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EFFECT ON DOPPLER RESONANCE FROM A NEAR-SURFACE SHEAR LAYER

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ABSTRACT

For waves generated by a wave source which is simultaneously moving and oscillating at a constant frequency ω, a resonance is well known to occur at a particular value τ_{res} of the nondimensional frequency $\tau = \omega V/g$ (V: source velocity relative to the surface, g: gravitational acceleration). For quiescent, deep water, it is well known that $\tau_{\text{res}} = \frac{1}{4}$. We study the effect on τ_{res} from the presence of a shear flow in a layer near the surface, such as may be generated by wind or tidal currents. Assuming the vorticity is constant within the shear layer, we find that the effects on the resonant frequency can be significant even for sources corresponding to moderate shear and relatively long waves, while for stronger shear and shorter waves the effect is stronger. Even for a situation where the resonant waves have wavelengths about 20 times the width of the shear layer, the resonance frequency can change by $\sim 25\%$ for even a moderately strong shear $VS/g = 0.3$ (S: vorticity in surface shear layer). Intuition for the problem is built by first considering two simpler geometries: uniform current with finite depth, and Couette flow of finite depth.

INTRODUCTION

We consider the classical problem of water waves generated by a wave-maker which advances at constant speed relative to the water surface, and is simultaneously oscillating at a constant frequency. A large body of literature has studied this topic going back more than half a century [1, 2]. The problem is closely related to the problem of seakeeping in regular waves, a problem that challenges researchers and engineers to this day. It is a common assumption that the response of a floating body to a general seastate can be approximated as the sum of responses to regular waves components of the wave spectrum [3].

The well-known phenomenon sometimes named Doppler resonance occurs in deep still water at the particular value of the non-dimensional frequency-velocity: $\tau = \omega V/g = \frac{1}{4}$. Here ω is the oscillation frequency, V the forward velocity, and g the gravitational acceleration. At resonance the forward-directed group velocity exactly equals V, and wave energy is unable to escape the vicinity of the wave-source, see e.g. [4, 5]. When $\tau < \frac{1}{4}$ wave solutions exist in front of as well as behind the moving source whereas for $\tau > \frac{1}{4}$ all waves are left behind by the ship. In a linear theory the Doppler resonance can result in infinite wave amplitudes, depending on the nature of the wave source. When the wave source is modelled as a moving, localised surface pressure, wave amplitudes diverge as $(\tau - \frac{1}{4})^{-1/2}$ in 2D, and like $\ln(\tau - \frac{1}{4})$ in 3D [6], but become finite once nonlinear wave components are included [7]. Also when the waves are created by a moving point-source of oscillating strength (a Green function in the theory of floating bodies), infinite wave amplitudes result in linear theory, but are rendered finite once such sources are used to create a body of nonzero volume [8, 9]. Particularly in numerical schemes the Doppler resonance requires particular care, and moreover, wave resistance can increase sharply near the critical frequency under some circumstances [10, 11].

Two of us recently showed how the presence of a sub-surface shear current can greatly enrich the complexity of the Dopper

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resonance question [12]. The simplest shear current was considered, depending linearly on depth as $U(z) = Sz$, S being the constant vorticity of the flow. Not only will the presence of even relatively weak shear significantly increase or decrease the resonant value of τ depending on the direction of motion relative to the current; for strong shear $\mathrm{Fr_s} > \frac{1}{3}$ several resonant values — as many as 4 – is possible. $\mathrm{Fr_s}$ is the "shear-Froude number" $\mathrm{Fr_s} = VS/g$. Variations of the resonant value τ_{res} was previously considered in 2D [13], in agreement with [12] as a special case.

Real-life shear currents in the coastal zone typically possess velocity profiles differing strongly from the simplest, linear form. Typical examples of shear flows that can strongly influence dispersion include wind-driven surface flows or tidal flows [14], where shear is concentrated near the surface. Useful though it is for allowing transparent analytical results, the ability of the simplest, strictly linear shear flow to make accurate predictions of the resonant frequency is restricted to relatively short wavelengths in the sense of [15]. Surface waves are affected by flow conditions within about half a wavelength's depth of the surface, hence sufficiently short waves will "see" an approximately linearly varying current, whereas longer waves are affected by the full velocity profile $U(z)$.

