

# Evaluation of Shipboard Wave Estimation Techniques Through Model-scale Experiments

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**Abstract**—The paper continues a study on the wave buoy analogy that uses shipboard measurements to estimate sea states. In the present study, the wave buoy analogy is formulated directly in the time domain and relies only partly on wave-vessel response amplitude operators (RAOs), which is in contrast to all previous works that either are formulated in the frequency domain and/or depend entirely on RAOs. Specifically, the paper evaluates a novel concept for wave estimation based on combined techniques using a wave frequency estimator, not dependent on RAOs, to detect wave frequency and, respectively, nonlinear least squares fitting to estimate wave amplitude and phase. The concept has been previously tested with only numerical simulations but in this study the techniques are applied to model-scale experiments. It is shown that the techniques successfully can be used to estimate the wave parameters of a regular wave train.

## I. INTRODUCTION

### A. Context

Operation of ships and other marine crafts at sea may profit from having knowledge of the prevailing sea state available. This knowledge can be used to improve both safety and efficiency of the particular operation that includes, e.g., vessels in transit, oil and gas production from floating dynamic positioned (DP) structures, general ship-to-ship actions, installation and maintenance of fixed or floating offshore structures such as wind farms.

### B. Sea State Estimation from Ships

Different means can be applied to estimate the sea state, which is often given in terms of characteristic parameters, e.g., significant wave height, mean wave period and mean wave direction. Since the 1970s (moored) directional buoys have been considered as the most reliable and accurate source for sea state estimation. However, traditional wave-rider buoys suffer from being subject to damage and/or loss. Further, and more importantly, an enormous geographical network would be required to cover all parts of the oceans. For marine operations, considering ships or other floating structures, a more practical and appealing approach is to use response measurements from shipboard sensors, so that the wave-induced responses from the vessel itself provide the basis for an estimate of the on-site sea state, e.g., [1]–[7]. Until recently, this approach - denoted the wave buoy analogy - has primarily

been explored in settings based in the frequency domain, and only in frameworks where the wave-vessel transfer function(s), equivalently response amplitude operators (RAOs), are fundamental in addition to the response measurements. A new concept for shipboard sea state estimation has been presented at OCEANS'15 MTS/IEEE by Nielsen et al. [8], where wave estimation is made directly in the time domain. Stepwise, the method provides, first, the (characteristic) wave period obtained solely from a measured response signal. In the second step, the method combines the use of corresponding RAOs and measurements to estimate wave amplitude and phase (of a regular wave train). In this context, the central and ultimate goal is to develop an estimation algorithm capable of estimating the complete set of sea state parameters, i.e. wave period, height and phase as well as wave direction, without the need for wave-vessel RAOs but relying on *some* other mathematical model relating vessel responses and wave excitations/conditions. The independency of RAOs would be representing a great advantage, as the use of those in practical operational cases is always associated with uncertainty due to incomplete knowledge of the input conditions, i.e. speed, wave heading, draft, etc.; not to mention (in)accuracies in the calculation of the RAOs themselves. To this point, the novel concept outlined in [8] still requires the input of RAOs but since the concept facilitates real-time wave estimation directly in the time domain, no assumption needs to be made about stationary conditions, as required for wave estimation techniques based in the frequency domain. In the frequency domain, stationary conditions are, in the strict sense, necessary to make reliable spectral calculations; in practice, however, changes in vessel speed and/or wave heading, not to mention changes in the sea state itself, imply that stationary conditions rarely occur.

### C. Content of Study

The present study is a direct continuation of Nielsen et al. [8] where new concepts for shipboard sea state estimation were presented. The concepts were tested and analysed for some highly theoretical conditions/examples addressing estimation of a regular wave train from numerical simulations of vessel responses. Specifically, conceptual examples showed that a regular, sinusoidal wave could be fully reconstructed in real time with negligible error on the estimated wave frequency,

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amplitude, and phase. The concepts need, indeed, to be (much) further developed to be applicable for handling real-case data, i.e. irregular ocean wave systems. Nonetheless, it will be valuable to test and evaluate how the concepts, as are, handle data obtained through experimental setups. Therefore, the theoretical examples [8] are re-evaluated using model-scale experiments, and the present study addresses this particular assessment. More precisely, wave parameters of regular ("sinusoidal") wave trains, generated in a model-basin, are estimated by the analysis of measurements of the wave-induced motions of a model-scale ship deployed in the basin.

#### D. Composition of Paper

In addition to the Introduction, the paper is organised into four sections numbered II-V. Section II presents the fundamental theoretical aspects of methods used for shipboard sea state estimation and the section gives the details of the particular estimation concept in focus in this study. In Section III, the experimental setup is outlined and a few facts about the RAOs of the model-scale ship are given. The results and associated discussions related to the model experiments appear in Section IV. Finally, the work is summarised and conclusions are drawn in Section V.

