

# Sensitivity analysis of sampling and clustering techniques in expansion planning models

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**Abstract**—Short and long-term power system planning models are becoming more complex in order to capture current and future market characteristics comprising more variability, uncertainty, and integration of geographically spread market areas. Dimension reduction methods can be used to keep the planning models tractable, e.g. time series sampling and clustering, but they represent a trade-off between model complexity and level of detail. The accuracy of dimension reduction methods can be measured both in terms of raw data processing and model output metrics, where the latter reveals how well a sampling technique fits that particular model instance. In this study, the robustness of several sampling and clustering techniques is quantified with different model instances by independently varying model parameters, such as e.g. the marginal cost of generation. As the obtained findings indicate that the performance of the considered techniques is, indeed, model-dependent, more insight into the performance of common dimension reduction techniques in power system planning applications is provided. The results are illustrated by a case study of the North Sea Offshore Grid (NSOG) for the scenario year 2030, using a bi-level mixed-integer linear optimization program. All things considered, systematic sampling and moment matching are shown to give the most robust results from the sensitivity analysis.

**Index Terms**—Clustering; Dimension Reduction; Sampling; Sensitivity Analysis; Time Series; Transmission Expansion Planning.

## I. INTRODUCTION

An increasing share of variable and non-dispatchable generation capacity is expected in many power systems over the coming decades, yielding more volatile variations in the net load (i.e. load subtracted by non-dispatchable power generation) which need to be balanced with conventional generation capacity and other means of system flexibility [1]. Hence, in order to maintain a reliable power system, it is important to incorporate adequate system characteristics in power system planning models capturing the value of both temporal and spatial flexibility [2]. For instance, larger geographical areas

should be considered due to wind speed smoothing effects [3], and different market states (operational resolution) due to interdependency among non-dispatchable generation and load, in addition to a variety of recurring power flow patterns [4].

Planning models with a multinational geographical scope are becoming more common and their size and complexity can easily lead to intractable optimization models [5]. For this reason, pre-processing of data input can be very beneficial in order to construct tractable optimization models, while, at the same time, trying to replicate the characteristics of the original data set for the problem [6]. Common approaches use load duration curves or other generic scenario reduction approaches, such as sampling and clustering methods, on the model's input data [7], [8], and [9]. Conversely, a reduction approach focused on the model's output data rather than the input data is shown in [10]. In general, these techniques produce more compressed time series and fewer operational time steps in the optimization problem, which can significantly reduce the computational complexity (solution time).

Transmission expansion planning (TEP) models are specifically sensitive to the aforementioned, multinational system characteristics since their lumpy and capital intensive investment decisions are dependent on regional- and national price differentials [11]. To illustrate, a high non-dispatchable production in one area with low demand could use a transmission line to transmit power to another area with high demand and low non-dispatchable generation. This is particularly relevant in the European context, where the European Union is pursuing a fully integrated internal energy market facilitating a free flow of electricity across its regions for a successful integration of renewable energy [1].

In this article, the focus is on TEP models with a case study of the North Sea Offshore Grid (NSOG) which has been identified as a strategic trans-European energy infrastructure priority in the EU Regulation No 347/2013. Sampling and clustering methods are used to reduce the number of time steps considered in this context, e.g. from a full year (8760 hours) to a set of representative hours in the range of 1-25 % of the full year time series (68 to 2190 hours).

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### A. Literature on comparative sampling and clustering

Comparative analyses dealing with a variety of sampling and clustering techniques are still not well established in the literature. One exception is the predecessor of this article which ranks the most common dimension reduction methods used for a multivariate set of time series input data in a power system planning model [6]. The presented methods are particularly suitable for applications in optimization models without temporal constraints, e.g. restrictions ensuring ramping limits, minimum up- and down-times, and energy storage. The results indicate that sampling methods performing best in terms of raw data fit, measured as the average normalized root-mean-square error (NRMSE), do not necessarily perform in the same order taking the model output into perspective, i.e. deviations in investment costs (CAPEX) and operational costs (OPEX).

In [6],  $k$ -means [12],  $k$ -medoids and hierarchical clustering [13] are studied, in addition to a simple systematic sampling and a statistical moment-matching technique [14]. In power system operation and planning, those methods have been used for a long time, but the literature falls short on comparisons of their overall performance. A comparison of different approaches for selecting representative days in generation expansion planning problems as well as a new optimization-based approach is presented in [15]. Other works such as [16] provide a comparison of different clustering techniques in the context of power system reliability assessments.