An oft-used model in such situations is to let $U(z)$ be a piecewise linear function of depth (e.g. [16]) so that vorticity S has one constant value S_1 for $0 > z > -h_1$, and S_2 for $-h_1 > z > -h_2$, with $h = h_1 + h_2$ the total depth. Hence two depth scales, h_1 and h, are involved. The model is a simple case of the so-called N-layer model [17, 18].

We concentrate herein on the dependence of the resonance frequency τ_{res} in a 2D setting . Our considerations are independent of what the source of the waves may be. Three special cases are analysed; finite water depth in the absence of shear, uniform vorticity over finite water depth, and the case where shear occurs only in a thin surface layer in otherwise deep water.

SYSTEM DESCRIPTION AND FORMALISM

Linear gravity surface waves generated by a moving, oscillating surface disturbance are considered in two dimensions atop a background shear flow. The fluid is assumed to be incompressible, of negligible viscosity and surface tension, and whose free surface is at $z = \zeta(x)$ where $z = 0$ denotes the undisturbed surface . Vertical coordinates z_1 and z_2 are defined within each layer such that the horizontal flow velocity in the upper layer can be written $U_1(z_1) = S_1 z_1$ and in the bottom layer as $U_2(z_2) = S_2 z_2 - S_1 h_1$. The geometry is shown in Fig. 1.

We assume that the surface disturbance oscillates at a single frequency ω and simultaneously advances at constant velocity V along the x direction. We choose a moving coordinate system with horizontal coordinate $\xi = x - Vt$, where t is the time. The disturbance is assumed to generate waves also oscillating at frequency ω in the moving reference frame. Though wave

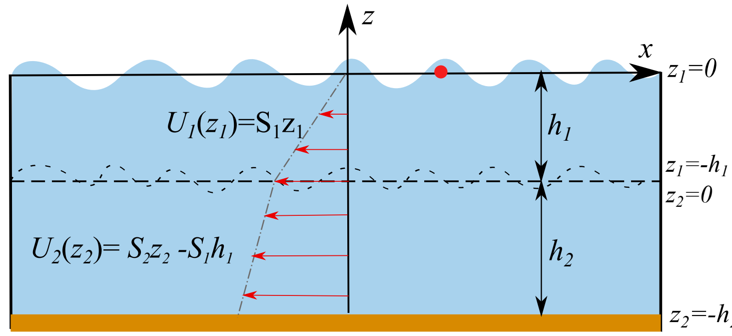

FIGURE 1. Geometry of the wave-current system: two layers of a linear shear current oriented along the x-direction are present, including the upper layer $U_1(z_1) = S_1 z_1$ and the lower layer $U_2(z_2) = S_2 z_2 - S_1 h_1$, vertical coordinates z_1, z_2 are defined within each layer; depths for the upper and lower layers are respectively h_1 and h_2.

amplitudes at a Doppler resonance can be quite large, we study only the onset of this resonance by considering the propagation of waves of infinitesimal amplitude. In this regime the wave-current interaction is unidirectional: waves are affected by the background flow but the opposite interaction is neglected. For the 2D geometry considered herein the wave disturbance is irrotational permitting the use of a velocity potential, yet we work with the fluid velocities $(\hat{u}, \hat{w})$ to readily allow extension to the generalized case where the motion of the disturbance is not parallel to the flow and the waves are rotational [19]. We choose to work in Fourier space of the horizontal axis, such that the physical perturbation quantities resulting from the oscillating disturbance can be written

$$[\hat{u}(z), \hat{w}(z), \hat{p}(z), \zeta] = \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} \mathrm{e}^{\mathrm{i}(k\xi - \omega t)} [u(z), w(z), p(z), B] \quad (1)$$

where k is the wave number, and $\hat{p}$ the dynamic pressure from the wave such that the total pressure $P = \hat{p} - \rho g z$ with fluid density ρ and gravitational acceleration g. Inserting Eq. (1) into the Euler and continuity equations leads to expressions for the vertical velocity and pressure within layer $j = 1, 2$:

$$k^{-1} w_j(z_j) = A_j(k) \sinh k(z_j + h_j) + C_j(k) \cosh k(z_j + h_j), \quad (2)$$

$$k^2 p_j(z_j)/\rho = \mathrm{i}[kV - kU_j(z_j) + \omega] w'_j(z_j) + \mathrm{i}kS_j w_j(z_j), \quad (3)$$

where A_j, and C_j are unknown constants determined by boundary conditions at the layer interface, bottom and free surface.

