Impact of Inertial Response Requirements on a Multi Area Renewable Network

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***Abstract*—Developments in renewable integration are continuously changing power system portfolios globally. Higher volatility of the networks might pose a threat to grid stability and thus increase the need for ancillary services. In this paper one such service - the provision of inertial frequency response (in short referred to as *inertia*) - is analyzed. An additional demand constraint is added to a SMIP (Stochastic Mixed Integer Problem) formulation of an interconnected two-area system consisting of wind, hydro and conventional thermal plants. Environmental stochastic influences - wind curtailment and hydrological inflow - as well as demand fluctuation, forecasting errors and inter-area congestion are incorporated. The potential of cross-border trade of inertial response such as the impact of inertia requirements on traditional scheduling is analyzed and discussed.**

***Index Terms*—Inertia, MIP, Stochastic Programming, Generation Scheduling, Ancillary Service, Cutting Plane**

I. Introduction

Growing integration of 'green' generation into power grids lead to an increase in demand for ancillary and balancing services. Those services are a necessity to stabilize grids with high shares of renewable energy, a form of generation more prone to deviation [1]. One factor describing grid quality - in terms of stability - is the reaction time to frequency deviations, also referred to as inertial (frequency) response or *'inertia'* [2], [3]. A wide range of research on this topic has been carried out over recent years, examples include the impact of frequency control on market dispatch [4] or the impact of wind power integration on grid stability [5]. However, no models to quantify the cost impact on a power system have been analyzed, which this paper aims to provide in a novel approach. The chosen method was a scheduling model, a method with long history, initially used in deterministic single unit systems [6] and recently focused on stochastic influences such as presented in [7], [8] and [9] for hydro-thermal, [10] for wind-hydro or [11] for wind-hydro. This paper aims to gather the ideas proposed here and add various components to show the impact of inertial response.

