



Norwegian University of  
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# Collision Avoidance for Autonomous Ships in Transit

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# Preface

This thesis is the result of my work on the compulsory master project at the Department of Engineering Cybernetics during the spring semester of 2018, ending the 5-year program for achieving a Master of Science (MSc) degree in Cybernetics and Robotics at the Norwegian University of Science and Technology. The initiative of this thesis came from ABB Marine and Ports by Dr Matko Barisic, who has been my supervisor during this project.

I would like to thank my supervisors Professor Kristin Y. Pettersen and Dr Matko Barisic for giving me the opportunity to work with this project as my master thesis.

A description of materials made available to me during this project, as well as the help I have received, is found in section 1.2 "Contributions and Background". A description of some of the difficulties which I encountered during the work on this thesis is also included.

Johanna Elisabeth Sandbakken  
*Trondheim, June 2018*



# Problem Description

The task of this assignment is to design an autonomously acting collision avoidance system of a commercial, detailed hydrodynamic model of a passenger ship while steaming. During steaming, the vessel is assumed to be at wide sea, i.e. without hazards to navigation other than ships. The system can assume measurement of the ship's own GPS coordinates, heading, bearing, and speed over ground. The system should be able to plan and execute a safe route circumnavigating other ships according to the international regulations for the avoidance of collisions at sea (COLREGS). The system may assume having perfect knowledge of the other ships' GPS coordinates, heading, and speed over ground from the AIS transponder system within a given range.



# Abstract

The interest in fully autonomous marine vessels has exploded in the last years, and as a result, many companies have developed and tested systems for autonomous marine vessels with great success. This thesis is part of a project where the goal is to develop a control system for large, autonomous marine vessels. The focus points in this thesis are collision avoidance and to follow the International Regulations for Preventing Collision at Sea (COLREGS). The three main scenarios for collision avoidance is overtaking, head-on and crossing situations between the controlled ship and other ships. The essential thing when controlling a large vessel is to take early action. It secures minimal alternating of the heading and the speed, actions that for a large vessel is highly power consuming.

In this thesis, both the Velocity Obstacle and the Virtual Potential Framework are tested as possible collision avoidance methods. Simulations of the three COLREGS situations mentioned above shows that the implemented systems manage to comply with the COLREGS rules and keep the desired safety distance to the obstacle ships. Problems regarding the two methods are highlighted and discussed, and suggestions for further development are given.





# Sammendrag

Interessen for fullt autonome marinefartøy har eksplodert de siste årene, noe som har resultert i stor vekst av selskaper som med stor suksess utvikler og tester systemer for slike marine fartøy. Denne oppgaven er en del av et prosjekt der målet er å utvikle et kontrollsystem for store, autonome skip. Fokuspunktene i avhandlingen er kollisjonsunngåelse og de internasjonale reglene for kollisjonsunngåelse på sjøen (COLREGS). De tre hovedscenariene for kollisjonsunngåelse er forbikjøring, front-mot-front og kryssing mellom det kontrollerte skipet og andre skip. Det viktigste med å kontrollere et stort fartøy er tidlig handling. Det sikrer minimal endring av kurs og hastigheten, handlinger som for et stort fartøy er svært energikrevende.

I denne oppgaven testes både "Velocity Obstacle" (VO) og "Virtual Potential Framework" (VPF) som mulige algoritmer for kollisjonsunngåelse. Simuleringer av de tre ovenfor nevnte COLREGS-situasjonene viser at de implementerte systemene klarer å overholde COLREGS-reglene og beholder ønsket sikkerhetsavstand til hindringsskipene. Problemene med de to metodene er diskutert, og forslag til videreutvikling er gitt.



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# Acronyms

**BODY** BODY frame.

**COLREGS** The International Regulation for Preventing Collisions at Sea.

**CPA** Closest Point of Approach.

**DOF** Degrees of Freedom.

**LOS** Line-of-Sight.

**NED** North, East,Down frame.

**PDF** Probability Density Function.

**RPM** Revolutions Per Minute.

**VO** Velocity Obstacle.

**VPF** Virtual Potential Framework.



# Symbols

$A_o$  Repulsion rating of potential away from obstacle.

$A_r$  Rotor rating of acceleration around the obstacle.

$A_w$  Attraction slope of potential towards the waypoint.

$P_\Sigma(\mathbf{x})$  Total virtual potential at point  $\mathbf{x}$ .

$R_k$  Circle of acceptance radius around waypoint.

$VO_{A|B}$  VO region for vessel A given vessel B.

$\Delta$  Look-ahead distance.

$\beta_a$  Constant bearing on the own vessel.

$\beta_b$  Constant bearing on the obstacle vessel.

$\Theta_{nb}$  Euler angles between  $n$  and  $b$ .

$v_c$  Velocity of current.

$\omega_{b/n}^b$  Angular velocity of  $b$  with respect to  $n$  expressed in  $b$ .

$\chi_d$  Desired course.

- $\chi_p$  Path-tangential angle.
- $\chi_r$  Velocity-path relative angle.
- $\hat{\mathbf{a}}_{rot}(\mathbf{p}_o)$  Direction of the rotor acceleration.
- $\mathbf{E}(\mathbf{x})$  Acceleration vector at point  $\mathbf{x}$ .
- $\mathbf{E}_r(\mathbf{p})$  Acceleration due to the rotor force.
- $\mathbf{E}_s(\mathbf{p})$  Acceleration due to the stator potentials.
- $\mathbf{f}_b^b$  Force about the point  $o_b$  expressed in  $b$ .
- $\mathbf{m}_b^b$  Moment about the point  $o_b$  expressed in  $b$ .
- $\mathbf{p}_{b/n}^n$  Position of point  $o_b$  with respect to  $n$  expressed in  $b$ .
- $\mathbf{v}_{b/n}^b$  Linear velocity of the point  $o_b$  with respect to  $n$  expressed in  $b$ .
- $\mathbf{w}_k$  Next waypoint  $[x_k, y_k]$ .
- $\omega$  Frequency constant in reference model.
- $\psi$  Heading of vessel.
- $\mathbf{C}(\mathbf{v})$  Coriolis and centripetal matrix.
- $\mathbf{D}(\mathbf{v})$  Damping matrix.
- $\mathbf{M}$  System inertia matrix.
- $\zeta$  Damping constant in reference model.
- $a_{rot}(\mathbf{p}_o)$  Magnitude of the rotor acceleration.
- $c_c(s)$  Curvature of the path.
- $c_i$  Cost of velocity vector  $i$ .
- $d$  Distance between two vessels.



$d_r$  Rotor force radius.

$d_w$  Distance to waypoint (outside this radius the acceleration towards the waypoint is constant).

$d_{min}$  Safety distance of vessel.

$e$  Error length from optimal path.

$r_d$  Desired yaw rate.

$r_d$  Safety distance.

$r_{sat}$  Maximal yaw rate of vessel.

$s$  Along-track distance.

$t_{max}$  Maximal time to CPA that still counts as a possible collision.

$u_d$  Desired surge speed.

$x_b$  x position in BODY frame.

$x_n$  x position in NED frame.

$y_b$  y position in BODY frame.

$y_n$  y position in NED frame.

$\eta$  Position and angle vector.

$\mathbf{v}$  Linear and angular velocity vector.

$\tau$  Force and moment vector.

$\mathbf{R}(\psi)$  Rotation matrix.



# Chapter 1

## Introduction

### 1.1 Motivation

The largest part of all world trade is today transported using marine shipping (Kim et al. (2015)), and a vast number of other marine services, such as passenger transportation and fishing boats, makes the oceans highly populated with marine vessels. A collision between vessels at sea is a major disaster and will lead to gigantic destruction and cost, and it occurs more often due to the growth in the number of ships, larger size, and higher speed. One of the greatest causes of collision originates in human errors and misjudgements (Zeng (2003), Kim et al. (2015), Statheros et al. (2008), Ziarati (2006)). Because of the high number of vessels at sea, it is impossible to plan collision-free routs for all the ships travelling the same waters (Kim et al. (2015)). To improve the marine industry one is now looking to autonomous ships, which will both prevent human errors and lower the cost due to minimisation of crew members on board, and optimisation of the paths travelled.

In the 80's the first research about path following algorithms was done for marine surface vessels, called track-keeping methods. Before this, course-keeping methods

were used, providing no compensation for weather disturbances that caused drifting without doing so manually (Zhang et al. (1996)). When track-keeping algorithms were developed for vessels operating on rivers and water channels, the system needed to be very precise to avoid any sideslip (Sandler et al. (1996), Wahl and Gilles (1998)), but also along the coast and in transit, precise path-following is useful as it provides the possibility for travelling along optimal paths and it improves safety (Messer and Grimble (1993), Morawski and Pomirski (1998), Encarnacao et al. (2000)). To generate the safe paths, collision avoidance systems are needed. Different strategies have been presented and tested, for instance using communication between ships (Kim et al. (2015)) and game theory (Lisowski (2014)). A fully autonomous collision avoidance system needs to operate without variation due to weather condition, the onboard equipment or the surrounding obstacles, and it needs to behave according to the International Regulations for the Avoidance of Collision at Sea (COLREGS).

Most often the map of the global space is neither static, a priori nor perfectly known. Therefore an active path planner that updates the best path while travelling is essential. This thesis is investigating two active approaches to the collision avoidance problem for vessels operating in transit. The first method is combining the Line-of-Sight (LOS) path-following algorithm and the Velocity Obstacle (VO) collision avoidance algorithm. The second method is the Virtual Potential Framework (VPF).

## 1.2 Contributions and Background

The main contributions of this thesis:

- Creating a low level controller for the ship model provided by ABB Marine and Ports.
- An analysis of the Velocity Obstacle (VO) algorithm as a collision avoidance method following COLREGS.
- An analysis of the Virtual Potential Framework (VPF) as a collision avoidance method, and adapt it to follow COLREGS.

- An evaluation of COLREGS as a regulation for autonomous vessels.

Following is a description of which materials that were made available to me during the work on this thesis, the modifications I have done to it and the help I have received, as well as some of the difficulties which I encountered:

The ship model in Simulink that is used in this thesis was provided to me by Mr Philipp Nguyen, responsible for the ship models in ABB Marine and Ports Finland. The documentation for this ship model is none-existing, so several weeks was used to understand the model and make it cooperate with the rest of the code. This model is the property of ABB Marine and Ports and is therefore only vaguely described in this thesis.

The Matlab code for the Potential Force Framework was provided to me by my supervisor Dr Matko Barisic, which he implemented during his doctor thesis (Barisic (2012)). The code needed to be modified a lot to work together with Simulink, and I have adapted the code to make it possible to change the global space during the simulation, so that the obstacles can move like ships, and changed how the potential and rotor force is applied.

The use of the Velocity Obstacle algorithm as a collision avoidance method was suggested to me by my supervisor Professor Kristin Y. Pettersen and the Matlab code for the Line-of-Sight and the Velocity Obstacle algorithms were roughly implemented by myself in a pre-project to this thesis and was further developed during this thesis.

During guidance meetings with Dr Barisic, he has given me the information about what is standard practice for regulating heading and speed of a marine vessel, how small run time that is needed in the controllers of a ship, the information about the thrusters in the ship mode, as well as introducing me to the speed-adaptation algorithm through the PhD of Dr Bibuli (Bibuli (2010)).

Since the ship model started out as a black box, I tried to find a suitable low-level controller and tune it based on measurements of the output from the ship model. This

was very time consuming and didn't go very well. A lot of time was then used to understand a bit of the model, making the trying and failing with the controller a bit more reasoned. 11 weeks, out of 20, into the project I had a functioning low-level controller, and I could finally start concentrating on investigating the collision avoidance algorithms.

Since I have learned that in a theoretical thesis like this it is unprofessional to use the word "I", and to me, it feels weird to use the word "we" about myself, everything in this thesis that is stated without having a reference added is solely coming from myself. Following is a list of the things that I have done entirely on my own:

- Introducing the fifth COLREGS situation,
- Creating the "shortest turn" calculations,
- Deciding the possible velocity vectors that the VO algorithm can choose from
- Figuring out how the rotor force should be applied
- Suggesting to introduce "breaking velocities" to the VO algorithm
- Introducing that avoiding collision is more important than to follow the COLREGS rules
- Introducing the problem occurring from the rotor force only being applied to the half side of the obstacle facing away from the waypoint
- Suggesting a look-out angle for the VPF
- Finding suggestings to the s-functions described in future work
- Finding the parameter values to the following:
  - reference model
  - PD and PI regulator
  - look-ahead distance

- radius of circle of acceptance
- closest point of approach
- safety distance
- cost function
- potential contour and rotor force

### **1.3 Assumptions**

The assumptions that are made in this thesis are:

- The vessel is in transit (open ocean), with only other vessels as possible obstacles
- All vessels are motorised ships
- The vessels position, heading and speed are always known
- The vessels operates in 2D space (x,y)

### **1.4 Outline**

After this introductory chapter, the outline of the thesis is as follows: Chapter 2 provides a literature study of active path planners. Chapter 3 presents the necessary theoretical background and chapter 4 explains the implementation of the system. In chapter 5, the results from the simulated scenarios are given. A discussion of the challenges is given in chapter 6 and possible extensions to the system in chapter 7. A conclusion is given in chapter 8.





# Chapter 2

## Literature Review

You can divide the path planning algorithms into two categories: The reactive and the active. The reactive path planners calculate the best path before executing the travel. The active path planners update the best path while travelling along the path, which is essential for a path planner that needs to handle moving obstacles. This chapter describes a selection of active path planners.

### 2.1 Virtual Force Field

Borenstein and Koren (1989) introduce the use of Virtual Force Field (VFF) as a collision avoidance algorithm for mobile robots. This method virtualize forces that drag the vessel towards the next waypoint and pushes it away from the obstacles. The desired force,  $F_d$ , that leads the vessel is shown in equation (2.1) and figure 2.1.

$$F_d = F_a + F_r \quad (2.1)$$

where  $F_a$  is the force pulling the vessel towards the next waypoint and  $F_r$  is the force pushing the vessel away from the obstacle.

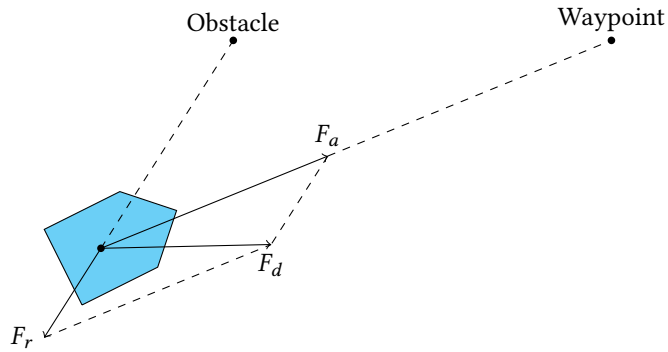


Figure 2.1: Virtual Force Field

Lee et al. (2004) introduce a track keeping force to the VFF. A force normal to a path will, in addition to making it possible for the vessel to follow the path, handle error due to environmental forces (Statheros et al. (2008)), such as wind, waves, and current. The Modified Force Field is shown in figure 2.2.