## II. THEORETICAL ASPECTS

### A. Wave Buoy Analogy

Most of today's marine vessels are instrumented with sensors to monitor, e.g., global motion components such as heave, pitch, and vertical acceleration at specific position(s) relative to the centre of gravity. In this sense, the vessels resemble classical wave rider buoys; although the latter typically have much simpler geometrical forms. Anyhow, the response measurements of marine vessels can be processed to facilitate estimation of the on-site sea state, making the analogy to classical wave rider buoys by relating the measurements and the sea state through a mathematical model, see Figure 1.

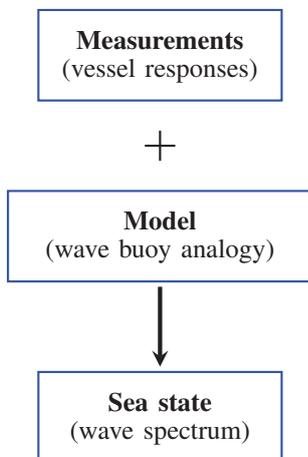


Figure 1: Combination of wave-induced response measurements and a mathematical model can be used to deduce information about the on-site sea state.

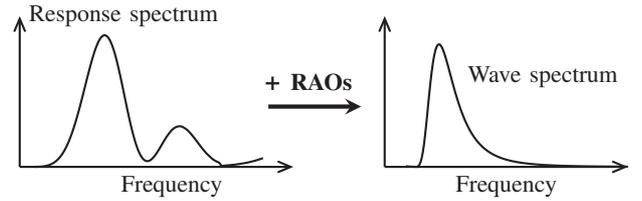


Figure 2: Main principle of the wave buoy analogy when it is formulated in the frequency domain.

As pointed out in the Introduction, the majority of previous work on the wave buoy analogy, e.g., [1]–[7], is based on a solution formulated entirely in the frequency domain. This is illustrated in Figure 2, where a response spectrum is combined with RAOs, using spectral analysis, so that an estimate of the sea state is given in terms of a wave (energy) spectrum. The *measured* response spectrum is derived by, e.g., fast Fourier transformation (FFT) of a corresponding time history recording, while the RAOs typically are calculated by applying strip theory and/or panel code procedures. In the literature, studies have shown that, in practice, wave estimation is improved by including a set of (optimally) three different vessel responses simultaneously.

The outcome of the wave buoy analogy, when formulated in the frequency domain, consists of the on-site wave system's complete energy distribution, with frequency and directional information, and thus the approach is applicable to general decision support systems for safe and efficient marine operations. Reasonable estimates of the wave spectrum can be expected [9]–[12], but the accuracy of the estimated sea state depends inherently on availability of accurate transfer functions. Furthermore, the reliability is highly dependent on the spectral (response) analysis, in which aspects of stationarity of the time history measurements influence the outcome [13], [14]. In principle, stationary operational conditions are necessary because a minimum time window, in the order 10-15 minutes, is needed to perform the spectral analysis. The reason is that if conditions are not stationary during the considered period, either because of a changing sea state or, more likely, as a result of speed and/or heading changes of the vessel, the sea state estimates are likely to be unreliable. Moreover, the need for a certain minimum time period has another consequence, as it implies that estimates, strictly speaking, will be backdated; which in turn may negatively influence response predictions

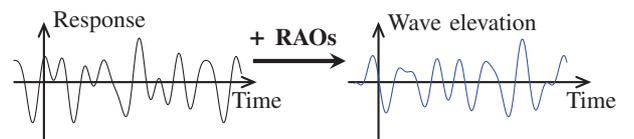


Figure 3: Conceptually, the wave buoy analogy can be formulated directly in the time domain to give the actual wave elevation at the site of the vessel.

made ahead of any measurements [15].

Instead of a solution formulated in the frequency domain, derived by use of spectral analysis and with relatively long processing time, it is suggested by [8] to make the fitting of the *measured* response and the corresponding *theoretically* calculated one directly in the time domain. In this sense, the approach is, to some extent, similar to a previous work by Pascoal and Guedes Soares [16], which is also formulated in the time domain. However, this method [16], based on Kalman filtering, relies completely on the RAOs, which is the main difference to the present work, including [8]. Moreover, the solution procedure of the present work yields the actual wave elevation process, as illustrated in Figure 3. Despite the similarity between the two procedures, the Kalman filtering approach is not mentioned any further and the remaining part of the paper addresses the new concept for sea state estimation, outlined previously by Nielsen et al. [8]. In the following, the fundamental steps of this estimation procedure are given.

### B. Estimation of Wave Peak Frequency

As mentioned above, the dependence on RAOs is not (necessarily) beneficial, and previous works have already addressed this problem with a concern for the characteristic wave frequency of a sea state. Thus, procedures have been successfully developed [17], [18] to estimate the encountered wave peak frequency  $\omega_p$  of a sea state. Particularly, the work by Belleter et al. [17] shows that the wave peak frequency can be estimated, real-time, solely from motion measurements of an advancing ship; without the use of RAOs and with no need for transformation from time to frequency domain allowing for nonstationary conditions.

