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Optimal Temperature Control of Rooms for Minimum Energy Cost

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Abstract

In this thesis was a room with direct heating in the floor and room modelled. The aim has been to minimize the energy cost of the room, assuming that the future energy price and weather forecast is known.

The constrained optimization problem turned out to be linear, and the solution of the problem will always be on the upper or lower bounds of the inputs or states. The idea is to store heat when the energy price is low, and use it when the energy price is high. A switching time that ensures that the model starts the heating of the system at an optimal time in order to save energy costs is thus of importance to find.

The problem was solved by using the matlab function fminsearch, and by assuming constant outdoor temperature. Two scenarios were analysed; 1) where only the floor heat is used to storage of heat, and 2) where both heaters are used to heat the system. In each scenario the length and starting point of the interval where the energy price is high was varied. This thesis show that storage of heat in the floor is preferred, apart from in the case where there is no time to heat before the peak interval begins, where both heaters in the floor and room should be used.

For comparison, the optimization problem was also solved by using PI controllers, where the two inputs control the temperature in the floor and room respectively. It turned out that the result of the control problem when using PI will resemble the solution of the optimal control problem when using fminsearch.

A couple of switching rules was derived in order to find the optimal switching time. This thesis show that the switching rules gives a good estimate of the switching time, apart from in the case where there is no time to heat. The switching rules was used in both methods (when using PI and fminsearch) and it was found that the obtained switching time is not far from the optimal solution in any of the methods.

The optimization problem when using fminsearch was tested with varying outdoor temperature. It was seen that the solution from the optimal control problem will take the disturbance into consideration if it is varied before the peak period. On the other hand, the model do not allow for a variation in the outdoor temperature after the peak period.

At last, the thesis show that the switching rule handle a variation in the disturbance before the peak period as good as the optimal control problem, but when the outdoor temperature becomes too cold will the result be poor.

Sammendrag

I denne oppgaven har et rom med varmeelement i rom og gulv blitt modellert. Formålet med oppgaven har vært og minimere energikostnadene til rommet, ved å anta at fremtidig energipris og værprognoser er kjent.

Det begrensede optimaliseringsproblemet viste seg å være lineært, og løsningen av problemet vil alltid være på den øvre eller nedre grensen til pådragene eller systemtilstandene. Ideen er å lagre varme når energiprisen er lav, og bruke den når energiprisen er høy. En optimal "switche" tid hvor varmen skal slås på er derfor viktig å finne. Denne tiden må ikke være for tidlig eller for sent for å kunne minimere energikostnadene.

Problemet ble løst ved å bruke matlabfunksjonen fminsearch, og ved å anta konstant utetemperatur. To senarioer ble analysert; 1) bare gulvvarmen brukes til lagring av varme, og 2) begge varmeapparatene i gulv og rom brukes til å varme opp rommet. I begge senarioene vil startpunktet og lengden av intervallet med høy energipris endres. Denne oppgaven viser at varmen bør lagres i gulvet, bortsett fra i tilfellet hvor systemet ikke har tid til å varme før energiprisen stiger, da bør begge varmeapparatene i gulv og rom benyttes.

For å kunne sammenligne ble optimaliseringsproblemet også løst ved bruk av PI-kontrollere, hvor de to pådragene kontrollerer temperaturen i henholdsvis rom og gulv. Det viste seg at løsningen av kontrollproblemet når man bruker PI og fminsearch ligner hverandre.

To switcheregler ble utledet for å finne den optimale switche tiden. Denne oppgaven viser at switchreglene gir et bra estimat av switchetiden, bortsett fra i tilfellet hvor systemet ikke har tid til å varme før energiprisen øker. Switcheregene ble brukt i begge metodene (ved bruk av PI og fminsearch), og det ble vist at den beregnede switchetiden ikke ligger langt fra den optimale løsningen for noen av metodene.

Optimaliseringsproblemet, løst ved fmisearch, ble også testet mot varierende utetemperatur. Det ble vist at den optimale løsningen tar en variasjon i utetemperatur i betraktning dersom variasjonen skjer før energiprisen øker. En variasjon av utetemperaturen etter perioden hvor energprisen øker vil ikke bli tatt hensyn til.

Til sist, denne oppgaven viser at switchereglene håndterer en variasjon i forstyrrelsen før energi prisen øker like godt som det optimale kontrollproblemet, men hvis utetemperaturen blir for lav blir resultatet dårlig.

Preface

This thesis was written as a final part of the masters degree at Norwegian University of Science and Technology, 2013.

I would like to thank my supervisor Sigurd Skogestad for giving me the opportunity to work with this project.

A special thanks go to my co-supervisor Chriss Grimholt who has showed a lot of interest in my project. I would like to thank him for all the valuable help and support he has given me.

Finally, I would like to thank my family and friends that I have got during the five years as a student, which have made these years at NTNU memorable.

Declaration of Compliance

I hereby declare that this is an independent work according to the exam regulations of the Norwegian University of Science and Technology (NTNU)

Siri Hofstad Trapnes   Date

Contents

1 Introduction

The focus of this work will be to minimize the energy cost of a room by storing energy in the floor when the energy price is low. This stored energy should then be used when the energy price is high.