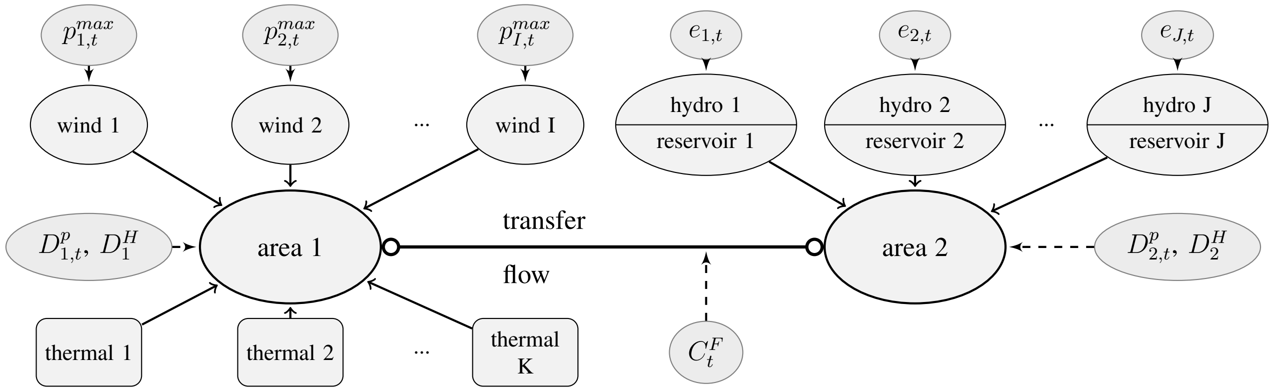
Nomenclature

$i = 1, ..., I$	wind units
$j = 1, ..., J$	hydro units
$k = 1, ..., K$	conventional (thermal) units
$t = 1, ..., T$	time period
$a = [1, 2]$	areas
$P = \{i, j, k\}$	total available plants
$P_1 = \{i, k\}$	available plants in area 1
$o_{P,t} \in \{0, 1\}$	plant state: off/on
$p_{P,t} \in \mathbb{R}^+$	unit output level
$r_{j,t} \in \mathbb{R}^+$	hydro inventory
$F_t \in \mathbb{R}$	(bidirectional) area transfer flow
τ	current time period
$p_h^{min/max}$, $p_k^{min/max}$	min/max output level hydro and thermal
p_P^n	nominal power plant output
p_i^{min}, $p_{i,t}^{max}$	min/max output level wind
r_j^{max}	hydro reservoir size
$e_{j,t}$	hydro reservoir inflow
$\theta_{k,t}$	current thermal runtime
θ_k^{max}	max thermal uptime (consecutive)
θ_k^{min}	min thermal downtime (consecutive)
α_P, β_P, γ_{P_1}	cost factors
$c_k^{up/down}$	thermal start/stop cost
$D_{a,t}^p$	electricity demand
C_t^F	flow capacity
H_P	inertia constant
D_a^H	inertia demand
Λ_t	hydro inventory coefficient
$W(t)$	social welfare function
$O_{t,k}^{up}$	minimum amount of starts over total time period T
$O_{t,k}^{downt}$	minimum amount of downtime over total time period T
$\mathbf{E}[D_{a,t}^p]$	electricity demand forecast
$\mathbf{E}[C_t^F]$	expected flow capacity
$\mathbf{E}[p_{i,t}^{max}]$	wind curtailment forecast
$\mathbf{E}[e_{j,t}]$	reservoir inflow forecast

II. Model Formulation

In the here presented model, the plants participating in the scheduling will be separated into two adjacent areas by a flow constraint. Fig. 1 illustrates this setup: I wind power

Fig. 1. Model Setup



plants and K conventional (thermal) power plants meet in a node, from here on referred to as *area*, and are connected by a transfer line to a second area with an array of J hydro power plants. The chosen representation for the reservoirs of the hydro power plants was the aggregated single reservoir form [12], [13]. This allowed to neglect interactions between the hydro facilities (one might think of outflow influencing the inflow of others) as well as different reservoir constellations which would otherwise increase the complexity of the model unnecessarily[1].

Objective function:

$$
\begin{aligned}
&\underset{o_{i/j/k,t}\ p_{i/j/k,t}\ r_{j,t}\ F_t}{\text{minimize}} W(t) = \sum_{t=\tau}^{T} -\Lambda_t \sum_{j=1}^{J} r_{j,t} \\
&+ \sum_{t=\tau}^{T}\sum_{i=1}^{I} \alpha_i o_{i,t} + \beta_i p_{i,t} \\
&+ \sum_{t=\tau}^{T}\sum_{j=1}^{J} \alpha_j o_{j,t} + \beta_j p_{j,t} + \gamma_j p_{j,t}^2 \\
&+ \sum_{t=\tau}^{T}\sum_{k=1}^{K} \alpha_k o_{k,t} + \beta_k p_{k,t} + \gamma_j p_{k,t}^2 \\
&+ \sum_{t=\tau}^{T}\sum_{k=1}^{K} o_{k,t}(1-o_{k,t+1}) c_k^{down} \\
&+ \sum_{t=\tau}^{T}\sum_{k=1}^{K} o_{k,t}(1-o_{k,t-1}) c_k^{up}
\end{aligned}
\tag{1}
$$

The chosen goal was to maximize welfare by fulfilling the inelastic demand in both areas. The objective function is shown in (1), it consists of the targets to minimize total cost (wind farms have linear cost curves, thermal and hydro quadratic curves) and maximimize reservoir inventory. Starting costs for conventional plants were included, but the low extend of those factors for hydro and wind power plants led to them not being included in the model. The initial plant states were considered as off ($o_{i/j/k,0} = 0$). As shown in [12], to incentivize saving water in hydro power plants, the value of the reservoir inventory (the so-called 'water value' Λ) can be assumed to have an increasing value over time. As shown in [12], the monetary difference in reservoir inventory ('water value' Λ_t) has to fulfill the condition $\Lambda_t < \Lambda_{t+1}$ in each period[2]. However, as the focus of this paper did not lie on determining this value, it was set to a static, very minor number($\sum_t \Lambda_t \to 0$). The aim of the here presented model was to schedule the generators in every single period t starting from the current period τ until the final period T, under consideration of the already committed resources in $t < \tau$ such as the reservoir levels and the total thermal uptime.

