



Risk Management in the Allocation of Sales for Salmon Farming Companies

Christer Nyrud and Christoffer Johansen Cock

Master of Science in Financial Economics

Submission date: 21.06.2017
Supervisor: Denis Becker

**Norwegian University of Science and Technology
Department of Economics**

Preface

This thesis represents the completion and last requirement of the 2-year Master program in Financial Economics at the Norwegian University of Science and Technology. This thesis is, in its entirety, a joint product completed by Christer Nyrud and Christoffer Johansen Cock. We would like to extend our appreciation and thanks to our supervisor Denis Becker for invaluable guidance and encouragement. In addition, we would like to thank Norway Royal Salmon represented by, CFO, Ola Loe and, Group Controller, Jan Pål Johannesen for graciously agreeing to meet with us and provide valuable insight into the operation of a Norwegian salmon farming company.

Trondheim, June, 2017

Christer Nyrud and Christoffer Johansen Cock

Abstract

This thesis describes the implementation of two new distinct decision tools for risk management for salmon farming companies. There are few tools salmon farming companies use when allocating sales between spot and forward contracts. Most decisions on the allocation of sales are based on “expert opinion” and loose policies. The aim of this thesis is to develop a decision tool which improve risk management in the allocation of sales for salmon farming companies. The focus is mainly on price risk. The thesis develops a Single-Period model in which practicability and usability has been emphasized. A framework for obtaining a policy regarding the allocation of sales between spot and one- through six-month forward contracts has been developed. In addition to the Single-Period model, the thesis develops a more complex Multi-Stage Stochastic Recourse model. The Multi-Stage model provide the salmon farming companies with a potential dynamic decision tool which can be incorporated into larger life-cycle optimization programs.

Table of Contents

Preface.....	ii
Abstract.....	iii
List of Figures.....	vi
List of Tables	viii
Introduction.....	1
The Salmon Industry	2
2.1 The Industry	2
2.2 Market Conditions in the Salmon Industry	3
2.2.1 Supply of Farmed Salmon	4
2.2.2 Demand for Farmed Salmon.....	4
2.3 Production Process and Facilities.....	5
2.4 The Fish Pool Marketplace	6
Literature Review	7
Financial Risk and Risk Management.....	10
4.1 Risk in Aquaculture and Salmon Farming.....	11
4.2 Financial Risk Management	11
4.3 Financial Risk Management in Salmon Farming	12
4.4 Discussion and Conclusion from Initial Research	13
Data Description.....	14
5.1 Salmon Spot Price.....	15
5.2 Descriptive Statistics.....	16
5.2.1 Normality	17
5.2.2 Autocorrelation	18
5.2.3 Spot Price Process.....	20
5.3 Spot-Forward Relationship.....	20
Methodology	23
6.1 Stochastic Programming	23
6.1.1 Single-Period Horizon: Policy Selection Problem.....	25
6.1.2 Recourse.....	26
6.1.3 Multi-Stage Recourse Problem.....	28
6.1.4 Scenario Trees.....	29

6.2	Portfolio Optimization and Quantitative Risk Measures.....	31
6.2.1	Modern Portfolio Theory	31
6.2.2	Risk measure	32
6.2.3	Utilized Risk Measures	33
6.3	Forecasting.....	34
Modelling		36
7.1	Data Simulation.....	37
7.1.1	Modelling Spot Prices.....	37
7.1.2	Modelling Forwards Prices	39
7.1.3	Modelling Production	40
7.2	Single-Period model and Portfolio Optimization.....	40
7.2.1	Optimization Structure.....	42
7.2.2	Using MPT on Forecasted Price Returns.....	43
7.3	Multi-Stage Stochastic Recourse Model	44
Model Application.....		49
8.1	Single-Period Horizon Model.....	49
8.1.1	Results.....	49
8.1.2	Analysis of an Unfavorable Market Scenario	53
8.1.3	Further work.....	56
8.2	Multi-Stage Model Application	57
8.2.1	Results.....	57
8.2.2	Model Completion/Development.....	59
Concluding Remarks		60
Bibliography:.....		62
Appendix A.....		68
Appendix B		70
Appendix C		74
Appendix D		76
Appendix E		81

List of Figures

Figure 5.2: 3D illustration of historic salmon price movement	16
Figure 5.3: Distribution of log-returns of spot prices with normal distribution superimposed.	17
Figure 5.4: Autocorrelation Functions: Salmon spot price.	18
Figure 5.5: Partial Autocorrelation Function: Salmon spot price.	18
Figure 5.6: Autocorrelation Function: Differenced salmon spot price.	19
Figure 5.7: Partial Autocorrelation Function: Differenced salmon spot price.....	19
Figure 5.8: Historic salmon price volatility.	20
Figure 5.9: Historic spot price, three-month forward price, and six-month forward price.....	21
Figure 5.10: absolute price differences between spot price and 6- and 3-month forward contract prices	22
Figure 6.1: Depiction of scenario tree of two-stage program (Popela et al.,2014).....	27
Figure 6.2: Depiction of scenario tree of multi-stage program (Loewe, n.d.).	30
Figure 7.1: Forecast of mean spot prices and 95% forecast intervals.....	38
Figure 7.2: Visualization of price simulation process.....	48
Figure 7.3: Visualization of decision nodes.....	48
Figure 8.1: Efficient Frontier of output from Multi-Stage Stochastic Recourse Model	58
Figure 10.1: Weekly export of salmon and a 52 week Moving Average	68
Figure 10.2: Market Threats (Bergfjord 2009.....	68
Figure 10.3: Risk assessment statements (Bergfjord, 2009).....	69
Figure 10.4: Correlation between spot, 3m forwards and 6m forwards	69
Figure 11.1: Scenario tree with numbered time periods and decision nodes.....	70
Figure 12.1: Visualization of Single-Stage Horizon Model interface	74
Figure 12.2: Overview of bank balance evolution in a set of scenarios	75
Figure 12.3: Close-up view of bank balance with risk metrics.....	75
Figure 13.1: Efficient Frontier: Expected Weekly Cash Flow with CVaR(95).....	76
Figure 13.2: Efficient Frontier: Expected Ending Bank Balance with CVaR(95).....	76
Figure 13.3: Efficient Frontier: Expected Weekly Cash Flow with Downside Deviation	77
Figure 13.4: Efficient Frontier: Expected Ending Bank Balance with Downside Deviation .	77
Figure 13.5: Efficient Frontier: Expected Weekly Cash Flow with CVaR(95) - Narrow margins.....	78

Figure 13.6: Efficient Frontier: Expected Ending Bank Balance with CVaR(95) - Narrow margins..... 78

Figure 13.7: Efficient Frontier: Expected Weekly Cash Flow with Downside Deviation - Narrow 79

Figure 13.8: Efficient Frontier: Expected Ending Bank Balance with Downside Deviation - Narrow margins 79

Figure 13.9: Efficient Frontier: Mean-Semi-deviation 80

List of Tables

Table 5.1: Descriptive statistics	16
Table 5.2: Betas.....	21
Table 5.3: Spot-Forward difference	22
Table 7.1: Result from AIC and BIC test.....	37
Table 8.1: Allocation of spot and contracts based on Expected Weekly Cash Flow using CVaR(95).....	50
Table 8.2: Allocation of spot and contracts based on Expected Ending Bank Balance using CVaR(95).....	50
Table 11.1: Result from example run of Multi-Stage Recourse Model.....	71

Chapter 1

Introduction

The salmon industry has grown into one of the most important industries in Norway. The growth of the industry coincides with the growth in prices of salmon. Throughout the 1990s the salmon price declined steadily, while from the 2000s and onward the price increased and fluctuated from peaks of NOK 45 and troughs of NOK 20. From 2015, the price increased dramatically and has seen peaks of NOK 80 and troughs of NOK 50. The increase in price from the early 2000s was also coupled with an increased price volatility (Oglend, 2013). In addition, production costs rose in the same period. The cost increase was driven by capital cost (Asche et al., 2013) and salmon lice related expenditures (Torrissen et al., 2013). The lice related expenditures are assumed to increase further, which would increase the total cost of production for salmon producers (Costello, 2009). Increased price volatility and future potential for significant increases in production cost due to lice problems has augmented the need for risk management tools to assist managers with sales decisions. Specifically, how to handle the spot price risk involved in selling salmon products.

The authors found that several optimization programs have been developed in previous research. The previous developments consider the optimization of life cycle production in the salmon industry, but most gloss over the sales and price modelling. The previous developments do either, not deal with the distinctive nature of forward contracts with different lengths or do not deal with the stochastic nature of price formulation at all. The motivations for this paper is to address the shortcomings of previous research and add to the application possibilities of said research. The objective of this paper is to create a decision tool for managers to use in allocation decisions between the sale of salmon over spot or forward contract of differing lengths. The paper will create two distinct models. One model is a policy decisions model set up as a single-period horizon model with two distinct objective functions for expected weekly cash flow and expected ending bank balance. The motivation for the development of the Single-period model is to create a functional tool that can easily be utilized by any salmon farming company. The second model will address the stochastic nature of price movements directly through a multi-stage stochastic recourse model from where the aim is to optimize revenues subject to a risk metric. The Multi-Stage model will take advantage of the value-adding application of the recourse option.

The thesis consists of nine chapters. Chapter 2 gives an introduction to the basic information regarding the salmon industry. Chapter 3 will follow with an overview of relevant literature regarding the salmon industry and methods. In chapter 4 an exposition of financial risk and risk management in terms of the salmon industry will be examined followed by chapter 5 where a description of the data used in this thesis will be outlined. Chapter 6 will detail the methodology used for the construction of both models followed by chapter 7 with a detailed description of the models. The application of the models will follow in chapter 8 with discussion of the results and suggestions for improvements. Chapter 9 consists of concluding remarks regarding the thesis.

In this thesis salmon farmers refers to anyone selling salmon using prevailing spot prices and forward prices. Throughout this thesis “forward contract”, or simply “forwards”, will be used to represent both forward contracts and future contracts. The difference between the two types of contracts are that future contracts are standardized forward contracts which can be traded financially. The thesis is based on Atlantic Salmon which throughout the thesis is referred to as “salmon”.

Chapter 2

The Salmon Industry

“Fish farming holds tremendous promise in responding to surging demand for food which is taking place due to global population growth” (“FAOs Director-General,” 2009). FAOs statement substantiate the growing importance of the salmon farming industry and accordingly its rising profitability. Over the last decade, salmon has grown to become one of Norway’s main commodities for export. In section 2.1 the industry with respect to the history of production and exports, and supply and demand of salmon is described. Section 2.2 review the salmon farming market, to give an insight to some of the drivers in the industry. Section 2.3 explore the operations of salmon farming companies more thoroughly. Finally, section 2.4 explores the sales side, specifically with focus on the Fish Pool marketplace.

2.1 The Industry

Experimental salmon farming began in the 1960s. However, it was not until the 1980s it became an established industry in Norway. In the 1990s Chile followed. Today, the largest global exporters of Atlantic Salmon in volume are Norway, Chile, Scotland, Canada and Faroe Islands.

Not many regions have the natural conditions for salmon farming. Regarding production, the water must be cold with temperature varying between 8°C and 14°C. In addition, the coastline must be sheltered and the biological conditions in the ocean must be optimal (Salmon Farming Industry, 2016).

As figure 2.1 illustrates, it has been high growth in export volume over the last 36 years. From 2012, the yearly growth stagnated and export has been fluctuating between 900.000 to 1.000.000 metric tons per year. Despite this, revenues from salmon farming have increased significantly over recent years due to record high prices for the commodity. Norway exported salmon for 61.4 billion NOK in 2016, a twenty-nine per cent increase from 2015. At the same time export volume fell by 5.2 per cent from 2015. (“Laks- og ørreteksporten,” n.d.)

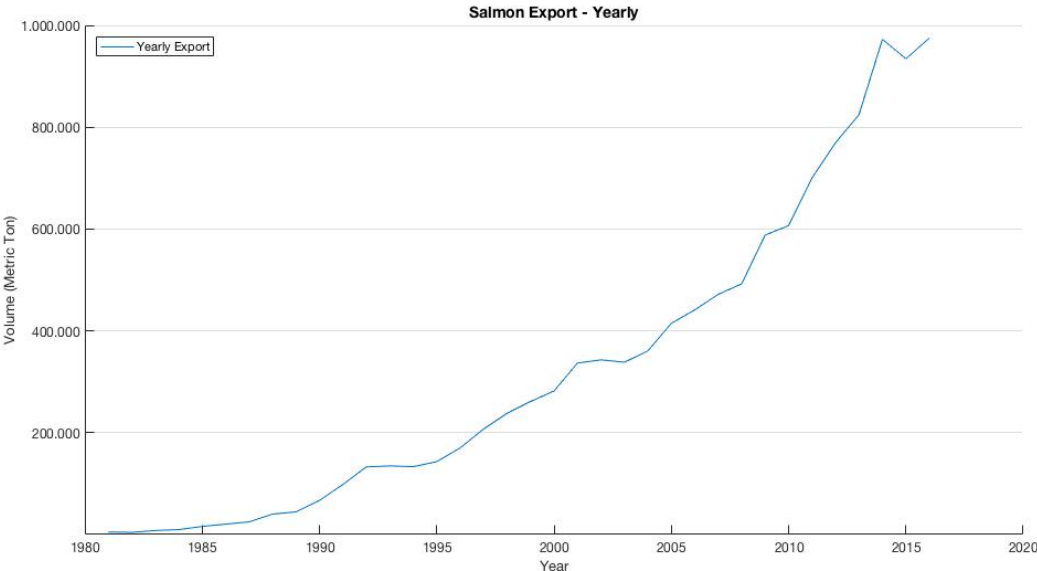


Figure 2.1: Yearly export of salmon

Figure 10.1 in Appendix A illustrate the variation in export throughout each year.

2.2 Market Conditions in the Salmon Industry

Economic theory describes supply and demand as the price determinants in a competitive market. Salmon is broadly considered to be homogenous product, despite some variation in size and quality segments.

2.2.1 Supply of Farmed Salmon

The growth in global production has been strong over the last twenty years. Globally, the total harvest quantity of Atlantic salmon has experienced a cumulative annual growth rate (CAGR) of seven percent since 1996 (“Salmon Farming Industry,” 2016) However, the growth rates are declining. CAGR was six percent from 2004 to 2015, and is projected at three percent from 2015 to 2020.

As previously mentioned, production is limited by the fact that few locations are suitable and optimal for salmon. This set a constraint on the supply. In addition, salmon farming companies experience other challenges that inhibit growth in production. Total production is now on a level where biological boundaries are being pushed. Lice and other deceases regularly cause extensive problems, resulting in mass slaughtering and lost profits, not to mention the impaired animal welfare. The Norwegian fish authorities have introduced strict regulations upon the industry and potential growth is closely associated with to which degree companies acquire licenses with rigid terms from the government (“Salmon Farming Industry,” 2016). Asche and Tveterås (2007) suggested that the long run supply elasticity was 1.5. However, government regulations combined with the long production cycle of salmon, results in a price elasticity close to zero in the short term. Production is expensive, complex and difficult to adjust. (Andersen & Tveterås, 2008)

2.2.2 Demand for Farmed Salmon

With the world population estimated at 9.7 billion in 2050, the need for increased global food production is an issue on which it is imperative to find a solution (“FAOs Director-General,” 2009). Farmed salmon is a highly efficient source of protein and may be a vital part of the future food production (“Protein production facts,” n.d.). In addition, salmon is recognized as a healthy meal choice, with high levels of Omega-3 and other vitamins and minerals. Health authorities worldwide encourage people to eat more fish and are increasingly promoting policies with advantages for healthier food (“What is salmon farming,” n.d.). Consequently, the demand for farmed salmon is increasing. In a master thesis from Lodhi (2015) results exhibited a price elasticity of -1.14 for fresh Norwegian Salmon to the EU. Interestingly, results on frozen salmon was significantly different (-0.39).

Historically, the key markets for Norwegian exporters have been the EU, Russia and Asia. The EU and North America are the largest markets globally. Transportation cost and time is a key

factor in exporting salmon, as most of the salmon sold is fresh. Closer markets are therefore preferred by producers, with some exceptions. The transport distance to Asia is approximately similar for all the producing regions and is therefore served by both Norway and Chile (“Salmon Farming Industry,” 2016). The global market has grown by CAGR of 8.2 per cent over the last twenty years. Asia and emerging markets have shown the strongest growth over the last ten years, with CAGR 19 per cent for Brazil for example (Salmon Farming Industry, 2016).

2.3 Production Process and Facilities

The companies with the largest salmon production in Norway are Marine Harvest (254.800), Salmar (136.400), Lerøy Seafood (135.000) and Cermaq (58.000), Norlaks (39.000) and Nova Sea (37.400). Top ten companies produce 70 percent of farmed salmon in Norway. All figures are in tones gutted weight equivalent (GWE). There has been an increase in consolidation in the industry during the last decade. However, Norway has a more fragmented industry compared to Chile due to policies of decentralized structures and local ownership from the Norwegian government (“Salmon Farming Industry,” 2016). Hordaland is the industry’s main county, with a high allocation of sea farming and smolt production licences and offices for many of the largest companies. Other key areas include Sør-Trøndelag, especially Frøya, followed by Nordland (“The Norwegian aquaculture,” 2016).

EY’s Norwegian Aquaculture Analysis 2016 define the salmon farming value chain as presented in figure 2.3. The value chain is complex and therefore the steps will give a rough impression of the process.

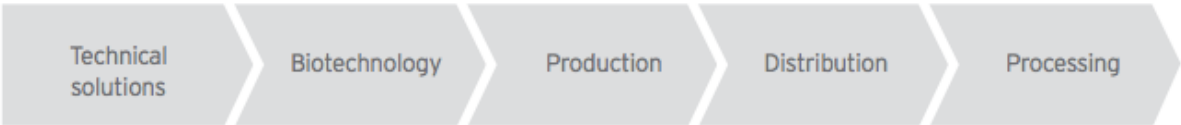


Figure 2.2: Salmon farming value chain (The Norwegian Aquaculture Analysis, 2016)

The supply side is essentially differentiated between biotechnology suppliers, technical solutions suppliers, and distributors. Biotechnology suppliers deliver feed, medicines, vaccines, and a large variety of other products, many related to fish health. Technical solutions suppliers are a key part at every stage in the value chain and deliver products such as barges, feeding systems, well boats, cages and software. Distribution include transport of smolt and transport

of harvestable fish. In addition, the farmed salmon is distributed to end consumers by exporters (“The Norwegian aquaculture,” 2016).

The production cycle of salmon farming is approximately three years from eggs to processing. First, the eggs are developed in to spawn. Next, the eggs are fertilized and the fish grow in controlled environments until it becomes smolt with a weight of 60 to 100 grams. This stage takes place in freshwater and the timeline is usually 10 to 16 months. In recent years, many companies have changed smolt production, adding more time to the phase in freshwater. Consequently, the smolt grows larger onshore, typically between 100 and 1000 grams. The risk for sea lice and other illnesses which is a substantial problem for the production in seawater is therefore reduced (“The Norwegian aquaculture,” 2016). The salmon must be transported into seawater cages for the next phase of production, which happen mainly twice a year in Norway. Within a period of 14 to 24 months the salmon grow in these cages and growth is heavily dependent on sea temperatures. Harvesting typically takes place when the weight reaches four to six kilograms and harvesting volume is approximately even throughout the year. The salmon is transported to shore, where it is slaughtered and gutted at primary processing plants (“Salmon Farming Industry,” 2016).

Primary processing concerns to the process which prepares the fish to be sold whole. It is then sold by weight measures such as Gutted Weight Equivalent (GWE), Head-on-Gutted (HOG) or Whole Fish Equivalent (WFE). Secondary processing produce value-added products and mainly refers to filleting, filet trimming, portioning and smoking. (“Salmon Farming Industry,” 2016: “The Norwegian aquaculture,” 2016).

2.4 The Fish Pool Marketplace

In 2006, several different marketplaces for financial trade of salmon was established. However, the only marketplace still in operation is Fish Pool. Fish Pool operates as a platform where salmon farmers can go to buy or sell contract for risk hedging or for speculative purposes. The products used at Fish Pool are bilateral contracts, cleared contracts, and options. Bilateral contract on Fish Pool are contracts where Fish Pool matches buyer and seller which then negotiates a contract. Cleared contracts are futures contracts where Fish Pool, in cooperation with the clearinghouse Nasdaq Clearing, acts as an intermediate and eliminates the counterparty risk. The option contract available at Fish Pool is an Asian option where the buyer can hedge the possibility of the price either going above or below a certain price level. Lattanzio (2015)

noted that the option market at Fish Pool suffer significant liquidity problems which would indicate a low application from the side of the salmon farmers. Salmon farmers do, however, take advantage of the bilateral contracts and cleared contracts provided by Fish Pool.

Chapter 3

Literature Review

There are several papers covering stochastic optimization of production and sales allocation. However, specific research into sales of salmon using stochastic spot prices versus sales at different forward contract lengths is still lacklustre, in both the use of Multi-Stage Stochastic Recourse modelling and Single-Period modelling. The aim of the literature review is to discover techniques and methods for addressing the modelling of sales of salmon at spot and forward contracts.

Previous research into stochastic modelling of the process of harvesting and selling salmon have showed that the value of using a stochastic multi-stage recourse model “is substantially higher” than using two-stage modelling or deterministic modelling (Hæreid, 2011). Hæreid, a master thesis, also concluded that the two-stage model used in his paper did not perform significantly better than the deterministic model. Hæreid uses stochastic programming to account for the myriad of uncertainties facing salmon producers. Hæreid addresses the planning a salmon farmer faces in terms of harvesting and sales. The model created in Hæreid addresses the stochasticity of biomass and uses a fixed spot price above the FHL, now Norwegian Seafood Federation, spot price.

Frøystein & Kure (2013) used stochastic optimization to model salmon production by minimizing the total expected cost related to smolt production. The approach is similar to that of Hæreid (2011) but focuses solely on the minimization of the expected cost related to smolt production. Inspired by Hæreid (2011), Denstad, et al. (2015) expanded Hæreids stochastic model to include more aspects of the value chain. Among the inclusions in the model are processed products and inventory management, as well as stochastic spot prices and fixed contract mark ups.

Research regarding stochastic optimization and the use of spot prices and forward contracts is present in several industries beside the salmon industry. An industry where there has been

significant research conducted of both topics is the energy industry. In their 2010 paper Kettunen et al. (2010) created a multi-stage stochastic optimization approach to help electricity retailers manage their contract portfolios. Kettunen et al. (2010) found, as Hæreid (2011) did for salmon producers, that “*stochastic optimization can be more efficient for risk management than periodic optimization or fixed allocation approaches.*” In line with Kettunen et al. (2010) and Hæreid (2011), Fleten et al. (2002) found that stochastic programming implementation have significant upside. Fleten et al (2002) created a stochastic programming model with the goal of coordinating electricity generation resources and risk reduction. The created stochastic optimization model included risk aversion, contract trading, and electricity operating decisions. Fleten et al. (2002) concluded that, in their example, risk could be reduced by 32%, with the same level of return, and they could increase returns by 1.1% for the same level of risk.

Topaloglou et al. (2008) studied the performance of dynamic stochastic models with regards to international portfolio management. The study showed that when including the possibility for recourse action at stages throughout the model the expected return increased significantly and it also contributed to higher stability of returns. The greatest effect of the recourse option was found in areas with the highest risk which indicate that recourse action is only marginally better and that for actors that have risk reduction/elimination as their main objective the potential cost of developing and using stochastic recourse modelling might outweigh the marginal benefit.

There are several risk measures that can be applied when evaluating stochastic optimization problems. Bjorgan et al. (1999) argues that one possible solution for risk management for energy producers can be the use of efficient frontiers to find the optimal or preferred portfolio of contracts. Efficient frontiers can be created by using several different risk measures, such as standard deviation, semi-deviation, downside deviation (Ogryczak & Ruszczyński, 1999), and Conditional Value at Risk. Rockafellar and Uryasev (2002) stated that the CVaR as a risk measure outperform other risk measures such as VaR and Variance. Mo et al. (2001) proposed an expansion of the modelling procedure in terms of risk management for hydro power scheduling and contract management. The standard before Mo et al. (2001) were to separately handle operation scheduling and contract management. Mo et al. (2001) proposed to use an integrated model that could model both tasks simultaneously. The risk measure used in the paper is a form of downside deviation. The authors set revenue targets from which they punished negative deviation. If the objective function fails to deliver a profit for the given period that is above the specified revenue target for the end of the period the deviation from that target

is classified as the risk. The punishment of negative deviations implicitly defined revenue utility function. Deviation from revenue target is a very good measure of risk in stochastic programming problems (Tauer, 1983).

In stochastic optimization where prices behave stochastically one need to be develop a forecasting model that models the price movements accurately. Guttormsen (2008) studied forecast methods for forecasting salmon spot prices. The study could not definitely determine a preferred forecast model but noted that several models performed well. Solibakke (2012) found that the variance of the salmon forward price exhibit stochastic volatility, meaning that the variance is time-varying. The found regarding the forward contracts of salmon can be taken to mean that spot prices of salmon also exhibit stochastic volatility. Similar to Solibakke (2012), Oglend (2013) found that the volatility of the salmon price has a time-varying mean. Specifically, the volatility has had an increasing trend since the start of the 2000s which was confirmed by modelling conditional variance of the price returns by a GARCH model.

There is some research on the use of contracts in the salmon industry. Larsen and Asche (2011) argues that the use of *“fixed price contracts primarily changes the profile of revenue flows”* and not the long-term revenues. The data used by Larsen and Asche was from the year 2006 which was the year with the highest price volatility up until 2011 (when the paper was written). One point to be made regarding Larsen and Asche (2011) is the limited timeframe of the study and the increased volatility exhibited over long periods of time in periods after the study, specifically after the 2010, might change the outcome of the study if the study were conducted now and with a larger timeframe in mind. The use of forward contract as a hedging tool was studied in Misund and Asche (2016). The results indicate that there is significant risk reduction potential in the use of forward contracts. Misund and Asche (2016) found that the risk reduction potential in the salmon market is higher than other seafood markets, but lower than agricultural markets. The use of forward contracts can potentially reduce the risk of salmon producers is the sales phase approximately 30-40%.

Liu and Wu (2007) constructed a portfolio optimization problem using Modern Portfolio Theory for optimal selection of sale of electricity in the electricity market using spot prices and bilateral contracts. Liu and Wu (2007) demonstrate the construction of a portfolio of contracts and spot sales. The main advantage of the use of MPT is the ease of understanding and its applicability. Martínez-de-Albéniz and Simchi-Levi (2006) conducted research into mean-

variance trade-off in supply contracts for manufacturers. Even though the research is focused on the purchase of inputs to be turned into finished products, known as the newsvendor problem, the methodology is interesting for pure sales situations as well. The situation described in Martínez-de-Albéniz and Simchi-Levi (2006) would be very similar in a larger production problem. Martínez-de-Albéniz and Simchi-Levi (2006) showed that there is an efficient frontier that is connected by the maximum expected return portfolio and the minimum variance portfolio. Every buyer (seller) would select a portfolio located on the efficient frontier.

