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ORIGINAL CONTRIBUTION

# Bingham's model in the oil and gas industry

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**Abstract** Yield stress fluid flows occur in a great many operations and unit processes within the oil and gas industry. This paper reviews this usage within reservoir flows of heavy oil, drilling fluids and operations, wellbore cementing, hydraulic fracturing and some open-hole completions, sealing/remedial operations, e.g., squeeze cementing, lost circulation, and waxy crude oils and flow assurance, both wax deposition and restart issues. We outline both rheological aspects and relevant fluid mechanics issues, focusing primarily on yield stress fluids and related phenomena.

Special Issue to celebrate the centennial anniversary of the seminal Bingham paper.

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## Introduction

This paper honors the contribution of E.C. Bingham to the oil and gas industry. In Bingham's initial work (Bingham 1916), the oil and gas industry does not feature, although many of the fluids discussed (suspensions, clays) play a role. He presents results of flow experiments through a capillary tube, measuring the flow rate and pressure drop for various materials of interest. Unlike viscous fluids, he records a "friction constant" (a stress) that must be exceeded by the pressure in order for flow to occur and, thereafter, postulates a linear relationship. This empirical flow law evolved into the Bingham fluid: the archetypical yield stress fluid. However, it was not until the 1920s that ideas of visco-plasticity became more established (Bingham 1922) and other flow laws were proposed, e.g., Herschel and Bulkley (1926). Inherent non-linearity in flow behavior slowed the evolution from geometry-specific flow laws and rheometry into a proper constitutive description until much later; see Oldroyd (1947) and Prager (1954).

Although mechanized oil well drilling dates from the 1850s, the modern industrial era started in the 1890s–1910s. In North America, many state-based oil companies became established in this period. In Azerbaijan, production grew to 200 MStb/d (>50% of global production), the first production pipelines were laid, foreign companies were granted mineral rights, and the Russian revolution then interrupted the party. European companies also first became active in the Middle East (initially in the present day Iran). Broad interest in oil-related technology and engineering, together with a perceived need to share this knowledge, resulted, in 1913, in the establishment in London of the Institution of Petroleum Technologists and in the USA of a standing committee on oil and gas within the American Institute of Mining Engineers (later evolving into the Society of Petroleum Engineers). Meetings, symposia, and a shared technical literature began to emerge.

Drilling muds and cements were already being used within the industry, but it was muds that attracted interest. There was a growing recognition of the importance of well-designed drilling muds to mitigate risks of blowouts, lost circulation, and stuck pipe, and to minimize erosion from cuttings. Concepts of viscosity and fluidity were still evolving generally, with the term "rheology" being introduced in the 1920s. As well as controlling mud density, there was a focus on viscosity and the need to measure and characterize in a repeatable way. For example, the Marsh funnel emerged (Marsh 1931) and is still in use today. Bingham's ideas on plastic flow found an audience within this technical community and he was invited to speak in 1933 at the first World Petroleum Congress (Bingham 1933). His main messages were the standardization of viscosity measurement/units and an introduction of plastic flow terminology, also with a view to standardization. It is from around this time that we see visco-plastic concepts taken up more widely in the oil and gas industry, both to characterize fluids and to measure their properties, e.g., Lewis et al. (1935) and Jones and Babson (1935).

In this paper, we skip forward from the above historical notes. The objective is to review why and how yield stress fluids are important in today's oil and gas industry: Bingham's rheological legacy. From the perspective of both fluid mechanics and rheology, the oil and gas industry is incredibly diverse: the different unit operations that involve fluid flow, the properties of the fluids used, the richness of flow phenomena that occur by design, or otherwise. Of course, not all oil and gas flows involve yield stress fluids; suspensions, granular, shear-thinning, thixotropic, and viscoelastic media are also common and most production flows are complex multi-phase flows. Thus, undoubtedly, our review will not cover all facets in the depth required.

In outline, our paper proceeds sequentially by reviewing those operations that involve yield stress fluids to an important degree. We cover in varying depths the following areas/operations: reservoir flows of heavy oil (in "Reservoir flows of visco-plastic heavy oils"); drilling fluids and operations (in "Drilling fluids and operations"); wellbore cementing (in "Wellbore cementing"); hydraulic fracturing and some open-hole completions (in "Fracturing and open-hole completions"); sealing/remedial operations, e.g., squeeze cementing, lost circulation ("Sealing operations"); waxy crude oils and flow assurance: both wax deposition and restart issues (in "Flow assurance"). The aim is to outline both rheological aspects and relevant fluid mechanics issues.

The above selection of topics admittedly is focused on operations upstream of the refinery. The ordering of topics in our paper in "Reservoir flows of visco-plastic heavy oils–Flow assurance" is based on the processes from reservoir to pipeline, e.g., we drill the reservoir, then cement/complete, then potentially fracture.

## Reservoir flows of visco-plastic heavy oils

In the 1950s, heavy oils exhibiting yield stress behavior were being extracted in the former USSR, leading to questions of how such fluids would flow in porous media. Fiber-bundle or capillary-tube models of yield stress fluids flowing through a porous media naturally lead to a limiting pressure gradient (LPG) that must be exceeded in order to flow. Thus, LPG generalizations of Darcy's law into nonlinear filtration/seepage laws were suggested and studied since the early 1960s, e.g., Sultanov (1960) and Entov (1967), and are attributed to Mirzadzhanzade (1959). An interesting feature of such models, even in homogeneous porous media and for simple flow settings, is the occurrence of dead zones in the reservoir, where the LPG is not exceeded and oil cannot be recovered. Taking a simple example of a single well in a 2D reservoir, the geometric configuration of dead zones depends strongly on the geometry and conditions imposed far from the well, as shown in elegant analytical solutions summarized in Barenblatt et al. (1989).

Resource depletion has led to increasing production of heavy oils worldwide and hence a renewed interest in reservoir flows. Rheological behavior in laboratory and reservoir shows wide geographical variation, from very viscous Newtonian to visco-plastic. Thus, LPG flow models are still employed, with flow laws fitted either to flow cell data or from closure approximations. A wider scientific interest is simply to understand the flow of yield stress fluids through porous structures. Without any Darcy-type closure, one may resolve the Stokes equations directly. 2D flows through uneven geometries (simulating porous channels) have been studied numerically by various authors (Balhoff and Thompson 2004; Roustaei and Frigaar 2013; Bleyer and Coussot 2014; Roustaei et al. 2015, 2016). Others have considered flow through packed beds or porous structures experimentally (Park et al. 1973; Al-Fariss and Pinder 1987; Chase and Dachavijit 2003, 2005; Clain 2010; Chevalier et al. 2013), which lead to both macroscopic closures and sometimes microscopic studies of the flow.

Fully 3D computations of yield stress fluid flows through digitized porous media geometries are still challenging (although manageable for Newtonian fluids), but macro-scale pore-throat network models have been developed

(Balhoff and Thompson 2004). In two dimensions, a range of different macro-scale models of porous media have been developed. Typically, a pore network or lattice is connected by capillary tubes along which one-dimensional flows (or similar closures) are assumed. Local heterogeneity can be introduced into the network via throat resistance or length, either systematically or stochastically, see Balhoff and Thompson (2004), Chen et al. (2005), Sochi and Blunt (2008), Balhoff et al. (2012), Talon et al. (2013), Talon and Bauer (2013), and Chevalier and Talon (2015). These approaches are beginning to understand macro-scale dynamics of the porous media flows.

Above, we have focused only on single-phase flows of yield stress fluids in a porous reservoir. There are also a number of multi-phase situations that involve interesting fluid mechanics/rheology with yield stress fluids or phenomena. These include (a) displacement flow of heavy oil by other fluids, (b) displacement of conventional oils by various polymer solutions, (c) creation of water/oil emulsions at the interface during water production, and (d) the use of hydrogels for water shutoff in mature reservoirs. For brevity, we do not review any of these here.

## Drilling fluids and operations

Drilling fluids are designed to perform several functions during drilling operations, including formation protection, pressure balancing (primary control), borehole stabilization, drill string and drill bit lubrication, and thermal management, as well as the transmission of information signals and energy. The primary rheological function of drilling fluids is the removal, transport, and separation of rock cuttings. Conventional water-base drilling fluids are thixotropic, shear-thinning yield stress fluids, dispersions of bentonite clay.

### Drilling fluid characterization

The classic Bingham relationship provides simplified characterization of drilling fluid rheology. Established American Petroleum Institute (API) standards for assessing drilling fluid rheology stipulate torque data acquisition at 300 and 600 rpm using a Fann® Model 35 viscometer, providing fitted values of the Bingham viscosity and yield stress. Quiescent wait times of 10 s or 10 min, followed by slow shear deformation, provide 10-s or 10-min gel strengths, respectively (American Petroleum Institute 1980). In addition to the common API protocols, yield points are also accessible using stress sweep or oscillatory amplitude sweep protocols.

Most service companies use additional viscometer readings in fluid design and laboratory characterization (typically 6 or 12, on a logarithmic scale), so that other rheological parameters may be fitted. Thus, the Herschel-Bulkley model has become a common standard, replacing and incorporating the earlier 2-parameter Bingham and power law models. The enduring popularity of these models operationally stems from the availability of analytical and semi-empirical closure expressions and approximations for hydraulic design calculations, dating from the late 1950s and onwards. These approaches are summarized in, e.g., Bourgoyne et al. (1986) and Govier and Aziz (1977), but are continually evolving. Many companies include internally researched results and/or geometry-specific approximations (e.g., the eccentric annulus) that make their predictions distinct, and such calculations are generally embedded within proprietary engineering design software that also calculate many other features relevant to drilling operations, e.g., torque and drag, hole cleaning parameters, swab/surge.

At the rig site, the drilling fluid is the responsibility of the mud engineer. The job here involves constant monitoring and adjustment. Mud weight (density) is the most important property controlled, followed by yield point and solid (cuttings) content. Drilling fluids in circulation are constantly changing, due to the incorporation of fine particles from the drill cuttings and due to mechanical degradation. Thus, initial designs of rheology in the lab are different from those that evolve on the rig. The mud engineer adjusts the drilling fluid rheology in response to monitoring and measurement. In the high-pressure operational environment, standardization of protocols and ease of application are the key. Continual rheological measurement is conducted using the Marsh funnel (Marsh 1931). Although the basic funnel design is still from the 1930s, efforts have been made to improve interpretation of the readings (Balhoff et al. 2011; Guria et al. 2013), and we should note that what is monitored with this apparatus is rheological change. Drilling rigs are mostly equipped with standard 6- or 12-speed viscometers, which are typically used daily to quantitatively characterize the mud shear rheology.

Conceptual simplicity and the above outline of design and operational procedures helps understand why models such as the Bingham fluid, traditionally, have played and will continue to play an important role in oil and gas well drilling. More complex rheological features (reviewed below) are certainly of importance and are incorporated in fluid design. Indeed, this industry is remarkably innovative rheologically. However, pragmatism and inertia, together with absence of clearly defined and widely accepted new standards, maintain Bingham's name. Another reason for the adoption of these models concerns the study of more complex fluid flows, beyond hydraulics, e.g., solid transport, conditioning and displacement flows, fluids loss. Where the flow itself is complex and there is a high degree of process uncertainty (geometry, in-situ rheology, etc), the first aim industrially is to understand the leading order effects of intuitively understood and accepted parameters, e.g., yield stress, shear-thinning, and viscosity.

### *Rheological objectives*

Rheologically, the yield stress is desirable in drilling as it aids the mechanical suspension of rock cuttings and co-formulated weighting material (e.g., barite, ilmenite, or $CaCO_3$ particles), preventing sedimentation in the borehole. Large ratios of yield point to plastic viscosity are generally thought to be desirable, serving to optimize the carrying capacity of the drilling fluid while simultaneously enabling reduced pumping rates and accompanying energy losses during circulation. Modern findings show that gel strength and low-shear-rate viscosity provide an improved measure of cutting removal performance (Becker et al. 1991).