The main contribution of this article is to follow up the previous work presented in [6] which quantified the performance of different dimension reduction methods in a transmission expansion planning model and compared it with the raw data sampling and clustering performance. However, the question as to whether the performance ranking is model- or instance-dependent, or not, for that matter, remains unanswered. In order to acquire more insights, a similar study with the same sampling and clustering techniques, the same sample sizes, and an almost identical model, but with different model instances in terms of sensitivity parameter variations is carried out. It is important to stress that by considering generation expansion in addition to transmission expansion and combining them both into the long-term expansion planning model a further dimension is added to the problem.

The remaining part is structured as follows. Section II outlines the overall methodology of the study and Section III provides an overview of the considered dimension reduction methods, the expansion planning model and its input data, as well as the investigated sensitivity parameters. The results are presented and discussed in Section IV and Section V concludes the study.

## II. METHODOLOGY

For a detailed overview and references of the different sampling and clustering techniques, as well as the compact mathematical model formulation of the long-term expansion planning model which is used in this study, the reader is referred to [6]. In the context of this article, only a brief overview will be given.

The approach of this study consists of three main steps: First, five different sampling and clustering techniques are used to reduce the size of a time series matrix containing information about hourly offshore wind, onshore wind, solar, and hydro generation, as well as hourly load levels in six different market areas, countries in this case. To that end, only a fraction of the total number of time steps in a full-year (8760 hours) is considered.

Secondly, the resulting reduced time series matrices are used as input for the expansion planning model which is then run for a range of possible model instances reflecting sensitivity variations, i.e. by individually varying six different model parameters.

Finally, the robustness and performance of all considered sampling and clustering techniques are assessed by evaluating the model output accuracy in terms of capital and operational expenses (CAPEX and OPEX) deviations for all analyzed model instances (sensitivity variations). The deviations are measured with respect to the results obtained by the model when using full-year time series. This allows for an estimate of whether there is reason to believe that the performance of each sampling or clustering method is model-dependent, or not.

## III. CASE STUDY FUNDAMENTALS

### A. Dimension reduction methods

In accordance with the dimension reduction methods discussed in [6], the candidate sampling and clustering techniques being employed here include

- Systematic sampling,
- $k$ -means clustering,
- $k$ -medoids clustering,
- Hierarchical clustering, and
- Moment-matching.

In order for the expansion planning model to identify efficient transmission investments for a multinational NSOG, it is important to capture the underlying values of its capability to provide both temporal and spatial flexibility for system operation. Moreover, in contrast to the study presented in [6], the investment model is able to expand generator capacity to incorporate power generators' response to transmission grid investments. Since generation expansion decisions add one more dimension to the problem, the sampling and clustering rankings obtained in [6] are likely to be affected by this model extension. However, the impact is limited as the generation expansion is restricted to 10% of the input data.

### B. Model and input data

The expansion planning model is a bi-level mixed-integer linear program (MILP) which is a common way to formulate TEP models [5]. It co-optimizes investment decisions and market operation in a power system consisting of several market areas bordering the NSOG: Norway (NO), Great Britain (GB), Denmark (DK), Belgium (BE), Germany (DE), and the Netherlands (NL).

The case study of a potential future offshore grid in the North Seas is based on one of ENTSO-E's scenarios for 2030 known as "Vision 4" [17]. This vision is a top-down scenario developed at the European level and it is designed to meet the objectives of the European Commission on market integration and climate mitigation. It is considered to be the most ambitious of the four visions in terms of share of renewable generation capacity. Therefore, the considered sampling and clustering methods' ability to capture extreme multivariate correlations across country borders becomes even more important in this context.

### C. Sensitivity parameters

To investigate the model-dependent and -independent effects of using dimension reduction methods, different model instances are created by varying the following sensitivity parameters:

- CO<sub>2</sub> price,
- Marginal cost of generation,
- Interest rate,
- Economic lifetime,
- Transmission infrastructure capital costs, and
- Annual energy inflow to hydro power plants.

For the purpose of this study, the sensitivity parameters in question are set to -50 %, -25 %, +25 %, and +50 % with respect to the input values used in [6].

## IV. RESULTS AND DISCUSSION

### A. Raw data sampling and clustering performance

In general, the sampling and clustering performance of raw data is independent of the model it is being applied to. Since the same data set of [6] for installed generation capacities and peak load levels, among others, are used here, the same raw data sampling and clustering results are obtained.