Based on the available literature the best approach would seem to be to focus on the use of stochastic programming. According to previous research the best model to use in stochastic programming seem to be a multi-period model, followed by a single period and two-stage model, respectively. There are several different risk measures available and previous research suggest a handful of useful risk measures which include Value at Risk, Conditional Value at Risk, standard deviation, semi-deviation, and downside deviation. There seems to be little to no research into the actual allocation of the sale of salmon over spot and forward contracts of different lengths.

Chapter 4

Financial Risk and Risk Management

Section 4.1 will outline the key risk factors in aquaculture and salmon farming with emphasis on price risk. As mentioned in chapter 2, it was not until 2006 that a forward market was introduced, and the use of financial instruments in salmon farming companies are still, to a degree, limited. In section 4.2 the field of ‘Risk Management’ will be introduced and the utilization of forwards and hedging methods in salmon farming will be discussed. Risk management is an established discipline within numerous industries. Extensive research has been published and various methods exist for managing both risk in general and price risk for commodities. However, within the salmon farming industry, financial risk management is a relatively immature field. Attitudes within the salmon farming companies are treated in section 4.3 and the path towards enhanced quantitative risk management for financial risk is explored. A short conclusion follows the review and two risk models will be introduced in section 4.4. The ambition for the models is to provide enhanced quantitative measures for handling risk within salmon farming companies, in line with the objective for this paper.

4.1 Risk in Aquaculture and Salmon Farming

“Risk is defined as uncertain consequences, usually unfavorable outcomes, due to imperfect knowledge. Hazards are tangible threats that can contribute to risk but do not necessarily produce risk. In aquaculture, the hazards can be broadly classified as production threats or market (or economic) threats” (Kam & Leung, 2008).

Production threats generally refer to threats that potentially give adverse impact on the saleable commodity, e.g. asset or equipment failure, environmental conditions or diseases. To mitigate production threats, companies rely heavily on the experience and knowledge of the personnel running the production. Market threats refer to factors such as regulations, sales price and prices of inputs (Kam & Leung, 2008). Decreasing commodity prices will lower revenues from sales and rising prices on input will diminish margins. Further examples of market threats are included in Figure 10.2 in Appendix A. In a survey from Bergfjord (2009), Norwegian salmon farming companies were asked questions with the intention of mapping risk attitudes and risk measures in the industry. Ideally, the survey would be more recent, but it still provides valuable information on many interesting and relevant aspects for risk management considerations today. First, Bergfjord asked the companies how they perceived the importance of various risk factors, with a scale of one through seven, one meaning “not important” and seven meaning “extremely important”. Results are included in table 10.1 in Appendix A. “Future salmon price” is perceived as the most important risk factor, with a score of 5.95. The second most important factor is “Uncertainty about market access/trade policy” (5.39) and the third is “Diseases” (4.97). The conclusion is clear: Salmon price is perceived as the most important risk factor by a relatively high margin. Furthermore, the standard deviation from responses on this factor is low. These results are not surprising when compared to sources of risk within agriculture. Several studies found price risk, production risk and institutional risk to be the most important sources of risk (Bergfjord, 2009).

4.2 Financial Risk Management

Quantitative Risk Management (McNeil et al., 2005) defines financial risk as “the quantifiable likelihood of loss or less-than-expected-returns”. However, no single sentence is comprehensive enough to be entirely satisfactory in all contexts. The nature of risk assessment can be qualitative or quantitative. When assessing factors with a qualitative perspective, they are often expressed in nonnumeric terms such as low, medium, high or negligible. The assessment is a logical and reasoned discussion of the relevant factors (Kam & Leung, 2008).

Quantitative risk assessment differ as monetary values can be assigned to a specific risk. It is widely believed that the value of a company can increase with appropriate financial risk management. There is extensive literature on “corporate risk management and shareholder value” within the subject of corporate finance. It is not the aim of this paper to give a full treatment of this subject, although some key relevant arguments will be presented in the following. First, risk management makes bankruptcy less likely, thus firm value can be increased by employing risk management in the presence of bankruptcy costs. A key issue is that bankruptcy costs may include liquidation costs, which can be considerable in the case of intangibles like research and development (R&D). In general, R&D spending is positively related to the use of RM (McNeil et al., 2005). Notably, salmon farming companies spend significant amounts on R&D. Moreover, there are typically adverse effects on employees, management and customer relations from increased likelihood of bankruptcy (McNeil et al., 2005). Second, risk management facilitates the realization of optimal investing. Therefore, it can lower the impact of costly external financing on firm value.

The utilization of forward and futures contracts for hedging purposes is common in various industries, e.g. in agriculture. Two fundamental features which are important for participants in a forward market are risk reduction potential and the role of future reference price, i.e. forward contracts “discover” future commodity prices (Berge, 2017)

Commodity producers automatically have a long position in the commodity, and are exposed to price fluctuation. A short position in the commodity forward- or futures contracts will hedge the position, and the extent to which the producer has secured his revenue is dependent on the number of contracts. Although risk management is meant to increase the overall value of the company, hedging itself should have a negative expected value. The motivation for such a strategy should be to reduce price risk (Kam & Leung, 2008).

4.3 Financial Risk Management in Salmon Farming

Figure 10.3 in Appendix A is from the research of Bergfjord (2009) and show the results of salmon farming companies responses on four statements regarding attitudes toward the futures market for salmon. Results show a mean on all four questions below the scale median of four. This indicate a limited interest in the futures market. In addition, the use of forward contracts receives a very low score from a question of “perceived importance of different risk management tools”, receiving only 3.18 out of 7. It appears that maintaining good liquidity

through multiple risk management tools is the most recognized risk management strategy, in line with similar research for agriculture (Kam & Leung, 2008). Furthermore, the research finds that liquidity as a risk management tool is more important for smaller companies, while larger companies naturally have more resources and can implement more advanced risk management strategies. As the survey is relatively old, it seems intuitive that forward contracts are more widely used today due to the rising activity within the industry over the last few years. However, the perceived importance of risk factors outlined in 4.1 are likely to be valid and approximately accurate today. The research concludes that fish farmers perceive themselves as moderately risk-averse and that this notion probably is correct.

An imperative question with regards to financial risk management in salmon farming is whether the two fundamental features of forward and futures markets outlined in section 4.2, risk reduction potential and reference price, are met at Fish Pool. First, as mentioned in Chapter 3, Misund and Asche (2016) concluded that the risk reduction potential in the salmon market is higher than other seafood markets, but lower than agricultural markets. A different paper from Asche et al. (2016) concluded that the use of forward and futures, in this case from Fish Pool, provide a good price hedge and that it has the possibility of becoming the standard risk handling tool in the salmon market. Second, research found inconsistent results regarding the suitability of salmon forward prices as a reference price. Results from Asche et al. (2016) indicate that Fish Pool futures does not have this feature. Conversely, research from Ankamah-Yeboah et al. (2016) indicate that some contracts may serve the role of discovering prices.

4.4 Discussion and Conclusion from Initial Research

Industry-, literature- and risk management research laid the foundation for informed choices on risk management for salmon farming companies. Although informal talks with some of the largest salmon farming companies in Norway has confirmed that use of financial instruments occur, comprehensive price risk models are yet to be incorporated into daily risk management. Consequently, the remainder of this paper will introduce, apply and discuss two financial risk management models. Most importantly, the models will be based on price risk, in line with perceived importance from industry players outlined in section 4.1. Based on the review in section 4.3, Fish Pool futures market is assumed to be sufficiently mature, and forward contracts will therefore be implemented in the models. Both models will optimize positions in salmon spot or any of the one- through six-month forward contracts. As mentioned in section 2.4 the

option market at Fish Pool has significant liquidity problems, thus options will not be included in the models.

The financial risk models differ in methodology. A key feature of first model will be simplicity in application, with steps in the method that are relatively intuitive to understand for an actor without extensive competence in finance. To achieve a dynamic model, production costs will also be included as it is a key factor for margins in salmon farming. In addition, negative bank balance is penalized to account for liquidity. The main output will be the allocation through the decision variables, with application of various risk measures to increase awareness of the relationship between revenues and risk.

As larger companies have more resources to pursue a more advanced quantitative model, the second model in this paper will contain key elements for development of such a risk management tool. The methodology used is stochastic programming which is based on the literature review. The model will use a revenue target and to account for risk it will use negative deviation from the set target, similar to the risk measure used in Mo et al. (2001) Necessary simplifications will be described and discussed.

Chapter 5

Data Description

Commodity data were extracted from Datastream and fishpool.eu. It covers historic salmon spot prices, one- through six-month forward prices, and spot prices for comparable commodities. The spot price data begins in January 2004 with weekly observations until March 2017. Forward price data begins in January 2006 due to lack of forward contracts in previous years. Salmon data is in Norwegian Kroner and data for other commodities are in US Dollar.

This chapter looks at how the data has developed historically in section 5.1 and examine salmon price characteristics. These characteristics are compared to the characteristics of similar commodities in section 5.2. Next, the relevant time series data are tested with regards to normality, serial correlation and other factors that provide an understanding of the price processes. Finally, the dynamics between salmon spot price and forward prices are investigated in section 5.3.

5.1 Salmon Spot Price

The salmon spot price development is presented in figure 5.1.

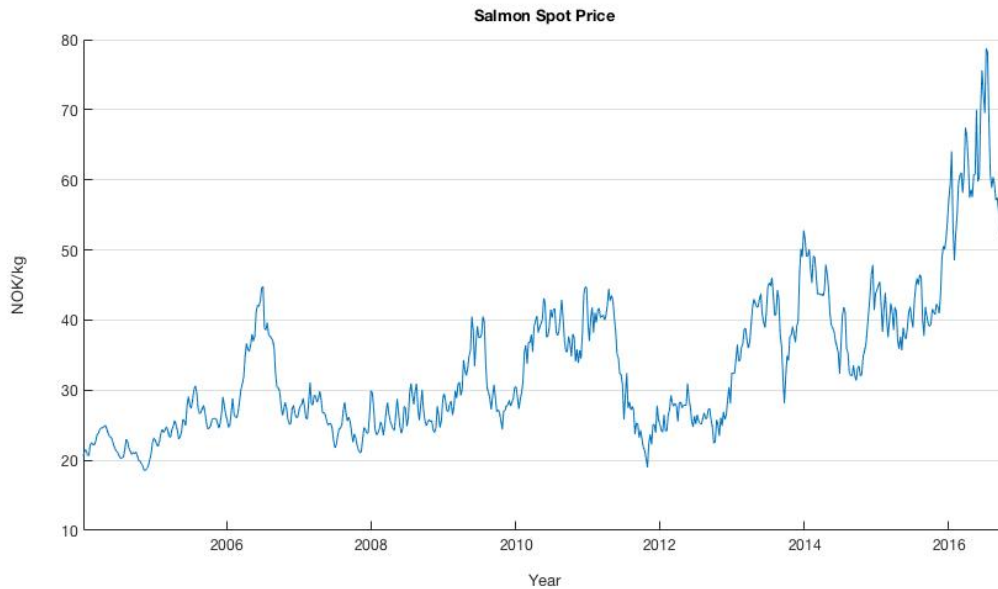


Figure 5.1: Historic salmon price movement

From 2004 to 2009 the salmon spot price was mostly in the interval 20 to 30 NOK except for one short period with higher price, peaking at approximately 45 NOK. From 2009, the spot price became more volatile and showed more distinct price shifts in the interval 20-45 NOK.

The strongest period in this time series is 2013 to present date where prices have been increasing remarkably. Figure 5.1 and 5.2 illustrate these movements in the 35-80 NOK interval with peaks at the 80 NOK threshold.

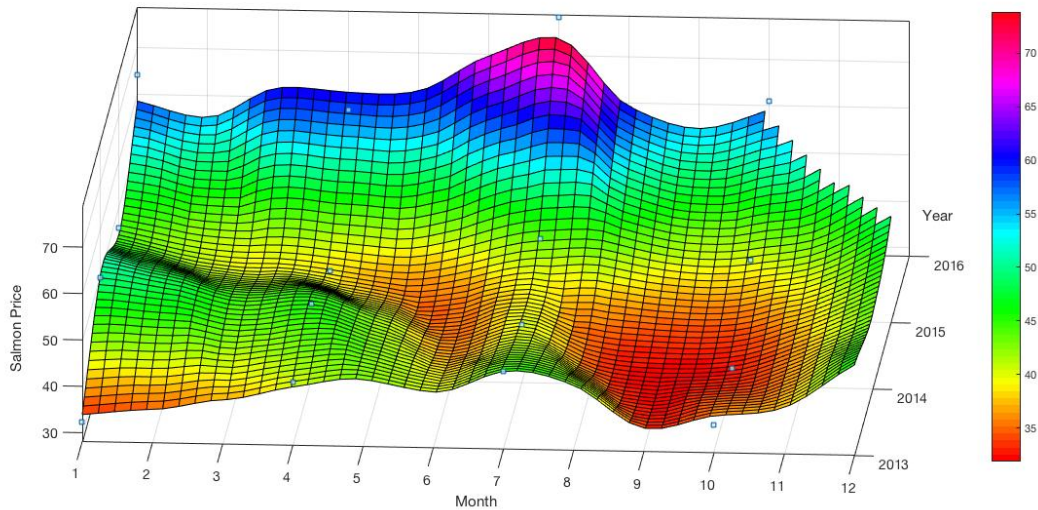


Figure 5.1: 3D illustration of historic salmon price movement

In recent years, we can see that prices tend to fall during the late part of the summer and the early part of the fall. Visually, stronger price periods seem to include mid-summer and around Christmas.

5.2 Descriptive Statistics

Table 4 displays mean, median, standard deviation, variance, kurtosis, skewness, minimum, maximum and number of observations for the various commodities and salmon forward contracts. Results on variance (0,326%) and standard deviation (5,710%) show that salmon spot is more volatile than sugar and corn. However, compared to the same commodities salmon spot exhibits less excess kurtosis and practically zero skewness. The minimum-maximum range is roughly identical for salmon, corn and sugar.

Table 5.1: Descriptive statistics

	Spot	1m fwd	2m fwd	3m fwd	4m fwd	5m fwd	6m fwd	Sugar	Corn
Mean	0,163 %	0,091 %	0,104 %	0,115 %	0,121 %	0,120 %	0,111 %	0,204 %	0,053 %
Median	0,000 %	0,000 %	0,000 %	0,000 %	0,000 %	0,000 %	0,000 %	0,161 %	0,477 %
Std. Dev	5,710 %	3,257 %	2,715 %	2,456 %	2,229 %	2,236 %	2,047 %	4,277 %	4,502 %
Variance	0,326 %	0,106 %	0,074 %	0,060 %	0,050 %	0,050 %	0,042 %	0,183 %	0,203 %
Kurtosis	0,24	3,21	3,69	3,33	3,09	4,63	5,20	1,48	1,59
Skewness	-0,02	-0,40	-0,10	-0,44	0,05	-0,52	-0,51	-0,07	-0,41
Minimum	-18,57 %	-14,35 %	-10,89 %	-12,26 %	-9,43 %	-11,03 %	-11,08 %	-17,60 %	-21,67 %
Maximum	16,51 %	13,48 %	11,86 %	8,57 %	9,77 %	7,68 %	9,53 %	17,69 %	15,86 %
Obs	667	558	558	558	558	558	558	668	668

All the salmon forward contracts exhibit less volatility than salmon spot. However, the results reveal higher kurtosis, indicating a distribution with “fatter tails” compared to the spot price. With respect to minimum and maximum the results show a significantly lower range.

5.2.1 Normality

The distribution of salmon spot returns is compared to a normal distribution in table 5.

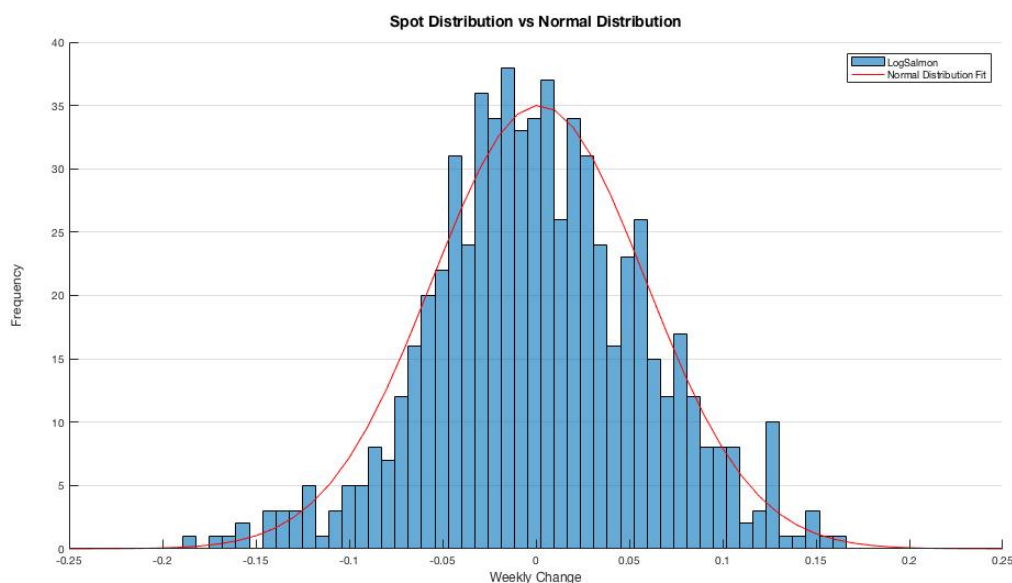


Figure 5.2: Distribution of log-returns of spot prices with normal distribution superimposed.

Normality of the returns were tested in MATLAB with Kolmogorov-Smirnov test, Lilliefors test, Jarque-Bera test and Shapiro-Wilk test. The Kolmogorov-Smirnov test reject normality of the returns for spot prices at the 5% significance level, indicating that returns are not normally distributed. However, Shapiro-Wilk test and Lilliefors test could not reject that the returns were normally distributed. An identical result was produced by the Jarque-Bera test, which tests if the data has the skewness and kurtosis that matches normal distribution. Thus, we cannot conclude on whether the data are normally distributed.

5.2.2 Autocorrelation

Salmon spot returns were analyzed with respect to autocorrelation and partial autocorrelation using 52 lags.

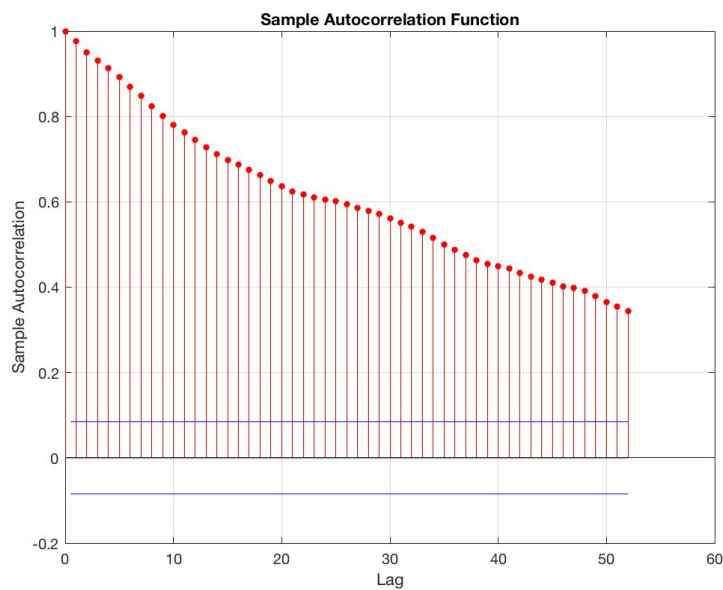


Figure 5.3: Autocorrelation Functions: Salmon spot price.

The autocorrelation function (ACF) of the salmon spot price is significant for a large number of lags. However, we want to make sure that the significance lags are not only due to the propagation of autocorrelation at some subset of lags. We do this by looking at the partial autocorrelation function.

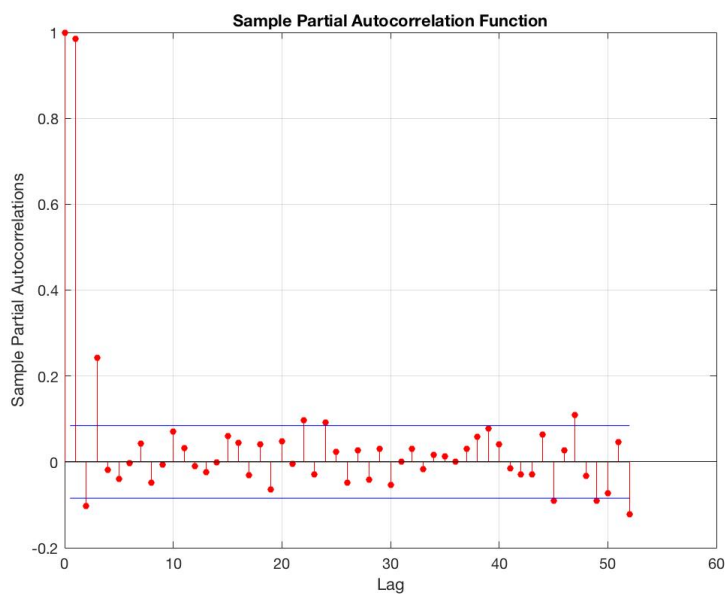


Figure 5.4: Partial Autocorrelation Function: Salmon spot price.

The partial autocorrelation function (PACF) indicate that after lag 3 the higher significant lags in the ACF is due to the propagation of autocorrelation at lag 1 through 3. The ACF and PACF looks to display an autoregressive (AR) signature where the PACF cuts off while the ACF has many significant lags. The ACF exhibit a slow linear decay which is typical of a nonstationary time series. Because of the indication of non-stationarity of the time series the data should be differenced. Below is the ACF and the PACF for the differenced data.

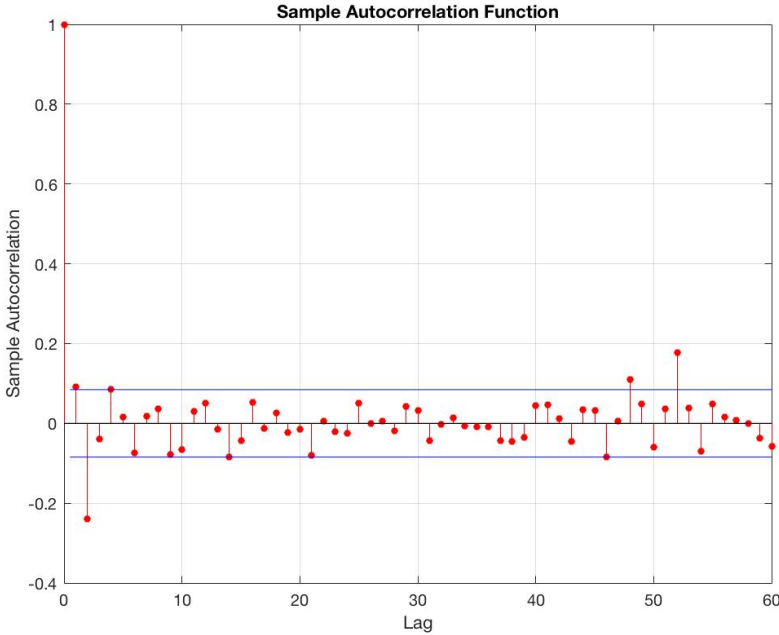


Figure 5.5: Autocorrelation Function: Differenced salmon spot price.

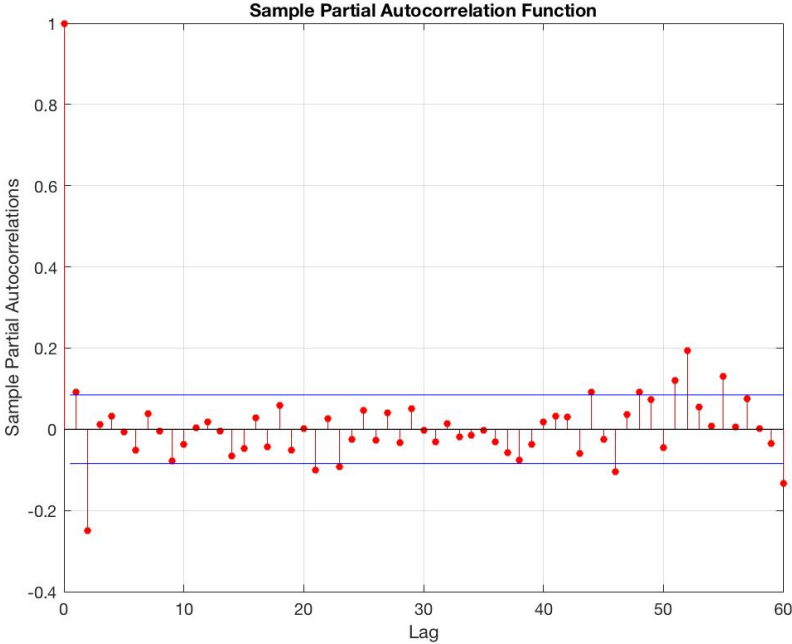


Figure 5.6: Partial Autocorrelation Function: Differenced salmon spot price.

We can observe that the non-stationarity that was indicated in the ACF of the original time series is no longer present. The time series presents indication of seasonality by having significant lags at around lag 52.

5.2.3 Spot Price Process

Figure 5.8 display the historic weekly price variations for salmon spot price:

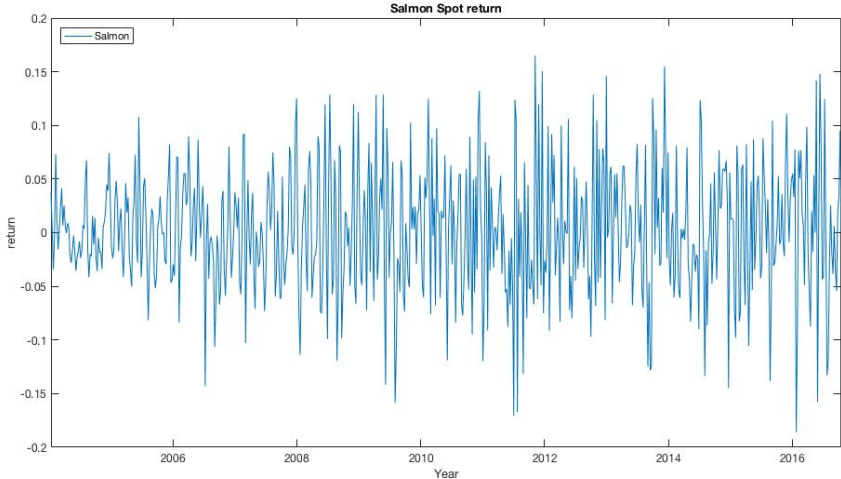


Figure 5.7: Historic salmon price volatility.