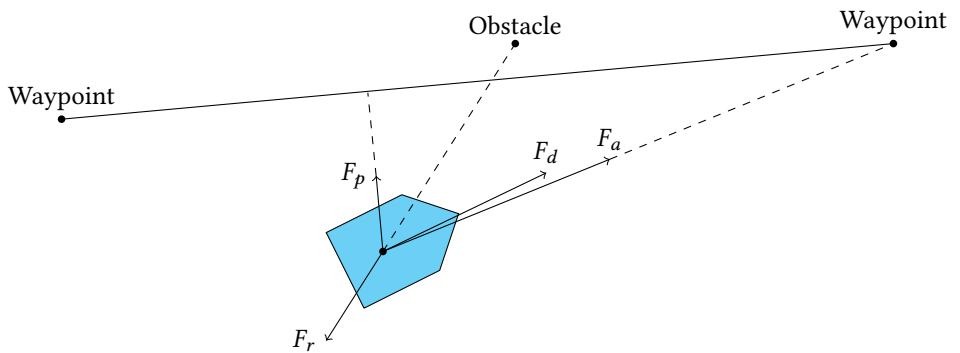


Figure 2.2: Modified Virtual Force Field

## 2.2 Artificial Potential Field

Naeem et al. (2016) is using the Artificial Potential Field (APF) Framework as a method for collision avoidance. The APF generates a framework with positive and negative charged objects. All the vessels will be positively charged, resulting in a potential force pushing the vessels away from each other, and the controlled vessel will be moving towards the waypoint which is negatively charged. The attraction potential that moves the controlled vessel towards the waypoint increases with the distance between the vessel and the waypoint. Here, the attraction potential is implemented as a quadratic function:

$$U(p)_{att} = \frac{1}{2}K_G d_G^2(p, p_{goal}) \quad (2.2)$$

where

$U(p)_{att}$  is the magnitude of the attractive potential field

$K_G$  is the attractive gain constant

$d_G$  is the Euclidean distance between the vessel and the waypoint.

$p$  is the position of the vessel

$p_{goal}$  is the position of the goal

With a quadratic function, the discontinuity problem at the singularity will be avoided, because the field space will have a parabolic space.

The repulsive potential that moves the controlled vessel away from the obstacle vessels increase as the distance between them decrease, and is non existing outside a predefined radius  $D_o$  around the obstacle:

$$U(p)_{rep} = \begin{cases} \frac{1}{2}K_{ob} \left( \frac{1}{d(p, p_{obs})} - \frac{1}{D_o} \right), & \text{if } d \leq D_o \\ 0, & \text{if } d > D_o \end{cases} \quad (2.3)$$

where

$U(p)_{rep}$  is the magnitude of the repulsive potential field

$K_{ob}$  is the repulsive gain constant

$d$  is the Euclidean distance between the vessel ans

$p_{obs}$  is the position of the obstacle

## 2.3 Distributed Local Search Algorithm

Kim et al. (2015) are using Distributed Local Search Algorithm (DLSA) to prevent collision between cooperating ships. All the neighbouring ships will calculate its own collision risk based on its own and the neighbour's information. DLSA allows the ship that can minimize the collision risk the most to first choose its next course. The problem with DLSA is that it can get stuck in a Quasi-Local Minimum (QLM). To solve this problem, Distributed Tabu Search (DTS) was developed, based on the Tabu Search algorithm by Glover (1986). Tabu Search will make a list of prohibited moves to counteract the system to return to an already visited state. This prevents the system to get trapped in a QLM. Including DTS in the cost minimization will also minimize the path to the given ship's destination point.

## 2.4 Case-based Reasoning Problem

Liu et al. (2008) investigates the possibility to use the experience from previous situations to solve a potential collision, a case-based reasoning problem. All the earlier cases are stored in a case base, which contains information about weather conditions, own position, velocity and heading, other ships relative to own ship and suggested actions, all for each timestep. When a new possible collision occurs, the case base is searched to find the most similar case. If this case is not similar enough to solve the current problem, the suggested case solution is modified to handle the new situation. If this new, modified solution manages to handle the collision problem perfectly, the case can be added to the case base.

## 2.5 Game Theory

Lisowski (2014) presents a solution using game theory to avoid collisions. The very basic of game theory is that each player wants to maximize own gain. To be able to do so, one needs to consider the worst-case scenario that the other players can choose (Dobrescu et al. (2017)). Looking at scenarios with up to ten obstacle ships, Lisowski calculates the collision risk with each ship. Solving this dual problem, it is assumed that the obstacle ship will choose a strategy that maximizes the collision risk (worst case scenario), while the own ship wants to minimize that risk. The velocity vector to follow is calculated by:

$$I_0^j = \min_{v_0} \max_{v_j} r_j \quad (2.4)$$

where  $r_j$  is the collision risk between the own ship and the obstacle ship  $j$ ,  $v_0$  is all the velocity vectors that the own ship can choose from, and  $v_j$  is the velocity vectors that the obstacle ship  $j$  can choose from. The collision risk  $r_j$  depends on the distance and time to the closest point of approach (CPA) (explained in chapter 3.5), the distance between the ships at the current point and the safety distance. The different factors are weighted depending on different circumstances: if the visibility is good or poor, if the water region is open or restricted, and speed, length, and beam of the ship. Of all the risks that are calculated for the available velocity vectors, the relation to the obstacle ship with the highest risk will influence the choice of velocity vector the most.



# Chapter 3

## Method

### 3.1 Coordinate Systems

For a marine surface vessel, one can assume small roll and pitch angles as well as constant height along the  $z$ -axis. Doing so gives a 2D problem that only depends on the surge, sway, and yaw, i.e. three Degrees of Freedom (DOF) (table 3.1). The different coordinate frames used in this project are the BODY frame and the NED frame:

The BODY frame is fixed to the body of the vessel, with the  $x$ -axis pointing straight forward and the  $y$ -axis pointing to the right (starboard). The  $z$ -axis is pointing down, but in this project, it is assumed that the height along the  $z$ -axis is constant and is therefore neglected. In figure 3.1, the BODY frame is spanned by the  $x_b$  and  $y_b$  vectors.

The NED frame, short for North, East, Down, lies as a tangent plane on the earth surface spanned by the  $x$  and  $y$ -axis. The  $x$ -axis points towards the True North and the  $y$ -axis points East. The  $z$ -axis points down, but is assumed constant and is therefore neglected. In figure 3.1, the NED frame is spanned by the  $x_n$  and  $y_n$  vectors.

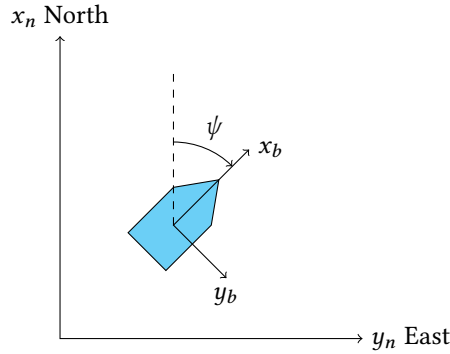


Figure 3.1: BODY frame spanned by  $x_b$  and  $y_b$ . NED frame spanned by  $x_n$  and  $y_n$ .  $\psi$  is the heading of the vessel.

The relationship between the BODY and NED frame in a 2D space is the heading angle of the body, the yaw angle  $\psi$ , as seen in figure 3.1.

## 3.2 Motion Variables

The following motion variables are vectors describing the position, speed and forces in linear and angular direction, expressed in the BODY frame,  $b$ , and the NED frame,  $n$  (Fossen (2011)):

$\mathbf{p}_{b/n}^n$  is the position of the point  $o_b$  with respect to  $n$  expressed in  $n$

$\mathbf{v}_{b/n}^b$  is the linear velocity of the point  $o_b$  with respect to  $n$  expressed in  $b$

$\Theta_{nb}$  is the Euler angles between  $n$  and  $b$

$\boldsymbol{\omega}_{b/n}^b$  is the angular velocity of  $b$  with respect to  $n$  expressed in  $b$

$\mathbf{f}_b^b$  is the force with line of action through the point  $o_b$  expressed in  $b$

$\mathbf{m}_b^b$  is the moment about the point  $o_b$  expressed in  $b$



$$\begin{array}{ll}
\text{NED position: } \mathbf{p}_{b/n}^n = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 & \text{Body-fixed linear velocity: } \mathbf{v}_{b/n}^b = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \in \mathbb{R}^3 \\
\text{Euler angles: } \boldsymbol{\theta}_{nb} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \in \mathbb{S}^3 & \text{Body-fixed angular velocity: } \boldsymbol{\omega}_{b/n}^b = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \in \mathbb{R}^3 \\
\text{Body-fixed force: } \mathbf{f}_b^b = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \in \mathbb{R}^3 & \text{Body-fixed moment: } \mathbf{m}_b^b = \begin{bmatrix} K \\ M \\ N \end{bmatrix} \in \mathbb{R}^3
\end{array}$$

where  $\mathbb{R}^3$  is the three dimensional Euclidean space and  $\mathbb{S}^3$  is the three angles defined on the interval  $[0, 2\pi]$  ( $\mathbb{S}^3$  is then shaped as a sphere).

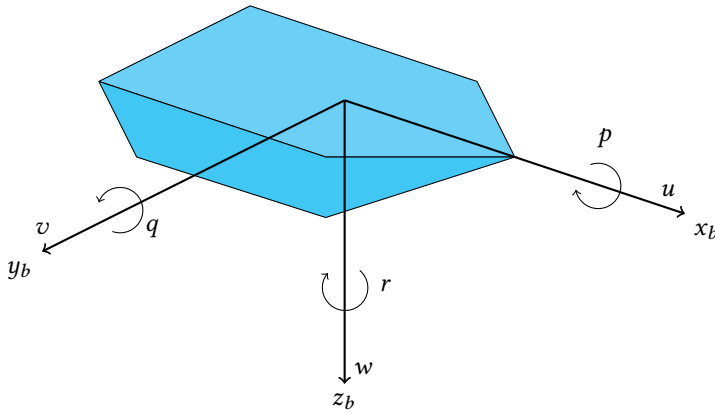
Grouping the position, speed and force vectors together gives the following notation:

$$\boldsymbol{\eta} = \begin{bmatrix} \mathbf{p}_{b/e}^n \\ \boldsymbol{\Theta}_{nb} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} \mathbf{v}_{b/n}^b \\ \boldsymbol{\omega}_{b/n}^b \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}, \quad \boldsymbol{\tau} = \begin{bmatrix} \mathbf{f}_b^b \\ \mathbf{m}_b^b \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \\ K \\ M \\ N \end{bmatrix} \quad (3.1)$$

The notation for marine vessels by SNAME (1950) is given in table 3.1, and figure 3.2 shows how the linear and angular velocities are defined in space.

Table 3.1: The notation by SNAME (1950) for marine vessels

| DOF |                                    | Forces and moments | Linear and angular velocities | Position and Euler angles |
|-----|------------------------------------|--------------------|-------------------------------|---------------------------|
| 1   | Motions in the x-direction (surge) | X                  | u                             | x                         |
| 2   | Motions in the y-direction (sway)  | Y                  | v                             | y                         |
| 3   | Motions in the z-direction (heave) | Z                  | w                             | z                         |
| 4   | Rotation about the x-axis (roll)   | K                  | p                             | $\phi$                    |
| 5   | Rotation about the y-axis (pitch)  | M                  | q                             | $\theta$                  |
| 6   | Rotation about the z-axis (yaw)    | N                  | r                             | $\psi$                    |

Figure 3.2: The linear velocities  $u$ ,  $v$  and  $w$ , and the angular velocities  $p$ ,  $q$  and  $r$ , in the BODY frame  $(x_b, y_b, z_b)$ .

### 3.3 Equation of Motion

#### 6 DOF Ship Model

The body of an object that can move along and rotate around all three axes in space has 6 DOF. The equation of motion expressed in BODY space with 6 DOF is given in

equation 3.2 (Fossen (2011)).

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \mathbf{J}_{\Theta}(\boldsymbol{\eta})\boldsymbol{v} \\ \mathbf{M}\dot{\boldsymbol{v}} + \mathbf{C}(\boldsymbol{v})\boldsymbol{v} + \mathbf{D}(\boldsymbol{v})\boldsymbol{v} + \mathbf{g}(\boldsymbol{\eta}) + \mathbf{g}_0 &= \boldsymbol{\tau} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{wave} \end{aligned} \quad (3.2)$$

where

$$\begin{aligned} \mathbf{M} &= \mathbf{M}_{RB} + \mathbf{M}_A \\ \mathbf{C}(\boldsymbol{v}) &= \mathbf{C}_{RB}(\boldsymbol{v}) + \mathbf{C}_A(\boldsymbol{v}) \\ \mathbf{D}(\boldsymbol{v}) &= \mathbf{D} + \mathbf{D}_n(\boldsymbol{v}) \end{aligned} \quad (3.3)$$

and

$$\mathbf{J}_{\Theta}(\boldsymbol{\eta}) = \begin{bmatrix} \mathbf{R}_b^n(\Theta_{nb}) & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{T}_{\Theta}(\Theta_{nb}) \end{bmatrix}, \text{ where } \mathbf{R}_b^n(\Theta_{nb}) \text{ is a rotation matrix around all three axes and } \mathbf{T}_{\Theta}(\Theta_{nb}) \text{ is a translation matrix along all three axes,}$$

$\mathbf{M}$  is the system inertia matrix, where  $\mathbf{M}_{RB}$  is the inertia due to the mass of the body and  $\mathbf{M}_A$  is the inertia due to the added mass (the inertia of the surrounding fluid),

$\mathbf{C}(\boldsymbol{v})$  is the Coriolis and centripetal matrix, where  $\mathbf{C}_{RB}(\boldsymbol{v})$  is the Coriolis and centripetal effect due to the body and  $\mathbf{C}_A(\boldsymbol{v})$  is the effect due to the added mass (rotation of  $b$  with respect to  $n$ ),

$\mathbf{D}(\boldsymbol{v})$  is the damping matrix where  $\mathbf{D}$  is the linear damping matrix caused by potential damping and possible viscous friction, and  $\mathbf{D}_n(\boldsymbol{v})$  is the nonlinear damping matrix due to quadratic viscous damping,

$\mathbf{g}(\boldsymbol{\eta})$  is the vector of gravitational/buoyancy forces and moments,

$\mathbf{g}_0$  is the vector used for pre-trimming (ballast control),

$\boldsymbol{\tau}$  is the vector of control input,

$\boldsymbol{\tau}_{wind}$  is the vector of wind forces,

$\boldsymbol{\tau}_{wave}$  is the vector of wave-induced forces.

The influence caused by the current is included by adding the current velocity  $\mathbf{v}_c$  to the body velocity:

$$\mathbf{v}_r = \mathbf{v} + \mathbf{v}_c \quad (3.4)$$

For more information about the equation of motion in 6 DOF see Chapter 6 in Fossen (2011).

### 3 DOF Ship Model

For a surface vessel, the equation of motion is simplified by assuming no vertical heave speed and no pitch or roll rotations. The model has then only 3 DOF left, and is only able to manoeuvre in the horizontal plane. A marine surface vessel modelled by 3 DOF can have the following equation of motion (Fossen (2011)):

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \mathbf{R}(\psi)\mathbf{v} \\ \mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} &= \boldsymbol{\tau} \end{aligned} \quad (3.5)$$

where

$$\boldsymbol{\eta} = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} u \\ v \\ r \end{bmatrix}, \quad \mathbf{R}(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.6)$$

## 3.4 Path Following

### 3.4.1 Line-of-Sight Guidance Method

One method to guide a vehicle along a straight path between two waypoints is the Line-of-Sight (LOS) guidance method (Fossen (2011)). Figure 3.3 shows the LOS problem, where  $\mathbf{w}_{k-1} = [x_{k-1}, y_{k-1}]$  is the last waypoint and  $\mathbf{w}_k = [x_k, y_k]$  is the next.  $\chi_d$  is the desired heading,  $\chi_r$  is the error angle,  $\chi_p$  is the angle of the path,  $e$  is the error length from the optimal path and  $\Delta$  is the predefined look-ahead distance. Equation 3.7 to 3.9 shows the LOS implementation.