Shear-assisted vs shear-inhibited propagation

For the geometry of Fig. 1, the much-used terminology "upstream" and "downstream" propagation is not well suited, since it depends on the reference system, for which there are at least two obvious options: either the surface velocity is chosen to zero, or the velocity bottom is zero. “Upstream” and “downstream” swap meanings depending on this choice. Defining vorticity as either positive or negative is another convention used in the literature, but this is less useful once the theory is extended to 3D. Instead we compare our system to that in which the flow is constant with depth, with the same surface velocity. Assuming a wave propagates along the positive x axis, any monotonous $U(z)$ can be said to assist the wave if $U'(z) \leq 0$ and to inhibit it if $U'(z) \geq 0$, relative to the constant U reference mentioned. Quantitaively, in the shear-assisted case the phase and group velocities are increased by the prescence of shear, and are decreased in the shear-inhibited case. The reader is referred to [20] for more detail on the effect of shear on the phase and group velocities.

Dispersion relation

Applying the boundary condition at the seabed gives $C_2 = 0$. At the layer interface $w(z)$ and $p(z)$ are continuous. At the surface the normal linearized conditions $\hat{w} = \dot{\zeta}$ and $P(\zeta) = 0$ apply, yielding

$$kA_1 \sinh kh_1 + kC_1 \cosh kh_1 = -\mathrm{i}(kV + \omega)B, \tag{4a}$$
$$\mathrm{i}(kV + \omega)(A_1 \cosh kh_1 + C_1 \sinh kh_1) + \mathrm{i}S_1(A_1 \sinh kh_1 + C_1 \cosh kh_1) - gB = 0, \tag{4b}$$
$$C_1 = A_2 \sinh kh_2, \tag{4c}$$
$$(kV + \omega + kS_1 h_1)A_1 + S_1 C_1 = (kV + \omega + kS_1 h_1)A_2 \cosh kh_2 + S_2 A_2 \sinh kh_2. \tag{4d}$$

If, as considered in [12], the disturbance is caused by an external surface pressure, the dynamic boundary condition is inhomogeneous with the applied pressure on the right-hand side. The eigenvalue problem given by the homogeneous linear system (4) then gives $\omega(k)$ via the implicit dispersion relation

$$\Delta[\omega(k), k] = 0, \tag{5}$$

here Δ denotes the determinant of the coefficient matrix for A_1, A_2, C_1, and B. In the present case it is not difficult to find Δ explicitly, yielding a somewhat unwieldy expression. Since the dispersion relation holds for all k, we must have $\mathrm{d}\Delta/\mathrm{d}k = 0$.

Doppler resonance

Physically, Doppler resonance occurs in the situations where the wave group velocity appears to be zero in a reference frame moving with the source, indicating that wave energy can-

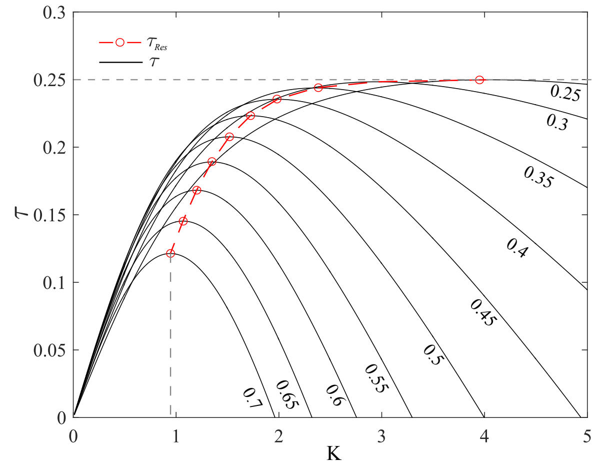

FIGURE 2. Nondimensional oscillating frequency τ in the absence of shear ($S_1 = S_2 = 0$) with respect to different $K = kh$ for Fr_h varying from 0.25 to 0.7 as indicated beneath each graph.