1.1 Motivation

Due to increasing energy prices and greater concerns about the greenhouse effects more efficient electricity production and usage is desirable. Electricity production based on renewable sources, like for example large wind turbines and solar-power, are preferred [Molderink et al., 2009]. These alternative energy technologies already improve the efficiency, but a major drawback is that they are strongly dependent on the weather conditions. This limitation is of importance since the energy production is expected to cover the energy demand at any given time [Oliveira]. Because of the increasing energy consumption and the growing amount of renewable energy sources is managing the gap between production and consumption, and thus the growing demand for electricity storage, becoming an important research topic [Molderink et al., 2009].

One possible solution to this problem would be to reduce the energy consumption by shifting the consumption from peak periods to more beneficial periods [Oliveira]. Field tests in the USA have showed that optimization of domestic energy consumption can significantly reduce peak periods [Hammerstrom et al., 2007]. Manipulating the energy price according to demand information and weather forecast is one way to achieve this. Electricity consumers are encouraged to use electricity more carefully in order to minimize their electricity bill [Oliveira].

1.2 Project scope

Assuming that future energy price and weather forecast is known, this project will focus on optimizing the energy cost of a room. The idea is to buy and store energy in the floor when the energy price is low, and use it during peak periods. The goal of the project is to find a simple switching strategy, that minimize the energy costs without breaking the constraints.

The dynamic optimization problem will be solved by using fminsearch, which is a gradient free method, and by using PI controllers. A comparison of the two methods will be given in the end. It will be shown that the result of the control problem when using PI will resemble the solution of the optimal control problem when using the gradient free method.

Some switching rules will be developed in order to find an optimal switching time. These rules will be tried in both methods described above.It will be shown that the switching rules give a good estimation of the switching time, but in the case where the system does not have time to heat will the switching rule give an infeasible solution.

The optimal control problem when using fminsearch will be tested against constant and varying outdoor temperature. This thesis will show that the solution from the optimal control problem will take the disturbance into consideration if it is varied before the peak period. On the other hand, the model do not allow for a variation in the outdoor temperature after the peak period.

2 Background

This chapter will give an introduction to the theory behind the calculations and methods that are used in this project.

2.1 Heat transfer

Heat transfer occur by one, or a combination of three basic mechanisms; conduction, convection or radiation [Geankoplis, 2003]. In this project will heat be transferred by a combination of the first two mechanisms.

Conduction is defined as heat that is transferred through solids, liquids and gas because of the energy of motion between adjacent molecules [Geankoplis, 2003]. Convection means transfer of energy between an object and its environment [Geankoplis, 2003]. Heat transfer by convection can also be divided into forced and natural/free convection. Forced convection transfer heat from one place to another because of a pump, fan or other mechanical devices, while the latter one is transfer of heat due to temperature differences in the fluid [Geankoplis, 2003]. The heat will in general be transferred from the gas or liquid with the highest temperature to the gas/liquid with the lower temperature [Geankoplis, 2003].

Heat transfer is described by Fourier's law [Geankoplis, 2003]:

$$\frac{q_x}{A} = -k\frac{\mathrm{d}T}{\mathrm{d}x} \tag{2.1.1}$$

where q_x represent the heat, A the cross-sectional area and k the thermal conductivity. The thermal conductivity is replaced by the convective coefficient, h_i, when the heat is transferred by convection. The thermal conductivity and the convective coefficient differs from one another in the units, $[\frac{W}{mK}]$ and $[\frac{W}{m^2K}]$ respectively [Geankoplis, 2003].

Heat transfer through a wall, or floor, where the area and thermal conductivity are constant is given in equation (2.1.2) [Geankoplis, 2003].

$$\frac{q}{A} = \frac{k}{\Delta x}(T_1 - T_2) \tag{2.1.2}$$

If the wall consist of more than one solid, for example three layers as given Figure 2.1.1 [Geankoplis, 2003] ,are the calculations as described in equation (2.1.3).

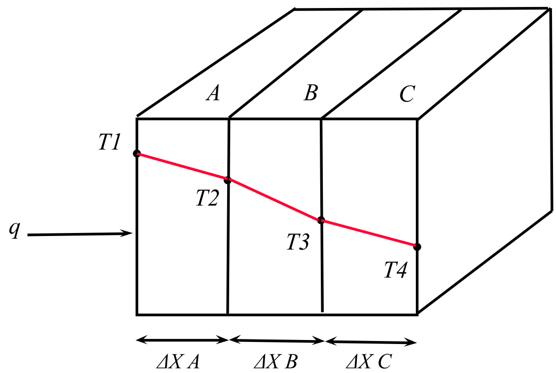

Figure 2.1.1: Heat flow through a plane wall with three layers

$$q = \frac{k_a}{\Delta x_a}(T_1 - T_2) = \frac{k_b}{\Delta x_b}(T_2 - T_3) = \frac{k_c}{\Delta x_c}(T_3 - T_4) \tag{2.1.3}$$