The figure gives an impression of how the volatility behaves. The data is consistent with Solibakke (2012) and Oglend (2013) findings of time-varying volatility. As in Oglend (2013), the volatility looks to have been increasing and show differences in volatility in different periods. Observation of the volatility of the price returns indicate that the price process exhibit heteroscedasticity. The Engle test and Ljung-Box-Q test for heteroscedasticity in the returns rejected the null hypothesis of homoscedasticity, indicating that there is heteroscedasticity in the residuals of the returns.

5.3 Spot-Forward Relationship

Naturally the salmon spot price and salmon spot expectations are the leading factors impacting forward price fluctuations. Table 7 exhibit three- and six-month forward contract prices compared to realized spot prices on delivery dates.

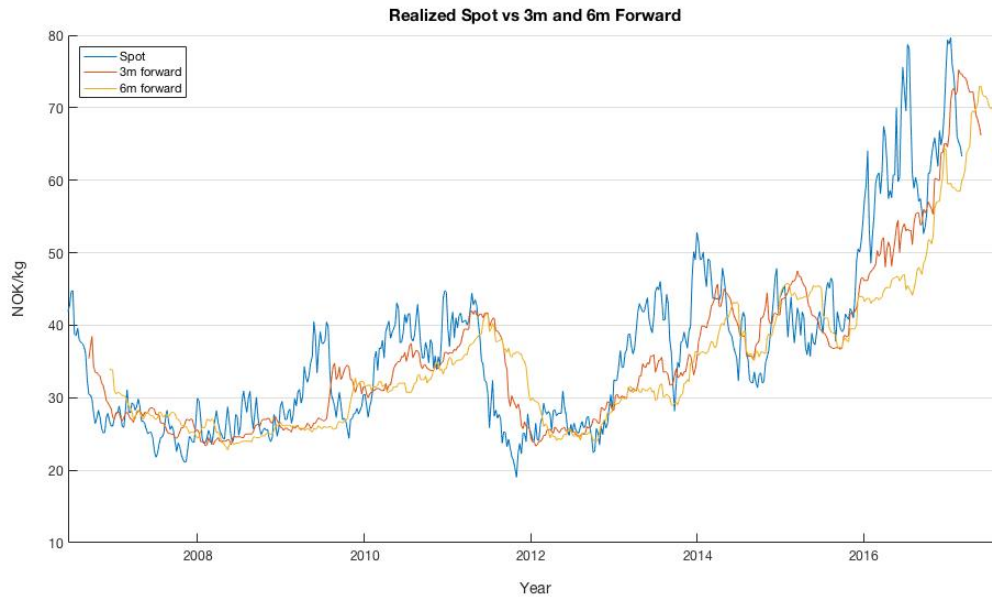


Figure 5.8: Historic spot price, three-month forward price, and six-month forward price.

Visually, the graphs indicate that the forward prices follow the spot price fluctuations relatively closely, although with a lag. The co-movement is characterized by a strong correlation, displayed in Table 0.3 in Appendix A. Furthermore, the graph clearly show that spot prices have stronger fluctuations and more ‘spikes’. As a representation of co-movement of the spot price with each forward contract we have used the beta β . The beta is calculated using classical linear regression:

$$\beta = \frac{Cov(Spot_t, fwd_t)}{Var(Spot_t)} \quad (5.1)$$

The betas for each forward contract is represented in table 5.2 below:

Table 5.2: Betas

Variables	1m fwd	2m fwd	3m fwd	4m fwd	5m fwd	6m fwd
Spot	0.9183	0.8844	0.8535	0.8315	0.8146	0.7901

Absolute price differences throughout the time series are presented in figure 5.10.

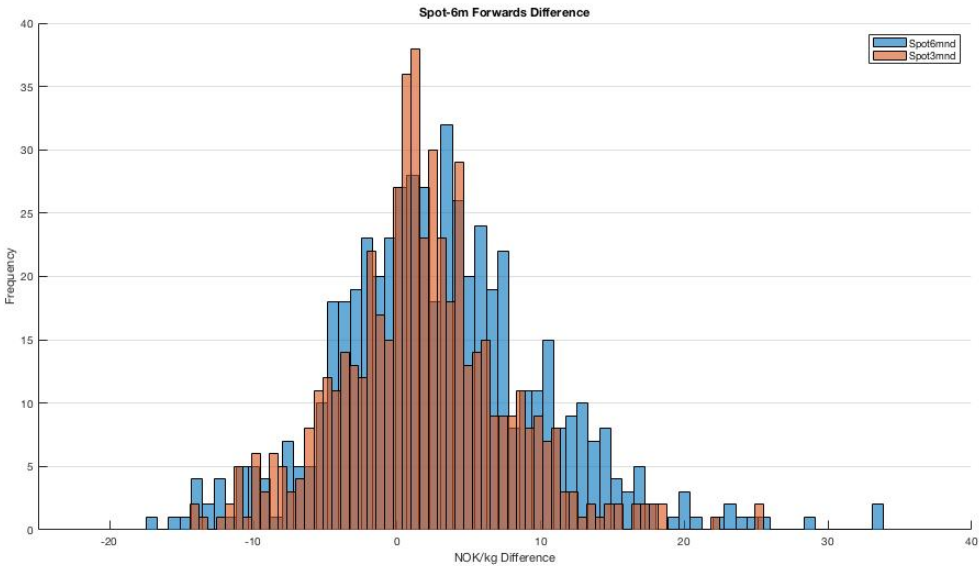


Figure 5.9: absolute price differences between spot price and 6- and 3-month forward contract prices

The distribution of price differences is distinctly broader for spot minus six-month forward contracts, which is not surprising considering weakening predicative power with increasing time horizon. Notably, the spot minus six-month forward distribution also display a higher number of ‘outliers’ in the tails, especially in positive values. The descriptive statistics in table 5.3 support the visual impression.

Table 5.3: Spot-Forward difference

	<i>Spot-3m fwd</i>	<i>Spot-6m fwd</i>
Mean	1,73	3,02
Median	1,47	2,65
Std. Dev	5,91	7,37
Variance	34,95	54,29
Kurtosis	1,10	1,21
Skewness	0,35	0,45
Minimum	-14,27	-17,31
Maximum	25,49	33,24
Obs	535	535

Standard deviation is higher when realized spot is compared to six-month forward prices and kurtosis is slightly higher. Furthermore, the results show a distinct difference for both mean and minimum-maximum range. On average the realized spot price was 3,02 NOK higher than the six-month forward price at delivery. Spot prices were 1,73 NOK higher than three-month forward prices on average. The minimum-maximum range show a wider span when the forward contracts have longer duration. In both cases the price differences are significantly higher upwards compared to downwards, but with higher upward differences for contracts with longer duration.

Chapter 6

Methodology

This chapter will introduce the reader to the most essential parts of stochastic programming as well as portfolio optimization and some applicable forecasting elements. In section 6.1 stochastic programming will be introduced. However, it is assumed that the reader is familiar with the basics of optimization and stochastic programming. For an indebt examination of stochastic programming the reader is directed to Birge and Louveaux (2011) and Kall and Wallace (1997). In section 6.1.1 we will discuss the single-stage horizon problem. Section 6.1.2 explore the features of recourse. Next, section 6.1.3 describe the multi-stage recourse problem and section 6.1.4 explain scenario tree theory. Section 6.2 will detail portfolio optimization using Modern Portfolio Theory along with some discussion regarding risk measure. Finally, section 6.3, will provide the reader with a walk-through of some applicable forecasting theory.

6.1 Stochastic Programming

The problem with managing a business, or anything for that matter, is that we need to make decisions before we have all the information that we would prefer. Because of the implication uncertainty has on future events several methods and application have been developed and designed to manage the uncertainty.

There are several decisions to be made before one even tries to deal with uncertainty. One of those decision is specifically how to model the uncertainty. There are several different methods in dealing with uncertainty. The easiest and most straightforward is the so-called deterministic approach which entails that the uncertainty variable is not included in the model

but rather is evaluated through one of the following techniques. Scenario analysis, what-if-analysis, or sensitivity analysis. In sensitivity analysis, we try to analyse the effect of changing different variables separately and without changing the fundamentals to measure the sensitivity of the outcome to changes in those specific variables which then give us an idea whether our solution is stable or unstable. In what-if-analysis, we try to analyse the effect of changing several variables where the fundamentals have changed to measure the change in outcome. In scenario analysis, we create several different scenarios based on estimation of possible outcomes. Then we analyse the solution of the model based on each scenario and combine the solutions and analyse the combined solutions based on some criteria. Based on for example mean and variance, we create a possible optimal solution (King & Wallace, 2012).

A more complex possibility to modelling uncertainty is to use stochastic programming. Stochastic programming accounts for uncertainty directly in the model formulation. In many cases the stochastic programming models would be better at dealing with uncertainty than the deterministic approaches (King & Wallace, 2012). Stochastic programming try to minimize or maximize a certain function subject to a set of constraints and by so seeking to extract a some possible solution that can serve as a policy in all future scenarios (Shapiro & Philpott, 2007). By taking uncertainty into account the decision maker might make other decisions than he would have done when disregarding uncertainty and the decisions which are made when acknowledging uncertainty most often prove to be better.

In stochastic programming, there are several different types of models. In this thesis, we will be looking at a one-stage stochastic model and a multi-stage stochastic model. A third common stochastic model, which is a special case of the multi-stage stochastic model, is the two-stage stochastic model. We will not focus on the two-stage model in this thesis and will therefore only state the essence of the model in a short manner. These models are explained in the sections that follows.

“A two-stage model is a model where the first decision is a major long-term decision, whereas the remaining stages represent the use of this investment. This could be something like building a factory for later production under uncertain demand, prices, or even products” (King & Wallace, 2012). Another variant is when someone must make a decision at stage one and afterwards there is a random event which affects the outcome of the decision. After stage one

we can add a recourse option to correct for, or take advantage of, the effect of the random event (Shapiro & Philpott, 2007).

6.1.1 Single-Period Horizon: Policy Selection Problem

A one-period horizon, or single stage, stochastic model is a model where we only have one decision stage and no recourse options. One example could be that we want to select the parameters for a specific model. For example, we want to minimize a cost function where we have some unknown event in the future that will affect the cost.

$$\zeta^* = \min_{x \in \mathcal{X}} \{f(x) = E[F(x, \xi)]\} \quad (6.1)$$

Where \mathcal{X} is the set of possible decision points and x is the specific decision. F is the cost function and ξ is the random event or information that become available after the decision. As $F(x, \xi)$ cannot be directly optimized because we have a stochastic variable we instead optimize the expectation of $F(x, \xi)$, $E[F(x, \xi)]$, which is why we have equation 6.1 above.

There are several methods for solving a single stage stochastic problem. Examples are, Sample Average Approximation, Stochastic Approximation, Response Surfaces, and Metamodels. The models have their roots from within different disciplines and therefore rest upon different assumptions about the data used in the problem. In this paper, for the single stage stochastic model, we are going to use the Sample Average Approximation method. For an overview of the assumptions regarding the problem considerations and description of the other three methods please see Hannah (2014).

The Sample Average Approximation is a method comprised of two parts. The two parts of the method is sampling and deterministic optimization. Normally the objective function cannot be directly calculated, however, the use of Monte Carlo simulations can in some cases approximate the objective function (Hannah, 2014). By using Monte Carlo simulations, we can approximate the objective function by taking the average of the realized simulations.

$$\mathbb{E}[F(x, \xi)] \approx \frac{1}{n} \sum_{i=1}^n F(x, \xi_i) \quad (6.2)$$

The n in equation X represents the number of realized simulations. The right-hand side of the equation is deterministic, and we can therefore use a deterministic optimization method to solve the approximation problem (Hannah, 2014):

$$\zeta_n^* = \min_{x \in \mathcal{X}} f_n(x) = \frac{1}{n} \sum_{i=1}^n F(x, \xi_i) \quad (6.3)$$

The restrictive assumption of a convex decision set \mathcal{X} and objective function $F(x, \xi)$ ensures that we will find the global optimum point. It is possible to relax this restriction. However, as a consequence, we would only be guaranteed to find a local optimal point (Hannah, 2014).

Normally, we have to assume that \mathcal{X} is convex and that “the objective function $F(x, \xi)$ is convex in x for any realization ξ ” (Hannah, 2014). If these assumptions are not met, we need to use a more specialized solution. We will not describe those methods here, however, we can refer the reader to Alrefaei & Andradottir (2001), Shi & Olafsson (2000), Norkin et al. (1998), Battiti & Tecchiolli (1996), and Glover & Laguna (1999) for a detailed exposition of the methods.

6.1.2 Recourse

One of the most thoroughly researched topics regarding stochastic programming deals with models with recourse. Recourse is closely related to Real Options (Wallace & King). Real options and recourse are used in many industries, if not all industries, to some extent. For example, the pharmaceutical industry uses real options and recourse during research and development where they have outlined several stages and will make a decision at each stage based on new information in terms of abandonment, increased investment, or simply a continuation of the investment (Gupta & Maranas, 2004). Other industries that real options and recourse are used include Airline industry, Oil and Gas industry, and the Utilities industry (Mun, 2006).

In a stochastic model recourse is the option to make a new decision to account for new observable information (Birge & Louveaux, 2011). In the first stage we make our initial decision while at some point before stage two an uncertain event takes place which provides us with information. In the second stage we use our recourse option and adjust our decision so as

to minimize any negative effects or maximize any positive effect of the uncertain event (Shapiro & Philpott, 2007). A depiction of the process of decision, uncertain event, and then new decision are depicted below in equation 6.4 where x is the first stage decision, $\xi(\omega)$ is the uncertain event, and $y(\omega, x)$ is the recourse decision based on information from the uncertain event (Birge & Louveaux, 2011).

$$x \rightarrow \xi(\omega) \rightarrow y(\omega, x) \tag{ 6.4 }$$

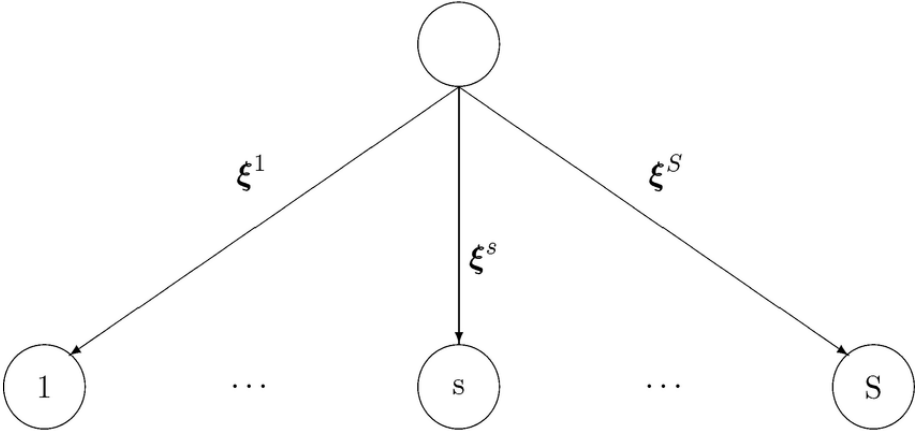


Figure 6.1: Depiction of scenario tree of two-stage program (Popela et al.,2014)

Figure 6.1 above visualize a scenario tree of a two-stage model.

A recourse model can be included in models from two-stage and up to, in theory, infinite stage modes, or by their more common classification of multi-stage models. The classic representation of a recourse model is the two-stage stochastic linear program with fixed recourse which was first proposed by Dantzig (1955) and Beale (1955) and is represented in equation 6.5 below:

$$\begin{aligned} \min z &= c^T x + E_{\xi}[\min q(\omega)^T y(\omega)] && (6.5) \\ \text{s.t} & \quad Ax = b, \\ & \quad T(\omega)x + Wy(\omega) = h(\omega), \\ & \quad x \geq 0, y(\omega) \geq 0. \end{aligned}$$

In the function the $c^T x$ is a deterministic term while $q(\omega)^T y(\omega)$ is the expectation of the second stage realization using all possible realization of the stochastic event ω . In the function we know c^T , x , A , and b at the first stage. After the first stage a stochastic event ω takes place and the variables $q(\omega)$, $h(\omega)$ and $T(\omega)$ become known. In the second stage when the stochastic variables have become know we make our second stage decision $y(\omega)$ which typically differ when the realization of the stochastic variables differ (Birge & Louveaux, 2011). The above formulation of the recourse model shows the information process directly in the decisions that are made, most often using scenario trees. This type of structure is often referred to as implicit formulation. Another formulation to the problem is to formulate a problem for each scenario and then add constraints to make sure that the information structure, coupled with the optimization process, is held. We formulate the model as such: (Higle, 2005).

$$\begin{aligned}
 & \min \sum_{\omega \in \Omega} (cx_{\omega} + q_{\omega}y_{\omega})p_{\omega} && (6.6) \\
 & \text{s.t. } T_{\omega}x_{\omega} + W_{\omega}y_{\omega} = h(\omega) \\
 & \quad x_{\omega} - x = 0 \quad \forall \omega \in \Omega \\
 & \quad x_{\omega}, y_{\omega} \geq 0.
 \end{aligned}$$

Where p_{ω} is the probability and $q_{\omega}y_{\omega}$ is the recourse part of the equation. The rest of the notation is as in equation 6.5. We see here that the distinction between the decision in the different stages in equation 6.6 is not as clear as in equation 6.5. In equation 6.6 the decision is dependent on the formulation of as a set of subproblem for each scenario. The structure equation 6.6 is in is often referred to as explicit formulation. Equation 6.6 includes a non-anticipatively constraint, $x_{\omega} - x = 0 \quad \forall \omega \in \Omega$, which ensures that decisions honour the information structure of the problem. When the non-anticipativity constraint is represented as in the explicit, or full form, equation the problem is sometimes referred to as the deterministic equivalent problem (Higle, 2005). For a detailed exposition on the deterministic equivalent the reader is referred to Wets (1974). In this thesis we will use explicit formulation for ease of understanding for the readers.

6.1.3 Multi-Stage Recourse Problem

The previous section laid out the recourse model in the classic two-stage model. However, in real life most decision problems have a long life span and have several points where new information becomes available and one can make corrective decisions. Higle (2005) describe it

as a “decide-observe-decide” pattern which forms a multi-stage stochastic recourse model. The problem in this paper is operational and continuous so we can therefore infer that our problem is “inherently multistage with, principally, an infinite number of stages as you have no plans to stop this activity” (King & Wallace, 2012).

We can generalize the information regarding recourse as detailed in the “recourse” section above to include several stages. As such, we can use the notation found in Higle (2005) to represent the general multi-stage stochastic model with recourse.

$$\begin{aligned}
 & \min \sum_{\omega} p_{\omega} c_{\omega} x_{\omega} && (6.7) \\
 \text{s.t. } & \sum_{j=1}^t A_{tj} x_j = b_t \quad t = 1, \dots, T \\
 & x_{\omega} \in X(\omega) \quad \forall \omega \in \Omega \\
 & \{x_{\omega}\}_{\omega \in \Omega} \in \mathcal{N}
 \end{aligned}$$

In this linear problem the \mathcal{N} denotes the set of non-anticipative solutions, c_{ω} is the objective function, x_{ω} is the decision variable, and p_{ω} is the probability of the uncertain event (Higle, 2005).

When the time horizon becomes very long it is advisable to include a discount factor to discount payments that are located in the far future. (King & Wallace, 2012)

6.1.4 Scenario Trees

In most cases, to find a solution for a stochastic program, one needs to discretize the probability distribution (King & Wallace, 2012). The discretization of a stochastic program along with the idea of recourse is very well represented in a scenario tree. A scenario tree is also useful when trying to understand the characteristics of the program and the non-anticipativity constraints. According to Higle (2005) a “scenario tree is a structured distributional representation of the stochastic elements and the manner in which they may evolve over the period of time represented in the problem.”

A scenario tree is represented in figure 6.2 below. The scenario tree divides into branches corresponding to different realizations of the evolution of the events (Birge & Louveaux, 2011).

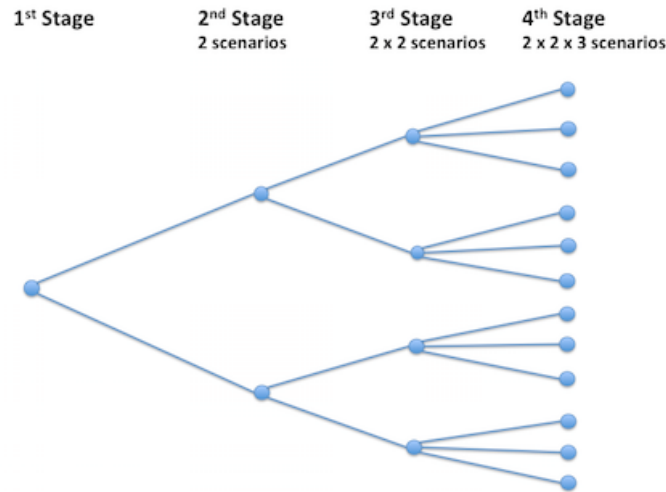


Figure 6.2: Depiction of scenario tree of multi-stage program (Loewe, n.d.).

The non-anticipativity constraint states in its simplest form that we cannot use information that we do not have and the decision must be based on the same history. When we simulate a scenario tree we technically have the information. However, in the real world we would not be able to see the realization beforehand and the non-anticipativity states the fact that we cannot utilize information revealed after the decision point.

There is often a difference between the term “period” and the term “stage”. The decisions are made at stages where new information becomes available while periods often represents time. A scenario is defined as the path from the beginning node at the start of the scenario tree through to the ending node at the end of the scenario tree. The number of branches at each node is decided by the modeller but is limited by the computational power of the software and/or hardware used to solve the problem. The branching does not necessarily need to be symmetrical, meaning that depending on the underlying problem, we can have a different number of branches both at different nodes at different stages and nodes at the same stage.

The importance of scenario generation for stochastic programming problems with the goal of real decision making cannot be overstated. King and Wallace (2005) state that the modeller “should be concerned that the solutions you are studying (and possibly implementing!) are driven not by how you make scenarios, but by the actual problem formulation.” However, the focus of this paper is to develop models / methodology to be used either in the development of a policy or in decision-making at each period for the sale of salmon over spot and forward

contracts. Keeping this in mind we will not elaborate too much on the theory of scenario generation but rather refer the reader to King and Wallace (2005) for a detailed exploration.

6.2 Portfolio Optimization and Quantitative Risk Measures

Here we are going to cover some of the theory behind portfolio optimization, specifically Markowitz optimization, and various quantitative risk measures. The classic Markowitz optimization comes from the Nobel Prize-winner Harry M. Markowitz's PhD. dissertation and article in 1952 and subsequently the publication of his book *Portfolio Selection* in 1959 and has now become what we refer to as Modern Portfolio Theory (MPT) (Markowitz, 1991). Section 6.2.1 outlines the use of modern portfolio theory. The paper will use MPT to compare the result obtained by applying Modern Portfolio Theory to the forecasted spot prices and forward prices to the result obtained from the single-period horizon model. Section 6.2.2 describes various risk metrics while section 6.2.3 details the risk metrics that are going to be used in this paper.

6.2.1 Modern Portfolio Theory

The revolutionary theory that Markowitz postulated was that there is a trade-off between risk and return in a given portfolio. In his original paper he compared the historic mean-return and the variance of those returns composed in a portfolio. When comprising several equities in a portfolio one has to calculate the mean return and the variance of the portfolio and not just for each equity. The return of a portfolio is just the weighted expected returns of each equity.

$$\sum_{i=1}^N X_i \mu_i = E \quad (6.8)$$

Where E is the expected or mean return of the portfolio, X_i is the weight of equity i and μ_i is the return of the set of equities.

The variance of a portfolio is a bit more complicated. We need to account for the covariance between each equity and the variance of each individual equity. The equation for evaluating the variance of a portfolio becomes as follows:

$$\sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij} \quad (6.9)$$

Where x_i is the weighting for equity i , x_j is the weighting of equity j , and σ_{ij} is the covariance matrix.

The main goal of the Markowitz mean-variance method is to create an efficient frontier where all optimal portfolio structures are represented. It may not be surprising then that the efficient frontier also represents a collection of all utility-maximizing functions. It can be shown that, up to second-order approximation, every utility function is represented in the efficient frontier. For a detailed explanation as to why that is the reader is referred to King and Wallace (2005) and Markowitz (1991).

The efficient frontier is developed by minimizing the risk measure for a given expected return. The normal procedure is to first calculate the mean excess return for each equity in the portfolio then calculate the covariance matrix. After the covariance matrix has been calculated we use equation 6.8 and 6.9 above to calculate the mean return and variance of the portfolio. There are several tools for optimization one can use but the most convenient for most people is to use Excel Solver to minimize the risk variable subject to a given return target. Based on several iterations of minimizing the risk measure subject to a specific return target we are able to construct the efficient frontier which represents the optimal set of portfolios for a given utility function.

6.2.2 Risk measure

There are several risk measures we can use, for example variance, standard deviation, Value at Risk (VaR), Conditional Value at Risk (CVaR), downside deviation, and semi-variance to name a few. Variance and standard deviation are measurements of volatility which gives you an idea of the distribution and dispersion of the data set. For more information regarding variance and standard deviation the reader is referred to Pfeiffer (1990). The semi-variance and semi-deviation, or downward deviation, as opposed to variance and standard deviation, does not measure the dispersion of the whole data set but only the dispersion below a certain critical value. Downside deviation is only focused on the downside risk. For further information on semi-variance and semi-deviation, or downward deviation, the reader is referred to Markowitz (1991). VaR and CVaR are two related risk measurements. Schachter (1997) define VaR as “a forecast of a given percentile, usually in the lower tail, of the distribution of returns on a portfolio over some period; similar in principle to an estimate of the expected return on a

portfolio, which is a forecast of the 50th percentile” and “an estimate of the level of loss on a portfolio which is expected to be equaled or exceeded with a given, small probability.” CVaR measures the average loss at a specified confidence level. CVaR is usually measured by taking the weighted average of the VaR and the losses that exceeded the VaR (Uryasev and Rockafellar, 2000). Uryasev and Rockfellar (2000) defines and contrasts VaR and CVaR as such “by definition with respect to a specified probability level β , and β -VaR, of a portfolio is the lowest amount such that with, probability β , the loss will not exceed α , whereas the β -CVaR is the conditional expectation of losses above that amount α .” For further details on the risk measure mentioned above the reader is referred to Uryasev and Rockfellar (2000), Sarykalin, Serraino, and Uryasev (2008), Schachter (1997), and Markowitz (1991).