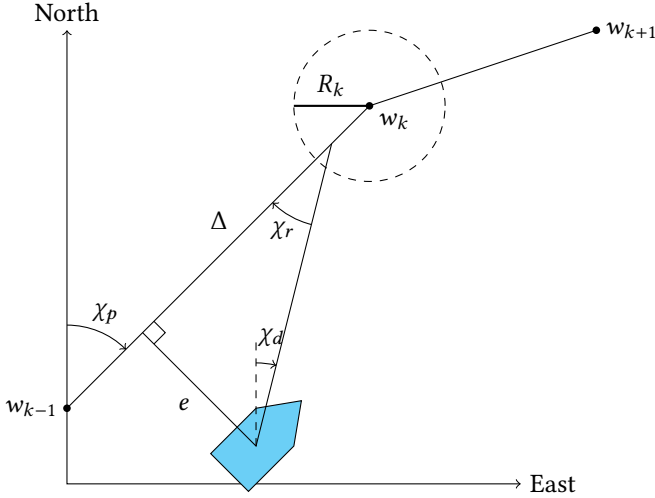


Figure 3.3: Line-of-Sight guidance between waypoints

Desired course:

$$\chi_d(e) = \chi_p + \chi_r(e) \quad (3.7)$$

Path-tangential angle:

$$\chi_p = \tan^{-1} \left( \frac{y_k - y_{k-1}}{x_k - x_{k-1}} \right) \quad (3.8)$$

Velocity-path relative angle:

$$\chi_r(e) = \tan^{-1} \left( \frac{-e}{\Delta} \right) \quad (3.9)$$

In case of any disturbance keeping the error length  $e$  from being zero, the velocity-path relative angle can be computed using integral action (Fossen (2011)):

$$\chi_r(e) = \tan^{-1} \left( -K_p e - K_i \int_0^t e(\tau) d\tau \right) \quad (3.10)$$

The integral action eliminates constant disturbances such as current forces.

### 3.4.2 Circle of Acceptance

To evaluate whether the vessel has reached the waypoint or not the circle of acceptance can be used (Fossen (2011)):

$$(x_k - x(t))^2 + (y_k - y(t))^2 \leq R_k^2 \quad (3.11)$$

where  $[x(t), y(t)]$  is the position of the vessel. Whenever the vessel is within a specified radius  $R_k$  of the waypoint, the waypoint is marked as reached, and the vessel will start to follow the path towards the next waypoint.

### 3.4.3 Speed-Adaptation

The goal of this speed-adaptation, taken from Bibuli (2010), is to reduce the sway effect from sharp turns. Because of the low friction in water, a vessel drifts sideways when altering its heading. Equation (3.12) gives the desired surge speed,  $u_d$ , which reduces the speed when the desired yaw-rate is high, and therefore the desired heading is reached faster.

$$u_d = \frac{U_{max} - U_{min}}{2} + \frac{U_{max} - U_{min}}{2} \left[ \cos \left( \frac{\pi r^*}{r_{sat}} \right) \right] \quad (3.12)$$

$U_{max}$  is the highest surge speed of the vessel,  $U_{min}$  is the lowest surge speed that still makes the vessel able to manoeuvre, and  $r_d$  is the yaw rate requested by the controller.  $U_{min}$ ,  $U_{max}$  and  $r_{sat}$  (the maximum applicable yaw-rate reference value) are the parameters of this adaptation law.

## 3.5 Closest Point of Approach

When two vessels meet out on the open ocean, the first thing to consider is whether they will collide or not. If both vessels keep their current speed and heading, the Closest Point of Approach (CPA) for vessel  $A$  and  $B$ , with positions  $\mathbf{p}_A$  and  $\mathbf{p}_B$ , and velocities  $\mathbf{v}_A$  and  $\mathbf{v}_B$ , can be found using equation (3.13) and (3.14). Equation (3.15)

defines the conditions for a collision situation (Stenersen (2015)).

The time to CPA:

$$t_{CPA} = \frac{(\mathbf{p}_B - \mathbf{p}_A)(\mathbf{v}_A - \mathbf{v}_B)}{\|\mathbf{v}_A - \mathbf{v}_B\|^2} \quad (3.13)$$

The distance between vessels at CPA:

$$d_{CPA} = \|(\mathbf{p}_A + \mathbf{v}_A t_{CPA}) - (\mathbf{p}_B + \mathbf{v}_B t_{CPA})\| \quad (3.14)$$

The vessels are in a collision situation if:

$$0 \leq t_{CPA} \leq t_{max} \quad \wedge \quad d_{CPA} \leq d_{min} \quad (3.15)$$

where  $t_{max}$  and  $d_{min}$  are predefined constants, and  $d_{min}$  equals the safety distance for the vessels.  $t_{max}$  sets the time distance for how early the vessel will act to solve a potential collision.

## 3.6 Velocity Obstacle

The Velocity Obstacle (VO) method divides all the possible velocity vectors that a vessel can take into those who will lead to a collision and those who will not. The set of velocity vectors that will lead to a collision lies inside a cone-like area which is located around the point where the other vessel will appear in the nearest future (see figure 3.4). All the velocity vectors that do not lie inside of this cone will not lead to a collision and is safe to choose. The VO region for vessel A given vessel B,  $VO_{A|B}$ , will be as follows (Stenersen (2015)):

$$VO_{A|B} = \{\mathbf{v}_A | \mathbf{v}_{BA} \frac{1}{\mathbf{p}_{norm,1}} \geq 0 \wedge \mathbf{v}_{BA} \frac{1}{\mathbf{p}_{norm,2}} \geq 0\} \quad (3.16)$$

where  $\mathbf{v}_{BA}$  is the velocity vector between the vessels:

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A \quad (3.17)$$

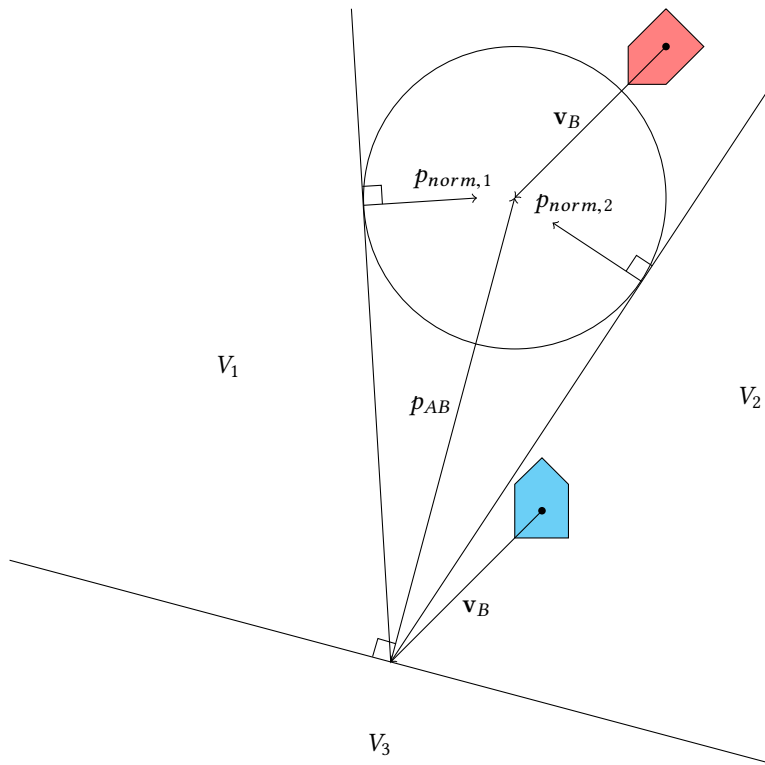


Figure 3.4: Velocity Object regions



and  $\mathbf{p}_{norm,1}$  and  $\mathbf{p}_{norm,2}$  are the normal vectors to the lines making the VO cone, shown in figure 3.4:

$$\mathbf{p}_{norm,1} = \mathbf{R}\left(-\alpha + \frac{\pi}{2}\right) \frac{\mathbf{p}_{AB}}{\|\mathbf{p}_{AB}\|} \quad (3.18)$$

$$\mathbf{p}_{norm,2} = \mathbf{R}\left(\alpha - \frac{\pi}{2}\right) \frac{\mathbf{p}_{AB}}{\|\mathbf{p}_{AB}\|} \quad (3.19)$$

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (3.20)$$

$$\alpha = \sin^{-1}\left(\frac{\max(r_{s,A} + r_{s,B})}{\|\mathbf{p}_{AB}\|}\right) \quad (3.21)$$

where  $r_{s,a}$  and  $r_{s,b}$  are the safety distance for the two vessels. The largest of these safety distances gives the radius of the circle in figure 3.4. The VO region is located inside the cone in figure 3.4.

The three other regions in figure 3.4,  $V_1$ ,  $V_2$  and  $V_3$ , are determined by:

$$V_1 = \{\mathbf{v}_A | \mathbf{v}_A \notin (VO_{A|B} \vee V_3) \wedge [\mathbf{p}_{AB} \times \mathbf{v}_{BA}]_z < 0\} \quad (3.22)$$

$$V_2 = \{\mathbf{v}_A | \mathbf{v}_A \notin (VO_{A|B} \vee V_3 \vee V_1)\} \quad (3.23)$$

$$V_3 = \{\mathbf{p}_{AB} \cdot \mathbf{v}_A < 0\} \quad (3.24)$$

where  $[\ ]_z$  denotes the z component of the vector.

### 3.7 Virtual Potential Framework

The Virtual Potential Framework (VPF) (Barisic (2012)) is built up by potentials that push the vessel away from the obstacles and towards the waypoints. The VPF is expressed as a finite sum over all the potentials that affects the vessel, and the acceleration

vector at point  $\mathbf{x}$  is found by the potentials derivative:

$$P_{\Sigma} = \sum_{j=1}^n P_j \quad (3.25)$$

$$\mathbf{E}(\mathbf{x}) = -\nabla P_{\Sigma}(\mathbf{x}) \quad (3.26)$$

where

$P_j$  is the virtual potential contribution from object  $j$ ,

$n$  is the number of objects, i.e. obstacles and waypoints, that affects the vessel,

$P_{\Sigma}(\mathbf{x})$  is the total virtual potential at point  $\mathbf{x}$ ,

$\mathbf{E}(\mathbf{x})$  is the acceleration vector at point  $\mathbf{x}$ .

By defining  $\mathbb{C}$  as the subspace where all the obstacles are excluded, calling this the navigable water space, it means that  $\mathbf{E}(\mathbb{C})$  is the vector field that includes all the accelerations for all points in  $\mathbb{C}$ .

The VPF is created by stator potentials, which generates a potential contour in the water space, and rotational forces, which prevents the vessel from being trapped in a local minimum. The acceleration vector that affects the vessel with position  $\mathbf{p}$  is given by:

$$\mathbf{E}(\mathbf{p}) = \mathbf{E}_s(\mathbf{p}) + \mathbf{E}_r(\mathbf{p}) \quad (3.27)$$

where  $\mathbf{E}_s(\mathbf{p})$  is the acceleration due to the stator potentials and  $\mathbf{E}_r(\mathbf{p})$  is the acceleration due to the rotor force.

### 3.7.1 Potential Contour Generator

The potential contour includes accelerations depending on the waypoints and the obstacles:

$$\mathbf{E}_s(\mathbf{p}) = \mathbf{a}_{wp}(\mathbf{p}) + \mathbf{a}_{obst}(\mathbf{p}) \quad (3.28)$$

as seen in figure 3.5.

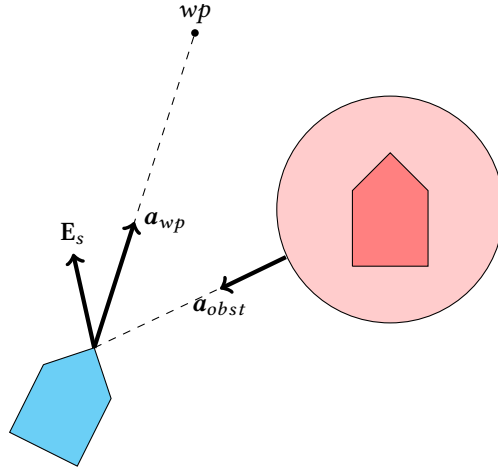


Figure 3.5: Acceleration vectors from obstacle and waypoint contributing to the potential contour

### Waypoint

To make the vessel approach the next waypoint the potential contour generator is creating an acceleration towards it. This acceleration is linearly decreasing as the distance,  $d$ , between the vessel and the waypoint increase. Outside radius  $d_w$  of the waypoint, the acceleration is constant, and will not go towards infinity as  $d$  increases. A linear decrease in the acceleration gives an asymptotically decrease in the potential. To meet the requirement mentioned above the potential of the waypoint is defined as:

$$p_{wp}(d) = \begin{cases} \frac{A_w}{2d_w} d^2, & \text{if } d \leq d_w \\ A_w d - \frac{A_w d_w}{2}, & \text{if } d > d_w \end{cases} \quad (3.29)$$

$$\frac{\partial}{\partial d} p_{wp}(d) = \begin{cases} \frac{A_w}{d_w} d, & \text{if } d \leq d_w \\ A_w, & \text{if } d > d_w \end{cases} = \min \left( \frac{A_w}{d_w} d, A_w \right) \quad (3.30)$$

where  $A_w$  is the attraction slope of the waypoint. This fulfils the requirements to the acceleration because:

$$\begin{aligned}
 \lim_{d \rightarrow \infty} p_{wp}(d) &= \infty \\
 \lim_{d \rightarrow 0^+} p_{wp}(d) &= 0 \\
 \lim_{d \rightarrow \infty} \frac{\partial}{\partial d} p_{wp}(d) &= A_w \\
 \lim_{d \rightarrow 0^+} \frac{\partial}{\partial d} p_{wp}(d) &= 0
 \end{aligned} \tag{3.31}$$

## Obstacle

To prevent the vessel from colliding with obstacles the potential contour generator creates an acceleration away from them. The acceleration is linearly increasing towards infinity as the distance,  $d$ , between the vessel and the obstacles, becomes zero. Likewise, the acceleration will decrease to zero as  $d$  increase towards infinity. The potential of an obstacle is therefore defined as:

$$p_{obst}(d) = \exp\left(\frac{A_o}{d}\right) - 1 \tag{3.32}$$

$$\frac{\partial}{\partial d} p_{obst}(d) = -\frac{A_o}{d^2} \exp\left(\frac{A_o}{d}\right) \tag{3.33}$$

where  $A_o$  is the repulsion rating of the obstacle. This fulfils the requirements to the acceleration because:

$$\begin{aligned}
 \lim_{d \rightarrow \infty} p_{obst}(d) &= 0 \\
 \lim_{d \rightarrow 0^+} p_{obst}(d) &= \infty \\
 \lim_{d \rightarrow \infty} \frac{\partial}{\partial d} p_{obst}(d) &= 0 \\
 \lim_{d \rightarrow 0^+} \frac{\partial}{\partial d} p_{obst}(d) &= \infty
 \end{aligned} \tag{3.34}$$