The heat flow is the same in each layer [Geankoplis, 2003], and T_1 and T_4 are the temperatures at the inside and outside layer respectively as shown in Figure 2.1.1. Rearranging with respect to temperature and adding the equations for the different solids give the following expression for the heat flow through the wall [Geankoplis, 2003].

$$q = \frac{T_1 - T_4}{(\frac{\Delta x_a}{k_a A}) + (\frac{\Delta x_b}{k_b A}) + (\frac{\Delta x_c}{k_c A})} \tag{2.1.4}$$

Consider a wall with fluid on both sides of the solid surfaces as given in Figure 2.1.2 [Geankoplis, 2003]. The figure show a hot fluid with temperature T_1 on the inside of the surface and a colder fluid on the outside surface with temperature T_4 [Geankoplis, 2003]. The convective coefficients for the inside and outside are h_i and h_o respectively. As explained before the heat flow can be written as

$$q = h_i A(T_1 - T_2) = \frac{k_a A}{\Delta x_a}(T_2 - T_3) = h_o A(T_3 - T_4) \tag{2.1.5}$$

A rearrangement of the expression above gives the overall heat transfer for combined convection and conduction [Geankoplis, 2003].

$$q = \frac{T_1 - T_4}{\frac{1}{h_i A} + \frac{\Delta x_a}{k_a A} + \frac{1}{h_o A}} \tag{2.1.6}$$

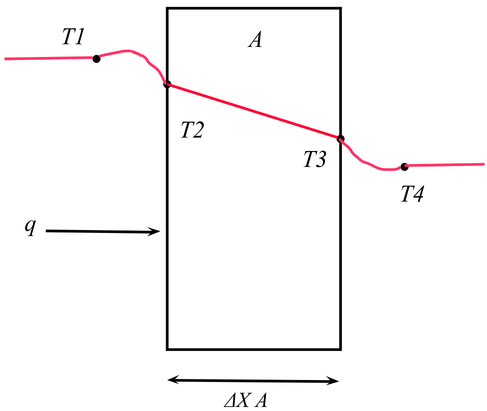

Figure 2.1.2: Heat flow through a plane wall with convective boundaries

The overall heat transfer is usually expressed in the following way

$$q = UA\Delta T \tag{2.1.7}$$

where U represents the overall heat transfer coefficient. The overall heat transfer coefficient for the above example is

$$U = \frac{1}{\frac{1}{h_i A} + \frac{\Delta x_a}{k_a A} + \frac{1}{h_o A}} \tag{2.1.8}$$

2.2 Linear state space model

A state space model gives a simple representation of a system of ordinary differential equations, ODEs. A general expression for the linear state space model is given in equation (2.2.1) and (2.2.2) below [Seborg, Dale E. , Edgar et al., 2011]

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{Ed} \tag{2.2.1}$$

$$\mathbf{y} = \mathbf{Cx} \tag{2.2.2}$$

In the above equations do **x** represent the state vector,**u** the input vector, **y** the vector with the output variables, and **d** the disturbances. The matrices **A**, **B**, **C** and **D** are matrices with constant values [Seborg, Dale E. , Edgar et al., 2011].

A linear system which vary with time are called a linear time varying, LTV, problem. LTV problems can be represented by the following state space model:

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{Ax}(\mathbf{t}) + \mathbf{Bu}(\mathbf{t}) \tag{2.2.3}$$

$$\mathbf{y}(\mathbf{t}) = \mathbf{C}\mathbf{x}(\mathbf{t}) \tag{2.2.4}$$

2.3 Hamilton's principle

The Hamilton principle describes the motion of a mechanical system from time t_1 to t_2 [Thornton, Stephen T., Marion, 2004]. Of all the possible paths from t_1 to t_2, is the actual path the one that minimizes the time integral of the difference between the kinetic (T) and potential (V) energy [Thornton, Stephen T., Marion, 2004]:

$$H = \int_{t_1}^{t_2} L(x, \dot{x}, t)\mathrm{d}t \tag{2.3.1}$$

The parameter L represent the Lagrangian, which is given by L=T-V [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978]. If the system contains any constraints, must the path be consistent with these [Thornton, Stephen T., Marion, 2004].

Considering a singe particle moving in a conservative force filed. The kinetic energy for such a particle will be a function of velocity, while the potential energy will be a function of the position of the particle [Thornton, Stephen T., Marion, 2004]. The Lagrangian is thus a function of position, x, and velocity, $\dot{x}$, of the particle. The Hamilton's Theorem sates that the Lagrangian must be minimized, which means that:

$$\int_{t_1}^{t_2} L(x_i, \dot{x}_i)\mathrm{d}t = 0 \tag{2.3.2}$$

To be able to solve these kind of problems must the Lagrangian satisfy the Lagrange equations of motion [Thornton, Stephen T., Marion, 2004]:

$$\frac{\partial L}{\partial x_i} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}_i} = 0 \tag{2.3.3}$$

