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A BÉZIER CURVE BASED SHIP TRAJECTORY OPTIMIZATION FOR CLOSE-RANGE MARITIME OPERATIONS

Guoyuan Li and Houxiang Zhang
Department of Ocean Space Operations and Building Technology
Norwegian University of Science and Technology
Ålesund, Norway
Email: {guoyuan.li, hozh}@ntnu.no

ABSTRACT

Ship maneuvering in close-range maritime operations is challenging for pilots, since they have to not only prevent the ship from collisions and compensate environmental impacts, but also steer it close to the target towards a proper heading. This paper presents a path planner to assist the pilots to foresee the optimal trajectory in the scenario. The path planning is formatted as an optimizing problem to minimize the turning variation fluctuation and the fuel consumption of the ship through ocean current while satisfying the constraint of orientations at the start and the end positions. Taking advantages of Bézier curves' smoothness and adjustability, feasible trajectories are divided into two categories based on the location of the intersection between the start and end directions, and are designed as a set of parameterized Bézier curves. The variables in the Bézier curves become the state space. By searching the space using an evolutionary technique, the candidate of the Bézier curve that has the best turning and the minimized fuel consumption can be obtained. Through two case studies, the feasibility and effectiveness of the proposed planner is verified.

INTRODUCTION

With the growth of maritime business in Norway, the complexity of ship maneuvering during maritime operations increases, as more constraints from position and heading accuracy, limited working space, and collision avoidance between vessels and floating structures need to be taken into consideration. Although advanced technologies in ship construction have been essentially oriented towards performing navigation system, ship accidents still frequently occur at sea [1]. Statistics show the majority of accidents are due to human errors, e.g., navigators are lack of full map of ship motion in mind and cannot foresee the operational consequence promptly. More importantly, no neglecting of environmental perturbations such as wind, wave and current increase the maneuvering difficulty and reduce the response time for decision-making [2]. To address the operational complexity and guarantee ship safety, new knowledge and technology for ship maneuvering tasks in complicated marine operations are urgently demanded.

Path planning is one of the possible ways to assist navigators to avoid accidents, indirectly improving the performance of ship maneuvering. In autonomous robotics domain, path optimization from an initial to a goal position in static or dynamic environments is a fundamental robotic issue. The goal position as well as location and dimensions of obstacles are pre-defined in the operational space. A feasible path can then be generated via path planning algorithms, such as potential field, elastic roadmaps and rapidly exploring random tree [3]. Besides collision avoidance, physical constraints of the robot, such as limits on velocity, acceleration and turning radius, are also of great concern in the development of path planners [4-6].

For ship navigation, although modern bridge systems contain limited trajectory planning functionality, they were designed to act as advisory devices, assisting to predict dangers of collision. Nevertheless, attempts from research perspectives, have been made on the premise of ship safety to optimize the route of marine vehicles. Tam and Bucknall developed a path planner for ships in close-range encounters. By modeling the dynamic obstacles and predicting their movements, an optimal navigation path can be generated based on evolutionary algorithms [7]. Norstad et al. presented a multi-start local search method to solve the fuel consumption problem for tramp shipping companies. They modeled the problem as a nonholonomic system and optimized the speed along a fixed single ship route to minimize fuel consumption [8]. Mizuno et al. designed a neural network based controller together with the sequential conjugate gradient-restoration method to generate the path and minimize the corresponding maneuvering time for parallel deviation ship maneuvering problem [9]. Tsou and Hsueh proposed a method to construct a collision avoidance path planning model via an ant colony algorithm. It combines navigational practices, a maritime laws/regulations knowledge base and real-time navigation information to plan a safe and economical collision avoidance path [10]. Besides the aforementioned work, some other ship routing systems are also developed, in which environmental impacts on ship trajectory generation, especially the ocean current are more concerned [11-14].

Although ship trajectory planning has been a hot topic in recent years, the emergence of new demands in marine operation, e.g., the requirement of ship orientation during marine operations, are seldom discussed. New constraints are not suitable for directly using off-the-shelf path planning methods. To this end, tailor-made algorithms for specific marine operations are necessary. This paper focuses on generating ship trajectory for close-range maneuvering, in which the initial and final ship orientation are strictly required. Bézier curves, to the best of our knowledge, are preferable to this kind of path planning task. Meanwhile, emphasis is placed on optimization in terms of energy saving. Ocean current is the most important factor we will focus in this paper. Some physical constraints like angular speed are also taken into consideration during the optimization. The result will be a near-optimal path that on the one hand, connects the initial and final points with a reasonable smooth curvature, and on the other hand, makes full use of ocean current for energy saving.

