METHOD OF REGULARIZING THE PROBLEM OF RECOVERY OF INPUT SIGNALS OF DYNAMIC OBJECTSANDRIY VERLAN

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Abstract. The task of signal recovery is one of the most important for automated diagnostics and control systems. This task is computationally complex, especially in the case of multiple heterogeneous errors in the signals and recovery is to be performed in real-time. The article deals with the application and investigation of a modified algorithm for the method of quadrature formulas for the numerical solution of Volterra integral equations of the I kind in solving the problem of signal recovery in real time. A method is proposed for selecting the parameters of regularizing links of computing means.

Key words: Signal reconstruction, interference, real time, Volterra equations of the I kind, regularization.

Introduction. The integral formulation of the problem of reconstructing the input signal of a stationary dynamic object is described by the Volterra equation of the I Kind

$$\int_0^t K(t-s)y(s)ds = f(t), \qquad (1)$$

where the functions $y(t)$ and $f(t)$ represent respectively the input and output signals, and the core (function $K(t)$) is the impulse response function of the object.

Problem of solving equation (1) belongs to a class of ill-posed problems, since the presence of errors in the right side and core usually causes numerical instability of the solution process, which makes it necessary to use regularization methods [1].

In many cases, in order to increase the stability of the solution process of the integral equation of type (1) A.N. Tikhonov's regularization method [2] is used, in which the problem is reduced to solving the integro-differential equation, one of the conditions for which is $K(0) \neq 0$. However in real objects usually $K(0) = 0$, which limits the possibilities of the method.

In this article to ensure stability of the computing process a possibility is considered of using the Lavrentiev's regularization method with a specific modernization of the regularization parameter's search process.

According to the Lavrentiev's method, instead of equation (1) the following equation is solved:

$$\alpha y(t) + \int_0^t K(t-s)y(s)ds = f(t). \qquad (2)$$

The problem of determining the regularization parameter α is time- and effort-consuming [3]. In the work [3] multiple ways of determining α are shown, including the model experiments (examples) method for Fredholm integral equation of the I kind.

Let us consider the possibility of applying the model experiments method to determine the regularization parameter α in the solution of Volterra integral equation of the I kind. Parameter α determination procedure consists of the following steps.

1. For the integral equation (1) to be solved a model equations is created

$$\int_0^t Q(t-s)y_Q(s)ds = f_Q(t), \qquad (3)$$

in which $Q(t)$ coincides with the predetermined function $K(t)$ and solution $y_Q(t)$ is given (selected) in such a way, that the right side $f_Q(t)$ is as close as possible to $f(t)$, i.e.

$$f(t) \approx f_Q(t) = \int_0^t Q(t-s)y_Q(s)ds. \qquad (4)$$

2. In practice, measurement mistakes are inevitable and instead of the equation with exactly known right side $\bar{f}(t)$ we have approximate right side, i.e.

$$f(t) = \bar{f}(t) + \Delta\bar{f}(t),$$

where $\Delta\bar{f}(t)$ – error. Having the law (e.g., normal) of $\Delta\bar{f}(t)$ distribution, we can record

$$f(t) = \bar{f}(t) + \xi\bar{f}(t),$$

where ξ – normally distributed random number. Therefore, the right part of the model equation (3) we perturb with an error, at which values $\|\Delta f_Q\| / \|f_Q\|$ and $\|\Delta f_Q\| / \|f_Q\|$ are approximately equal, i.e. instead of equation (3) we have

$$\int_0^t Q(t-s)\bar{y}_Q(s)ds = f_Q^*(t), \qquad (5)$$

where $f_Q^*(t) = f_Q(t) + \xi f_Q(t)$.

Applicating Lavrentev's regularization method allows to record of equation (3) as

$$\alpha y_{Q\alpha}(t)+\int_0^t Q(t-s)y_{Q\alpha}(s)ds = f_Q^*(t). \quad (6)$$

3. By repeated numerical solutions of (6), e.g. by using quadrature formulas, for a row of values of α $\alpha_{\text{opt}Q}$ is determined, at which

$$\sum_{i=0}^{m}\left|y_{Q\alpha}(t_i)-y_Q(t_i)\right|^2 = \min, \quad i=1,m, \quad (7)$$

where m – the number of sampling points.