Often misunderstood conceptually is the role of the yield stress in cutting transport flows. Conventional drill strings rotate rapidly during drilling and the (annular) drilling geometry can vary due to both unconsolidated formation and to changing drill string position (temporally as well as axially). Thus, the notion of a rigid unyielded plug moving along a uniform annulus carrying suspended cuttings is false. A dense particle induces shear stresses in the surrounding fluid which can yield the fluids allowing the particle to settle under its own weight. The critical ratios of yield stress to buoyancy stress have been long known for simple geometries, e.g., Beris et al. (1985). However, in simple flows such as a Poiseuille flow, the shear stress varies linearly, reducing locally the amount of yield stress available to rigidly suspend particles. Thus, the transition between rigidly suspended transport or settling depends critically on the particle positioning, as shown by Merkak et al. (2009). Such distinctions become more important in horizontal drilling.

In geometries with slow streamwise variation, extensional stresses also act to yield the otherwise uniform plug, resulting in large pseudo-plug regions within which to leading order the yield stress is just exceeded (Putz et al. 2009). Thus, a more accurate picture of how the yield stress influences cutting transport is via viscous drag (to which the yield stress significantly contributes), from a fluid that in laminar regimes will have strong transverse gradients due to shear and extension. At higher flow rates as the drilling fluid becomes turbulent, the viscous stresses become progressively less important.

The above situation is quite different when the pumps are stopped, as it frequently occurs operationally. Now, the yield stress is vital for suspending solids, preventing sedimentation within the wellbore. Here, thixotropy generally imparts beneficial mechanical properties to the drilling mud. In stagnant conditions, the effective yield stress (gel strength) provides suspension of rock cuttings and this is a thixotropic effect. Conversely, during continuous drilling and pumping operations, shear-induced viscosity reduction allows for higher flow rates that facilitate efficient transport of rock cuttings to the surface.

## Thixotropy

Thixotropy is natural in many drilling fluids due to their composition. Although it might be thought that rapid aging and development of a large static gel strength would be ideal for solid suspension in static conditions, this also makes re-establishing mud circulation and pipe movement difficult, so that in practice, a compromise is sought and the net benefits of thixotropy to drilling are under constant review.

In short and medium distance wells, thixotropy generally benefits drilling operations. During static conditions, which occur during breaks in fluid circulation, thixotropic structural buildup prevents barite sag and provides suspension of the rock cuttings. In conventional water-base drilling fluids formulated with bentonite clay, attractive forces arise between opposing electric charges located on the basal and edge surfaces of the bentonite platelets, driving assembly of a colloidal gel structure at quiescent and low-shear-rate conditions (similarly with sepiolite, laponite, or montmorillonite particles). The colloidal structure imparts a yield stress to the fluid. Thixotropic structural buildup allows a strong gel to form with a relatively low clay content. Upon resumption of shearing, the colloidal gel structure undergoes fragmentation, driving a thixotropic reduction in viscosity. During continuous circulation, low viscosity facilitates efficient removal and transport of cuttings as well as efficient energy transfer to the mud motor. Thixotropic viscosity reduction thereby facilitates high drilling penetration rates by reducing energy losses associated with the drilling fluid in contact with the drill string and bit. Thixotropic viscosity reduction also facilitates efficient separation of rock cuttings and entrained gas in surface separation units where fluid agitation is maintained. In sum, thixotropic structural buildup and viscosity reduction facilitate efficient drilling operations in conventional wells.

However, in extended reach and deepwater wells, the balance shifts. Thixotropy contributes to detrimental pressure swings (surge/swab pressures) arising in the borehole during operations such as casing insertion, drill string positioning, cementing, and the initiation of circulation. Thermal disparities along the flow path of the drilling fluid exacerbate the pressure swings. Deepwater wellbores are particularly vulnerable to pressure fluctuations. In deepwater reservoirs, the envelope between local pore pressure and local fracture pressure is often narrow, as it is with extended reach horizontal wells. In order to prevent formation fracturing and intrusion of formation fluids into the wellbore, downhole pressure conditions must be maintained within the pore-frac pressure envelope. Pressure variations exceeding these limits may ultimately compromise the integrity of the well or formation. Formation fractures usually lead to substantial loss of drilling fluid to the formation. Conversely, large pressure reductions may lead to hole collapse or invasion of reservoir fluids (loss of primary well control). In deepwater drilling, conventional drilling fluid formulations are typically unable to maintain borehole pressures within the respective limits, due to a combination of thixotropic and temperature-dependent fluid rheology.

### *Classical thixotropic models*

Ideal thixotropy denotes a time-dependent viscous response to imposed changes in shear rate, originating from flow-driven alteration in the fluid structural state (Larson 2015). Ideal thixotropic fluids exhibit instantaneous stress dissipation upon flow cessation, indicating an absence of elastic recoil effects. Ideal thixotropy may readily incorporate explicit yielding functionality, as quantified by a Bingham-like yield stress parameter. Conversely, non-ideal thixotropic fluids exhibit a viscoelastic response at timescales shorter than the thixotropic response. In a general description of thixotropy provided by Moore (1959), the structural state of a thixotropic fluid is ascribed to a structural parameter $\lambda(t)$ which adheres to the following dynamic relation:

$$\frac{d\lambda}{dt} = k_+(1-\lambda) - k_-\dot{\gamma}\lambda \tag{1}$$

where $k_+$ and $k_-$ denote buildup and breakdown coefficients, establishing an equilibrium $\lambda_e$ value at each specified shear rate. Upon changes in shear rate conditions, the structural parameter $\lambda$ exhibits a characteristic relaxation time of $T = 1/(k_+ + k_-\dot{\gamma})$. A typical constitutive rheological equation of state, incorporating explicit yielding as well as shear thinning phenomena, is a modified Cheng-Evans relation (Tehrani and Popplestone 2009)

$$\tau(t) = \lambda(t)\tau_y + (\eta_\infty + c\lambda(t))\dot{\gamma}^m. \tag{2}$$

Analytical incorporation of a transient response for $\lambda(t)$ provides a unified description of thixotropy, yielding, and shear-thinning phenomena, thereby maintaining explicit yielding functionality while neglecting elastic recoil responses.

In principle, thixotropic parameters are extractable from any prescribed variation in shear rate, allowing experimental corroboration with diverse protocols such as imposed stress ramps, hysteresis loops, and shear rate step changes (Tehrani and Popplestone 2009). However in practice, delineating yielding, shear-thinning, and thixotropic rheology require tailored protocols, due to co-occurrence of multiple rheological phenomena, including viscoelastic responses. Prescribed shear rate step changes establish rate coefficients for thixotropic structural buildup and breakdown, which in conjunction with steady state shear rate curves provide comprehensive rheological predictions in shear mode. Herzhaft et al. (2006) established a unique measurement regimen in which pre-sheared fluid is subjected to two consecutive rest and shearing intervals, rigorously delineating $k_+$ and $k_-$ coefficients.

In an alternate thixotropic approach, a constitutive rheological equation of state formalism has been developed to theoretically capture very slow shearing at applied stresses lower than the nominal yield stress value. An apparent viscosity approximation is implemented, quantified as Livescu (2012)

$$\eta = \eta_0(1 + \beta\lambda^n), \tag{3}$$

providing asymptotic creeping flow predictions at low shear rates for $n > 1$, thereby circumventing an explicit shear stress threshold for flow initiation while retaining an applicable thixotropic functional response. Such models are driven by dynamic relation for $\lambda(t)$, such as the toy model of Coussot et al. (2002) and Moller et al. (2006). The apparent viscosity equation of state then inherently carries an implicit shear history-dependent shear stress threshold that delineates the two bifurcating shear regimes, highlighting the modelling limitations of explicit stress threshold formalisms. Abandoning the explicit yield stress while formalizing an implicit yield stress in this way can provide improved versatility in modelling deterministic thixotropic processes occurring at very low shear rates, while retaining a relevant stress threshold for large-scale flow initiation. Such a modelling approach re-establishes continuous deformation at applied stresses less than the nominal yield stress, successfully reproducing avalanche behavior and a demonstrable bifurcation in steady state viscosity. Rheological modelling of these phenomena has led to improved understanding of complex processes such as barite sag (gravitational separation of weighting material) and swab/surge pressures (transient pressure troughs and peaks arising during drill string positioning movements).

### *Stress-driven thixotropic models*

Many models adopt a Herschel-Bulkley-like constitutive equation to describe the yield stress, e.g., Eq. 2. The archetypical thixotropic model that incorporates shear-thinning and yield stress behavior within the classical framework outlined above is that of Houska (1981) (also used in modelling waxy crude oils).

$$\begin{cases} \tau = \tau_y(\lambda) + K(\lambda)\dot{\gamma}^{n(\lambda)} & \text{when} \quad \tau \geq \tau_y(\lambda) \\ \dot{\gamma} = 0 & \text{otherwise} \end{cases} \tag{4}$$

where $\tau_y(\lambda)$, $K(\lambda)$, and $n(\lambda)$ are, respectively, the structure-level-dependent yield stress, consistency index, and behavior index. The structure-level-dependent yield stress is invariably assumed to vary linearly with λ, i.e., commonly $\tau_y(\lambda) = \lambda\tau_{y,0}$, where $\tau_{y,0}$ is the yield stress of the fully structured material. Therefore, $\tau_{y,0}$ is the yield stress in the classical sense. Thus, $\tau_y(\lambda)$ is maximum when the material is fully structured ($\lambda = 1$) and and decreases monotonically as the material becomes less structured, reaching zero only when the material becomes completely unstructured ($\lambda = 0$).

The problem with constitutive equations of the form of Eq. 4 is that they predict a behavior that is in clear disagreement with experimental evidence. Specifically, according to Eq. 4, when $\tau \leq \tau_y(\lambda)$, the material retains a solid-like behavior throughout the whole range of λ. However, real yield stress materials display a solid-like behavior before yielding only when $\lambda = 1$. The yielding process typically consists of a dramatic rupture of the percolated microstructure that was responsible for conferring a solid-like behavior to the material. After yielding ($\lambda < 1$), the structure typically consists of flocs or aggregates suspended in a continuous phase, i.e., liquid-like suspension behavior is observed. Therefore, the assumption that the yield stress depends on λ is questionable. In other words, the viscosity is infinite at $\lambda = 1$ but becomes finite after yielding ($\forall\ \lambda < 1$), *regardless of the applied stress*. Moreover, the viscosity decreases monotonically as the structuring level is decreased.

A different approach that borrows partly from the dynamical approach of Coussot et al. (2002) and Moller et al. (2006) has been advanced recently in the series of articles (de Souza Mendes 2011; de Souza Mendes and Thompson 2012, 2013; Van Der Geest et al. 2015). The main features are as follows.

