The Vehicle Routing Problem with Dynamic Occasional Drivers

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Abstract. Technological advances, such as smart phones and mobile internet, allow for new and innovative solutions for transportation of goods to customers. We consider a setting where a company not only uses its own fleet of vehicles to deliver products, but may also make use of ordinary people who are already on the road. This may include people who visit the store, who are willing to take a detour on their way home for a small compensation. The availability of these occasional drivers is naturally highly uncertain, and we assume that some stochastic information is known about their appearance. This leads to a stochastic vehicle routing problem, with dynamic appearance of vehicles. The contribution of this paper is a mixed integer programming formulation, and insights into how routes for the company vehicles could be planned in such a setting. The results of the stochastic model are compared with deterministic strategies with reoptimization.

Keywords: Vehicle Routing, Occasional Drivers, Stochastic Programming

1 Introduction and Literature

Transportation can be a significant cost for last-mile and same-day delivery, which has prompted many companies to seek creative and innovative solutions to lower their costs. One such solution, considered by among others Walmart and Amazon, is *crowdshipping*, i.e. getting ordinary people who are already en route to pick up and deliver packages [5, 6]. Better utilization of vehicles that are already on the road can be profitable both for the company and the occasional drivers, and help lower emissions.

Walmarts vision of having in-store customers to help deliver goods ordered by online customers, gives rise to new variants of the dynamic vehicle routing problem (DVRP). In a recent survey and taxonomy of the DVRP [11], the authors state that "some 80% of the problems in the taxonomy involve the dynamic appearance of customers, some 10% involve dynamic travel times and some 3% consider vehicle breakdowns. In our search we were not able to find papers handling other types of dynamic events (...)". Since then, some research has been done on the effects of occasional drivers [3, 4], where [3] introduces crowdshipping and study a static version of the problem, and [4] looks at a deterministic approach of matching dynamically appearing customers and drivers. A crowdshipping platform would naturally contain a lot of uncertainty with respect to the availability of drivers. Our problem is similar to [3, 4], but extends these works by introducing stochasticity and studying how this uncertainty affects the problem.

The related body of literature for this paper can be split into two parts. Firstly, the work done on dynamic vehicle routing problems (see, for instance, the surveys in [10, 11]) is relevant for models and solution methods for the DVRP. Secondly, the various innovative variants of urban logistic problems, such as ride-sharing [1, 7], transporting people and parcels simultaneously through taxi networks [8] or public buses [9], together with the aforementioned papers on crowdshipping, are relevant to put this paper into a larger frame of the environmental direction of our research community.

Here we study a setting in which a company not only uses its own vehicles to deliver a set of small parcels from a warehouse to customers, but may also use dynamically appearing occasional drivers (ODs) that arrive at some point in time during the day. This is a new variant of the DVRP, with one central depot, a set of customers, one set of company vehicles, and a set of stochastically appearing occasional drivers, see Fig. 1 for an illustration. We assume that some stochastic information related to the ODs are known, and exploitable. The objective of the problem is to generate routes for the regular vehicles that minimize the total expected cost throughout the day, with the knowledge that ODs may appear later in the day.

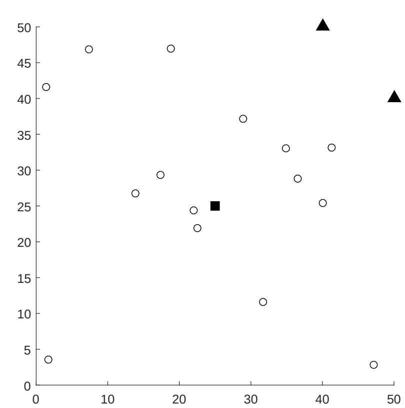

Fig. 1. Example of a stochastic vehicle routing problem with dynamic occasional drivers. The square located in the center of the graph is the warehouse, which also is the origin and destination of the company vehicles, and the origin of the ODs. Customers are circles, and the destinations of ODs are depicted as triangles in the upper right corner. The availability of the ODs is revealed while the routes for regular vehicles are executed.

To model this problem, a two stage stochastic problem is proposed. The first stage models decisions that must be made before information about the ODs become available, and the second stage models decisions after. Customer delivery locations are known in advance, together with a planning horizon starting in T_0 and ending in T_4. At a point in time T_1, information related to ODs arrive, and they may be used between T_2 and T_3. The company vehicles may start to deliver goods before T_1, or wait until the information is revealed. See Fig. 2 for the flow of a day of planning.

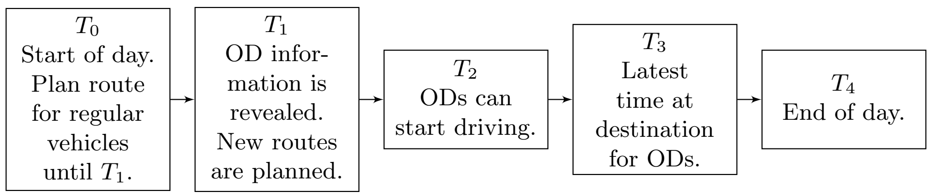

Fig. 2. Structure of the problem for one day of planning. Note that new information is revealed at T_1, so decisions are made at T_0 and T_1.