4. The obtained value $\alpha_{\text{opt}Q}$ is used to solve integral equation (1).

Numerical simulations show that using the modeling experiments method in the development of a numerical equation solving algorithm for signal restoration problem (1) allows to determine the effective values of parameter α, which regularizes the problem.

When numerically solving the considered problem it is important to understand possible results' errors.

Note that for a linear integral equations the error in solutions can be expressed using the errors' fundamental formulas. Indeed, the machine solution can be represented as depending on a number of quantities q_1, q_2, ..., q_n, characterizing the model's parameters, input actions, etc., deviations in which cause result's error. If there are deviations, real solution can be decomposed in the limited Taylor series and can be represented as

$$\begin{aligned} &Y(t,q_1+\Delta q_1,\ldots,q_n+\Delta q_n)\approx Y(t,q_1,\ldots,q_n)+ \\ &+u_1(t)q_1+u_2(t)q_2+\ldots+u_n(t)q_n, \end{aligned} \quad (8)$$

where $u_1(t)$, $u_2(t)$, ..., $u_n(t)$ – influence (or sensitivity) coefficients.

Subtracting from (8) the exact solution of $Y(t, q_1, q_2, ..., q_n)$, we obtain

$$\Delta Y(t)+u_1(t)\Delta q_1+u_2(t)\Delta q_2+\ldots+u_n(t)\Delta q_n.$$

To determine the sensitivity coefficients, it is possible to obtain the corresponding equations. We assume that the parameters q_1, q_2, ..., q_n are determined by the internal properties of the model, i.e. they are part of the core of the solved machine equation, which in this case has the form

$$\alpha Y(t)+\int_0^t K_M(t-s,q_1,\ldots,q_n)Y(s)ds = f(t). \quad (9)$$

($Y(t)$ – sought approximate solution)

Differentiating both sides of (9) by the parameter q_i (i = 1, ..., n), we obtain

$$\alpha\frac{\partial Y(t)}{\partial q_i}+\int_0^t \frac{\partial}{\partial q_i}K_M(t-s,q_1,\ldots,q_n)Y(s)ds=0$$

or

$$\alpha\frac{\partial Y(t)}{\partial q_i}+\int_0^t\left[\frac{\partial K_M(t-s,q_1,\ldots,q_n)}{\partial q_i}Y(s)+ + K_M(t-s,q_1,\ldots,q_n)\frac{\partial Y(s)}{\partial q_i}\right]ds=0.$$

Introducing the notation

$$\frac{\partial Y(s)}{\partial q_i}=u_i(t),$$

$$\frac{\partial K_M(t-s,q_1,\ldots,q_n)}{\partial q_i}=K'_{Mq_i}(t-s,q_1,\ldots,q_n),$$

we obtain the sought equations

$$\begin{aligned} &\alpha u_i(t)+\int_0^t K_M(t-s,q_1,\ldots,q_n)u_i(s)ds = \\ &=-\int_0^t K'_{Mq_i}(t-s,q_1,\ldots,q_n)Y(s)ds. \end{aligned} \quad (10)$$

As the function $Y(s)$, on the right side of (10) approximate solutions can be used. As seen from (10), to determine the sensitivity coefficients we can decide to use the basic solved equation, since the core of equation (10) coincides with its core.

As in the case of differential equations, for linear integral equations one can obtain an equation for the error. We assume that while solving (2), equation actually being solved looks like this

$$\tilde{y}(t)+\int_0^t\tilde{G}(t-s)y(s)ds=\tilde{\varphi}(t), \quad (11)$$

where $\tilde{G}(t-s)=\tilde{K}(t-s)/\alpha$, $\tilde{\varphi}(t)=f(t)/\alpha$.