The rest of the paper is organized as follows. First, the problem formulation will be presented. Then, the whole methodology from Bézier curve design to optimization will be introduced in detail, followed by simulation of case studies. Conclusion and future work are drawn in the end.

CLOSE-RANGE MARITIME OPERATION PROBLEM FORMULATION

Ship maneuvering for close-range maritime operation often occurs when maneuvering the ship towards floating structures like the oil platform and wind turbines [15]. For example, Fig.1 depicts that a ship is approaching a floating platform to load/unload goods from the start point (SP) with the heading angle θ_0 to the end point (EP) with the heading angle θ_T. To guarantee ship safety during the fine maneuvering and ensure there is enough space for the subsequent operation, the pilot is very concerned about the following aspects:

- **Speed**: the ship in this case is suggested to have a lower speed to avoid collision.

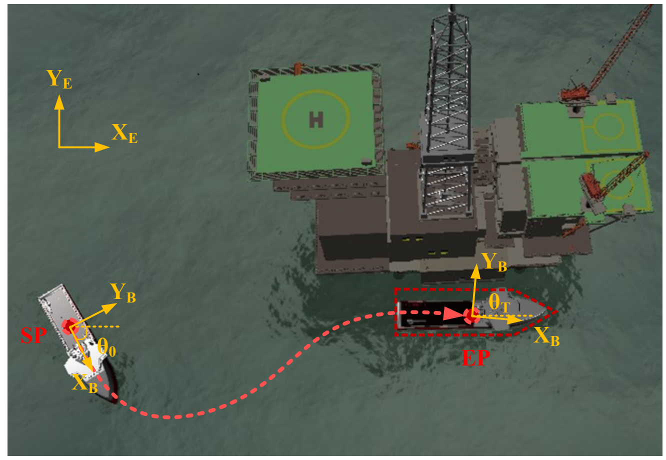

Fig. 1 Example of ship maneuvering for close-range maritime operation.

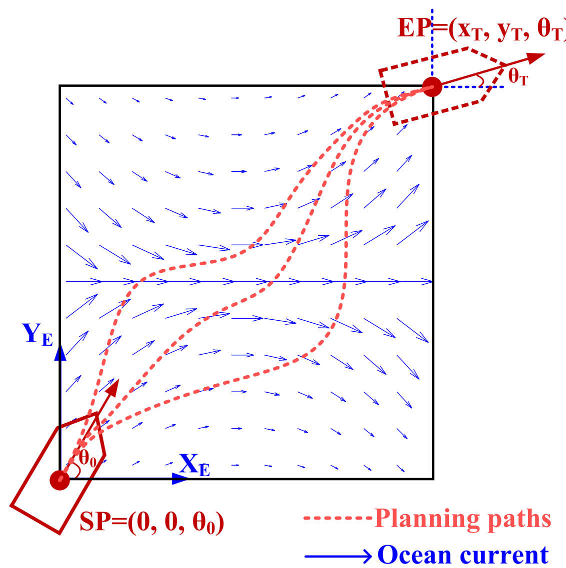

Fig. 2 Problem formation for path planning in close-range maritime operation.

- **Turning**: it is essential to perform a smooth turning to prevent unexpected ship motion like rolling from capsizing.
- **Orientation**: ship heading plays the key role in maneuvering. In particular, close-range maneuvering requires strict ship orientation to avoid collision.
- **Environment**: wave, wind and current inevitably affect the operation of the ship. It is critical to compensate the resultant ship motion resulted from environment. Alternatively, it is potential to take advantages of environmental impact to achieve energy-saving during maneuvering.

According to the concerns for ship maneuvering in close-range maritime operation, a path planner that can optimize all the above-mentioned factors is useful for facilitating the fine maneuvering task. In this paper, we aim to develop such a path planner.