The purpose of this paper is to study the effects of uncertainty in planning of routes when ODs can appear later in the day. The contribution is a presentation of a new vehicle routing problem, the vehicle routing problem with dynamic occasional drivers. A mathematical formulation is proposed, together with an extended formulation, symmetry breaking constraints and valid inequalities. This allows us to solve large enough instances such that we can show how the uncertainty of this problem affects the routes. The results are compared with the solutions from deterministic models with different risk profiles, showing the strength of a stochastic model.

The remainder of the paper is organized as follows. In Sect. 2, we formally define the stochastic vehicle routing problem with occasional drivers and present a mixed integer programming formulation. The formulation is strengthened with an extended formulation, valid inequalities and symmetry breaking constraints in Sect. 3, and a computational study is presented in Sect. 4. Finally, in Sect. 5, we present some final remarks and discuss future research directions.

2 Mathematical Formulation

The stochastic vehicle routing problem with dynamic occasional drivers consists of a set of nodes $\mathcal{N} = \{1, \ldots, n\}$. A homogeneous fleet of regular vehicles $\mathcal{K}^R$, and a fleet of occasional drivers $\mathcal{K}^O$, are available to service these nodes. Vehicle $k \in \mathcal{K} = \mathcal{K}^R \cup \mathcal{K}^O$ has an origin $o(k)$ at the depot and a destination $d(k)$. For all regular vehicles, the destination is at the depot, while for the ODs, the destination is at a different location. Let $\mathcal{N}_k \subseteq \mathcal{N} \cup \{o(k), d(k)\}$ be the set of nodes a vehicle k can visit, and $\mathcal{A}_k \subset \mathcal{N}_k \times \mathcal{N}_k$ be the set of possible arcs for vehicle k, and denote the arc from node i to node j as (i, j).

All vehicles have time windows for their origin node $[\underline{T}_{o(k)}, \overline{T}_{o(k)}]$ and destination node $[\underline{T}_{d(k)}, \overline{T}_{d(k)}]$. For the regular vehicles this spans the entire planning horizon, while the ODs are only available for parts of the day. There is a cost of C_{ijk} and travel time of T_{ijk} to travel from node i to node j with vehicle k.

Let $\mathcal{W}$ be the set of all scenarios and let p^ω be the probability of scenario ω. The binary variables x_{ijk} and z^ω_{ijk} denote if vehicle k uses arc (i, j), in respectively the first or second stage in scenario ω. The variable t_{ik} denotes the time when vehicle k starts service at node i in the first stage, if a node is visited in a scenario in the second stage then u^ω_{ik} denotes start of service. The parameter α^ω_k is 1 if occasional driver k is available in scenario ω and 0 otherwise; for regular vehicle k, $\alpha^\omega_k = 1$.

The ODs can be used to serve one or more of the customers. Customers can be assigned to OD k and a compensation $f_k(z^\omega)$ is given to this OD in scenario ω, where z^ω denotes the arcs used in scenario ω. The binary variable y^ω_i is 1 if customer i is not served in scenario ω, and 0 otherwise. If customer i is not served, a penalty γ_i is given. The objective is to design a set of routes, one for each vehicle, such that the average cost, consisting of the routing cost plus the compensation to the ODs and the penalties of not serving a customer, is minimized.

$$\begin{aligned}\min \sum_{k\in\mathcal{K}^R} \sum_{(i,j)\in\mathcal{A}_k} C_{ijk} x_{ijk} + \sum_{\omega\in\mathcal{W}} p^\omega \Big(\sum_{k\in\mathcal{K}^R} \sum_{(i,j)\in\mathcal{A}_k} C_{ijk} z^\omega_{ijk} \\ + \sum_{k\in\mathcal{K}^O} \alpha^\omega_k f_k(z^\omega) + \sum_{i\in\mathcal{N}} \gamma_i y^\omega_i \Big) \qquad (1)\end{aligned}$$

subject to

$$\sum_{j\in\mathcal{N}\cup\{d(k)\}} (x_{o(k)jk} + z^\omega_{o(k)jk}) = \alpha^\omega_k \qquad \omega\in\mathcal{W}, k\in\mathcal{K} \quad (2)$$

$$\sum_{i\in\mathcal{N}\cup\{o(k)\}} (x_{ijk} + z^\omega_{ijk}) - \sum_{i\in\mathcal{N}\cup\{d(k)\}} (x_{jik} + z^\omega_{jik}) = 0 \qquad \omega\in\mathcal{W}, k\in\mathcal{K}, j\in\mathcal{N} \quad (3)$$

$$\sum_{i\in\mathcal{N}\cup\{o(k)\}} (x_{id(k)k} + z^\omega_{id(k)k}) = \alpha^\omega_k \qquad \omega\in\mathcal{W}, k\in\mathcal{K} \quad (4)$$

$$\sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{N}\cup\{d(k)\}} (x_{ijk} + z^\omega_{ijk}) + y^\omega_i = 1 \qquad \omega\in\mathcal{W}, i\in\mathcal{N} \quad (5)$$

$$(t_{jk} - t_{ik} - T_{ijk}) x_{ijk} \geq 0 \qquad \omega\in\mathcal{W}, k\in\mathcal{K}^R, (i,j)\in\mathcal{A}_k \quad (6)$$

$$(u^\omega_{jk} - u^\omega_{ik} - T_{ijk}) z^\omega_{ijk} \geq 0 \qquad \omega\in\mathcal{W}, k\in\mathcal{K}, (i,j)\in\mathcal{A}_k \quad (7)$$