Assuming that $\tilde{G}(t-s)$ accounts for initial modeling errors (methodical and instrumental errors), and is the sum

$$\tilde{G}(t-s)=G(t-s)+\Delta G(t-s).$$

Right side of equation (11) $\tilde{\varphi}(t)$ contains the external disturbance error and equals

$$\tilde{\varphi}(t)=\varphi(t)+\Delta\varphi(t);$$

$\tilde{y}(t)$ – approximate solution determined by the relation

$$\tilde{y}(t)=y(t)+\Delta y(t).$$

Where $\Delta y(t)$ –solution's total error. Then, by subtracting expression (2) from (11) we obtain

$$\begin{aligned} &\Delta y(t)+\int_0^t\{[G(t-s)+\Delta G(t-s)][y(s)+\Delta y(s)]- \\ &-G(t-s)y(s)\}ds=\Delta\varphi(t). \end{aligned}$$

Expanding brackets under the integral and considering errors $\Delta G(t-s)$ and $\Delta y(t)$ so small, that their product can be neglected, we obtain the sought equation

$$\Delta y(t)+\int_0^t G(t-s)\Delta y(s)ds=\Delta\varphi(t)-\int_0^t\Delta G(t-s)y(s)ds$$

or

$$\alpha\Delta y(t)+\int_0^t K(t-s)\Delta y(s)ds=\Delta f(t)-\int_0^t \Delta K(t-s)y(s)ds.$$

This equation is difficult to use to calculate the error $\Delta y(t)$ because of the uncertainty generally occurring in primary errors, as well as due to the fact that rather than true solution $y(s)$ on the right side we must use read only approximate. However, it is applicable for a qualitative study of errors, since in particular, shows that various components of the total error can be defined separately (leaving in the right side only $\Delta f(t)$, we can determine the result's inherited error, and leaving only the integral – numerical algorithm's error). In addition, the equation for the error allows us to make its assessment. Let us give an example of such an assessment.

If (t, s) belongs to the region D, $0\le t\le\delta, 0\le s\le t$ and you can set constraints

$$\max_{(t,s)\in D}\left|K(t-s)\right|\le K,\quad \max_{(t,s)\in D}\left|\tilde{K}(t-s)\right|\le\tilde{K},$$

$$\max_{(t,s)\in D}\left|\Delta K(t-s)\right|\le\delta,\quad \max_{t\in[0,\delta]}\left|\tilde{f}(t)\right|\le f,$$

$$\max_{t\in[0,\delta]}\left|\Delta f(t)\right|\le\eta,$$

then, using the results of [4], we obtain the estimate

$$\Delta y(t)\le\left[f\delta\frac{e^{\frac{1}{\alpha}(K-\tilde{K})t}-1}{K-\tilde{K}}+\mu\right]e^{\frac{1}{\alpha}\tilde{K}t},$$

Conclusion. Thus, the use of the Lavrentiev's regularization method in solving Volterra integral equations of the I kind provides required stability of the signal recovery process, and model experiments method allows determining the values of the regularization parameter. Expressions, obtained on basis of the accuracy analysis of solved equations, are the basis of deterministic and probabilistic error estimates of the sought solution.

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СПОСІБ РЕГУЛЯРИЗАЦІЇ ЗАДАЧІ ВІДНОВЛЕННЯ ВХІДНОГО СИГНАЛУ ДИНАМІЧНОГО ОБ'ЄКТУ.

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Задача відновлення сигналів є однією з першочергових для автоматизованих систем діагностики і управління. Це обчислювально складна задача, особливо при наявності в сигналах великої кількості гетерогенних завад і необхідності проводити відновлення в реальному часі. В статті розглядаються питання застосування та дослідження модифікованого алгоритму методу квадратурних формул чисельного рішення інтегральних рівнянь Вольтерра першого роду при вирішенні задачі відновлення сигналів в реальному часі. Пропонується спосіб вибору параметрів регуляризуючих зв'язків моделюючих ланцюгів обчислювальних засобів.

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