$$
\begin{aligned}
D_{1,t}^{p} &\le \sum_{i=1}^{I} p_{i,t} + \sum_{k=1}^{K} p_{k,t} + F_t \ \ \forall t = \tau \\
\mathbf{E}[D_{1,t}^{p}] &\le \sum_{i=1}^{I} p_{i,t} + \sum_{k=1}^{K} p_{k,t} + F_t \ \ \forall t = \tau+1,...,T
\end{aligned}
\tag{2}
$$

(2) and (3) shows the electricity demand fulfillment constraints. The flow in between was restricted through the time variable flow capacity constraint presented in (4)[3].

$$
\begin{aligned}
D_{2,t}^{p} &\le \sum_{j=1}^{J} p_{j,t} - F_t \ \ \forall t = \tau \\
\mathbf{E}[D_{2,t}^{p}] &\le \sum_{j=1}^{J} p_{j,t} - F_t \ \ \forall t = \tau+1,...,T
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
-C_t^F &\le F_t \le C_t^F \ \ \forall t = \tau \\
-\mathbf{E}[C_t^F] &\le F_t \le \mathbf{E}[C_t^F] \ \ \forall t = \tau+1,...,T
\end{aligned}
\tag{4}
$$

[1] pumping was not included in the here presented model

[2] other cases like the risk of spilling in high inflow scenarios which were chosen to be neglected in this model

[3] It has to be noted that the flow capacity has a negative lower bound. In solvers unable to compute this, two variables with opposing directions achieve a similar outcome in summation.

$$\sum_{i=1}^{I} o_{i,t} \times p_i^n \times H_i + \sum_{j=1}^{J} o_{j,t} \times p_j^n \times H_j$$
$$+ \sum_{k=1}^{K} o_{k,t} \times p_k^n \times H_k \geq D^H \quad \forall t = \tau, ..., T \tag{5a}$$

in case the inertial response cannot be shared:

$$\sum_{i=1}^{I} o_{i,t} \times p_i^n \times H_i$$
$$+ \sum_{k=1}^{K} o_{k,t} \times p_k^n \times H_k \geq D_1^H \quad \forall t = \tau, ..., T \tag{5b}$$
$$\sum_{j=1}^{J} o_{j,t} \times p_j^n \times H_j \geq D_2^H \quad \forall t = \tau, ..., T$$

In addition to the demand fulfillment, the market areas also impose inertia requirements on the plant schedules, as shown in (5). The calculation method was derived from the formulation of total system inertia in [14]. Both cases of shared eqrefeq:inertiademandA and separated fulfillment (5b) can be studied with the model. This separation solely focuses on catering to an inertial response requirement in an AC network, demand fulfillment is considered pooled in any of the presented cases. Inertia was considered to be able to be provided by all involved means of generation, in case of wind plants through additional curtailment [5].

$$\begin{aligned} o_{i,t} \times p_i^{min} \leq p_{i,t} & \\ p_{i,t} \leq o_{i,t} \times p_{i,t}^{max} & \quad \forall i = 1, ..., I;\ t = \tau \\ o_{i,t} \times p_i^{min} \leq p_{i,t} & \\ p_{i,t} \leq o_{i,t} \times \mathbf{E}[p_{i,t}^{max}] & \quad \forall i = 1, ..., I;\ t = \tau + 1, ..., T \end{aligned} \tag{6}$$

(6) realizes the wind capacity constraints, whereas the maximum possible output is variable over time. The reason lies in the curtailment of wind.

$$\begin{aligned} o_{j,t} \times p_j^{min} \leq p_{j,t} & \\ p_{j,t} \leq o_{j,t} \times p_j^{max} & \quad \forall j = 1, ..., J;\ t = \tau, ..., T \end{aligned} \tag{7}$$

$$\begin{aligned} o_{k,t} \times p_k^{min} \leq p_{k,t} & \\ p_{k,t} \leq o_{k,t} \times p_k^{max} & \quad \forall k = 1, ..., K;\ t = \tau, ..., T \end{aligned} \tag{8}$$