$$z^\omega_{ijk} + x_{jlk} + y^\omega_j \leq 1 \qquad \omega\in\mathcal{W}, k\in\mathcal{K}^R, (i,j),(j,l)\in\mathcal{A}_k \quad (8)$$

$$u^{\omega}_{ik} - t_{ik} \geq 0 \qquad \omega \in \mathcal{W}, k \in \mathcal{K}^R, i \in \mathcal{N}_k \quad (9)$$

$$u^{\omega}_{ik} \geq T_1 \qquad \omega \in \mathcal{W}, k \in \mathcal{K}, i \in \mathcal{N}_k \quad (10)$$

$$\underline{T}_i \leq t_{ik} \leq \overline{T}_i \qquad k \in \mathcal{K}^R, i \in \{o(k), d(k)\} \quad (11)$$

$$\underline{T}_i \leq u^{\omega}_{ik} \leq \overline{T}_i \qquad \omega \in \mathcal{W}, k \in \mathcal{K}, i \in \{o(k), d(k)\} \quad (12)$$

$$z^{\omega}_{ijk} \in \{0, 1\} \qquad \omega \in \mathcal{W}, k \in \mathcal{K}, (i, j) \in \mathcal{A}_k | \alpha^{\omega}_k = 1 \quad (13)$$

$$x_{ijk} \in \{0, 1\} \qquad k \in \mathcal{K}^R, (i, j) \in \mathcal{A}_k \ . \quad (14)$$

The objective function (1) minimizes the here-and-now routing costs in the first stage, plus the expected costs of the second stage, namely routing costs, compensations offered to ODs and penalties. The compensation is set to make up for the detour of the occasional driver, times a compensation parameter P, such that $f_k(z^{\omega}) = P(\sum_{(i,j)\in\mathcal{A}_k} C_{ijk} z^{\omega}_{ijk} - C_{o(k),d(k),k})$. To increase readability, the sums in constraints (2)-(5) are made over both x_{ijk} and z^{ω}_{ijk} for all vehicles, even though the first stage variables x_{ijk} do not exist for the ODs. Constraints (2) and (4) make sure that a vehicle exits its origin and enters its destination, and for the company vehicles this may happen in the first or second stage. Constraints (3) ensure that the flow is balanced from origin to destination. Further, (5) force every delivery to be performed either by a regular vehicle in stage one, or any vehicle in stage two, or a penalty is paid if the customer is not served. Constraints (6) and (7) are scheduling constraints, and ensure that time passes when an arc is traversed, and waiting is allowed. Constraints (8) ensure that the first stage arc variables are no longer used, after a second stage arc variable has been used. The term y^{ω}_j in (8) is added to strengthen the constraints. Constraints (9) and (10) couple the first and second stage time variables. Constraints (10) require that the second stage variables cannot be used before T_1, while (9) enforce that the second stage variables cannot be used for a regular vehicle k that is on its way to a customer i at T_1, before it has visited that customer at $t_{ik} > T_1$. Constraints (11) and (12) set time windows on origin and destination nodes. Finally, the binary restrictions for the arc variables are given in (13) and (14). To increase readability, we have not included that several of the constraints are only necessary when $\alpha^{\omega}_k = 1$.

3 Strengthening Formulation

In the following we show an extended formulation, symmetry breaking constraints for the homogeneous vehicles and when to shift from first to second stage variables, and valid inequalities. Additionally, as there are time windows on the origin and destination of each vehicle, there are implicitly time windows on all deliveries, which we strengthen to the earliest possible arrival and latest possible departure. This is not further explained.

3.1 Extended Formulation

To exploit the structure of the problem, we extend the formulation. Extended formulations may create tighter relaxations, at the cost of adding more variables and constraints [2]. The flow variable f_{ijdk}^{ω} is equal to 1 only if vehicle k traverses arc (i,j) on the way to d in scenario ω. Let $\mathcal{F}_k \subset \mathcal{N} \times \mathcal{N} \times \mathcal{N}$ be the set of all possible flows (i,j,d) on arc (i,j) on its way to node d for vehicle k. Then we add the following constraints to obtain an extended formulation,

$$f_{ijdk}^{\omega} \leq 1 - y_d^{\omega} \qquad \omega \in \mathcal{W}, k \in \mathcal{K}, (i,j,d) \in \mathcal{F}_k \quad (15)$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}} f_{o(k)jdk}^{\omega} = 1 - y_d^{\omega} \qquad \omega \in \mathcal{W}, d \in \mathcal{N} \quad (16)$$

$$\sum_{i \in \mathcal{N} \cup \{o(k)\}} f_{ijdk}^{\omega} - \sum_{i \in \mathcal{N} \cup \{d(k)\}} f_{jidk}^{\omega} = 0 \qquad \omega \in \mathcal{W}, k \in \mathcal{K}, j,d \in \mathcal{N} | j \neq d \quad (17)$$

$$\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}} f_{iddk}^{\omega} = 1 - y_d^{\omega} \qquad \omega \in \mathcal{W}, d \in \mathcal{N} \quad (18)$$

$$f_{ijdk}^{\omega} \leq x_{ijk} + z_{ijk}^{\omega} \qquad \omega \in \mathcal{W}, k \in \mathcal{K}, (i,j,d) \in \mathcal{F}_k \quad (19)$$

$$f_{ijdk}^{\omega} \geq 0 \qquad \omega \in \mathcal{W}, k \in \mathcal{K}, (i,j,d) \in \mathcal{F}_k \ . \quad (20)$$

Constraints (15) ensure that no flow for delivery d occurs if d is not serviced. Constraints (16) and (18) ensure that if the delivery is serviced, then the flow of delivery d is one out of the depot and one into the delivery. Constraints (17) make sure that the flow is balanced through all nodes, except the depot and the delivery node. Constraints (19) ensure that there is no flow on arcs that are not used. Constraints (20) define the variables. Note that due to the time windows, several of these variables and constraints may in some instances be excluded from the problem.