In the present work, a simpler version of the procedure presented in [17] has been implemented, and this restricts the analysis to responses generated by regular waves (sinusoidal signals). The simplifications could quite easily be relaxed to consider irregular responses like in [17], but as the subsequent estimation of wave amplitude and phase, in the second step, is restricted (so far) to regular signals, there is no need for the robustness properties that the estimator presented in [17] provides. Basically, the idea is to establish a method that can be used to track the (dominant) frequency of a given signal. A solution, in terms of a filter, was initially derived by Aranovskiy et al. [19]. However, herein a slightly modified version of the filter is applied and the main points are repeated below:

A sine wave with unknown constant amplitude  $A_y$ , frequency  $\omega_e$  and phase  $\varepsilon$  is given

$$y(t) = A_y \sin(\omega_e t + \varepsilon) \quad (1)$$

and the objective is to estimate the frequency  $\omega_e$  on the basis of only noisy measurements of  $y(t)$ .

Basically, any sinusoidal signal represents the solution to the problem of an undamped harmonic oscillator

$$\ddot{y} = -\omega_e^2 y = \varphi y \quad (2)$$

and thus  $\varphi = -\omega_e^2$  is the parameter to be estimated. The equivalent critically-damped mass-spring system with forcing was studied in [19], wherein it was shown that the auxiliary filter:

$$\dot{\xi}_1 = \xi_2 \quad (3)$$

$$\dot{\xi}_2 = -2\xi_2 - \xi_1 + y \quad (4)$$

with the equivalent second order transfer function

$$\xi_1(s) = \frac{1}{(s+1)^2} y(s) \quad (5)$$

tracks the measured sinusoid; until a cut-off frequency at 1 rad/s. For any wave with higher frequency the filter can be modified as follows

$$\xi_1(s) = \frac{\omega_f^2}{(s+\omega_f)^2} y(s) \quad (6)$$

where the cut-off frequency  $\omega_f$  should be chosen such that  $\omega_f > \omega_e$  to ensure that the auxiliary filter is sufficiently fast to keep track of the wave.

The frequency estimator thus becomes, cf. [19]:

$$\dot{\xi}_1 = \xi_2 \quad (7)$$

$$\dot{\xi}_2 = -2\omega_f \xi_2 - \omega_f^2 \xi_1 + \omega_f^2 y \quad (8)$$

$$\dot{\hat{\varphi}} = k_a \xi_1 (\dot{\xi}_2 - \hat{\varphi} \xi_1) \quad (9)$$

$$\hat{\omega}_e = \sqrt{|\hat{\varphi}|} \quad (10)$$

where  $k_a > 0$  is the constant estimation gain. The frequency estimator in Eq. (10) with  $\omega_f = 1$  was first presented in [19]. Belleter et al. [17] extended this work by including gain-scheduling of the estimation gain  $k_a$  and testing it on model- and full-scale vessel motion data. Furthermore, a stability proof for global exponential stability of the estimator is given in [17].

### C. Estimation of Wave Amplitude and Phase

The frequency estimator can be directly applied to real-time vessel response measurements and facilitates determination of the (peak) frequency of the signal without any further modelling. In this way, the estimation of the wave peak frequency can be said to be entirely *signal-based*. The

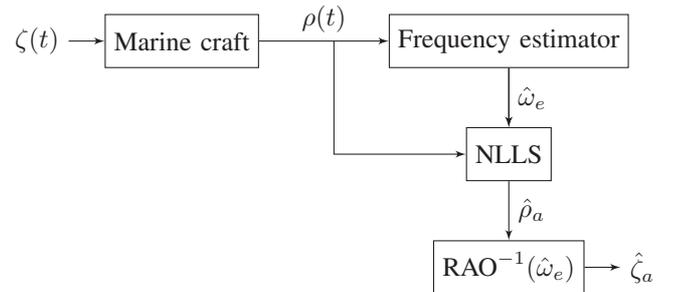


Figure 4: Wave amplitude estimation using nonlinear least squares fitting (NLLS).

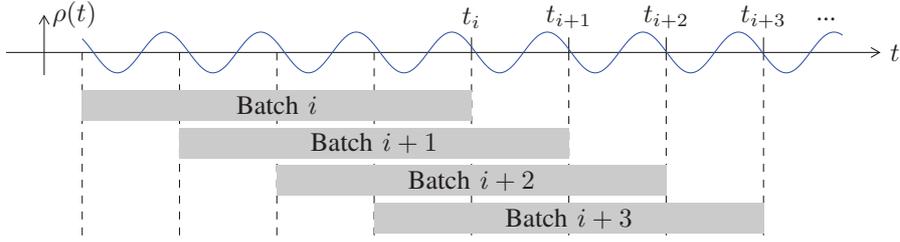


Figure 5: Batch data with 75% overlap. Batch  $i$  is processed at time  $t_i$ . [20]

estimation of wave amplitude and phase, on the other hand, needs a model to relate the response measurement and the wave excitation. The typical way to obtain the estimates of amplitude and phase, by the wave buoy analogy, is to conduct spectral analysis on the measured vessel response whereafter the obtained response spectrum is compared to a theoretically calculated one obtained by combined use of RAOs and a *guessed* wave spectrum; but iteratively improving the guess by some mathematical technique. The consequence of this approach is that wave amplitude and phase are not directly estimated, since the solution is given in terms of wave spectral ordinates in the frequency domain. As discussed previously, the necessity of spectral analysis and associated transformation to frequency domain by, e.g., standard FFT, or parametric methods [21], [22], implies that the wave estimation is backdated and can be unreliable in case of nonstationary conditions. These disadvantages are ever present, to smaller or larger degree, and efforts should/could be introduced to mitigate them; for instance, spectral procedures to handle nonstationary conditions can be introduced/developed [13], [23], [24].

