

EERA DeepWind'2014, 11th Deep Sea Offshore Wind Conference

Cross-Border Transfer of Electric Power under Uncertainty: A Game of Incomplete Information

Phen Chiak See ^{a*}, Olav Bjarte Fosso ^a, Kuan Yew Wong ^b

^aDepartment of Electric Power Engineering, Norwegian University of Science and Technology, 7491-Trondheim, Norway.

^bFaculty of Mechanical Engineering, Universiti Teknologi Malaysia, 81310, UTM Skudai, Johor, Malaysia.

Abstract

Cross-border transfer of electric power promotes collaboration in power generation between integrated electricity markets. It resolves grid reinforcement issues in existing transmission networks. Because of that, researchers have given higher attention to this field and conducted various studies on the subject using technical simulation approaches. Yet, substantial work has to be done for quantifying the socioeconomic benefits of the mechanism. This paper intends to fill the gap by introducing a method for analyzing the mechanism by representing it as a game of incomplete information. The subject is modeled as a Bayesian game in which the type of marginal generators located within one (or more) external market area is not known. Based on that, the Bayesian equilibrium which represents the state where all marginal generators would incline to converge is found. The authors suggest that the method is robust and can be used for quantifying the performance of a market coupling mechanism because it realistically considers all marginal generation scenarios.

© 2014 Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/3.0/>).

Selection and peer-review under responsibility of SINTEF Energi AS

Keywords: Electricity markets, game of incomplete information, Bayesian equilibrium.

1. Introduction

Previously, transmission lines that span across regions within Europe are primarily used for stabilizing the transmission network [1]. Because of the recommendation in EU Directive 96/92 for creating a harmonious Internal Electricity Market (IEM), these transmission lines are increasingly used in the trade of electric power across borders. Consequently, the transmission bottlenecks become a major concern. These issues are treated in the successive legislations: (i.) the EU Regulation 1228/2003 which promotes development of market-based congestion management to appropriately reflect price signals and incentive of investments; and (ii.) Commission Decision 2006/770/EC which suggests implementation of flow-based allocation governed by physical flow of electric power. These directives further promote the sharing of power generation and implementation of Flow Based Market Coupling (FBMC) mechanisms.

* Corresponding author. E-mail address: phen.chiak.see@elkraft.ntnu.no

Generally, cross-border transfer of electric power is a known mechanism which encourages the sharing of power generation resources. It promotes liquidity and competition among stakeholders of connected electricity markets, while reducing the tendency of price hoarding [2]. This further enhances the welfare and security of power supply in interconnected market areas. Additionally, it is viewed as a viable means for resolving grid reinforcement issues. Nonetheless, the implementation of FBMC is not without its challenges. Most noticeably, the tie lines connecting many European countries possess limited transfer capacity. This complicates the quantification of operational parameters (i.e., transfer capacity of flow-gates) within the mechanism.

Therefore, it is crucial for developing a viable analytical method for quantifying the extent of improvement that FBMC could bring to an electricity market. The model should simultaneously consider the transmission network limitations and the strategic trading behavior of stakeholders. By doing so, we have to deal with many unknown variables in the electricity market model. Particularly, the exact setup that the market (and transmission network) would eventually evolve has yet to be publicly announced. Furthermore, the details of marginal cost of generators located within relevant market areas are not publicly accessible. Fortunately, game theory offers several analytical methods that could be applied for serving the purpose in the absence of clear price information [3].

In this paper, the authors outline a game theoretic approach for quantifying the expected performance of FBMC scenarios. The mechanism is modeled as a game of incomplete information played by active generators located within market areas which implement FBMC. This approach adopts the Bayesian game theory [4,5,6], which is known as a robust means for deriving the expected utility of a game [7]. The remainder of this paper is organized as follows. Section II describes the equilibriums in strategic interactions; Section III outlines strategic interactions and equilibriums in FBMC; Section IV describes the methods and steps implemented in the analysis; Section V shows and discusses the results obtained in the analysis; and Section VI concludes the paper.