To summarize, the normalized root-mean-square error (NRMSE) was calculated as an average for all time series, i.e. load, onshore wind, offshore wind, solar, and hydro, for each dimension reduction method and sample size. The average NRMSE measure suggested that the  $k$ -means clustering performed best for all sample sizes when processing the raw data. It therefore stood to reason that the  $k$ -means method also yields the most accurate results when using it in the expansion planning model.

### B. Model output performance: fixed model instance

However, the  $k$ -means method does not perform as well as expected. In fact, it exhibits a poor performance in the model output regarding the average deviation in investment strategy and performs only slightly better than the systematic sampling when considering total cost deviations [6]. Compared to the full-year optimization, hierarchical, and  $k$ -medoids clustering resulted in the most accurate model output behavior. Note that those results are based on transmission expansion, only.

By including generation expansion into the model instance presented in [6], similar performance rankings are obtained for the considered sampling and clustering techniques. However,

there is a minor change worth highlighting: on average, the  $k$ -medoids is outperformed by the moment-matching technique. This means that the ranking was affected by the inclusion of generation expansion.

The scope of this work is not the ranking in terms of how well different dimension reduction methods perform on one model instance, as discussed above and in [6], but rather to analyze how robust a given ranking is over a range of model instances created by varying sensitive model parameters. A summary of the corresponding findings is the subject of the following part of this analysis.

### C. Model output performance: instance sensitivity

1) *Operational cost performance:* Fig. 1 shows the model output performance for different parameter variations ranging from -50 % to +50 %. Each diagram shows the isolated impact of increasing or decreasing the value of one particular parameter. For instance, the upper left diagram exhibits the impact of varying the marginal cost of generation with -50 %, -25 %, +25 %, and +50 %, for all sampling and dimension reduction methods with sample sizes of 68, 137, 274, 548, 1095, and 2190 time steps. The deviation in OPEX is calculated in relative terms to the corresponding full-year benchmark optimizations (for each sensitivity variation). Focusing on the smallest sample size, 68, it becomes obvious that hierarchical clustering gives the smallest deviation in OPEX for the +50 % case (marked with a big upward-facing triangle). If this performance is model-independent, then the same should hold for the other variations in marginal costs, i.e. -50 %, -25 %, and +25 %. It is shown that a +25 % increase in marginal costs (marked with a small upward-facing triangle) yields the same ranking, but for -25 %, it seems that moment-matching is the best performing dimension reduction method. That said, for -50 % it rather is systematic sampling.

Based on the operational cost impact in Fig. 1, it becomes clear that the considered sampling and clustering techniques do not exhibit a consistent ranking across all model instances. However, even though there is an inconsistent ranking, some parameters yield a smaller impact and variability of results than others. For instance, the sensitivities of economic lifetime and interest rate result in a smaller range of OPEX variations than e.g. changes in marginal costs or energy inflow to hydropower plants.

One particularly interesting observation from Fig. 1 is the impact of varying the infrastructure investment cost. A +25 % increase in investment costs returns about the same OPEX as the full-year simulation, while a -25 % decrease in investment costs induces a significant impact on the performance of the dimension reduction methods. In the context of the conducted case study, this could imply that cheaper market integration gives larger exposure to multivariate correlations and regional characteristics in power generation- and load levels. Hence, the sampling and clustering techniques' ability to capture those correlations becomes more important. Surprisingly, a -50 % decrease in investment costs offsets some of the extreme deviations from the full-year simulation. Hence, there is reason