Other approaches, all formulated directly in the time domain, have been investigated in [20], and one concept was given special attention by Nielsen et al. [8]. The particular technique is based on nonlinear least squares (NLLS) fitting of a batch of time-series data of a response; but emphasising that calculation of response spectra are unnecessary. It is noteworthy that the use of *recursive* NLLS methods might be able to provide real-time estimates without the need of using batch data, although this is outside the scope of the current paper.

In the following the proposed solution is discussed and for matters of convenience a general response denoted by  $\rho(t)$  is considered. The solution process is illustrated as a block diagram in Figure 4. The frequency estimator, Eq. (10), is used to provide the frequency estimate to simplify the nonlinear fitting since global convergence otherwise has been found unreliable due to both local minima in the nonlinear cost function and regions with small gradients. Herein, the nonlinear optimisation is implemented with the Levenberg-Marquardt algorithm which is an iterative least squares algorithm addressed specifically to nonlinear minimisation problems. The actual fitting is done using a batch process as shown in Figure 5, where each batch contains measurements from 512 samples

equivalent to 51.2 seconds (for a response sampled at 10Hz). A batch overlap of 75% has been used and an estimate is thus calculated every  $\frac{51.2}{4} = 12.8$  seconds. Obviously, this practice leaves room for a bit of 'tuning' depending on the physical problem; however, in this work such a parameter study has not been considered.

The batch data is fitted with the regression function

$$\rho(t) = \hat{\rho}_a \cos(\hat{\omega}_e t + \hat{\varepsilon}) \quad (11)$$

where the independent variables to be fitted are the response amplitude estimate  $\hat{\rho}_a$  and phase estimate  $\hat{\varepsilon}$ , respectively. In order to avoid erroneous fitting results, it has been found necessary to split the fitting into two subsequent steps: First fitting the phase  $\hat{\varepsilon}$  using a fixed initial amplitude guess  $\hat{\rho}_a = \max(\rho)$  and then fitting the amplitude  $\hat{\rho}_a$  using the previously determined phase estimate. This strategy requires the algorithm to be followed twice, thus increasing the computing time. On the other hand, calculations have been found more robust against local minima if the following trigonometric relation is used

$$\begin{aligned} \rho(t) &= \hat{\rho}_a \cos(\hat{\omega}_e t + \hat{\varepsilon}) \\ &= \hat{\rho}_a \cos(\hat{\varepsilon}) \cos(\hat{\omega}_e t) - \hat{\rho}_a \sin(\hat{\varepsilon}) \sin(\hat{\omega}_e t) \\ &= a_1 \cos(\hat{\omega}_e t) - a_2 \sin(\hat{\omega}_e t) \end{aligned} \quad (12)$$

and fit for both  $a_1$  and  $a_2$  simultaneously where  $a_1 = \hat{\rho}_a \cos(\hat{\varepsilon})$  and  $a_2 = \hat{\rho}_a \sin(\hat{\varepsilon})$ . The response amplitude estimate  $\hat{\rho}_a$  and phase estimate  $\hat{\varepsilon}$ , respectively, are thus given by

$$\hat{\rho}_a = \sqrt{a_1^2 + a_2^2} \quad (13)$$

$$\hat{\varepsilon} = \text{atan2}(a_2, a_1); \quad -\pi < \text{atan2}(\dots) \leq \pi \quad (14)$$

Consequently, using Eq. (12) as the regression function, it is sufficient to run the NLLS algorithm once.

The final step in the wave estimation by the NLLS concept, see Figure 4, is to combine the response amplitude estimate and the (inverse of the) RAO to obtain the wave amplitude. The wave phase, on the other hand, is determined by subtracting the total phase, given by the NLLS fitting, and the phase, set by the RAO, between wave and response.

### III. EXPERIMENTAL SETUP AND MOTION RAOs

The outlined techniques for sea state estimation have been previously tested by [8] for some theoretical examples based on numerical simulations of vessels exposed to sinusoidal



Figure 6: *Cybership 3* deployed in the model-basin at NTNU. [18]

wave trains. The examples considered below, in Section IV, also focus on a ship exposed to regular (“sinusoidal”) wave trains but, herein, results are derived from model-scale experiments. This section gives a brief introduction to the experimental facilities, where the experiments were conducted. Furthermore, an overview of the test cases is included and, finally, the section provides some basic information about the RAOs of the studied vessel.

#### A. Laboratory Facilities

Facilities at the Marine Cybernetics Laboratory (MCLab) at NTNU, Trondheim, were used for the experiments. It includes a basin with dimensions  $40 \text{ m} \times 6.45 \text{ m} \times 1.41 \text{ m}$  ( $L \times B \times D$ ), a vision-based positioning system that provides position and orientation measurements to a dynamic positioning (DP) system, and a wave flap<sup>1</sup> for generating waves from different wave spectra. Figure 6 shows the model vessel in action.