### 3.7.2 Rotor Force

The rotor force is also a force from the obstacle on the vessel. It is perpendicular to the normal of the obstacle surface and will guide the vessel around the obstacle the way that is closest to the next waypoint (figure 3.6). This acceleration is applied when the distance between the vessel and the obstacle is less than some radius  $d_r$ . The acceleration is defined as:

$$\begin{aligned}
 \mathbf{a}(\mathbf{p}_o) &= a_{rot}(\mathbf{p}_o)\hat{\mathbf{a}}_{rot}(\mathbf{p}_o) \\
 a_{rot}(\mathbf{p}_o) &= -\frac{A_r}{d^2} \exp\left(\frac{A_r}{d}\right) \\
 \hat{\mathbf{a}}_{rot}(\mathbf{p}_o) &= \hat{\mathbf{r}}(\mathbf{p}_o) \times \hat{\mathbf{n}}_{vessel}(\mathbf{p}_o) \\
 r(\mathbf{p}_o) &= \hat{\mathbf{n}}_{wp}(\mathbf{p}_o) \cdot \hat{\mathbf{n}}_{vessel}(\mathbf{p}_o) \\
 \hat{\mathbf{r}}(\mathbf{p}_o) &= \begin{cases} [0 & 0 & -1]^T, & \text{if } r(\mathbf{p}_o) = 0 \\ \text{sign}[\hat{\mathbf{n}}_{vessel}(\mathbf{p}_o) \times \hat{\mathbf{n}}_{wp}(\mathbf{p}_o)], & \text{otherwise} \end{cases} \quad (3.35) \\
 \hat{\mathbf{n}}_{wp}(\mathbf{p}_o) &= \left[ \frac{\mathbf{w}_k - \mathbf{p}_o}{\|\mathbf{w}_k - \mathbf{p}_o\|} \middle| 0 \right]^T \\
 \hat{\mathbf{n}}_{vessel}(\mathbf{p}_o) &= \left[ \frac{\mathbf{p} - \mathbf{p}_o}{\|\mathbf{p} - \mathbf{p}_o\|} \middle| 0 \right]^T
 \end{aligned}$$

where

$A_r$  is the rotor rating, a predefined scale parameter,

$a_{rot}(\mathbf{p}_o)$  is the magnitude of the rotor acceleration,

$\hat{\mathbf{a}}_{rot}(\mathbf{p}_o)$  is the direction of the rotor acceleration (clockwise or counter-clockwise around the obstacle),

$\mathbf{w}_k$  is the current waypoint,

$\mathbf{p}_o$  is the position of the obstacle,

$\mathbf{p}$  is the position of the vessel,

$r(\mathbf{p}_o)$  is the rotor direction discriminator, a value used to resolve the case when the vector cross-product returns  $\mathbf{0}$ , meaning the two unit vectors are parallel, and that the vessel is on the very opposite side of the obstacle compared to the waypoint,

$\hat{\mathbf{r}}(\mathbf{p}_o)$  is the unit rotor direction generator, being either  $[0 \ 0 \ 1]^T$  (clockwise) when the waypoint is closest on the left side of the obstacle, or  $[0 \ 0 \ -1]^T$  (counter-clockwise) when the waypoint is closest on the right side of the obstacle,

$\hat{\mathbf{n}}_{vessel}(\mathbf{p}_o)$  is the unit vector in the outward normal direction of the obstacle, pointing towards the vessel,

$\hat{\mathbf{n}}_{wp}(\mathbf{p}_o)$  is the unit vector in the outward normal direction of the obstacle, pointing towards the waypoint.

See Barisic, Vukic and Miskovic (2008) and Barisic, Vukic and Omerdic (2008) for more information about the Virtual Potential Framework.

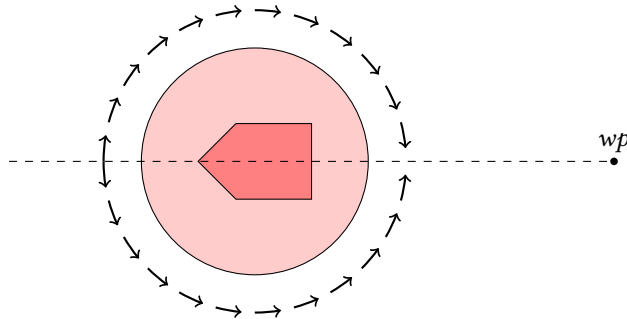


Figure 3.6: Rotor Force around an obstacle

### 3.8 COLREGS

The International Regulation for Preventing Collisions at Sea (COLREGS) was in 1972 established by the International Marine Organization (International Marine Organization (2005)). It is mandatory for all vehicles at sea to follow these rules.

### 3.8.1 COLREGS Rules

The next subsections are copied from International Marine Organization (2005). The rules relevant to this thesis are the following:

#### **Rule 8 - Action to avoid collision**

*(b). Any alteration of course and/or speed to avoid collision shall, if the circumstances of the case admit, be large enough to be readily apparent to another vessel observing visually or by radar; a succession of small alterations of course and/or speed should be avoided.*

*(c). If there is sufficient sea-room, alteration of course alone may be the most effective action to avoid a close-quarters situation provided that it is made in good time, is substantial and does not result in another close-quarters situation.*

*(d). Action taken to avoid collision with another vessel shall be such as to result in passing at a safe distance. The effectiveness of the action shall be carefully checked until the other vessel is finally past and clear.*

#### **Rule 13 - Overtaking**

*(a). ... any vessel overtaking any other shall keep out of the way of the vessel being overtaken.*

*(b). A vessel shall be deemed to be overtaking when coming up with another vessel from a direction more than 22.5 degrees abaft her beam, that is, in such a position with reference to the vessel she is overtaking, that at night she would be able to see only the sternlight of that vessel but neither of her sidelights.*

*(d). Any subsequent alteration of the bearing between the two vessels shall not make*

*the overtaking vessel a crossing vessel within the meaning of these Rules or relieve her of the duty of keeping clear of the overtaken vessel until she is finally past and clear.*

**Rule 14 - Head-on situations**

*(a). When two power-driven vessels are meeting on reciprocal or nearly reciprocal courses so as to involve risk of collision each shall alter her course to starboard so that each shall pass on the port side of the other.*

*(b). Such a situation shall be deemed to exist when a vessel sees the other ahead or nearly ahead and by night she could see the masthead lights of the other in a line or nearly in a line and/or both sidelights and by day she observes the corresponding aspect of the other vessel.*

**Rule 15 - Crossing situations**

*When two power-driven vessels are crossing so as to involve risk of collision, the vessel which has the other on her own starboard side shall keep out of the way and shall, if the circumstances of the case admit, avoid crossing ahead of the other vessel.*

**Rule 16 - Action by give-way vessel**

*Every vessel which is directed to keep out of the way of another vessel shall, so far as possible, take early and substantial action to keep well clear.*

**Rule 17 - Action by stand-on vessel**

*(a). (i). Where one of two vessels is to keep out of the way the other shall keep her course and speed. (ii). The latter vessel may however take action to avoid collision by her manoeuvre alone, as soon as it becomes apparent to her that the vessel required to keep out of the way is not taking appropriate action in compliance with these Rules.*

*(b). When, from any cause, the vessel required to keep her course and speed finds herself so close that collision cannot be avoided by the action of the give-way vessel alone, she*



*shall take such action as will best aid to avoid collision.*

*(c). A power-driven vessel which takes action in a crossing situation in accordance with subparagraph (a)(ii) of this Rule to avoid collision with another power-driven vessel shall, if the circumstances of the case admit, not alter course to port for a vessel on her own port side.*

### 3.8.2 Act According to the COLREGS Rules

To determine which COLREGS situation is taking place between two vessels, the relative bearing angle (the position of one vessel relative to the other vessels heading) must be considered. Equation 3.36 and 3.37 (Stenersen (2015)) gives the relative bearing angle,  $\beta_b$ , and figure 3.7 shows the angles used in the equations.

$$\psi_{AB} = \tan^{-1}\left(\frac{y_A - y_B}{x_A - x_B}\right) \quad (3.36)$$

$$\beta_b = \psi_{AB} - \psi_B \quad (3.37)$$

The angle  $\psi_{AB}$  is the angle in NED frame of the line between the two vehicles position,  $\psi_B$  is the heading of vehicle B, and  $\beta_b$  is the relative bearing angle deciding the COLREGS situation.

The angels differing a crossing situation from a head-on situation are not given in the COLREGS, but Stenersen (2015) suggests a separation at  $\pm 15^\circ$  behind the heading, based on the work of Loe (2008), Benjamin et al. (2006) and Colito (2007). The COLREGS situations with corresponding relative bearing angles are given in table 3.2.

The situations mentioned above only apply to situations when the vessel is on its way towards the obstacle. However, in the situation where the obstacle is the one overtaking, another set of actions should occur for the vessel now being overtaken. The fifth COLREGS situation is defined as in table 3.3, computing the relative bearing angle for the vessel instead of the obstacle.

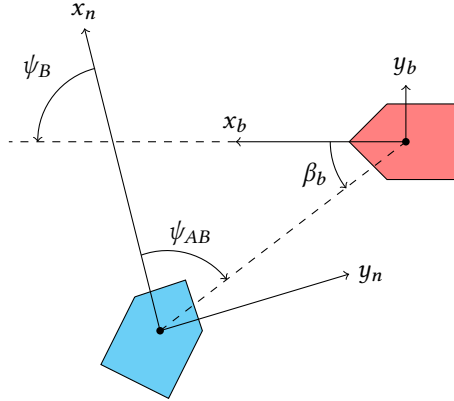


Figure 3.7: Angles used to determine which COLREGS situation the vehicles are in

Table 3.2: COLREGS Situations

| State               | Angles   |
|---------------------|--|
| Head on             | $\beta_b \in [-15^\circ, 15^\circ)$                                      |
| Crossing from left  | $\beta_b \in [15^\circ, 112, 5^\circ)$                                   |
| Overtaking          | $\beta_b \in [112, 5^\circ, 180^\circ) \vee [-180^\circ, -112, 5^\circ)$ |
| Crossing from right | $\beta_b \in [-112, 5^\circ, -15^\circ)$                                 |

Table 3.3: COLREGS Extra Situation

| State           | Angles   |
|-----------------|--|
| Being overtaken | $\beta_a \in [112, 5^\circ, 180^\circ) \vee [-180^\circ, -112, 5^\circ)$ |

# Chapter 4

## Implementation

The whole system is implemented as seen in the block diagram in figure 4.1. The following sections each describes the blocks in this figure. Table 4.1 describes the signal variables.

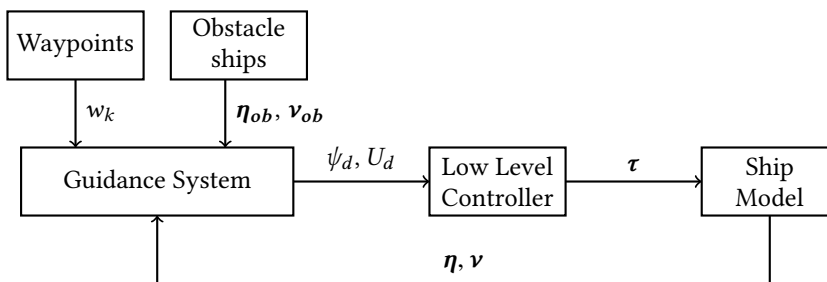


Figure 4.1: Block diagram implementation

Table 4.1: Signals in block diagram figure 4.1

| Signal      | Description                        |
|-------------|------------------------------------|
| $\psi_d$    | Desired heading                    |
| $U_d$       | Desired surge speed                |
| $\tau$      | Desired force                      |
| $\eta$      | Position vector of vessel          |
| $\nu$       | Velocity vector of vessel          |
| $w_k$       | Current waypoint                   |
| $\eta_{ob}$ | Position vector of obstacle vessel |
| $\nu_{ob}$  | Velocity vector of obstacle vessel |

## 4.1 Ship Model

The ship model block in figure 4.1 is the ship model from ABB Marine and Ports, and its structure is shown in figure 4.2. Following is a description of the blocks in figure 4.2:

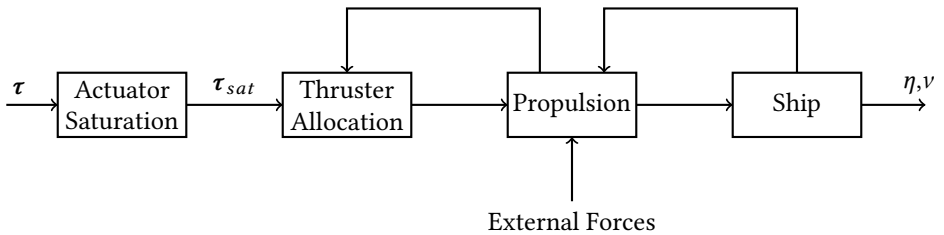


Figure 4.2: Ship Model

### Actuator Saturation

The actuator saturation makes sure that the desired forces from the controller are kept within the limits of what the system can handle.

### Thruster Allocation

The thruster allocation's task is to distribute the force to the available thrusters. In this model there are two azipods and one bow thruster. An azipod is an azimuth thruster

produced by ABB, which is a propeller placed on a pod that can be rotated. A bow thruster is used for precise manoeuvring at low speed, which will not be of particular use in this thesis.

### Propulsion

The propulsion block combines the desired forces from the thrusters and any external forces, like current and wind disturbances.

### Ship

The ship block receives the force from the thrusters and environmental forces combined and calculate the behaviour of the ship after being exposed to these forces. Equation 3.2 is implemented in this block, but only the 1<sup>st</sup>, 2<sup>nd</sup> and 6<sup>th</sup> DOF is used (see table 3.1). Table 4.2 gives some basic information about the ship.

Table 4.2: Ship model parameters

| Ship Model       | Value  |
|------------------|--------|
| Length of vessel | 294 m  |
| Beam             | 37.9 m |
| Speed max        | 10 m/s |

## 4.2 Low-Level Controller

The low-level controller consists of a reference model and a regulator, which is implemented in Simulink. It also includes an algorithm that finds the shortest turn to reach the desired heading.

### 4.2.1 Reference Model

A reference model is used for both the heading and the speed controller to extract the desired yaw rate  $r_d$  and the desired surge acceleration  $\dot{u}_d$ . The reference model will also low-pass filter the signals  $\psi_{ref}$  and  $u_{ref}$ , coming from the LOS and speed-adaptation

algorithms, which is needed since these signals can have step changes. The equation for the reference model used is as follows (Fossen (2011)):

$$\dot{r}_d = -2\zeta\omega r_d - \omega^2(\psi_d - \psi_{ref}) \quad (4.1)$$

$$\ddot{u}_d = -2\zeta\omega\dot{u}_d - \omega^2(u_d - u_{ref}) \quad (4.2)$$

where  $\zeta$  is a damping constant and  $\omega$  is a frequency constant. The values used for the constants are given in table 4.3.

Table 4.3: Values of constants in reference model

| Constant | Value |
|----------|-------|
| $\zeta$  | 1     |
| $\omega$ | 0.1   |

## 4.2.2 PID Regulator

The most commonly used regulator for marine vessels is the PID regulator and is given in equation (4.3) (Fossen (2011)).

$$\tau = -K_p\tilde{x} - K_d\dot{\tilde{x}} - K_i \int_0^t \tilde{x}dt \quad (4.3)$$

where  $\tilde{x} = x - x_d$ , which gives the difference between the actual value  $x$  and the desired value  $x_d$ .

For the heading controller, only the proportional and the derivative part is used, giving a regulator on the form:

$$\tau = -K_{p,h}\tilde{\psi} - K_{d,h}\dot{\tilde{\psi}} \quad (4.4)$$

In regulation of marine vessels it is common to not use the integration part when regulating the heading. The integration part of a PID regulator is used to deal with

constant errors, and because of the thrust allocation in the ship model this will not be a problem.