- In this approach, thixotropy is described by a dynamical system whose equilibrium locus is the flow curve, which is thus an important input of the model. Therefore, by construction, these models always predict the correct flow curve. Such an equilibrium is also present in models of Houska type but has not been given much attention. This issue plays a major role in describing the mechanical behavior of thixotropic materials, and neglecting this fact is expected to lead to unphysical predictions. This is discussed in detail elsewhere (de Souza Mendes and Thompson 2012).
- The key difference with those models considered in the previous section is that it is assumed that the agent that breaks the microstructure is the current stress, instead of the shear rate. Since the microstructure exists due to bonds between structural units, it is easy to see that it is the action of external forces (or imposed stress) that can break these bonds. At first, it may seem that this is an irrelevant detail, because shear rates are caused by stresses, and so the two quantities would be equivalent as far as this matter is concerned. However, this is by no means the case: it is not difficult to invoke real situations of non-zero stress with zero shear rate (e.g., the avalanche effect in a viscoplastic fluid) and others in which the stress is zero or very small but the shear rate is arbitrarily large (e.g., the onset of a constant shear rate flow of an elasto-viscoplastic gel) (de Souza Mendes and Thompson 2012).
- The classical concept of yield stress—namely the stress below which no unrecoverable strain is observed—is preserved. Indeed, these new thixotropic models can be seen as a wider class of constitutive equations that can reduce neatly to the classic viscous, visco-plastic, or elasto-viscoplastic non-thixotropic models, as the timescales for structural changes become small.

As an illustrative example, we briefly describe the elasto-viscoplastic thixotropic model proposed in de Souza Mendes and Thompson (2013).

The constitutive equation is a generalized Jeffreys model given by:

$$\dot{\gamma} + \theta_2\ddot{\gamma} = \frac{\theta_2}{\eta_\infty}\left(\frac{\tau}{\theta_1} + \dot{\tau}\right) \tag{5}$$

where

$$\theta_1 = \left(1 - \frac{\eta_\infty}{\eta_v(\lambda)}\right)\frac{\eta_v(\lambda)}{G_s(\lambda)} \quad ; \quad \theta_2 = \left(1 - \frac{\eta_\infty}{\eta_v(\lambda)}\right)\frac{\eta_\infty}{G_s(\lambda)} \tag{6}$$

$$G_s = G_o e^{m\left(\frac{1}{\lambda} - \frac{1}{\lambda_o}\right)} \tag{7}$$

$$\eta_v(\lambda) = \eta_\infty e^{\lambda} \tag{8}$$

where $\theta_1$ and $\theta_2$ are, respectively, the relaxation and retardation times; $\eta_\infty$ is the infinite-shear-rate viscosity; $\eta_v(\lambda)$ is the viscosity; and $G_s(\lambda)$ is the shear modulus, which we note both depend upon the structural parameter λ. The shear modulus of the fully structured material is $G_o$ and $m$ is a parameter to be determined experimentally. For the case of inelastic materials ($G_o \to \infty$ and hence $\theta_1 = \theta_2 = 0$), Eq. 5 reduces to the following generalized Newtonian equation, namely

$$\tau = \eta_v(\lambda)\dot{\gamma}, \tag{9}$$

but otherwise is viscoelastic.

The evolution equation for λ is

$$\frac{d\lambda}{dt} = \frac{1}{t_{eq}}\left[\left(\frac{1}{\lambda} - \frac{1}{\lambda_o}\right)^a - \left(\frac{\lambda}{\lambda_{eq}(\tau)}\right)^b\left(\frac{1}{\lambda_{eq}(\tau)} - \frac{1}{\lambda_o}\right)^a\right] \tag{10}$$

$$\lambda_{eq}(\tau) = \ln\left(\frac{\eta_{eq}(\tau)}{\eta_\infty}\right) \tag{11}$$

where $\lambda_o$ is the value of $\lambda$ corresponding to the fully structured material. Note that in this model, $0 \leq \lambda \leq \lambda_o$, $\lambda_o$ being infinite for yield stress materials and large but finite for apparent yield stress fluids. Here, $\lambda_{eq}(\tau)$ corresponds to the equilibrium structure level evaluated at the current stress $\tau$; $\eta_{eq}(\tau)$ is the corresponding equilibrium viscosity evaluated at the current stress $\tau$; $t_{eq}$ is the microstructure buildup time; and $a$ and $b$ are parameters to be determined experimentally. Thus, for yield stress materials, the evolution equation simplifies to

$$\frac{d\lambda}{dt} = \frac{1}{t_{eq}}\left[\left(\frac{1}{\lambda}\right)^a - \left(\frac{\lambda}{\lambda_{eq}(\tau)}\right)^b \left(\frac{1}{\lambda_{eq}(\tau)}\right)^a\right]. \tag{12}$$

It is worth noting that when $t_{eq} = 0$, meaning instantaneous microstructure buildup (or equivalently, zero thixotropy), Eqs. 10 and 12 both reduce to $\lambda = \lambda_{eq}(\tau)$, as expected.

The equilibrium viscosity $\eta_{eq}$ (flow curve) is given by

$$\begin{aligned}\eta_{eq}(\dot\gamma) = &\left[1 - \exp\left(-\frac{\eta_o\dot\gamma}{\tau_y}\right)\right] \\ &\times\left\{\frac{\tau_y - \tau_{yd}}{\dot\gamma}e^{-\dot\gamma/\dot\gamma_{yd}} + \frac{\tau_{yd}}{\dot\gamma} + K\dot\gamma^{n-1}\right\} \\ &+\eta_\infty\end{aligned} \tag{13}$$

where $\eta_o = \eta_\infty e^{\lambda_o}$ is the viscosity of the fully structured material; $\tau_y$ and $\tau_{yd}$ are, respectively, the static and dynamic yield stresses; $K$ is the consistency; and $n$ is the power law index. It is not difficult to see that Eq. 13 reduces to the Herschel-Bulkley viscosity function in the case of yield stress materials ($\lambda_o \to \infty \Rightarrow \eta_o \to \infty$) that possess a single yield stress ($\tau_{yd} = \tau_y$).

A drawback shared by all thixotropy models available to date is the excessive number of parameters which are hard to determine experimentally, rendering rather the difficult usage in practical applications. In addition, the functional forms of the buildup and breakdown terms of the evolution equations for $\lambda$ are often arbitrarily defined with the motivation of mathematical simplicity, which undermines the predictive capability.

### *Flat rheology*

Flat rheology drilling fluids were developed in order to resolve the operational issues related to pressure management in extended reach and deepwater boreholes. In addition, the new formulations offer improved cutting removal performance in remote high-temperature wells where significant thinning otherwise occurs with conventional drilling fluids. Flat rheology fluids have stable rheological properties across extended temperature and pressure ranges. Well-defined yielding characteristics, attributable to minimal thixotropy, are also provided in flat rheology drilling fluids. The gel strengths of flat rheology fluids are therefore relatively stable with respect to static time interval; this property is often referred to as non-progressive gel strengths.

Flat rheology drilling fluids are specifically tailored synthetic- or oil-base fluid formulations containing emulsified water. Bentonite is not inherently dispersible in oil, due to a lack of swelling and platelet delamination. Prior to application in non-aqueous fluid formulations, bentonite clay is modified with quaternary fatty acid amines in order to disperse the platelets. When organophilic clay (amine-treated bentonite) is applied in non-aqueous drilling fluid formulations, electrostatic interactions are minimal. Nevertheless, dispersed organophilic clay imparts significant yielding, thixotropy, and temperature-dependent rheology to the fluid. In order to obtain flat rheology, the clay content is generally reduced and counteracted by rheological modifiers and viscosifiers. Several strategies are available to provide rheology modification. Modifying polymers undergo coil expansion and retraction at high and low temperatures, respectively. Changes in polymer conformation serve to modulate the fluid rheology, counteracting the inherent temperature-dependent rheology of organophilic clay dispersions in oil (Mullen et al. 2005). In another modification strategy, thermally activated surfactants interact with organophilic clay at high temperatures, driving additional structural buildup to counterbalance the inherent thinning of organophilic clay dispersions at increasing temperatures (Mullen et al. 2005). Formulation strategies may also involve manipulating the role and functional activity of the emulsifier. Effective emulsifiers ensure thermally stabile emulsions, extending the flatness of the rheology profile to increased temperatures. Designated emulsifiers may also reduce structural buildup of organophilic clay at low temperatures, counteracting the inherent thickening of clay at low temperatures (Shursen 2014). A reduction in thixotropic structural buildup provides non-progressive gel strengths. In all modification strategies, the total balance of rheological character stemming from clay and modifier results in temperature-insensitive and pressure-insensitive yielding properties. Thermal and baric stability, along with low thixotropy, meet the broadest definition of flat rheology.

A distinct strategy for obtaining flat rheology is to eliminate clay and exploit the emulsion structure to impart gel strength and yielding characteristics to the fluid. Emulsion gels are usually fragile, but show well-defined yielding characteristics that are advantageous during drilling of remote high-temperature wells. Clay-free synthetic-based drilling fluids were first developed in 2001, formulated using a synthetic ester-internal olefin blend (Burrows et al. 2004). In a recent development, a clay-free oil-based drilling fluid formulation was introduced with combined chemical and particulate stabilization (Carbajal et al. 2009). Thermal and baric stability in yielding characteristics is complemented by non-progressive gel strengths. Rapid rheological transitions associated with the emulsion are characterized by minimal thixotropy. Rapid structural buildup upon flow cessation leads to excellent resistance against barite sag. Similarly, rapid viscosity reduction upon shearing application serves to minimize surge and swag pressures, facilitating downhole pressure management. Clay-free fluids have additional benefits for drilling operations. Clay-free fluids do not undergo significant thinning at high-temperature and high-pressure downhole conditions, providing fluid suspension characteristics without imparting increased viscosity, which benefits cutting removal and transport performance as well as downhole pressure management. Clay-free fluids do not demand the involved on-site logistics related to conditioning of clay-containing fluids and tolerate extended static periods in the borehole. Finally, clay-free drilling fluids provide excellent formation protection as quantified by return permeability measurements.

An alternate means of eliminating most solids from drilling fluids is to utilize highly concentrated formate brines (Downs 1993). Highly soluble cesium formate imparts a relative density as high as 2.3 without utilizing weighting material, although low $CaCO_3$ contents are often retained as filtercake material. Mixtures of potassium/cesium formate may be employed, often formulated together with biopolymers (xanthan gum, polyanionic cellulose, or starch) as viscosifying and fluid loss control agents. Formate brine formulations offer favorable toxicity, biodegradation, anti-microbial, anti-oxidative, anti-hydrolytic, anti-corrosivity, and elastomeric compatibility properties, and also stabilize biopolymers at high temperatures via a distinct salting-out phenomenon. Formate brine formulations mitigate formation impairment risks by minimizing insoluble solids and ensuring compatibility with reservoir sulfate ions and carbonate ions. Formate brines are distinctly applicable for mechanically stabilizing shale formation wellbores by (1) increasing filtrate viscosity and (2) generating osmotic backflow of pore water, serving to reduce pore pressures and thereby stabilizing the well. Temperature stability and low plastic viscosity values are provided with low MW polyanionic cellulose, providing effective hydraulic energy transmittance to the mud motor, while minimizing frictional losses (“drag reduction”) in turbulent flows. Hence, formate brine formulations provide many of the same performance benefits as designated “flat rheology” fluids.

## Wellbore cementing

All oil and gas wells undergo multiple cementing operations during their lifetime. During construction, a steel casing is inserted into newly drilled sections of borehole and is cemented into place (primary cementing). As the well descends deeper into the earth, the operation is repeated as successive casings are cemented into place. Objectives of this operation include (i) mechanical support for the well, (ii) hydraulically sealing the annular region outside the casing, (iii) preventing fluid migration along the well, and (iv) preventing corrosive formation brines from reaching the casing. Additionally, at various times during well construction, remedial operations must be executed and at the end-of-life stage, wells are permanently abandoned. Here, cement plugs are commonly used. Both operations are outlined and discussed in depth by Nelson and Guillot (2006).