3.2 Symmetry Breaking Constraints

As the regular vehicles are homogeneous, the symmetry caused by any permutation of their routes can be broken, and hopefully decrease solution time. This is done by requiring that the lowest indexed delivery that is served by a regular vehicle, is served by the lowest indexed regular vehicle in either the first stage or in the second stage in a chosen scenario ω_1. The following constraints are added,

$$x_{ijk} = 0, z_{ijk}^{\omega_1} = 0, \qquad k \in \{2 \ldots |\mathcal{K}^R|\}, i \in \{1 \ldots k-1\}, (i,j) \in \mathcal{A}_k \quad (21)$$

$$\sum_{j \in \mathcal{N}_k} (x_{ijk} + z_{ijk}^{\omega_1}) \leq \sum_{p=k-1}^{i-1} \sum_{s=k-1}^{\min\{p, |\mathcal{K}^R|\}} \sum_{j \in \mathcal{N}_s} (x_{pjs} + z_{pjs}^{\omega_1}), \quad (22)$$
$$i \in \mathcal{N} \backslash \{1\}, k \in \{2 \ldots \min\{i, |\mathcal{K}^R|\}\},$$

where ω_1 can be any scenario. We set ω_1 to be the scenario with no ODs in the computational study. Constraints (21) enforce that the i-th delivery is not done by a higher indexed regular vehicle in the first stage or second stage in ω_1.

Constraints (22) force the set of regular vehicles that can deliver to node i, to be equal to the set of regular vehicles that can deliver to node $i-1$, plus one extra regular vehicle if available. In effect, the set of possible regular vehicles for a delivery gets smaller if a lower indexed delivery is served by an OD, a lower indexed regular vehicle or not delivered at all.

Different constraints can be used to decide when the first stage variables x_{ijk} and t_{ik} should no longer be used, and the second stage variables z_{ijk}^{w} and u_{ik}^{ω} should take over. The following constraints make the change directly based on T_1, such that a first stage arc variable x_{ijk} can only be used if $t_{ik} \leq T_1$,

$$t_{ik} \leq T_1 + (T_4 - T_1)(1 - \sum_{j \in \mathcal{N}_k} x_{ijk}) \qquad i \in \mathcal{N}_k, k \in \mathcal{K}^R \ . \tag{23}$$

Note that this does not enforce that $t_{jk} \leq T_1$. If arc (i,j) is part of an optimal solution, where i is serviced before T_1 and j is serviced after T_1, then (6) together with $t_{jk} \leq T_1$ would make that solution infeasible. Thus we need to allow t_{jk} to be greater than T_1 when there are no first stage arcs out of node j.

An alternative way of changing between stages is to make the change when decisions become different for a vehicle, i.e. if the same arc is traversed in all scenarios with the same vehicle just after the first stage, then this can be forced to be stated with the first stage variables instead. This leads to an alternative way of breaking symmetry,

$$\sum_{w \in \mathcal{W}} z_{ijk}^{\omega} \leq |\mathcal{W}| - \sum_{l \in \mathcal{N}_k} x_{lik} \qquad (i,j) \in \mathcal{A}_k, k \in \mathcal{K}^R \ . \tag{24}$$

These constraints enforce that if a first stage arc variable is used into a node i, then all second stage arc variables for (i,j) out of that node cannot be used. This causes x_{ijk} to be one, instead of letting z_{ijk}^{ω} be one for all scenarios ω. In effect this can cause the first stage variables to be used, even after T_1, as long as all scenarios lead to the use of the same arcs by the same vehicles. This makes (23) and (24) incompatible. Constraints (24) do not apply to the occasional drivers, as they are not modelled with first stage variables.

3.3 Valid Inequalities

To further exploit the structure of the problem, valid inequalities have been developed to strengthen the LP relaxation and in turn reduce the solution time.

Firstly, the total amount of time used in the second stage for each vehicle and scenario can be limited. By studying Figure 2, we see that these limits are different for the regular and occasional drivers, and valid inequalities may be expressed as,

$$\begin{aligned} \sum_{(i,j) \in \mathcal{A}_k} T_{ijk} z_{ijk}^{\omega} &\leq T_4 - T_1 \qquad \omega \in \mathcal{W}, k \in \mathcal{K}^R \\ \sum_{(i,j) \in \mathcal{A}_k} T_{ijk} z_{ijk}^{\omega} &\leq T_3 - T_2 \qquad \omega \in \mathcal{W}, k \in \mathcal{K}^O \ . \end{aligned} \tag{25}$$