For the speed controller, only the proportional and the integration part is used:

$$\tau = -K_{p,s}\tilde{u} - K_{i,s} \int_0^t \tilde{u} dt \quad (4.5)$$

The derivative part will increase the responsiveness of a system, which here means the revolution of the propeller. A rapid increase in the propellers RPM (revolutions per minute) may cause cavitation, which makes the power of the thrusters ineffective, it is damaging to the propeller, and hence something we want to avoid. This is why it's normal to use a PI regulator in the speed controller for marine vessels.

The values used for the gain constants are given in table 4.4.

Table 4.4: Values of gains in regulator

| Gain Constant | Value  |
|---------------|--------|
| $K_{p,h}$     | 16000  |
| $K_{d,h}$     | 800000 |
| $K_{p,s}$     | 100    |
| $K_{i,s}$     | 1      |

### 4.2.3 Shortest Turn

If the difference between the heading and the desired heading,  $\tilde{\psi} = \psi - \psi_d$ , is greater than  $\pi$ , it is shorter to turn "away" from the desired heading and go around  $\pm 180^\circ$  on the way. The following calculation finds the alternative difference between the

heading and the desired heading:

$$\begin{aligned}\tilde{\psi} &= -\text{sign}(\psi)(\alpha + \alpha_d) \\ \alpha &= \pi - \text{abs}(\psi) \\ \alpha_d &= \pi - \text{abs}(\psi_d)\end{aligned}\tag{4.6}$$

If the desired heading is greater than the heading, i.e.  $\psi < 0$  and  $\psi_d > 0$ , the desired rotation must be counter-clockwise, i.e.  $\tilde{\psi} > 0$ . If the heading is greater than the desired heading, the desired rotation must be clockwise, i.e.  $\tilde{\psi} < 0$ . Figure 4.3 shows the angles used in equation 4.6.

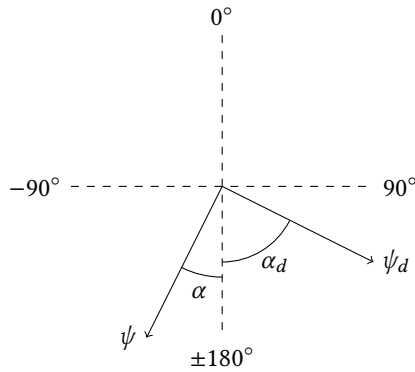


Figure 4.3: Angles used to find the shortest turn around the vessels own body z-axis

### 4.3 Guidance System - Velocity Obstacle

This block consists of mainly two algorithms: The LOS path following algorithm and the VO collision avoidance algorithm. Both implemented in Matlab code. The block works in the following way:

The LOS algorithm, as described in section 3.4.1, is implemented with a look-ahead distance  $\Delta$  of 1800 meters, six times longer than the vessel. Fossen (2011) recommends



the look-ahead distance to be three times the length of the vessel, as a rule of thumb, but since travelling in the direction of the path in this case is more important than being on top of the path line, a longer look-ahead distance was chosen. The circle of acceptance, as described in section 3.4.2, is used to decide when to switch to the next waypoint. The radius  $R_k$  of the circle is chosen to be 800 meters.

The speed-adaptation algorithm, as described in section 3.4.3, is implemented with a maximum surge speed of 10 m/s, a minimum surge speed of 0 m/s and a maximum yaw rate of 1.6 rad/s, which are given by the ship model. The reason that the minimum surge speed can be 0 m/s is because the vessel is equipped with azipod thrusters and can therefore turn without having any surge speed.

To determine whether the vessel is on a collision course or not, the closest point of approach, as described in section 3.5, is used.  $t_{max}$  is chosen to be 500 seconds, meaning that there is no possible collision unless there is less than 500 seconds until the possible collision will occur.  $d_{min}$  is chosen to be 1000 meters, meaning there is no possible collision unless the distance at CPA is less than 1000 meters.

The safety distance, which decides the size of the VO cone,  $r_d$ , is chosen to be 500 m.

All the parameter values mentioned earlier in this section are found in table 4.5.

### 4.3.1 Possible Velocity Vectors

When finding the desired collision-free velocity vector using the cost function in section 4.3.2, the possible velocity vectors to choose from has the following angle, in the BODY frame, and magnitude:

The different heading angles are spaced with 0.1963 radians and are 21 in total. This gives the values:  $[-1.9630, -1.7667, 1.5704, -1.3741, -1.1778, -0.9815,$

Table 4.5: Parameter values for Path following and CPA

|                                   |           |           |
|-----------------------------------|-----------|-----------|
| <b>Line of sight:</b>             |           |           |
| Look-ahead distance               | $\Delta$  | 1800      |
| Circle of acceptance radius       | $R$       | 800       |
| <b>Speed adaptation:</b>          |           |           |
| Minimum surge speed               | $U_{min}$ | 0 m/s     |
| Maximum surge speed               | $U_{max}$ | 10 m/s    |
| Yaw rate saturation               | $r_{sat}$ | 1.6 rad/s |
| <b>Closest point of approach:</b> |           |           |
| CPA time parameter                | $t_{max}$ | 500 s     |
| CPA distance parameter            | $d_{min}$ | 1000 m    |
| <b>VO cone:</b>                   |           |           |
| Safety distance                   | $r_s$     | 500 m     |

$-0.7852, -0.5889, -0.3926, -0.1963, 0, 0.1963, 0.3926, 0.5889, 0.7852, 0.9815, 1.1778, 1.3741, 1.5704, 1.7667, 1.9630]$ .

The different surge speeds are spaced with 2 m/s and are 6 in total. This gives the values: [0 m/s, 2 m/s, 4 m/s, 6 m/s, 8 m/s, 10 m/s].

The total number of possible velocity vectors is 106 (only 1 vector with magnitude 0 m/s). Figure 4.4 illustrates the different velocity vectors described above.

### 4.3.2 Cost Function

If a collision will occur in the near future, within  $t_{max}$  seconds, the velocity vector that is most similar to the one given by LOS and also outside the forbidden VO regions will be chosen as the desired heading and speed vector. The velocity vector is chosen using a cost function:

A cost is applied to every possible velocity vector according to (inspired by Sten-

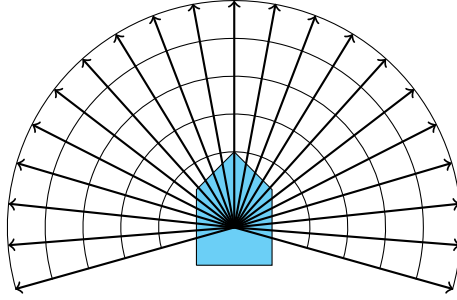


Figure 4.4: Possible velocity vectors

ersen (2015):

$$c_i = \alpha \tilde{d}_i^2 + \beta \tilde{u}_i^2 + \gamma f_i + \zeta g_i \quad (4.7)$$

where

$c_i$  is the cost of velocity vector  $i$ ,

$\tilde{d}_i$  [rad] is the angle that differs velocity vector  $i$  from the desired LOS velocity vector,

$\tilde{u}_i$  [m/s] is the difference between the magnitude of velocity vector  $i$  and the desired surge speed given by the speed adaptation algorithm,

$f_i$  is a boolean that equals 1 if the velocity vector is inside the velocity obstacle cone,

$g_i$  is a boolean that equals 1 if the velocity vector violates the COLREGS rules,

$\alpha, \beta, \gamma, \zeta$  are scaling constants.

The values used for the scaling constants are given in table 4.6. The large values of  $\gamma$  and  $\zeta$  will prevent the velocity vectors that are inside the velocity obstacle cone or that conflicts with the COLREGS rules from being chosen as the desired velocity vector. The parameters  $\alpha$  and  $\beta$  are weighted so that a change in speed will have a higher cost than

a change in heading. This is due to the statement given in COLREGS Rule 8c in section 3.8.1, stating that it is more efficient to alter the heading than to change the surge speed, which in general is true for a large vessel. The value chosen for  $\alpha$  comes from  $1/0.1963$ , giving a cost of 1 for every angular step between the possible velocity vectors. If the angular difference is  $100^\circ$ , this gives a cost that is almost 9. Since there is 2 m/s between the possible magnitudes of the velocity vector, a  $\beta$  value of 4.5 makes it preferable to lower the surge speed one step only if the difference in heading is  $100^\circ$  or more.

Table 4.6: Values of constants in the cost function (4.7)

| Parameter | Value  |
|-----------|--------|
| $\alpha$  | 5.0942 |
| $\beta$   | 4.5    |
| $\gamma$  | 1000   |
| $\zeta$   | 1000   |

Whether a velocity vector violates the COLREGS rules or not is decided using the velocity obstacle method as described in section 3.6, and the COLREGS rules: In a head-on and a crossing from right situation, velocity vectors that are inside the VO region  $V_1$  will violate the COLREGS rules, but in other situations, all the VO regions, as described in equation (3.22) to (3.24), can be travelled. Table 4.7 sums up which VO regions that violates the COLREGS rules in the different COLREGS situations.

Table 4.7: Which VO regions, equation (3.22) to (3.24), violates the COLREGS rules

| COLREGS situation   | VO region that violates COLREGS  |
|---------------------|----------------------------------|
| Head-on             | $V_1$                            |
| Crossing from right | $V_1$                            |
| Crossing from left  | None. Stand-on vessel            |
| Overtaking          | None. Can overtake on both sides |

### 4.3.3 Passed COLREGS Situation

To decide when the controlled vessel has passed the obstacle vessel, meaning the COLREGS situation is over, the relative bearing angle limits in table 4.8 are used, as suggested by Stenersen (2015). These limits implies that for a head-on situation, it is not over until the relative bearing indicates that the vessel is in the overtaking zone, an overtaking situation is not over until the vessel is in the head-on zone and a crossing situation is not over until the vessel is in the overtaking zone.

Table 4.8: Relative bearing angle limits that ends COLREGS situations

| State               | Angle Limits            |
|---------------------|-------------------------|
| Head-on             | $\beta_b < -112.5$      |
| Crossing from left  | No action is taken      |
| Overtaking          | $\beta_b \in (-15, 15)$ |
| Crossing from right | $\beta_b < -112.5$      |

## 4.4 Guidance System - Virtual Potential Framework

When using the virtual potential framework method (section 3.7), both the guiding towards the waypoint and the guidance around the obstacles are done due to the potentials. The method is implemented in Matlab.

### 4.4.1 Obstacle Representation and Parameter Values

An obstacle vessel is represented as a circle with centre coordinate  $\mathbf{p}_o = [x_o, y_o]$  and radius  $R_o = 500$  meters. The values of the predefined parameters in the equations for the potential contour and the rotor force in section 3.7.2 and 3.7.1, is given in table 4.9.

### 4.4.2 Rotor Force and COLREGS

In a head-on and crossing from right situation the rotor acceleration needs to follow counter-clockwise around the obstacle (see figure 4.5), forcing the vessel to alter its

Table 4.9: Parameter values of the potential contour and rotor force

| Parameter | Value |
|-----------|-------|
| $A_w$     | 80    |
| $d_w$     | 4     |
| $A_o$     | 3000  |
| $A_r$     | 55000 |
| $d_r$     | 1000  |

course to starboard when approaching the obstacle, independently of the position of the waypoint (which differs from the description in section 3.7.2). By altering the unit rotor direction generator,  $\hat{\mathbf{r}}$ , in equation (3.35) to be  $[0 \ 0 \ -1]^T$  in the head-on and crossing from right situations, the direction of the rotor acceleration,  $\hat{\mathbf{a}}_{rot}$ , will be in the counter-clockwise direction around the obstacle in these situations.

The rotor force is implemented to only affect the vessel on the half circle of the obstacle facing away from the waypoint, see figure 4.5, by looking at the two unit vectors  $\hat{\mathbf{n}}_{wp}(\mathbf{p})$  and  $\hat{\mathbf{n}}_{vessel}(\mathbf{p})$  in equation (3.35). If the angle between them

$$\cos^{-1}(\hat{\mathbf{n}}_{vessel}(\mathbf{p}) \cdot \hat{\mathbf{n}}_{wp}(\mathbf{p})) \in \left[ \frac{\pi}{2}, 3\frac{\pi}{2} \right) \quad (4.8)$$

then the rotor acceleration force is applied to the total potential framework of the vessel (Barisic (2012)).

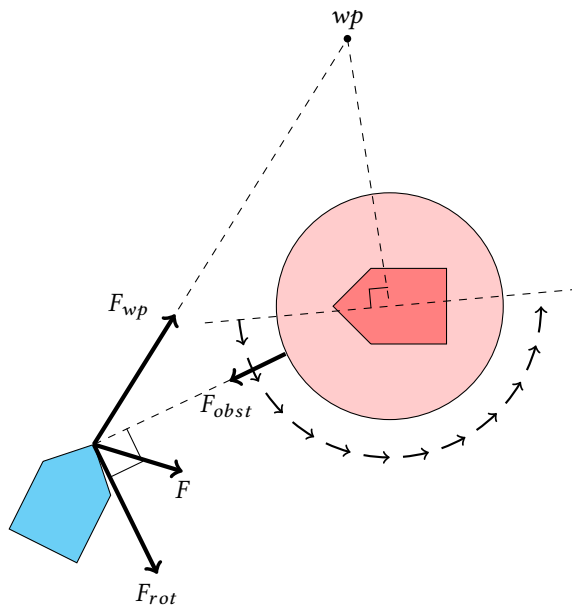


Figure 4.5: Forces applied to the potential framework of the vessel. The rotor force is counter clockwise to fulfil the COLREGS rules, and is only applied on the half side of the obstacle that is opposite to the waypoint.  $F$  is the sum of all forces.





# Chapter 5

## Simulation

In this section, simulations of the COLREGS situations head-on, right crossing and overtaking is presented. In the following simulations there will be two vessels operating. Note that both of the vessels are controlled by respectively the VO algorithm and the VPF. The security distance of 500 m should be larger for real life operating vessels, but works for this demonstration purpose.

### 5.1 Simulations Using VO

In the following simulations both the vessels are controlled by the VO algorithm. Table 5.1, 5.2 and 5.3 shows the initial conditions for the simulations, and figure 5.1, 5.2 and 5.3 shows the corresponding simulations.