2.4 Lagrange Multipliers

The Lagrange multiplier, λ, helps to find the local minimum (or maximum) of a function with constraints [Weir, Maurice D., Hass, Joel, Giordano, 2008]. In order to find the minimum of the function f(x,y,z) subject to the constraint g(x,y,z) = C, must the following system of equations be solved simultaneously [Weir, Maurice D., Hass, Joel, Giordano, 2008]:

$$\bigtriangledown f(x, y, z) = \lambda \bigtriangledown g(x, y, z) \tag{2.4.1}$$

$$g(x, y, z) = C \tag{2.4.2}$$

It is assumed that f(x,y,z) and g(x,y,z) are differentiable and $\bigtriangledown g \neq 0$ when $g(x, y, z) = 0$ [Weir, Maurice D., Hass, Joel, Giordano, 2008]. Equation (2.4.1) can be written as

$$f_x(x, y, z) = \lambda g_x(x, y, z) \tag{2.4.3}$$

$$f_y(x, y, z) = \lambda g_y(x, y, z) \tag{2.4.4}$$

$$f_z(x, y, z) = \lambda g_z(x, y, z) \tag{2.4.5}$$

when remembering that $\bigtriangledown f(x, y, z)$ and $\bigtriangledown g(x, y, z)$ are vectors [Weir, Maurice D., Hass, Joel, Giordano, 2008]. Once the critical points are found, can these be implemented in f. The minimum is found at the point where f is at the smallest [Weir, Maurice D., Hass, Joel, Giordano, 2008].

If the system has more than one set of constraints will multiple Lagrange multipliers be introduced [Weir, Maurice D., Hass, Joel, Giordano, 2008].

$$\bigtriangledown f(x, y) = \lambda \bigtriangledown g(x, y)_1 + \mu \bigtriangledown g(x, y)_2 \tag{2.4.6}$$

2.5 Pontryagin's minimum principle

The Pontryagin's minimum principle is used to find the minimum of a dynamic optimization problem that has constraints on the states and/or inputs [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978]. This section will give an introduction to the minimum principle.

Given a process on the form

$$\dot{x} = f(x, u, t) \tag{2.5.1}$$

with an optimization criteria defined as [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978]

$$J = \int_{t_1}^{t_2} L(x, u, t)\mathrm{d}t \tag{2.5.2}$$

where the parameter L represent the objective function. In equation (2.5.2) is t_1 known while t_2 will be given or free [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978].

A new function, the Hamilton function is introduced [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978]:

$$H(x, p, u, t) = L(x, u, t) + \lambda^T f(x, u, t) \tag{2.5.3}$$

where $\lambda(t)$ represent the Lagrange multiplier. The derivative of the Lagrangian multiplier is given as [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978]

$$\dot{\lambda} = -\Big[\Big(\frac{\partial f}{\partial x}\Big)^T \lambda + \frac{\partial L}{\partial x}\Big] = -\frac{\partial H}{\partial x} \tag{2.5.4}$$

Similar, the derivative of x can be written as [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978]:

$$\dot{x} = \frac{\partial H}{\partial \lambda} = f(x, u, t) \tag{2.5.5}$$

Taking the Hamilton function in equation (2.5.3) and deriving it with respect to u, will the following expression be obtained [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978]:

$$\frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \Big(\frac{\partial f}{\partial u}\Big)^T \lambda = 0 \tag{2.5.6}$$

According to the minimum principle is the optimal manipulated variable determined by

$$\min_{u \in U} H(t_1 \leq t \leq t_2) \tag{2.5.7}$$

where U is the area where the manipulated variable is. This means that for every $t_1 \leq t \leq t_2$ should $u(t) \in U$ be chosen such that the Hamilton function, H, get a minimum value [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978].

The minimum of H can either take place in the inner area of U where $\frac{\partial H}{\partial u} = 0$, or it can be defined at the boarder of U, where $\frac{\partial H}{\partial u} \neq 0$ [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978].

In some cases will one in addition to the constraints on the inputs, have constraint on the states [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978]. A subspace, X, of the n-dimensional state space, E^n, will then be introduced. In case the system is inside X will the normal minimum principle be considered, while if the system is on the border of X will a special form of the minimum principle be used. The transition between the boarder and inner area of X will be subjected to special conditions [Bachen, Jens G., Fjeld, Magnus, Solheim, 1978].

2.6 Linear programming

The solution of a constrained optimization problem that has a linear objective function and linear constraints are obtained via linear programming, LP [Seborg, Dale E. , Edgar, 2004]. The constraints can be both equalities and inequalities in such problems.

Consider a multivariable process with two inputs, (u_1,u_2) and two outputs (y_1,y_2), and where the linear model is defined as [Seborg, Dale E. , Edgar, 2004]

$$\mathbf{y} = \mathbf{K}\mathbf{u} \tag{2.6.1}$$

The upper and lower bounds of the constraints **u** and **y** define the operating window for the process, as shown in Figure 2.6.1 [Seborg, Dale E. , Edgar, 2004]. A linear cost function will have an optimal operating condition where