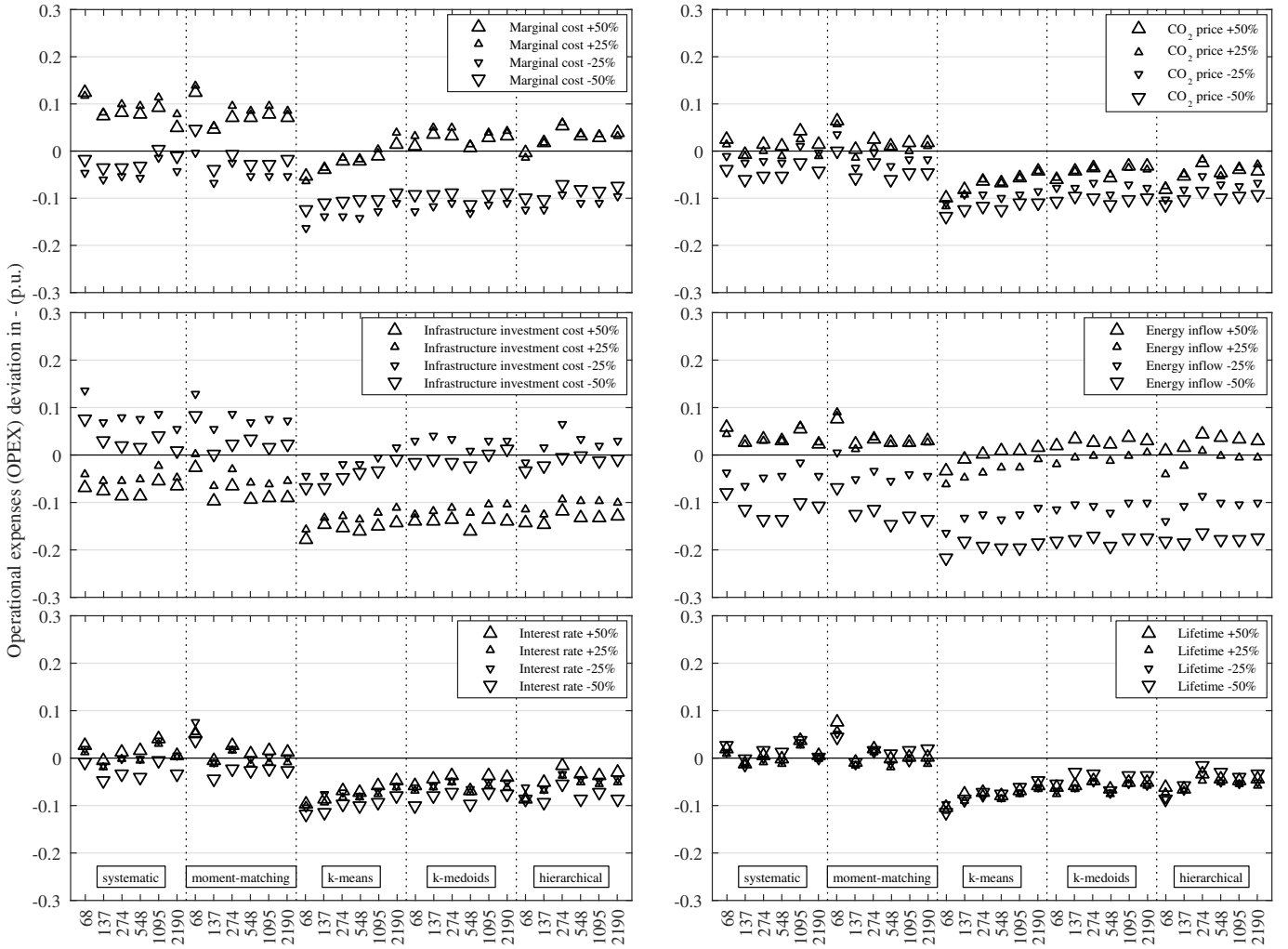


Fig. 1. Operational expenses (OPEX) performance for six different model parameters, each using reduced time series matrices for five different sampling and clustering techniques with sample size varying from 68 to 2190 times steps. Their performance is measured in relative terms to a full-year analysis (benchmark) and marked with upward- and downward-facing triangles indicating the direction of sensitivity parameter change with the size indicating the level of change.

to believe that some model instances, or sensitivities, have a more global than regional impact, which, in this case, is reflected by the operational flexibility in the system. In the -25% case, this means that only some candidate lines are built, yielding an asymmetric integration of the system through a few candidate transmission branches, compared to a more symmetric integration in the -50% case where it is economic efficient to build even more transmission lines. Those type of situations are difficult to account for with model-independent sampling and clustering methods.

Moreover, the diagrams in Fig. 1 show two occurring trends with respect to OPEX deviations from their full-year benchmark: 1) systematic sampling and moment-matching seem to give a more symmetric distribution of deviations around zero compared with the other approaches being prone to underestimate the true OPEX, and 2), *k*-medoids outperforms *k*-means in terms of robustness and deviations which, in turn, is outperformed by hierarchical clustering. As the

cluster centroids of *k*-medoids contain actual data points, correlations might endure slightly better than with the *k*-means method, resulting in smaller deviations from the reference case. Overall, a clear distinction can be made between the sampling and clustering robustness from Fig. 1: the sampling methods perform symmetrical around zero, while the clustering methods tend to consistently underestimate the operational costs.