(7) and (8) show the capacity constraints for the other plant types.

$$\begin{aligned} \theta_{k,t} \geq o_{k,t} + o_{k,t} \times \theta_{k,t-1} & \quad \forall k = 1, ..., K;\ t = 1, ..., T \\ 0 \leq \theta_{k,t} \leq \theta_k^{max} & \quad \forall k = 1, ..., K;\ t = 1, ..., T \end{aligned} \tag{9}$$

As mentioned above, there has to be a maximum thermal uptime imposed, realized through (9). This constraint is based on the potential of overheating of units, which has to be avoided through forcing the units to stop to cool off. It has to be mentioned that the nested variable $\theta_{k,t-1}$ in itself is limited by $o_{k,t-1} + o_{k,t-1} \times \theta_{k,t-2}$ with the emerging row of constraints proceeding until $o_{k,0} = 0$.

$$\theta_{k,t} = \sum_{y=1}^{T} \prod_{t=y}^{T} o_{k,t} \quad \forall k = 1, ..., K \tag{10}$$

The series notation shown in (10) demonstrates that the constraint is of quadratic nature and thus would transform the existing problem from a MIP to a MIQCP (Mixed Integer Quadratic Constrained Problem), thus result in avoidable complications (as described in [15]) - as the conventional plants constitute the most expensive plants and thus will be run in support (as a peak load plant) to the other means of production, thus in most cases already fulfilling the required downtime constraint without the need to have it imposed strictly [1]. Thus a cutting plane algorithm, as shown in Fig. 2 and the necessary dynamic capacity constraint (11) was determined a fitting solution and introduced to the model to enable it to deal with and decrease the level of complexity.

$$\sum_{t=\tau}^{T} o_{k,t} \times (1 - o_{k,t-1}) \geq O_{\tau,k}^{up} \quad \forall k = 1, ..., K \tag{11}$$

In a similar manner, the minimum downtime restriction (12) - required to give the thermal units the necessary time to cool down - had to be implemented.

$$\sum_{t_2=0}^{\theta_k^{min}-1} o_{k,t+t_2} \leq 0 \quad \forall t = 3, ..., T - \theta_k^{min} \tag{12}$$
$$\text{where } o_{k,t-2} \times (1 - o_{k,t-1}) = 1$$

This was realized by adding another dynamic constraint with the aim of increasing the total downtime (or, in other words, decreasing the total uptime) of the units[4] until either a feasible solution was reached or infeasibility determined, as seen in (13).

$$\sum_{t=\tau}^{T} o_{k,t} \leq T - O_{\tau,k}^{downt} \quad \forall k = 1, ..., K \tag{13}$$

$$\begin{aligned} r_{j,t} \leq r_{j,t-1} + e_{j,t} - p_{j,t} & \quad \begin{array}{c} \forall j = 1, ..., J; \\ t = 1, ..., \tau \end{array} \\ r_{j,t} \leq r_{j,t-1} + \mathbf{E}[e_{j,t}] - p_{j,t} & \quad \begin{array}{c} \forall j = 1, ..., J; \\ t = \tau + 1, ..., T \end{array} \end{aligned} \tag{14}$$

(14) displays the inventory function of the reservoirs. Spilling is indirectly considered through the opportunity costs of losing one unit of Λ for each unit of $e_{j,t}$ spilled. (15) defines the reservoir size.

$$r_{j,t} \leq r_j^{max} \forall j = 1, ..., J;\ t = 1, ..., T \tag{15}$$

Determining a price for Inertia through dual values of the inertia constant demand constraint (5) is restricted by the fact that Integer Problems (IPs) do not offer a straight forward approach for the determination of shadow prices. In consideration of the constraint setup however, it can be seen that there only exists one direct impact lever for an increase of inertial response - constraint (set) (5), meaning that Inertia as such should have a marginal cost function of 0, as there are no variable cost components necessary to consider in increasing total system inertia, causing the sole existance of