Table 5.1: Head-on initial values VO

| <b>Vessel</b>   | <b>Position (x,y)</b> | <b>Heading (<math>\psi</math>)</b> | <b>Velocity (u)</b> | <b>Waypoints</b>   |
|-----------------|-----------------------|------------------------------------|---------------------|--------------------|
| Vessel A (blue) | (0,3500) m            | $-\pi/2$ rad                       | 10 m/s              | (0,3500) (0,-3500) |
| Vessel B (red)  | (0,-3500) m           | $\pi/2$ rad                        | 10 m/s              | (0,-3500) (0,3500) |

Table 5.2: Crossing initial values VO

| <b>Vessel</b>   | <b>Position (x,y)</b> | <b>Heading (<math>\psi</math>)</b> | <b>Velocity (u)</b> | <b>Waypoints</b>            |
|-----------------|-----------------------|------------------------------------|---------------------|-----------------------------|
| Vessel A (blue) | (500,0) m             | 0 rad                              | 10 m/s              | (500,0) (8000,0)            |
| Vessel B (red)  | (5000,4800) m         | $-\pi/2$ rad                       | 10 m/s              | (5000,4800)<br>(5000,-3000) |

Table 5.3: Overtaking initial values VO

| <b>Vessel</b>   | <b>Position (x,y)</b> | <b>Heading (<math>\psi</math>)</b> | <b>Velocity (u)</b> | <b>Waypoints</b> |
|-----------------|-----------------------|------------------------------------|---------------------|------------------|
| Vessel A (blue) | (-2500,0) m           | 0 rad                              | 10 m/s              | (0,0) (5000,0)   |
| Vessel B (red)  | (150,0) m             | 0 rad                              | 2 m/s               | (0,0) (5000,0)   |

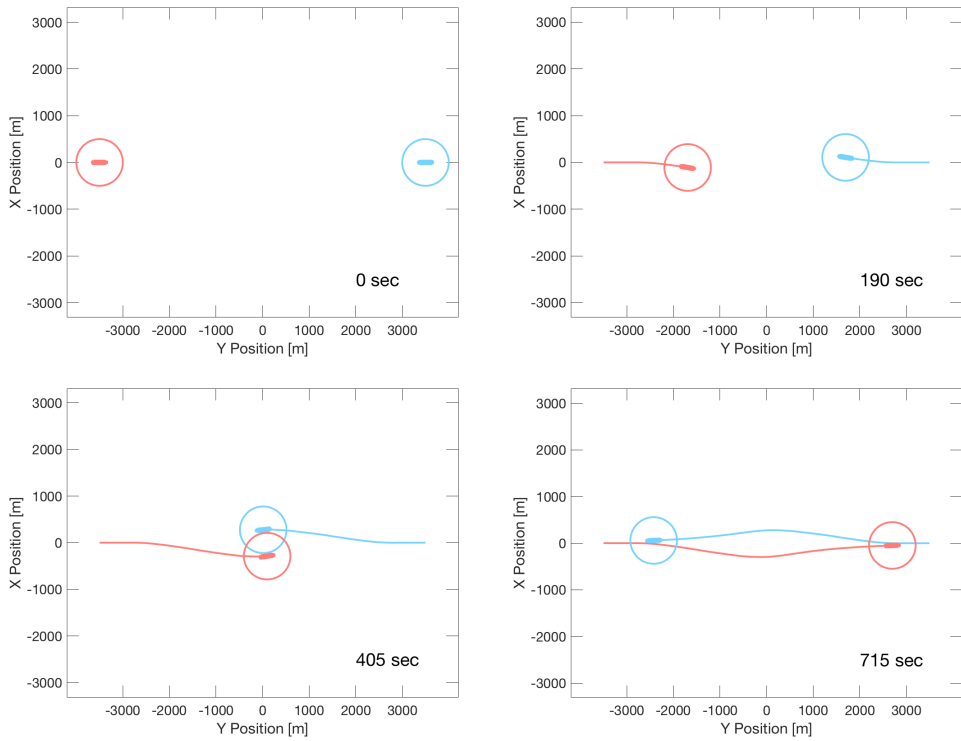


Figure 5.1: Head-on situation with two vessels controlled using the VO algorithm. Both vessels alter their course to starboard side and continue to follow the guidance line after passing the other vessel.

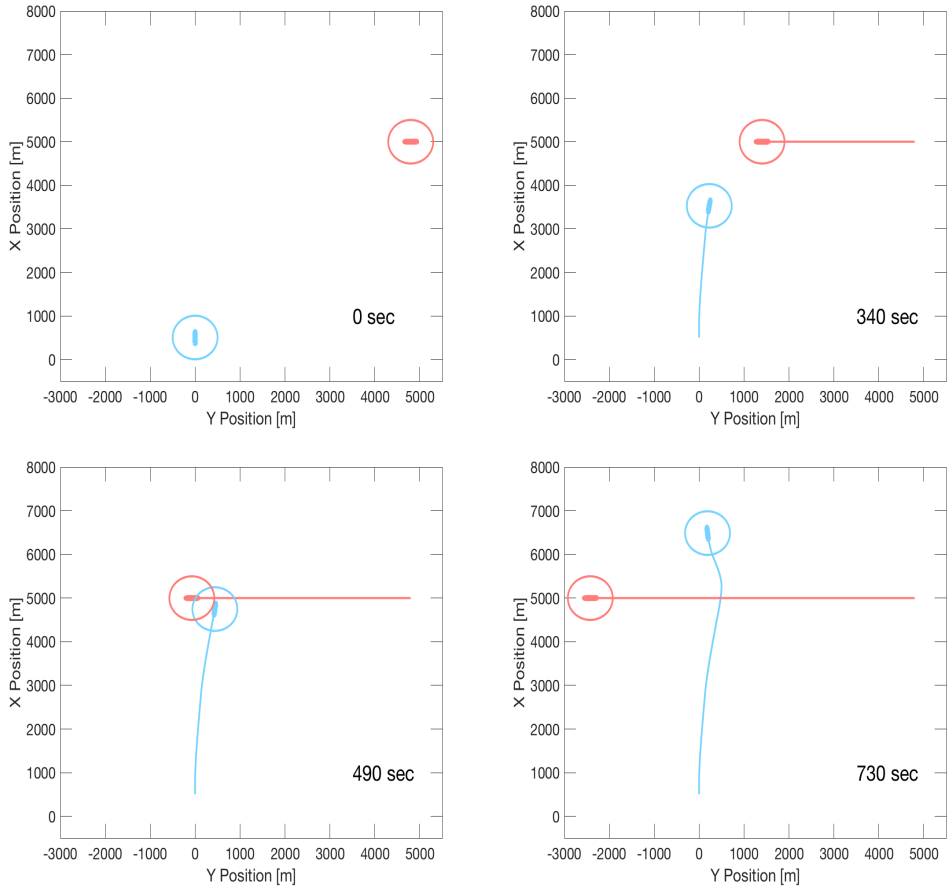


Figure 5.2: Crossing situation with two vessels controlled using the VO algorithm. The give-way vessel alter its course to starboard to pass behind the other vessel.

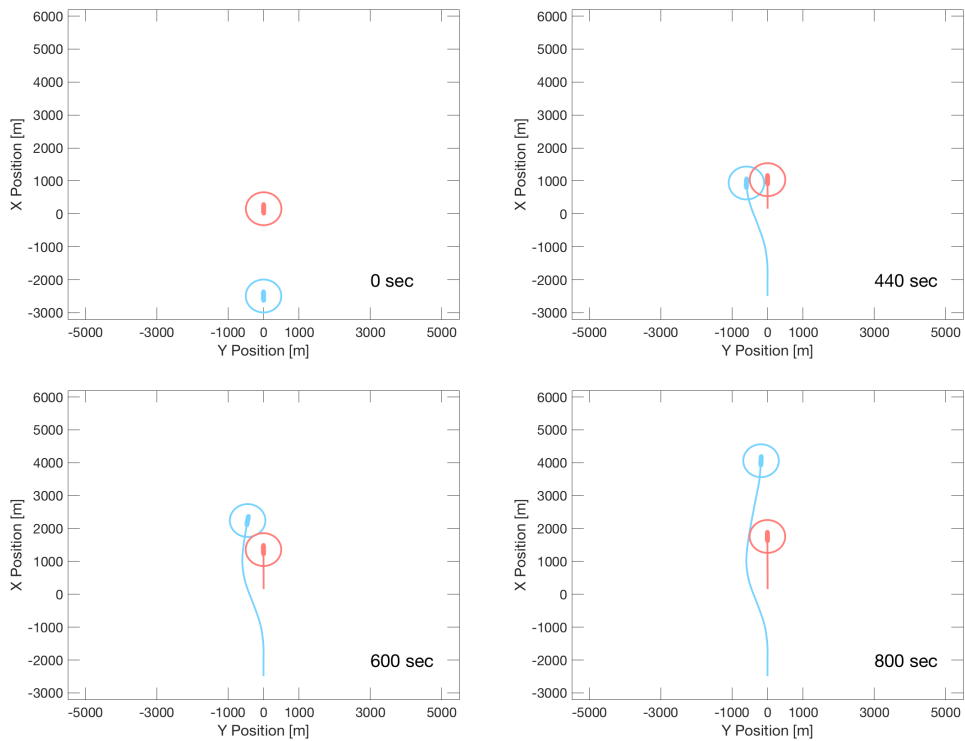


Figure 5.3: Overtaking situation with two vessels controlled using the VO algorithm. The overtaking vessel keeps enough distance so that the vessel being overtaken does not have to take action.

## 5.2 Simulations Using VPF

In the following simulations both the vessels are controlled by the VPF. Table 5.4, 5.5 and 5.6 shows the initial conditions for the simulations, and figure 5.4, 5.5 and 5.6 shows the corresponding simulations.

Table 5.4: Head-on initial values VPF

| Vessel          | Position (x,y) | Heading ( $\psi$ ) | Velocity (u) | Waypoints |
|-----------------|----------------|--------------------|--------------|-----------|
| Vessel A (blue) | (0,3500) m     | $-\pi/2$ rad       | 10 m/s       | (0,-3500) |
| Vessel B (red)  | (0,-3500) m    | $\pi/2$ rad        | 10 m/s       | (0,3500)  |

Table 5.5: Crossing initial values VPF

| Vessel          | Position (x,y) | Heading ( $\psi$ ) | Velocity (u) | Waypoints    |
|-----------------|----------------|--------------------|--------------|--------------|
| Vessel A (blue) | (-2500,0) m    | 0 rad              | 10 m/s       | (12000,0)    |
| Vessel B (red)  | (5000,7000) m  | $-\pi/2$ rad       | 10 m/s       | (5000,-5000) |

Table 5.6: Overtaking initial values VPF

| Vessel          | Position (x,y) | Heading ( $\psi$ ) | Velocity (u) | Waypoints |
|-----------------|----------------|--------------------|--------------|-----------|
| Vessel A (blue) | (-6000,0) m    | 0 rad              | 3 m/s        | (12000,0) |
| Vessel B (red)  | (-1000,0) m    | $-\pi/2$ rad       | 10 m/s       | (12000,0) |

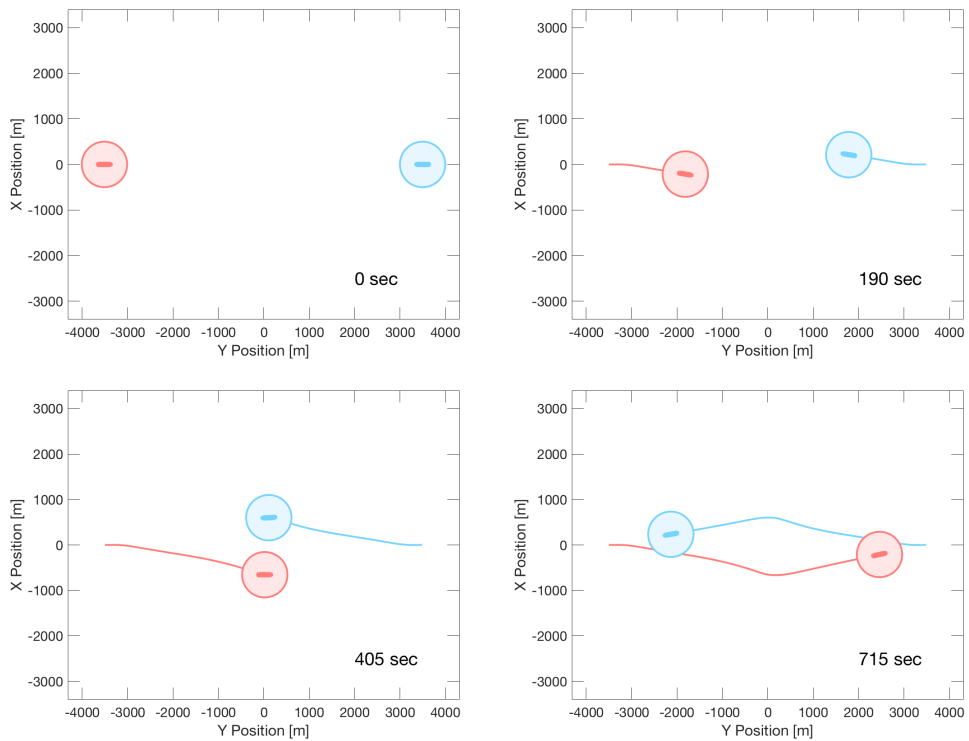


Figure 5.4: Head-on situation with two vessels controlled using the VPF algorithm. Both vessels alter their course to starboard side and continue towards the waypoint after passing the other vessel.

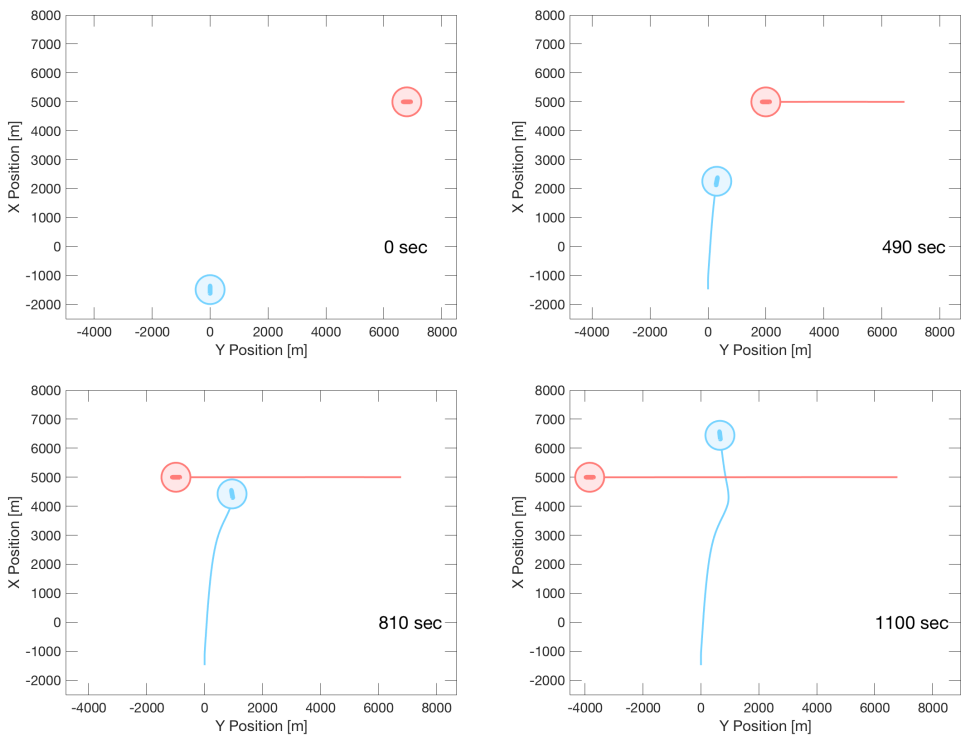


Figure 5.5: Crossing situation with two vessels controlled using the VPF algorithm. The give-way vessel alter its course to starboard to pass behind the other vessel.



