where  $\xi_t$  is the normalized inflow. The matrices  $W, G, H, B, C, D$  are of suitable dimensions, representing the given hydropower system. The time-linking constraint (2) includes all reservoir balances and generator state time-couplings. The energy balance is given by (3) and the constraints describing the system's ability to provide capacity reserves is given in (4) and (5). The generation function and other miscellaneous constraints, such as bypass limits, spillage limits and other system dependent constraints are included in (6).

### 3.1. Generation function

The generation function describes how the hydropower station's discharge relates to the output power, usually modelled as a piecewise linear function. The function is typically computed when the hydropower station is built. This function provides data input to the scheduling models and as a control that the station meets the efficiency as promised by the contractors. As the efficiency may deteriorate over time, the more recent the measurements, the better. Advantageously, a system with continuous measurement could give the best result in terms of describing the generation function most accurately. The methods proposed in this work will work irrespective of the measurement approach, it would just require some preprocessing of the data.



**Figure 1:** Illustration showing the mathematical notation of the generation function.

The line between two points in the generation function is referred to as a segment. Extending on the notation from previous section, the generation function is modelled as follows

$$0 \leq q_i(Q_i^{\text{Max}} - Q_i^{\text{Min}})x_i \quad \forall i \in S \quad (8)$$

$$p_i = P_i^{\text{min}}x_i + \eta_i q_i \quad \forall i \in S \quad (9)$$

$$\sum_{i \in S} x_i \leq 1 \quad (10)$$

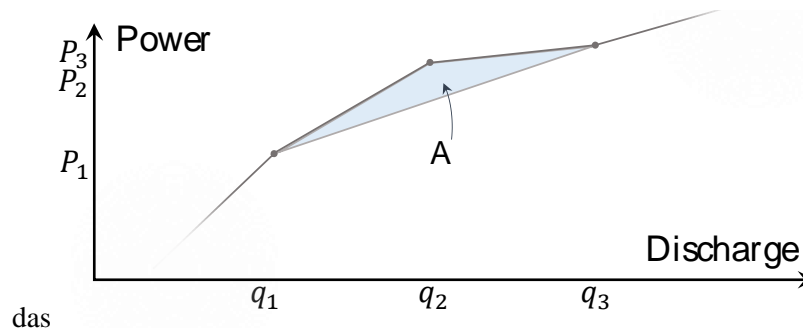
$$p = \sum_{i \in S} p_i, \quad (11)$$

where  $q_i$  is the discharge (in  $Mm^3$ ) for segment  $i$ ,  $Q_i^{\text{Min}}$  and  $Q_i^{\text{Max}}$  are the minimum and maximum discharge for each segment,  $p_i$  is the generation (in MW) for each segment,  $P_i^{\text{Min}}$  is the minimum generation for each segment,  $x_i$  is a binary variable indicating whether the segment  $i$  is active or not,  $p$  and  $p_i$  is, respectively, the overall generation and generation for the individual segments in the set  $S$  of segments. An illustration of the generation function is given in Figure 1. Observe that even though the illustration depicts a concave function the method also applies to the nonconcave case, which is an important aspect in this paper. A concave generation function would imply that the first segment has the best efficiency, i.e. best power output for a given discharge, while the other segments are decreasingly

worse. This case would need no binary variables to ensure that the efficiency of each segment is used correctly, since the model will always use the segment with the best efficiency first and the rest followingly.

