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A New Keynesian Framework and Wage and Price Dynamics in the US

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Abstract

This paper investigates theoretical implications from a new Keynesian model focusing on the labor market, by imposing them as testable restrictions in an estimated vector error correction model on US data from 1982Q3 to 2016Q1. By this, I conduct an important, but rarely addressed, step in assessing the empirical relevance of a theoretical new Keynesian model. Another advantage of this approach is that the cycle and trend components of the data are separated when imposing the testable restrictions, such that there is no need to filter the data series prior to estimation. The results show that most of the properties pertaining to the theoretical model cannot be rejected when imposed as restrictions. The new Keynesian model on the labor market is thus found to be empirically relevant. Furthermore, the estimated econometric model explains a large degree of the wage and price dynamics in the US, such that the paper also provides an estimated macroeconomic model. Cointegration new Keynesian models steady state wages prices unemployment error correction C32 E24 E31

1 Introduction

Estimating new Keynesian (NK) models (also referred to as dynamic stochastic general equilibrium (DSGE) models) by probabilistic methods such as maximum likelihood is often problematic due to local maxima and minima and nearly flat surfaces (Fernández-Villaverde, 2010). NK models are therefore to a large degree estimated by Bayesian estimation in order to overcome these problems.

A disadvantage with both of these approaches is that the non-stationary macroeconomic data series usually are filtered by a statistical univariate method, such as the Hodrick-Prescott filter, prior to estimation. This is common when estimating NK models, and is done in order to isolate the cycle components of the data from the trend (see DeJong and Dave (2007)). A NK model is linearized around its theoretical steady state, such that a potential mismatch between the theoretical steady state in the NK model and the trends in the data series is not taken into account when pre-filtering.

If we estimate a vector error correction model (VECM), the theoretical steady state from the NK model may be tested by imposing it as restrictions on the long-run properties of the data. This enables us to assess the empirical relevance of the NK model, and avoids the need for estimating under the null hypothesis of the correct model. Hence, we are able to assess the empirical relevance of the theoretical model. If certain parts of the model is rejected by the testing procedure, we may modify the model such that it becomes in line with the empirical observation. By performing this analysis, we may therefore combine deductive and inductive inference (see Juselius (2006)), improving the modeling procedure.

Even though testing NK models in a VECM has been done in other studies¹, the NK model that I test here focus on the labor market rather than the usual focus on monetary policy. I can thereby assess the empirical relevance of this theoretical NK model for the labor market through the VECM framework. VECMs focusing on the labor market and wage and price dynamics are also estimated and analyzed in Bårdsen et al (2007) for Australia, Bårdsen and Fisher (1999) for the UK, and Bårdsen et al (1998) for Norway and the UK. However, the theoretical model tested in these papers is mainly a model of wage bargaining and not a NK model as here.

¹See e.g. Juselius and Franchi (2007) and Kivedal (2014) who test restrictions from the model in Ireland (2004) and Iacoviello (2005), respectively.

Additionally, a NK model may be represented as a vector autoregressive (VAR) model of the log-linear relationships such that there is a direct link between the VECM model (which is a reparameterization of a VAR model with non-filtered data) and the NK model. Hence, restrictions on the short-run structure may also be tested in this framework. This testing procedure is outlined in Bårdsen and Fanelli (2015).

In this paper, I adopt a NK model from the models in Blanchard and Galí (2007) and Blanchard and Galí (2010), in order to test its properties in the VECM. The purpose of this paper is thereby to assess the empirical relevance of a NK model from a labor market perspective. Additionally, the paper provides an estimated dynamic model that explains wage and price dynamics in the US for the sample period.

The next section presents the theoretical NK model, while the VECM is estimated in section 3 together with testing the restrictions from the NK model. Section 4 concludes.

2 The Theoretical New Keynesian Model

The model that I use here is adopted from the framework in Blanchard and Galí (2007) and Blanchard and Galí (2010). This involves a NK model which includes extensions such as labor market frictions, real wage rigidities and staggered price setting. Even though these extensions are added to the basic NK model, the model is relatively simple and transparent. It consists of households which maximizes their utility, and perfectly competitive firms who each produce an intermediate good that is transformed into differentiated final goods by monopolistically competitive firms. Labor market frictions are included using hiring costs which depend on labor market tightness, such that there is a relationship between unemployment and wage and price dynamics.

2.1 Households

A representative households maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - \chi \frac{N_t^{1+\phi}}{1+\phi} \right\}, \quad (1)$$

where C_t is composite consumption (a composition of several goods with elasticity of substitution between them of ε) and N_t is employment or hours

of work. β is the discount factor for the households, ϕ measures the inverse of the Frisch elasticity of labor supply, and χ is a preference parameter. E_0 denotes the expectations operator given the information set at time period 0. This is maximized s.t. the budget constraint

$$P_t C_t + Q_t T_t \leq T_{t-1} + W_t N_t, \quad (2)$$

where W_t is the nominal wage and P_t is the price of the bundle of consumption goods. Furthermore, T_t denotes the quantity of a one-period, nominally riskless discount bond purchased in period t at price Q_t and maturing in period $t + 1$, paying one unit of money. Maximizing (1) w.r.t (2) for labor/leisure, consumption and risk-free bonds gives

$$\frac{W_t}{P_t} = \chi N_t^\phi C_t, \quad (3)$$

which shows the labor supply as a function of the real wage and consumption, and

$$Q_t = \beta E_t \left\{ \frac{C_t P_t}{C_{t+1} P_{t+1}} \right\}, \quad (4)$$

which may be interpreted as the stochastic discount factor² for one period ahead.

2.2 Intermediate goods firms

The j intermediate goods firms are assumed perfectly competitive and each of them produce $X_t(j)$ using the production function

$$X_t(j) = A_t N_t^{1-\eta}(j) M_t^\eta(j), \quad (5)$$

where A_t measures productivity, assumed to be equal for all firms, $N_t(j)$ is employment in firm j , and $M_t(j)$ is the amount of a non-produced input used in firm j following Blanchard and Galí (2007). η is a parameter measuring the relative size of the two input factors in the production function. Capital is ignored as a productive factor, following the simplification in Blanchard and Galí (2007) and much of the literature on NK models. Hence, capital can be viewed as a fixed factor in the production function, normalized to unity.