Secondly, as the flow balance constraints (3) include both first and second stage variables, the flow is not necessarily balanced in the first and second stage variables separately. Except for through the node where we change from first to second stage, the flow should be balanced in both the first and second stage variables over all nodes in $\mathcal{N}$. As the node where this shift is done is not known in advance, these valid inequalities instead state that the flow of the second stage variables increase through every node, and that the first stage flow through nodes decrease. This is stated as,

$$\begin{aligned} \sum_{j\in\mathcal{N}_k} z^{\omega}_{jik} &\leq \sum_{j\in\mathcal{N}_k} z^{\omega}_{ijk} \qquad && \omega\in\mathcal{W}, k\in\mathcal{K}, i\in\mathcal{N} \\ \sum_{j\in\mathcal{N}_k} x_{ijk} &\leq \sum_{j\in\mathcal{N}_k} x_{jik} \qquad && k\in\mathcal{K}^R, i\in\mathcal{N} \ . \end{aligned} \tag{26}$$

Thirdly, the time windows of the vehicles can lead them to be able to visit two nodes separately, but not in the same route. If this is the case for node i and j, then the following holds,

$$\sum_{l\in\mathcal{N}_k} (x_{ilk} + z^{\omega}_{ilk} + x_{jlk} + z^{\omega}_{jlk}) \leq 1 \qquad \omega\in\mathcal{W}, k\in\mathcal{K}, i,j\in\mathcal{N}_k \ . \tag{27}$$

Lastly, subtour elimination constraints between two nodes are written as,

$$x_{ijk} + z^{\omega}_{ijk} + x_{jik} + z^{\omega}_{jik} + y^{\omega}_i \leq 1 \qquad \omega\in\mathcal{W}, k\in\mathcal{K}, (i,j)\in\mathcal{A}_k \ . \tag{28}$$

4 Computational Study

All instances of our mathematical programming models are solved using Mosel Xpress in Windows 7 Enterprise, on a Dell Precision M4800 with Intel(R) Core(TM) i7-4940MX CPU @ 3.10GHz, 3.30GHz and 32 GB RAM. Note that Xpress solves LP problems integrated with the IP solution procedure. Due to the use of Presolve in Xpress, the LP bounds that are reported in this section may be higher than if the LP relaxation of the IP problem was solved explicitly. All figures of routes in this section break the symmetry between stages by (24), such that equal decisions in all scenarios are assigned to first stage variables.

4.1 Instance Generation

Four main instances with 20 delivery locations each were randomly generated on a square of 50×50, and named A, B, C and D. These are each divided into four sizes of the first 5, 10, 15 and 20 deliveries (S, M, L and XL). A concatenation of these are used as reference, such that AL refers to the instance with the first 15 destinations in A.

The instances have either two or three ODs, with destinations at $(40, 50)$ and $(50, 40)$ for two ODs, and $(30, 50), (50, 50)$, and $(50, 30)$ for three. To show the effect of the location of the destinations of the ODs, these are in some of the examples rotated 90, 180 and 270 degrees around the center of the 50×50 square.

From preliminary tests we have chosen several parameters of the problem that seems to be suitable to demonstrate the effects of ODs and the uncertainty of the ODs. The planning horizon starts at $T_0 = 0$ and ends at $T_3 = T_4 = 100$, and ODs can be used as soon as the information is revealed at $T_1 = T_2 = 50$. The ODs are compensated $P = 1.3$ times the cost of their detour. The penalty of not serving a customer is set to 50, and two regular vehicles are available in all instances. All possible realizations of ODs are used in the scenarios, giving $|\mathcal{W}| = 2^{|\mathcal{K}^O|}$ number of scenarios, with equal probability $p^\omega = \frac{1}{|\mathcal{W}|}$ for each scenario.

4.2 Effect of Strengthening the Formulation

In this section, the results from the testing of the model, the valid inequalities and symmetry breaking constraints are presented. We have tested the valid inequalities and symmetry breaking constraints independently, as well as in some promising combinations. Table 1 shows results from these tests for a subset of the instances. The compact formulation (1)-(14) is noted by C, and the extended formulation (1)-(20) is noted by E. The columns of the table show these formulations with different symmetry breaking constraints and valid inequalities. C+X and E+X give respectively C and E with (21), (24) and all valid inequalities. A maximum of 2 hours CPU time is allowed, and the linear relaxation bound, best bound, best integer solution and used CPU time is reported.

E provides tighter LP bounds than C, but due to the added complexity it struggles to improve the lower bound for larger instances. Even though C has a weaker LP bound, it manages to improve the lower bound more than E during the solution process. Due to this, C outperforms E in solution time for several of the instances, while for the XL instances the bound of C never reaches the LP bound of E. The symmetry breaking constraints and valid inequalities improve the bounds and solution time for most instances, both separately and together, for both C and E. We note that adding constraints (22) to C+(21) did not improve the performance significantly, neither did adding constraints (27) and (28) to the compact formulation. Both formulation C+X and E+X solve up to 15 customers with 2 and 3 ODs, with C+X being slightly faster on the M and L instances. For all instances, we see that E+X gives an LP bound that is at most 12% from the best known integer solution. Thus, E+X may be useful to get a dual bound for larger instances together with heuristics for primal bounds.