6.2.3 Utilized Risk Measures

The following is the risk measured that are going to be used in the single-stage horizon model. The first risk measure is the Conditional Value at Risk. The second risk measure is downside deviation. In this paper, for the single-period model, the critical level for the downside deviation has been defined as zero. Below follows the equation for the minimization of the CVaR (6.13) and downside deviation (6.15), respectively.

$$\text{For } \alpha \in]0,1[\quad \min CVaR_{\alpha}(X) = \int_{-\infty}^{+\infty} z dF_X^{\alpha}(z) \quad (6.10)$$

$$\text{Where } F_X^{\alpha}(z) = \begin{cases} 0 & \text{When } z < VaR_{\alpha}(X) \\ \frac{F_X(z) - \alpha}{1 - \alpha} & \text{When } \geq VaR_{\alpha}(X) \end{cases} \quad (6.11)$$

$$\min E\sigma_p^d = \left[\frac{1}{S} \sum_{F_{T,s} < \bar{F}_T} (F_{T,s} - \bar{F}_T)^2 \right]^{\frac{1}{2}} \quad (6.12)$$

$E\sigma_p$ represents the standard deviation of the portfolio, $F_{T,s}$ represents the portfolio ending balance. \bar{F}_T is the critical value and S represents the number of scenarios (“Downside Deviation,” n.d.).

In addition to the above mentioned risk measures we will also use semi-variance and the related semi-deviation. Semi-deviation is represented in equations 6.13

$$\min \Sigma_{pB} = \left[\sum_{i=1}^n \sum_{j=1}^n x_i x_j \Sigma_{ijB} \right]^{\frac{1}{2}} \quad (6.13)$$

Where Σ_{pB} is the semi-deviation, x_i is the weight of equity i, x_j is the weight of equity j, and Σ_{ijB} is the semi-covariance matrix associated with the set of equities which in this paper consist of the spot price of salmon and forward contract of salmon for one through six months.

6.3 Forecasting

Uncertainty is inherent in all decisions and is a problem for many organizations and individuals. As a means to ameliorate the problems inherent with uncertainty several different forecasting techniques has been developed. Depending on the specific circumstances surrounding the desired forecast, such as available data, reason for forecast, accuracy desired, and other characteristics of the forecast, the specific forecast technique, or method, is chosen (Chambers et al., 1971). Forecasting is inherently difficult and uncertain within its own right.

Before one starts to forecast there are some questions that one should ask oneself. *“What is the purpose of the forecast—how is it to be used? This determines the accuracy and power required of the techniques. What are the dynamics and components of the system for which the forecast will be made? This clarifies the relationships of interacting variables, and how important is the past in estimating the future?”* Chambers et al., 1971). There are three common approaches to forecasting, they include the qualitative approach, time series analysis and projection, and causal methods. In this paper time series analysis with projection is going to be used. The reader is referred to the Harvard Business Review article on forecasting by Chambers, Mullick, and Smith for further details on causal methods and qualitative approaches.

A popular way of choosing the appropriate methods for forecasting a univariate time series model is the Box-Jenkins three-stage method. The three stages in the method are 1. Identification stage, 2. Estimation stage, and 3. Diagnostic stage. In the identification stage the forecaster would want to look at the data by plotting it in a time series, looking at the autocorrelation function (ACF), and partial autocorrelation function (PACF). In this first stage the forecaster is trying to identify potential problems that might affect a forecast model and correct these potential problems. The forecaster is checking if the data series has any outliers, missing values, and structural breaks. Evaluating the ACF and PACF the forecaster can get compare several plausible models to that of several theoretical ARMA models. One would also

like to check the stationarity of the time series. One can check for stationarity using the sample correlogram. However, a more precise way to establish whether or not a time series is stationary is to use the Dickey-Fuller test. The Dickey-Fuller test tries to establish if $\alpha_1 = 1$ in the equation $y_t = \alpha_1 y_{t-1} + \varepsilon_t$. One starts by subtracting y_{t-1} from each side of the equation to get $\Delta y_t = \gamma y_{t-1} + \varepsilon_t$ where $\gamma = \alpha_1 - 1$. We then test the hypothesis $\gamma = 0$ which is that the time series contain a unit root which indicate that the time series is nonstationary. One can apply the Dickey-Fuller test by using OLS and evaluating the t-statistics of the resulting data with the associated critical value found in a Dickey-Fuller table we can determine whether to reject the null hypothesis or not. Most econometrics packages today come with a built-in Dickey-Fuller test for easy evaluation. The Dickey-Fuller test can also account for deterministic elements such as a constant or intercept and linear time trends as shown below in equation 6.14 through 6.16 where equation 6.14 is a pure random walk, equation 6.15 have an intercept or drift included, and equation 6.16 has both an intercept or drift and a linear time trend (Ender, 2014).

$$\Delta y_t = \gamma y_{t-1} + \varepsilon_t \quad (6.14)$$

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \varepsilon_t \quad (6.15)$$

$$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 + \varepsilon_t \quad (6.16)$$

We can also use a so-called Augmented Dickey-Fuller test which can account for higher-order equations. For further information of the Augmented Dickey-Fuller test the reader is referred to Ender (2014).

In the second stage the forecaster fit a selection of potential models to the data and evaluate those models on a set of criteria. The goal would be to select a “*stationary and parsimonious model that has a good fit*” (Ender, 2014). R^2 and the average of the residual sum of squares are common measures of goodness-of-fit for OLS. However, the goodness-of-fit tend to improve by adding more parameters to the model. As a goodness-of-fit evaluation that accounts for the importance of parsimony the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC) are good models to capture the overall goodness-of-fit of the model (Ender, 2014). The AIC and BIC formulas are as follow:

$$AIC = T \ln(\sum_{i=1}^n r_i^2) + 2n \quad (6.17)$$

$$BIC = T \ln(\sum_{i=1}^n r_i^2) + n \ln(T) \quad (6.18)$$

Where n = number of parameters estimated ($p + q +$ possible constant term) and T = number of usable observations (Ender, 2014).

One would want to choose the model with the smallest AIC and BIC. In cases where the AIC and BIC gives contradiction opinions one has to keep in mind the purpose of the model. AIC is said to be better for choosing among models for forecasting purposes. Studies show that AIC and BIC has advantages over other model selection methods such as the hierarchical likelihood ratio test. For details regarding the advantages the reader is referred to Posada & Buckley (2004).

The third stage of the Box-Jenkins method is evaluating the chosen model. There are several ways to evaluate a model. A common practice is to plot the residuals to look for outliers and possible periods where the model does not fit the data well. There are several tests developed to measure how good the forecasts are. Some of these tests include the Mean Square Prediction Error, the Granger-Newbold test, and the Diebold-Mariano test. For a detailed exposition of these test please see Ender (2014). The main purpose of this paper is not to create a good forecast for salmon price but rather to create a set of plausible forecasts to be used in the model creation and evaluation.

Chapter 7

Modelling

We are going to explain the modelling of the two models, the Single-Period model along with the portfolio optimization used and the multi-stage stochastic recourse model. In section 7.2.1 the forecast model for the spot prices will be detailed followed by section 7.2.2 with the forecast model for the forward prices, and section 7.2.3 with the modelling of salmon production. In section 7.2 the One-Period Horizon model with accompanying objective function as well as the use of MPT will be detailed. Lastly, in section 7.3 the multi-stage model with accompanying objective function will be explained.

7.1 Data Simulation

7.1.1 Modelling Spot Prices

Neither spot prices nor forward prices are based solely on previous data points. However, since the forecast model is not the key part in this paper and to keep the forecast model simple enough to be practical we have decided to use an autoregressive ARIMA model with GARCH innovations. We found, as Solibakke (2011) discovered, that the data exhibit time-varying volatility indication a stochastic volatility structure which are represented by a GARCH model. Alexander (2008) and McNeil et al. (2005) provide a detailed exploration of ARIMA and GARCH. The main objective of the forecast model in this paper is not to simulate “correct” price movements but rather to forecast plausible price movements to allow us to run the optimization models using realistic real world price movements.

Based on the data characteristics we have chosen to use a ARIMA-GARCH model to create the forecasted spot prices. The best-fitted model was chosen by using the AIC and BIC model selection strategy in combination with analysis of the ACF, PACF and the Augmented Dickey-Fuller test of the original time series data and the differenced time series data. The ARIMA model has an order of (2,1,0) with a t-distribution while the GARCH model has an order of (2,1) with a t innovation distribution. Based on visual analysis of the ACF and PACF and observing that the time series presents indication of seasonality by having significant lags at around lag 52 and that the lags are positive which indicating that a Seasonal Autoregressive (SAR) process is appropriate, as opposed to and Seasonally Moving Averages (SMA) process. To account for the believed seasonality present in the data we include an SAR term at lag 52. The model was chosen by using forward stepwise AIC and BIC test. Below is the AIC and BIC outputs for the assessed models.

Table 7.1: Result from AIC and BIC test

Models	AIC	BIC
ARIMA(1,1,0)	2.5260	2.7598
ARIMA(2,1,0)	2.4898	2.7279
ARIMA(1,1,1)	2.5110	2.7492
ARIMA(3,1,0)	2.4914	2.7339
ARIMA(3,1,1)	2.4914	2.7338
ARIMA(3,1,2)	2.4951	2.7462
ARIMA(3,1,3)	2.4967	2.7522

Table 7.1 represents the AIC and BIC test for the ARIMA-GARCH model with different number of parameters. Both test chose the ARIMA(2,1,0)-GARCH(2,1) model with a Seasonally Autoregressive term at lag 52 as the optimal model, as can be gleaned from table 7.1. The seasonality was included because the Augmented Dickey-Fuller test using equation 6.14 and 6.15 could not reject the null hypothesis of a unit root process while using equation 6.16 we rejected the null hypothesis of a unit root. After considering the results of the Augmented Dickey-Fuller tests and after examining the historic prices movements, as can be seen in chapter 4, and industry characteristics, as can be seen in chapter 3, we believe we have seasonal trends in the time series.

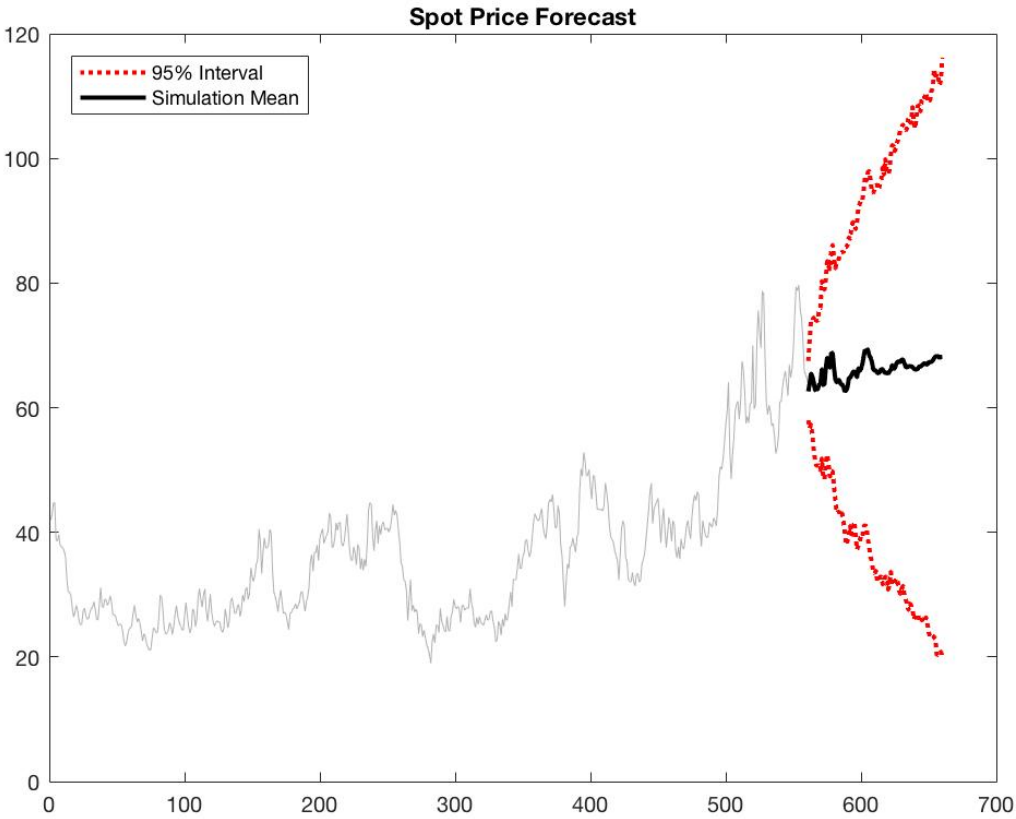


Figure 7.1: Forecast of mean spot prices and 95% forecast intervals

Figure above depicts mean forecast and 95% interval for the forecast of the spot price as used in the single-period horizon model along with portfolio optimization.

7.1.2 Modelling Forwards Prices

There is little research into the forecast of forward contracts in general. A reason for the lack of specific research into the forecast of forward contracts is that a forecast of forward contracts is basically a forecast of the spot price. If we generalize the statement of Kaye, Outhred, and Bannister (1990) we can state:

$$P_{t+q} = E\{Spot_{t+q} \mid \text{system condition at } q\} \quad (7.1)$$

Where P_{t+q} is the set forward price set to be realized at time $t+q$ which is equal to the expected spot price at time $t+q$ conditional on all the available information. However, forward prices coming due at time t will rarely equal the spot price at time t .

Forward contract prices and, for that matter, future contract prices are based on several different aspects of the commodity being sold such as, the spot price, the expectation of the future, supply and demand, storage cost, and other factors to establish an equilibrium price for the forward or future price in terms of the spot price. For an equilibrium to hold for two set of prices the price movements of one price, in this case the forward price, must move in the same direction as the other price, in this case the spot price (Stein, 1961). It is natural to state that the forward price is, in some form, dependent on the spot price.

Asche et al. (2016) studied the relationship between spot prices and forward prices of salmon. Asche et al. (2016) found that, contrary to many empirical studies of the spot-forward relationship in many commodities market, the lead-lag relations in the salmon market is that of a leading spot price and a lagging forward price. Asche et al. (2016) found that the spot and forward prices were cointegrated for up to 6-month forward contracts and that the causality was one-directional, leading from spot to forward price. A reason for the result could be that the forward market for salmon is still immature (Asche et al., 2016) but this could change as time goes on.

For our model we need to account for the fact that in the real world forward prices change continually as spot prices change. The changes in spot price within the model necessitates that we also create a model that represents the changes in forward prices as the spot prices change. Based on the data characteristics, Stein (1961), and Asche et al. (2016) we develop a simplistic

forward price forecasting model that is conditional on the forecasted spot prices and the relationship between the forward prices and spot prices, which is represented by β_q .

$$P_{t+q,s} = Spot_{t-3,s} + ((Spot_{t,s} - Spot_{t-3,s}) * \beta_q) \quad (7.2)$$

Where $Spot_{t,s}$ is the simulated spot price at time t in scenario s. $P_{t+q,s}$ is the forecasted forward price for forward contract with length q in scenario s while β_q represents the historic joint movement between the spot price at t and forward price at time t.

7.1.3 Modelling Production

The assumption in this paper is that the optimizing salmon producer is a medium sized company. The total sales volume is based on other salmon producers in the Norwegian market and then adjusted for size. The comparable salmon producers are Salmar AS, Norway Royal Salmon, and Marine Harvest where Salmar AS and Marine Harvest are classified as large salmon producers while Norway Royal Salomon is a small to medium salmon producer.

Salmar sold 128 100 tons of salmon 2016 (“Salmar ASA,” 2017) while Marine Harvest sold 125 400 (“Marine Harvest ASA,” 2017) and Norway Royal Salmon sold 66 808 (“Norway Royal Salmon,” 2017). Based on these sales numbers we have decided that our producers’ weekly production will produce and sell 1700 tons of salmon. Salmon production is not certain in terms of weekly production so we have included fluctuations around the mean of 1700 tons’ with a standard deviation of 100 tons’ as such:

$$X_{t,s} \sim \mathcal{N}(\mu, \sigma) \quad (7.3)$$

Where $X_{t,s}$ is the production in tons, μ is the mean, and σ is the standard deviation.

7.2 Single-Period model and Portfolio Optimization

We will use a Single-period model to select the optimal policy of sales of salmon over contract and spot at the end of the simulated period based on a selection of assumptions. The goal of the model is to set a policy at time t that will hold for all future periods as to optimize the selected assumptions. Assume a fixed planning horizon T that is split up into discrete equidistant points

of time. Assume that we can hedge each months production volume $X_{t,s}$ (given in tons) where t represents the month and $s = 1, \dots, S$ denotes the scenario. Let ω_t denote the quantity to be sold forward to period $t = 1, \dots, T$ at given forward prices p_t . Let the spot price be denoted by $q_{t,s}$. Furthermore, let $F_{t,s}$ be cash flow from decisions made before the planning horizon. Assume that idle cash is hold on a bank account with a monthly interest rate r_B . Furthermore, cash can be drawn from a credit line with a monthly interest rate of r_C , which include a significant penalty percentage for negative cash flows. The motivation behind including a significant penalty for negative cash flow is to account for the risk of involved in the lack of liquidity, which are especially important for smaller producers. In addition, a variable cost k_v and a fixed cost k_F is subtracted from each period.

The model uses scenario analysis constructed in MATLAB by using the forecast models for the spot prices and forward prices to forecast 100 observations and 1000 scenarios. The forecasted scenarios, along with the salmon production of 100 observations and 1000 scenarios, are transferred to Excel where the optimization is calculated using GRG Nonlinear Solver and Simple LP Solver. In calculating the optimization problem based on the set of selected assumptions in the one-period horizon model we use 100 weeks of forecasted observations. However, we start the optimization at week 24 to include all forecasted forward contracts in each period-calculation. The optimization will be based on the both the expected ending bank balance and the expected weekly cash flows resulting from salmon sales.

In the Single-Period model several optimizations will be conducted based on distinctive optimization objectives and restrictions. We have 2 distinct optimization problems that are set to be optimized based on whether we are optimizing expected weekly cash flow or expected ending bank balance.

The following is the expected weekly cash flow in each intermediate period and distinctive scenario:

$$\sum_{\tau=1}^t \omega_{\tau,s}^t \cdot p_{\tau,s}^t + (X_{t,s} - \sum_{\tau=1}^t \omega_{\tau,s}^t) \cdot q_{t,s} - X_{t,s} k_v - k_F = F_{t,s} \quad (7.4)$$

In period $t = 0$ we have the following constraint for the financial balance:

$$-b_{0,s} + c_{0,s} = F_{0,s} \quad \text{for all } s = 1, \dots, S \quad (7.5)$$

The financial balance constraint indicates that there is no uncertainty in period $t = 0$. This is because the first forward contract only comes due after one month.

The expected ending bank balance in each intermediate step and distinctive scenarios is represented below:

$$\begin{aligned} \sum_{\tau=1}^t \omega_{\tau,s}^t \cdot p_{\tau,s}^t + (X_{t,s} - \sum_{\tau=1}^t \omega_{\tau,s}^t) \cdot q_{t,s} - X_{t,s} k_v - k_F + \\ b_{t-1,s} \cdot (1 + r_B) - b_{t,s} - c_{t-1,s} \cdot (1 + r_C) + c_{t,s} = F_{t,s} \end{aligned} \quad (7.6)$$

s.t.

$$c_{t,s} \geq 0$$

$$b_{t,s} \geq 0$$

$$c_{t,s} \geq b_{t,s}$$

$$x_{its} \geq 0$$

The restrictions state that both the interest on credit and the interest rate on bank deposits must be positive, the credit interest rate must be larger than the interest received on bank deposits, and that the decision variables has to be positive. The restriction on the decision variables limits the actor from buying in the market. For a visual representation of the model interface please see appendix C.

7.2.1 Optimization Structure

The structure of the policy-decision can be sketched as following using optimization objectives and restriction. The following are the optimization structure used for the expected weekly cash flow and the expected ending bank balance.

$$\max E[F_T] = \frac{1}{S} \left(\frac{1}{T} \left(\sum_{\tau=24}^t \omega_{\tau,s}^t \cdot p_{\tau,s}^t + (X_{t,s} - \sum_{\tau=24}^t \omega_{\tau,s}^t) \cdot q_{t,s} - X_{t,s} k_v - k_F \right) \right) \quad (7.7)$$

F_T represents or the expected weekly cash flow, or Cash, at the end of the period and where the variables in the equation is described above in section 7.2.

$$\max E[B_T] = \frac{1}{S} \left(\begin{array}{l} \sum_{\tau=24}^t \omega_{\tau,S}^t \cdot p_{\tau,S}^t + (X_{t,S} - \sum_{\tau=24}^t \omega_{\tau,S}^t) \cdot q_{t,S} - k_F \\ -X_{t,S}k_v + b_{t-1,S} \cdot (1 + r_B) - b_{t,S} - c_{t-1,S} \cdot (1 + r_C) + c_{t,S} \end{array} \right) \quad (7.8)$$

\overline{B}_T represents the expected ending bank balance, or Balance, at the end of the period and where the variables in the equation is described above in section 7.2.

The optimization is conducted with respects to both the expected weekly cash flow and the expected ending bank balance. The optimization follows Markowitz theory of portfolio optimization and an efficient frontier where all utility functions are represented is constructed.

7.2.2 Using MPT on Forecasted Price Returns

The semi-deviation described in section 6.2.3 will use the returns of the forecasted spot and contract prices to establish both the return of each “equity” and semi-covariance matrixes. The returns are established by averaging each period forecasted price by scenarios and then calculating the changes over the forecasted time period as follows.

$$E[r_p] = \frac{\sum_{t=24}^T \ln(\frac{F_{a+1}}{F_a})}{T} \quad (7.9)$$

Where

$$F_a = \frac{\sum_{s=1}^S F_{t,S}}{S} \quad (7.10)$$

$$F_{a+1} = \frac{\sum_{s=1}^S F_{t+1,S}}{S} \quad (7.11)$$

Where $E r_p$ is the expected return, or change, of the specific price path, F_a is the average forecasted price for all scenarios at time t, while F_{a+1} is the average forecasted price at time t+1. Using standard deviation and semi-deviation along with the return data we are able to account for the theoretical covariance between the spot price movements and each of the foreword contract price movements which might produce different distribution between spot sales and contract sales than the distribution found from the objective function.

7.3 Multi-Stage Stochastic Recourse Model

We will use a multistage stochastic recourse model that maximizes the expected final wealth at the end of the planning horizon for a typical medium salmon producer. The model will use scenario analysis where we forecast a set of scenarios for the planning horizon. The price movement leading up until the beginning of our model is deterministic while the price movement in the future scenarios is considered to be uncertain. However, the future price movements are dependent on past realizations of price movements up until time T through the use of the forecast model ARIMA-GARCH.

In our model we start at time t equal 0. At time 0 we will have production of $X_{0,s}$ which all will be sold at spot. We will also sell forward contract for future production at time 1 through the end of the planning horizon, T , which will produce an income at time 1 through T . This means that we will have to make a decision of how much salmon should be sold at time 1 through T at the specified forward price at time 0 and at each new time period. The allocation decision will be repeated every month which is represented by the time fragments in this model.

We assume a discretization of future scenarios. Let the scenarios be denoted by $s = 1, \dots, S$. Let $X_{t,s}$ be the production at point of time t . The produced salmon are not storable so at each period every salmon either will be sold on spot or delivered at forward contracts previously sold. Let $\omega_{t_1,s}^{t_2}$ be the quantity which is sold at point in time t_1 and delivered at point of time t_2 (forward contract of length $t_2 - t_1$) in scenario s . Let $p_{t_1,s}^{t_2}$ be the forward price that is negotiated upon in t_1 and scenario s . Let the spot price be $q_{t,s}$. Let k_p represent the transaction cost on the sale of forward contracts. Furthermore, let $F_{t,s}$ be cash flow from decisions made before the planning horizon which continues to grow (be reduced) throughout the planning horizon. The cash flow in $F_{t,s}$ include cash from sale of forward contracts at $t-1$ through $t-6$ and spot at time t . Let $b_{t,s}$ represent cash inflows and $c_{t,s}$ be a credit line. Assume that idle cash is hold on a bank account with a monthly interest rate r_B . Furthermore, cash can be drawn from a credit line with a monthly interest rate of r_C . The decision variables are the $\omega_{t_1,s}^{t_2}$ variables which is the quantity of salmon to be sold on forward contract in future periods.

In period $t = 0$ we have the following constraint for the financial balance:

$$-b_{0,s} + c_{0,s} = F_{0,s} \quad \text{for all } s = 1, \dots, S \quad (7.12)$$

The financial balance constraint has the same justification as the financial balance constraint in the single-period model.

In each intermediate period (and every scenario) we have defined the financial balance in the following way:

$$\begin{aligned} \sum_{\tau=1}^t \omega_{\tau,s}^t \cdot p_{\tau,s}^t + (X_{t,s} - \sum_{\tau=1}^t \omega_{\tau,s}^t) \cdot q_{t,s} - \sum_{\tau=1}^t \omega_{\tau,\square}^t \cdot k_p + \\ b_{t-1,s} \cdot (1 + r_{B,s}) - b_{t,s} - c_{t-1,s} - c_{t-1,s} \cdot r_{C,s} + c_{t,s} = F_{t,s} \end{aligned} \quad (7.13)$$

or simply

$$\begin{aligned} \sum_{\tau=1}^t \omega_{\tau,s}^t \cdot p_{\tau,s}^t + (X_{t,s} - \sum_{\tau=1}^t \omega_{\tau,s}^t) \cdot q_{t,s} - \sum_{\tau=1}^t \omega_{\tau,s}^t \cdot k_p + \\ b_{t-1,s} \cdot (1 + r_B) - b_{t,s} - c_{t-1,s} \cdot (1 + r_C) + c_{t,s} = F_{t,s} \end{aligned} \quad (7.14)$$

for all $s = 1, \dots, S$ and all $t = 1, \dots, T - 1$

s.t.

$$c_{t,s} \geq 0$$

$$b_{t,s} \geq 0$$

$$c_{t,s} \geq b_{t,s}$$

$$X_{t-1,s} \leq x_{its} \geq 0$$

The restrictions state that both the interest on credit and the interest rate on bank deposits must be positive, the credit interest rate must be larger than the interest received on bank deposits, and that the decision variables has to be between zero and the expected production which is represented by the production $X_{t-1,s}$ in the previous period. The restriction on the decision variables limit the actor from buying in the market and selling more salmon than are being produced.

In the last period of the planning horizon we would have the following:

$$\begin{aligned} \sum_{\tau=1}^T \omega_{\tau}^T \cdot p_{\tau}^T + (X_T - \sum_{\tau=1}^T \omega_{\tau,s}^T) \cdot q_T - \sum_{\tau=1}^t \omega_{\tau,s}^t \cdot k_p + \quad (7.15) \\ b_{T-1,s} \cdot (1 + r_B) - c_{T-1} \cdot (1 + r_C) - W_T = F_{T,s} \end{aligned}$$

for all $s = 1, \dots, S$ and all $t = 1, \dots, T - 1$

We maximize the expected final wealth, which is the cash flow generated during the planning horizon subtracted by the credit line at the end of the planning horizon:

$$\bar{W}_T = \frac{1}{S} \sum_{s=1}^S W_{T,s} \quad (7.16)$$

The non-anticipativity constraints are as follows:

$$x_{its} = x_{its'} \quad \forall i \in N, \forall t \in T, \forall s \in S, \forall s' \in S_s^t \quad (7.17)$$

Where x_{its} represents the decisions i at time t in scenario s based on historic variables, while $x_{its'}$ represents the decisions i at time t in scenario s' based on historic variables. $\forall i \in N$ are all decision variables from i to N , $\forall t \in T$ are all time period from t to T , $\forall s \in S$ are all scenarios from s to S , and $\forall s' \in S_s^t$ are all scenarios from s' to S_s^t which are equivalent to s at time t .