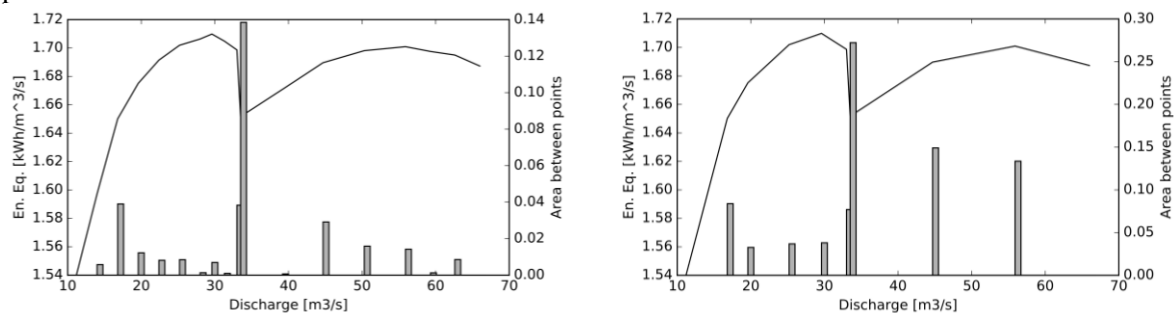
### 3.1.1. Area removal algorithm

The following section describes the greedy algorithm used to reduce the number of segments in the generation function. The idea is taken from [12], where the authors apply the method on bid curves for a hydropower producer. The underlying idea is that you should remove points in the function that alter its geometric shape as little as possible. Illustration given in Figure 2.



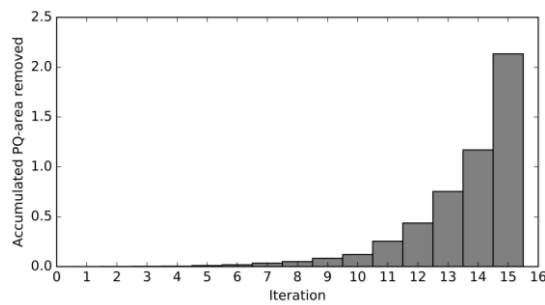
**Figure 2:** The figure shows a part of the generation function and how the area (A) is computed between three adjacent points.

The algorithm computes the area between two adjacent point for all the points on the curve. The point that has the lowest computed area is removed. This procedure is repeated until a desired amount of points are removed.



**Figure 3:** Illustration of the generation function. Right axis indicates the area of the triangle computed with the two adjacent points. Note that we are illustrating the energy equivalent [ $\text{kWh}/\text{m}^3$ ]. This is done to get a better visualization of the curve. Left: The original generation function. Right: The generation function after eight points have been removed by the area removal technique.

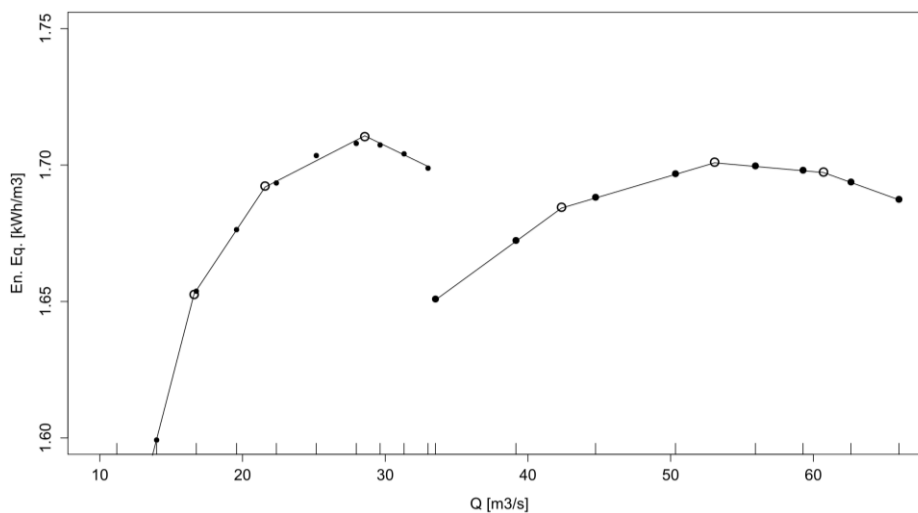
Figure 3 shows the generation function and the area between the adjacent points for the original function and for the function when eight points have been removed. As expected the geometric shape of the function is very much kept, even though half of the points on the curve has been removed. Below, in Figure 4, the accumulated area that is removed from the function is plotted. It is evident that there are many points on the curve that can be removed with almost no impact. After a certain threshold the accumulated area increases significantly, giving an initial indication for how much the function can be reduced.



**Figure 4:** Bar graph of the accumulated area between adjacent points in the generation function that are removed. As illustrated, there are many points that can be removed with little impact on the shape of the curve, following the more points removed, the more significant this impact becomes.