The path planning task is considered as a two-point boundary value problem, in which the SP and EP together with their heading angles θ_0 and θ_T are given in advance as the constraints, as shown in Fig.2. For easiness of description, the SP is fixed to the origin, whereas the EP is located upper right of the boundary. Assumptions have been made to simplify the problem:

1. The planning paths are within the boundary;
2. θ_0 and θ_T are in the scope of [0˚, 90˚];
3. The velocity of the ship over the ground is constant;
4. The ocean current is the only environmental perturbations and observable for path planning;
5. The ship will follow the path with its heading towards the tangent direction.

A kinematic model of the ship under ocean currents in a two-dimensional plane can be described as:

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} v\cos(\theta) + v_{cx}(x,y) \\ v\sin(\theta) + v_{cy}(x,y) \\ (\arctan(y'/x'))' \end{bmatrix} \tag{1}$$

where $[x, y, \theta]$ is the ship status including position and orientation information; v is the ship speed in still water; v_{cx} and v_{cy} are the current velocities in the x_E and y_E directions at the ship's position. Note $u=\sqrt{x'^2+y'^2}$ is the constant ship speed over ground from assumption 3.

Given a feasible path, the forward speed u and the above assumptions, the path length, the sailing time T and the turning θ can be determined. Moreover, the ship velocity in still water v can be calculated by (1), which can be eventually converted into fuel consumption. The objective is to find a near-optimal trajectory that satisfies the constraints of ship status between SP and EP, while minimizing the turning and the fuel consumption.

PATH PLANNING USING BÉZIER CURVES

A Bézier curve is a parametric smooth curve defined by a set of control points [16]:

$$B(n_i) = \sum_{j=0}^{M} b_{j,M}(n) P_j \tag{2}$$

where i is the index of point on the Bézier curve; $n_i \in [0, 1]$ is the parameter describing the interpolation point $B(n)$ between the control point P_j; M is the order of the Bézier curve, i.e., the total number of control points; $b_{j,m}(n)$ is the blending coefficient. The

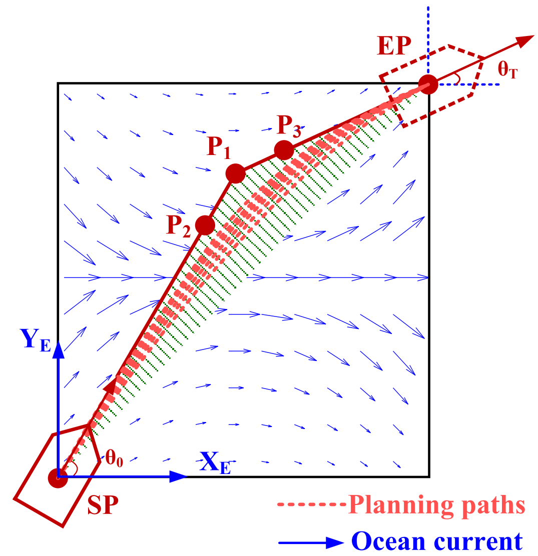

Fig. 3 The case for intersection P1 from SP to EP falling into the boundary.

Bézier curve is within the convex hull of the control points and passes through the first and last control points. Another interesting property of Bézier curves is that the tangents to the curve at the first and last control points are the directions to the adjacent control points. This exactly satisfies the constraints described in previous section where the position and orientation of SP and EP in close-range maneuvering are determined before path planning. For path planning of close-range maneuvering problem, the question now turns to find out the proper control points of the Bézier curve to achieve the optimizing goals.

In this study, in order to decrease the computational complexity, we investigate the cubic Bézier curves with an order of three ($M=4$). Considering a simple case when the orientations from SP to EP intersect at P_1 inside the boundary, as shown in Fig. 3, it is intuitive to use a quadratic Bézier curve (SP, P_1 and EP) to describe the path. However, it loses the variability for searching other possible Bézier curves in the dashed area in Fig. 3. Therefore, another two control points P_2 and P_3 are added to form a cubic Bézier curve (SP, P_2, P_3 and EP). Note P_2 and P_3 are variable control points that can move along the line segment SP-P_1 and P_1-EP, respectively.