The following depict the evolution of the optimization equation from period $t = 0$ through period $t = 6$

$$\mathbf{t = 0:} \quad X_{0,s} \cdot q_{0,s} + b_{-1,s} \cdot (1 + r_B) - b_{0,s} - c_{-1,s} \cdot (1 + r_C + ip_{C,t}) + c_{0,s} = F_{0,s} \quad (7.18)$$

$$\begin{aligned} \mathbf{t = 1:} \quad \omega_0^1 \cdot p_{0,s}^1 + (X_{1,s} - \omega_0^1) \cdot q_{1,s} - \omega_0^1 \cdot k_p + b_{0,s} \cdot (1 + r_B) - \quad (7.19) \\ b_{1,s} - c_{0,s} \cdot (1 + r_C + ip_{C,t}) + c_{1,s} = F_{1,s} \end{aligned}$$

$$\begin{aligned} \mathbf{t = 2:} \quad \omega_0^2 \cdot p_{0,s}^2 + \omega_1^2 \cdot p_{1,s}^2 + (X_{2,s} - \omega_0^2 - \omega_1^2) \cdot q_{2,s} - \omega_0^2 \cdot k_p - \quad (7.20) \\ \omega_1^2 \cdot k_p + b_{1,s} \cdot (1 + r_B) - b_{2,s} - c_{1,s} \cdot (1 + r_C + ip_{C,t}) + c_{2,s} = F_{2,s} \end{aligned}$$

$$\begin{aligned}
t = 6: & \omega_0^6 \cdot p_{0,s}^6 + \omega_1^6 \cdot p_{1,s}^6 + \omega_2^6 \cdot p_{2,s}^6 + \omega_3^6 \cdot p_{3,s}^6 + & (7.21) \\
& \omega_4^6 \cdot p_{4,s}^6 + \omega_5^6 \cdot p_{5,s}^6 + (X_{6,s} - \omega_0^6 - \omega_1^6 - \omega_2^6 - \omega_3^6 - \omega_4^6 - \omega_5^6) \cdot q_{6,s} - \\
& \omega_0^6 \cdot k_{\square} - \omega_1^6 \cdot k_p - \omega_3^6 \cdot k_p - \omega_4^6 \cdot k_p - \omega_5^6 \cdot k_p - \omega_6^6 \cdot k_p + \\
& b_{5,s} \cdot (1 + r_B) - c_{5,s} \cdot (1 + r_C) = F_{6,s}
\end{aligned}$$

After the calculation of the solution of the stochastic program an efficient frontier is created to represent all optimal portfolios for actors with different risk appetite. The risk measure used is based on the risk measure outlined in Mo, et. al. (2001). The risk measure used negative deviation from a set revenue target. The risk aversion of the actors which outline the efficient frontier is reprinted by lambda λ which range from 0 to 1 with 0.01 intervals. The risk adjusted objective function is represented as follows:

$$\bar{W}_T \lambda - (1 - \lambda)R \quad (7.22)$$

Where \bar{W}_T is the expected ending wealth, λ is the risk aversion where 0 represents perfect risk aversion and 1 represents no risk aversion. R represents the risk measure which in this model is the negative deviation from the revenue target

$$R = - \sum_{s=1}^S \mathbb{P}_s \min\{w_{T,s}; \psi\} \quad (7.23)$$

Where \mathbb{P} represents the probability of the outcome, which in this model is the number of distinct scenarios. ψ represents the set revenue target.

When calculating a multi-stage stochastic recourse model a problem that often occurs is the problem of dimension where each period the number of new branches grows exponentially leading to a problem too large for today's technological computing power to compute. Therefore, the tree-structure we have constructed is based on a monthly decision variable with

weekly price updates for the simulations which reduces the exponential growth of the tree-structure by only branching at every fourth node.

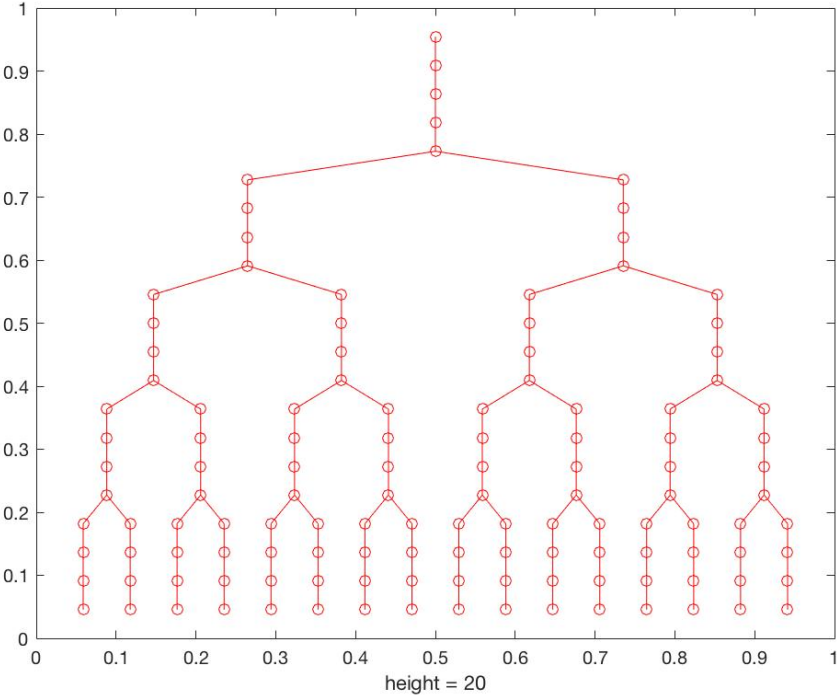


Figure 7.2: Visualization of price simulation process

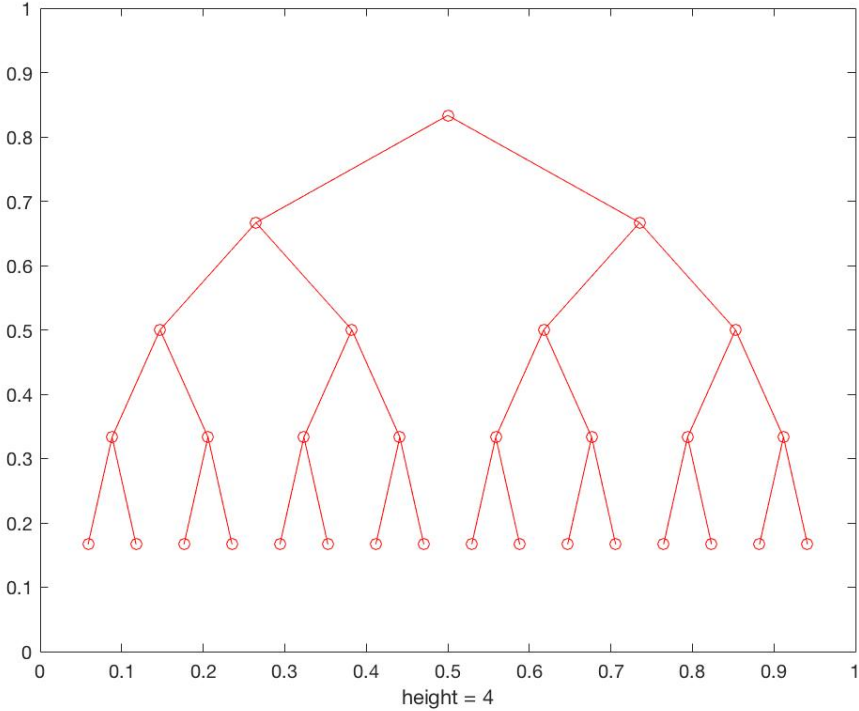


Figure 7.3: Visualization of decision nodes

The reduction in the branching in the tree-structure makes it possible for a personal computer to compute a more demanding computation of the model that we otherwise would have needed a special computer with extreme computing power to compute for the same number of time periods and scenarios.

Chapter 8

Model Application

This chapter contains the results from the risk models introduced in Chapter 7. Section 8.1 present and discuss output from the single-period model, with comparison of various approaches for utilization. In section 8.2 the application of the multi-stage model will be presented. This is a more sophisticated model where the focus will be on “hypothetical” results and steps for a complete model.

8.1 Single-Period Horizon Model

Section 8.1.1 present results from the application of the single-period model. The model use different risk measures/metrics to substantiate the inferences that are drawn from the results. In addition, an optimization based on Modern Portfolio Theory is presented and compared to the use of objective functions. Next, an application of the model with a more “negative” market scenario is presented and discussed in section 8.1.2. Finally, in section 8.1.3, further development of the model is discussed.

8.1.1 Results

In the following we present the decision variables output from optimizing the two objective functions presented in section 7.2. As the first function use expected weekly cash flow and the second function use expected ending bank balance, we compare tables from use of these respective functions. In the first presentation of tables, the model use CVaR as a risk metric. For the same purpose, the second presentation of tables use downside deviation.

The non-stochastic input variables used in the model are:

$$k_v = 35 \text{ NOK per kg} \quad (8.1)$$

$$k_F = 15\,000 \text{ NOK weekly} \quad (8.2)$$

$$r_C = 45\% \text{ yearly} \quad (8.3)$$

$$r_B = 2\% \text{ yearly} \quad (8.4)$$

The tables consist of a set of optimizations subject to different constrictions. The ‘Min Var’ in the table is minimization of the risk measure used in the respective table. Because CVaR is negative, it is actually maximized by the model when applied. ‘Max Return’ is maximization of the expected return of the objective functions. The optimization providing the ‘Optimal’ allocation of decision variables is conducted by maximizing the division of expected return on the risk measure for the risk measure downside deviation. Equivalently, we minimize the division when CVaR is applied.

Table 8.1 and 8.2 show model output from use of respective objective functions and CVaR:

Table 8.1: Allocation of spot and contracts based on Expected Weekly Cash Flow using CVaR(95)

	Min Var	Optimal					Max Return
Mean ret	37801	37801	38300	38500	38700	38900	38979
CVaR (95)	-8242	-8242	-13464	-14592	-16672	-20157	-21561
Slope	-4,59	-4,59	-2,84	-2,64	-2,32	-1,93	-1,81
spot	0 %	0 %	0 %	0 %	14 %	76 %	100 %
1m forw	0 %	0 %	0 %	45 %	0 %	0 %	0 %
2m forw	0 %	0 %	18 %	19 %	86 %	24 %	0 %
3m forw	0 %	0 %	51 %	19 %	0 %	0 %	0 %
4m forw	0 %	0 %	0 %	2 %	0 %	0 %	0 %
5m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %
6m forw	100 %	100 %	31 %	15 %	0 %	0 %	0 %

Table 8.2: Allocation of spot and contracts based on Expected Ending Bank Balance using CVaR(95)

	Min Var	Optimal					Max Return
Mean ret	2 912 106	2 912 106	2 924 999	2 940 000	2 954 996	2 970 000	2 986 938
CVaR (95)	-740 565	-740 565	-840 179	-1 106032	-1 325719	-1 467293	-2 024878
Slope	-3,93	-3,93	-3,48	-2,65	-2,22	-1,83	-1,47
spot	0 %	0 %	0 %	0 %	23 %	36 %	100 %
1m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %
2m forw	0 %	0 %	0 %	0 %	15 %	2 %	0 %
3m forw	0 %	0 %	13 %	58 %	17 %	62 %	0 %
4m forw	0 %	0 %	12 %	0 %	25 %	0 %	0 %

5m forw	0 %	0 %	20 %	0 %	5 %	0 %	0 %
6m forw	100 %	100 %	55 %	42 %	15 %	0 %	0 %

The columns represent steps on the efficient frontier and explicitly show allocations of sale.

Table 8.3 and 8.4 show model output from use of respective objective functions and Downside Deviation:

Table 8.3: Allocation of spot and contracts based on Expected Weekly Cash Flow using Downside Deviation

	Min Var	Optimal				Max Return
Mean ret	37801	37801	38100	38400	38700	38979
DD	12115	12115	13025	14174	15343	16726
Slope	3,12	3,12	2,93	2,71	2,52	2,33
spot	0 %	0 %	25 %	51 %	58 %	100 %
1m forw	0 %	0 %	0 %	0 %	0 %	0 %
2m forw	0 %	0 %	0 %	0 %	13 %	0 %
3m forw	0 %	0 %	0 %	0 %	16 %	0 %
4m forw	0 %	0 %	0 %	0 %	0 %	0 %
5m forw	0 %	0 %	0 %	0 %	0 %	0 %
6m forw	100 %	100 %	75 %	49 %	13 %	0 %

Table 8.4: Allocation of spot and contracts based on Expected Ending Bank Balance using Downside Deviation

	Min Var	Optimal					Max Return
Mean ret	2 912105	2 912105	2 925000	2 940000	2 955000	2 970000	2 986938
DD	1088772	1088772	1 150944	1 239007	1 342388	1 457123	1 632591
Slope	2,67	2,67	2,54	2,37	2,201	2,03	1,82
spot	0 %	0 %	15 %	23 %	0 %	37 %	100 %
1m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %
2m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %
3m forw	0 %	0 %	0 %	19 %	56 %	63 %	0 %
4m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %
5m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %
6m forw	100 %	100 %	85 %	58 %	44 %	0 %	0 %

We can clearly see from all the tables that the model mainly allocate sale to either spot or the six-month forwards, with some minor variations along the frontier. By aggregating the use of the different risk measure we can draw some inferences from the single-period model. The first

clear-cut inference is that that for an actor who seeks to maximize profits the portfolio he should choose is a portfolio consisting of 100% spot sales while an actor seeking to minimize risk should allocate 100% sales to forward contracts.

The model seems to prefer a parsimonious allocation policy in which the allocation is centered around the use of the least amount of different sales distributions. There are significant differences in allocations of spot and contracts when using the different risk measures. Downside deviation seem to be most sensitive to risk and the model will therefore allocate a larger proportion of sales to forward contracts when downside deviation is used. The differences highlight the importance of choosing the right risk measure for the individual salmon companies.

The inclusion of credit rate significantly changes the risk profile of the cash flow. The difference in risk profile is highlighted in the different allocations between the two objective functions. Where the credit rate is included, in the bank balance, the model prefers an allocation with significantly larger portion of sales allocated to forward contracts. The difference can be seen in the comparison of table 8.1 with table 8.3 and table 8.2 with table 8.4.

8.1.1.1 Output from the use of MPT with semi-deviation

An alternative to the objective functions is to optimize the allocation of decision variables with the ‘Mean Variance’ approach from Modern Portfolio Theory. This is based on forecasted returns of the spot price and forward contracts. We apply semi-deviation as a risk measure, which is approximately identical to the risk measure in the original theory. Results from this optimization follows in table 8.5.

Table 8.5: Allocation of spot and contracts based on MPT using Semi-Deviation

			Min Var						Optimal			Max return
Mean ret	0,053%	0,055 %	0,057 %	0,061 %	0,065 %	0,068 %	0,071 %	0,075 %	0,075 %	0,076 %	0,076 %	0,076 %
Semi-SD	0,403%	0,402 %	0,402 %	0,403 %	0,408 %	0,417 %	0,428 %	0,446 %	0,450 %	0,463 %	0,463 %	0,475 %
Slope	0,132	0,137	0,142	0,152	0,159	0,163	0,166	0,167	0,167	0,164	0,164	0,160
spot	46 %	48 %	50 %	54 %	62 %	67 %	73 %	80 %	86 %	94 %	94 %	100 %
1m forw	37 %	36 %	35 %	33 %	23 %	16 %	9 %	0 %	0 %	0 %	0 %	0 %
2m forw	10 %	7 %	5 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
3m forw	7 %	9 %	10 %	13 %	15 %	17 %	18 %	20 %	14 %	6 %	0 %	0 %
4m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
5m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
6m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %

The output from use of Modern Portfolio Theory show a more diversified set of portfolios than was present in results from the use of the objective functions.

Equivalently to the results from using the objective functions with CVaR and downside deviation, the results in table 8.5 indicate that a profit maximizing actor should place all his sales on the spot market. Contrary to the results provided when the objective functions are used, the results in table 8.5 does not indicate that the actor seeking to *minimize* risk should allocate 100% in contracts.

The reason for the difference between the use of the objective functions and that of MPT could be that MPT accounts indirectly for the risk associated with contracts of longer longevity. The fact that MPT accounts for the covariance between the spot price and the forward contracts of different length might provide an indirect appreciation for the risk associated with being overexposed in spot price or in one single forward contract length. The issue of overexposure will be explored further in later sections.

An interesting feature from the MPT approach with semi-deviation is that the use of five- and six-month contracts is non-existent. The result from the use of MPT is consistent with reality in the sense that the range of the efficient portfolios is within the range that salmon producers operate today, which is between 20 and 60 percent allocation to contracts.

8.1.2 Analysis of an Unfavorable Market Scenario

As mentioned earlier in the paper, salmon prices are at historic heights and costs have increased significantly as well. However, costs have not grown at the same rate as prices. The currently large price margins will probably have a significant impact on the results of the single-period horizon model. Consequently, we will analyze a more unfavorable market scenario. We implement higher production cost which lead to tighter margins. Note that we use the original model from the first part of this chapter, with the original objective functions. Equivalently, risk measures are CVaR and Downside Deviation. Below are the updated non-stochastic inputs:

$$k_v = 55 \text{ NOK per kg} \quad (8.5)$$

$$k_F = 15\,000 \text{ NOK weekly} \quad (8.6)$$

$$r_C = 45\% \text{ yearly} \quad (8.7)$$

$$r_B = 2\% \text{ yearly}$$

(8.8)

Notice that production cost per kg of salmon has been adjusted to 55 NOK per kg.

Table 8.6 and 8.7 show model output from use of objective functions and CVaR:

Table 8.6: Margin adjusted allocation of spot and contracts based Expected Weekly Cash Flow using CVaR(95)

	Min Var	Optimal						Max Return
Mean ret	3 356	3 356	3 600	3 800	4 000	4 200	4 400	4 534
CVaR (95)	-38 097	-38 097	- 41 382	-44 138	-46 970	-50 202	-53 652	-56 039
Slope	-0,088	-0,088	-0,086	-0,086	-0,085	-0,083	-0,082	-0,080
spot	0 %	0 %	0 %	0 %	0 %	0 %	69 %	100 %
1m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
2m forw	0 %	0 %	0 %	0 %	0 %	94 %	31 %	0 %
3m forw	0 %	0 %	36 %	65 %	95 %	6 %	0 %	0 %
4m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
5m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
6m forw	100 %	100 %	64 %	35 %	5 %	0 %	0 %	0 %

Table 8.7: Margin adjusted allocation of spot and contracts based on Expected Ending Bank Balance using CVaR(95)

	Min Var	Optimal				Max Return
Mean ret	155 524	155 524	160 000	165 000	170 000	171 231
CVaR (95)	- 3 579 127	- 3 579 127	- 3 807 823	- 4 138 802	- 4 568 712	- 4 935 654
Slope	-0,043	-0,043	-0,042	-0,040	-0,037	-0,035
spot	0 %	0 %	0 %	0 %	0 %	10 %
1m forw	0 %	0 %	0 %	0 %	0 %	0 %
2m forw	0 %	0 %	0 %	0 %	0 %	86 %
3m forw	0 %	0 %	23 %	57 %	98 %	4 %
4m forw	0 %	0 %	0 %	0 %	0 %	0 %
5m forw	0 %	0 %	0 %	0 %	0 %	0 %
6m forw	100 %	100 %	76 %	43 %	1 %	0 %

Table 8.8 and 8.9 show model output from use of objective functions and Downside Deviation:

Table 8.8: Margin adjusted allocation of spot and contracts based on Expected Weekly Cash Flow using Downside Deviation

	Min Var	Optimal						Max return
Mean ret	3 356	3 356	3 600	3 800	4 000	4 200	4 400	4 534
DD	12 127	12 127	13 078	13 888	14 709	15591	16 546	17 248
Slope	0,276	0,276	0,275	0,273	0,271	0,269	0,265	0,262
spot	0 %	0 %	0 %	0 %	0 %	33 %	71 %	100 %
1m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
2m forw	0 %	0 %	7 %	0 %	0 %	0 %	6 %	0 %
3m forw	0 %	0 %	27 %	65 %	95 %	67 %	23 %	0 %
4m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
5m forw	0 %	0 %	0 %	0 %	0 %	0 %	0 %	0 %
6m forw	100 %	100 %	66 %	35 %	5 %	0 %	0 %	0 %

Table 8.9: Margin adjusted allocation of spot and contracts based on Expected Ending Bank Balance using Downside Deviation

	Min Variance	Optimal				Max Return
Mean ret	155 524	155 524	160 000	165 000	170 000	171 231
DD	1 138 983	1 138 983	1 207 736	1 306 511	1 434 095	1 538 240
Slope	0,137	0,137	0,132	0,126	0,119	0,111
spot	0 %	0 %	0 %	0 %	0 %	10 %
1m forw	0 %	0 %	0 %	0 %	0 %	0 %
2m forw	0 %	0 %	0 %	0 %	0 %	86 %
3m forw	0 %	0 %	23 %	57 %	99 %	4 %
4m forw	0 %	0 %	0 %	0 %	0 %	0 %
5m forw	0 %	0 %	0 %	0 %	0 %	0 %
6m forw	100 %	100 %	77 %	43 %	1 %	0 %

With every factor except for production cost equal to the initial analysis, we find significantly different results compared to a ‘wide margin scenario’. The results suggest that the actor should increase the proportion of forward contracts as margins shrink. The shift in allocation can be seen especially succinct in the result from optimization with the objective function ‘Expected

Ending Bank Balance tables reflecting the expected ending bank balance in table 8.7 and table 8.9 above allocate more sales through contracts compared to the allocations in table 8.6 and 8.8.

The analysis indicate that the model takes account of the increased risk associated with closer price margins. Specifically, the model exhibit more dynamic features when based on expected ending balance. It adjusts the decision variable allocations to minimize the risk of negative ending bank balance by penalizing negative ending balance through the credit interest rate.

For salmon farming companies the usefulness of the Single-Stage Model comes through the application of the model as a guiding or decision tool. Adaptation to changing price margins makes the model reliable in a cyclical industry. The elements most significant to the dynamic properties are the production cost and credit interest rate. The model provides the companies with a policy for allocation decisions for a selected timeframe. Each individual salmon farming company must decide the risk measure to be used, the timeframe, provide a forecast for the future spot price, set credit rate, and include cost. Based on changing industry fundamentals and the salmon company's needs, the model can be updated and run again to provide the salmon farmer with an updated allocation policy.

8.1.3 Further work

A potential shortcoming with the Single-Stage Horizon Model, as highlighted by comparison of MPT, is the fact that the model does not, to a satisfactory degree, capture the risk involved in overexposure in one single forward contract. Further research can improve the objective functions in the single-period model by discovering the risk factors of forward contracts of different lengths and their contribution to risk levels of a portfolio consisting of spot sales and various forward contracts.

The importance of forecasting in the model indicate that research specifically into the “correct” forecasting model should be conducted. In addition to the forecasting of spot prices, the relationship between spot prices and forward contract price should be examined. Since the single-period model is implemented in excel and solved by Excel Solver GRG nonlinear we are not guaranteed to find the global optimum, only the local optimum. The recommendation of the authors is to use Global Optimization theory to address the issue of local optimums. One

possibility would be to create a similar MATLAB scrip as the one created for the multi-stage stochastic recourse model discussed in this paper.

8.2 Multi-Stage Model Application

Section 8.2.1 present hypothetical results from application of the single-period model. The efficient frontier based risk appetite will be presented, as well as explanation of how the model should be used when completed. Shortcomings and further steps for the model will be presented in section 8.2.2. The main purpose of the results section is to illustrate how the model work in practice and how any practitioner would deal with application of the model when completed.

8.2.1 Results

Due to size limitation within MATLAB the maximum number of branches that could be used in the model when running the program was 3 branches. Lacking computational power also set a constraint on the scope of the model as simulation time grow exponentially with increased number of variables, branches and time periods. The maximum number of time periods that could be used was 14 and the maximum combination of branches and time periods was three and eight, respectively. The optimal solution was to run the model with 14 future time periods ($t = 14$) and two branches ($B=2$) as it proved to be an acceptable compromise between simulation time and quality of the output.

As presented in chapter 7.4, the model optimizes the stochastic program with maximization of expected final wealth as the objective, with deviation from revenue target as the risk measure. The multi-stage model is highly dependent on the inclusion and setting of the revenue target. The target within the model is the variable which creates the risk variable.

The efficient frontier from model optimization is represented in figure 8.1:

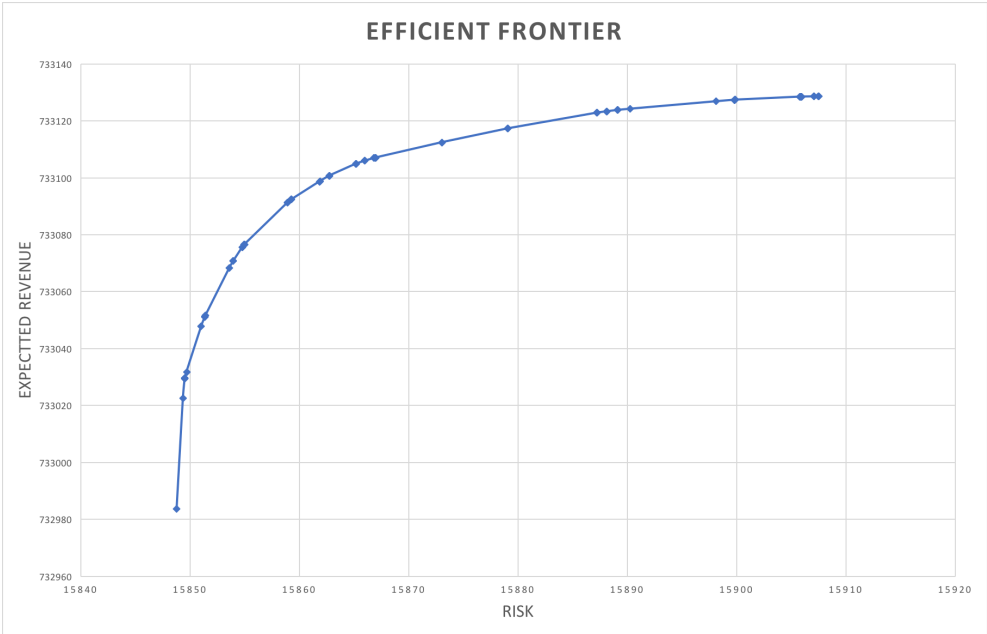


Figure 8.1: Efficient Frontier of output from Multi-Stage Stochastic Recourse Model

The efficient frontier is created by using the risk profile of individual salmon producers and combining them with their expected revenue and exposure to deviation from expected revenue, as can be seen from equation 7.22.