Having said that, it is the investment decision which matters most for expansion planning applications. Hence, minor deviations in operational cost performance could be tolerable as long as the resulting investment strategies remain consistent. It must be noted, however, that those two elements are strongly related, as already became clear from the investment cost sensitivity in Fig. 1. Then again, the bulk nature of transmission infrastructure investments features some “buffers” since the number of cable investments is determined by integer variables (not continuous variables as for generation expansion). This

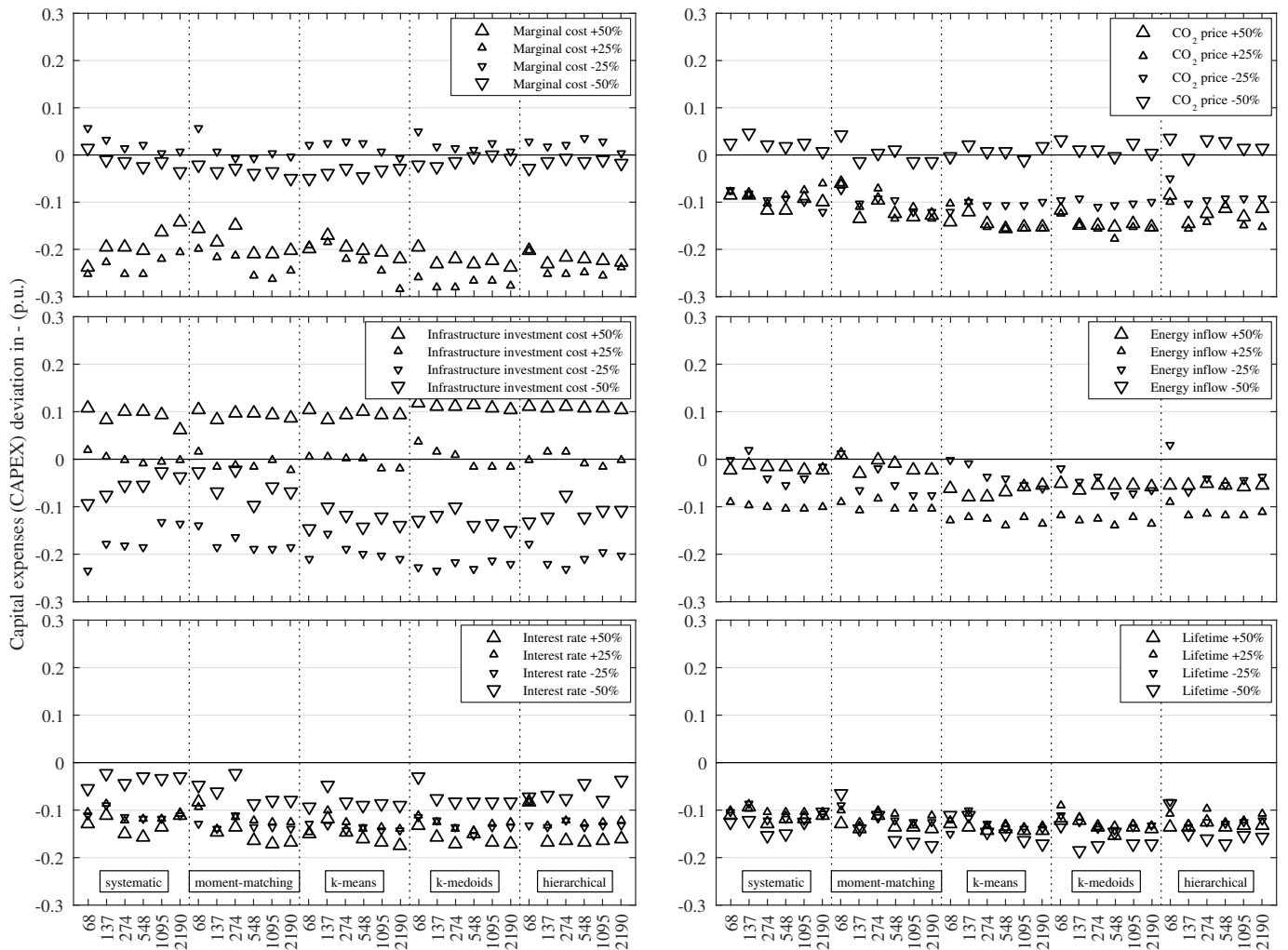


Fig. 2. Capital expenses (CAPEX) performance for six different model parameters, each using reduced time series matrices for five different sampling and clustering techniques with sample size varying from 68 to 2190 times steps. Their performance is measured in relative terms to a full-year analysis (benchmark) and marked with upward- and downward-facing triangles indicating the direction of sensitivity parameter change with the size indicating the level of change.

implies that minor deviations in operational costs might not necessarily lead to new infrastructure investments.