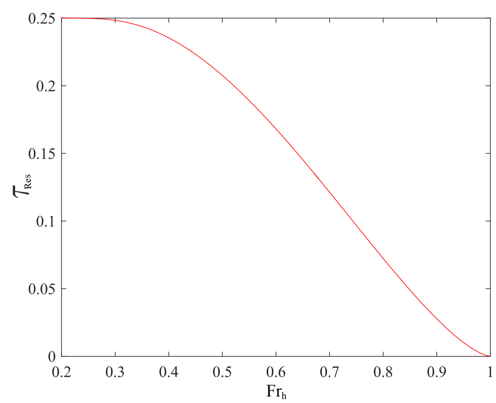

FIGURE 3. Resonance frequency τ_{res} with respect to Fr_h in the absence of a shear current.

not be radiated away, i.e.,

$$c_g^R = \frac{\mathrm{d}\omega}{\mathrm{d}k} = -\frac{\partial\Delta(\omega,k)/\partial k}{\partial\Delta(\omega,k)/\partial\omega} = 0. \tag{6}$$

It is convenient to define a non-dimensionalized frequency $\tau = \omega V/g$ which is well-known to naval architects. Eqs. 5 and 6 then yield a resonant wave number k_0 and frequency ω_0 as a function of velocity V, depth h, and background shear current strengths S_1 and S_2. Hence, we have the resonant frequency $\tau_{\mathrm{res}} = \omega_0 V/g$, which is well-known to equal $\frac{1}{4}$ when $h = h_1 + h_2 = \infty$ and $S_1 = S_2 = 0$.

LIMITING CASES AND RESULTS

Finite water depth in the absence of a shear current

When $S_1 = S_2 = 0$ one finds from a non-dimensionalized dispersion relation (5)

$$\tau(K) = -K\mathrm{Fr}_h^2 + \mathrm{Fr}_h\sqrt{K\tanh K}. \quad (7)$$

where $K = kh$ and $\mathrm{Fr}_h = V/\sqrt{gh}$. The nondimensional group velocity is now defined as $C_g^R = \mathrm{d}\tau/\mathrm{d}K$. The resonant value is found as $\tau_{\mathrm{res}} = \tau(K_0)$ where from (6) or $\mathrm{d}\tau/\mathrm{d}K = 0$, K_0 solves

$$2\mathrm{Fr}_h\sqrt{K_0\tanh K_0} = \tanh K_0 + K_0\sec^2 K_0. \quad (8)$$

We choose $K_0 \geq 0$ by convention. One readily verifies that $\tau_{\mathrm{res}} \to \frac{1}{4}$ when $h \to \infty$. This is the geometry considered in [21], where no quantitative discussion of the dependence of τ_{res} on h is given.

Fig. 2 shows $\tau(K)$ and the resonance frequency τ_{res} as a function of the wave number K at various Fr_h. The figure shows how the Doppler resonance occurs at wavenumbers where $\mathrm{d}\tau/\mathrm{d}K = 0$. It is straightforward from Fig.2 that, for a particular Fr_h, the group velocity C_g^R decreases from positive to negative as the wavelength decreases from ∞. In addition, shallower depth (larger Fr_h) tends to decrease C_g^R for a particular K (wavelength) , requiring a longer resonant wavelength (smaller K_0) to satisfy (8). The resonance frequency also decreases with increasing Fr_h, as shown explicitly in Fig. 3. τ_{res} drops to zero at $\mathrm{Fr}_h = 1$, which corresponds to the critical situation where transverse waves from a moving ship vanish in finite water depth. This is the simplest manifestation of a more general conclusion drawn in [12] that the Doppler resonance decreases to zero at the critical value of s-hear Froude number; the critical velocity in the presence of finite depth and shear was discussed in detail in Ref. [22].

Uniform vorticity over water depth

When one layer of a linear shear profile of uniform vorticity S is considered, we obtain

$$\tau(K) = -K\mathrm{Fr}_h^2 - \tfrac{1}{2}\mathrm{sgn}(S)\mathrm{Fr}_s\tanh K + \sqrt{\mathrm{Fr}_h^2 K\tanh K + (\tfrac{1}{2}\mathrm{Fr}_s\tanh K)^2}, \quad (9)$$

where $\mathrm{Fr_s} = V|S|/g$. Again $\tau_{\mathrm{res}} = \tau(K_0)$ where K_0 is the positive root (if such exists) of the equation obtained by $C_g^R = \mathrm{d}\tau(K)/\mathrm{d}K = 0$. By convention we choose $K_0 \geq 0$, i.e., a wave moving in the positive x direction. The situation $S > 0$ is denoted “shear inhibited”, and $S < 0$ we call “shear assisted”.