²See e.g. Cochrane (2001) for an explanation of the stochastic discount factor approach, or Galí (2008) for applications of the stochastic discount factor in a NK framework.

Real profits for the intermediate goods firm are defined as

$$\Pi_t(j) = X_t(j) \frac{P_t(j)}{P_t} - G_t H_t(j) - \frac{W_t}{P_t} N_t(j) - \frac{V_t}{P_t} M_t(j), \quad (6)$$

where $P_t(j)$ is the price of the intermediate good sold to the final goods producers, G_t denotes real hiring costs, and is defined as

$$G_t = A_t B x_t^\alpha, \quad (7)$$

where we have the constants $B > 0$ and $\alpha \geq 0$, and x_t is a labor market tightness index defined below. $H_t(j)$ is the hiring in firm j at time t , W_t is the nominal wage and V_t is the nominal price of the non-produced input, which is assumed equal across firms. Accordingly, profits in firm j are given as income from sales of the intermediate good X_t minus total hiring costs, wage costs and costs for the non-produced input.

Hiring in firm j evolves according to

$$H_t(j) = N_t(j) - (1 - \delta) N_{t-1}(j), \quad (8)$$

where δ is a separation rate which measures the fraction of the employed in period $t - 1$ who leaves their job in firm j prior to period t . Hence, hiring consists of the change in employment and the job separation. Furthermore, the labor market tightness index is defined as the ratio between aggregate hires and unemployment U_t ,

$$x_t \equiv H_t/U_t = \frac{N_t - (1 - \delta) N_{t-1}}{1 - (1 - \delta) N_{t-1}},$$

such that low unemployment or a high degree of hiring relatively to each other increase the labor market tightness, which leads to a higher hiring cost G_t in (7). The hiring costs are taken as given by the intermediate goods firms (following Blanchard and Galí (2010)).

As indicated by the dynamics of hiring, employment in firm j evolves according to

$$N_t(j) = (1 - \delta) N_{t-1}(j) + H_t(j),$$

i.e. employment is made up of workers that stays in the firm and hiring.

For perfect competition when the intermediate goods firms maximize their profits by determining the optimal level of employment, the following expression must hold:

$$\frac{P_t(j)}{P_t} = (1 - \eta)^{\eta-1} \left(\frac{1}{\eta}\right) \frac{1}{A_t} \left(\frac{V_t}{P_t}\right)^\eta \left(G_t + \frac{W_t}{P_t} - Q_t(1 - \delta) E_t G_{t+1}\right). \quad (9)$$

$Q_t = \beta E_t\{(C_t P_t)/(C_{t+1} P_{t+1})\}$ is the stochastic discount factor for one period ahead as shown in (4). This is also used for the intermediate goods firms since they are owned by the households.

2.3 Final goods firms

Final goods firms are monopolistically competitive, such that they set the optimal price of the final good by adding a markup over the price of the intermediate good (which is equivalent of adding a markup over their marginal cost). We define this markup as in Blanchard and Galí (2010);

$$\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1},$$

such that

$$P_t = \mathcal{M} P_t(j).$$

Using this markup and inserting for the stochastic discount factor $Q_t = \beta E_t\{(C_t P_t)/(C_{t+1} P_{t+1})\}$, we may rewrite the optimality condition in (9) to

$$\frac{1}{\mathcal{M}} = (1-\eta)^{\eta-1} \left(\frac{1}{\eta}\right)^\eta \frac{1}{A_t} \left(\frac{V_t}{P_t}\right)^\eta \left(G_t + \frac{W_t}{P_t} - \beta(1-\delta)E_t\left\{\frac{C_t P_t}{C_{t+1} P_{t+1}}\right\} G_{t+1}\right)^{1-\eta}. \quad (10)$$

Inserting for the hiring costs defined in (7), we get the steady state relationship

$$\frac{1}{\mathcal{M}} = \frac{1}{1-\eta} \left(\frac{1}{\eta}\right)^{\frac{\eta}{1-\eta}} A^{\frac{1}{\eta-1}} \left(\frac{V}{P}\right)^{\frac{\eta}{1-\eta}} \left(ABx^\alpha(1-\beta(1-\delta)) + \frac{W}{P}\right),$$

which may be approximated to (assuming $Bx^\alpha(1-\beta(1-\delta))$ is small)

$$p = w - \frac{1}{1-\eta}a + \frac{\eta}{1-\eta}v - \log(1-\eta) - \frac{\eta}{1-\eta}\log\eta - \frac{1}{\eta-1}\log\mathcal{M}, \quad (11)$$

where lower case letters denote logarithms of the respective variable and v is the log of the *real* price of the non-produced input. This shows that we have a linear relationship between the price, wage, productivity and the real price of the non-produced input in steady state. Even though the labor market tightness index is included in the price equation (before approximating it), it is taken as given by the firm when they maximize their profits since hiring

costs are taken as given as in Blanchard and Galí (2010). Labor market tightness is therefore considered a given parameter in the price setting.

Furthermore, Calvo pricing (Calvo, 1983) introduces nominal rigidities, which yields the New Keynesian Phillips curve (NKPC),

$$\pi_t = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \widehat{mc}_t + \beta E_t \pi_{t+1}, \quad (12)$$

where θ measures the fraction of the final goods producers who are not able to reset their prices in each period, π_t is inflation and \widehat{mc}_t is the log deviation from the steady state value of the real marginal cost (for the final goods producers). Hence, we have a NKPC which describes inflation as a function of the real marginal cost and future expected inflation. The real marginal cost for the final goods producers is given by the right hand side of (10) and thus includes the real price of the non-produced input, such that this may be considered as a variable in the NKPC. This is in line with e.g. Hooker (2002) who includes the growth rate for crude oil relative to the inflation rate, and Roberts (1995) who includes the log difference of the real oil price in the NKPC, since the non-produced input in (5) may be oil.