The fluid flows that occur in cementing operations are characterized by the pumping of multiple fluid stages along a flow path. The volumes are such that normally each fluid stage interacts only with those before/after. The in situ fluid is typically a drilling mud, which must be removed and replaced with the cement slurry, ensuring an adequate bond of the cement to both casing and formation. Drilling fluids have been described above. Due to cement-mud incompatibility, a number of pre-flushes are pumped ahead of the cement slurry. These are loosely classified into washes and spacers. Cement slurries are fine colloidal suspensions that react (relatively slowly) during hydration. The rheology of cement slurries is discussed below in “Rheology of cement slurries” section. All these fluids are generally of different densities and are typically characterized rheologically as shear-thinning yield stress fluids, although this is of course a pragmatic simplification.

The function of washes is to thin and disperse the mud. The wash is usually water-based (or simply water) and becomes turbulent due to its low viscosity. Washes contain similar dispersants as in cement slurries and may also contain surfactants if oil-based fluids are to be removed. Spacers are viscous fluids custom designed to prevent mud-cement contact/contamination and aid mud removal. The term spacer includes relatively low viscosity suspensions that may follow the wash in turbulent flow, fluids such as scavenger slurries (low density cement) but in more recent years has increasingly meant fluids that are sufficiently viscous to generally be pumped in inertial laminar regimes. These fluids are varied and proprietary, but commonly include a combination of viscosifiers (e.g., polyacrylamides, cellulose derivatives, xanthan/bio-polymers, clays such as bentonite); dispersants (e.g., polynapthalene sulfonate); fluid loss agents; weighting agents (e.g., barite, fly-ash, hematite), surfactants, and other optional chemicals, e.g., NaCl/KCl, to inhibit dissolution/damage of certain formations. In general, the idea of a laminar spacer is to have density and effective viscosity intermediate between the cement slurry and drilling mud, eliminating chemical incompatibilities. Examples and more information may be found in Nelson and Guillot (2006).

The main fluid mechanical focus of primary cementing is on removing the drilling mud from the annulus, replacing it with cement slurry that can bond to both the outside of the casing and inside of the borehole, setting hard. Detrimental effects arise if either the mud is not removed or if there is excessive mixing of the cement slurry with other fluids. The former can result in porous hydraulic pathways along the well, caused by dehydration of the mud as the cement sets. The latter can result in contamination that can prevent the hydration reactions from completing and the cement from hardening. The risk in either case is that reservoir gases can migrate along the cemented borehole, leaking to surface.

Thus, cementing flows of interest tend to be fluid-fluid displacement flows. The regular flow geometries are the pipe or eccentric annulus, both of which are inclined relative to gravity. Pump rates used can place the flows anywhere in the laminar to fully turbulent range. Generally speaking, considering a two-fluid displacement: six dimensional and two dimensionless parameters describe the fluids; two to four parameters describe the geometry, plus an inclination angle, plus gravitational acceleration and a flow rate. Following a dimensional analysis, 10–12 dimensionless groups describe the full range of flows, meaning that exhaustive study of these flows is practically impossible. This physical and parametric complexity is part of the challenge of understanding cementing. The other aspect that makes cementing flows difficult is that unlike drilling, these are single volume flows, by which we mean that the in situ fluids are to be replaced by the cement slurry and other fluids pumped. There is no continual circulation to allow monitoring of the flows, there is generally little downhole instrumentation/monitoring during the operation, and post-placement evaluation of job effectiveness is limited.

The importance of the yield stress to primary has been acknowledged for at least 60 years, since the possibility of a mud channel forming on the narrow side of the annulus was first identified (McLean et al. 1966). This occurs if the axial pressure gradient is insufficient to move the mud, which leads to a simple operational rule. In the 1970s–1980s, cementing companies developed their own systems of design rules, purported to mobilize drilling mud and to ensure a steady displacement front advancing along the well, e.g., Jamot (1974), Lockyear and Hibbert (1989), Lockyear et al. (1990), Guillot et al. (1990), and Couturier et al. (1990). The physical reasoning behind such systems was based largely on developing simplified hydraulic analogies. These methods were generally targeted at laminar displacements in near-vertical wells (with turbulent displacements being regarded as anyway effective).

Since the 1990s, these methods have been re-examined and improved. Firstly, the advent of highly deviated and horizontal wellbores in the 1990s led to the identification of new problems for primary cementing; see Keller et al. (1987), Crook et al. (1987), and Sabins (1990). Among the fluid mechanics issues, large density differences tend to cause slumping towards the lower side of the annulus in highly deviated sections and settling effects in cement slurries are amplified. Secondly, computational fluid mechanics models have become a valuable predictive tool, and thirdly, there have been a number of concerted laboratory scale experimental studies of displacement flows. Below, we review those studies of flows in the different cementing geometries.

## Pipe flow displacements

Most cementing operations involve a pipe flow from surface down the well. Cement slurries are usually denser than drilling fluids, so that this displacement process is frequently mechanically unstable. Efforts are made to separate fluids physically with rubberized plugs, but operational constraints mean that these are frequently missing or only separate one or two interfaces. In plug cementing and remedial operations, smaller diameter tubing is common and separating plugs are not common. Consequently, it is of interest to study density unstable displacement flows of miscible fluids in long inclined pipes.

Miscible Newtonian displacement flows in pipes have been studied for many years. High Péclet number flows at low-moderate Reynolds numbers have been studied computationally (Chen and Meiburg 1996) and experimentally (Petitjeans and Maxworthy 1996), for limited ranges of pipe inclination and density differences. Effects of flow rate and viscosity ratio were studied in vertical displacement flows by Scoffoni et al. (2001), identifying stable finger, axisymmetric and corkscrew modes. Other experimental studies of vertical displacement flows include (Kuang et al. 2004; Balasubramaniam et al. 2005) investigating instabilities due to viscosity and density effects. All these flows are more structured than those found in cementing, which although laminar are significantly inertial, buoyant and include non-Newtonian effects.

A systematic extension of these studies towards cementing displacements is ongoing, focusing initially on Newtonian fluids, buoyancy, viscosity differences, effects of pipe inclination, and flow rate. The effects of increasing the mean flow velocity ($\hat{V}_0$) on near-horizontal displacement flows are studied in Taghavi et al. (2010), identifying three main regimes as $\hat{V}_0$ was increased from zero. At low $\hat{V}_0$, the flow resembles the exchange flows of Seon et al. (2005). As $\hat{V}_0$ is increased, the front velocity $\hat{V}_f$ was found to vary linearly with $\hat{V}_0$. The first two of these regimes may be either viscous or inertial-dominated. When the mean speed is further increased, we enter the turbulent regime where $\hat{V}_f = \hat{V}_0$. The behavior of the trailing displacement front was studied in Taghavi et al. (2011). A synthesis of the results on iso-viscous nearly horizontal displacement flows is presented in Taghavi et al. (2012c), based on a mix of experimental, numerical, and analytical results. These studies have been extended to the full range of pipe inclinations (Alba et al. 2013a), partly also to density stable displacements (Alba et al. 2012). Ongoing work is focused on studying viscosity ratio effects and shear-thinning behavior, where a variety of interesting instabilities are found.

Regarding yield stress effects, the field is less well explored. When the displaced fluid has a yield stress, it is possible for the flow to leave behind residual fluid layers stuck to the wall, which remain permanently. These are illustrated in the elegant study of Gabard-Cuoq (2001) and Gabard-Cuoq and Hulin (2003) in which vertical displacement of Carbopol solutions by glycerin results in beautifully uniform stationary residual layers. More recent work has focused on the case of a dominant yield stress (e.g., a drilling mud that is *hard* to displace) and displacing with density unstable Newtonian fluids; see Taghavi et al. (2012b), Alba et al. (2013), and Alba and Frigaard (2016). These flows result in two primary flow types: central displacement and slump displacements, distinguished parametrically by an Archimedes number. The slump displacements show a wonderful range of complex flow patterns, including those that rupture the displaced fluid and spiral patterns; see, e.g., Fig. 1. The stratified viscous regimes of Taghavi et al. (2010) and Taghavi et al. (2012c) have been modelled for two Herschel-Bulkley fluids; see Moyers-Gonzalez et al. (2013), but experimental reality in cementing regimes rarely conforms to the strict model assumptions. Ongoing research has studied the central regime extensively (in the absence of any density difference; Moises 2016) and studied vertical pipes with a range of positive and negative density differences.

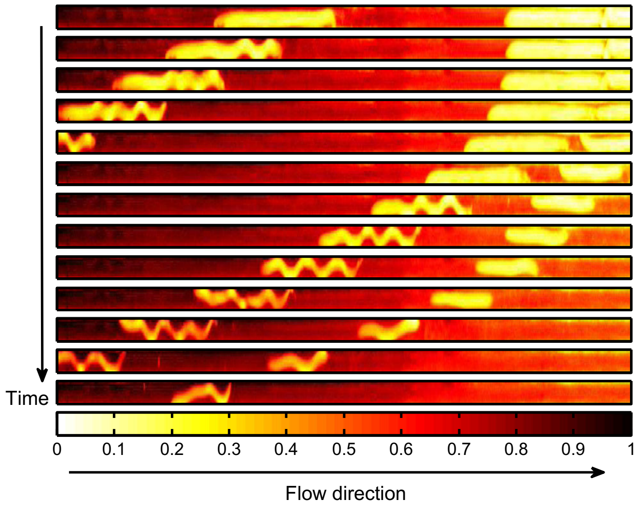

**Fig. 1** Time sequence from a downward density unstable displacement of 0.1125% Carbopol solution (yield stress fluid, *colorbar* = 0) with weighted water (*colorbar* = ‘) with ≈3.2% density difference at mean velocity $\hat{V}_0 = 9.4$ mm/s: images at 3-s intervals

## Narrow annular displacements

The second and most critical displacement geometry is the annular space formed by the outside of the steel casing and the inside of the borehole. Typically, the mean annular gap is in the range 1–3 cm, but even when wells are vertical, the annulus is eccentric. Modern wells typically start with a vertical section (surface casing) and end up aligning directionally with the reservoir (production casing). Cemented sections are typically many hundreds of meters long, and the diameters of the steel casings decrease with depth. The annulus is initially filled with drilling mud which should be pre-circulated for conditioning prior to the displacement. Displacing fluids enter the annulus at the bottom and move upwards to surface: the detrimental unstable density difference inside the casing is now stabilizing. Whatever mixing has occurred inside the casing between fluids is now transferred to the annular displacement.

The majority of fluid mechanic studies have focused on laminar displacement flows. A popular approach has been to average the velocity field across the narrow annular gap, thus reducing the flow to a 2D problem for the gap-averaged velocity field. The earliest developments were by Martin et al. (1978). A further-simplified pseudo-2D approach was developed and validated against a series of experiments in Tehrani et al. (1992, 1993), and this style of model was also derived and solved computationally in Bittleston et al. (2002). Fully 2D computations, a rigorous analysis of the model and comparisons with some of the rule-based systems can be found in the series of papers (Pelipenko and Frigaard 2004a, b, c), targeted at near-vertical displacements. For example, in Pelipenko and Frigaard (2004c), it is shown that rule-based systems such as the earlier (Couturier et al. 1990), although physically sensible, can be extremely conservative in the requirements needed for an effective displacement. Near-vertical experiments and model comparisons were made in Malekmohammadi et al. (2010). Strongly inclined and horizontal wells have been studied in Carrasco-Teja et al. (2008a, b) and more recently, the effects of casing rotation have been studied in Carrasco-Teja and Frigaard (2009, 2010) and Tardy and Bittleston (2015). Qualitatively, this level of modelling is adaptable to rather complex wellbore geometries and has been shown to identify bulk features of the flow, such as mud channels remaining stuck on the narrow side of the wellbore, see Fig. 2, for an example. Such models are appropriate for process design and predict well the dominant effects of wellbore eccentricity, rheology, density differences, and inclination. Variants of this approach are increasingly widely used in industry, e.g., Tardy and Bittleston (2015), Guillot et al. (2007), Chen et al. (2014), and Bogaerts et al. (2015). It is interesting to reflect that the above approach is mathematically analogous to the LPG reservoir flows outlined in "Reservoir flows of visco-plastic heavy oils," with varying annular gap width corresponding varying permeability.