As the frontier represent all optimal portfolios for companies with different risk appetite, we can extract decision variable allocations for any given risk profile. Specifically, the points which represent risk profiles are found at lambda (λ) 0.01 through 0.99. Lambda 0.01 represent the most risk averse companies and conversely lambda 0.99 represent the least risk averse companies. The reader is now referred to Appendix B for an example of model output. Note that the results are hypothetical in principle and should therefore not be used for actual allocation decisions.

In terms of methodical characteristics, it is important to emphasize that the model put a significant value in flexibility. Consequently, the model tends to avoid allocations with a high

degree of long term contracts. As it has the possibility of making new decisions in the future, it will often make use of that possibility by waiting. Albeit not an optimal example in terms of representing models in general, the output in Appendix B illustrate this point. In summary, there are more extensive allocation to short term contracts. Optimally, we would want to automate the re-running of the model several times and then aggregated the output for a better result. Unfortunately, this was not possible due to computing power limitations.

Based on current features, the most essential information that the model would provide for salmon producers is the first-stage decisions. Decision variable x_{its} , as exemplified in Appendix B, will provide allocation in terms of volume sold through the various forward contracts. The model would function as a rolling-horizon model where for each new time-period the model would be re-run and update a new first-stage decision.

An alternative approach would be to adjust the model to produce a long-term allocation policy suitable for every period in a year. This is not something that we will explore further in this paper. However, with a higher number of branches and time periods, it will be possible to implement such a policy.

8.2.2 Model Completion/Development

The most obvious shortcoming of the current model, in terms of practical utilization for a salmon farming company, is that it relies heavily on spot- and forwards forecast. However, an acquired forecast that is more sophisticated can easily replace the forecast we produced.

Secondly, computational power is mentioned as a critical bottleneck and is linked with competence in advanced programming. Any company with sufficient resources can overcome this shortcoming, in terms of attracting skilled personnel and investing in computational software and hardware. Programmers can develop algorithms which make it possible to handle sparse matrices. Furthermore, greater resources would enable companies to incorporate a cost element in the model which might prove beneficial in terms of accuracy and versatility. Currently, the multi-stage model only includes a transaction cost variable. A fixed or variable production cost element, or both, would produce a more dynamic model which take shifting market conditions more into account. Considering that a bank balance element and a deposit- and credit interest rate element is included in the objective function, the framework is already in place for additional variables.

Chapter 9

Concluding Remarks

The prosperity in the salmon farming industry has generated an interest within many fields, especially among scientific communities and financial actors. Accordingly, the growth in scientific papers and financial services have been exponential. However, the field of price risk management is relatively untouched. Our literature review discovered that multiple scientific papers have been written dealing with hedging, but the number of papers regarding quantitative price risk models for salmon farming are few or zero. From chapter 4 regarding financial risk management, we learned that price risk is perceived to be the most important risk factor for salmon farmers. Consequently, the need for price risk management within the salmon forming industry exists. Furthermore, in talks with multiple salmon companies, we learned that quantitative risk models are not incorporated into risk management decision processes. To accommodate the potential benefit for the industry, we decided to develop two risk models. Both models include ARIMA-GARCH forecasting for salmon spot prices and one- through six-month forward contracts.

Essentially, the two models mainly diverge with regards to the degree of sophistication. The first model is a one-period horizon stochastic model, in which the optimization set a policy for the allocation of sales throughout the time horizon. The model has a fixed- and variable cost element incorporated, most importantly to take increasing production cost into account. The ambition of the model is to adjust to changing market conditions, specifically changing margins. Results indicate that the model use contracts more actively under unfavorable market conditions when optimization is based on expected ending balance. The credit interest rate penalizes negative bank balances, and the model adjust accordingly.

In term of application properties, the single-period model is practical, can be applied without profound competence and is dynamic regarding shifting margins. It can be improved with more advanced forecasts, separation of credit rates based on thresholds, or with incorporation of additional variables such as currency. However, it is basically a complete model, ready to be utilized.

The second model is a multi-stage stochastic recourse model, in which stochastic- and linear programming are used for the optimization of sales. The model produce output for decision variables representing sales allocation in the *present* point in time ($t=0$). Consequently, it must be applied at every decision point in time. Some additional steps are imperative for the multi-stage model to qualify for actual risk management utilization. A key initial criterion is computational power to increase the number of branches and time periods. Next, the model should incorporate production cost to be sufficiently dynamic under shifting market conditions. With the framework developed in the objective function, incorporation of additional variables should be relatively straightforward. Equivalently to the one-period horizon model, more sophisticated spot- and forward price forecasts would be preferable. The multi-stage model is aimed at companies with resources to seek a sophisticated quantitative risk model. It will be natural for such companies to hold employees with sufficient financial competence to utilize the model.

In conclusion of this paper, the key question is to which degree the solution accommodates the aim of the paper from the introduction: “The aim is to develop a decision tool which improve risk management in salmon farming companies” Although there can be made various improvements to both risk models, central elements are founded on thorough review of existing financial literature and theory, and should therefore be solid. Furthermore, the models are built to be valid in a long-term horizon, and easily adjustable for incorporation of new elements. Based on careful examination of results of the single-period model, companies should be able to make some useful inferences for allocation decisions. Regarding the multi-stage model, we hope it may contribute to an enhanced focus on quantitative models in the salmon farming industry, and potentially be a foundation for further work. Example of a next step could be to integrate the Multi-Stage Stochastic Recourse model from this paper with the comprehensive life-cycle models presented in Hæreid (2011) or Denstad et al. (2015).

Bibliography:

- Alexander, C. (2008). Market risk analysis. Practical financial econometrics. Chichester: Wiley.
- Alrefaei, M. H. & Andradottir, S. (2001), 'A modification of the stochastic ruler method for discrete stochastic optimization', *European Journal of Operational Research* 133(1), 160– 182.
- Andersen, T. B., Roll, K. H., & Tveterås, S. (2008). The Price Responsiveness of Salmon Supply in the Short and Long Run. *Marine Resource Economics*, 23(4), 425-437. doi:10.1086/mre.23.4.42629673
- Ankamah-Yeboah, I., Nielsen, M., & Nielsen, R. (2016). Price formation of the salmon aquaculture futures market. *Aquaculture Economics & Management*, 1-24. doi:10.1080/13657305.2016.1189014
- Asche, F., Guttormsen, A. G., & Nielsen, R. (2013). Future challenges for the maturing Norwegian salmon aquaculture industry: An analysis of total factor productivity change from 1996 to 2008. *Aquaculture*, 396-399, 43-50. doi:10.1016/j.aquaculture.2013.02.015
- Asche, F., Misund, B., & Oglend, A. (2016). The spot-forward relationship in the Atlantic salmon market. *Aquaculture Economics & Management*, 20(2), 222-234. doi:10.1080/13657305.2016.1156192
- Asche, F., Tveteras, S. (2007). Five success stories in aquaculture. 2. Salmon aquaculture: production growth and new markets: Session 1: Aquaculture growing strength. *Global trade conference on aquaculture*, 1, Qingdao (China), 29-31 May 2007, Issue 9, pp.53-57.
- Battiti, R. & Tecchiolli, G. (1996), 'The continuous reactive tabu search: blending combinatorial optimization and stochastic search for global optimization', *Annals of Operations Research* 63(2), 151–188.
- Berge, A. (2017, January 05). Fish Pool-priser – fantasi eller virkelighet? Retrieved April 6, 2017, from <https://ilaks.no/fish-pool-priser-fantasi-eller-virkelighet/>
- Bergfjord, O. J., (2009). Risk perception and risk management in Norwegian aquaculture, *Journal of Risk Research*, 12:1, 91-104, DOI: 10.1080/13669870802488941
- Bessembinder, H. and Lemmon, M. L. (2002), Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets. *The Journal of Finance*, 57: 1347–1382. doi:10.1111/1540-6261.00463

- Birge, J. R., & Louveaux, F. (2011). Introduction to stochastic programming. New York: Springer Science.
- Bjorgan, R., Liu, C., & Lawarree, J. (1999). Financial risk management in a competitive electricity market. *IEEE Transactions on Power Systems*, 14(4), 1285-1291. doi:10.1109/59.801886
- Chambers, J. C., Mullick, S. K., & Smith, D. D. (1971, July). How to Choose the Right Forecasting Technique. Retrieved March 20, 2017, from <https://hbr.org/1971/07/how-to-choose-the-right-forecasting-technique>
- Costello, M. J. (2009). The global economic cost of sea lice to the salmonid farming industry. *Journal of Fish Diseases*, 32(1), 115-118. doi:10.1111/j.1365-2761.2008.01011.x
- Denstad, A. G., Lillevand, M., & Ulsund, E. A., (2015). Production Planning and Sales Allocation in the Salmon Farming Industry. Master's thesis. Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management.
- Downside Deviation | Zephyr Associates, Inc. (n.d.). Retrieved April 5, 2017, from <http://www.styleadvisor.com/resources/statfacts/downside-deviation>
- Ender, W. (2014). Applied Econometric Time Series, 4th Edition, Wiley Series in Probability and Statistics.
- FAOs Director-General on How to Feed the World in 2050. (2009). Population and Development Review, 35(4), 837-839. doi:10.1111/j.1728-4457.2009.00312.x
- Fleten SE., Wallace S.W., Ziemba W.T. (2002) Hedging Electricity Portfolios via Stochastic Programming. In: Greengard C., Ruszczyński A. (eds) Decision Making Under Uncertainty. *The IMA Volumes in Mathematics and its Applications*, vol 128. Springer, New York, NY
- Frøystein, K. & Kure, E. H. (2013). Optimal Salmon Production - Producing Smolt Under Uncertainty. Master's thesis. Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management.
- Glover, F. & Laguna, M. (1999), Tabu Search, Springer, New York.
- Gupta, A., & Maranas, C. D. (2004). Real-Options-Based Planning Strategies under Uncertainty. *Industrial & Engineering Chemistry Research*, 43(14), 3870-3878. doi:10.1021/ie034164a
- Guttormsen, A. G. (1999). Forecasting weekly salmon prices: Risk management in fish farming. *Aquaculture Economics & Management*, 3(2), 159-166. doi:10.1080/13657309909380242

- Hannah, L. A. (2014). Stochastic Optimization. from <http://www.stat.columbia.edu/~liam/teaching/compstat-spr15/lauren-notes.pdf>
- Higle, J. L. (2005). Stochastic Programming: Optimization When Uncertainty Matters. *Emerging Theory, Methods, and Applications*, 30-53. doi:10.1287/educ.1053.0016
- Hæreid, M. B. (2011). Allocating Sales in the Farming of Atlantic Salmon: Maximizing Profits Under Uncertainty. Master's thesis, Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management.
- Kall, P., & Wallace, S. W. (1997). Stochastic programming. Chichester: Wiley.
- Kam, L. E., & Leung, P. (2008). Financial risk analysis in aquaculture. Understanding and applying risk analysis in aquaculture, 153-209. doi:10.18411/a-2017-023
- Kaye, R., Outhred, H., & Bannister, C. (1990). Forward contracts for the operation of an electricity industry under spot pricing. *IEEE Transactions on Power Systems*, 5(1), 46-52. doi:10.1109/59.49085
- Kettunen, J., Salo, A., & Bunn, D. (2010). Optimization of Electricity Retailers Contract Portfolio Subject to Risk Preferences. *IEEE Transactions on Power Systems*, 25(1), 117-128. doi:10.1109/tpwrs.2009.2032233
- King, A. J., & Wallace, S. W. (2012). Modeling with stochastic programming. New York: Springer.
- Laks- og ørreteksporten tredoblet på 8 år – 65 milliarder i 2016. (n.d.). Retrieved April 3, 2017, from <http://www.mynewsdesk.com/no/seafood/pressreleases/laks-og-oerreteksporten-tredoblet-paa-8-aar-65-milliarder-i-2016-1720079>
- Larsen, T. A., & Asche, F. (2011). Contracts in the Salmon Aquaculture Industry: An Analysis of Norwegian Salmon Exports. *Marine Resource Economics*, 26(2), 141-150. doi:10.5950/0738-1360-26.2.141
- Lattanzio, G. (2015). Options and futures in the fishpool market: a brief analysis. Master's thesis. Norwegian School of Economics.
- Liu, M., Wu, F., F. (2007). Portfolio optimization in electricity markets *Electric Power Systems Research.*, 77 (8), 1000–1009, doi.org/10.1016/j.epr.2006.08.025
- Lodhi, M., (2015). Import demand elasticities for farmed salmon in the European Union and United States. Master's thesis, The Arctic University of Norway.
- Loewe, M. (n.d.). Scenario tree for a stochastic model with four stages [Digital image]. Retrieved from <https://www.gams.com/latest/docs/solvers/de/index.html>

- Marine Harvest ASA. (2017). 2016 annual rapport of Marine Harvest ASA. Retrieved from: <http://marineharvest.no/investor/annual-reports/>
- Markowitz, H. M. (1991). *Portfolio selection: Efficient diversification of investments*. Oxford: Blackwell.
- Martínez-de-Albéniz, V. and Simchi-Levi, D. (2006), Mean-variance trade-offs in supply contracts. *Naval Research Logistics*, 53: 603–616. doi:10.1002/nav.20186
- McNeil, A. J., Frey, R., & Embrechts, P. (2005). *Quantitative risk management: Concepts, techniques and tools*. Princeton, N.J: Princeton University Press.
- Misund, B., & Asche, F. (2016). Hedging efficiency of Atlantic salmon futures. *Aquaculture Economics & Management*, 20(4), 368-381. doi:10.1080/13657305.2016.1212123
- Mo, B., Gjelsvik, A., & Grundt, A. (2001). Integrated risk management of hydro power scheduling and contract management. *IEEE Transactions on Power Systems*, 16(2), 216-221. doi:10.1109/59.918289
- Norkin, V. I., Pflug, G. C. & Ruszczyński, A. (1998), ‘A branch and bound method for stochastic global optimization’, *Mathematical Programming* 83(1-3), 425–450.
- Norway Royal Salmon ASA. (2017). 2016 annual rapport of Norway Royal Salmon. Retrieved from: <http://norwayroyalsalmon.com/no/investor/Finansielle-rapporter>
- Mun, J. (2006). *Modeling Risk : Applying Monte Carlo Simulation, Real Options Analysis, Forecasting, and Optimization Techniques*. Hoboken, N.J.: Wiley.
- Nøkkeltall. (n.d.). Retrieved March 1, 2017, from <http://seafood.no/markedsinnsikt/nokkeltall/>
- Oglend, A. (2013). Recent trends in salmon price volatility. *Aquaculture Economics & Management*, 17(3), 281–99. doi:10.1080/13657305.2013.812155
- Pfeiffer, P. E. (1990). Variance and Standard Deviation. *Springer Texts in Statistics Probability for Applications*, 355-370. doi:10.1007/978-1-4615-7676-1_19
- Popela, P., Novotny, J., Roupec, J., Hrabec, D., & Olstad, A. (2014). Scenario tree for two-stage program [Digital image]. Retrieved from https://www.researchgate.net/publication/273064107_Two-stage_Stochastic_Programming_for_Engineering_Problems
- Posada, D., & Buckley, T. R.; Model Selection and Model Averaging in Phylogenetics: Advantages of Akaike Information Criterion and Bayesian Approaches Over Likelihood Ratio Tests. *Syst Biol* 2004; 53 (5): 793-808. doi: 10.1080/10635150490522304

- Protein Production Facts. (n.d.). Retrieved April 5, 2017, from <https://globalsalmoninitiative.org/en/sustainability-report/protein-production-facts/>
- Salmar ASA. (2017). 2016 annual rapport of Salmar ASA. Retrieved from: <http://www.salmar.no/arsrapporter>
- Salmon Farming Industry Handbook 2016, Marine Harvest. (n.d.). Retrieved February 21, 2017, from <http://marineharvest.com/investor/industry-handbook/>
- Sarykalin, S., Serraino, G., & Uryasev, S. (2008) Value-at-risk vs. conditional value-at-risk in risk management and optimization. *Tutorials in operations research, INFORMS*, p. 270–94.
- Schachter, B. (1997). 'An irreverent guide to Value-at-Risk', *Financial Engineering News*, 1, 1997.
- Shapiro, A., & Philpott, A. (2007). A Tutorial on Stochastic Programming, from http://www2.isye.gatech.edu/people/faculty/Alex_Shapiro/TutorialSP.pdf
- Shi, L. & Olafsson, S. (2000), 'Nested partitions method for stochastic optimization', *Methodology in Computing and Applied Probability* 2(3), 271–291.
- Solibakke, P. B. (2012). Scientific Stochastic Volatility Models for the Salmon Forward Market: Forecasting (Un-)Conditional Moments. *Aquaculture Economics & Management*, 16(3), 222-249. doi:10.1080/13657305.2012.704618
- Stein, J. (1961). The Simultaneous Determination of Spot and Futures Prices. *The American Economic Review*, 51(5), 1012-1025. Retrieved from <http://www.jstor.org/stable/1813846>
- Tauer, L. W. (1983). Target MOTAD. *American Journal of Agriculture Economics*, 65 (3), 606-610. doi: 10.2307/1240515
- The Norwegian Aquaculture Analysis 2016. (2016). Retrieved March 9, 2017, from [http://www.ey.com/Publication/vwLUAssets/EY_The_Norwegian_Aquaculture_Analysis/\\$File/EY-The-Norwegian-Aquaculture-Analysis-web.pdf](http://www.ey.com/Publication/vwLUAssets/EY_The_Norwegian_Aquaculture_Analysis/$File/EY-The-Norwegian-Aquaculture-Analysis-web.pdf)
- Topaloglou, N., Vladimirou, H., & Zenios, S. A. (2008). A dynamic stochastic programming model for international portfolio management. *European Journal of Operational Research*, 185(3), 1501-1524. doi:10.1016/j.ejor.2005.07.035
- Torrissen, O., Jones, S., Asche, F., Guttormsen, A., Skilbrei, O. T., Nilsen, F., Horsberg, T. E. and Jackson, D. (2013), Salmon lice – impact on wild salmonids and salmon aquaculture. *J Fish Dis*, 36: 171–194. doi:10.1111/jfd.12061

- Uryasev, S., & Rockafellar, R. T. (2001). Conditional Value-at-Risk: Optimization Approach. *Applied Optimization Stochastic Optimization: Algorithms and Applications*, 411-435. doi:10.1007/978-1-4757-6594-6_17
- What is salmon farming and why do we need it? (n.d.). Retrieved March 21, 2017, from <https://globalsalmoninitiative.org/en/what-is-the-gsi/what-is-salmon-farming-and-why-do-we-need-it/>
- Wets, R. J. (1974). Stochastic Programs with Fixed Recourse: The Equivalent Deterministic Program. *SIAM Review*, 16(3), 309-339. doi:10.1137/1016053

Appendix A

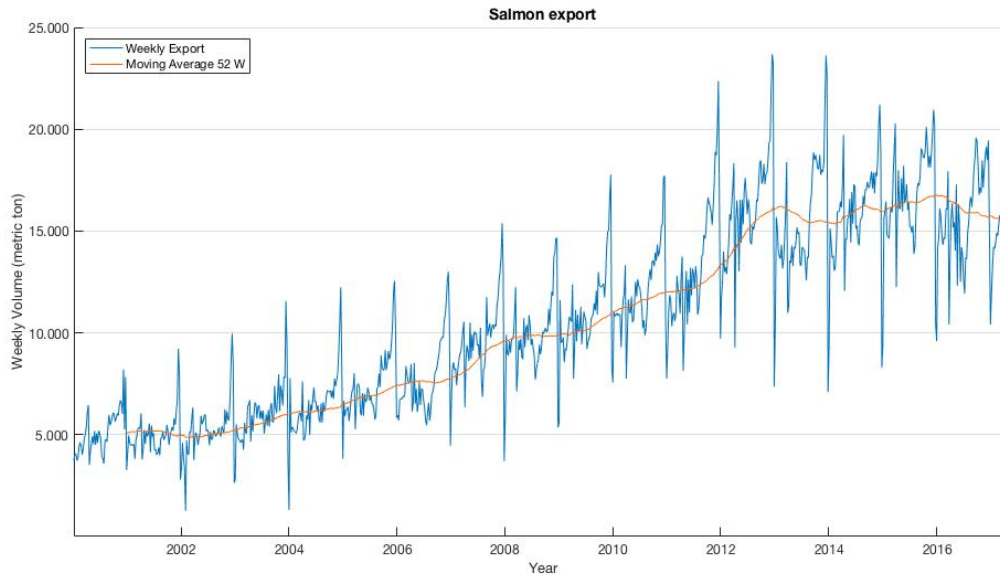


Figure 10.1: Weekly export of salmon and a 52 week Moving Average

Market threats

- Decreasing sale prices (prices of outputs)
- Increasing production costs (prices of inputs)
- Availability (scarcity) of inputs
- Escalating interest rates
- Decreasing market demand
- Limited market access
- Creditor instability

Figure 10.2: Market Threats (Bergfjord 2009)

Table 10.1: Percieved importance of risk factors (Bergfjord, 2009).

Risk source	Mean	SD	Rated among top three
Future salmon price	5.95	1.31	13
Uncertainty about market access/trade policy	5.39	1.26	26
Diseases	4.97	1.70	21
Future demand for salmon (for instance, substitution effect)	4.89	1.75	7
Market regulation measures	4.89	1.52	7
Sufficient sea area access	4.84	1.60	8
Future exchange rates	4.45	1.62	2
Future changes to license system	4.39	1.59	3
Future interest rates	4.05	1.43	0
Price of feed	3.97	1.62	5

Statement	Mean	SD
Our firm would actively use a futures market	3.61	1.79
We can predict future prices well enough to profit from such a market	2.89	1.62
Our company knows how to use a futures market	2.94	1.80
Using a futures market could substantially reduce our risk	3.69	1.53

Figure 10.3: Risk assessment statements (Bergfjord, 2009)

Realized spot vs forwards prices - on delivery

	Spot	3 month fwd	6 month fwd
Spot	1		
3 month fwd	0,877139163	1	
6 month fwd	0,814135241	0,953762025	1

Figure 10.4: Correlation between spot, 3m forwards and 6m forwards

Appendix B

Below in figure 11.1 is an example-representation of the decision result of running the Multi-Stage Stochastic Recourse Model. In table X the first four decision points are represented. There are 15 decision nodes which are disbursed throughout the time period of 0 through 3.

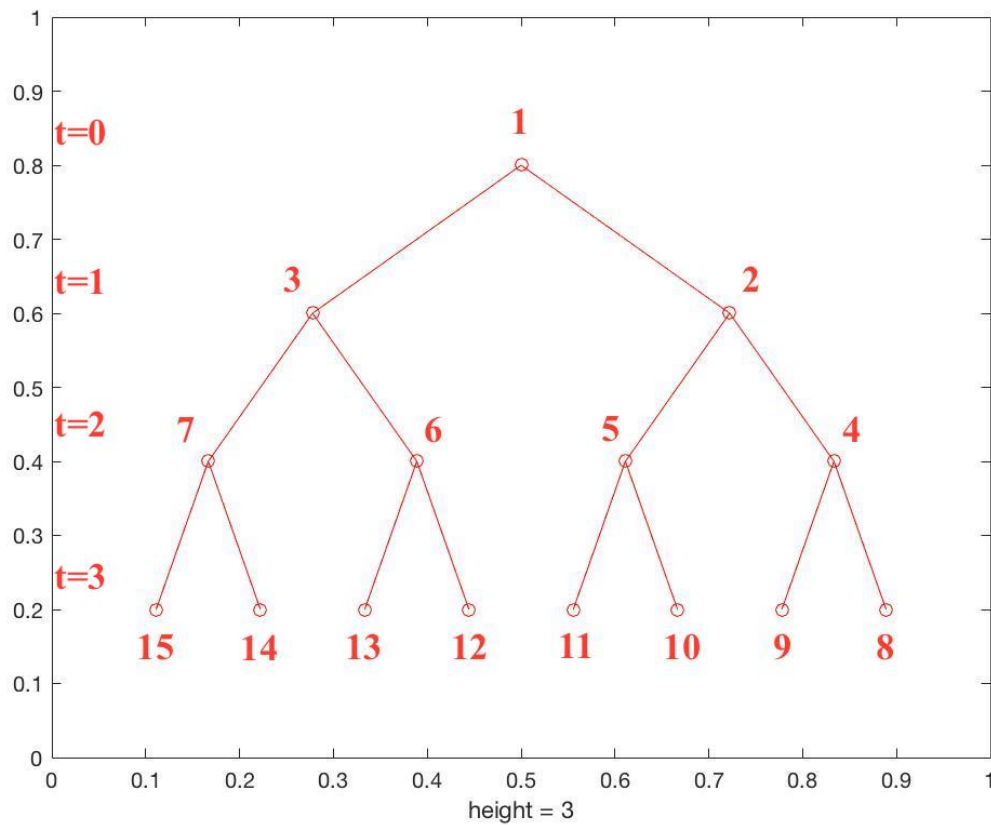


Figure 11.1: Scenario tree with numbered time periods and decision nodes

Figure 11.1 visualize to what time-period each decision node belongs. Table 11.1 represents the node in which the data and decision is locate, the forecasted spot price for each node, the projected production at each node, in addition to the decision at each node.

Table 11.1: Result from example run of Multi-Stage Recourse Model

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Price	63	65	68	62	63	68	67	62	64	62	65	67	68	69	67
Prod	1700	1611	1819	1691	1752	1756	1795	1776	1562	1695	1635	1707	1824	1643	1639

λ	One-month forward														
0,00	0	1691	1692	0	0	1469	32	0	0	0	0	18	35	37	20
0,01	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,02	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,03	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,04	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,48	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,49	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,50	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,51	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,52	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,96	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,97	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,98	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
0,99	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0
1,00	0	1691	1752	0	0	1636	0	0	0	0	0	0	0	0	0

λ	Two-month forward														
0,00	599	348	314	229	157	266	697	229	135	217	143	302	1143	264	87
0,01	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,02	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,03	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,04	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,48	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,49	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,50	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,51	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,52	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,96	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,97	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,98	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
0,99	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0
1,00	0	9	0	54	0	0	1724	0	0	16	0	33	1642	0	0

λ	Three-Month Forward															
0,00	0	0	6	0	0	16	11	0	0	0	0	22	30	42	21	
0,01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,02	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,97	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,98	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,99	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1,00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

λ	Four-Month Forward															
0,00	0	0	6	0	0	15	11	0	0	0	0	23	27	42	22	
0,01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,02	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,97	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,98	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,99	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1,00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

λ	Five-Month Forward															
0,00	0	0	6	0	0	15	12	0	0	0	0	20	17	30	22	
0,01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,02	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,97	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,98	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,99	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1,00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

λ	Six-Month Forward															
0,00	0	0	6	0	0	11	11	0	0	0	0	12	10	15	16	
0,01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,02	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,03	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,97	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,98	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0,99	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1,00	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

Appendix C

In this appendix a visualization of the application and interface of the Single-Period Horizon Model is presented. In the top half of figure 12.1 we can see the decision variables, spot and forward allocations, and both of the objective functions along with their risk metrics. The bottom half of figure 12.1 a representation of the optimized Expected Weekly Cash Flow objective function and its accompanying efficient frontier.