2. Equilibriums in strategic interactions

The Bayesian game theory developed by John C. Harsanyi [4,5,6] is a field of non-cooperative game theory which was initially established by John F. Nash [8,9]. In 1949 [10], Nash (by generalizing the two-player zero-sum games developed by von Neumann and Morgenstern [11]) proved that every multi-player, non-cooperative game possesses at least one equilibrium state where all of the players would incline to converge. The states are known as Nash equilibrium (NE) where the players receive no incentive for changing their incumbent strategy.

Essentially, game theorists study the strategic interactions among players participating in a strategic game and how NE would be achieved. The strategic interactions are defined as a situation in games where each player intends to maximize his respective payoff while his payoff depends on the strategies taken by his opponents [7]. Such interdependence of outcome is indicated with a Cartesian product in player i 's utility function, $u_i = A_i \times \dots \times A_N \rightarrow \mathbb{R}$, where $A_i = \{a_1, \dots, a_N\}$ represents the set of actions taken by player i .

Broadly, A_i consists of the following types: (i.) player i may execute an action, a_k which he thinks is the best response to his opponents (i.e., he may bid in the market using his true marginal cost); and (ii.) player i may consecutively execute A_i following a probability distribution (i.e., he may bid in the market in a random manner, with probability x at high price, and probability $1 - x$ at true marginal cost). The players achieve a pure-strategy NE if they reached the equilibrium state while all of them consistently execute an action (following the former type). Otherwise, they achieve a mixed-strategy NE if they reached the equilibrium state while some (or all) of them execute a finite list of actions in a random manner (following the latter type).

Based on that, we can mathematically denote the strategy taken by player i with $S_i \equiv \{s_1, \dots, s_N\}$, where s_k is the probability for executing a_k (the subset of A_i) owned by player i . A mixed-strategy NE is therefore, an NE state achieved when each of the players executes a list of his actions following $s_k : a_k \rightarrow [0, 1)$ and $\sum s_k(a_k) = 1$. Pure-strategy NE is the degenerated case of mixed-strategy NE, in which all players execute only one a_k with probability $s_k(a_k) = 1$.

In its original form, NE possesses some limitations related to its strict rationality assumptions. Although the game is assumed as imperfect (each player does not know exactly the action that his opponents are about to take), it assumes that all of them know exactly the type of their opponents [12]. This assumption is not always true. For instance, in coupled electricity market, a generator could neither know the type of marginal generators (thermal, hydro, etc) located in other market areas that would participate in short-term bidding nor their exact location within the transmission network. Even

though he may guess the marginal cost of generation owned by these generators based on past experience (empirical information) [12], he cannot decide any action because of the incomplete knowledge on the type of his opponents.

Although NE does not apply well in such cases, we can use Bayesian game theory for solving the problem. This method allows a player to form first-order belief on the type of his opponent before determining his best response in a game [13]. The best responses are determined based on expected utility functions. As an extension of NE, the method allows us to consider all possible types of each player in game based on the probability of occurrence. For instance, suppose a generator located within an area is proposing a bid price at hour t . Meanwhile, its operator believes that another connected market area possesses two generators which have the following similarities: (i.) marginal cost, (ii.) remaining generation capacity; and (iii.) ramping characteristics. Also, he knows that both of them share the same possibility of being the marginal generator. Then, he can include this perception by modeling the marginal generator in that area as a Bayesian player, with $p = 0.5$ for being either the first or the second unit.

Formally, let θ_i be the type of player i in a game, and $p(\theta_{-i}|\theta_i)$ represents the first-order belief owned by player i towards the type of his opponent (given that his type is θ_i , which is known only to himself). The set of all types of player i is Θ_i and $\theta_i \in [0, 1)$ for all $\theta_i \in \Theta_i$. Under such conditions, a player would choose his action based on his type, and different actions may be assigned to different types. Based on that, he owns a strategy, s_i that maps Θ_i to A_i . Hence, $s_i : \Theta_i \rightarrow A_i$. Since Bayesian game theory suggests that the choice of a player's action follows θ_i and $p(\theta_{-i}|\theta_i)$, the expected payoff for player i in the game becomes:

$$E[u_i(s_i|s_{-i}, \theta_i)] = \sum_{\theta_{-i} \in \Theta_{-i}} u_i(s_i, s_{-i}(\theta_{-i}), \theta_i, \theta_{-i}) p(\theta_{-i}|\theta_i) \quad (1)$$

where, $s_{-i}(\theta_{-i})$ is the strategy taken by players except player i , given that the type of player i is θ_i . A Bayesian equilibrium (BE) is the Nash equilibrium of the Bayesian game, formulated as follows.

$$E[u_i(s_i|s_{-i}, \theta_i)] \geq E[u_i(s'_i|s_{-i}, \theta_i)] \quad (2)$$

Upon achieving BE, player i receives a lower expected utility if he uses a strategy other than s_i (denoted by s'_i). The existence of BE is guaranteed because of the proven existence of NE.