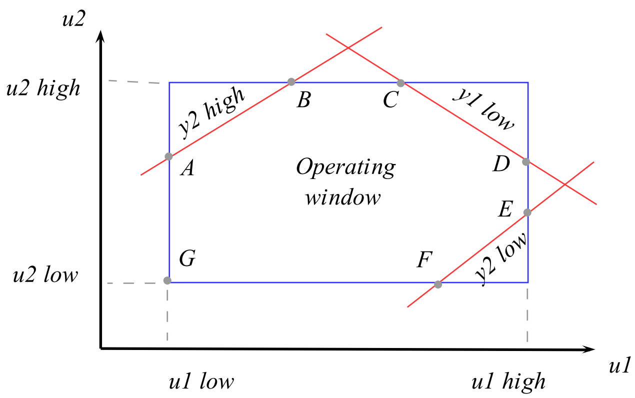

Figure 2.6.1: Operating window for a 2 × 2 linear optimization problem

constraints intersects, in one of point A to G in Figure 2.6.1 [Seborg, Dale E. , Edgar, 2004]

A standard LP problem where the objective function is minimized can be stated as follows:

$$min f = \sum_{i=1}^{N_V} c_i x_i \tag{2.6.2}$$

subject to

$$x_i \geq 0 \quad i = 1, 2, ..., N_V \tag{2.6.3}$$

$$\sum_{j=1}^{N_V} a_{ij} x_j \geq b_i \quad i = 1, 2, ..., N_I \tag{2.6.4}$$

$$\sum_{j=1}^{N_V} \tilde{a}_{ij} x_j = d_i \quad i = 1, 2, ..., N_E \tag{2.6.5}$$

In the above expression does N_I represent the number of inequality constraints [Seborg, Dale E. , Edgar, 2004].

The number of independent variables are found by a degree of freedom analysis. Assuming no constraints, will the number of independent variables, N_F, be [Seborg, Dale E. , Edgar, 2004]

$$N_F = N_V - N_E \tag{2.6.6}$$

where N_V and N_E is the number of process variables and independent equations respectively. One have three possible solutions to a DOF problem [Seborg, Dale E. , Edgar, 2004]:

1. $N_F = 0$: the process is exactly specified, the set of equation has a solution

2. $N_F > 0$: the process is underspecified, there are more process variables than equations. The equation has an infinite number of solutions since the process variables can be specified arbitrarily.

3. $N_F < 0$: the process model is overspecified. The set of equation has no solution. One or more additional independent equations must be developed for the model to have an exact solution.

The above solution show that N_F can be specified independently to maximize/minimize the objective function. Adding inequality constraints to the problem gives a different solution. The number of process variables cannot be chosen arbitrarily any more, they must satisfy all of equality and inequality constraints [Seborg, Dale E. , Edgar, 2004].

2.7 Tuning

The expression for a proportional-integral-derivative, PID, controller is given in equation (2.7.1) below [Seborg, Dale E. , Edgar et al., 2011].

$$G_c = K_c(1 + \frac{1}{\tau_I s} + \tau_D s) \tag{2.7.1}$$

In the above equation does K_c represent the controller gain while τ_I and τ_D is the integral and derivative time respectively. The expression for a proportional-integral, PI, controller is the same as for a PID controller except that the derivative time is omitted.

It is not easy to find good values for the parameters in equation (2.7.1) without using systematic procedures [Skogestad, 2003]. The SIMC rules, Skogestad's IMC rules, [Skogestad, 2003] is such a procedure. In contrast to the IMC rules, will the SIMC rules propose only one tuning rule for tuning of PI controllers. The tuning rule for a first-order-plus-time-delay, FOPTD, model

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \tag{2.7.2}$$

are given in equation (2.7.3) and (2.7.4) [Skogestad, 2003]. The term θ in equation (2.7.2) represent the effective delay.

$$K_c = \frac{1}{k'}\frac{1}{\theta + \tau_c} \quad \text{where} \quad k' = \frac{K}{\tau_I} \text{ and } \quad \tau = \tau_I \tag{2.7.3}$$

$$\tau_I = \min(\tau_I, 4(\tau_c + \theta)) \tag{2.7.4}$$

The tuning parameter, τ_c, must be in the range -$\theta < \tau_c < \infty$ to get a positive and non-zero controller gain [Skogestad, 2003]. There are two main possibilities for the optimal value of τ_c [Skogestad, 2003]:

1. Tight control
   - Fast response with good robustness
   - $\tau_c = \theta$
2. Smooth control
   - Slow control with acceptable disturbance rejection
   - $\tau_c > \theta$