4.3 Routes under Uncertainty

In this section two figures are included to show the effect of uncertainty on the routes of the regular vehicles. In Fig. 3, the destinations of the ODs are rotated clockwise to clearly illustrate the effect of uncertainty on the first stage routes of the regular vehicles. The trend is that the regular vehicles wait to serve the

Table 1. Results for a subset of the instances with different methods of strengthening the formulation. z_{LP}, $\underline{z}$ and $\overline{z}$ give objective value of the LP bound, and the best bound and integer solution after two hours, respectively. A - indicates no integer solution was found. Bold font is used to indicate the quickest formulation, or the best bounds for those instances that were not solved to optimality.

Inst.	Info	C	C+(21)	C+(23)	C+(24)	C+(25)	C+(26)	C+X	E	E+X
AS	z_{LP}	60.8	60.8	60.8	60.8	60.8	60.8	79.9	85.6	87.7
2OD	$\underline{z}$	96.2	96.2	96.2	96.2	96.2	96.2	96.2	96.2	96.2
	$\overline{z}$	96.2	96.2	96.2	96.2	96.2	96.2	96.2	96.2	96.2
	t	1.4	1.0	1.1	**0.5**	1.7	1.0	0.6	1.1	0.6
BM	z_{LP}	113.3	114.4	115.4	114.4	113.2	114.7	120.0	124.5	128.7
2OD	$\underline{z}$	134.5	134.5	134.5	134.5	134.5	134.5	134.5	134.5	134.5
	$\overline{z}$	134.5	134.5	134.5	134.5	134.5	134.5	134.5	134.5	134.5
	t	3643.7	52.3	283.0	112.6	71.1	21.1	**8.8**	54.4	26.7
CL	z_{LP}	160.1	160.1	160.1	160.1	160.1	160.1	163.0	198.1	199.1
2OD	$\underline{z}$	197.7	211.4	211.4	211.4	211.4	211.4	211.4	199.9	211.4
	$\overline{z}$	211.4	211.4	211.4	211.4	211.4	211.4	211.4	211.4	211.4
	t	7200.0	1361.0	359.7	2184.4	766.8	100.1	**19.4**	7200.0	185.2
DXL	z_{LP}	104.1	104.8	104.1	104.1	104.1	104.1	105.8	159.6	**162.5**
2OD	$\underline{z}$	119.3	126.7	119.5	120.3	134.2	121.5	139.4	160.1	**163.8**
	$\overline{z}$	273.7	185.9	220.2	-	**173.5**	256.8	176.4	602.8	182.9
	t	7200.0	7200.0	7200.0	7200.0	7200.0	7200.0	7200.0	7200.0	7200.0
AS	z_{LP}	56.8	57.3	56.8	56.8	56.8	56.8	70.5	76.5	78.4
3OD	$\underline{z}$	89.0	89.0	89.0	89.0	89.0	89.0	89.0	89.0	89.0
	$\overline{z}$	89.0	89.0	89.0	89.0	89.0	89.0	89.0	89.0	89.0
	t	3.5	4.7	5.1	16.3	5.4	6.2	4.4	3.2	**1.3**
BM	z_{LP}	106.7	107.6	105.9	109.9	106.0	107.3	106.8	118.3	121.7
3OD	$\underline{z}$	117.6	128.1	114.6	122.4	128.1	118.1	128.1	128.1	128.1
	$\overline{z}$	128.1	128.1	128.1	128.1	128.1	128.1	128.1	128.1	128.1
	t	7200.0	673.8	7200.0	7200.0	765.0	7200.0	**39.7**	6319.2	62.4
CL	z_{LP}	155.1	155.2	155.1	155.1	155.1	155.1	157.6	197.2	197.6
3OD	$\underline{z}$	209.5	209.5	192.9	209.5	209.5	209.5	209.5	197.9	209.5
	$\overline{z}$	209.5	209.5	209.5	209.5	209.5	209.5	209.5	211.1	209.5
	t	3376.6	2933.5	7200.0	3959.7	3460.9	1015.9	**340.9**	7200.0	4985.9
DXL	z_{LP}	102.4	103.1	102.4	102.4	102.4	102.4	104.0	157.0	**158.9**
3OD	$\underline{z}$	112.0	111.8	112.0	112.3	114.2	113.0	115.7	157.7	**161.0**
	$\overline{z}$	299.2	390.9	340.1	-	**175.3**	175.7	278.1	-	180.5
	t	7200.0	7200.0	7200.0	7200.0	7200.0	7200.0	7200.0	7200.0	7200.0

deliveries that can be taken by ODs, until this information gets revealed. We also see that at the end of the first stage, the vehicles tend to position themselves to make good second stage decisions possible. This is especially obvious in the two graphs to the left and the upper right graph. In the upper right graph, only one of the the company vehicles are used in the first stage, but this vehicle is well positioned to serve the cluster of customers to the right of the depot if necessary. Finally, in the lower right, we see one of the regular vehicles moving close to the destination of the ODs, which might seem like a bad decision at first glance, but the second stage solutions that follow show this to be a good decision.