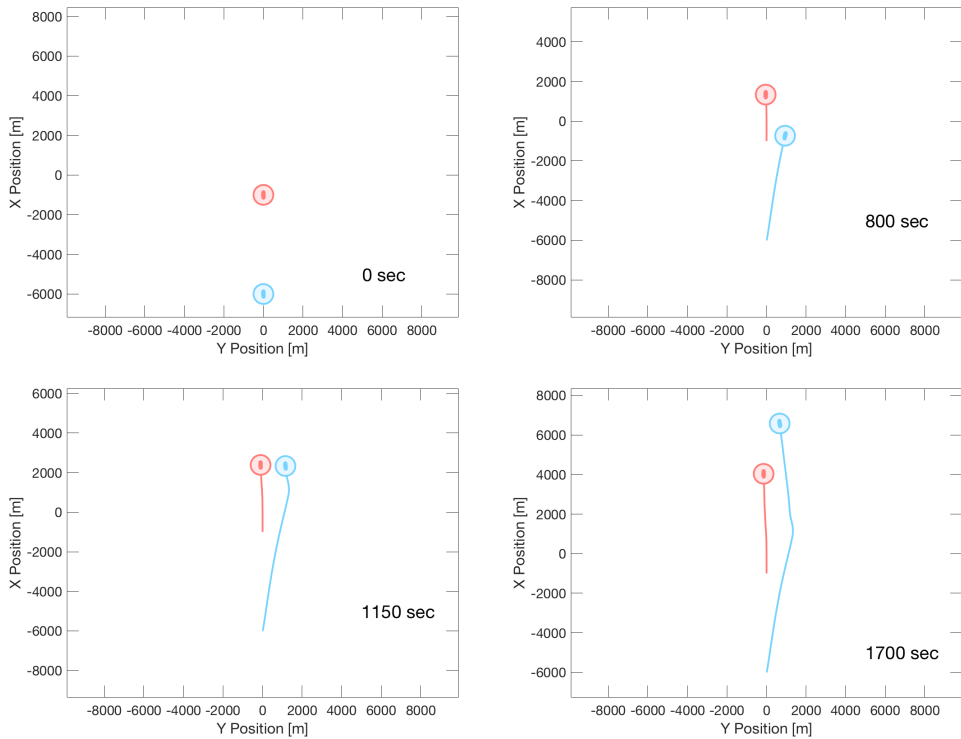


Figure 5.6: Overtaking situation with two vessels controlled using the VPF algorithm. The overtaking vessel keeps enough distance so that the vessel being overtaken does not have to take action.

### 5.3 Comments to the Simulations

Every situation has successfully been managed by both the VO algorithm and the VPF. This subsection includes some comments on the simulations:

In the overtaking situation, the VPF chooses to alter the heading to starboard, and the VO algorithm chooses to alter the heading to the port side. For the VPF, this is because the algorithm is implemented to have a counter-clockwise rotor force when both ways around the obstacle towards the waypoint are the same length. The VO algorithm is implemented to traverse the possible velocity vectors from the smallest to the largest angle, i.e. from the most port to the most starboard side. If two velocity vectors have the same cost, the one first in the iteration will be used.

In all the situations, the VPF is keeping a more significant distance to the obstacle vessel compared to the VO algorithm. It also starts the crossing and the overtaking situation much earlier. This is because of the difficulty of tuning the VPF parameters to make the vessel act early enough to prevent a high alteration of the heading.

# Chapter 6

## Discussion

The strength of the VO algorithm lies in its simplicity, but this is also its weakness, as it will struggle with more complex situations than straightforward head-on, crossing and overtaking situations. The VPM is also quite simple when it comes to selecting a desired velocity vector, but it will not always pick the most efficient path. Either way, it is complicated to create a good controller for such a slow system as a large vessel. Early action is essential to create a smooth anti-collision path.

### 6.1 Imprecise COLREGS Rules

#### 6.1.1 Overtaking or Crossing?

Even though there are no difficulties determining the COLREGS situation in a possible collision situation, the preferred handling of the situation may change. What started as an overtaking situation can become an event where the overtaking vessel needs to pass behind the obstacle vessel, similar to a crossing from right situation (Stenersen (2015)). A situation like this is shown in figure 6.1. The situation is clearly starting out as an overtaking situation, where the blue vessel has a waypoint in (8000, 0), and the shortest way around the red obstacle vessel is on the port side. Because of the

obstacle vessels waypoint in  $(8000, -2000)$ , it will create a situation where the blue vessel should be acting like it is a crossing situation and cross behind the obstacle vessel.

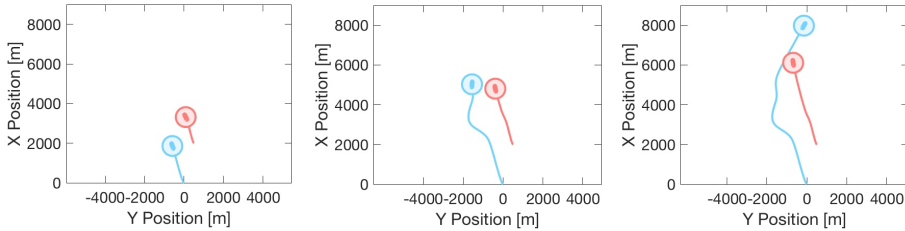


Figure 6.1: A situation that starts as an overtaking situation but where the give-way vessel should act as if it was a crossing situation.

### 6.1.2 New COLREGS Rules for Autonomous Vessels

As per today, the COLREGS rules are overall too subjective. If autonomous vessels shall travel the sea, we are in need of more precise rules that are not written for humans. In the current rules there is room for personal interpretation regarding whether a vessel is in a head-on or a crossing situation. Rule 14b in section 3.8.1 states that a head-on situation exists if the meeting vessels see the other one ahead, or nearly ahead, which is too indistinct for an autonomous vessel. The rules also state several times that the described action to obey the rules is to be executed "... if the circumstances of the case admit...". Expressions like "in good time" and "a safe distance" should be stated more precisely, as this will make the situations more predictive for all vessels involved. Nevertheless, as the first autonomous vessels start to operate, they need to interact with manned vessels, which follows the current COLREGS rules.

## 6.2 Rotor Force Problem

The rotor force is only applied when the vessel is on the opposite side of the obstacle compared to the waypoint, and this is because when the vessel has reached around to

the half circle of the obstacle that is facing the waypoint, it can be defined as passed the obstacle. If the rotor force were applied all around the obstacle, it would continue to force the vessel to move along the side of the obstacle, and the vessel would not only be affected by the force towards the waypoint. However, the absence of the rotor force in areas around the obstacle may in some situations cause COLREGS violation problems. In the situation shown in figure 6.2, where the vessel is in a head-on situation with an obstacle vessel, with the waypoint in (4000, 2000), it should alter its course to starboard. Since the waypoint and the vessel is seen by the obstacle at an angle less than 90 degrees, the rotor force will not be applied to the vessel, and the potential acceleration forces caused by the waypoint and the obstacle will make the vessel alter its heading to the pass in front of the obstacle vessel.

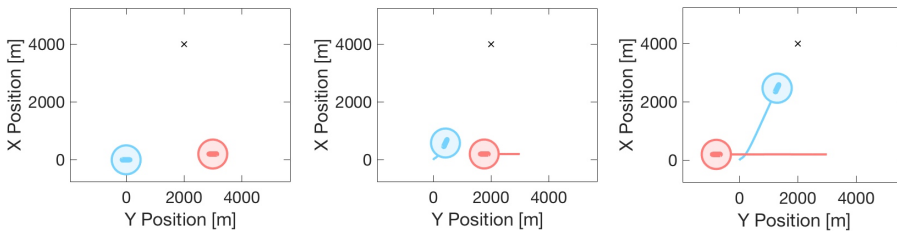


Figure 6.2: A problem with the rotor force when it is only applied when the vessel is on the half side of the obstacle that is facing away from the waypoint. In this plot only the blue vessel is controlled by the VPF and the red vessel is travelling in a straight line

### 6.3 Waypoint Switching Criteria

Fossen (2011) presents two methods on how to decide when to switch to the next waypoint; the circle of acceptance and along-track distance. One problem with the circle of acceptance is that an obstacle vessel could prevent the vessel from reaching the waypoint, which then needs to turn around after passing the obstacle vessel. This is not necessary when travelling in transit, where reaching every specific waypoint is not the goal, but rather to use a minimum of power and travelling shortest distance.

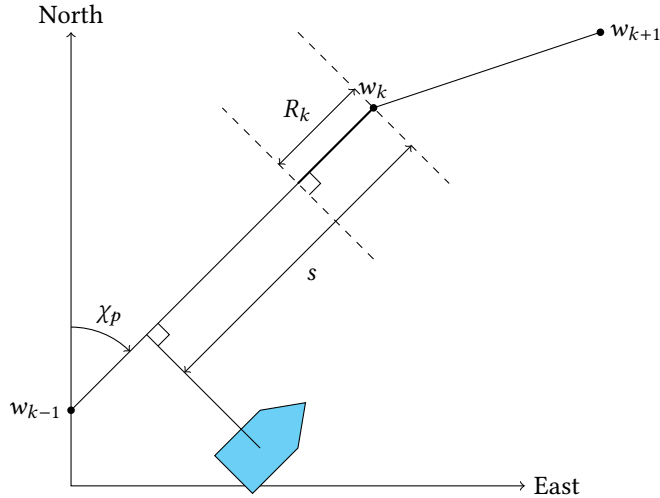


Figure 6.3: Along-track distance switching criteria.

Instead, using the along-track distance should fit much better for the purpose of transit paths. This allows the LOS path follower to change to the next waypoint when the vessel has passed the current waypoint, i.e. when the vessel has crossed the line normal to the path line, as given in equation (6.1).

$$s = (x_k - x(t))\cos(\chi_p) + (y_k - y(t))\sin(\chi_p) \leq R_k \quad (6.1)$$

where  $s$  is the along-track distance between the vessel and the current waypoint, and  $R_k$  is the switching criteria as seen in figure 6.3.

## 6.4 Local Minimum

For both the VPF and the VO algorithm, local minimums are located at the opposite side of the obstacle relative to the waypoint. The VPF is using a rotor force to guide the vessel out of and away from the local minima, which solves the problem completely. For the VO algorithm in an overtaking situation, where two vessels are travelling along

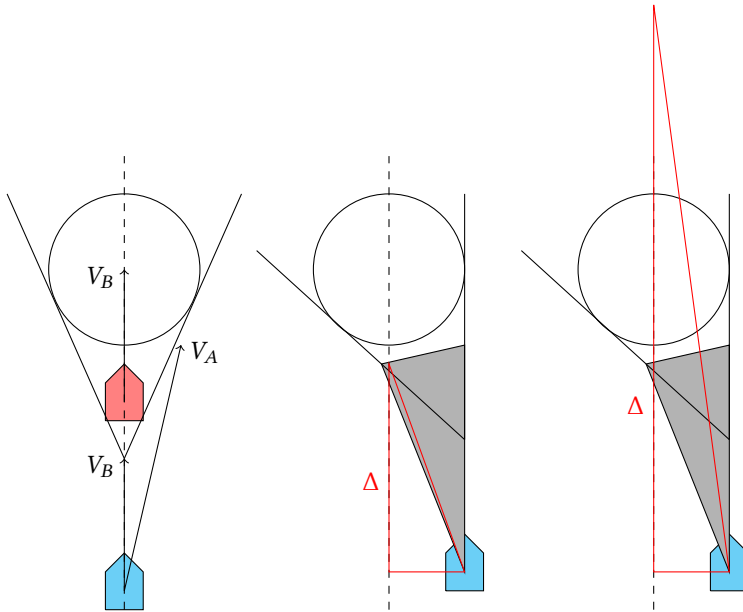


Figure 6.4: Unstable equilibrium while overtaking another vessel can be solved with a larger look-ahead distance.  $V_A$  is here the desired velocity vector.

the same line, there will be an unstable equilibrium for the vessel that is overtaking. As the desired velocity vector is chosen to be the vector outside the VO cone that is closest to the LOS desired vector, the desired heading will shift to the other side of the cone when crossing the path. This will only happen in an overtaking situation as there are no restrictions on which side to choose when overtaking.

One solution to this problem, inspired by the solution for a static obstacle proposed by Stenersen (2015), is to have a longer look-ahead distance in the LOS path-following algorithm when overtaking another vessel. A large enough look-ahead distance will result in an optimal heading that doesn't oscillate between the two VO regions on the two sides of the cone. Figure 6.4 shows the velocity cone for an obstacle vessel and two different look-ahead distances  $\Delta$ . In the middle subfigure the look-ahead distance

so short that the velocity vector that is closest to the LOS generated velocity will be on the left side of the cone. The subfigure most right has a look-ahead distance that is long enough for the next velocity vector to be chosen from the right side of the cone. A look-ahead distance that is the double of the distance between the vessels would be long enough to prevent the oscillation.

Another solution would be to pick a side, forcing the controller to pick a velocity vector from the same VO region as the previous vector was chosen from (Stenersen (2015)). However, this limits the freedom of being able to change side in the situations where this is required due to other obstacle vessels.

## 6.5 Run Time

Table 6.1 shows the run time of different elements in the Matlab and Simulink code. The times given for "Control system" are the amount of time from the signal leaves the ship model (figure 4.1) until its back at the ship model again. This includes the low-level controller. For the "LOS", "Velocity Obstacle" and "Virtual Potential Framework" only the run time of their respective Matlab code is measured. The times specified are all measured when the vessel is in a collision situation. Simulink is a tool that is good for exploring and testing different methods, changing parameters build up an easy-to-read structure, but it is not efficient when it comes to run time. That is why it has such a long run time compared with the individual Matlab scripts.

Table 6.1: Run time

|                             | <b>Maximum Run Time</b> |
|-----------------------------|-------------------------|
| Control system VO           | 0.7 – 1.2 seconds       |
| LOS                         | 0.008 seconds           |
| Velocity Obstacle           | 0.002 seconds           |
| Control system VPF          | 0.6 – 1.2 seconds       |
| Virtual Potential Framework | 0.0002 seconds          |



The VPF has a lower run time than the VO algorithm. The VO algorithm iterates through many potential velocity vectors and VPM calculates only one (no iteration through potential vectors). By reducing f.ex. the number of possible speed values the amount of possible velocities to iterate through will be smaller.

The low level controllers, regulating heading and speed, need a run time of about 100 milliseconds. The higher level controller can have a longer run time, f.ex. the set point change can use longer time. If the algorithms are implemented in a way that better exploits the available memory, and structures it in a different way, both algorithms will possibly be able to run within a lower time limit. For a better performance, the completed method can be translated closer to machine code using the integrated Simulink Functions in Simulink.

## 6.6 Avoid Collisions

### 6.6.1 Crash Stop

Halting a large vessel in cruising speed needs an extreme amount of power. It is not ideal to use energy to slow down the vessel when operating in transit, as the goal is to take the vessel to the last waypoint using as little power as possible. However, in some situations, this might be necessary to avoid collisions. Regarding the VPF, this task will be handled well as long as the ship model has a thrust allocation and is equipped with thrusters that is capable of this manoeuvre. When the vessel is heading straight towards an obstacle, the potential field creates an acceleration force in the opposite direction, away from the obstacle, forcing the vessel to halt, or even reversing. The VO algorithm does not take into account that the vessel should be able to stop, as the possible velocity vectors to choose from only can alter the course or slow down the speed. A velocity vector with zero length means applying zero energy to the thrusters, and will not make the vessel stop due to the low friction from the water. The VO algorithm can be altered to include "breaking velocities", but it is essential to make

sure that the cost function only choose these velocity vectors if it is necessary.

### 6.6.2 Avoid Collision vs Break the COLREGS Rules

The cost function with the parameters given in section (4.3.2) excludes all velocity vectors leading to a possible collision or that violates the COLREGS rules. Choosing a velocity vector that conflicts with the COLREGS rules may in some situations be the only way to avoid a collision, which is the top priority to avoid at all times. Therefore the parameters in the cost function should be balanced to satisfy the demand of punishing the velocity vectors that lead to a collision more than those causing a violation of the COLREGS rules, at all times. Not doing so may lead to a situation where the vessel is drive straight into the obstacle vessel because not altering the speed or heading has the lowest cost.