When the intersection P_1 from SP to EP is outside of the boundary, the above control point selection strategy cannot be directly adopted due to assumption 1. Instead, the case has to be decomposed into two independent sub cases and then the above simple strategy can be applied. Fig. 4 illustrates the strategy:

1. **Feasible region partition**: there will be a feasible region of polygon that is consisted of SP, EP, and their intersections with the boundary (the dashed area in Fig. 4). The resultant Bézier curves will be generated

within this area. The rest area in the boundary is not taken into consideration due to the cost in terms of distance and curvature.

2. **Joining point**: in order to connect Bézier curves, joining points, as a special kind of control points, are defined. The case in Fig. 4 contains one joining point P_4. Joining points are restricted in the feasible region and considered as tunable variables that will combine with optimization methods to search for optimal paths.

3. **The slope of the joining point**: when connecting two Bézier curves, the slope will determine the connecting direction. The slope will have two intersections with the line section from SP and EP within the boundary, respectively, such as P_5 and P_6 in Fig. 4. It will be the other variable used in searching optimal path.

4. **Decomposition**: the joining point along of its slope will divide the feasible region into two parts. Each part can then be treated as a simple case. The joining point will become the EP for one of the part while the SP for the other part. For instance, in Fig. 4, P_4 is the EP for the simple case SP-P_5-P_4. Likewise, it becomes the SP when dealing the simple case P_4-P_6-EP.

As a result, we smoothly connected the SP and EP using Bézier curves for the close-range maneuvering problem. At the same time, the initial and final orientation constraints are satisfied.

PATH OPTIMIZATION VIA GENETIC ALGORITHM

From the previous section, it has shown that there are several variables, such as the control points, the joining point and its slope, that are closely related to the construction of the resultant Bézier curve. Evolutionary techniques are needed to search for the Bézier curve which minimizes the turning and the fuel consumption in close-range maneuvering through ocean current.

Here, generic algorithms (GA) which is a heuristic search algorithm simulating the survival of the fittest among individuals over consecutive generations is applied [17]. In a generation, the GA maintains a population of chromosomes. The chromosome is coded as an equal length vector of variables or genes, representing a possible solution to the given problem. Each chromosomes is ranked via a fitness function. The chromosomes having high fitness scores are sought and their genes are allowed to be passed to the next generation. Through crossover and mutation operation over these genes, a new generation is produced towards the evolution that better solutions will thrive while the least fit solutions die out. GA repeats the process and terminates until there is no noticeable difference between successive generations, i.e., GA has converged to the global (or near-global) optimum.

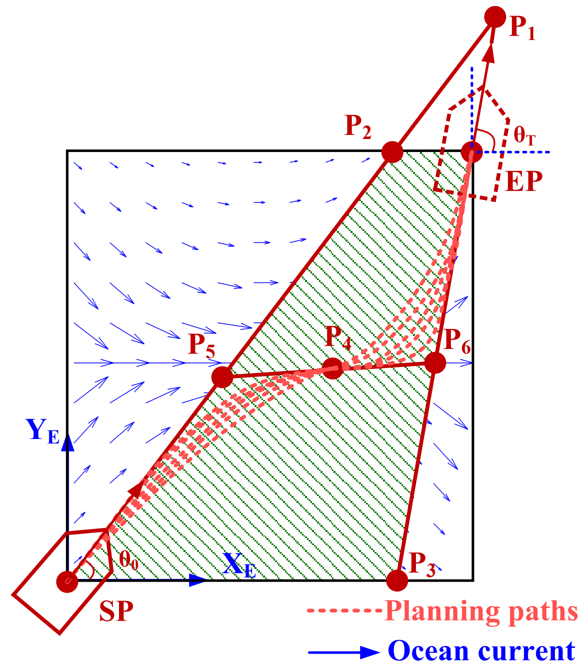

Fig. 4 The case for intersection P1 from SP to EP located out of the boundary.

Table 1 summarizes the variables that are treated as genes in GA. Note λ is normalized as the percentage of a line segment. For example, λ for P_2 in Fig. 3 is 0.19. It means line segment P_2-P_1 accounts for 19% length of the line segment SP-P_1, from which the coordinates of P_2 are easy obtained as long as the coordinates SP and P_1 are known. Furthermore, there would be at most four variables of λ, which are applicable to the case when the intersection is outside the boundary. Likewise, the normalization process is also performed for the coordinates of the joining point (P_x and P_y). The slope of the joining point k is not always fully in the range [-π, π], but rather depends on its intersections with the lines from SP or EP to the points on the boundary — just like P_5 and P_6 in Fig. 4.