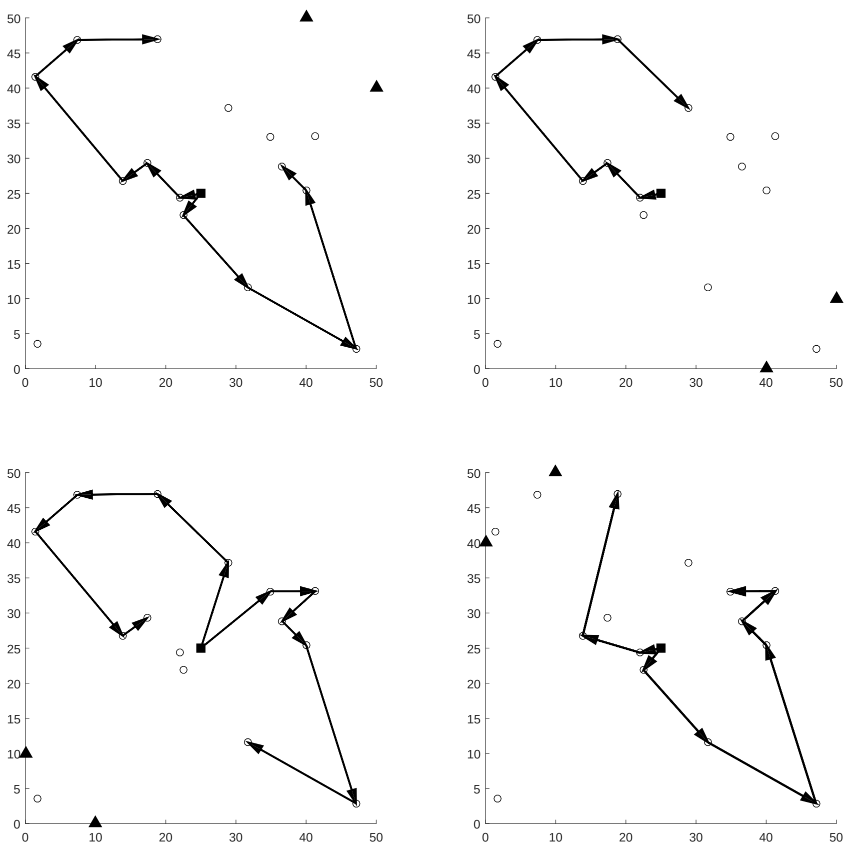

Fig. 3. Four examples of how the first stage routes for the company vehicles change when only the destinations of the ODs are altered, for the instance CL. Notice that the customers are the same in all graphs. Starting in the first graph with the destinations of the two ODs in the upper right corner, and rotating the destinations 90 degrees around the center for each graph.

Second stage solutions for the lower right graph from Fig. 3 is shown in Fig. 4. This shows how the behaviour of the regular vehicles in the first stage fits well with the different scenarios. The route of the regular vehicle that served a delivery close to the destination of the ODs in the first stage now seems more reasonable. All the deliveries in this area are served in all scenarios, with ODs when they are available and through rerouting of the company vehicles when no

ODs appear. Notice also how the route crosses itself when no ODs are available. This would obviously be suboptimal if we knew beforehand that no ODs occur, but this is part of the repositioning that happens due to the information flow of the problem.

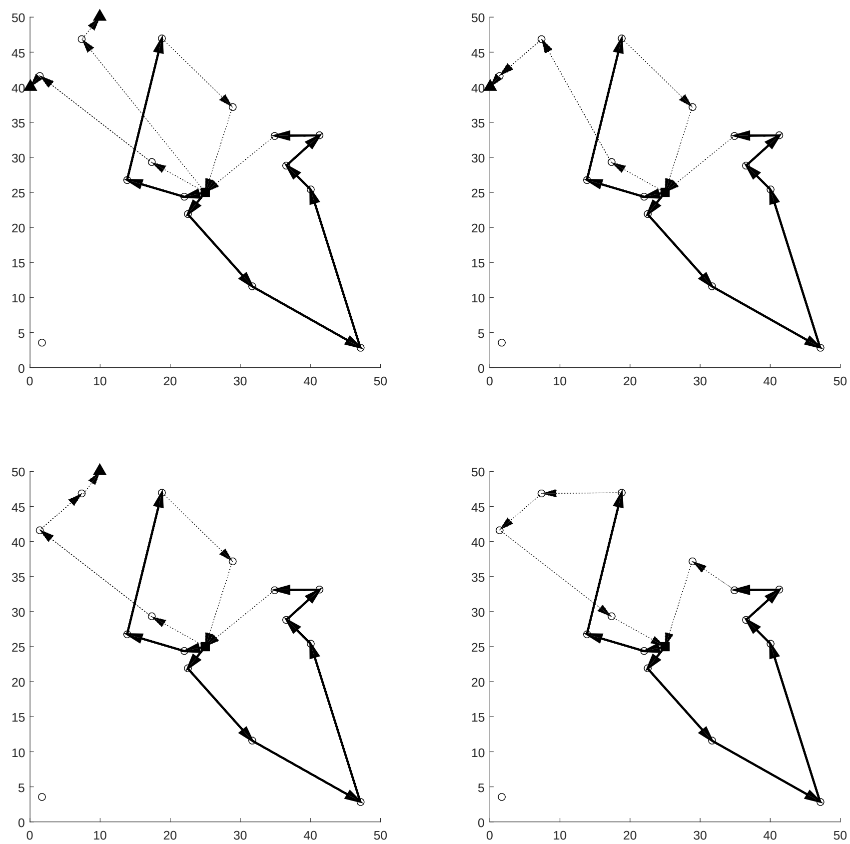

Fig. 4. Comparison of second stage solutions of different scenarios. Notice that the first stage solution is the same in all graphs, and allows for good second stage solutions in all the scenarios.