The experiments were conducted with *Cybership 3*, a 1:30 scale model of a platform supply vessel (PSV) with dimensions  $L_{pp} = 1.971 \text{ m}$  and  $B = 0.437 \text{ m}$ . It is equipped with three azimuth thrusters; two at stern with fixed angles of  $\pm 30^\circ$  and one in the bow at  $90^\circ$ , see Figure 7. The vessel has eight 12 V batteries supplying power to the thrusters and a National Instruments CompactRio (cRIO) where the DP control system is running. The operator supplies setpoints and specifies controller gains from a laptop, and communication between the camera system, operator laptop and cRIO is via Ethernet.

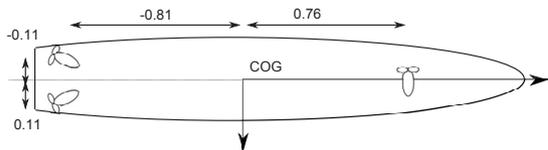


Figure 7: *Thruster configuration of Cybership 3*. Measures are in meters. [18]

#### B. Test Cases

In total, five test cases were run and the associated wave parameters (in model-scale) are given in Table I. All cases

<sup>1</sup>DHI Wave Synthesizer, [www.dhigroup.com](http://www.dhigroup.com).

were made with zero-forward speed of the vessel, and the recorded motion responses, relative to the ship’s centre of gravity (COG), include among others heave and pitch.

Table I: *Test cases and corresponding wave parameters.*

Case	Wave heading	Wave amplitude	Wave period
A	Head sea	1.5 cm	0.9 s
B	Head sea	2.0 cm	1.2 s
C	Head sea	3.0 cm	1.5 s
D	Beam sea	2.0 cm	1.2 s
E	Beam sea	3.0 cm	1.7 s

#### C. Motion RAOs

For matters of convenience the RAOs of the motion components, relative to COG, have been calculated using closed-form expressions [25], [26] to enable “continuous” updates of the RAOs, without the need to interpolate for different frequencies; as given by the frequency estimate from Eq. (10). It is noteworthy that the closed-form expressions are derived for a box-shaped hull form, as this assumption generally applies to the expressions [25]. The approximation with a box-shaped hull form means that pitching motions are neglected, equivalently taken to be zero, in any beam sea condition and, hence, the closed-form expressions cannot be used in beam sea. Results of the closed-form expressions for heave and pitch have been compared to corresponding results obtained by in-house software<sup>2</sup> *ShipX* used at MARINTEK. The comparisons for *Cybership 3* are seen in Figure 8 for heave and pitch at zero-forward speed.

<sup>2</sup><https://www.sintef.no/en/marintek/software/maritime/shipx-vessel-responses/>

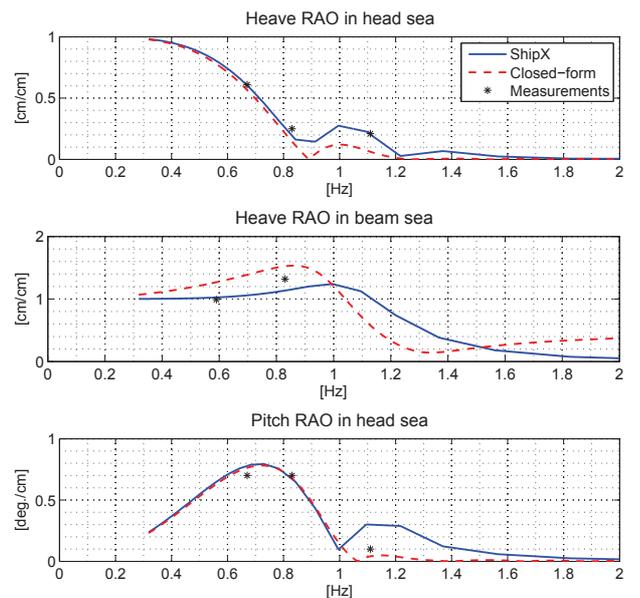


Figure 8: *Heave RAOs of Cybership 3 in head and beam waves, the upper and middle plot, respectively, and pitch RAO in head waves.*

As noted from the plots in Figure 8 the agreement between the two sets of results is reasonable, and both sets, too, agree fairly well with experimental results derived from the model tests. In the following examples it the pitch response is used for wave estimation in the head sea cases (A, B, and C), whereas heave will be used in the beam sea cases (D and E).

#### IV. RESULTS AND DISCUSSIONS

##### A. Response Measurements

The recorded motion responses considered in the test cases, cf. Table I, can be seen in Figure 9 as the upper and lower plots for pitch and heave, respectively, with the individual cases indicated by square boxes. It is noted that the measurements were made without interruptions during the transition periods in which the wave parameters were changed. Note also that, although the lack of zoom leaves few details to be seen, the motion measurements are clearly not sinusoidal and/or regular in the strict sense, as the amplitude level can be seen to fluctuate during all test cases.