Figs. 4 and 5 show the resonance frequency τ_{res} in different combinations of the parameters $\mathrm{Fr_s}$ and Fr_h, respectively in the 'shear-inhibited' and 'shear-assisted' situations. Interplays of the shear current and finite water depth are quite obvious in both situations. For shear-inhibited propagation both stronger shear and shallower depth will tend to decrease the group velocity, hence Fig. 4 shows that τ_{res} decreases as either $\mathrm{Fr_s}$ or Fr_h increases. In accordance with [22], we ascertain that $\tau_{\mathrm{res}} = 0$ when $\mathrm{Fr}_h = \sqrt{1-\mathrm{Fr_s}}$ if $S > 0$, corresponding to the criterion for critical velocity of ship waves ($\omega = 0$). Conversely, stronger shear tends to increase τ_{res} for shear assisted propagation, $S < 0$, shown in Fig. 5 where increasing shear strength tends to increase the group velocity.

Effects of surface vorticity

We now consider a two-layer fluid as depicted in Fig. 1 with constant vorticity S_1 in the upper layer and zero vorticity in the bottom layer ($S_2 = 0$). We assume the thickness h_2 of the bottom layer to be infinite for all practical purposes, such that finite depth effects are omitted. The piecewise constant vorticity flow defined here is approximately representative of more realistic profiles such as those driven by wind and tidal currents, where the vorticity is greatest near the surface and decays rapidly with depth. Such flows can have very strong near-surface shear which will significantly affect waves whose length is in the order of a few times h_1, or less.

Eqs. 4-6 can be solved to find the Doppler resonance frequency τ_{res} as a function of slightly re-defined parameters $K = kh_1$, $\mathrm{Fr}_{s1} = V|S_1|/g$ and $\mathrm{Fr}_{h1} = V/\sqrt{gh_1}$. Similar to the previous section we consider both the shear inhibited ($S_1 > 0$) case in Fig. 6 and shear assisted ($S_1 < 0$) in Fig. 7. As there are no depth effects, $\tau_{\mathrm{res}} \to \frac{1}{4}$ as $\mathrm{Fr}_{s1} \to 0$ independent of Fr_{h1}.

The trends as a function of shear strength are similar to Figs. 4-5 with τ_{res} decreasing for the shear-inhibited case due to weakened dispersion, while increasing for the shear-assisted case due to the opposite effect.

The striking conclusion seen in Fig. 6 is that even very moderate values of Fr_{s1} change the value of τ_{res} quite significantly. The effect is strongest for small values of Fr_{h1}, corresponding to higher K_0. These are short waves which only “see” the shear layer, not the uniform flow below, and are well described by the simplest, linear profile studied above. Going by the rule of thumb that a wave can “see” about half its wavelength into the deep, the profile will appear linear to waves for which $K_0 \gtrsim \pi$.

Yet perhaps surprisingly, increasing Fr_{h1} to 0.8, the rate of change for increasing S is reduced only by about a factor 2. This we find striking, since now $K_0 \ll \pi$ (by almost a factor 10). The wave “sees” the flow down to about 10 times the width of the shear layer, and yet a relatively modest $\mathrm{Fr}_{s1} = 0.3$ reduces τ_{res} by more than 25% for shear-inhibited propagation. The change in τ_{res} with Fr_{s1} for shear-assisted flow (Fig. 7) is similar, but tending to increase rather than decrease τ_{res}.