3. Strategic interactions and equilibriums in coupled electricity markets

Interactions between strategic players (marginal generators) participating in a wholesale electricity market could be defined as a non-cooperative strategic game. Ideally, we assume that the players do not resort to collusions and treat each of their counterparts as an opponent. They too have small influence on the outcome of the bid [14]. They compete in the two-sided auction process by means of selecting a marginal cost from a finite range of values and propose it to the market organizer [15]. Their aim is to receive the highest possible payoff after the market is cleared. They are said to behave strategically if they propose a price other than their true marginal cost in the bidding process [16]. They have market power if they receive increased profit by implementing strategic bidding.

Many important assumptions adopted in non-cooperative game theory are applicable in analyzing such scenarios. Most importantly, bidding activities in an electricity market are an iterated game. This suits the mass action principle proposed by Nash [17,18,19]. Subsequently, the rationality assumptions in NE also apply well in this area [7]. The mass action theorem states that players in an iterative game would continually adapt their strategy following the direction of a better reply to whatever is being played by the entire population of players. The adjustment stops only when an NE state is achieved. The theorem fits well with the operation of an electricity market because (i.) the main structure of the transmission network does not change in short time; and (ii.) the load demand within the market follows a consistent cyclical trend over weeks, months, quarters and years. This forms an iterative environment where active generators gain empirical information on the relative consequence of their strategies [19]. Later, they are able to make correct judgments hence stabilizing themselves in NE.

The use of NE theory alone is limited in the analysis of coupled electricity markets because of its strict rationality assumptions. Before the implementation of market coupling, active generators located within a market area only possess empirical information leading to the formation of strategies within their local market. Nonetheless, they do not have sufficient information for dealing with strategic bidding of active generators located in other market areas. After

the implementation of market coupling, they have to observe various uncertainties in the operation of the market while each of them only has experience dealing with active generators located in their local market. They would not know the type of marginal generator they shall compete with, under various load levels. They become ex-interim players (who know about their own type but not the type of the other players [20]) and fail to immediately form exact common priors.

Literally, BE theory can be applied for addressing this issue. Based on incomplete information, it is able to quantify the potential influence of market coupling on the expected payoffs received by generators located in each area. This helps in justifying the overall performance of a proposed market design involving market couplings. As mentioned in Port Royal Logic [21], we should consider the probabilities of all scenarios, and put them in geometrical proportions when considering a combined effect of the scenarios. BE theory deals with incomplete information problems in that manner.

In the following sections, the authors explore the implementation of FBMC. Generally, FBMC is an extension of the zonal pricing scheme which is intended for use in organizing the bidding of cross-border transfer of electric power within a common Day-Ahead (DA) market [22]. The dispatch within the entire network is cleared based on an approximated transmission network that closely represents the physical flow of electric power. Hence, three unknown variables may affect the expected utilities received by the generators within each market area: (i.) the transmission network configuration characteristics (especially, the Power Transfer Distribution Factors (PTDFs)) of an external market; (ii.) the possibility that an external market may divide into smaller market areas in order to deal with internal congestion issues or to allow better involvement of generators in cross-border DA bidding; and (iii.) the type of marginal generators located in external market areas for all load instances.

The authors show a case study of FBMC in a transmission network. The implementation intends to cover the mentioned unknowns and outline the method implemented for computing the expected payoffs received by all marginal generators located in each market area at BE.