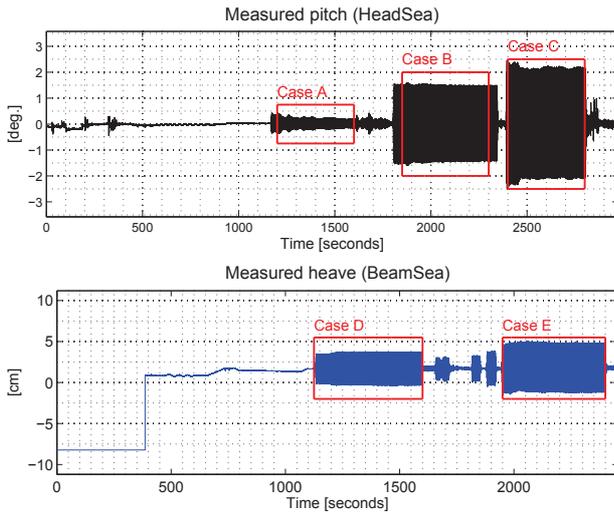


Figure 9: Overview of time history recordings of the response measurements. Cases A, B, and C appear from the upper plot which shows the pitch motion, whereas the lower plot represents the heave motion, i.e. Cases D and E.

##### B. Wave estimation

The underlying wave trains of the single cases have been estimated based on 200 seconds long time history measurements of the pitch and heave responses and the results are seen in Figure 10. The individual plots represent a zoom on an arbitrary time window selected within the full duration, 200 sec., of the considered case. It is noteworthy that in any case the response measurement has been averaged to zero-mean. Usually, the wave elevation is measured in the model-basin by a wave probe but, unfortunately, the probe was malfunction on the days when the experiments were taking place. Consequently, no comparisons can be made between

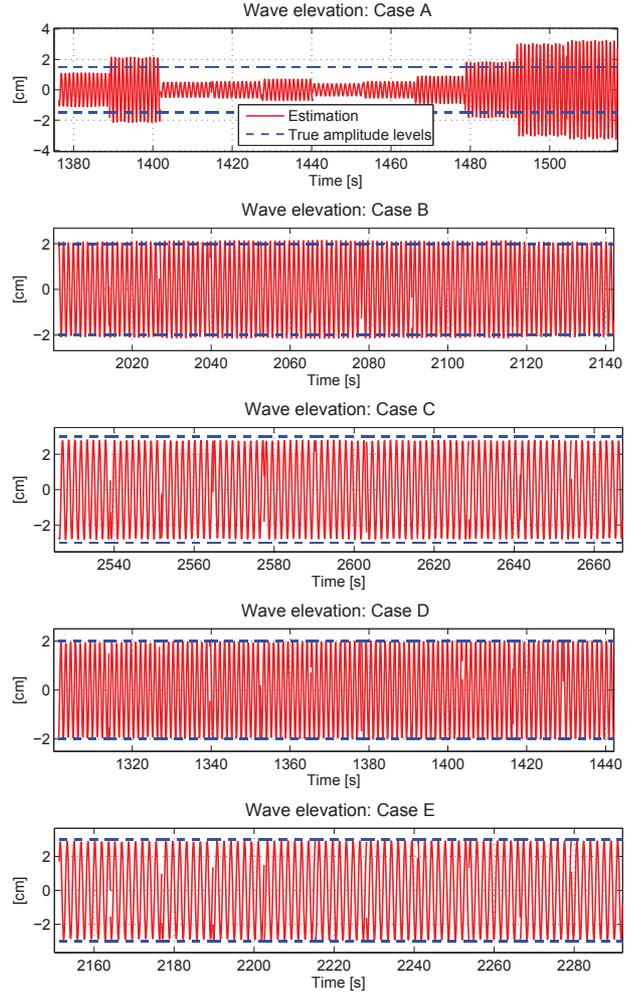


Figure 10: Estimations of wave elevations in Cases A (upper plot) to E (lower plot). Note the difference in scales on the y-axis.

the estimated wave elevation and the actual, true one in the model-basin.

The outcome of the estimations, except that of Case A, looks reasonable as judged by visual inspections of the plots in Figure 10. In order to study the quality of the estimations in more detail, statistics have been produced from the set of batches comprising the single cases (A to E), and the results

Table II: Statistics of estimated wave parameters derived from the batches of data comprising the single cases.

Case	Amplitude [cm]		Period [s]	
	Mean	Std	Mean	Std
A	1.36 (1.5)	1.04	1.03 (0.9)	0.05
B	2.11 (2.0)	0.03	1.21 (1.2)	0.01
C	2.78 (3.0)	~ 0	1.52 (1.5)	0.01
D	1.99 (2.0)	0.02	1.21 (1.2)	0.01
E	2.90 (3.0)	0.01	1.71 (1.7)	0.01