### 3.1.2. Segmented regression method

Segmented regression is applied to reduce the number of points in the generation function. This is done with the R library *segmented* [13]. It is an iterative approach where the user suggests starting breakpoints. Being a statistical approach, it would be beneficial with a lot of data to fit the curve. In such sense, the approach would be well suited for a system that does continuous measurements. Nonetheless, the approach provides results well fitted to the original data, as seen in Figure 5.



**Figure 5:** The generation function is given by the filled dots. The line going through the circles are the results from the segmented regression. Observe that there are two generators in the power station, each fitted individually.

## 4. Case Study

Two power stations are investigated in the study. The first, Station A, consists of a single generator unit with 45 MW installed capacity. The second, Station B, consists of two generator units, each with 210 MW installed capacity. Both stations only have Francis turbines installed.

The two systems are solved with a time horizon of two years with weekly uncertainty of inflow. For simplicity, the energy prices and capacity reserve prices are modelled as deterministic. The week is then separated into 21 time periods, three for each day indicating morning, mid-day and evening.

The SDDiP approach is applied, described in [10] [11], to solve the hydropower scheduling problem. The problem is solved with the original generation function and an EFP function is obtained, which in turn is used to simulate with the different reductions of the generation function. For each simulation 100 different scenarios are generated that are used to evaluate the expected results, for all simulations. To compare the results to what is normal approach for this problem type in operative models today, a case

where the generation function is concave is performed. Lastly, some tests are performed on how Station A would be operated given that there is a time period in the summer months that requires a minimum discharge, due to environmental constraints.

## 5. Results and Discussion

The following section outlines some results from the case studies and discusses its implications.

Table 1 and Table 2 shows some of the results obtained from the different simulations. The area removal algorithm (A. Rem.), segmented regression method (Seg. Reg.) and Concave case (Conc.) was run. The tables depict the computation time of each simulation, the expected objective value, deviation in shape of the duration curve and total generation from the original problem, given by the first column. The second row of the table indicates how many points on the curve has been removed from an original total amount of 18 and 11 in System A and B, respectively.

**Table 1:** Results from System A. Simulation time is given in seconds, whereas the other values are given as a percentage. Val. represents the expected objective value, Dur. indicates the percentage deviation of the duration curve compared to the original problem. Gen. is the expected generation for the two years.

	A. Rem					Seg. Reg.				Conc.	
	0	2	4	6	8	10	8	9A	9B		10
Time	920	895	763	668	614	523	555	522	521	475	59
Val.	0.00	0.00	0.01	0.00	0.00	-0.03	-0.01	-0.02	-0.01	-0.01	2.22
Dur.	0.00	0.67	0.69	0.67	1.33	2.62	3.10	4.39	4.77	4.28	13.02
Gen.	0.00	-0.01	0.02	-0.01	-0.30	-0.15	-0.14	0.46	-0.24	-0.11	0.34

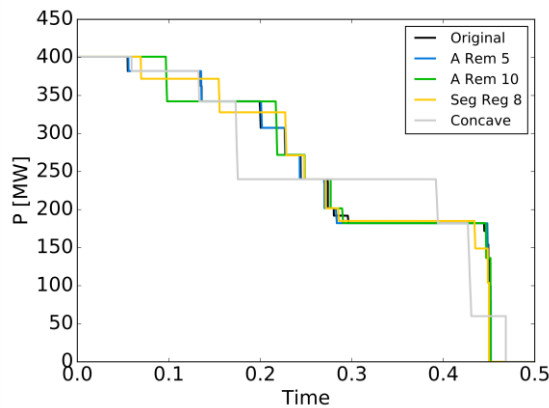
As expected the computation time drops significantly with the number of points being removed. This is especially evident for the concave case where there are less constraints and binary variables in the problem. Moreover, one can observe that the expected objective is not changed much between the cases. This is due to the fact that points on the generation curve are being removed regardless whether it results in an increase or decrease in overall efficiency. The changes in shape of the duration function increases the more points are removed from the generation function. This is especially clear for the segmented regression case, but not unexpected as seen from Figure 5, where the best operating points shift slightly. Lastly, another expected result occurs where there is a positive change in generation for the concave case.