[4]not to be mistaken with the total startups/stops

Fig. 2. Cutting Plane Algorithm for Time Period t

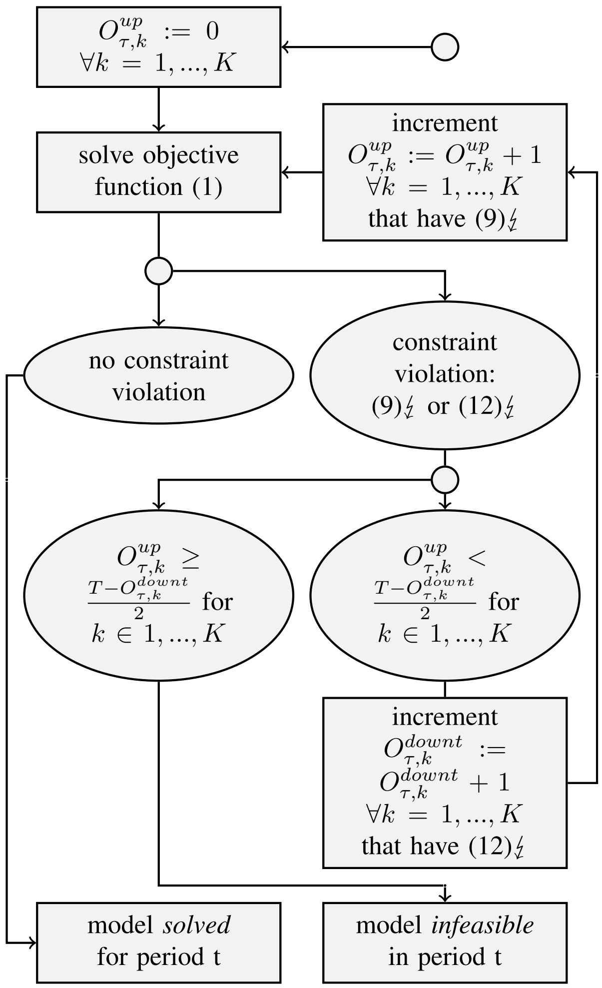


Fig. 3. Inertia Demand Scenario with $RH = LH$

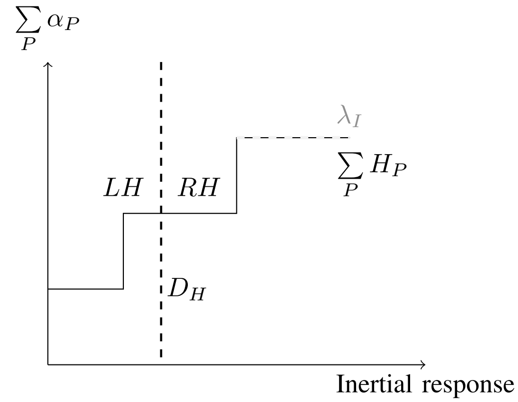
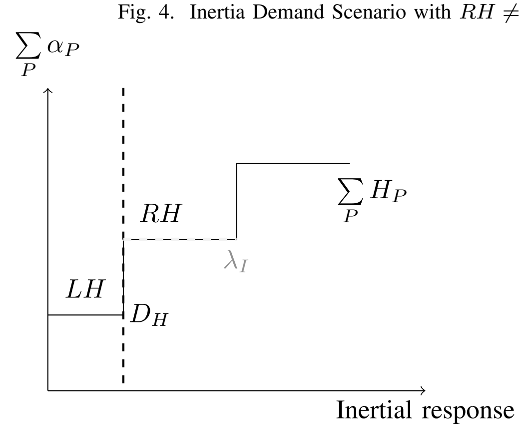


Fig. 4. Inertia Demand Scenario with $RH \neq LH$



long term implications of inertia requirements, which should be included in the investment decision rather than the day to day expenses. Another implication demonstrated in [16], the fallacy of strongly inelastic demand - as assumed in the here proposed model - is given by the (potential) difference in right-hand (RH) and left-hand (LH) side values, as shown in Fig. 3 and Fig. 4.