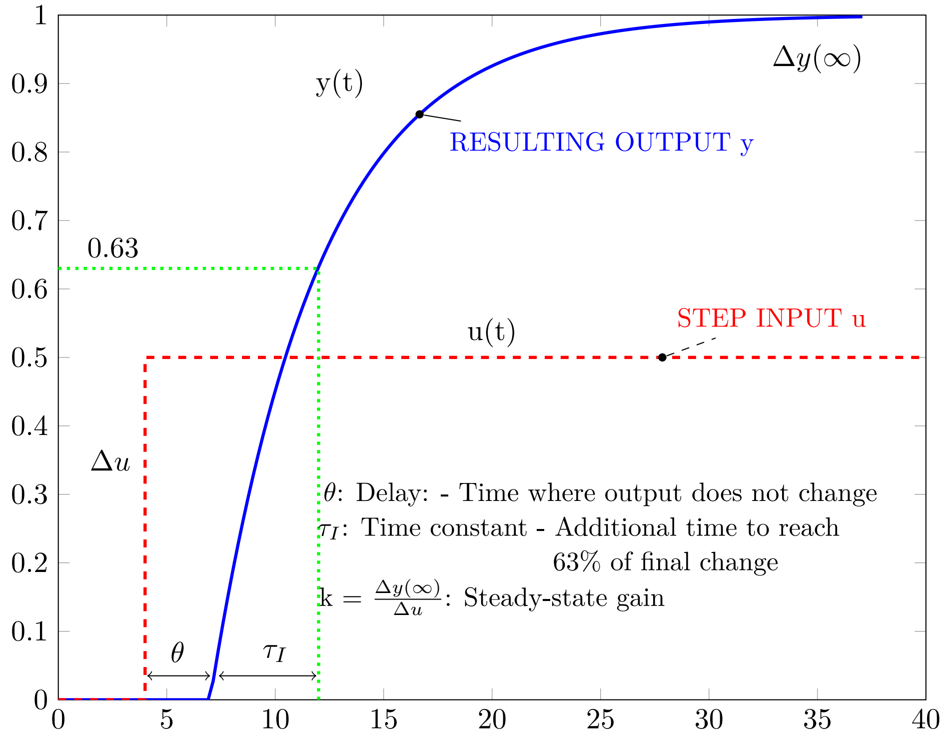

Figure 2.7.1: Open-loop step response experiment to obtain parameters k,τ_I and θ in a FOPTD model

It depends on the system if tight or smooth control is the best choice.

In practise, the tuning parameters for a first-order model are often obtained from a step response experiment. Figure 2.7.1 represent such an open-loop experiment [Skogestad and Grimholt]. A rule of thumb is that the experiment does not need to run for longer than about 10 times the effective delay in order to find the tuning parameters [Skogestad and Grimholt]. The system can then be approximated as an integrating model:

$$\frac{Ke^{-\theta s}}{\tau_I s + 1} \approx \frac{k' e^{-\theta s}}{s} \tag{2.7.5}$$

where $k' = \frac{k}{\tau_I}$ is the slope [Skogestad and Grimholt]. Figure 2.7.2 gives a representation of such an integrating model. Equation (2.7.6) describe how the slope of the process is calculated [Skogestad and Grimholt].

$$k' = \frac{\Delta y}{\Delta t \Delta u} \tag{2.7.6}$$

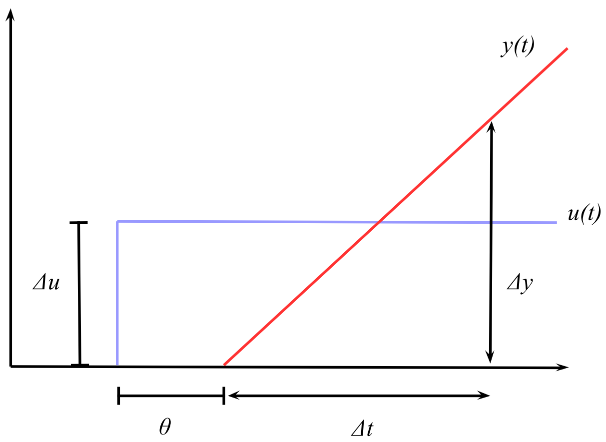

Figure 2.7.2: Open-loop step response experiment to obtain the tuning parameters k' and θ in an integrating model

3 Modelling

A description of the system will be given in this chapter together with the development of the dynamic energy and mass balance. Matlab was used to solve the problem, the Matlab script are given in Appendix E.

3.1 Description of system

The system consists of a single room without any windows, and a schematic overview is given in Figure 3.1.1. The room is simulated with a floor area

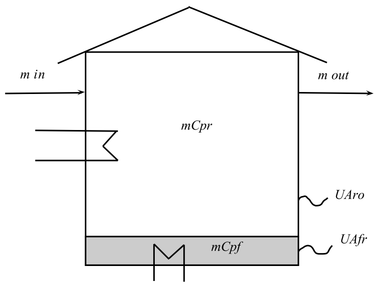

Figure 3.1.1: Schematic overview of the system

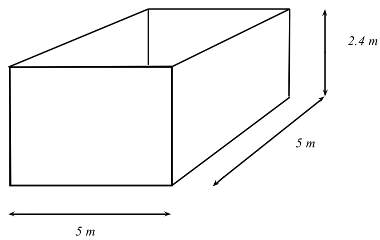

Figure 3.1.2: Dimension of the room

of 25 m^2 with a floor heating device, a radiator and a ventilation system. The dimensions of the room are given in Figure 3.1.2. Table 3.1.1 lists the wall and floor compositions.

Table 3.1.1: Building specifics

Parameter	**Value**
Walls	1.5 cm oak on both sides of a 25 cm rock wool layer
Floor	10 cm concrete with 1.5 cm of oak on the top

3.2 Mass and energy balance

It is assumed that all the heat lost by the floor is transferred to the air in the room. The heat in the air can be lost through the walls or through the ventilation system. Figure 3.2.1 visualizes the energy and mass flow in the system.