Aside from Tehrani et al. (1992, 1993) and Malekmohammadi et al. (2010), other experimental studies include that of Jakobsen et al. (1991) that investigated a subset of density and rheology differences, eccentricity, inclination, and Reynolds number. A number of authors have studied the annular flows in 3D computationally. For example, Szabo and Hassager (1995, 1997) studied Newtonian displacements in eccentric annular geometries. Comparisons between the 3D computational fluid dynamics (CFD) results of Vefring et al. (1997) and earlier experiments of Jakobsen et al. (1991) are generally favorable. In a modern era of massively parallel computation, one might ask why 3D CFD has not had more impact? The first point here is that advantages over the 2D models come from resolving the scale of the annular gap (cm scale). 3D meshes at that resolution become unmanageable over circumferential distances of ~0.5 m and wellbore lengths of many hundreds of meters (e.g., $\gtrsim 10^9$ mesh nodes). Secondly, many of the critical features of mud removal displacements concern the yield stress and the residual fluid left behind in the annulus. Reliable implementation of yield stress models into CFD codes, in a way that resolves the unyielded regions properly, results in considerable additional computational iteration compared to a Newtonian fluid flow. Thirdly, there is a question of resolution, data processing, and analysis: the coarse-graining of an averaged approach leads to fairly simple interpretations of displacement results, in much of the annulus nothing much is happening, etc.. Most critical however is certainly the large dimensionless parameter space discussed earlier (10–12 parameters). This rules out systematic study on the scale of the wellbore. Experiments also have issues of scale. In lab scale displacements, the annular lengths used are limited (typically <10 m), which makes interpretation of these studies harder in comparison to 2D models on the wellbore scale.

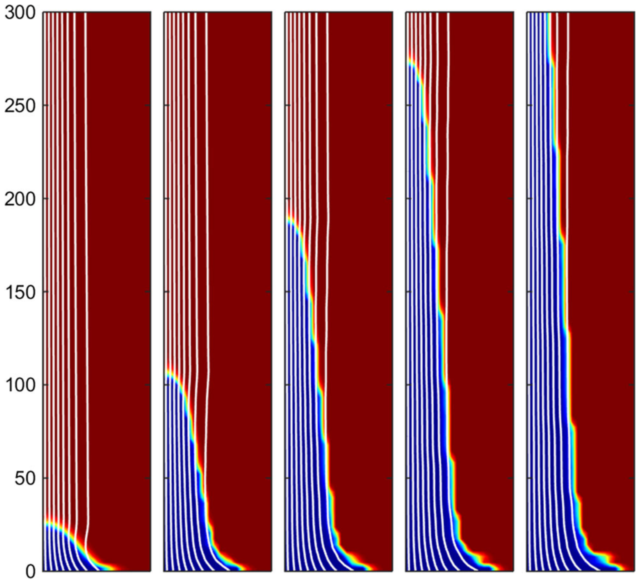

**Fig. 2** Displacement using approach of Bittleston et al. (2002) and Pelipenko and Frigaard (2004b). Images show half (*wide-narrow side*) of an unwrapped vertical annulus (310 ft long, 7 in. ID, 8.9 in. OD, 30% eccentric): 1.68 SG mud (*red*) with 50 Pa yield stress, displaced by 2.0 SG spacer fluid (*blue*) with 0.41 Pa yield stress (*white* = streamlines). Static mud remains

## Micro-annuli and washouts

A different way of resolving the through-gap distribution of fluids is to consider longitudinal sections of the narrow annulus, i.e., as a plane channel displacement. Firstly, lubrication/thin-film approaches have been used, giving a simplified pseudo-2D prediction. This approach dates back to Beirute and Flumerfelt (1977), but with errors in the derivation. Symmetric displacements were considered by Allouche et al. (2000) and inclined channels by Taghavi et al. (2009). The latter work has been extended to include weak inertial effects (Alba et al. 2013b) and more recently to converging-diverging 2D sections by Mollaabbasi and Taghavi (2016). These models allow one to predict the maximal layer of drilling mud that may remain stationary on the wall of the channel (=annulus) and to predict qualitative behaviors of the displacement fronts. Fully 2D simulations and analysis can be found in Taghavi et al. (2012a, c), Allouche et al. (2000), Wielage-Burchard and Frigaard (2011), and Alba et al. (2014). These simulations cover a limited subspace of parameters, which is being currently extended. Figure 3 shows an example of a displacement of a Bingham (drilling) fluid by a Newtonian (spacer) fluid at along a uniform channel of width $\hat{D}$ at mean imposed velocity $\hat{V}_0$. Two different viscosity ratios are considered: thicker

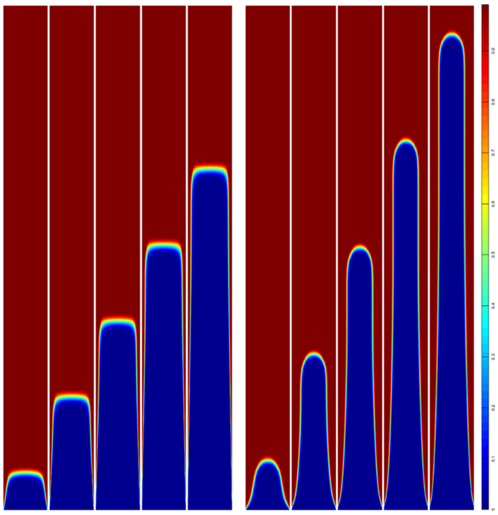

**Fig. 3** Example channel displacement of a Bingham fluid by a Newtonian fluid at Reynolds number, $Re = 0.1$; denimetric Froude number, $Fr = 0.1$; and Bingham number $B = \hat{\tau}_Y \hat{D}/(\hat{\mu}\hat{V}_0) = 5$. *Left*: viscosity ratio (Bingham plastic viscosity/Newtonian viscosity) $m = 0.1$; *right*: $m = 10$. Images at time intervals of $4\hat{D}/\hat{V}_0$

static layers are evident for the more viscous displaced fluid. The focus of these studies is to predict the so-called *micro-annuli*, i.e., annular wall layers of undisplaced mud extending along the wellbore. As the cement eventually hydrates, these layers dry into porous longitudinal conduits, compromising the annular seal integrity.

Many boreholes are drilled into unconsolidated formations. The combination of drill string vibration, jetting through the drill bit and geological weakness, often results in washout sections, i.e., where the annular geometry has a local expansion into the rock formation. These features are largely unpredictable geometrically although they are increasingly measured using caliper logs prior to cementing. It is of interest that some of the earliest experimental studies considered the effects of sudden expansions on the annular geometry, e.g., Clark and Carter (1973) and Zuiderwijk (1974), but this approach was then abandoned experimentally until quite recently, e.g., Kimura et al. (1999). However, although studied experimentally, these works are largely in the form of yard tests: using limited ranges of realistic fluids but not allowing one to draw more general fluid mechanic understanding.

The main issue with irregular washout shapes is that fluids with yield stress (e.g., drilling muds) are known to have regions of zero strain (plugs) and irregular geometries can promote regions of low shear stress close to wall, which result in static zones. In primary cementing, it is common to pre-circulate drilling mud prior to pumping cement, to condition the mud. Thus, it becomes operationally important to estimate the flowing volume of the annulus, particularly washouts. Although single phase, the requirement now is to determine the yield surface bounding immobile mud. Static wall regions also occur in regular uniform ducts, e.g., with cross-sections having corners, (Mosolov and Miasnikov 1965, 1966). Mitsoulis and coworkers (e.g., Mitsoulis and Huilgol 2004) studied both planar and axisymmetric expansion flows, showing significant regions of static fluid in the corner after the expansion. Flow of yield stress fluids through an expansion-contraction has been studied both experimentally and computationally by de Souza Mendes et al. (2007), Naccache and Barbosa (2007), and Nassar et al. (2011). In de Souza Mendes et al. (2007), Carbopol solutions were pumped through a sudden expansion/contraction, i.e., narrow pipe–wide pipe–narrow pipe, with yield surfaces visualized by particle seeding. Stagnant regions first appear in the corners of the expansion, grow with increasing yield stress, and become asymmetric with increasing Reynolds number. In Roustaei and Frigaard (2013), large amplitude wavy-walled channel flows were studied numerically, predicting the onset of stationary fluid regions, which occur initially at the walls in the widest part of the channel. A more comprehensive study of geometrical variation was carried out in Roustaei et al. (2015). Yield stress fluid becomes trapped in sharp corners and small-scale features of the washout walls and fills the deepest parts of the washout as the depth ($\hat{H}$) is increased. For sufficiently large yield stress ($\hat{\tau}_Y$) and sufficiently deep washouts ($\hat{H}$), the actual washout geometry has little effect on the amount of fluid that is mobilized: for a deep washout, the flowing fluid "self-selects" its geometry. Figure 4 shows an example of this flowing area invariance. Having established stationary regions within the depths of the washout, further increasing $\hat{H}$ does not significantly affect the position of the yield surface. In Roustaei and Frigaard (2015), inertial effects were considered, for similar flows as in Roustaei et al. (2015). Surprisingly, moderate Reynolds numbers (but laminar) can in fact result in a reduction in flowing area, contrary to industrial intuition that pumping faster is better.

## Plug cementing

Plug cementing occurs principally when abandoning wells, although sometimes also earlier in construction. In this process, *plugs* of ∼100 m of cement are placed along the wellbore to seal it permanently. Before around 2000, it was relatively uncommon to provide any mechanical support to the cement, with the result that the heavy cement slurry frequently exchanged places with the less dense fluids below, in a destabilizing exchange flow. These flows (heavy fluid over light fluid in a pipe with zero net flow) have received considerable attention in the scientific literature (exchange flows), for Newtonian fluids. In plug cementing, the fluids have a yield stress, which can prevent this mechanically unstable motion, and some features of these flows have been

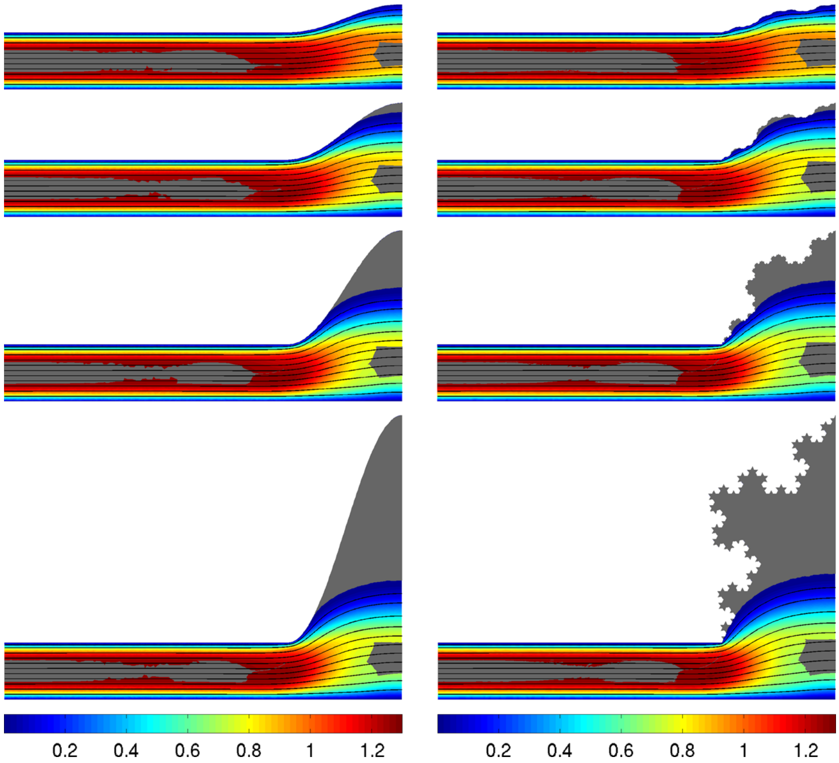

**Fig. 4** Example Stokes flows computed through washout geometries of increasing depth, imposed on a uniform channel of width $\hat{D}$. *Speed colour map* (normalized with mean velocity $\hat{V}_0$), *streamlines*, and *gray plug regions*: Bingham number $B = \hat{\tau}_Y \hat{D}/(\hat{\mu}\hat{V}_0) = 5$. Flow is from left to right and the washouts are assumed symmetric (left-right) so that only half the domain is computed

studied. In more recent years, it has become common to use a mechanical support under each cement plug, removing the interesting buoyancy-driven exchange flow. However, the actual plug placement still contains many of the features of the primary cementing displacement: downward flow of fluid stages through a pipe and removal (displacement) of the wellbore fluids around the outside of the tubing.