$$
\begin{aligned}
\lambda_{I,\tau} =& \min\{\frac{\alpha_P}{p^n_P \times H_P} + \psi_\tau \,|\, o_{P,\tau} = 0\} \\
& \text{with new objective function:} \\
W(t) +& \begin{vmatrix} \psi_1 \\ ... \\ \psi_T \end{vmatrix}^T \begin{vmatrix} C^F_T - F_1 \\ ... \\ C^F_T - F_T \end{vmatrix} \\
\text{and} & \quad \psi_t \geq 0
\end{aligned}
\tag{16a}
$$

$$
\begin{aligned}
& \text{or in case of inertia-segregated areas:} \\
\lambda_{I,\tau,1} =& \min\{\frac{\alpha_{P_1}}{p^n_{P_1} \times H_{P_1}} \,|\, o_{P_1,\tau} = 0\} \\
\lambda_{I,\tau,2} =& \min\{\frac{\alpha_j}{p^n_j \times H_j} \,|\, o_{j,\tau} = 0\}
\end{aligned}
\tag{16b}
$$

However, it was still deemed possible to give a quantiative estimation for the value of the short term impact of inertia. The 'shadow price' of inertial response - $\lambda_{I,\tau}$ (or $\lambda_{I,\tau,1/2}$ in case of separated areas) - was thus selected to appear in the next committed unit with the lowest fixed cost component. It has to be noted that the distribution shown in Fig. 3 and Fig. 4 does not have to follow a merit order, as the units on the RH side might carry a lower fixed cost portion than on the LH, as the units are committed in regards of their lowest total cost and not only their fixed cost. However, the next unit committed solely for the purpose of providing inertial response has to show the lowest fixed component of the available units. (16) depicts this. In the case of (16a) with the possibility of transfer of inertial response, the shadow prices ψ_τ of the line flow, i.e. the dual value of (4) have to be defined. A two-stage approach (solving the original problem and then assigning the resulting schedule as deterministic parameters to the binary variables $o_{P,t}$) proved successful. Due to the nature of dual values, no congestion would therefore set the shadow price to 0 and the price for inertial response to a fraction of the fixed cost of the cheapest extra-marginal unit, such as in the case of two areas without shared inertia as shown in (16b).