4.4 Comparison to Deterministic Strategies

To test the quality of the stochastic solutions, we compare them to the solutions of three strategies where deterministic planning and reoptimization are used. The strategies differ in their risk profiles, where the no risk, medium risk and high risk profile relate to planning with respectively zero, one or all ODs available. A plan is created at T_0 for the entire day, with the assumption that either zero, one or all ODs are available. All decisions from the plan that are taken before T_1 are considered fixed, and reoptimization is done at T_1 for each scenario. An average over all scenarios is considered the expected *actual cost* of the strategy for a given instance, while the cost of the initial plan is referred to as the *objective value*.

Table 2 shows that the objective value from the deterministic planning with high risk gives an optimistic value when we compare with the actual cost of that solution, and that the no risk solution gives a pessimistic objective value. The deterministic models never give a better actual cost than the solution from the stochastic model.

The objective values of the no risk profile corresponds to solving the VRP without ODs, and thus give us results for the potential savings by using crowdshipping in these instances. In a paper with only deterministic models, the objective value of the no risk profile is often compared to the objective values that are found in the medium or high risk profiles. This could for our instances show savings of up to almost 50 %. A more realistic comparison is however to compare the no risk objective value to the objective value of the stochastic models. These savings are on average 13% for our instances and up to 23%. Further, a comparison of the actual costs of the deterministic strategies to the costs of the stochastic solution, gives us the value of solving a stochastic model over a deterministic model. This shows that the stochastic model gives 2-3% better solutions than the medium and high risk deterministic profiles, and 12% better solutions than the no risk profile.

Deterministic planning gives optimal solutions for the scenarios that match their risk profile, while the stochastic solution plans for the uncertainty and thus performs better on average. The table does however also show the problem of the stochastic model, where the XL instances are not solved in reasonable time and therefore omitted. The deterministic models are faster to solve, and can be used as heuristics to solve the stochastic model.

5 Final Remarks

In this paper, we develop a stochastic mixed integer programming formulation for a new vehicle routing problem, where occasional drivers appear dynamically. Symmetry breaking constraints and valid inequalities are proposed, and some of them are shown to decrease solution time substantially. The LP bounds are strengthened by an extended formulation, while the compact formulation slightly outperforms it with respect to solution time. Several figures are included to show the effects of the uncertainty in the problem. This shows that the company vehicles focus on first delivering to the customers that are unlikely to be served by the ODs. The solutions from the stochastic model are compared to solutions from deterministic models, showing that the stochastic model performs 2-3% better than planning with some ODs and reoptimizing when information became

Table 2. Planned deterministic objective function value for different risk profiles, compared to actual expected cost of implementing the solutions from these profiles. The rightmost column gives the stochastic optimal value. The avg. ratio gives the average ratio between the stochastic solution and the solutions in that column. Italics are used for best integer solution when the optimal solution is not found in 2 hours.

		Objective value			Actual cost			Stoch
	Instance \ Risk	No	Medium	High	No	Medium	High	
2OD	AS	113.9	78.9	70.0	105.9	102.1	100.2	96.2
	AM	169.7	140.5	119.3	169.7	163.7	158.9	158.9
	AL	223.1	170.0	163.1	223.1	200.0	195.9	194.8
	BS	107.3	96.4	82.4	107.3	93.3	101.6	93.3
	BM	160.4	142.8	102.0	159.2	143.4	153.1	134.5
	BL	172.6	155.1	123.7	172.6	170.3	180.0	165.5
	CS	81.9	68.3	52.0	81.9	68.9	68.9	68.9
	CM	134.3	113.7	96.7	134.3	112.7	112.7	112.7
	CL	219.3	215.0	204.1	219.3	212.0	212.9	211.4
	DS	133.3	112.2	112.0	133.3	118.2	132.0	118.2
	DM	171.7	149.1	149.1	171.7	160.5	161.9	156.5
	DL	176.5	151.8	151.8	176.5	163.2	163.2	160.1
3OD	AS	113.9	73.5	42.7	107.9	94.5	89.0	89.0
	AM	169.7	135.1	106.7	169.7	156.2	148.8	148.8
	AL	223.1	164.8	152.2	220.1	191.0	196.3	188.1
	BS	107.3	90.2	70.5	107.3	85.1	89.7	85.1
	BM	160.4	112.4	89.5	160.4	132.7	134.8	128.1
	BL	172.6	144.0	104.0	170.8	164.2	157.2	*157.2*
	CS	81.9	62.9	45.8	81.9	64.0	63.1	63.1
	CM	134.3	108.3	92.6	134.3	107.0	107.0	107.0
	CL	219.3	209.6	199.7	219.3	214.7	215.5	209.5
	DS	133.3	110.6	100.7	133.3	121.1	131.1	118.4
	DM	171.7	147.5	147.5	171.7	158.0	158.0	156.8
	DL	176.5	150.3	149.7	160.0	160.8	160.7	*160.0*
	Avg. ratio	0.87	1.06	1.26	0.88	0.98	0.97	

available, and 12% better than planning without ODs and reoptimizing. For our instances, the average cost savings of using ODs are 13%. Creating better solution methods, e.g. through scenario generation, heuristics and decomposition algorithms, together with testing on real data, is future research.

Acknowledgments. The work presented in this paper was partly funded by the Research Council of Norway, Distribution Innovation AS, and the Norwegian Public Roads Administration, in the context of the DynamITe project [Contract 246825/O70, SMARTRANS].

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