However, the pipe/tubing used to place the plugs is generally smaller than the casing in primary cementing. Thus, the annular placement geometry is no longer narrow. Indeed, some jurisdictions require the existing casing to be milled out into the surrounding rock formation. The fluids within the well may then be either old production fluids, possibly weighted brines, or drilling muds from the milling operation. Undoubtedly, this all makes the annular displacement problem harder. As a further complication, while the cement is pumped, the tubing is often slowly withdrawn from the hole, which leads to buoyancy-driven motion re-balancing of the static pressures between tubing and annulus.

## Rheology of cement slurries

A comprehensive introduction to cement chemistry, oilfield additives, and slurry rheology may be found in Nelson and Guillot (2006).

Fresh cement slurries are essentially concentrated suspensions that possess yield stress, thixotropy, and sometimes elasticity.

Cement is composed of calcium silicate and calcium aluminate phases. At the moment cement particles and water come into contact during mixing to form the slurry, chemical reactions begin. These reactions are collectively called *hydration*. The hydration products of silicate phases are CHS (calcium hydrosilicate) and $Ca(OH)_2$ (calcium hydroxide). The calcium aluminate phases react rapidly with water causing rapid hardening, and hence, the addition of calcium sulfate is needed to avoid early setting (Taylor 1997).

In the early stages, the reactions go through a dormant period (the induction stage) of typically a few hours, after which setting initiates and the slurry progressively hardens. During the dormant period, the slurry is said to be *fresh*. A fresh slurry can be pumped and flow to the region where it is supposed to harden later on. Therefore, a reliable design of cementing operations requires a thorough understanding of the mechanical behavior of the fresh cement slurries (Banfill 1997). In well cementing, retarders are used to control the length of the induction stage, allowing a safety margin for pumping operations to complete.

The rheology of fresh cement slurries is a strong function of the mixing method (Yang and Jennings 1995), because hydration kinetics will depend on the mixing efficiency. At the moment mixing is started, a suspension of aggregates of cement particles forms. The particles are held together in the aggregate by action of an enveloping membrane of hydrated minerals that forms instantaneously. The strength of this membrane is quite high, approaching that of a typical chemical bond between atoms, whereas links between particles—due to van der Waals attraction force—are one order of magnitude weaker (Banfill 1997). Therefore, hydration efficiency will depend directly to what extent the mixing process is successful in rupturing the membranes and thus breaking the initially formed aggregates.

Other factors also have important effect on the rheology of fresh cement slurries, namely the water/cement ratio, temperature, cement fineness, cement type, and the content of admixtures, polymer latexes, flyash, slag, limestone, microsilica, and so on Banfill (1997).

Rheological measurements with cement slurries are rather difficult, due to many potential sources of measurement error. Therefore, good laboratory data requires sophisticated rheometers operated by experienced rheologists. In practical applications of the oil and gas industry, however, it is seldom possible to employ advanced laboratory rheometers, and the usual consequence is lack of reproducibility. The main experimental difficulties and suggested cures are now briefly discussed. A thorough discussion about this topic is found elsewhere (Roussel 2012).

- The sample preparation requires a rigid protocol for the quality of water and cement, mixing method, and sample loading in the rheometer.
- The choice of geometry and gap should take into account:
    - The presence of solid particles, which requires gaps at least 10 times the characteristic particle size. This requirement typically precludes the usage of the cone-plate geometry.
    - The possibility of wall slip, demanding roughened surfaces.

  In general, surface-roughened Couette and parallel-plate geometries with large enough gaps perform satisfactorily.
- Due to the highly thixotropic and sometimes elastic nature of fresh cement slurries, in flow curve and oscillatory experiments, it is of central importance to make sure that all (non-periodic) transient effects have faded out before any data point is registered.
- Shrinkage due to drying is likely to occur, introducing important measurement error. It may be avoided by providing a water-saturated atmosphere around the sample, i.e., using the so-called solvent trap and cap.

Sedimentation is one of the great challenges found in the rheometry of cement slurries. The large density difference between the dispersed phase and water often leads to sedimentation, especially in the high end of the range of water/cement ratio. To reduce and control sedimentation, chemical additives are often included in the slurry composition (Al-Yami 2015). The additives are selected to perform satisfactorily for application purposes. However, even for a slurry that does not exhibit significant settling problems when pumped downhole, sedimentation may still undermine the quality of rheological data. For example, for the parallel-plate geometry, a depleted layer is formed adjacent to the upper plate, leading to grossly underestimated viscosities.

For the Couette geometry, sedimentation causes a stratified viscosity distribution, and the measured value again does not correspond to the viscosity of the homogeneous sample. When it is not possible to obtain reliable data before appreciable settling occurs, one remedy to circumvent sedimentation includes the usage of a modified bob in the Couette geometry that possesses helical grooves which help maintaining homogeneity. The grooves cause a significant departure from the purely tangential flow assumed in the rheometer theory, and therefore, an error is introduced. It is important to estimate the effect of the grooves and re-calibrate, e.g., by running preliminary tests with standard oils.

An interesting alternative to reduce sedimentation is to increase the viscosity of the continuous phase with the aid of some additive and then present the data in the form of relative viscosity, namely the viscosity of the slurry divided by the viscosity of the thickened continuous phase. Therefore, to obtain the viscosity of the original slurry (without the additive), it suffices to multiply the measured relative viscosity by the viscosity of water. Of course, this method is not free of artifacts and should be used cautiously. The viscosity thus obtained will to some extent deviate from the correct one due to possible qualitative changes of the interactions between the continuous and dispersed phase.

Rheological measurements are also useful to characterize the evolution of viscosity due to setting. The performance of chemicals used to control the setting time can be evaluated nicely with the aid of rotational rheometry. In the industry, a consistometer is used for this purpose.

## Fracturing and open-hole completions

The broad range of fluids used hydraulic fracturing and open-hole completions such as gravel-packing are similar, although flow rates and solid loading may be different. We do not intend a thorough review here, as this is recently available in Barbati et al. (2016). Briefly, many of the fluids used in fracturing are non-Newtonian, but a large fraction show no yield stress characteristics. In particular, low permeability reservoirs are often fractured using slickwater slurries, where the focus is on drag reduction at high speeds.

The so-called *viscous* slurries are used elsewhere and these typically have shear rate-dependent rheology and sometimes a yield stress, but also show strong viscoelastic behavior (and potentially other traits such as shear-banding, degradation, and thixotropy). Shear-thinning and yield stress models, such as the power law, Bingham, and Herschel-Bulkley fluid, are still commonly used in oilfield rheological characterizations, even though other rheological behaviors are widely acknowledged as important. It is simply that these models provide a common descriptive language and allow design calculations. A wide range of fluids are used in the industry, according to operation and company, often with proprietary formulation, e.g., typically aqueous polymer gels (guar, hydroxypropyl guar HPG, etc.), either linear gels or cross-linked (e.g., with Borate). Addition of small fibers is sometimes used to influence yield stress (e.g., Bivins et al. 2005) which has application in recent innovations in the pulsed delivery of proppant, e.g., Gillard et al. (2010), as well as control of settling.

Rather than focusing on specific fluids, it is perhaps clearer to focus on particular parts of the fracturing operation where a yield stress (or gelling behavior) is important. Some interesting flows in this context are (i) transverse settling of proppant particles through a pressure-driven channel flow, (ii) dispersion and migration of proppant across and along the fracture and the effects of the yield stress, and (iii) study of flowback and clean-up operations, e.g., how much of a yield stress fluid (or gel) is removed from a fracture at the end of the operation. Other flow features such as granular jamming during screen out (i.e., where the frac fluid leaks off to such an extend that the proppant particles jam before reaching their desired position) are not classical yield stress phenomena although potentially could be modelled using granular flow models that mathematically have a similar yield stress structure, e.g., Boyer et al. (2011).

## Sealing operations

In squeeze cementing, a section of cased well is isolated temporarily above and below the section needing repair. The steel casing is perforated at intervals along this section and thin cement (or other sealing fluids) are forced under pressure into the casing cement, sealing cracks, and fissures. This operation occurs for a variety of reasons: to cure annular gas migration, to correct a drop in well productivity, to repair corroded spots in the casing, etc.. Although studied and practiced since at least the 1950s (Howard and Fast 1950; Binley et al. 1958), quantitative understanding of the process is lacking.

Typically, the sealing fluids are significantly more viscous than any gases or formation brines that must be displaced. Hence, the *displacement* aspect is not problematic. Instead, these flows are analogous to a *filling* flow. A large pressure is applied at the wellbore driving the fluid into the perforation/crack, which is presumably at a reservoir pressure. The perforation/crack/fissure geometry is of course largely unknown, and this is where the main predictive difficulty lies.

It is interesting that whereas yield stress fluids are routinely used in models for other forms of well cementing, they are not prevalent in squeeze cementing. Many designs are based on variants of filtration style models that date back to Binley et al. (1958). In these models, the cement slurry is regarded as a separable suspension and the solute (water) filters away through the walls of the perforation. Models predict the buildup of a cement filtercake on the walls of the perforation and these are used to help estimate operational times and volumes to be pumped.

Typically, squeeze cementing pressures are below the fracture pressure of the formation. However, the nature of the operation is that cracks and fissures are to be filled, as well as closed perforations. From the process perspective, one would like to estimate how far a given sealing fluid can penetrate under a fixed differential pressure, into a network of cracks/fissures of unknown geometry. There are some simple estimates of penetration using axisymmetric models and yield stress fluids, e.g., Dai and Bird (1981) and El Tani (2012). While these are clearly gross simplifications, the difficulty is to specify a meaningful pressure gradient at which the flow stops, for more representative ranges of geometry. Essentially, this is a similar problem to those of determining limiting pressure gradients for porous media flows, as discussed earlier in "Reservoir flows of visco-plastic heavy oils."

Lost circulation flows are similar in physical scope, but frequently occur in an unplanned way. In these situations, typically during drilling, fluid losses from the wellbore become severe, i.e., far above those due to general filtration losses. Fluids are pumped that will stem the flow into the formation, e.g., fibrous or other suspensions, cements, viscous pills, and emulsions. Generally, this is determined by the materials available quickly at rig site.

## Flow assurance

Flow assurance is a quite broad research area in the oil and gas industry that is concerned with the phenomena that potentially cause flow problems during production. A number of these phenomena, perhaps the most critical ones, involve yield stress materials. Examples include gelation of waxy crude oils, formation of hydrates, and formation of water-in-oil emulsions (Jamaluddin and Kabir 2012; de Oliveira et al. 2012; de Oliveira and Goncalves 2012). The boundaries of thermodynamic envelopes for the precipitation of different solids are given qualitatively in Fig. 5.