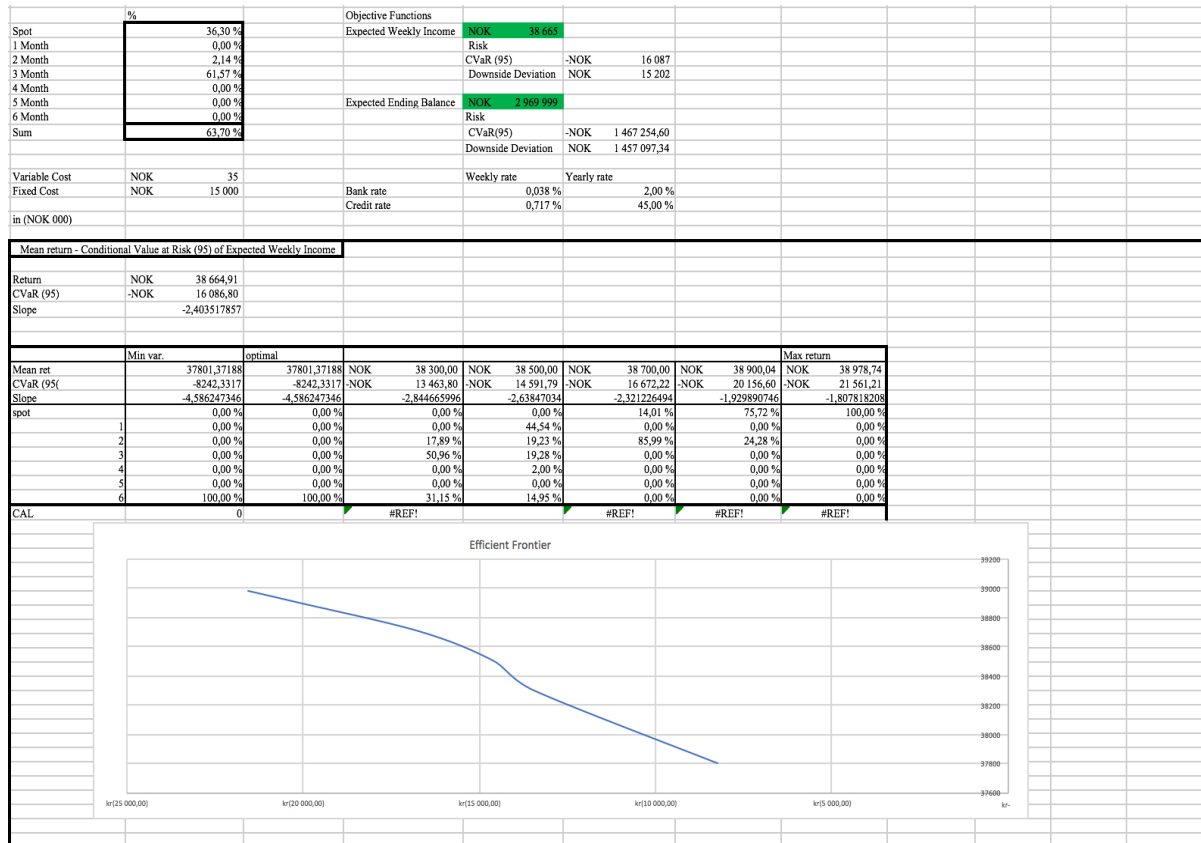


Figure 12.1: Visualization of Single-Stage Horizon Model interface

Figure 12.2 depicts an overview of the evolving bank balance of the 100 observations in each of the 1000 scenarios.

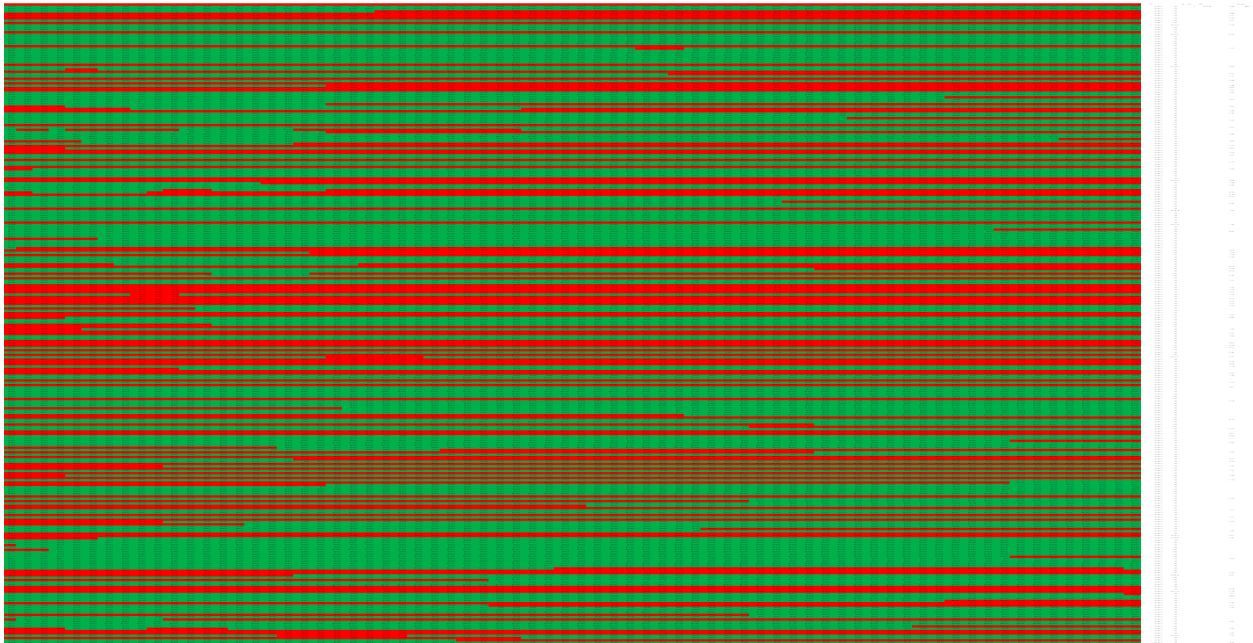


Figure 12.2: Overview of bank balance evolution in a set of scenarios

Figure 12.3 gives us a close-up view of the ending of the bank balance evolution along with the resulting risk metrics

	98	99	100	Var(95)	Count	Sum Count	CVaR (95)	Downside Deviation
-NOK	2 187 793,34	-NOK	2 219 052,33	-NOK	4 186 470,31	-	-	-
-NOK	2 894 773,66	-NOK	3 035 316,86	-NOK	4 186 470,31	-	-	-
-NOK	1 046 401,01	-NOK	1 077 453,64	-NOK	4 186 470,31	-	-	-
-NOK	679 799,31	-NOK	694 784,08	-NOK	4 186 470,31	-	-	-
-NOK	3 997 233,82	-NOK	4 058 150,25	-NOK	4 186 470,31	-	-	-
-NOK	2 737 743,86	-NOK	2 781 202,82	-NOK	4 186 470,31	-	-	-
-NOK	1 344 235,93	-NOK	1 352 006,57	-NOK	4 186 470,31	-	-	-
-NOK	2 179 133,72	-NOK	2 220 239,05	-NOK	4 186 470,31	-	-	-
-NOK	4 169 392,57	-NOK	4 222 279,36	-NOK	4 186 470,31	-	-	-
-NOK	2 532 002,96	-NOK	2 570 260,15	-NOK	4 186 470,31	-	-	-
-NOK	3 489 093,67	-NOK	3 593 287,74	-NOK	4 186 470,31	-	-	-
-NOK	1 795 515,05	-NOK	1 780 145,35	-NOK	4 186 470,31	-	-	-
-NOK	8 171 089,20	-NOK	8 334 362,56	-NOK	4 186 470,31	-	-	-
-NOK	541 488,09	-NOK	532 756,36	-NOK	4 186 470,31	-	-	-
-NOK	1 243 654,25	-NOK	1 264 200,45	-NOK	4 186 470,31	-	-	-
-NOK	213 949,54	-NOK	204 135,13	-NOK	4 186 470,31	-	-	-
-NOK	3 403 966,20	-NOK	3 510 249,10	-NOK	4 186 470,31	-	-	-
-NOK	2 575 310,08	-NOK	2 615 654,27	-NOK	4 186 470,31	-	-	-
-NOK	1 513 281,30	-NOK	1 559 876,91	-NOK	4 186 470,31	-	-	-
-NOK	502 479,29	-NOK	522 267,76	-NOK	4 186 470,31	-	-	-
-NOK	1 376 026,04	-NOK	1 401 616,98	-NOK	4 186 470,31	-	-	-
-NOK	2 648 344,47	-NOK	2 672 166,71	-NOK	4 186 470,31	-	-	-
-NOK	404 148,85	-NOK	397 252,65	-NOK	4 186 470,31	-	-	-
-NOK	1 679 840,21	-NOK	1 699 330,60	-NOK	4 186 470,31	-	-	-
-NOK	2 242 221,72	-NOK	2 280 190,30	-NOK	4 186 470,31	-	-	-
-NOK	3 484 453,81	-NOK	3 535 468,78	-NOK	4 186 470,31	-	-	-
-NOK	4 915 042,98	-NOK	5 008 245,45	-NOK	4 186 470,31	-	-	-
-NOK	3 752 124,71	-NOK	3 830 888,87	-NOK	4 186 470,31	-	-	-
-NOK	1 265 818,46	-NOK	1 326 985,42	-NOK	4 186 470,31	-	-	-
-NOK	2 225 878,61	-NOK	2 260 650,00	-NOK	4 186 470,31	-	-	-
-NOK	719 117,68	-NOK	745 715,38	-NOK	4 186 470,31	-	-	-
-NOK	8 024 927,20	-NOK	8 100 444,31	-NOK	4 186 470,31	-	-	-
-NOK	2 957 825,98	-NOK	3 020 694,98	-NOK	4 186 470,31	-	-	-
-NOK	1 359 844,12	-NOK	1 354 168,46	-NOK	4 186 470,31	-	-	-
-NOK	121 077,62	-NOK	131 744,68	-NOK	4 186 470,31	-	-	-
-NOK	43 846,23	-NOK	33 849,98	-NOK	4 186 470,31	-	-	-
-NOK	2 474 236,89	-NOK	2 513 502,57	-NOK	4 186 470,31	-	-	-
-NOK	3 319 757,65	-NOK	3 441 821,55	-NOK	4 186 470,31	-	-	-
-NOK	2 225 024,61	-NOK	2 340 359,87	-NOK	4 186 470,31	-	-	-
-NOK	423 208,15	-NOK	413 090,69	-NOK	4 186 470,31	-	-	-
-NOK	862 633,48	-NOK	961 865,41	-NOK	4 186 470,31	-	-	-
-NOK	1 334 752,50	-NOK	1 372 116,03	-NOK	4 186 470,31	-	-	-
-NOK	1 375 707,34	-NOK	1 389 369,11	-NOK	4 186 470,31	-	-	-
-NOK	1 759 926,32	-NOK	1 812 472,92	-NOK	4 186 470,31	-	-	-
-NOK		-NOK		-NOK	4 186 470,31	50	6 033 186,81	-2252064,998
								-1 880 390,01

Figure 12.3: Close-up view of bank balance with risk metrics

Appendix D

The following are the Efficient Frontiers for the tables included in chapter 8. Figure 13.1 through 13.4 show the efficient frontier for the original wide price margins and figure 13.5 through 13.8 show the efficient frontier for the model using the scenario with narrow price margins. Figure 13.9 show the efficient frontier for the mean-semi-deviation.

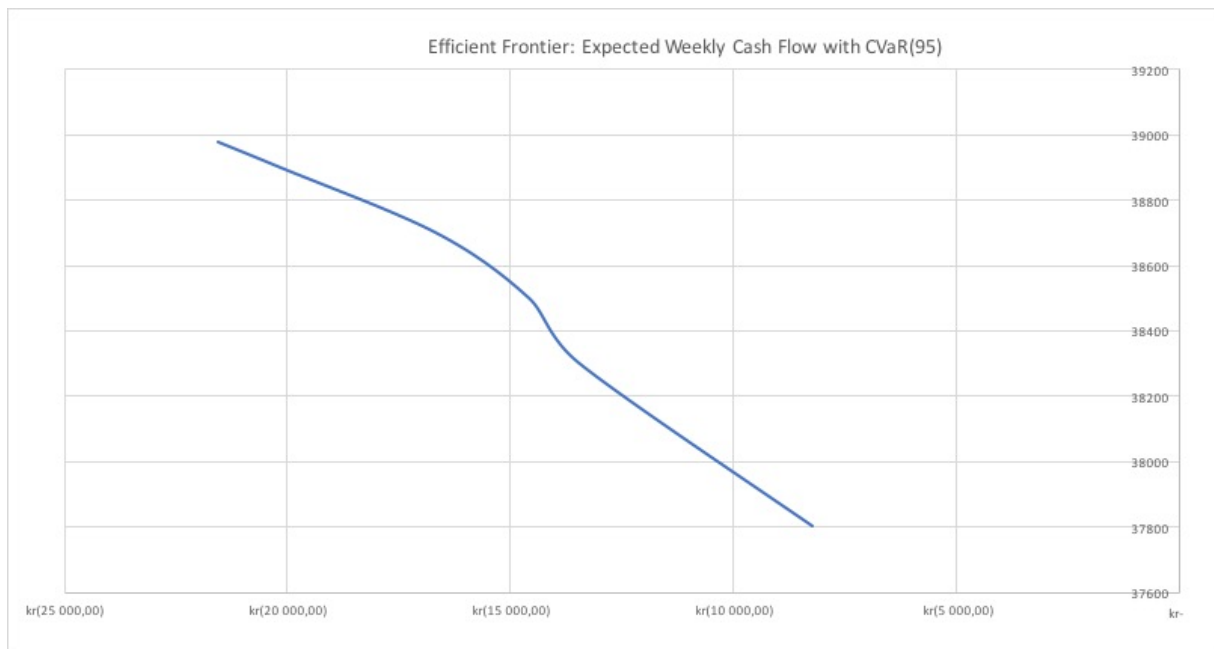


Figure 13.1: Efficient Frontier: Expected Weekly Cash Flow with CVaR(95)

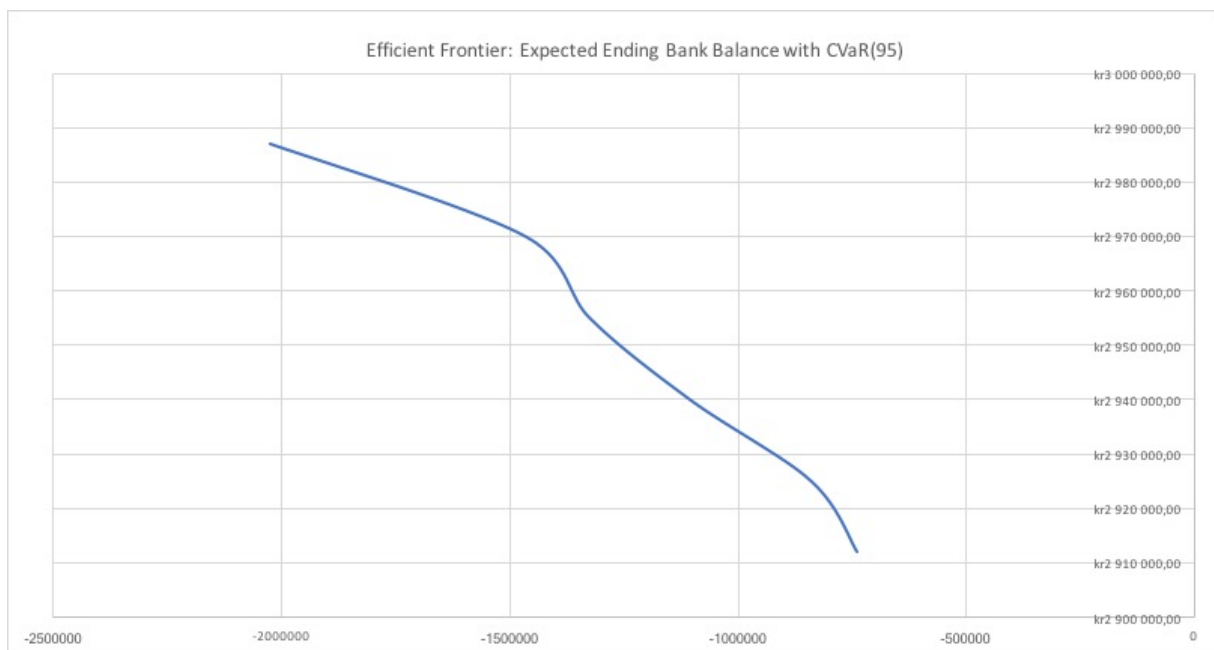


Figure 13.2: Efficient Frontier: Expected Ending Bank Balance with CVaR(95)

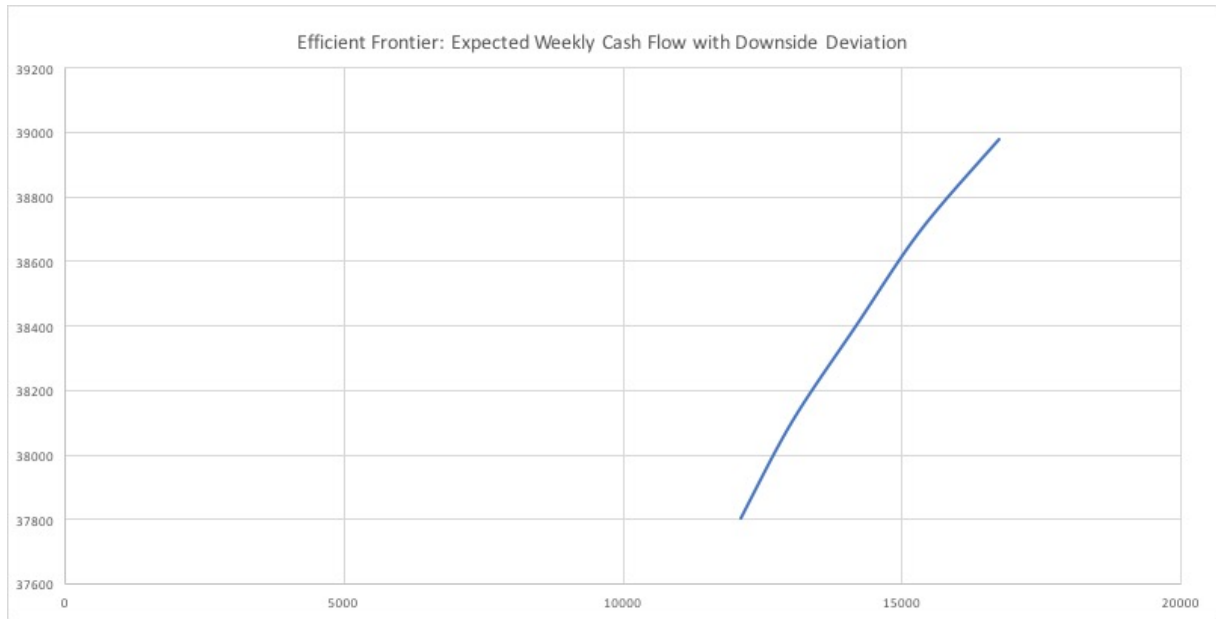


Figure 13.3: Efficient Frontier: Expected Weekly Cash Flow with Downside Deviation

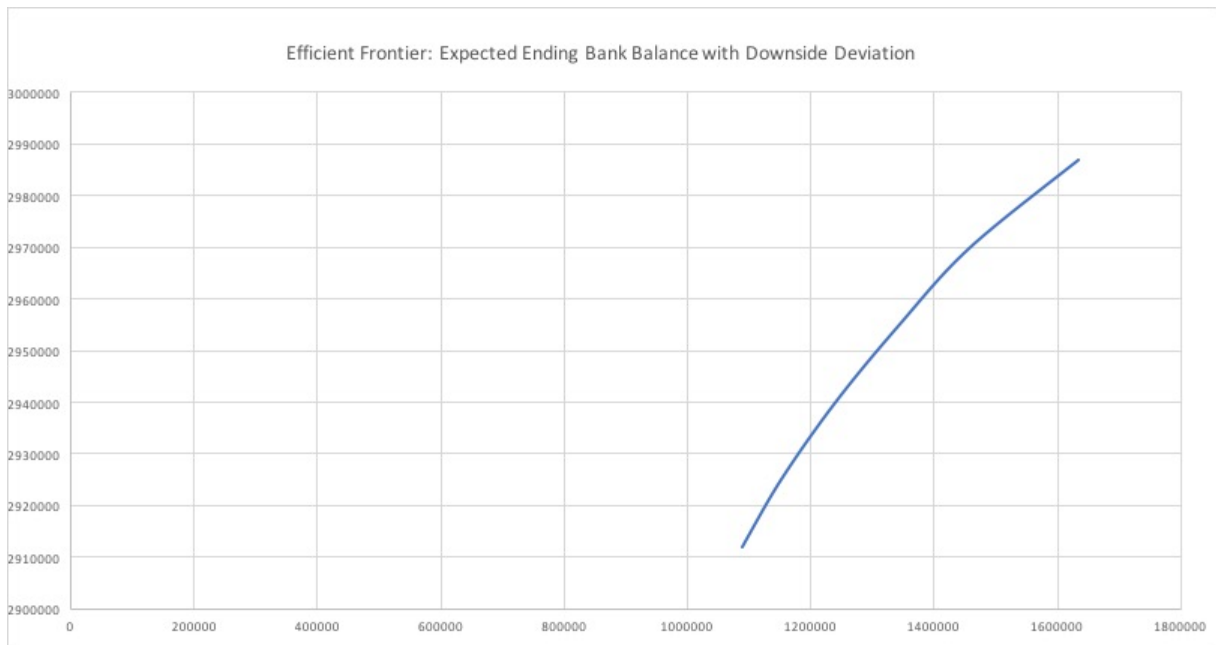


Figure 13.4: Efficient Frontier: Expected Ending Bank Balance with Downside Deviation

The following figures show the efficient frontier for the narrow margin scenario.

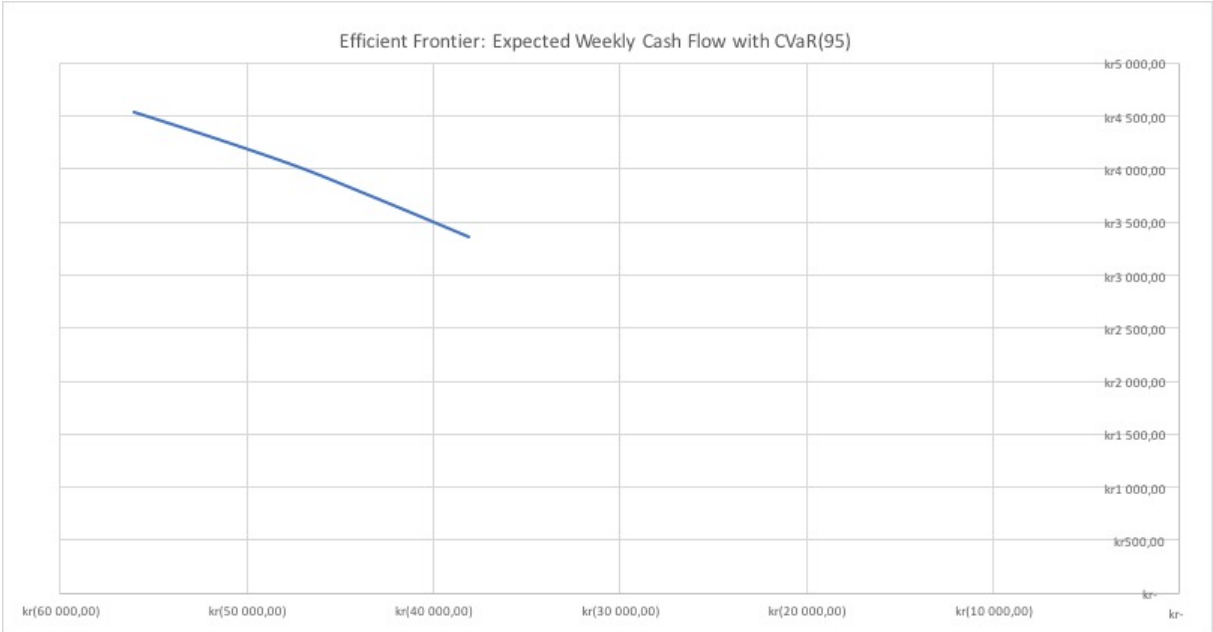


Figure 13.5: Efficient Frontier: Expected Weekly Cash Flow with CVaR(95) - Narrow margins

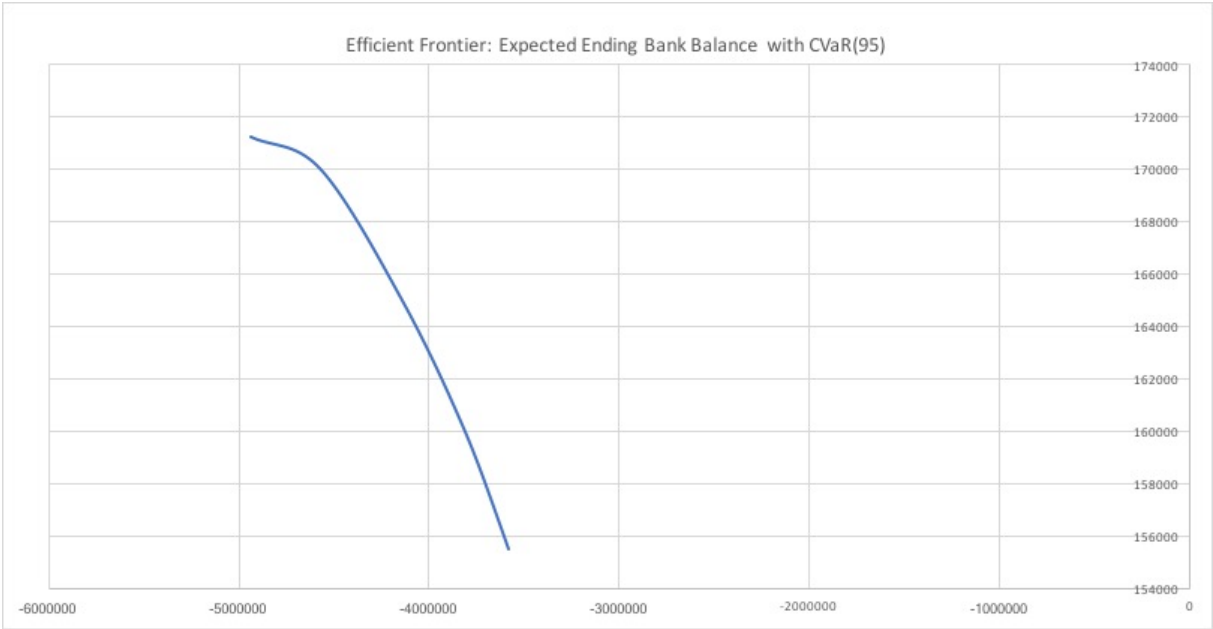


Figure 13.6: Efficient Frontier: Expected Ending Bank Balance with CVaR(95) - Narrow margins