The effect of oil price movements on core inflation in the US has been shown to be much larger before 1981 than after (see Hooker (2002)). This indicates that the parameter for the effect of price of unprocessed goods' movements on inflation may be non-constant. However, the sample used in the estimation conducted in section 3 starts in 1982Q3 such that this problem is less relevant. Additionally, the change in the (log) usage of unprocessed goods (Δm_t) is assumed to be a white noise process in Blanchard and Galí (2007). Hence, M_t is treated as a constant parameter M in the analysis here in order to simplify calculations.

2.4 Real wage rigidities

I model the real wage ($w_t - p_t$ or W_t/P_t) through a partial adjustment model for the log of the real wage following Blanchard and Galí (2007);

$$(w_t - p_t) = \gamma(w_{t-1} - p_{t-1}) + (1 - \gamma)mrs_t. \quad (13)$$

$0 \leq \gamma \leq 1$ measures the degree of real wage rigidity, such that the real wage is assumed to respond slowly to changes in the labor market. Hence, changes in the marginal rate of substitution, mrs_t , is not reflected fully in

real wages as long as $\gamma > 0$. The inclusion of wage rigidities could also have been done in line with Galí (2011), who models nominal wage rigidities through a Calvo wage setting framework as in Erceg et al (2000), but in order to simplify calculations, I use the partial adjustment model above. The equilibrium wage is therefore defined to be equal to the households' marginal rate of substitution between labor and consumption. This marginal rate of substitution is shown in (3), such that $MRS_t = \chi C_t N_t^\phi$. Inserting for this gives

$$(w_t - p_t) = \gamma(w_{t-1} - p_{t-1}) + (1 - \gamma)\chi(c_t + \phi n_t).$$

In steady state, the rigidities are absent since they are assumed to represent distortions. Hence, this amounts to the same steady state relationship as the steady state of (3):

$$\frac{W}{P} = \chi C N^\phi.$$

Inserting for the aggregate resource constraint (as defined in Blanchard and Galí (2010) and consistent with the first part of the model outlined above),

$$C_t = A_t(N_t^{1-\eta}M_t^\eta - Bx_t^\alpha H_t), \quad (14)$$

we get the steady state relationship

$$\frac{W}{P} = \chi A(N^{1-\eta}M^\eta - Bx^\alpha H)N^\phi.$$

The steady state expression for employment is $N = 1 - u$ (the sum of aggregate employment and aggregate unemployment rate is defined to equal unity), for hiring $H = \delta N = \delta(1 - u)$ and for labor market tightness $x = \delta(1 - u)/(u + \delta(1 - u))$. Inserting these, we have the steady state relationship

$$\frac{W/P}{A} = \chi \left((1 - u)^{1-\eta} M^\eta - B \left(\frac{\delta(1 - u)}{1 - (1 - \delta)(1 - u)} \right)^\alpha \delta(1 - u) \right) (1 - u)^\phi.$$

Taking logs and assuming that u and $B \left(\frac{\delta(1-u)}{1-(1-\delta)(1-u)} \right)^\alpha \delta(1-u)^\eta M^{-\eta}$ are small yields

$$w = p + a - \Phi u + \eta \chi \log M - \frac{B\chi}{M^\eta} \delta, \quad (15)$$

where $\Phi \equiv \chi \left(1 - \eta + \phi + \frac{B}{M^\eta} (\eta + \alpha (\delta - (\delta - 1))) \right)$. This implies that there is a linear relationship between the log of the wage, log of the price log of

productivity and the unemployment rate in steady state. We denote this relationship the long-run wage equation.

Furthermore, the log-linearized model may be written as

$$\mathbf{X}_t = \hat{A}\mathbf{X}_{t-1} + \hat{B}E_t\mathbf{X}_{t+1} + \hat{C}\mathbf{X}_t + \hat{D}e_t, \quad (16)$$

where \mathbf{X}_t is a vector containing log-linearized real wage, inflation, productivity, unemployment, and the real price of the non-produced input. e_t is a vector of error terms, and \hat{A} , \hat{B} , \hat{C} and \hat{D} are matrices of parameters. This may be solved (at least numerically) in order to yield a backward-looking solution such as

$$\mathbf{X}_t = \hat{E}\mathbf{X}_{t-1} + \hat{F}e_t, \quad (17)$$

where \hat{E} and \hat{F} are matrices of parameters.

3 Estimation

3.1 Data

The solution of the theoretical NK model is given by (17) in the previous section, where the vector \mathbf{X}_t contains five variables (in log-linearized form). I also find empirical observations for these five variables, and use them in order to estimate the vector error correction model (VECM) below. These are variables for the unemployment rate, price, wage, productivity, and the real price of the non-produced input. Hence, all of the variables in (11) and (15) (as well as the variables in (16) and (17)) are observed.

The data set includes the civilian unemployment rate for persons 16 years of age and older from the Bureau of Labor Statistics (BLS), the non-farm business sector's implicit price deflator, compensation per hour for the non-farm business sector, output per hour of all persons for the non-farm business sector from BLS' Productivity and Costs release, and the Producer Price Index (PPI) by Commodity for Intermediate Demand by Commodity Type: Unprocessed Goods for Intermediate Demand (from BLS' Producer Price Index release). The real price of unprocessed goods is calculated as the nominal unprocessed goods price index deflated by the implicit price deflator used in the data set. The sample used is from 1982Q3 to 2016Q1, and is chosen because of the inflation regime break found for 1982Q3 by Caporale and Grier (2005), which cites other works that also finds a break close to this date.

3.2 Estimation and testing

In order to avoid imposing the entire structure of the theoretical NK model prior to estimation and estimate under the null of the model being the actual data generating process, as is usually done when estimating NK models, we may use the more general VECM framework for estimating the economy. Features of the theoretical NK model may then be imposed as testable restrictions on a general econometric model. Two relationships that we may impose on the long-run structure of the VAR model are the long-run relationship for the price in (11) and for the wage in (15). This entails a general-to-specific approach, starting as a purely statistical VAR model and ending with a dynamic simultaneous equations model identified by non-rejected restrictions from the theoretical model.