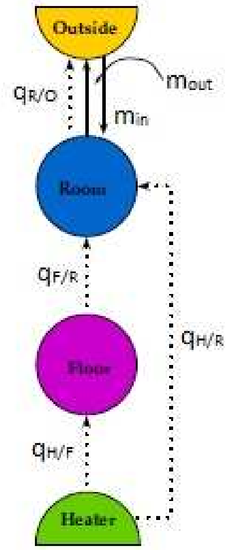

Figure 3.2.1: The topology of the system

The energy balance for the floor is

$$(mCp)_F\dot{T}_F = q_{H/F} - q_{F/R} \tag{3.2.1}$$

where the transfer of energy from the floor to the air, $q_{F/R}$, is given by

$$q_{F/R} = (UA)_F(T_F - T_R) \tag{3.2.2}$$

A combination of equation (3.2.1) and (3.2.2) give an expression for the temperature in the floor:

$$\boxed{\dot{T}_F = \frac{-(UA)_{F/R}}{(mCp)_F}(T_F - T_R) + \frac{q_{H/F}}{(mCp)_F}} \tag{3.2.3}$$

Similarly, the energy balance for the room can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}(mCpT)_R = q_{H/R} + q_{F/R} - q_{R/O} + m_{in}CpT_O - m_{out}CpT_R \tag{3.2.4}$$

where the transfer of energy from the room is given by

$$q_{R/O} = (UA)_R(T_R - T_O) \tag{3.2.5}$$

The mass of air is defined as

$$\frac{\mathrm{d}m}{\mathrm{d}t} = m_{in} - m_{out} \tag{3.2.6}$$

From the assumption that the mass of air in equals the mass of air out, $m_{in} = m_{out}$, is the following expression for the temperature in the room obtained

$$\boxed{\dot{T}_R = \frac{q_{H/R}}{(mCp)_R} - \frac{(UA)_{R/O}}{(mCp)_R}(T_R - T_O) + \frac{(UA)_{F/R}}{(mCp)_R}(T_F - T_R) + \frac{m_{in}}{m_R}(T_O - T_R)} \tag{3.2.7}$$

The values necessary to find T_F and T_R are given in Table 3.2.1, the calculations of the parameters are given in Appendix A.

Table 3.2.1: Constant parameters used in the simulation

Parameter	**Unit**	**Value**
$UA_{R/O}$	$[\frac{kW}{K}]$	0.007
$UA_{F/R}$	$[\frac{kW}{K}]$	0.350
mCp_R	$[\frac{kJ}{K}]$	70
mCp_F	$[\frac{kJ}{K}]$	4000
m_R	$[kg]$	70

3.3 Abstraction

The system of equations derived in the above section can be written on a general form as given below

$$\dot{x_1} = \frac{u_1}{\theta_1} - \frac{\theta_2}{\theta_1}(x_1 - x_2) \tag{3.3.1}$$

$$\dot{x_2} = \frac{u_2}{\theta_3} + \frac{\theta_2}{\theta_3}(x_1 - x_2) + \left(\frac{-\gamma_2(t)}{\theta_3} - \frac{\theta_4}{\theta_3}\right)(x_2 - \gamma_1(t)) \tag{3.3.2}$$

where

$$x = \begin{bmatrix} T_F \\ T_R \end{bmatrix} \tag{3.3.3}$$

$$u = \begin{bmatrix} q_{H/F} \\ q_{H/R} \end{bmatrix} \tag{3.3.4}$$

$$\theta = \begin{bmatrix} (mCp)_F \\ (UA)_{F/R} \\ (mCp)_R \\ (UA)_{R/O} \end{bmatrix} \tag{3.3.5}$$

$$\gamma(t) = \begin{bmatrix} T_o(t) \\ m_{in}Cp(t) \end{bmatrix} \tag{3.3.6}$$

are the state variables, manipulated variables, parameters and disturbances respectively. The system can be further simplified by rearranging it into state space

$$\dot{\mathbf{x}} = A(t)\mathbf{x}(t) + B\mathbf{u}(t) + K(t) = f(x, u, t) \tag{3.3.7}$$

$$\mathbf{y} = C\mathbf{x}(t) \tag{3.3.8}$$

where C is an identity matrix and A(t),B and K(t) are defined as

$$A(t) = \begin{bmatrix} -\frac{\theta_2}{\theta_1} & \frac{\theta_2}{\theta_1} \\ \frac{\theta_2}{\theta_3} & \frac{-\theta_2-\theta_4-\gamma_2(t)}{\theta_3} \end{bmatrix} \tag{3.3.9}$$

$$B = \begin{bmatrix} \frac{1}{\theta_1} & 0 \\ 0 & \frac{1}{\theta_3} \end{bmatrix} \tag{3.3.10}$$

$$K(t) = \begin{bmatrix} 0 \\ \frac{(\theta_4+\gamma_2(t))\gamma_1(t)}{\theta_3} \end{bmatrix} \tag{3.3.11}$$

Equation (3.3.7) illustrates that the system is a linear time varying, LTV, problem.