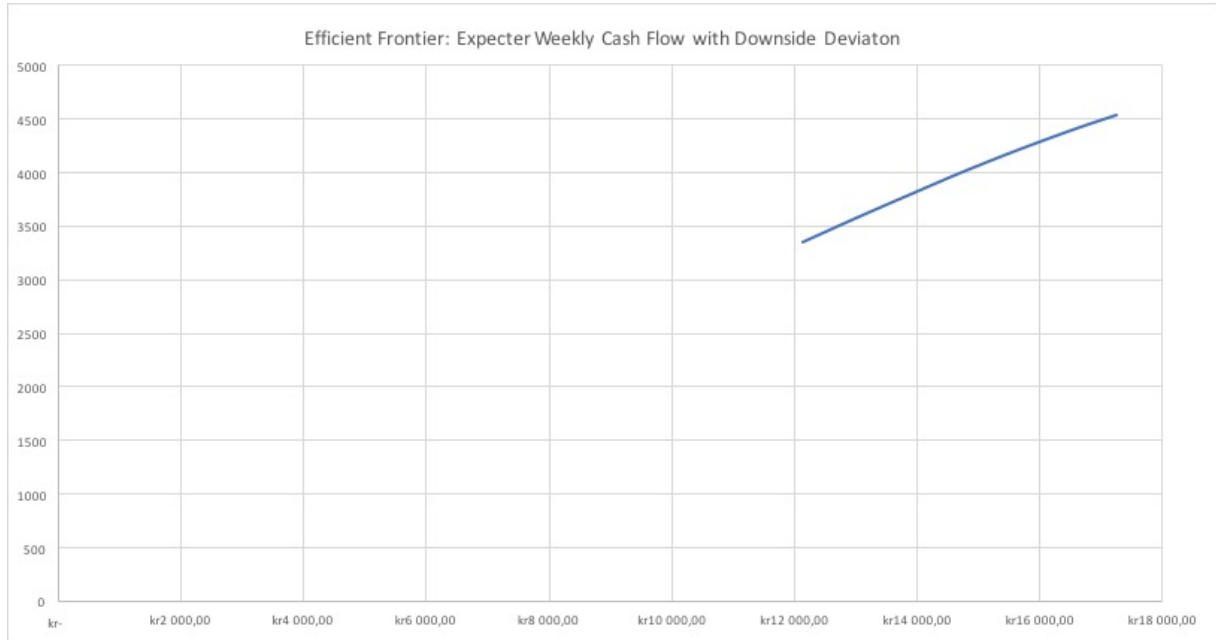


Figure 13.7: Efficient Frontier: Expected Weekly Cash Flow with Downside Deviation - Narrow margins

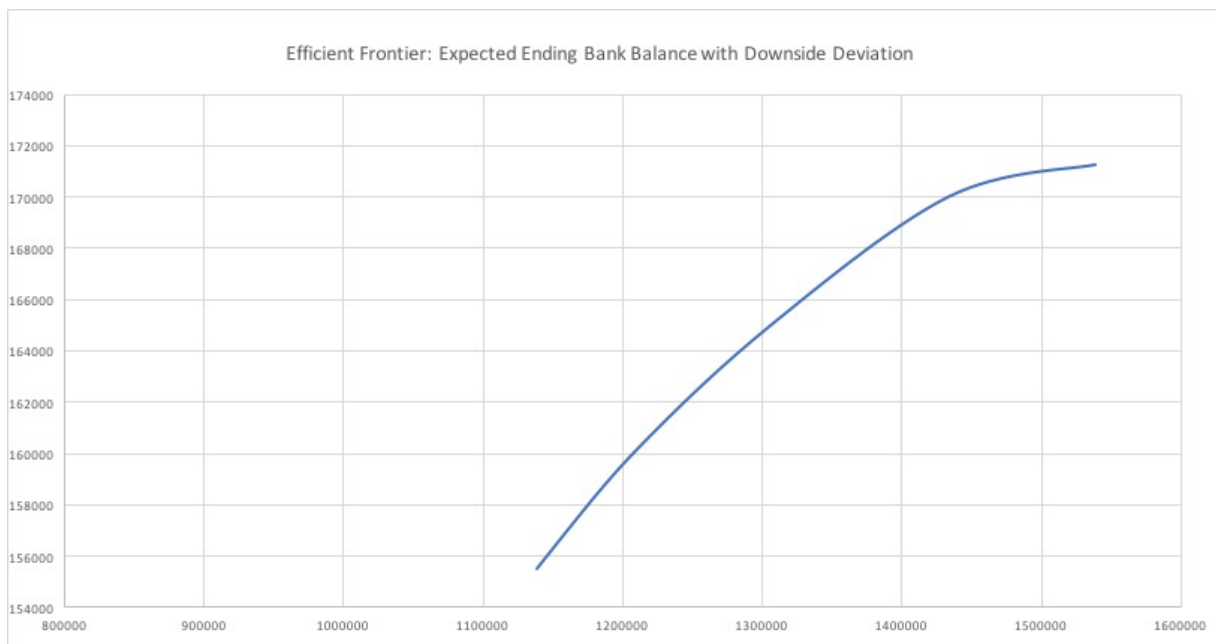


Figure 13.8: Efficient Frontier: Expected Ending Bank Balance with Downside Deviation - Narrow margins

Table 32 below display the efficient frontier for the mean-semi-deviation optimization.

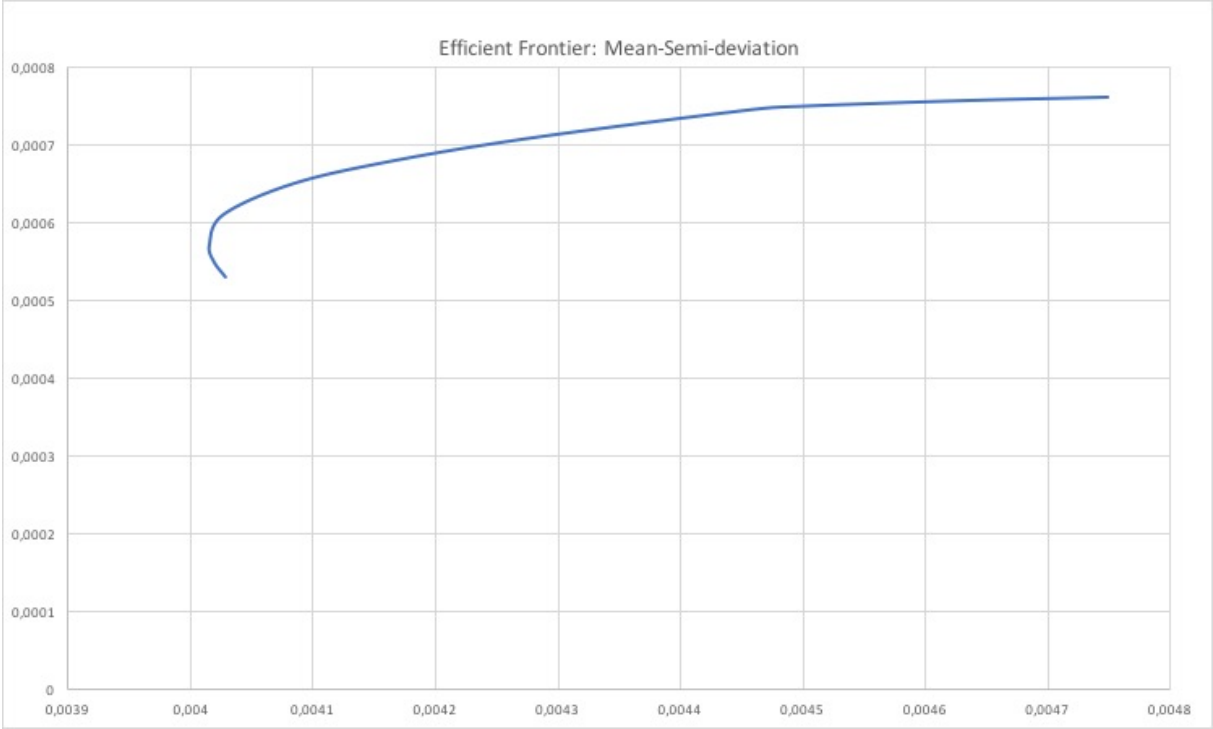


Figure 13.9: Efficient Frontier: Mean-Semi-deviation

Appendix E

The MATLAB SCRIPT created for the Multi-Stage Stochastic Recourse Model follows below:

```
clear all
clc

tic

BankInterest=0.02;
CreditInterest=0.1;
TransCosts=0.02;      % Transaction costs forward
Target=1500000;

D=53; % Here we specify the numer of lags of the stochastic process.
      % D has to be greater equal 1

ForwLength=6; % Lenght of the longest forward contract to be considered
              % important to know how many predecessors in tree

U=4; % Here we specify the subperiods from one to the next decision point

T=ForwLength+7; % The number of decision points
                % At least 1 pluss length of longest forward

B=2; % Branches in decsion points

%-----
% ESTIMATION BASED TREE STRUCTURE
%-----

node=1; % This variable represents the current node
parent=0; % This variable represents the current parent node

% Number of nodes in simulation tree
% D-1 is the pre-decision period
% (B-1)+(U-1)*(B^T-B)/(B-1) is the decision period

VS=(D-1)+(B^T-1)/(B-1)+(U-1)*(B^T-B)/(B-1);

% Number of nodes in decision based tree

VD=(B^T-1)/(B-1);

PS=zeros(D,VS); % vector for the simulation-based scenario tree
PD=zeros(ForwLength,VD); % vector for decision-based scenario tree
IndPD=zeros(1,VS);
```

```

for t=1:D-1
    node=node+1;
    parent=node-1;
    PS(1,node)=parent;
end

for t=1:T-1
    for i=1:B^(t-1)
        parent=parent+1;
        for j=1:B
            node=node+1;
            PS(1,node)=parent;
        end
    end
    for i=1:(U-1)*B^t
        node=node+1;
        parent=parent+1;
        PS(1,node)=parent;
    end
end

figure(1);
treepplot(PS(1,:));

% First degree predecessors unambiguously determine higher degree
% predecessors as follows:
for d=2:D
    for v=1:VS
        if PS(1,v)==0
            PS(d,v)=0;
        elseif PS(d-1,PS(1,v))==0
            PS(d,v)=0;
        else
            PS(d,v)=PS(d-1,PS(1,v));
        end
    end
end

%-----
% SIMULATION OF TREE
%-----

% Now the tree can be simulated with the desired stochastic model
% In what following we use an AR(2) process coupled with GARCH(2,1)
% Initial parameters AR(2):
alpha0=0.0421;
alpha1=0.1276;
alpha2=-0.2807;
SAR52=0.2485;

% Initial parameters GARCH(2):
beta0=0.1454;
beta1=0.4780;
beta2=0.3525;
beta3=0.1453;

% Here we create the variables that contain all simulated values
DeltaSpot=zeros(1,VS);
Noise=zeros(1,VS);
Noise2=zeros(1,VS);

```

```

Variance=zeros(1,VS);
SpotS=zeros(1,VS);
SpotD=zeros(1,VD);

% Previously observed/estimated variances and noise
Variance(52)=0.2;
Variance(53)=0.1;
Noise(52)=0.8;
Noise(53)=0.5;
Noise2(52)=-1;
Noise2(53)=1;

% First difference of spot
DeltaSpot(1)=0.4; DeltaSpot(2)=-2.9; DeltaSpot(3)=3; DeltaSpot(4)=6.33;
DeltaSpot(5)=-1.25; DeltaSpot(6)=-3.52;DeltaSpot(7)=-5.24;
DeltaSpot(8)=1.17; DeltaSpot(9)=-1.05; DeltaSpot(10)=3.18; DeltaSpot(11)=0;
DeltaSpot(12)=9.26; DeltaSpot(13)=-10.22; DeltaSpot(14)=0.6299;
DeltaSpot(15)=9.63; DeltaSpot(16)=5.58; DeltaSpot(17)=-3.24;
DeltaSpot(18)=-2.86; DeltaSpot(19)=9.22; DeltaSpot(20)=-0.5699;
DeltaSpot(21)=-9.73; DeltaSpot(22)=-7.69; DeltaSpot(23)=-1.88;
DeltaSpot(24)=1.52; DeltaSpot(25)=-1.06; DeltaSpot(26)=-2.23;
DeltaSpot(27)=0.3599; DeltaSpot(28)=-1.9; DeltaSpot(29)=-2.94;
DeltaSpot(30)=0.92; DeltaSpot(31)=1.91; DeltaSpot(32)=5.5299;
DeltaSpot(33)=0; DeltaSpot(34)=2.27; DeltaSpot(35)=1.61;
DeltaSpot(36)=1.02;DeltaSpot(37)=-1.9899;DeltaSpot(38)=-1.99;
DeltaSpot(39)=4.96; DeltaSpot(40)=-2.02; DeltaSpot(41)=1.34;
DeltaSpot(42)=3.2599; DeltaSpot(43)=5.46; DeltaSpot(44)=4.47;
DeltaSpot(45)=-0.62; DeltaSpot(46)=0.9399; DeltaSpot(47)=-3.86;
DeltaSpot(48)=-1.4599; DeltaSpot(49)=-3.15; DeltaSpot(50)=-5.26;
DeltaSpot(51)=-0.6899; DeltaSpot(52)=-0.6099; DeltaSpot(53)=-1.37;

% First spot price
SpotsS(1)=60.62; SpotsS(2)=61.02; SpotsS(3)=58.12;SpotsS(4)=61.12;
SpotsS(5)=67.45; SpotsS(6)=66.20; SpotsS(7)=62.68; SpotsS(8)=57.44;
SpotsS(9)=58.61; SpotsS(10)=57.56; SpotsS(11)=60.74; SpotsS(12)=60.74;
SpotsS(13)=70.00;SpotsS(14)=59.78; SpotsS(15)=60.41; SpotsS(16)=70.04;
SpotsS(17)=75.62; SpotsS(18)=72.38; SpotsS(19)=69.52; SpotsS(20)=78.74;
SpotsS(21)=78.17; SpotsS(22)=68.44; SpotsS(23)=60.75; SpotsS(24)=58.87;
SpotsS(25)=60.39; SpotsS(26)=59.33; SpotsS(27)=57.46; SpotsS(28)=55.56;
SpotsS(29)=52.62; SpotsS(30)=53.54; SpotsS(31)=55.45; SpotsS(32)=60.98;
SpotsS(33)=60.98; SpotsS(34)=63.25; SpotsS(35)=64.86; SpotsS(36)=65.88;
SpotsS(37)=63.89; SpotsS(38)=61.90; SpotsS(39)=66.86; SpotsS(40)=64.84;
SpotsS(41)=66.18; SpotsS(42)=69.44; SpotsS(43)=74.90; SpotsS(44)=79.37;
SpotsS(45)=78.75; SpotsS(46)=79.69; SpotsS(47)=75.83; SpotsS(48)=74.37;
SpotsS(49)=71.22; SpotsS(50)=65.96; SpotsS(51)=65.27; SpotsS(52)=64.66;
SpotsS(53)=63.29;

% Here the simulation of spot prices starts:

% %-----
% % Simulation of spot price changes
% %-----
for v=D+1:VS
Variance(v)=max(0,beta0+beta1*Variance(PS(1,v))+beta2*Variance(PS(2,v))+bet
a3*Noise(PS(1,v)));
    Noise(v)=normrnd(0,sqrt(Variance(v)));

DeltaSpot(v)=alpha0+alpha1*DeltaSpot(PS(1,v))+alpha2*DeltaSpot(PS(2,v))+SAR
52*DeltaSpot(PS(52,v))+Noise(v);
    SpotS(v)=SpotS(PS(1,v))+DeltaSpot(v);
end

```

```

% Here is the calculation of forward prices
ForwS=zeros(ForwLength,VS);

roh(1)=0.9183;
roh(2)=0.8844;
roh(3)=0.8535;
roh(4)=0.8315;
roh(5)=0.8146;
roh(6)=0.7901;

for i=1:ForwLength
    for v=D+i:VS

        ForwS(i,v)=SpotS(PS(4+4*i,v))+(SpotS(PS(4*i,v))-
SpotS(PS(4+4*i,v)))*roh(i);

    end
end

%-----
% DECISION BASED TREE STRUCTURE
%-----

% Reducing the tree to decision/result points
% First we make a vector that tells which nodes are decision and result
% nodes

node=D-1;
for t=1:T
    for j=1:B^(t-1)
        node=node+1;
        IndPD(node)=1;
    end
    node=node+(U-1)*B^t;
end

% Now we create the decision based scenario tree.
% First we have to create the structure of this reduced tree.

% We now define the scenario tree structure.
% first line/row in matrix are first order predecessors, second line second
% degree predecessors etc.
% We start with predecessors of first degree
counter=0;
for v=2:B:VD
    counter=counter+1;
    for b=0:B-1
        PD(1,v+b)=counter;
    end
end
end

```

```

% First degree predecessors unambiguously determine higher degree
% predecessors as follows:

for d=2:ForwLength
    for v=1:VD
        if PD(d-1,v)==0
            PD(d,v)=0;
        else
            PD(d,v)=PD(d-1,PD(1,v));
        end
    end
end

figure(2);
treeplot(PD(1,:));

% Then we have to copy the values from the large tree to the decision based
% tree by means of IndPD.

node=0; % We reuse the variable node
for v=D:VS
    if IndPD(v)==1
        node=node+1;
        % Here we need to copy all the info from the tree above to the
        % reduced tree
        SpotD(node)=SpotS(v);

        for i=1:ForwLength
            ForwD(i,node)=ForwS(i,v);
        end
    end
end

%-----
% Simulation of expected and realized production quantities
%-----

% In the following we assume uniformly distributed variables
QuantityD(1)=1700;
QuantExp=1700;
QuantVariation=400;

for v=2:VD

    QuantityD(v)=QuantExp+rand*QuantVariation-QuantVariation/2;
    ExpQuantD(v)=QuantExp;

end

```

```

% Now we plot the scenario tree
figure(1)
treepplot(PS(1,:))
[x,y] = treelayout(PS(1,:));
for i=1:length(x)
    text(x(i),y(i),num2str(SpotS(i)))
    text(x(i),y(i),num2str(IndPD(i)))
end

% Now we plot the scenario tree
figure(2)
treepplot(PD(1,:))
[x,y] = treelayout(PD(1,:));
for i=1:length(x)
    text(x(i),y(i),num2str(SpotD(i)))
end

% Now we plot the scenario tree
figure(3)
treepplot(PD(1,:))
[x,y] = treelayout(PD(1,:));
for i=1:length(x)
    text(x(i),y(i),num2str(QuantityD(i)))
end

% Now we plot the scenario tree
figure(4)
treepplot(PD(1,:))
[x,y] = treelayout(PD(1,:));
for i=1:length(x)
    text(x(i),y(i),num2str(ForwD(1,i)))
end

% Now we plot the scenario tree
figure(5)
treepplot(PD(1,:))
[x,y] = treelayout(PD(1,:));
for i=1:length(x)
    text(x(i),y(i),num2str(ForwD(2,i)))
end

% -----
% TRANSITION TO LINEAR PROGRAM
% -----

% Here we determine the sizes of submatrices: one submatrix per forward
% length
for i=1:ForwLength
    BlockSizeForw(i)=(B^(T-i)-1)/(B-1);
end

ColsForw=sum(BlockSizeForw); % Number of columns
ColsBank=(1-B^(T-1))/(1-B);
ColsCredit=(1-B^(T-1))/(1-B);
ColsNegDev=B^(T-1);

Cols=ColsForw+ColsBank+ColsCredit+ColsNegDev;

```

```

L=zeros(VD,Cols); % This is the matrix for the financial constraints
L3=zeros(VD,Cols); % This is the matrix for the Quantity constraints

% -----
% 1. Financial Balance and Quantity Constraints for Forwards
% -----

BlockStart=0;

% Forward Contracts

for i=1:ForwLength

    % In all periods from t=1 to t=T-1 we have equalities
    for t=1+i:T-1
        for v=(B^(t-1)-1)/(B-1)+1:(B^t-1)/(B-1)
            L(PD(i,v),BlockStart+PD(i,v))=-TransCosts;
            L(v,BlockStart+PD(i,v))=ForwD(i,v)-SpotD(v);
            L3(v,BlockStart+PD(i,v))=1;
        end
    end

    % In the last period we have a reversed sign

    for t=T
        for v=(B^(t-1)-1)/(B-1)+1:(B^t-1)/(B-1)
            L(PD(i,v),BlockStart+PD(i,v))=-TransCosts;
            L(v,BlockStart+PD(i,v))=-ForwD(i,v)+SpotD(v);
            L3(v,BlockStart+PD(i,v))=1;
        end
    end

    BlockStart=BlockStart+BlockSizeForw(i);

end

% LIQUIDITY ACCOUNT

L(1,BlockStart+1)=-1;

for v=2:(B^(T-1)-1)/(B-1)
    L(v,BlockStart+v)=-1;
    L(v,BlockStart+PD(1,v))=(1+BankInterest);
end

for v=(B^(T-1)-1)/(B-1)+1:VD
    L(v,BlockStart+PD(1,v))=-(1+BankInterest);
end

BlockStart=BlockStart+ColsBank; % BlockStart refers to end of Last Block

```

```

% CREDIT ACCOUNT

L(1,BlockStart+1)=1;

for v=2:(B^(T-1)-1)/(B-1)
    L(v,BlockStart+v)=1;
    L(v,BlockStart+PD(1,v))=-(1+CreditInterest);
end

for v=(B^(T-1)-1)/(B-1)+1:VD
    L(v,BlockStart+PD(1,v))=(1+CreditInterest);
end

BlockStart=BlockStart+ColsCredit; % BlockStart refers to end of previous
block

% DEVIATIONS

counter=0;
for v=(B^(T-1)-1)/(B-1)+1:VD
    counter=counter+1;
    L(v,BlockStart+counter)=-1;
end

BlockStart=BlockStart+ColsNegDev;

% We will here split up the matrix L into L1 (equality constraints) and
% L2 (inequalities)

L1=L(1:(B^(T-1)-1)/(B-1),:); % Equality Constraints
L2=L((B^(T-1)-1)/(B-1)+1:VD,:); % Inequality Constraints

% -----
% GENERATION OF R.H.S
% -----

b=zeros(VD,1);

b(1)=0;

% All periods from t=2 to T-1

for t=2:T-1
    for v=(B^(t-1)-1)/(B-1)+1:(B^t-1)/(B-1)
        b(v)=-SpotD(v)*QuantityD(v);
    end
end

% In the last period we have a reversed sign

for t=T
    for v=(B^(t-1)-1)/(B-1)+1:(B^t-1)/(B-1)
        b(v)=SpotD(v)*QuantityD(v)-Target;
    end
end

```



```

% Split b like L

b1=b(1:(B^(T-1)-1)/(B-1)); % Equality Constraints
b2=b((B^(T-1)-1)/(B-1)+1:VD); % Inequality Constraints

% -----
% RHS of Quantity Constraints
% -----

b3=zeros(VD,1);

for v=2:VD
    b3(v)=QuantityD(v-1);
end

% Now we add b3 to b2 and L3 to L2

b2=[b2
    b3];

L2=[L2
    L3];

count99=0
counter1000=0;
for lambda = 0:0.01:1

counter1000=counter1000+1;

% -----
% OBJECTIVE FUNCTION
% -----

c=zeros(1,Cols);
cExp=zeros(1,Cols);
cRisk=zeros(1,Cols);
Prob=1/(B^(T-1));
% ub=zeros(1,Cols);

% -----
% OBJECTIVE: FORWARDS
% -----

BlockStart=0;

for i=1:ForwLength

    % In the last period we have a reversed sign

    for t=T
        for v=(B^(t-1)-1)/(B-1)+1:(B^t-1)/(B-1)
            c(BlockStart+PD(i,v))=c(BlockStart+PD(i,v))-
            Prob*lambda*(ForwD(i,v)-SpotD(v));
        end
    end
end

```

```

cExp(BlockStart+PD(i,v))=cExp(BlockStart+PD(i,v))+Prob*(ForwD(i,v)-
SpotD(v));
    end
end

    BlockStart=BlockStart+BlockSizeForw(i);

end

% BlockStart=ColsForw;

% -----
% OBJECTIVE: LIQUIDITY ACCOUNT
% -----

for v=(B^(T-1)-1)/(B-1)+1:VD
    c(BlockStart+PD(1,v))=c(BlockStart+PD(1,v))-
    Prob*lambda*(1+BankInterest);

cExp(BlockStart+PD(1,v))=cExp(BlockStart+PD(1,v))+Prob*(1+BankInterest);
end

BlockStart=BlockStart+ColsBank;

% -----
% OBJECTIVE: CREDIT ACCOUNT
% -----

for v=(B^(T-1)-1)/(B-1)+1:VD

c(BlockStart+PD(1,v))=c(BlockStart+PD(1,v))+Prob*lambda*(1+CreditInterest);
    cExp(BlockStart+PD(1,v))=cExp(BlockStart+PD(1,v))-
    Prob*(1+CreditInterest);
end

BlockStart=BlockStart+ColsCredit;

% -----
% OBJECTIVE: DEVIATIONS
% -----

for counter=1:B^(T-1)
    c(BlockStart+counter)=Prob*(1-lambda);
    cRisk(BlockStart+counter)=Prob;
end

BlockStart=BlockStart+ColsNegDev;

toc

```

```

% -----
% SOLVE LINEAR PROGRAM
% -----

% Now we solve the linear programming problem.

lb=c*0; % Note that all decision variables are non-negative.

[x,fval,exitflag,output]=linprog(c,L2,b2,L1,b1,lb);

toc

WholeMatrix=[x'    0
              c    c*x
              cExp cExp*x
              cRisk cRisk*x
              L1    b1
              L2    b2];

Decisions(counter1000,:)=x;
Risk(counter1000)=cRisk*x;
Wealth(counter1000)=cExp*x;
count99 = count99+1
end

```