4. Implementation

The construction of the FBMC model begins with representation of each market area in the transmission network as a node (copper-plate) [22]. The combined characteristics of transmission lines (also known as the interfaces connecting them) are represented as fictitious transmission lines which possess flow characteristics computed as close as possible to their physical value. The method neither modifies the existing network configurations nor the market area arrangements. The optimal dispatch can be computed with Direct Current Optimal Power Flow (DC-OPF) computation. It simultaneously clears the generation dispatch and physical flow of power within the entire network.

The key to implementing a successful FBMC model is the correct representation of power flow between market areas (nodes in the approximated transmission network model) through the interfaces. Any deviation from the real characteristics would create an inefficient (suboptimal) generation dispatch plan [2]. Specifically, the Generation Shift Factors (GSFs) at all interfaces have to be properly computed. Various methods have been used by researchers to achieve this. For instance, Krause [22] represented each of the interfaces as a line of equivalent reactance, and Kurzidem [23] used Interface-GSF (I-GSF) for representing the total power flow on each of the interfaces, when one unit of power is generated within a market area while another unit of power is consumed in a reference market area. Domestic congestion within each market area is ignored because FBMC concerns only cross-border transfer of power. Furthermore, it is built upon the existing arrangement of market areas, which has already resolved the internal congestion issues.

The use of I-GSF is practical because Transmission System Operators (TSOs) of connected market areas are able to estimate this variable without knowing the details of the transmission network located in an external market area. Nevertheless, the method too has limitations. Due to the aggregation of buses in a market area as a single node, we have difficulties in choosing the right reference bus for establishing the I-GSF matrices. The I-GSF computed using different reference bus varies, thus creating an asymmetric power flow in the network. Also, DC-OPF would compute power flow with the assumption that in the market area where the reference bus is located, power is only consumed at the reference bus. In most cases, this does not represent the real consumption of power within that area because power retrieved from buses other than the reference bus (within the same area) is not taken into account.

In view of that, the authors redefine I-GSF as the extent of changes in power flow on all interfaces caused by the generation of one unit of power in the marginal bus located within a market area, followed by the consumption of

one unit of power in the reference market area. The method does not lose its generality because market areas in large transmission networks (i.e., pan-European transmission network) may cover a substantially wide geographical region. In that case, it is reasonable to assume that the load is distributed evenly to all buses within a market area. Furthermore, the market areas are arranged such that the load at all enclosed buses does not differ significantly from the average value (especially if rural areas and urban areas form different market areas because of the transmission bottlenecks).

Let's apply the method on a case study using the transmission network model shown in Fig. 1. The model is constructed by slightly modifying the standard IEEE 30-bus test system. Each generator in the model comprises two units, with a generation capacity, and marginal cost of generation, C_g shown in Table 1. The transfer limit of transmission lines other than L12, L14, L15, L26 and L36 is infinite. The transfer limit of these lines (L12, L14, L15, L26 and L36) is considered as a binding constraint in the power flow computation (the same technique is applied in [24], where the transfer limit of some transmission lines in the model is regarded as binding constraint). The power generated is dispatched in two consecutive stages. First, 60% of the total load demand (170.04 MW) is cleared with DC-OPF computation. This forms the base case of dispatch, which represents the arrangement to supply the base load and intermediate load pre-defined in long-term markets. It is assumed that the generators do not interact strategically at this stage.

Afterwards, the remaining transfer capacity in each of the interfaces is used in FBMC. Three parameters are required in this stage of computation:

- i. The marginal generator located in each area
The marginal generator located in each area is determined after the implementation of DC-OPF at 60% total load level. A generator is selected from each area, to supply the next unit of power.
- ii. The computation of I-GSF
The I-GSF computation begins with the selection of the reference area. Basically, it either comprises one or multiple areas selected within the model. They are chosen based on the following criteria: (i.) areas possessing insufficient generation capacity, where the remaining generation capacity of the marginal generator is inadequate for supplying the local load demand; and (ii.) the area with the marginal generator that possesses the highest C_g , if all areas do not meet the former criterion. Next, I-GSF is computed by summing the total GSF created, when one unit of power is generated within each area (by their respective marginal generators) followed by withdrawal of one unit of power evenly distributed to all buses within the reference area. If the reference area consists of more than one area, the load is first proportionately distributed to these areas before the average value of load is computed for all enclosed buses.
- iii. The transfer capacity at the interfaces
The transfer capacity at the interfaces is computed by means of determining the maximum value of distributed load demand in the reference area (as in the case of I-GSF) that could be supported by the transmission network. The load demand at the reference area is incrementally increased, and DC-OPF is implemented after each increment until an infeasible solution is found (this indicates that a transmission constraint is violated). At that point, the total power flow within each interface is used as the available transfer capacity.