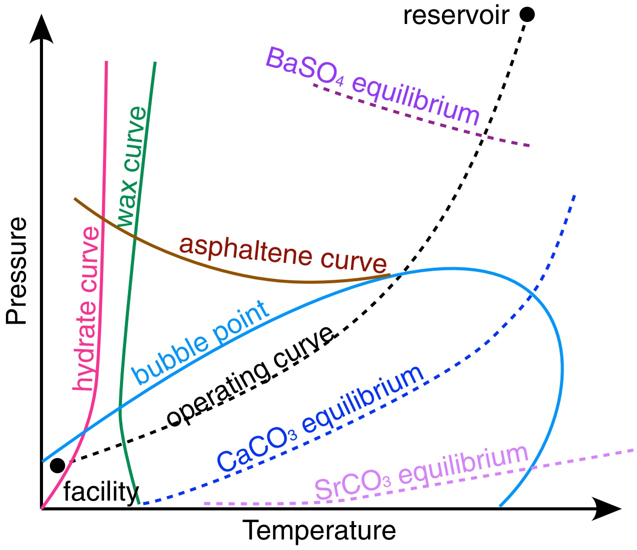

**Fig. 5** Schematic phase behavior characteristics, according to Jamaluddin and Kabir (2012)

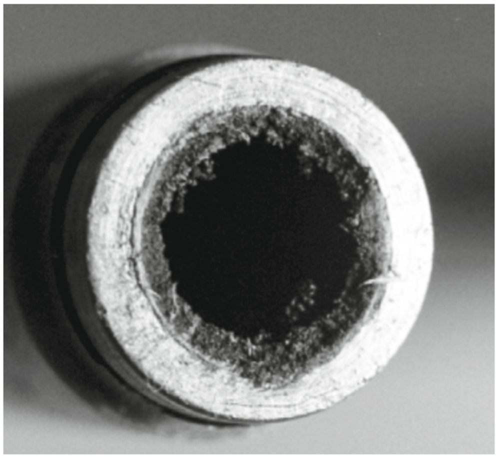

**Fig. 6** A typical wax deposit (Ribeiro et al. 1997)

## Waxy crude oils

Waxy crude oils are characterized by a large amount of *n*-paraffins in their composition. At high enough temperatures, the paraffins are fully dissolved, and the oils behave as Newtonian fluids. When the temperature falls below the so-called *wax appearance temperature* (WAT), wax crystals nucleate in the bulk and can lead to a sol-gel transition when the mass of wax solids exceeds 1–2% (Vignati et al. 2005).

### *Wax deposition*

While cooling during the flow through pipelines, the negative radial temperature gradient causes a concentration gradient that is responsible for molecular diffusion of the paraffins towards the wall (de Souza Mendes and Braga 1996). This process results in wax deposits on the wall that may drastically reduce or even stop production (Fig. 6). The wax deposits are themselves oil gels whose microstructure undergoes aging due to diffusion of paraffins of different carbon numbers, making them harder closer to the wall as time elapses (Azevedo and Teixeira 2003; Aiyejina et al. 2011).

In the operation of subsea pipelines, the waxy oil enters at reservoir temperature (65–85 °C) and cools along its way to the platform, due to the low temperature of the sea floor ($\simeq$ 4° in deep water conditions). Whenever feasible, the operation is designed to ensure that the oil exits the pipeline still above or just slightly below the WAT, to avoid wax deposition problems.

In the situations in which wax deposition cannot be avoided, it is important to be able to model the deposition process, to allow a reliable pipeline and operation design. Different attempts to model wax deposition are reported in the literature. The commonly accepted mechanisms of wax deposition are molecular diffusion and Brownian motion of solid crystals. Thus, there are simple integral models based on one or more of these mechanisms that are able to predict the thickness of the deposited layer as a function of time and axial position (de Souza Mendes and Braga 1996; Ribeiro et al. 1997). These simple models typically ignore the rheological properties of the deposited gel and thus the possibility of yielding due to the shear stress exerted by the flow on the deposit surface.

Thermodynamic models have also been tried, leading to more rigorous but rather complex formulations. These models are limited to the extent that on the one hand the detailed composition of the waxy oils is not known, and on the other hand, taking into account a too large number of components render unfeasible the thermodynamic approach (Azevedo and Teixeira 2003; Aiyejina et al. 2011).

A promising alternative to model wax deposition is to solve the mass, momentum, energy, and species balance equations simultaneously, in conjunction with a constitutive model to describe the rheological behavior of the waxy oil below and above the WAT (Benallal 2008; Benallal et al. 2008a, b; Minchola et al. 2010). In this manner, the deposit consists of a region adjacent to the wall within which the stress level remains below the yield stress.

### *Startup flow of gelled crudes in pipelines*

Flow shutdowns are not uncommon, due either to accidents or to periodic maintenance. Depending on the time duration of the shutdown, the oil may gel and in this case flow restart may be a problem. Therefore, the occurrence of shutdowns must be accounted for in the design stage, i.e., the pump and pipeline dimensions must be selected to ensure restartability in the worst scenarios.

Such design requires rheological information that is not at all simple to obtain. Waxy crude oil gels are rather complex yield stress materials, whose mechanical behavior is a strong function not only of their composition but also of the cooling rate and shear rate histories. For static cooling(zero shear rate), low cooling rates tend to yield stronger gels (higher yield stresses and viscosity levels), and vice-versa (Marchesini et al. 2012.

For a given set of cooling rate and shear rate histories, the oil gel typically behaves like a (complex) viscoelastic solid when exposed for a long enough time to stresses always below the yield stress. When above the yield stress (for a long enough time), elasticity is typically unimportant, and in steady state, the yielded oil behaves like a (highly) shear-thinning liquid. Its flow curve is thus reasonably well represented by either the Herschel-Bulkley or the Robertson-Stiff functions (Marchesini et al. 2012).

However, in general, a strong time dependence is observed, meaning that the microstructure responds slowly to changes in stress. When there is reversibility in the sense that a stress increase causes a viscosity decrease and vice-versa, and in addition when the final, equilibrium viscosity depends on the imposed stress only (and not on the stress history), then the oil is said to be *thixotropic*. Although nearly always time-dependent, waxy crude oil gels are often not thixotropic, essentially because high stress levels cause typically irreversible changes in the gel microstructure. This of course adds significantly to the complexity of the mechanical behavior of these materials.

As a consequence of the above described complex rheology, there is to date no theory available that describes accurately enough and in its entirety the mechanical behavior of waxy crude oil gels. Therefore, the modelling of the startup problem that is needed in the design stage of subsea pipelines is a great challenge. The lack of thorough understanding of the material behavior explains the absence of well-established protocols for rheological characterization of these materials. Even the most fundamental and undisputedly important material properties, like the yield stress for example, are quite difficult to measure, and hence, there is controversy regarding several measurement strategies usually employed.

Thus, the alternative of design engineers has been to rely on simple (and thus necessarily inaccurate) models to describe the startup flow problem and then add large safety margins. This strategy generally leads to overdesign and hence higher cost. As an extreme example, the elasticity and time dependence of the oil gel may be neglected, and then a simple force balance within a pipeline of length $L$ and diameter $D$ gives the minimum pressure gradient $\Delta p/L|_{\min}$ required for flow startup, as a function of the yield stress $\tau_y$:

$$\left.\frac{\Delta p}{L}\right|_{\min} = \frac{4}{D}\tau_y \tag{14}$$

This simple approach is known to give minimum pressure gradients that are several times higher than the ones actually observed in the field, thus leading to major overdesign.

The reason for the poor prediction of this approach is not precisely known, but it may be related to neglecting one or more of the rheological characteristics that are known to exist, namely time dependence/thixotropy and elasticity. It may also be at least partially due to overprediction of the yield stress due to a poor choice of the experimental technique. For example, it is known that the technique involving crossover of the $G'$ and $G''$ curves in stress amplitude sweep tests tends to grossly overpredict the yield stress.

The failure of the simple approach given by Eq. 14 may also be due to other causes such as apparent wall slip or to thermal shrinkage after gelation. A basic assumption to account for shrinkage that has been adopted by several authors (Cawkwell and Charles 1987, 1989; Sestak et al. 1987; Frigaard et al. 2007; Vinay et al. 2006, 2007, 2009; Wachs et al. 2009; de Oliveira and Negraode 2015; Kumar et al. 2015, 2016) is that the appearance of gas voids confers to the gelled crude a kind of compressibility, which is introduced by assuming that the material is barotropic and possesses a constant isothermal compressibility coefficient, i.e., $(\partial\rho(p,T)/\partial p)/\rho = (d\rho(p)/dp)/\rho =$ constant.

Most of the articles that use the just mentioned weakly compressible fluid formulation also consider time-dependent rheological effects by using modified versions of the thixotropy model proposed by Houska (1981). Since the Houska-type thixotropy models assume that the microstructure breakdown agent is the shear rate, then no flow startup can be predicted unless some initial non-zero shear rate is present to trigger the breakdown process. The weakly compressible fluid formulation provides such initial shear rate even at shear stresses below the yield stress, which explains the popularity of this formulation, despite its obvious disparity from the actual physics. In addition, the weakly compressible fluid approach does not account for loss of contact with the wall due to thermal shrinkage, which is an effect that may be responsible for the lack of success of Eq. 14. The aforementioned thixotropy models that assume that the microstructure breakdown agent is the stress instead of the shear rate (de Souza Mendes 2009, 2011; de Souza Mendes and Thompson 2012, 2013; Van Der Geest et al. 2015) do not require the compressibility assumption to trigger the breakdown process. In this case, the resulting (incompressible) formulation of the startup problem becomes much simpler and with clearer underlying physics (de Souza Mendes et al. 2012).

Commercial codes are often used that contain coarse models and are “calibrated” with field data. As expected, however, the level of reliability of the strategy adopted in these codes clearly cannot be as high as the one that would be achieved had a thorough knowledge of the physics involved been available.

In terms of the fluid mechanics, it is worth commenting that most approaches taken are essentially 1D or pseudo-1D approaches (albeit complicated by thixotropy, elasticity, and/or compressibility, with or without displacement models). There are many other interesting fluid mechanics problems that merit attention in considering startup flows. One of these is the inhomogeneity of thermal shrinkage, both along the pipeline and in any particular section. It is self evident that the yielding behavior of a relatively homogeneous (bubbly) distribution of gas voids will be different from the that of larger consolidated voids (or slugs) at the same volume fraction, and that loss of contact with the wall will play a significant role in a restart. However, these features are not well studied, nor the physical conditions that produce them.

## Hydrates

Clathrate hydrates are ice-like inclusions that—under appropriate thermodynamic conditions—form at the interface between water and hydrocarbons or low molecular weight gases, by trapping the guest species within cages of hydrogen-bonded water (Sloan and Koh 2008; Leopércio et al. 2016).

Specifically, they are typically formed at high pressures and low temperatures. Consequently, they often represent a severe flow assurance problem in deep and ultradeep water oil production, when the content of dispersed water in the produced oil is high enough. The hydrates form as the gas (e.g., methane) dissolved in the oil migrates to the water-oil interface. In steady-state production, hydrates can be avoided by pipe insulation, which prevents the temperature to fall within the so-called hydrate thermodynamic envelope.

The insulation solution, however, is not effective for the case of a long enough unexpected production shutdown, because in this case, the temperature may reach the one of water at the sea floor, attaining the necessary conditions for hydrate formation and maybe pipeline blockage. An alternative is to inject additives such as ethanol, which move the thermodynamic envelope so that the current state becomes outside it.