Table 1 The variables to be optimized consisting of the chromosome in GA.

Variable	Description	Range
λ_i (i=1,…4)	Control point selection between the intersection and the end control points.	[0, 1]
P_x	The x coordinate of the joining point	[0, 1]
P_y	The y coordinate of the joining point	[0, 1]
k	The slope of the joining point	[-π, π]

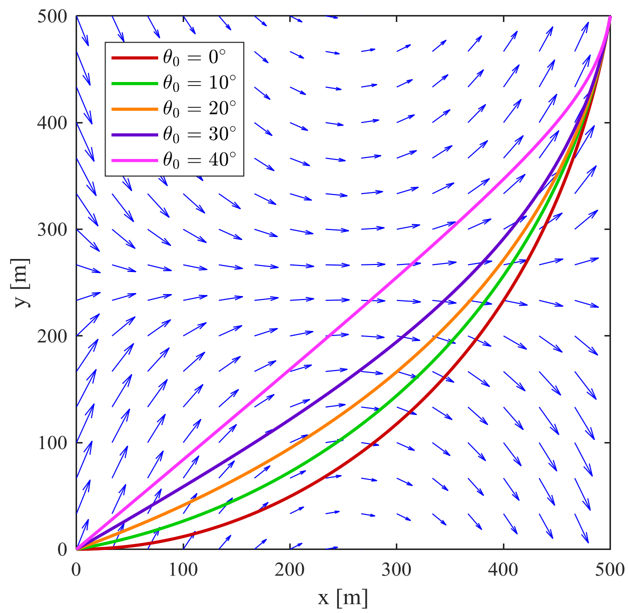

Fig. 5 The variation of ship trajectories with respect to different initial orientations.

The fitness function in GA is defined as the multiplication of the evaluation for both the turning and the fuel consumption, respectively:

$$J = \sigma \cdot f \tag{3}$$

where σ stands for the standard deviation of θ' in (1); f donates the summation of fuel consumption in the transition of adjacent points of the Bézier curve [11]:

$$f = \sum_i c \cdot \|v\|^3 \cdot \frac{\|B(n_{i+1}) - B(n_i)\|}{\|u\|} \tag{4}$$

where c is a constant only related to the ship considered; v and u are the ship speed in still water and over the ground, respectively; $B(n)$ is the point on the Bézier curve.

Regarding to the fitness function in (3), we did not use weight average to add both σ and f together since it is hard to normalize them. Instead, considering each of them is positive from definition, multiplying them is the alternative. Performing GA leads to search the variable space in Table 1 for the Bézier curve, which consequently optimizes the fitness function by minimizing both σ and f simultaneously.

SIMULATION RESULTS

We have carried out experiments to verify the proposed ship trajectory planner. In the experiments, the scenario was assumed within a 500 m×500 m water area. The location of SP and EP were set at (0 m, 0 m) and (500 m, 500 m). The current in the water area was designed to be:

$$\begin{cases} v_{cx}(x,y) = \dfrac{8}{(2y/y_{\max}-1)^2} \\ v_{cy}(x,y) = \dfrac{15(2x/x_{\max}-1)(2y/y_{\max}-1)}{(1+(2y/y_{\max}-1)^2)^2} \end{cases} \tag{5}$$

in which the maximum magnitude of the current is about 7.98 m/s and 4.86 m/s in x and y directions, respectively.

A real coded GA was implemented: the population size was set to 200; the GA operation including the roulette selection, the uniform crossover with a rate of 0.8 and the random number mutation with a rate of 0.1, were applied; the termination criteria was designed as the convergence of 0.99 in 50 successive generations.

The first experiment is to investigate the feasibility of the planner in dealing with the case where the intersection is inside the boundary. The ship speed over ground was fixed at $u=1$ m/s. The EP orientation θ_T was set to 80˚, whereas the SP orientation θ_0 varied from 0˚ to 40˚. For simplicity, Fig. 5 only shows the result of five different initial orientations. The generated path is smooth and satisfies the SP and EP constraints. Furthermore, All the trajectories are the optimized results through GA. It is interesting that among these curves, the lowest fitness score is found at θ_0=10˚. This makes sense because on the one hand, higher values of θ_0 are more or less consistent with the ocean current direction, resulting in a relative faster ship speed over ground. Then extra effort has to be made to decrease u to 1 m/s. The ocean current for lower values of θ_0, on the other hand, cannot provide enough ship speed over ground. Therefore, compensation is needed to rise u up to 1 m/s.