The marginal generators that serve the load demand in the DA market (after 60% of the total load demand is served in the base case) are shown in Table 2. The remaining load demand in areas 1, 2, and 3 is 71.48 MW, 19.4 MW, and 22.48 MW respectively. In addition, four generators are identified as marginal generators in the DA market, which are G11 (in area 1), G41 and G61 (in area 2) and G31 (in area 3). Two marginal generators are identified for area 2 because they possess similar C_g . Area 1 is selected as the reference area (in the establishment of I-GSF) because it possesses the marginal generator with the highest C_g .

The marginal generators located in area 2 could exist in three different forms, causing area 2 to exhibit different types of unknown to marginal generators in other areas. Most importantly, the actual type in area 2 is unknown to the marginal generator in area 3 which strategically interacts with it for serving the load demand in the net importing area (area 1). The types are listed as below:

- i. Only G41 is used as the marginal generator in area 2 for serving the load demand in the DA market (type *a*).

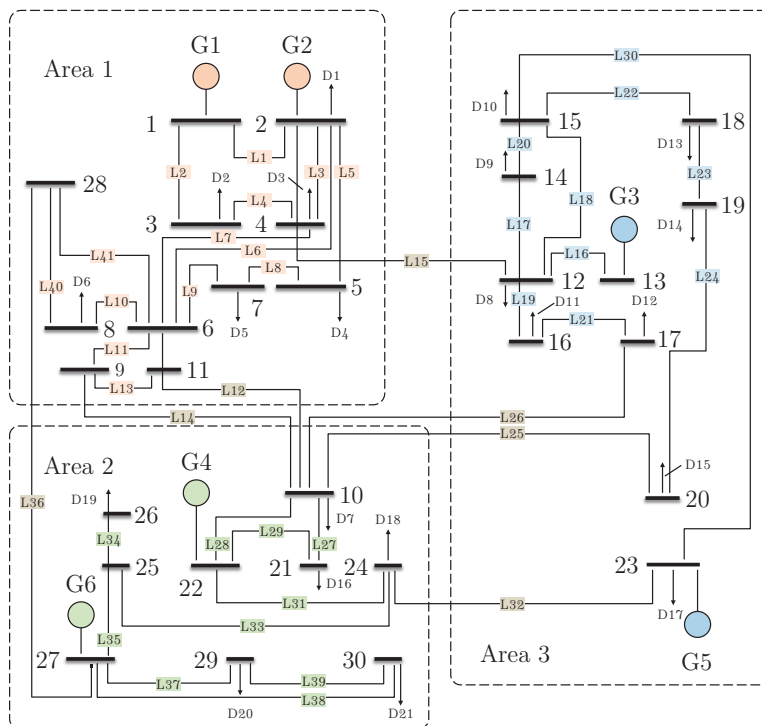


Fig. 1. A slightly modified IEEE 30-bus test system.

Table 1. Generators in the transmission network model (Fig. 1).

Name	Generator	Capacity [MW]	Cost [\$/h]
G1	1	35	55
	2	20	50
G2	1	30	55
	2	25	50
G3	1	35	25
	2	20	30
G4	1	40	30
	2	30	10
G5	1	35	25
	2	30	10
G6	1	35	30
	2	35	10

Table 2. Load served, DA load demand, and marginal generator for use in the DA market.

Area	Load served	DA load demand	Marginal unit
A1	107.22	71.48	G11
A2	29.1	19.4	G41 and G61
A3	33.72	22.48	G31

- ii. Either G41 or G61 is used as the marginal generator in area 2 (type *b*). Each of them may create different patterns of power flow on the interfaces. This is because of their relative location within the transmission network, which creates different I-GSFs.
- iii. Both G41 and G61 are used as marginal generators in area 2 (type *c*). This is possible if area 2 divides itself into two smaller local areas either for resolving internal congestion issues or for allowing both generators to simultaneously participate in the DA market. The I-GSF for this case is again, different compared with types *a* and *b*.