The five observable variables outlined in section 3.1 are used in the estimation below. This implies that we have observable variables for the nominal wage, w_t , the price level, p_t , the real price of the non-produced input, v_t , productivity, a_t and the unemployment rate, u_t . Using the log of the four former series and the level of the unemployment rate is in line with the theoretical variables in the steady state relations shown in (11) and (15). These series are plotted in figure 1 for the sample used in the estimation (1982Q3-2016Q1).

The system outlined in (16) can be solved such that we get a restricted VAR(1) model as shown in (17). Additionally, an estimated unrestricted VAR may be restricted in order to obtain the VAR(1) representation in (17), i.e. by imposing the cross-equation restrictions that follows from the matrices \hat{E} and \hat{F} in (17). However, we then need to obtain the analytical solution of the model and thereby all of non-linear combinations of parameters in all of the cells in the parameter matrices. Since only a numerical solution was possible to obtain here, imposing and testing these cross-equation restrictions is not possible on this NK model. Hence, I only execute the first two out of three steps in the test procedure in Bårdsen and Fanelli (2015) here. The first step in their procedure is the cointegration rank test and the second is the cointegration matrix test (i.e. restrictions on the cointegrating vectors). The third step, which is the cross-equation restrictions test, is not carried out here.

The system expressed as a VAR model with k lags may be written as

$$Z_t = \Pi_1 Z_{t-1} + \dots + \Pi_k Z_{t-k} + \Phi D_t + \varepsilon_t. \quad (18)$$

Alternatively, (18) may be reformulated to a vector equilibrium correction

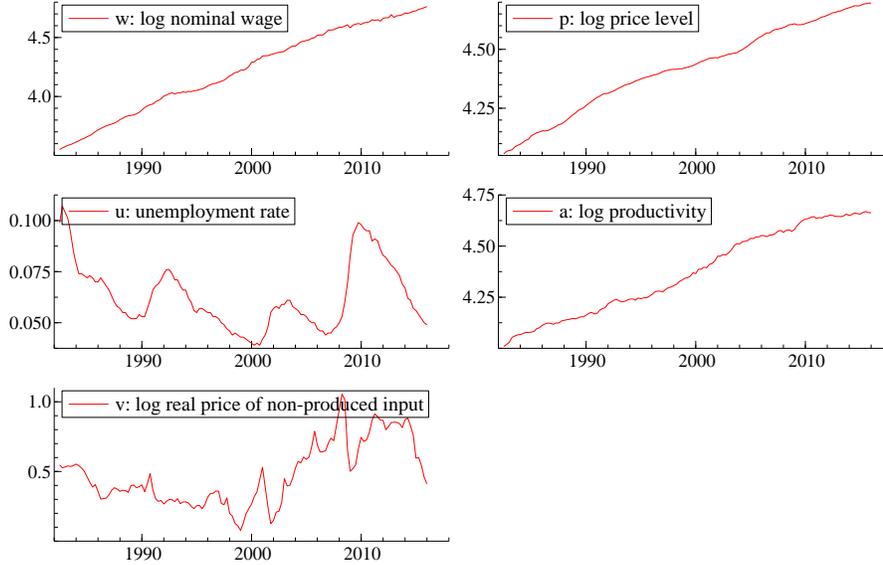


Figure 1: Logarithms of the data series (except the unemployment rate in levels).

model, such as

$$\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + \dots + \Gamma_{k-1} \Delta Z_{t-k+1} + \alpha \tilde{\beta}' \tilde{Z}_{t-1} + \Phi D_t + \gamma_0 + \gamma_1 t + \varepsilon_t, \quad (19)$$

where $Z_t = [w_t, p_t, u_t, a_t, v_t]'$, $\tilde{\beta}' = [\beta, \beta_0, \beta_1]$, $\tilde{Z}_{t-1} = [Z_{t-1}, 1, t]'$, $\varepsilon_t \sim IN(0, \Omega)$ for $t = 1, \dots, T$ and Z_{-1}, Z_0 is given. D_t is a vector of dummy variables, and γ_0 and γ_1 are constants. The trend is restricted to be in the cointegrating space in order to prevent quadratic trends (i.e. $\beta_1 \neq 0$ and $\gamma_1 = 0$).

First, I estimate an unrestricted VAR model, where the number of lags are chosen to be as few as possible while still having a well specified model. Additionally, dummy variables pertaining to periods with extraordinary events not explained by the model may need to be added to account for residual outliers from the normal distribution such that we get a well specified model. The relevant dummies are found by Autometrics (Doornik, 2006). These quarters are 1985Q1 (dummy significant in the price equation), 2000Q1 (dummy significant in the wage equation) and 2008Q4 (dummy significant in the productivity and unprocessed goods equations). The dummy for 1985Q1

accounts for the rather large increase in price growth this quarter. When interest rates declined sharply in December 1984, the effect of this may have occurred at least one month later (Litterman and Todd, 1985). This large drop in the interest rate may be the explanation for the large price increase in 1985Q1 which the model is not able to capture. The dummy for 2000Q1 is significant in the wage equation, and controls for the large increase in wages in this period which may be due to the inclusion of employee stock options in the wage compensation variable in the late 1990s (Mehran and Tracy, 2001). Finally, the dummy for 2008Q4 is added to take account for the financial crisis starting in this period. The estimated VAR with dummies is well specified w.r.t. autocorrelation and normality at a significance level of 5.7% as shown in table 1. Although the model contains autoregressional conditional heteroskedasticity (ARCH), the VAR results should be robust to moderate ARCH effects (Rahbek et al, 2002). I set the lag length to $k = 2$, as indicated by various information criteria and an F test of reducing the lag length from $k = 3$ to $k = 2$, as shown in table 2.