**Table 2:** Results from System B. Simulation time is given in seconds, whereas the other values are given as a percentage. Val. represents the expected objective value, Dur. Indicates the percentage deviation of the duration curve compared to the original problem. Gen. is the expected generation for the two years.

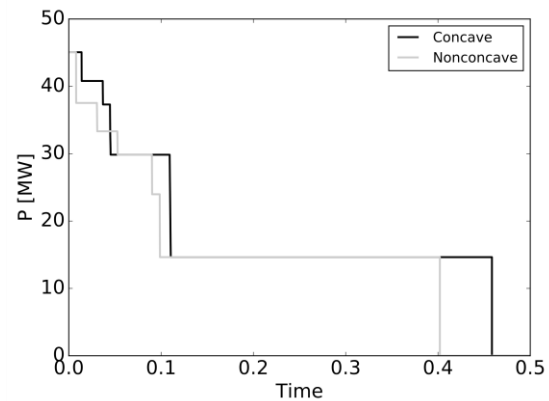
	A. Rem.					Seg. Reg.		Conc.
	0	2	4	6	8	6	7	
Time	335	284	238	218	180	197	178	77
Val.	0.00	-0.03	-0.01	-0.05	-0.58	0.01	0.09	2.05
Dur.	0.00	0.16	0.50	0.52	9.99	1.04	2.95	15.09
Gen.	0.00	0.00	-0.04	0.06	-0.76	0.09	0.01	1.02

The duration curve for selected simulations are shown in Figure 6 illustrates some duration curves for System A. The degree of regulation, i.e. the system's ability to store water for usage at a later stage, is very high for the system. Thus, the model will shift as much of the generation as possible to the hours in the winter where the prices are highest. Also, considering that the efficiency of the station is low at low power outputs the model will tend to avoid such operation. The best efficiency of the station is obtained around 180 MW, where a large portion of the generation comes from. Note that the concave

generation function is not able to represent the low efficiency at low power outputs, some generation is therefore observed around 60 MW for this simulation. The yellow line shows the duration curve for the simulations with the segmented regression method applied. As expected the generation points are somewhat shifted from the original problem. This illustrates the importance of the generation function modelling on the operation of the power station. This method would, as above-mentioned, get a much more robust representation of the generation function had there been more measurements to fit.



**Figure 6:** Duration curve for a selection of the results from System A. A Rem. refers to the Area removal method and how many points are remove. Seg Reg 8 is for the segmented regression and the grey line is for the concave generation function.



**Figure 7:** Duration curve for a selection of the results from System B. Minimum discharge limit during summer months.

Figure 7 shows the duration curve when Station B ran with a minimum generation requirement during the summer months. This would result in much less flexibility in terms of storing the water for the hours of the year with highest prices. The simulations were performed as a test to investigate how the system reacts to generation on lower power outputs. The minimum power output for the station is 15 MW, as seen in the figure where most of the power station operation is performed. The tests were performed with the original generation function and the concave one. It can be seen that the shape of the duration curves is very much similar, but with much less generation for the original, nonconcave, problem. This shows a significant drawback with the concave generation function, it overestimates the efficiency of the station and therefore assume that it can generate more electricity than what is possible. For the given case the simulation with concave generation function had an expected generation of 14.1% more than the original one, indicating how much the generation can be overestimated for simplifications of the generation function. For the other simulations this number was lower (1.02% and 0.34%), showing that how the station is operated has an impact on how much detail should be included in the generation function. Since it is expected that the market for ancillary services will grow and increased environmental constraints will occur, more operation of power stations at low power output can be more common in the future.

## 6. Conclusion and Further Work

In the given paper a segmented regression approach and a greedy algorithm have been proposed and tested to simplify the piecewise linear generation function used to model hydropower stations. Impacts from environmental constraints and how they alter the operation has been examined, demonstrating the importance of a more detailed modelling of the generation function.

The assumption that the generation function is only dependent on discharge is reasonable for stations with high head and less regulation of the reservoirs. On the contrary, if this is not the case further work will investigate methods that can include the head dimension in the generation function.



### Acknowledgments

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