Based on that, we can define the strategic interactions between marginal generators in areas 2 and 3 as a Bayesian game (see Fig. 2). Conventionally, a virtual player called “nature” is included in this game, which assigns the probability of area 2 in exhibiting each of the mentioned types. The assignment depends on the first-order belief (on the type of marginal generator in area 2) owned by the marginal generator located within area 3. Let’s assume that the probability of area 2 to appear respectively as types *a*, *b* and *c* is 0.4, 0.3, and 0.3. Hence, we assume that the likelihood for area 2 in using G41 as the marginal generator is higher.

Four non-singleton information sets exist in the game, each related to the first-order belief of the marginal generators on the types of their opponents. The first information set which includes node 2 to 5 of the game tree shown in Fig. 2 relates to the generators located within area 2. The remainder of the information sets relates to the response of G31 located within area 3, based on the perceived types of marginal generators in area 2.

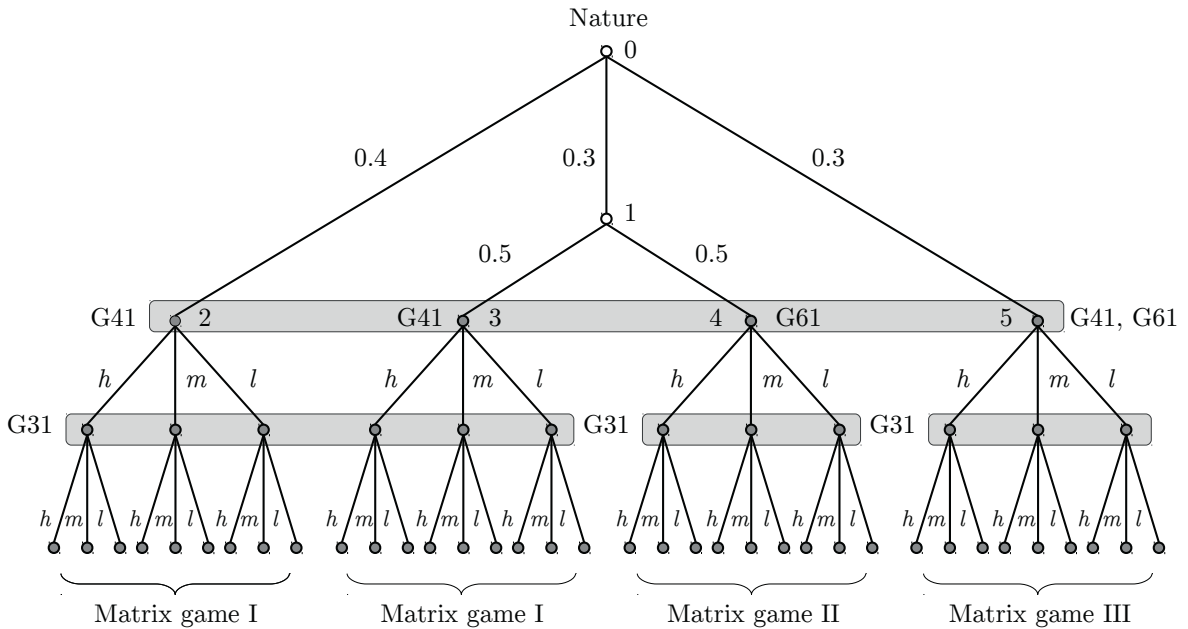


Fig. 2. The Bayesian game in cross-border trade of electric power simulated in this work.

Three matrix games (subgames in normal form) are established based on the perception of G31 on the types of marginal generators in area 2. First, the matrix game I is played when G41 is used in area 2 as the marginal generator. This happens when area 2 is perceived as either type *a* or *b*. Second, the matrix game II is played when G61 is used as the marginal generator in area 2 (type *b*). Third, the matrix game III is played when both G41 and G61 are used as marginal generators in area 2 (type *c*). In these games, the marginal generators interact with each other by adjusting their proposed C_g , so as to gain the highest payoff through the bidding process. Three levels of C_g can be proposed by each of them, which are low (*l*, equal to true C_g), medium (*m*, equal to true $C_g + 5$) or high (*h*, equal to true $C_g + 10$) values. For instance, the list of candidate C_g for G31 is {25, 30, 35}. The payoffs received by the marginal generators in

the matrix games are obtained through DC-OPF computation implemented on the approximated transmission network model. The payoff values are shown in Tables 3, 4, and 5, respectively.