### 6.6.3 The Fifth COLREGS Situation

In a possible collision situation, the vessel will calculate the constant bearing,  $\beta_b$ , of the obstacle vessel to determine which COLREGS situation that should be followed. If the vessel is the one being overtaking, only looking at the constant bearing of the obstacle vessel will result in a believed head-on situation since it is right in front of the obstacle. The vessel will therefore alter its heading to starboard, when it should have been acting as a stand-on vessel. The fifth COLREGS situation, as given in table 3.3, will prevent this from happening.

When being a stand-on vessel, like in a crossing from left and being overtaken situation, the vessel must take action if the vessel that is supposed to give way is not handling the situation good enough. For these cases it should exist a safety radius that indicates that actions must be taken when the distance to the give-way vessel is smaller than this radius. Such an approach may be compromising with the efficiency of the new paths, making the give-way vessel a large detour around the obstacle for this second safety radius to be possible. Another approach is to monitor the behaviour of the obstacle vessel to detect whether or not it takes action at all. This is more complicated,

and the fact that the obstacle vessel alter its heading or speed doesn't necessarily mean that it will lead to a collision free passing.



# Chapter 7

## Future Work

### 7.1 Velocity vectors

The velocity vectors provided to the VO algorithm doesn't reflect how the vessel is going to act, but if the linear velocities are replaced with curved velocities, as demonstrated in figure 7.1, it will provide a path that the vessel is actually able to follow (Loe (2008)). The VO algorithm as it is implemented in this thesis is very simple since it requires no information about the vessel. To create curved velocities that reflects how the vessel will act requires knowledge about the dynamics of the vessel, which makes it much more complicated. However, just a slightly upgrade from the linear velocities will probably create a significant difference. The goal with using more appropriate velocities is to increase the accuracy of the algorithm, since the current implementation only points out an approximately heading and need more safety precautions.

A suggestion from Stenersen (2015) is to use the VO algorithm as a filter, where another well proven method can suggest possible paths for the vessel and the VO algorithm tests if any of the paths will conflict with the COLREGS rules.

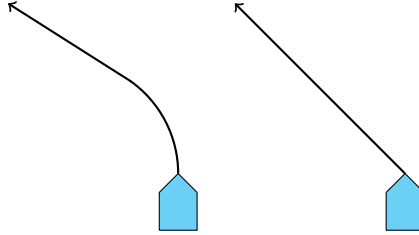


Figure 7.1: Curved velocity vs linear velocity

## 7.2 Surge Speed

The speed-adaptation that is used in this implementation is of a very simple kind. Bibuli (2010) includes two additional approaches that would be interesting to test together with this system. One method includes the curvature of the path and the other has a kinematic approach.

The desired surge speed calculated considering the curvature  $c_c(s)$  of the path is given as follows:

$$u_d = U_{min} + (U_{max} - U_{min})[1 - \tanh^2(k_u c_{max})] \quad (7.1)$$

where  $k_u$  is a free parameter and  $c_{max}$  is the maximum value of the curvature  $c_c(\bar{s})$ , where  $\bar{s}$  is on the interval  $[s, s + h]$  with  $h$  as the step size along the path.

The approach using the dynamics of the vessel is relying on a kinematic model that reflects the behaviour of the vessel. This is much more complicated than just using the maximum yaw-rate value as done in this thesis. See Bibuli (2010) for details about this method.

## 7.3 Observe Angle

The biggest struggle with tuning the potential acceleration forces and the rotor force in the VPF was to have a large enough force to make the vessel act early but not make it turn too much. Because of the delay in such a large vessel, it will start to alter its course but will still continue to get closer to the obstacle. Figure 7.2 illustrates this issue in an overtaking scenario where the obstacle vessel is almost standing still. As the distance between the vessel and the obstacle is at its smallest when the vessel is on its way around an obstacle (see the middle image in figure 7.2), the acceleration force from the obstacle that affects the vessel is growing larger, even though the vessel is on the right path and almost around the obstacle. This causes the vessel to take a large turn away from the vessel, when it could have started to turn back behind the obstacle. To solve this problem an observe angle can be introduced, applying the potential caused by an obstacle only if the obstacle is seen by the vessel, i.e. inside an observe angle, as seen in figure 7.3. It is important that the observe angle only affects the potential force from the obstacle and not the rotation force. If not, the vessel will be completely unable to take action to situations regarding an obstacle outside the observe angle, and the rotation force is not contributing to the problem that the observe angle is trying to solve. The observe angle only fully fills its task when the vessel is close to an obstacle, so when the vessel is far away from an obstacle, there is no need to use the observe angle. This problem is not of critical concern as it doesn't lead to a dangerous situation, but it is rather a question of effectiveness when overtaking an obstacle vessel.

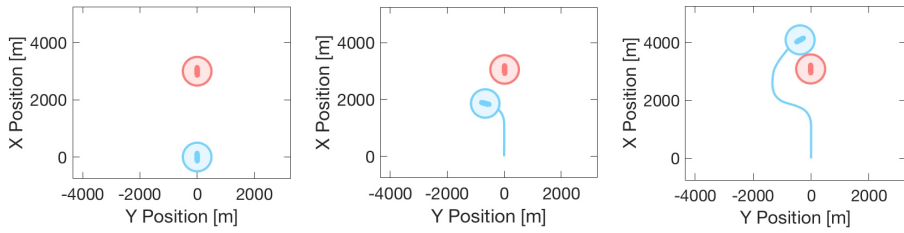


Figure 7.2: Overtaking situation where the overtaking vessel could have selected a more efficient route

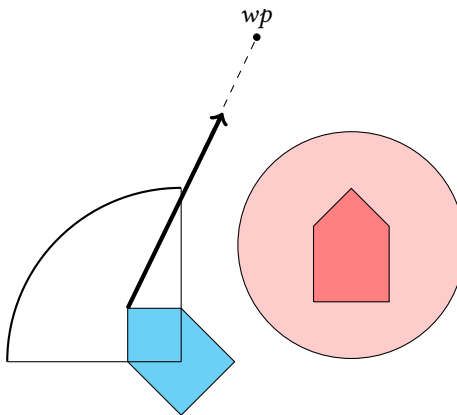


Figure 7.3: Observe angle of  $\pm 45$  degrees



## 7.4 S-function

The introduction of an observe angle, as described in section 7.3, will cause a step in the VPF contour when an obstacle goes in and out of observation. To prevent this, a sigmoid function or similar should be used over the observe angle. One suggestion is to implement the logistic function in equation (7.2) (Wikipedia (2018a)) with parameters given in table 7.1. Figure (7.4) shows the function.

$$f(x) = \frac{L}{1 + e^{-k(x-x_1)}} - \frac{L}{1 + e^{-k(x-x_2)}} \quad (7.2)$$

Table 7.1: Parameter values logistic function

| Parameter | Value |
|-----------|-------|
| $k$       | 0.15  |
| $x_1$     | -30   |
| $x_2$     | 30    |
| $L$       | 1     |

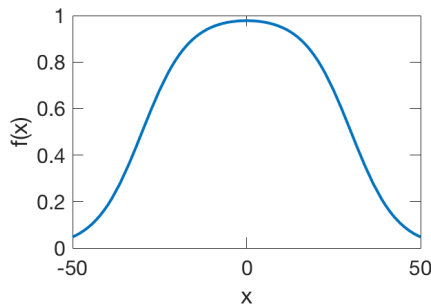


Figure 7.4: Logistic function

Another suggestion is to implement the Probability Density Function (PDF) in equation

(7.3) (Wikipedia (2018b)) with parameters given in table 7.2. Figure (7.5) shows the function.

$$f(x) = L \frac{e^{-\frac{x-\mu}{\beta}}}{\beta \left( e^{-\frac{x-\mu}{\beta}} + 1 \right)^2} \quad (7.3)$$

Table 7.2: Parameter values PDF

| Parameter | Value |
|-----------|-------|
| $\beta$   | 6     |
| $\mu$     | 0     |
| $L$       | 24    |

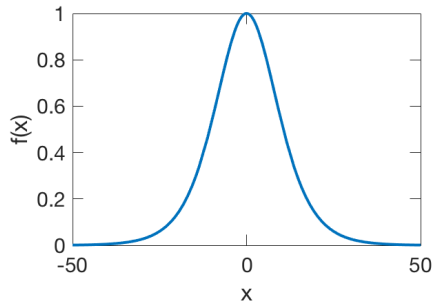


Figure 7.5: Probability density function

## 7.5 Expand the Obstacle in VPF

Applying the rotor force as described in section 4.4.2 makes the VPF follow the COLREGS rules. However, this may not be enough, as seen in section 6.2. In addition to the rotor force, changing how the obstacles affect the potential contour, as suggested in Naeem et al. (2016), will hopefully prevent the VPF from conflicting with the COLREGS rules. In Naeem et al. (2016), COLREGS zones are created by extending the obstacle in

the direction where the vessel is not supposed to drive (see figure 7.7), by for instance defining the obstacle as an ellipse. Finding the added potential from an ellipse is quite similar to the added potential from a circle, which is used in this thesis, but the challenge is to find the vector between the obstacle and the vessel, which gives the distance and the direction of the potentials.

### Distance to an Ellipse

The representation of an ellipse is as follows (Barisic (2012)):

$a$  is the semi-major axis,

$b$  is the semi-minor axis,

$\mathbf{p}_e = (x_e, y_e)$  is the centre coordinate,

$\psi_e$  is the angle that rotates the ellipse in the NED frame.

All the parameters are define in figure 7.6.

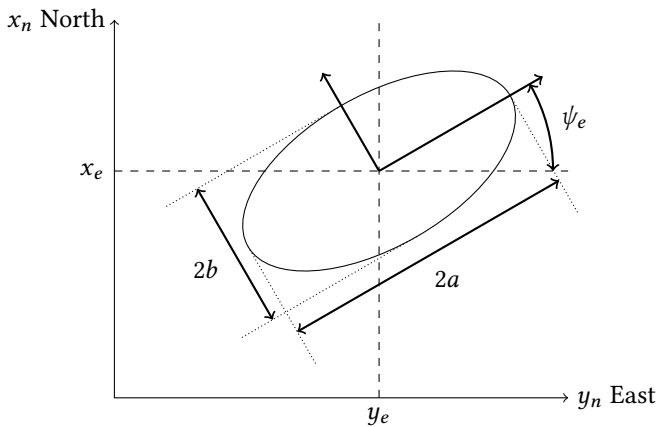


Figure 7.6: Representation of an ellipse in a 2D plane

An ellipse in its own BODY frame is defined by (Barisic (2012)):

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1 \quad (7.4)$$

where  $(x', y') = \mathbf{p}'$  is a point on the ellipse. To transform the vessels position  $\mathbf{p}^n$  from the NED frame to the BODY frame of the ellipse, there needs to be a rotation and a transition as given in equation (7.5):

$$\mathbf{p}^b = \mathbf{R}(-\psi_e)(\mathbf{p}^n - \mathbf{p}_e) \quad (7.5)$$

where  $\mathbf{R}(-\psi_e) = \begin{bmatrix} \cos(\psi_e) & \sin(\psi_e) \\ -\sin(\psi_e) & \cos(\psi_e) \end{bmatrix}$ . The shortest distance between the vessel and an ellipse is perpendicular to the tangent in the closest point on the ellipse. This vector needs to follow the line:

$$\mathbf{p}^b = \mathbf{k}_n t + \mathbf{p}' \quad (7.6)$$

where  $\mathbf{k}_n$  is the direction vector of the line, which is the derivative of equation (7.4):

$$\mathbf{k}_n = \nabla \left( \frac{x'^2}{a^2} + \frac{y'^2}{b^2} - 1 \right) \implies \left[ \frac{x'}{a^2} \quad \frac{y'}{b^2} \right]^T \quad (7.7)$$

Including equation (7.7) in equation (7.6) gives:

$$\begin{aligned} [x - x' \quad y - y']^T &= \left[ \frac{x' t}{a^2} \quad \frac{y' t}{b^2} \right]^T \\ [x \quad y]^T &= \left[ x' \frac{a^2 + t}{a^2} \quad y' \frac{b^2 + t}{b^2} \right]^T \\ [x' \quad y']^T &= \left[ \frac{a^2 x}{t + a^2} \quad \frac{b^2 y}{t + b^2} \right]^T \end{aligned} \quad (7.8)$$

Substituting equation (7.8) into equation (7.4) gives:

$$\begin{aligned} \left(\frac{ax}{t+a^2}\right)^2 + \left(\frac{by}{t+b^2}\right)^2 &= 1 \\ (t+b^2)^2 a^2 x^2 + (t+a^2)^2 b^2 y^2 &= (t+a^2)^2 (t+b^2)^2 \quad (7.9) \\ (t+a^2)^2 (t+b^2)^2 - (t+b^2)^2 a^2 x^2 - (t+a^2)^2 b^2 y^2 &= 0 \end{aligned}$$

The greatest root of this polynomial gives the distance between the vessel and the ellipse. The direction of this vector, rotated back to the NED frame, is given as:

$$\hat{\mathbf{n}} = \mathbf{R}(\psi_e) \frac{\mathbf{p}_e - \mathbf{p}^n}{\|\mathbf{p}_e - \mathbf{p}^n\|} \quad (7.10)$$

where  $\mathbf{R}(\psi_e) = \begin{bmatrix} \cos(\psi_e) & -\sin(\psi_e) \\ \sin(\psi_e) & \cos(\psi_e) \end{bmatrix}$ .

### Expansion of Obstacle

In a crossing from right situation the vessel is not allowed to cross in front of the obstacle vessel. The obstacle should therefore be expanded along the velocity vector of the obstacle vessel. In a head-on situation the vessel is not allowed to pass the obstacle vessel on the obstacles starboard side. The obstacle should therefore be expanded along the perpendicular of the velocity vector of the obstacle vessel, towards its starboard side. Figure 7.7 demonstrates a crossing from right and a head-on situation where the obstacle is extended.

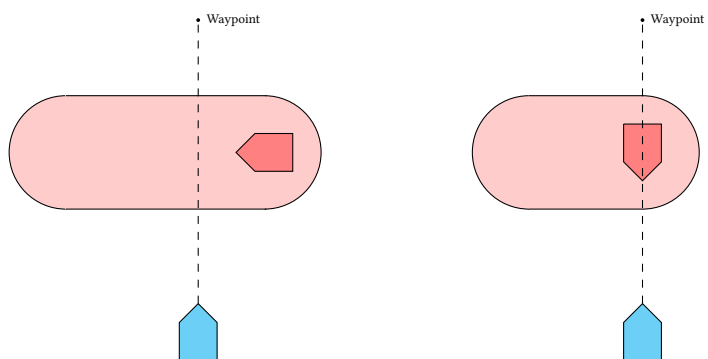


Figure 7.7: The obstacle is extended in the direction where the vessel should not be driving

# Chapter 8

## Conclusion

The thesis has presented two different methods for collision avoidance that achieves to navigate according to the COLREGS rules; The VO algorithm that creates a cone around the obstacle and tests different velocities to exclude those making the vessel travel inside the cone, and a modified VPF that applies potential forces to the vessel that are pushing it away from the obstacles and dragging it towards the waypoint. The simulations of the COLREGS situations head-on, crossing and overtaking, shows that both the VPF and the VO algorithm handles these situations correctly.

More general, the VPF proves to be overall robust at avoiding collisions, but there are some difficulties regarding the COLREGS rules. The VO algorithm, on the other hand, doesn't handle every situation well enough to avoid a collision. However, it is more robust when it comes to following the COLREGS rules.

A low-level controller was created for the ship model provided by ABB Marine and Ports, and also a discussion on the inaccurate formulations made in the COLREGS and how it will conflict with the new era of autonomous vessels is provided.





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