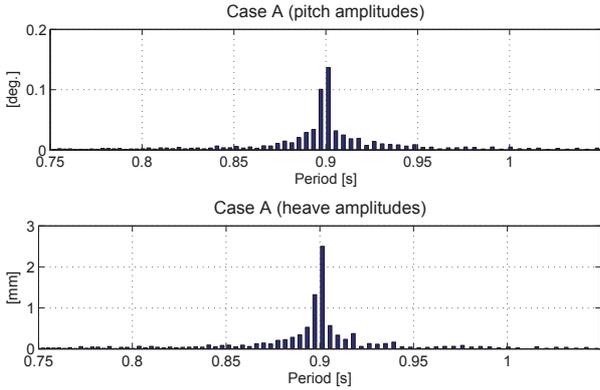


Figure 11: *Distribution of response amplitudes in Case A.*

are given in Table II. As can be read from the table, the estimations in terms of mean values are relatively good as compared to the true input values, included in parenthesis, see Table I. This observation applies to both wave amplitude and wave period with the relative deviation to the true value being at most 9% and 14% for amplitude and period, respectively. However, it is important to note that the outcome of Case A distinguishes itself from the others, since the standard deviations, considering both wave amplitude and wave period, are relatively large. This point is clearly illustrated by the upper plot in Figure 10. The reason for the poorer results, and rather large standard deviations, in the estimation of wave parameters in Case A cannot be explained by (wrong) values of the RAO, cf. Figure 8. Instead, the explanation should be sought in the pitch response measurement itself. Thus, Figure 9 reveals (with good eyes...) that the amplitude level in Case A varies more than it does in the other cases. This is indeed evident by the upper plot in Figure 11, which shows the components of pitching amplitudes, in Case A, for different (wave) periods. The plot has been produced by running an FFT on the measured pitch time series, yielding the power spectrum of pitch and, afterwards, the amplitude components are obtained. Ideally, the plot should show an amplitude component at only *one* (wave) period corresponding to  $T_A = 0.9$  s. Obviously, this is not so for the specific case, since amplitude components are non-zero for many periods distributed around  $T_A$ . Consequently, the estimated wave elevation, based on the fitting of parameters on batches of data, see Figure 4, will be no better than the actual response measurement allows. In the particular case, it makes no difference to use the heave motion as the response for wave estimation, since heave also shows a rather wide distribution of non-zero amplitudes, which is seen from the lower plot in Figure 11. It is noteworthy that similar plots, although not shown, of the other cases (B to E) show less distribution of non-zero response amplitudes, which explains the better estimates in those cases.

### C. General Comments

The (theoretical) examples addressed in [8] were studying also the capability of the estimation techniques to handle non-

stationary conditions, such as stepwise and sudden changes to the (true) wave parameters of the wave train to be estimated. The same kind of experiments could not be made in the model testing facility, since it was possible only to change the control mechanism of the wave generator after a full stop.

Another missing part in this study is the absence of the wave elevation recording in the model basin. Unfortunately, it means that the estimated wave elevation cannot not be directly compared with the actual one and, consequently, it is not possible to check the accuracy of the estimated phase, cf. Eq. (12).

## V. SUMMARY AND CONCLUDING REMARKS

New techniques for shipboard sea state estimation have been studied. Altogether, the combined procedure relies on a frequency estimator [19] to detect the (encountered) wave peak frequency and, subsequently, a nonlinear least squares (NLLS) fitting used to estimate the wave amplitude and phase. The method has been previously studied [8] for some theoretical cases given in terms of numerical simulations. In the present work, similar cases were evaluated, not by simulations but by model experiments, and it was shown that the techniques comprising the combined method again could successfully estimate the wave parameters of a wave train by analysis of wave-induced vessel responses. So far, the method is limited to handle only the estimation of regular wave trains from corresponding response measurements on a ship without forward speed.

Based on the present work and the initial study [8] the following points should be noted:

- The frequency estimator, Eq. (10), requires the gain to be tuned properly, and efforts could be made to allow the gain tuning to be completely automated, which is not the case in the procedure as is. Work in this direction has been explored already and one feasible approach is developed in [17].
- Until now, cases of zero-forward speed have been considered only and, obviously, the procedure should also be capable to handle data from advancing marine crafts.
- The extension to consider regular wave trains composed by two wave components could be beneficial, as it would provide knowledge about how to handle estimation of an irregular wave train made up by a (very) large number of regular wave components. Specifically, work could address the use of several notch or bandpass filters to select individual harmonic components from a wave spectrum, and then use regular wave estimators in parallel for each component. In the end, this would make the method applicable to real (full-scale) data.
- The combination/consideration of several responses simultaneously, e.g., {heave; roll; pitch} could possibly be used to estimate also the relative wave heading.