Table 3. Payoffs received by G41 and G31 if marginal generator in area 2 consists of type *a* or *b*.

G41 – G31	Low bid	Medium bid	High bid
Low bid	700, 1000	700, 800	700, 600
Medium bid	525, 1000	525, 800	525, 600
High bid	350, 1000	350, 800	350, 600

Table 4. Payoffs received by G61 and G31 if marginal generator in area 2 consists of type *b*.

G61 – G31	Low bid	Medium bid	High bid
Low bid	700, 875	700, 700	700, 525
Medium bid	525, 875	525, 700	525, 525
High bid	350, 875	350, 700	350, 525

Table 5. Payoffs received by i.) G41 and G61, and ii.) G31 if marginal generator in area 2 consists of type *c*.

G41, G61 – G31	Low bid	Medium bid	High bid
Low bid	1400, 465.83	1400, 372.66	1400, 592.12
Medium bid	1050, 465.83	1050, 372.66	1050, 600
High bid	700, 465.83	700, 372.66	700, 279.5

5. Results and discussion

The Bayesian game is solved with Gambit solver [25]. In the process, it is found that a pure-strategy NE exists in each of the matrix games. These NEs happen when (i.) G41 and G31 simultaneously propose a low bid in matrix game I (Table 3); (ii.) G61 and G31 simultaneously propose a low bid in matrix game II (Table 4); and (iii.) G41 and G61 propose a low bid while G31 proposes a high bid in matrix game III (Table 5). A BE is as well identified in the game, which happens when G31 proposes a low bid in matrix games I and II (if marginal generator in area 2 is of types *a* and *b*), and a high bid in matrix game III (if marginal generator in area 2 is of type *c*). Meanwhile, the marginal generators in area 2 (G41 and G61) propose a low bidding price in all of the matrix games. When BE is reached, the marginal generators in area 2 (G41 and G61) collectively earn 910 as payoff. Simultaneously, the marginal generator in area 3 (G31) earns 858 as payoff.

Consequently, it is known that the players in the game would propose bidding prices to the DA market following the strategy suggested by BE. This is because, BE is the state where all of them would incline to converge after interacting iteratively with each other within the same market framework. This certainty is assured as long as they make rational decisions at all time. For instance, G31 receives less incentive if he deviates from the BE strategy and proposes a medium bidding price in matrix game I (Table 3). In that case, he would receive 800 instead of 1000 as payoff. In view of that, we can apply Bayesian game analysis for determining the expected payoffs received by all market areas when an equilibrium state is achieved within a proposed market design. In other words, BE can be used as a criterion for quantifying the impact and performance of a market design option. As in the case study, it is clearly shown that the expected payoffs received by the generators within areas 2 and 3 do not differ substantially. This indicates that the market mechanism works well.

Also, BE is guaranteed to exist in all strategic games of incomplete information. This is well supported by practical reasons. BE is the state where all of the marginal generators receive the best award as they depend on their opponents for determining the power dispatch.

6. Conclusions

This paper presents the Bayesian game theory and its application in analyzing cross-border transfer of electric power. It explains how the method could be used in dealing with cross-border transfer within a market mechanism that contains market players possessing uncertain types of marginal generators. The method described in this paper is demonstrated with a case study that comprises a simple three-area transmission network implementing FBMC. It is shown that BE could be a good indicator for representing the impact and performance of a market design framework. Based on that, the authors suggest (at least in the planning stage) that the Bayesian game theory can be applied for creating baseline performance indicators that take all possible types (that may exist within external market areas) into account. These indicators can as well be used as a means for comparing market design proposals, which lead to the selection of the best possible option for implementation.

7. Acknowledgments

The authors are grateful to the Norwegian Center of Offshore Wind Technologies for funding this research.