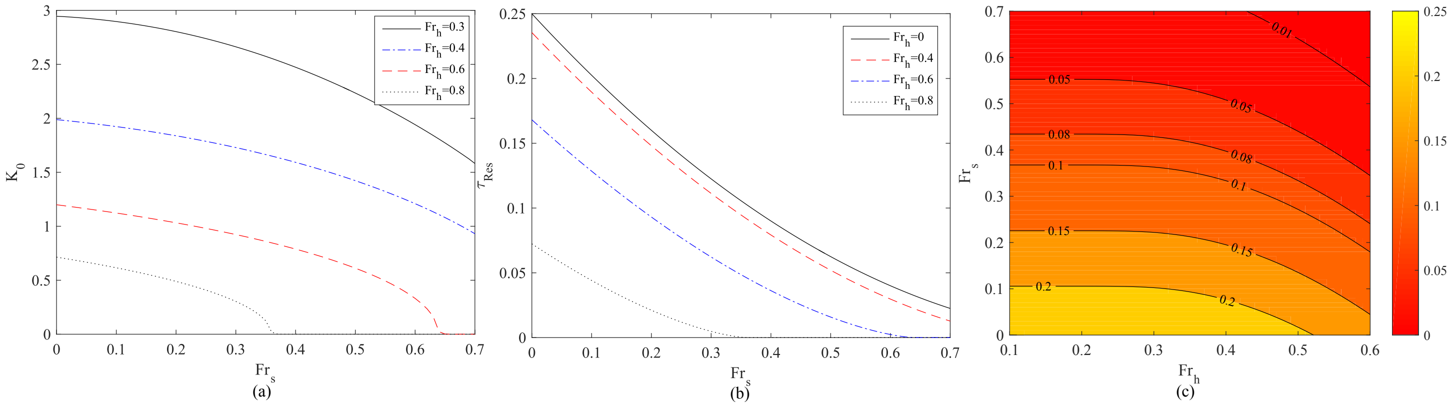

FIGURE 4. K_0 (a) and resonance frequency τ_{res} (b) with respect to different combinations of Fr_s and Fr_h in the 'shear-inhibited' $(S > 0)$ situations: (c) contour plot of τ_{res}.

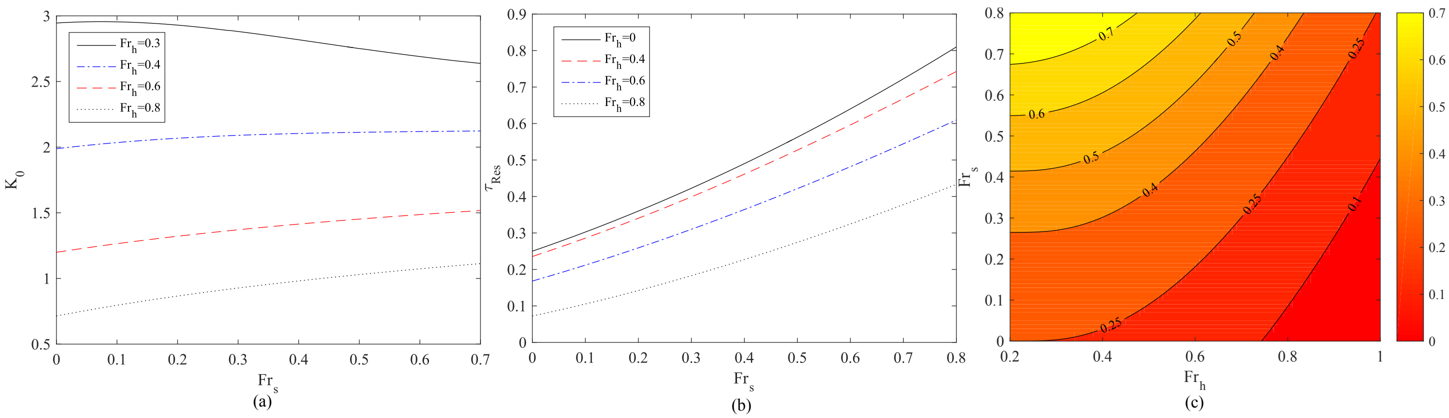

FIGURE 5. K_0 and resonance frequency τ_{res} with respect to different combinations of Fr_s and Fr_h in the 'shear-assisted' $(S < 0)$ situations : (c) contour plot of τ_{res}.

Conclusions

We have studied the Doppler resonances from an oscillating and moving wave source in a 2D fluid with a free surface, considering the effects of finite depth and background shear currents. We are particularly interested in the effect of a shear layer near the surface of otherwise quiescent (or uniformly flowing) fluid, typical of wind and tide driven currents.