2) *Investment cost performance:* The diagrams in Fig. 2 follow the same logic as in Fig. 1 with the key difference of depicting the model output behavior regarding capital expenses (CAPEX). First, note that the relative spread for marginal costs and infrastructure investment costs is somewhat larger than for OPEX deviations in Fig. 1. This is because both transmission and generation expansion are considered by the model. By only accounting for transmission expansion, the range of variations would have been smaller, as was shown in [6]. In contrast, the spread, or impact, resulting from changes in energy inflow is smaller than for the latter. Recall that variations in energy inflow mainly affect hydropower in market area NO, where its installed hydropower capacity is considerably larger than in the continental mainland market areas [17]. Due to the fact that those variations are only “visible” for the system through a limited set of candidate transmission lines (e.g. from NO to DE and GB), minor

changes are reasonable.

There are a few patterns in Fig. 2 worth discussing. First, note the small deviations that occur for reductions in marginal costs and for -50% CO<sub>2</sub> price. Most of those cases give robust results by not deviating too far from the full-year benchmark simulations, leading to about the same level of investments. As those parameter reductions mainly impact thermal units, one explanation could be that arbitrage opportunities are canceled out due to small price differentials between market areas bordering the NSOG. Hence, there is only small room for deviations in investment strategies. In turn, this could also explain the opposite effect becoming obvious for increased marginal costs and CO<sub>2</sub> prices since those scenarios consistently result in under-investments, i.e. there might be room for more investment opportunities than the model manages to identify with the reduced time series data.

Compared to the OPEX findings in Fig. 1, the same level of consistent trends or patterns is not as clear for CAPEX deviations in Fig. 2. This might be due to the “bulky” nature of

grid investments which represent the largest share of CAPEX, making it harder to distinguish the different results. However, all things considered, the most robust methods which are performing well over a wide range of model sensitivities seem to be the systematic sampling and moment-matching technique. One reason for this might be the fact that the more sophisticated clustering techniques build clusters in which the most extreme data points are represented by a cluster centroid. In other words, a minimum power feed-in from e.g. wind or solar can be higher than in the original data set, see [6].

## V. CONCLUSION

Motivated by the concern of growing model complexity and increasing computational challenges, this article investigates the impact of dimension reduction methods for power system models. To that end, a selection of dimension reduction methods is analyzed and used to sample from hourly full-year time series data including load and renewable generation. The robustness of these techniques is evaluated by running a sensitivity analysis on model parameters, and thereby different model instances.

This article shows that the considered techniques vary in terms of robustness, while, at the same time, revealing how sensitive the model is for particular parameters. In fact, the study also implies that some sensitivities have more local than global impacts in operation conditions in the expansion planning model with recurring effects in the investment strategies.

Finally, the core of this research suggests that the performance of different sampling and clustering techniques are model-dependent, implying that some methods might yield better results for particular expansion planning models than others. For this reason, model-dependent dimension reduction techniques might lead to more robust solutions than the most common, model-independent techniques which are investigated in this study.

### A. Shortcomings and future work

First, the sampling and clustering methods considered in this study are independent of the model itself. For instance, it could be worthwhile to use other methods such as importance sampling which identifies the cost elements being most crucial in a model. According to expectations, this would offset some of the volatility being observed in this study.

Secondly, the expansion planning model has no inter-temporal constraints. Including that kind of constraints would add another element to the problem, see [6], and allow for more dimension reduction methods to be evaluated.

Finally, the inclusion of generation expansion complicated the interpretation of the results. Moreover, because the dimension reduction techniques relied on the installed generation capacity to cluster and weigh data points, enabling the model to expand generation capacity distorted the “true” performance of the sampling and clustering techniques to some extent. Then again, it should be noted that the generation expansion was limited to 10% of the input data.

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