References

- [1] S. Oruc, S. W. Cunningham, Strategic management of transmission access to the electricity grid, in: Proceedings of the Technology Management in the Energy Smart World, Portland, OR, 2011, pp. 1–6.
- [2] Booz & Co., D. Newberry, G. Strbac, Physical and financial capacity rights for cross-border trade, Tech. Rep. R01071, Booz and Company, London, UK (September 2011).
- [3] R. Gibbons, An introduction to applicable game theory, *The Journal of Economic Perspectives* 11 (1) (1997) 127–149.
- [4] J. C. Harsanyi, Games with incomplete information played by Bayesian players, part I, *Management Science* 14 (3) (1968) 159–183.
- [5] J. C. Harsanyi, Games with incomplete information played by Bayesian players, part II, *Management Science* 14 (5) (1968) 320–334.
- [6] J. C. Harsanyi, Games with incomplete information played by Bayesian players, part III, *Management Science* 14 (7) (1968) 486–502.
- [7] M. Risse, What is rational about Nash equilibria?, *Synthese* 24 (3) (2000) 361–384.
- [8] J. F. Nash, Equilibrium points in n-person games, *Proceedings of the National Academy of Sciences* 36 (1950) 48–49.
- [9] J. F. Nash, Non-cooperative games, *Annals of Mathematics* 54 (1951) 286–295.
- [10] H. W. Kuhn, J. C. Harsanyi, R. Selten, J. W. Weibull, E. van Damme, J. F. Nash, P. Hammerstein, The work of John Nash in game theory, *Nobel Seminar* (1994) 1–31.
- [11] J. von Neumann, O. Morgenstern, *Theory of games and economic behaviour*, Princeton University Press, 1944.
- [12] E. van Damme, T. W. Weibull, Equilibrium in strategic interaction: the contributions of John C. Harsanyi, John F. Nash and Reinhard Selten, *The Scandinavian Journal of Economics* 97 (1) (1995) 15–40.
- [13] R. B. Myerson, Comments on “games with incomplete information played by Bayesian players, I-III”, *Management Science* 50 (12) (2004) 1818–1824.
- [14] J. C. Harsanyi, Games with incomplete information, *The American Economic Review* 85 (3) (1995) 291–303.
- [15] S. Stoft, Using game theory to study market power in simple networks, in: *Game Theory Tutorial*, IEEE Power Engineering Society, Pasippany, New Jersey, 1998, pp. 33–40.
- [16] N. Lucas, P. Taylor, Characterizing generator behaviour: bidding strategies in the pool: a game theory analysis, *Utilities Policy* 3 (2) (1993) 129–135.
- [17] J. F. Nash, Non-cooperative games, Ph.D. thesis, Princeton University (1950).
- [18] K. Binmore, Commentary: Nash’s work in economics, *Games and Economic Behavior* 71 (2011) 2–5.
- [19] E. van Damme, On the contributions of John C. Harsanyi, John F. Nash and Reinhard Selten, *International Journal of Game Theory* 24 (1995) 3–11.
- [20] S. Ceppi, N. Gatti, N. Basilico, Computing Bayes-Nash equilibria through support enumeration methods in Bayesian two-player strategic-form games, in: *Proceedings of the IEEE/WIC/ACM Joint Conferences on Web Intelligence and Intelligent Agent Technologies*, Milan, Italy, 2009, pp. 541–548.
- [21] A. Arnauld, P. Nicole, *Logic or the art of thinking*, Cambridge University Press, 1996.
- [22] T. Krause, Evaluating congestion management schemes in liberalized electricity markets applying agent-based computational economics, Ph.D. thesis, Swiss Federal Institute of Technology (2007).
- [23] M. J. Kurzidem, Analysis of flow-based market coupling in oligopolistic power markets, Ph.D. thesis, ETH Zurich (2010).
- [24] T. Mount, L. Anderson, R. Zimmerman, J. Cardell, Coupling wind generation with controllable load and storage: a time-series application of the superopf, Tech. Rep. 12-29, Power Systems Engineering Research Center, USA (November 2012).
- [25] R. D. McKelvey, A. M. McLennan, T. L. Turocy, *Gambit: software tools for game theory*, version 13.1.0, <http://www.gambit-project.org> (2013).