Table 1: Residual analysis for the VAR(2) with dummy variables

Multivariate tests (p-values in brackets)	
Residual autocorrelation	$F(125, 452) = 1.0899$ [0.2631]
Test for normality	$\chi^2(10) = 17.863$ [0.0573]
Test for heteroskedasticity	$F(110, 519) = 1.8316$ [0.0000]

Table 2: Determination of lag length

Model	T	p		log-likelihood	SC	HQ	AIC
VAR(3)	135	80	OLS	2490.75	-33.99	-35.02	-35.72
VAR(2)	135	55	OLS	2475.54	-34.67	-35.38	-35.86
Tests of lag reduction:							
VAR(3) \rightarrow VAR(2): $F(25,428) = 1.0724$ [0.3712]							

As suggested by the results from the trace test and the eigenvalues in table 3, I set the rank to $r = 2$, i.e. two cointegrating relations. This is in

Table 3: Eigenvalues of the model and the trace test of the cointegration rank.

eigenvalue	log-likelihood	rank	$H_0 : r \leq$	Trace test	[p-value]
	2462.241	0	0	116.41	[0.000]**
0.27109	2483.586	1	1	73.725	[0.005]**
0.21943	2500.307	2	2	40.281	[0.089]
0.12681	2509.460	3	3	21.975	[0.143]
0.10205	2516.726	4	4	7.4439	[0.310]
0.053647	2520.448	5			

line with the NK model, where the steady state of the model comprises into the two relations shown in (11) and (15). Even though cointegration is not modeled explicitly in the NK model, near unit roots should be approximated by unit roots in order to approximate the finite sample distribution by the standard asymptotic distribution (Johansen and Colander, 2006). Hence, the two equations for the steady state in the NK model, (11) and (15), may be considered as cointegrating relationships since the common stochastic trends should cancel through these relationships.

Testing for weak exogeneity given a rank of $r = 2$ suggests that productivity and the real price of unprocessed goods is jointly weakly exogenous.³ The price of unprocessed goods and productivity should be assumed weakly exogenous according to the theoretical model in section 2 since they do not depend on any of the other variables in the NK model.

I impose weak exogeneity of these variables, and use a partial VAR model as introduced by Johansen (1992). The partial VAR(2) model in equilibrium correction form then becomes

$$\Delta ZZ_t = \alpha \tilde{\beta}' \tilde{Z}_{t-1} + \Gamma_1 \Delta Z_{t-1} + \Phi D_t + \gamma_0 + \gamma_1 t + \varepsilon_t, \quad (20)$$

where $ZZ_t = [w_t, p_t, u_t]'$ and \tilde{Z}_{t-1} is as defined above. The trend is restricted to be in the cointegrating space.

When estimating the partial VAR model, Autometrics is run again in order to find dummy variables needed to have a well specified model. The

³Restricting the α matrix to have weak exogeneity for productivity and the real price of unprocessed goods suggests that the hypothesis cannot be rejected at a 22.7% level of significance (test value of $\chi^2(4) = 5.648$).

dummy variables needed now are 1983Q1 and 2000Q1, such that the previously used dummy variables for 1985Q1 and 2008Q4 are not needed in order to obtain a well specified partial VAR model. However, a dummy variable for 1983Q1, significant in the unemployment equation, is now needed. This dummy may account for the change in the estimation of the data series for the unemployment rate by BLS (see Bregger (1982)), as well as the economic recovery tax act of 1981 that to a large degree came into action 1983 (Eissa, 1996). Additionally, the Social Security Amendments of 1983 (recommended in January 1983 and signed in April 1983, see Svahn and Mary (1983)) may have affected the unemployment dynamics in this period.

Imposing restrictions according to the steady state relations, i.e. the price equation in (11) and the wage equation in (15), yields the results in table 4. The trend is not excludable from the first β vector such that this vector is trend-stationary. The two (stationary) cointegration relations in the restricted partial VAR are shown in figure 2, where the first vector represents the long-run wage equation and the second the price equation.

Table 4: Estimated long run structure of restricted partial VAR for the α and β vectors. Standard errors in parentheses below estimated coefficients. Test of over-identifying restrictions: $\chi^2(4) = 7.42$ (p-value 0.116).

	β_1	β_2		α_1	α_2
w	1.00	-1.00	w_t	-0.187 (0.037)	-0.005 (0.037)
p	-1.00	1.00	p_t	-0.015 (0.012)	-0.065 (0.012)
u	1.104 (0.199)	0.00	u_t	0.003 (0.011)	-0.038 (0.011)
a	-1.00	0.984 (0.019)			
v	0.00	-0.112 (0.017)			
trend	0.001 (0.00009)	0.00			

According to the long-run price equation in the NK model shown in (11), the coefficients on productivity and the price of unprocessed goods should sum to one. Additionally, the NK model suggests $0 \leq \eta < 1$, which is not possible since the results in table 4 indicates $1/(1 - \eta) = 0.984$ and $\eta/(1 - \eta) = 0.11$ (i.e. yielding $\eta = -0.016$ and $\eta = 0.099$ respectively). Hence, this suggests that the restrictions on the production function (i.e.

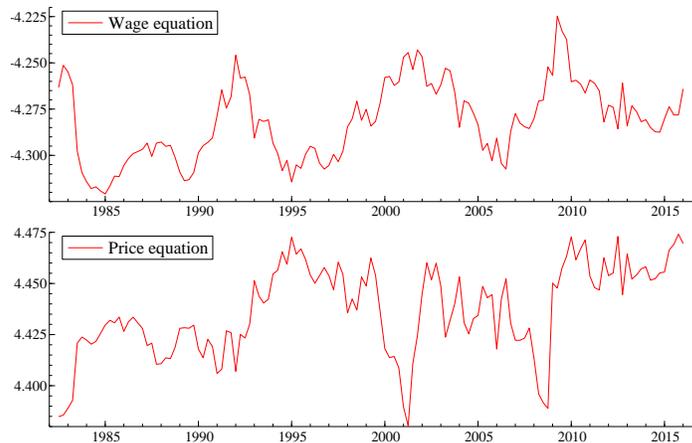


Figure 2: Cointegration relations, with restrictions on both vectors.

that the coefficients on employment and the non-produced input should sum to one) may be too strong. However, the estimated coefficients in the beta matrix suggests that the price of unprocessed goods has a relatively small but significant positive effect on the general price level in the long run (a long-run elasticity of 0.112), and that productivity has a near one-to-one negative effect on prices.