Fig. 5. Hydro Plant Reservoir Inflow Forecast Accuracy

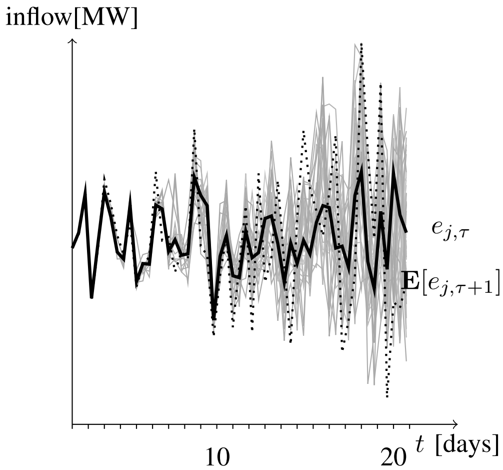


Fig. 6. Wind Power Forecast Accuracy

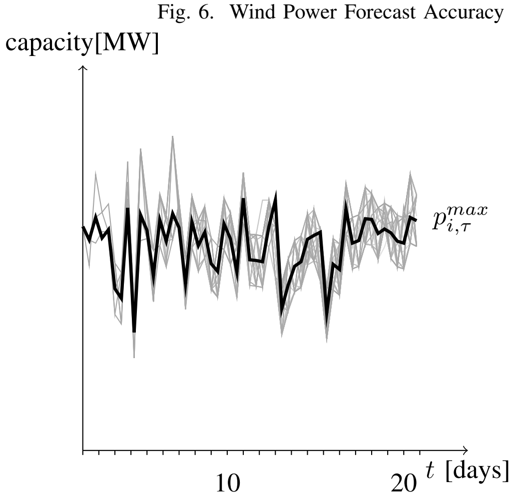


Fig. 7. No Inertia Requirement

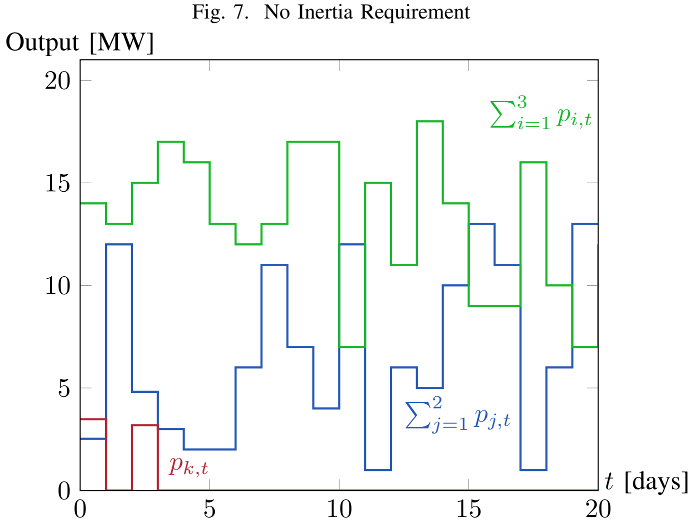


Fig. 8. Shared Inertia Requirement

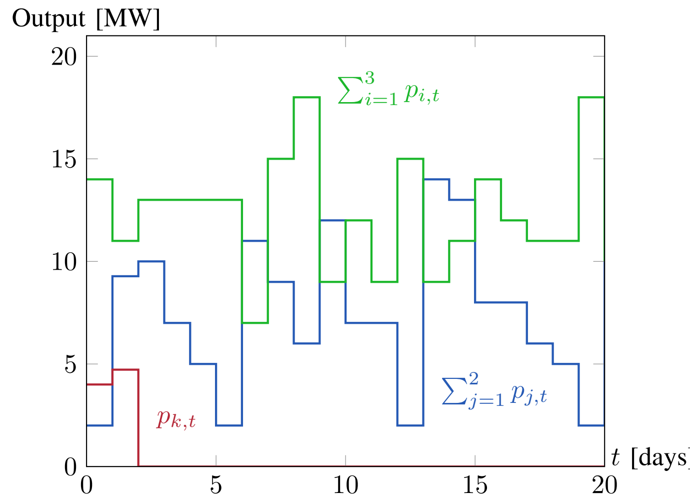
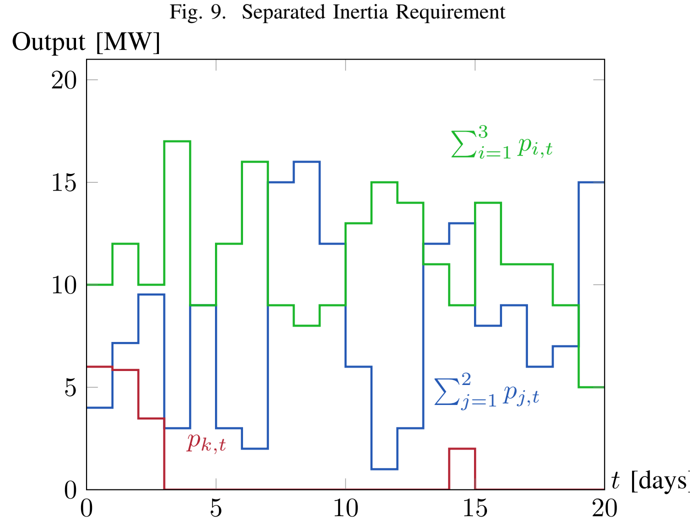