Another experiment is designed for the case when the intersection locates outside the boundary of the water area. The orientations for SP and EP were fixed at θ_0=60˚ and θ_T=80˚, respectively. Table 2 lists the result of the optimized variables regarding to different ship speeds over ground from $u=1$ m/s to $u=5$ m/s. The corresponding paths are drawn in Fig .6. Again, the generated smooth curves not only satisfy the two ends constraints, but also minimize the multiplication of the turning and the fuel consumption. Combining Table 2 and Fig. 6, there are several interesting findings:

- The variables λ_1 to λ_4 have closed values for different u in table 2, which indicates the curves in Fig. 6 have similar patterns.
- The joining points P_x and P_y (the dot in Fig. 6) for different u are approximately located at the center of the water area. This is reasonable because the ocean current distribution is symmetric there.
- The slopes of these joining points k are very similar, with a mean of 0.520 and a standard derivative of 0.009. It is considered that k is closely related to the orientations of SP and EP.

Table 2 The optimized variables via GA.

Variables	u [m/s]				
	1	2	3	4	5
λ_1 [%]	0.953	0.947	0.936	0.941	0.939
λ_2 [%]	0.255	0.263	0.272	0.284	0.344
λ_3 [%]	0.225	0.231	0.242	0.257	0.264
λ_4 [%]	0.961	0.955	0.950	0.948	0.943
P_x [m]	248.3	251.9	258.1	266.0	272.6
P_y [m]	216.2	220.1	226.5	235.2	243.5
k [rad]	0.527	0.525	0.524	0.519	0.504

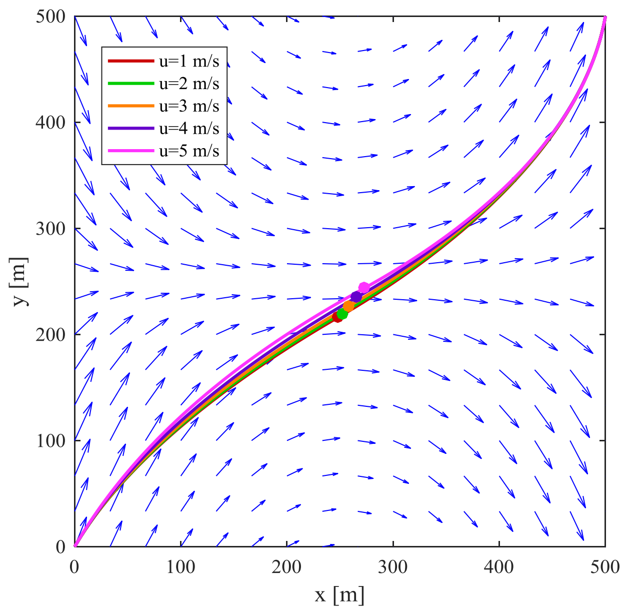

Fig. 6 The variation of ship trajectories with respect to different ship speeds over ground.

In summary, the resultant curves have little differences to each other regardless of the changes of u. This means u plays minor role in finding the optimal trajectory.

As a result, the two experiments show the proposed method is efficient to generate optimized Bézier curves for path planning of close-range maneuvering problem.

CONCLUSION

This paper focuses on addressing the path optimization problem for ship maneuvering in close-range maritime operations. First, we format this type of ship maneuvering as a two-point boundary value problem and add additional assumptions to make it solvable. Then, the Bézier curve is applied to generate the trajectory. By deploying the variables including the control points, the joining point and its slope, the Bézier curve satisfies the two ends constraints. Next, GA is used to optimize these variables, aiming to minimize the turning and fuel consumption of the ship along the generated trajectory. Last, two case studies verify the proposed method is efficient in searching for near-optimal trajectory for ships when maneuvering in maritime operations.

For future work, attention will be paid in two aspects. First, higher-order Bézier curves are taken into consideration to ensure the generated path more flexible. Second, some assumptions such as assumption 2 and 3 will be removed to make the problem be more close to reality.

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