Furthermore, the estimated long-run parameter for unemployment indicates that an increase in unemployment by one unit (e.g. from 5% to 6% – one percentage point) decreases wages by 1.104%.⁴ Hence, for an unemployment rate of 5%, this implies an elasticity of 0.055. The long-run elasticity of the unemployment rate estimated in Bårdsen et al (2007) for Australia, Bårdsen and Fisher (1999) for the UK or Bårdsen et al (1998) for Norway is 0.1, 0.065 and 0.08, respectively.⁵ Hence, the elasticity found in the US data

⁴The percentage change in the dependent variable following a one unit (i.e. one percentage point for the unemployment rate which is measured in percent of the labor force in the data set) increase in the independent variable is measured approximately as $100 \cdot \beta\%$ if the dependent variable is in logs and the independent variable is in levels. If both the dependent and the independent variable are in logs, the coefficient measures the elasticity; i.e. the percentage change in the dependent variable if the independent variable increase by one percent. See e.g. Stock and Watson (2015).

⁵The log of unemployment is used as the independent variable in Bårdsen et al (2007), Bårdsen and Fisher (1999), and Bårdsen et al (1998), while the unemployment rate is used here.

is smaller (given an initial unemployment rate of 5%), such that the effect of unemployment on wages is smaller in the US than in Australia, the UK and Norway. However, the sample period in this paper is different from the other studies such that this may indicate that the effect of unemployment on wages is smaller in the less distant past.

Additionally, the results in the first beta vector suggests that in the long run, productivity is fully reflected in wages (unity coefficient on productivity), and the wage share $((W/P)/A)$ is a function of unemployment.

The small estimated values in the alpha matrix may suggest rigidities, such that the results are in line with the rigidities implied by the theoretical model, such as real wage rigidities introduced in (13), price rigidities modeled by Calvo pricing and labor market frictions modeled by hiring costs and job separation. Additionally, wages adjust faster to disequilibrium than prices and unemployment, *ceteris paribus*, according to the estimated alpha coefficients. This indicates that wages are not as rigid as prices and unemployment, all other variables held constant. A rapid adjustment of wages to disequilibrium may be anticipated in the US, since the labor market in the US is considered to be quite fluid.

The estimated long-run relationships are then imposed on the model as equilibrium expressions in a vector error correction model (VECM). They may be expressed as

$$\begin{aligned} ciw_{t-1} &= w_{t-1} - [p_{t-1} + a_{t-1} - 1.104u_{t-1} - 0.0102t] + constant \\ cip_{t-1} &= p_{t-1} - [w_{t-1} - 0.984a_{t-1} + 0.112v_{t-1}] + constant, \end{aligned} \quad (21)$$

where t is the trend.

These cointegrating relationships are then used as long-run values for the system $\{\Delta w_t, \Delta p_t, \Delta u_t\}$ which is estimated conditional on $\{ciw_{t-1}, cip_{t-1}, \Delta w_{t-1}, \Delta p_{t-1}, \Delta u_{t-1}, \Delta a_{t-1}, \Delta v_{t-1}\}$ as well as the dummy variables. This yields the following system, where standard errors of the coefficients are in parenthesis below the estimated parameter values:

$$\begin{aligned}
\Delta w &= - \underset{(0.0795)}{0.245} \Delta w_{t-1} + \underset{(0.253)}{0.0178} \Delta p_{t-1} + \underset{(0.241)}{0.306} \Delta u_{t-1} + \underset{(0.1)}{0.304} \Delta a_t \\
&+ \underset{(0.0984)}{0.342} \Delta a_{t-1} - \underset{(0.0105)}{0.0045} \Delta v_t + \underset{(0.0108)}{0.0273} \Delta v_{t-1} \\
&+ \underset{(0.00714)}{0.0283} D_{00Q1} + \underset{(0.00729)}{0.00258} D_{83Q1} + \underset{(0.00707)}{0.000963} D_{85Q1} \\
&- \underset{(0.213)}{0.771} - \underset{(0.0375)}{0.186} \text{ciw}_{t-1} - \underset{(0.0371)}{0.00443} \text{cip}_{t-1} \\
\Delta p &= \underset{(0.0252)}{0.021} \Delta w_{t-1} + \underset{(0.0803)}{0.254} \Delta p_{t-1} - \underset{(0.0764)}{0.178} \Delta u_{t-1} - \underset{(0.0317)}{0.0721} \Delta a_t \\
&- \underset{(0.0312)}{0.00569} \Delta a_{t-1} + \underset{(0.00334)}{0.00967} \Delta v_t - \underset{(0.00343)}{0.000249} \Delta v_{t-1} \\
&+ \underset{(0.00227)}{0.000657} D_{00Q1} - \underset{(0.00231)}{0.00334} D_{83Q1} + \underset{(0.00225)}{0.00876} D_{85Q1} \\
&+ \underset{(0.0676)}{0.24} - \underset{(0.0119)}{0.00727} \text{ciw}_{t-1} - \underset{(0.0118)}{0.0603} \text{cip}_{t-1} \\
\Delta u &= - \underset{(0.0231)}{0.0105} \Delta w_{t-1} - \underset{(0.0737)}{0.0926} \Delta p_{t-1} + \underset{(0.0701)}{0.675} \Delta u_{t-1} - \underset{(0.0291)}{0.052} \Delta a_t \\
&- \underset{(0.0286)}{0.0242} \Delta a_{t-1} - \underset{(0.00306)}{0.00284} \Delta v_t - \underset{(0.00315)}{0.00641} \Delta v_{t-1} \\
&+ \underset{(0.00208)}{0.000198} D_{00Q1} - \underset{(0.00212)}{0.00912} D_{83Q1} - \underset{(0.00206)}{0.00124} D_{85Q1} \\
&+ \underset{(0.062)}{0.178} + \underset{(0.0109)}{0.00173} \text{ciw}_{t-1} - \underset{(0.0108)}{0.0383} \text{cip}_{t-1}.
\end{aligned} \tag{22}$$

After identifying the system, removing insignificant variables, respecifying the dating of the variables in the error correction terms, and impose some additional restrictions, we end up with the following more parsimonious dynamic model:⁶

⁶The modeling process is shown in appendix A.