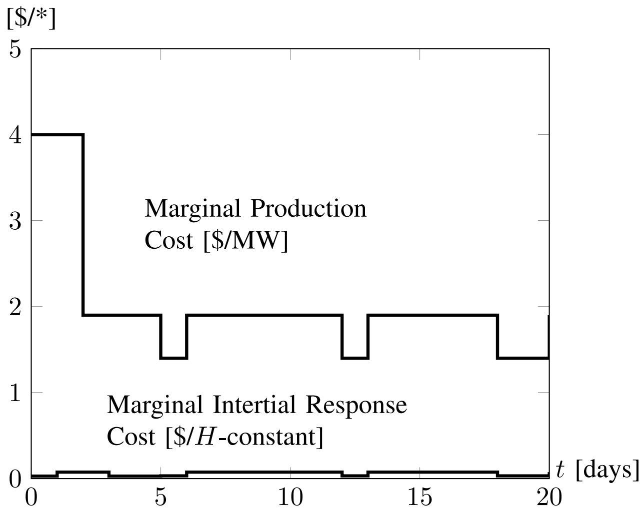
Fig. 9. Separated Inertia Requirement



III. Case Study

A system consisting of two small and one medium wind power plant, two medium sized hydro power plants and a single thermal plant was used to evaluate the model(inertial response impact was held constant for the different plant types.). The total potential power output without wind curtailment and with full reservoirs was set to 25 MWh in total. The hydro reservoirs were sized to be able to cater demand of one area for 3 periods and the transmission line was considered big enough to transmit the nearly the full demand of one area to another (nearly no congestion). There was a seasonality in form of a sinus curve with a 20% change over the 3 week period applied to both areas. To create expected values used in the system forecast, an exponential error term was added to the deterministic data set. The expected value of a function f in time $t2$, as observed in time t was defined as $E[f(t2)]_t = \min[f(t2) \times \Gamma(t), 0]$ where the uniformly distributed error term used in the equation was defined as $\Gamma(t) \in \mathrm{U}\{2 - e^{(t2/s)}, e^{(t2/s)}\}$ and $-\Gamma(t)^{max} <= \Gamma(t) <=$

Fig. 10. Cost Curves for Shared Inertia Obligation (Fig.8)



$\Gamma(t)^{max}$. Thus, being closer in time to a period gave a more accurate depiction of the situation to incorporate into the planning - Fig. 5 and Fig. 6 show the simulated forecast scenarios for wind and water [1]. Fig. 7 shows the scheduled output aggregated on form of generation for a period of three weeks and in case of no inertia requirements. Wind power provided the main generation to the system, supported by hydro in periods of low wind supply. Thermal production was used to supply the demand in the initial filling period, balancing the starting level of the hydro plants. Adding a shared inertia constraint on both areas, increased the on-time of the hydro plants, as shown in 8. Decoupling of the trade of inertia as shown in 9 lead to the necessity of starting the thermal plant to supply the necessary inertia and thus to an even greater total price increase. Consequently, it can be stated that increasing the inertia requirement causes plants to start redundantly (and thus to pay their fixed cost portion α_P), which would otherwise not have been scheduled. Fig. 10 shows the difference in *Market Cutoff Price* and price of inertia $\lambda_{I,\tau}$ for the case of an area with shared inertia fulfillment[5].

IV. Conclusion

In a novel modeling approach, a 'demand' for inertial response was imposed on the system and its impact quantified. This is based on the fact that as this inelastic requirement was realized through a cut in the solution set, a quantitative difference to the initial, optimal social welfare situation can be expected. Furthermore, this paper analyzed the impact of inertial response requirements on a two-node/are system characterized by renewable generation forms. It was demonstrated, that inertia intuitively behaves like a traditional capacity payment but differs on short term from the price of capacity as the cost of providing inertial response are carried by the cost of production; however, still influenced by area congestion. Thus follows, that on a long term average, capacity price should be considered $\geq$ inertia price. For future work, a long term analysis via a rolling time horizon and more (both in number and generation form diversity) areas could be included in the model, as well as additional work on the process of inertia-pricing might be advised.

[5]the price curve of the test system for separated areas of inertia was omitted as it showed a nearly steady level due to the low amount of participating plants

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