The resonance occurs when the group velocity equals zero in the reference frame of the moving source. In quiescent, deep waters the resonance is well known to occur at $\tau_{\text{res}} \equiv V\omega/g = \frac{1}{4}$ (V: source speed relative to surface, ω: oscillation resonant frequency). Intuition is built by considering first two simpler cases: finite depth without shear, and finite depth with constant shear (vorticity). Finite depth reduces the resonance frequency due to weakened dispersion, up to a critical value $\text{Fr}_\text{h} = 1$ where the resonance frequency is zero. A background shear current results in directionally dependent dispersive properties, where the resonance frequency is decreased for the case of positive vorticity (shear-inhibited waves) and increased in the opposite (shear-assisted) case.

We finally consider a shear layer of thickness h_1 near the surface, assumed to have constant vorticity S_1, while the fluid beneath the layer is presumed to be deep and at rest. The situation is governed by two Froude numbers: $\text{Fr}_{h1} = V/\sqrt{gh_1}$ and the shear-Froude number $\text{Fr}_{s1} = VS_1/g$. We find that the effect of the surface shear layer can be surprisingly high. Even for waves of wavelength $\lambda \sim 20h_1$, the resonant frequency is reduced by more than 25% for moderate shear $\text{Fr}_{s1} = 0.3$. We conclude that the Doppler resonanc e frequency for ships and vessels can differ significantly from $\tau_{\text{res}} = \frac{1}{4}$ in the presence of surface currents from wind and tides.

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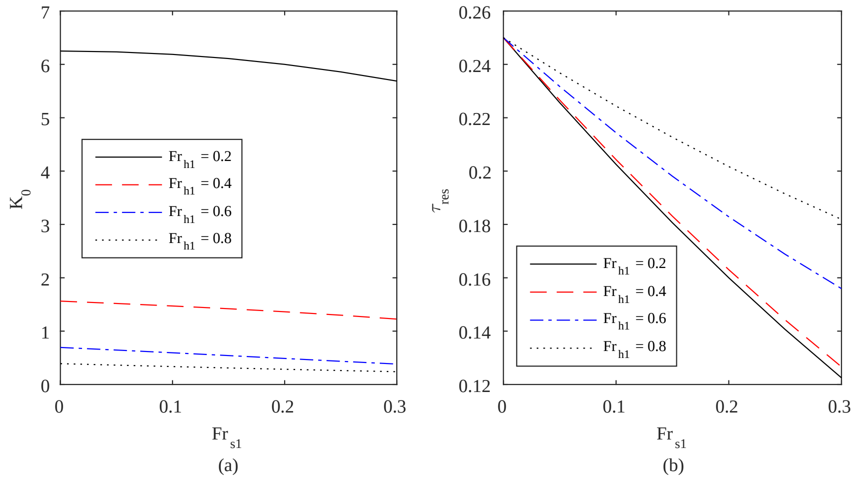

FIGURE 6. K_0 and resonance frequency $\tau_{\rm res}$ with respect to different combinations of Fr_{s1} and Fr_{h1} in the 'shear-inhibited' situations.

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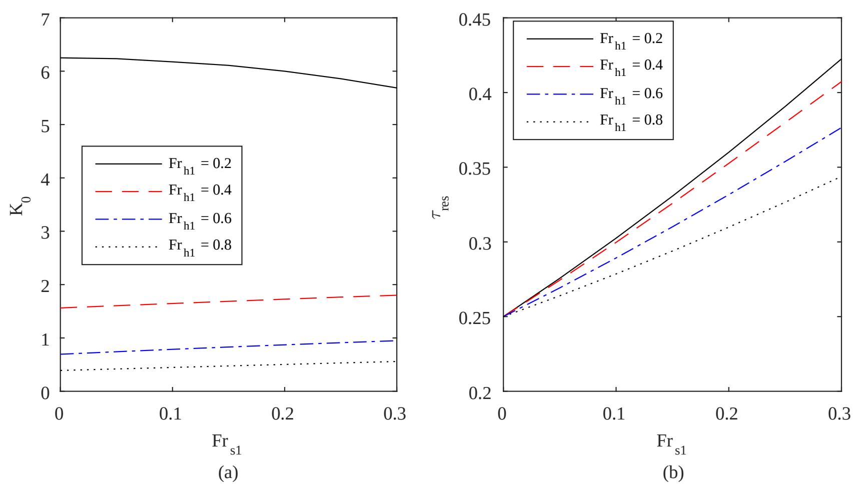

FIGURE 7. K_0 and resonance frequency τ_{res} with respect to different combinations of Fr_{s1} and Fr_{h1} in the 'shear-assisted' situations.