$$\begin{aligned}
\Delta w &= - 0.409 \Delta w_{t-1} + 0.274 \Delta a_t + 0.548 \Delta a_{t-1} + 0.0226 \Delta v_{t-1} \\
&\quad (0.0768) \quad (0.0431) \quad (0.0863) \quad (0.00931) \\
&\quad + 0.0273 D_{00Q1} - 0.706 - 0.167 \text{ecw}_t \\
&\quad (0.00689) \quad (0.132) \quad (0.0309) \\
\Delta p &= 0.219 \Delta p_{t-1} - 0.283 \Delta u_{t-1} + 0.00879 D_{85Q1} + 0.243 - 0.054 \text{ecp}_t \quad (23) \\
&\quad (0.0791) \quad (0.0657) \quad (0.00229) \quad (0.0469) \quad (0.0105) \\
\Delta u &= - 0.131 \Delta p_{t-1} + 0.689 \Delta u_{t-1} - 0.0573 \Delta a_t - 0.0287 \Delta a_{t-1} \\
&\quad (0.0704) \quad (0.0623) \quad (0.0244) \quad (0.0122) \\
&\quad - 0.00716 \Delta v_{t-1} - 0.00943 D_{83Q1} + 0.171 - 0.0382 \text{ecp}_t \\
&\quad (0.00281) \quad (0.00206) \quad (0.0415) \quad (0.00932)
\end{aligned}$$

This model cannot be rejected when tested against the dynamic unrestricted model in (22) at a 33.2% significance level ($\chi^2(21) = 23.24$). The model is also well specified as shown by the residual analysis in table 5 at a significance level of 3.3%, and explains the data quite well as indicated by the fit of the model in figure 3. However, some parts of the dynamics in the data is not captured by the model, given that the p-value for the residual autocorrelation test is quite low. This is also indicated by figure 3 where there are some gaps between the actual and fitted values around 1990 for the wage equation and the price equation, as well as for the price equation around 2005. Since autocorrelation in the model indicates that the conditional expectations of the VAR model deviates from the observed realizations (Juselius, 2006, p. 74), the model may be too simple in explaining the economic reality. Including other relevant variables could be considered in order to increase the fit of the model.

Table 5: Residual analysis for final restricted dynamic model.

Multivariate tests (p-values in brackets)	
Residual autocorrelation	$F(45, 333) = 1.452$ [0.036]
Test for normality	$\chi^2(6) = 4.293$ [0.637]
Test for heteroskedasticity	$F(132, 606) = 1.294$ [0.033]

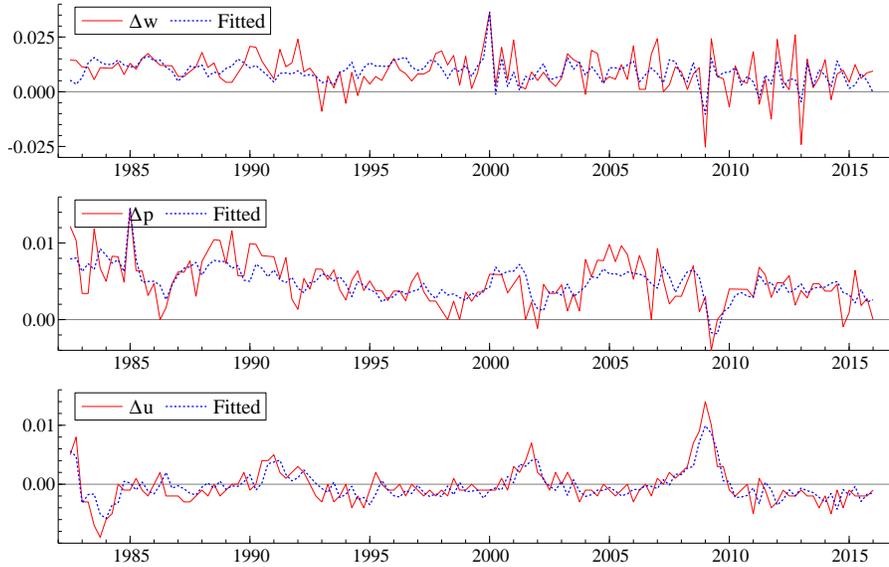


Figure 3: Actual and fitted values for the three equations.

The final model shows that wages and prices have no short run effects on each other since the price is not an explanatory variable in the wage equation and vice versa. This may be a sign of rigidities, in line with the theoretical model which suggests real wage rigidities and price rigidities. Hence, there is no short-run homogeneity between wages and prices, even though there is long-run homogeneity.

The low values for the estimated parameters of the speed of adjustment indicates rigidities, and is in line with the estimates in the α matrix in the VECM. The speed of adjustment is highest for wages, which also was indicated by the α matrix as shown in table 4. This effect is *ceteris paribus*, implying that if the wage relationship is in disequilibrium, wages will return quickly to equilibrium given that no other variables change. Prices will return slower to equilibrium if all other variables are fixed, since the coefficient on the error correction term is higher. This is similar to the results found in Australia (Bårdsen et al, 2007), the UK (Bårdsen and Fisher, 1999) and in Norway and the UK (Bårdsen et al, 1998). The high value of the autocorrelation parameter for unemployment change (0.69) also indicates that the speed of adjustment to disequilibrium for unemployment is low.

In the short run, there is no significant effect of changes in the price of

unprocessed goods on the general price, but there is an effect in the long run as shown in the estimated β vector. Since an increase in the price of unprocessed goods affects the cost for the firms, it takes time before firms change their prices and in effect influences the general price level. This difference for the effect of changes in the price of unprocessed goods in the short and long run as seen here should therefore be expected.

Furthermore, the final dynamic model shown in (23) is on the same form as the log-linearized NK model shown in (17) – except that the variables in (17) are expressed as log deviation from their steady state value while the variables in (23) are log first differences (first difference for unemployment). The long run cointegrating vector (i.e. the imposed steady state of the NK model) is controlled for in the VECM by being added as explanatory variables in the dynamic model. Hence, they are both dynamic systems conditional on the steady state or estimated long-run properties. If the analytical solution of the NK model had been obtained, it would have been possible to compare the two and identify the structural parameters in the NK model, i.e. conduct the third step of the analysis in Bårdsen and Fanelli (2015).