Interestingly, it has been observed that hydrates may form in some oil emulsions without plugging the pipeline. That is, in some cases, the formed hydrates do not agglomerate to form a percolated structure. Rather, they remain disperse forming a slurry. This phenomenon may be due to the presence of indigenous anti-agglomerants in the oil (de Oliveira et al. 2012; de Oliveira and Goncalves 2012). Or else, it may be due to a small water content such that water depletion occurs and the hydrate formation is halted before the hydrate crystals have a chance to touch each other and agglomerate.

For pipeline and operation design purposes, it is important to know the rheological properties of the hydrate slurries. The rheological characterization of this kind of material requires elaborate rheometrical experiments. A rheometer equipped with a pressure cell is needed, in order to provide the appropriate thermodynamic conditions during the measurements. These cells are not easy to operate. To allow pressurization and eliminate leaks, the cell connects to the shaft of the rheometer motor by means of a magnetic coupling. This coupling has a non-negligible residual torque that hinders measurements at low stresses, thus severely limiting the range of accurate rheological measurements.

The rheology of hydrate slurries formed from a water-in-oil emulsion has been investigated (Peixinho et al. 2010; Webb et al. 2012a, b). Viscosity and pressure of the slurry were measured during hydrate formation, growth, aggregation, and dissociation, using an interesting experimental procedure. In addition to the dependence on shear rate, water content, and temperature, the slurry viscosity was shown to increase rapidly with time when hydrates form and then decay after going through a maximum as hydrate aggregates breakup or rearrange. The yield stress initially increases with time and then levels off. The authors also observed that the slurries are shear thinning and that both viscosity and yield stress increase with the water content.

## Water-in-oil emulsions

It is not uncommon the presence of solid particles in the oil, such as rock fines, clays, and especially asphaltenes. When the oil becomes in contact with formation water, the asphaltenes and other particles work as indigenous surfactants that adsorb onto the water-oil interface to help forming highly viscous and very stable water-in-oil emulsions (Sjöblom et al. 2003). Wax and hydrate crystals, and naphthenic acids can also stabilize oil-in-water emulsions, in the same fashion as the asphaltenes and other particles. Water-in-oil emulsions are present in different stages of oil production and constitute a major flow assurance problem (de Oliveira et al. 2012; de Oliveira and Goncalves 2012).

In order to devise strategies to circumvent the problems associated with stable water-in-oil emulsions, one of the key aspects is to understand the mechanisms that control the rheological properties of the water/oil interface, because the interfacial rheology controls the emulsion stability and affects the bulk rheological properties. Therefore, a thorough knowledge of the properties and behavior of the different indigenous surfactants and understanding how they affect the interfacial rheology is necessary (Sjöblom et al. 2003). For example, the interface activity of asphaltenes is a strong function of their solubility in the bulk of the oil. As the solubility decreases, the concentration of asphaltenes at the interface is increased, leading to higher interface activity. However, a too low solubility tends to cause a too large amount of asphaltenes at the interface, leading to aggregation and ultimately to a low surface activity.

Water-in-crude oil emulsions generally undergo a sol-gel transition as temperature is decreased, especially for waxy oils (Hemmingsen et al. 2005; Visintin et al. 2008; Paso et al. 2009; Maia Filho et al. 2012; Haj-shafiei et al. 2013; Barbato et al. 2014; Sun et al. 2014). The yield stress of these gelled emulsions may be very high (of the order of 1000 Pa), and an accurate and reproducible method for its measurement still lacks (Barbato et al. 2014). To the large number of difficulties already found in the determination of the rheological properties of waxy crude oils (Marchesini et al. 2012), several other arise, due to many additional factors that influence the gel structure and hence the bulk rheology (Visintin et al. 2008; Paso et al. 2009; Maia Filho et al. 2012; Sun et al. 2014). Among these, we can mention the oil composition and molecular weight, drop size distribution, interfacial tension and rheology, salinity, and pH, just to name but a few. Many of these emulsion characteristics are not easily accessible, which severely undermines the controllability and reproducibility of the rheological measurements.

## Irreversible non-ideal thixotropy

After crystallization and structural buildup of a wax-oil gel, the colloidal solid wax crystals exist in a volume-spanning physical network that entrains the remaining liquid oil among the crystals. Van der Waals interactions between the individual wax crystals collectively impart a yield stress to a wax-oil gel. The yield stress consists of a degradable portion and a non-degradable portion. The degradable portion of the yield stress maps linearly to the structural parameter λ and the non-degradable portion of the yield stress is constant. Similarly, the overall Bingham viscosity term consists of a degradable portion and a non-degradable portion. The degradable portion of the Bingham viscosity maps linearly to the structural parameter λ, and the non-degradable portion of the Bingham viscosity is constant. The structural parameter λ follows a modified thixotropic dynamic relation in which an exponent $n$ is placed on the structural parameter in the breakdown term, as shown in the following equation

$$\frac{d\lambda}{dt} = \underbrace{0}_{\text{Buildup}} - \underbrace{k_{-}\dot{\gamma}\lambda^{n}}_{\text{Breakdown}} \tag{15}$$

In a well-justified first approximation, the buildup term is negligible after initial structural buildup of the gel has been completed, due to an irreversible nature of the gel rupture process that is consistent with irreversible crystal-crystal bond rupture. Post-rupture buildup effects such as Ostwald ripening occur on a timescale substantially larger than the relevant rheological timescales, justifying negligence of the thixotropic buildup term. The thixotropic rate equation is simplified as follows

$$\frac{d\lambda}{dt} = -k_{-}\frac{d\gamma}{dt}\lambda^{n}.$$

and on implementing the chain rule

$$\frac{d\lambda}{d\gamma} = -k_{-}\lambda^{n}. \tag{16}$$

This indicates that the structural state of the wax-oil gel, λ, follows a point function of the imposed shear deformation $\gamma$ and is otherwise time-independent. This revealing transformation effectively removes the $\dot{\gamma}$ parameter (shear rate) from informing the structural breakdown dynamics (which is otherwise viewed as problematic in many applications). The exponent $n$ indicates the kinetic order of the breakdown rate with respect to shear deformation $\gamma$.

It is important to note that Eq. 16 applies only to the irreversible rheological breakage phenomena of a wax-oil gel. When shear stress is initially imposed upon an unperturbed wax-oil gel, the first rheological response is a conventional viscoelastic response derived from straining the crystal network, which involves bending of crystal-crystal contact junctions and possibly subtle structural rearrangement of crystals in the colloidal network. Subsequent to network straining and/or network rearrangement up to the yield strain, rheological breakage ensues. Therefore, rheological breakage is the second sequential rheological response, which necessarily occurs at a timescale larger than the initial low-deformation viscoelastic response, in accordance with the rigorous definition of non-ideal thixotropy provided by Larson (2015). Furthermore, when (16), with an absence of a buildup term, is applied analytically to inform the trajectories of the degradable terms within the extended Bingham formalism, the overall rheology description rigorously meets the definition of *irreversible non-ideal thixotropy*. This categorization of the governing rheology of wax-oil gels is applicable to gelled pipe restart applications and has been rigorously confirmed for model wax-oil gels in which crystal-crystal contact junctions are of a crystallographic nature (Paso et al. 2009). Adsorption of heavy polar colloidal components onto the wax crystal interfaces may alter the nature of the junction sites to a partially amorphous nature. However, even for waxy crude oil gels containing resins and asphaltenes, the primary rheological response remains deformation-dependent, lending credence to the dynamic thixotropic relation in Eq. 16.

The irreversible thixotropic nature of wax-oil gels carries several implications for gelled pipeline restart operations. Deformation-driven rupture of waxy crude oil gels results in localization of applied stress in the axial direction when flow is restarted in gelled oil pipelines. Rheological yielding occurs primarily at the pressure wave front, while flow rate variations govern the evolving pressure profile behind the pressure front. Another implication of deformation-driven rheology for pipeline restart applications is that gel breakage occurs sequentially along the axial length of the pipeline, allowing flow to restart in substantially longer pipe segments as compared to the incompressible case where gel breakage must occur simultaneously at all axial locations. A significant time delay is required in order for sustained flow to restart in pipe sections longer than the conventional critical pipe length established by the linear force balance methodology. This time delay allows the gel to break sequentially along the axial length of the pipeline commensurate with the pressure propagation process. Finally, because the gel breaks with deformation instead of time, repetitive pressure swinging remains a highly viable option to restart flow in difficult plugging cases. Pressure swings may be applied at one or both ends of the pipeline, and internal pressure monitoring may inform the appropriate pressure swing intervals.

## Wax rheology characterization

Rheometric geometries used for assessment of waxy crude oil gels include vane and vane-like geometries, cone and plate geometries, and concentric cylinders. Vane and vane-like measuring geometries ensure fixation of the gel to the measuring surface, precluding artifacts related to adhesive breakage (slippage of the gel past the measuring surface). Disadvantages of the vane and vane-like measuring geometries are that deformation fields and flow fields are non-uniform in the geometry volume during measurement, resulting in artificial broadening of the observed gel breakage peak accompanied by artificial reduction in the peak height. Uniform deformation fields and flow fields are provided by cone and plate geometries. However, the disadvantage of cone and plate geometries is that adhesive rupture effects are prevalent. Gel fixation may be promoted by sandblasting the contact surfaces. Cone and plate geometries are also vulnerable to micro-sedimentation of wax crystals at the upper cone surface during gel formation, resulting in a locally reduced wax content and reduced strength of the gel in contact with the upper cone surface, affecting non-uniform gel breakage with shearing localized at the upper surface. The micro-sedimentation phenomenon is consistent with a noticeably detached gel at the upper cone surface after an experiment. Upright concentric cylinder geometries ensure representative wax content of the fluid in contact with the measuring surface, precluding micro-sedimentation artifacts while providing relatively uniform flow fields. However, upright concentric cylinder geometries are vulnerable to conventional adhesive breakage effects.

Occurrence of adhesive breakage must be properly accounted for during selection of measuring geometry. In general, for low-wax-content fluids in which cohesive breakage inherently occurs, vane-like geometries underestimate the yield stress; therefore, cone/plate or concentric cylinder geometries should be selected in order to obtain representative yield stress values. Conversely, for high-wax-content fluids where adhesive rupture is prevalent, vane or vane-like geometries should be used, cognizant of the fact that yield stress peaks are somewhat underestimated. In a recent development in Brazil, a grooved coaxial geometry was introduced for waxy crudes that combines the benefits of vane and concentric cylinder geometries (Barbato et al. 2014), confirming occurrence of slippage in comparative concentric cylinders. Implementation of grooved coaxial geometries likely provides the most representative values of true yield stress for the high waxy crude oil gels that otherwise exhibit adhesive breakage.

Various rheometric protocols are useful to assess waxy gel rheology. Optimally, a waxy crude sample should be loaded into the geometry at a temperature above the wax solubility limit, and a representative thermal profile should be imposed prior to measurement. The yield stress value may be accurately and rapidly attained using a linear imposed stress ramp with time. However, rapid acceleration of shearing after rupture precludes meaningful rheological analysis. Linear imposed shear rate ramps are used for obtaining equilibrium flow curves of waxy oil. In order to properly assess the entire deformation-dependent rheology of wax-oil gels, constant shear rate protocols should be used, which allow proper spacing of acquired data points with respect to the deformational state. Within a constant shear rate protocol, prescribed step changes in shear rate allow inference of the full deformation-dependent rheology of a wax-oil gel sample, including trajectories of yield and plastic viscosity values with respect to imposed deformation.

## Final remarks

The materials used within the oil and gas industry and the processes used to produce these resources are continually changing. Here, we have simply attempted to provide a snapshot of the present-day usage of yield stress fluids within the industry 100 years after E.C. Bingham's seminal work (1916).

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