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## REFERENCES

- [1] J. Hua and M. Palmquist, "Wave Estimation through Ship Motion Measurement," Naval Architecture, Department of Vehicle Engineering, Royal Institute of Technology, Tech. Rep., 1994.
- [2] T. Iseki and K. Ohtsu, "Bayesian estimation of directional wave spectra based on ship motions," *Control Engineering Practice*, vol. 8, pp. 215–219, 2000.
- [3] E. A. Tannuri, J. V. Sparano, A. N. Simos, and J. J. D. Cruz, "Estimating directional wave spectrum based on stationary ship motion measurements," *Applied Ocean Research*, vol. 25, pp. 243–261, 2003.
- [4] U. D. Nielsen, "Estimations of on-site directional wave spectra from measured ship responses," *Marine Structures*, vol. 19, pp. 33–69, 2006.
- [5] ———, "Introducing two hyperparameters in Bayesian estimation of wave spectra," *Probabilistic Engineering Mechanics*, vol. 23, pp. 84–94, 2008.
- [6] R. Pascoal, C. G. Soares, and A. J. Sørensen, "Ocean Wave Spectral Estimation Using Vessel Wave Frequency Motions," *Journal of Offshore Mechanics and Arctic Engineering*, vol. 129, pp. 90–96, 2007.
- [7] N. Montazeri, U. D. Nielsen, and J. J. Jensen, "Estimation of wind sea and swell using shipboard measurements - A refined parametric modelling approach," *Applied Ocean Research*, vol. 54, pp. 73–86, 2016.
- [8] U. D. Nielsen, M. Bjerregård, R. Galeazzi, and T. I. Fossen, "New Concepts of Shipboard Sea State Estimation," in *Proc. of MTS/IEEE OCEANS'15*, Washington, DC, USA, 2015.
- [9] U. D. Nielsen and D. C. Stredulinsky, "Sea state estimation from an advancing ship - A comparative study using sea trial data," *Applied Ocean Research*, vol. 34, pp. 33–44, 2012.
- [10] U. D. Nielsen, I. M. V. Andersen, and J. Koning, "Comparisons of Means for Estimating Sea States from an Advancing Large Container Ship," in *Proc. 12th PRADS*, Changwon, South Korea, 2013.
- [11] A. N. Simos, J. V. Sparano, E. A. Tannuri, and V. L. F. Matos, "Directional Wave Spectrum Estimation Based on a Vessel 1st Order Motions: Field Results," in *Proc. of 17th International Offshore and Polar Engineering Conference*, Lisbon, Portugal, 2007.
- [12] J. V. Sparano, E. A. Tannuri, A. N. Simos, and V. L. F. Matos, "On the Estimation of Directional Wave Spectrum Based on Stationary vessels 1st Order Motions: A New Set of Experimental Results," in *Proc. of OMAE'08*, Lisbon, Portugal, 2008.
- [13] C. Møgster, "Bayesian methods for estimating non-stationary ship response spectra," Master's thesis, Norwegian University of Science and Technology, Trondheim, Norway, 2015.
- [14] T. Iseki and U. D. Nielsen, "Study on a Short-term Variability of Ship Responses in Waves," *Journal of Japan Institute of Navigation*, vol. 132, pp. 51–57, 2015.
- [15] U. D. Nielsen and T. Iseki, "Prediction of First-Order Vessel Responses with Applications to Decision Support Systems," in *Proc. of 5th World Maritime Technology Conference*, Providence, RI, USA, 2015.
- [16] R. Pascoal and C. G. Soares, "Kalman filtering of vessel motions for ocean wave directional spectrum estimation," *Ocean Engineering*, vol. 36, pp. 477–488, 2009.
- [17] D. J. W. Belleter, R. Galeazzi, and T. I. Fossen, "Experimental verification of a global exponential stable nonlinear wave encounter frequency estimator," *Ocean Engineering*, vol. 97, pp. 48–56, 2015.
- [18] A. H. Brodtkorb, U. D. Nielsen, and A. J. Sørensen, "Sea State Estimation Using Model-scale DP Measurements," in *Proc. of MTS/IEEE OCEANS'15*, Washington, DC, USA, 2015.
- [19] S. V. Aranovskiy, A. A. Bobtsov, A. S. Kremliev, and G. V. Lukyanova, "A robust algorithm for identification of the frequency of a sinusoidal signal," *Journal of Computer and Systems Sciences International*, vol. 46, pp. 371–376, 2007.
- [20] M. Bjerregård, "Methods for sea state estimation," Master's thesis, Technical University of Denmark, Kgs. Lyngby, Denmark, 2014.
- [21] H. Akaike and T. Nakagawa, *Statistical Analysis and Control of Dynamic Systems*. KTK Scientific Publishers, 1988.
- [22] A. Neumaier and T. Schneider, "Estimation of Parameters and Eigenmodes of Multivariate Autoregressive Models," *ACM Transactions on Mathematical Software*, vol. 27, pp. 27–57, 2001.
- [23] X. Jiang and G. Kitagawa, "A Time Varying Coefficient Vector AR Modeling of Nonstationary Covariance Time Series," *Signal Processing*, vol. 33, no. 3, pp. 315–331, 1993.
- [24] T. Iseki and D. Terada, "Study on Real-time Estimation of the Ship Motion Cross Spectra," *Journal of Marine Science and Technology*, vol. 7, pp. 157–163, 2002.
- [25] J. J. Jensen, A. E. Mansour, and A. S. Olsen, "Estimation of ship motions using closed-form expressions," *Ocean Engineering*, vol. 31, pp. 61–85, 2004.
- [26] A. Mansour, J. Jensen, and A. Olsen, "Fast Evaluation of the Reliability of Container Securing Arrangements," in *Proceedings of PRADS'04*, Travemünde, Germany, 2004, pp. 577–585.