4 Conclusion

This paper performs a general-to-specific approach to estimating a new Keynesian model focusing on the labor market. I do this by estimating a vector error correction model where the properties of the new Keynesian model, such as the steady state and the implied weak exogeneity in the model, is tested through imposing over-identifying restrictions. The theoretical properties of the new Keynesian model can mainly not be rejected. We then get a dynamic simultaneous equations model that explains the dynamics of wages, prices, and unemployment. The results also indicates rigidities, which is a substantial part of the new Keynesian model.

By combining a new Keynesian model with an econometric framework, this paper takes an alternative approach to estimating the log-linearized model using maximum likelihood or Bayesian estimation methods. The advantage of using the method outlined here is that properties of the theoretical model may be tested, while maximum likelihood and Bayesian estimation of the log-linearized model assumes the theoretical model being the data generating process prior to estimation. An additional advantage is that we overcome the problem of a potential mismatch between the trend found by

filtering methods such as the Hodrick-Prescott filter and the trend suggested by the theoretical model, since I impose and test the steady state of the model as cointegrating relationships.

The approach in this paper is important when assessing the empirical relevance for a new Keynesian model. The results also provide an econometric model that explains dynamics in wages and prices in the US, and indicate that prices and wages are determined simultaneously. Furthermore, this shows that modeling the labor market by a new Keynesian model is relevant in order to explain the dynamics in the economy.

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A The modeling process

Starting from (22), ciw_{t-1} and unemployment are removed from the price equation, and cip_{t-1} and price of unprocessed goods are removed from the wage equation in order to identify the equations. Additionally, insignificant explanatory variables are removed in order to provide a simpler and more

robust system. This gives

$$\begin{aligned}
\Delta w &= - \underset{(0.0714)}{0.249} \Delta w_{t-1} + \underset{(0.095)}{0.337} \Delta a_t + \underset{(0.0914)}{0.348} \Delta a_{t-1} + \underset{(0.00931)}{0.0229} \Delta v_{t-1} \\
&\quad + \underset{(0.00701)}{0.0283} D_{00Q1} - \underset{(0.133)}{0.714} - \underset{(0.0309)}{0.168} \text{ciw}_{t-1} \\
\Delta p &= \underset{(0.0735)}{0.273} \Delta p_{t-1} - \underset{(0.0657)}{0.284} \Delta u_{t-1} + \underset{(0.00229)}{0.00878} D_{85Q1} \\
&\quad + \underset{(0.0469)}{0.243} - \underset{(0.0105)}{0.054} \text{cip}_{t-1} \\
\Delta u &= - \underset{(0.0658)}{0.092} \Delta p_{t-1} + \underset{(0.0623)}{0.689} \Delta u_{t-1} - \underset{(0.0278)}{0.0573} \Delta a_t - \underset{(0.0269)}{0.028} \Delta a_{t-1} \\
&\quad - \underset{(0.00282)}{0.00716} \Delta v_{t-1} - \underset{(0.00206)}{0.00943} D_{83Q1} + \underset{(0.0418)}{0.171} - \underset{(0.00938)}{0.0383} \text{cip}_{t-1}.
\end{aligned} \tag{A.1}$$

Additionally, the variables in the steady state solutions should be dated at their longest lag, see e.g. Bårdsen (1992), Bårdsen and Fisher (1999) or Bårdsen et al (2005). This is done in order to be able to facilitate the interpretation of the short-run parameters. It also provides the possibility of reducing the model further since this often enables removing additional insignificant dynamic terms. The equilibrium correction relations will then be defined as

$$\begin{aligned}
ecw_t &= w_{t-2} - [p_{t-1} + a_{t-2} - 1.104u_{t-1} - 0.001t] + constant \\
ecp_t &= p_{t-2} - [w_{t-1} - 0.984a_{t-1} + 0.112v_{t-1}] + constant,
\end{aligned} \tag{A.2}$$

since the longest lag of productivity and wages in the wage equation and of prices in the price equation is $t - 2$ in (A.1). This gives

$$\begin{aligned}
\Delta w &= - \underset{(0.0774)}{0.417} \Delta w_{t-1} + \underset{(0.095)}{0.337} \Delta a_t + \underset{(0.0966)}{0.516} \Delta a_{t-1} + \underset{(0.00931)}{0.0229} \Delta v_{t-1} \\
&\quad + \underset{(0.00701)}{0.0283} D_{00Q1} - \underset{(0.133)}{0.713} - \underset{(0.0309)}{0.168} ecw_t \\
\Delta p &= \underset{(0.0791)}{0.219} \Delta p_{t-1} - \underset{(0.0657)}{0.284} \Delta u_{t-1} + \underset{(0.00229)}{0.00878} D_{85Q1} + \underset{(0.0469)}{0.243} - \underset{(0.0105)}{0.054} ecp_t \\
\Delta u &= - \underset{(0.0706)}{0.13} \Delta p_{t-1} + \underset{(0.0623)}{0.689} \Delta u_{t-1} - \underset{(0.0278)}{0.0573} \Delta a_t - \underset{(0.0269)}{0.028} \Delta a_{t-1} \\
&\quad - \underset{(0.00282)}{0.00716} \Delta v_{t-1} - \underset{(0.00206)}{0.00943} D_{83Q1} + \underset{(0.0418)}{0.171} - \underset{(0.00938)}{0.0383} ecp_t.
\end{aligned} \tag{A.3}$$

Additionally, two constraints may be added on the parameters in order to further simplify the model. These are restrictions on the parameters for the two lags of productivity in the wage and the unemployment equation. The coefficient on period $t - 1$ productivity is restricted to be twice the size of that on period t in the wage equations and half the size in the unemployment equation, such that we have $\beta_i \Delta a_t + 2\beta_i \Delta a_{t-1} = \beta_i \Delta_2 a_t + \beta_i \Delta a_{t-1}$ and $2\beta_i \Delta a_t + \beta_i \Delta a_{t-1} = \beta_i \Delta_2 a_t + \beta_i \Delta a_t$, where $\Delta_2 a_t = a_t - a_{t-2}$. This implies that wage and unemployment growth (Δw_t and Δu_t) is affected by the productivity growth in a “smoothed manner”. These restrictions are added, and the estimated model is shown in (23).

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