4 Optimal control problem

This chapter presents the formulation of the dynamic optimization problem and the approach to solve it. The section starts with a formulation of the optimization problem, while a description of the main disturbances will be given in the end.

4.1 Formulation of problem

The objective of the optimization is to minimize the energy costs over a finite horizon. Assuming that the future energy price, P, and weather forecast is known, is the idea to store the energy when the energy price is low, and use it when the price is high.

Both the temperature and heat in the floor and room are limited by upper and lower bounds, as given in Table 4.1.1.

Table 4.1.1: Upper and lower boundaries for the temperature and heat

Parameter	**Upper bound**	**Lower bound**	**Unit**
T_F	25	20	[°C]
T_R	25	19	[°C]
$q_{H/F}$	2.5	0	$[kW]$
$q_{H/R}$	2	0	$[kW]$

The constrained dynamic optimization problem can be formulated as:

$$\min_{u} \int_{0}^{t_f} L(u,t)\mathrm{d}t = \int_{0}^{t_f} p^T u\mathrm{d}t \tag{4.1.1}$$

subject to

$$\dot{x} = A(t)x(t) + Bu(t) + E(t) = f(x,u,t); \quad x(0) = x_o \tag{4.1.2}$$

$$x_{lb} \leq x \leq x_{ub} \tag{4.1.3}$$

$$u_{lb} \leq u \leq u_{ub} \tag{4.1.4}$$

From equation (4.1.2) it can be seen that the cost function is linear on the form $J = p^T \times u$. The constraints and equation (4.1.2) are also linear, and the system can thus be solved by a linear programming method.

Equation (4.1.2) can be expressed in multivariable form as given below:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}a_{1,1} & a_{1,2}\\ a_{2,1} & a_{2,2}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}b_{1,1} & b_{1,2}\\ b_{2,1} & b_{2,2}\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} + \begin{pmatrix}k_{1,1}\\ k_{2,2}\end{pmatrix} \tag{4.1.5}$$

where the elements in $a_{i,j}$,$b_{i,j}$ and $k_{i,j}$ correspond to the parameters in matrix A(t), B and K(t) in equation (3.3.9), (3.3.10) and (3.3.11) respectively.

By rearranging equation (4.1.4) and (4.1.3) it can be seen that the control and state bounds can be written as

$$C_i(u_i,t) = \begin{pmatrix}u_i - u_{ub,i}\\ u_{lb,i} - u_i\end{pmatrix} \leq 0 \tag{4.1.6}$$

$$S_i(x_i,t) = \begin{pmatrix}x_i - x_{ub,i}\\ x_{lb,i} - x_i\end{pmatrix} \leq 0 \tag{4.1.7}$$

respectively, where u and x are vectors. To get the state constraint as a function of the inputs one derive the state bounds once.

$$\frac{\mathrm{d}S_i(x_i,t)}{\mathrm{d}t} = S_i^{(1)}(x_i,u_i,t) = \begin{pmatrix}\dot{x}_i\\ -\dot{x}_i\end{pmatrix} = \begin{pmatrix}f_i(x_i,u_i,t)\\ -f_i(x_i,u_i,t)\end{pmatrix} \leq 0 \tag{4.1.8}$$

The minimum of the Hamiltonian with respect to the inputs is given in equation (4.1.9). According to [Bryson, Arthur E., Ho, 1975] should the state constraints be included in the Hamiltonian.

$$\min_{u_i \in \mathcal{U}} H_i = p_i u_i + \lambda_i f_i + \mu_i S_i^{(1)} \tag{4.1.9}$$

The multiplier μ will be different from zero when the inputs are at the constraints, and zero when the inputs do not have any active constraints, as summarized in Table 4.1.2. To find the solution of the Hamiltonian one

Table 4.1.2: Value of the constraint and multiplier when one are at the constraints and not

Constraint	**multiplier**
$S_i(x_i,t) = 0$	$\mu_i \neq 0$
$S_i(x_i,t) \neq 0$	$\mu_i = 0$

need to look at the case where one are at the state constraint and where one are not.

Case I: No state constraint

In the case where one does not have any active state constraints will $\mu_i = 0$ and $u_{lb,i}$ and $u_{ub,i} \geq 0$. This means that the Hamiltonian can be simplified to

$$\min_{u_i \in \mathcal{U}} H_i = p_i u_i + \lambda_i f_i \tag{4.1.10}$$

In order to find the minimum of the Hamiltonian one have two possible solutions. If

$$p_i + \lambda_i(b_{1,i} + b_{2,i}) > 0 \tag{4.1.11}$$

will the optimal input be at the lower bound, $u_i = u_{lb,i}$, because the Hamiltonian is to be minimized with respect to u_i. Similar, if

$$p_i + \lambda_i(b_{1,i} + b_{2,i}) < 0 \tag{4.1.12}$$

will the optimal input be at the upper bound, $u_i = u_{ub,i}$. This result demonstrate that the input will be either at the lower or upper bound if one does not have any active state constraints. It should be pointed out that one input can be at the upper bound while the other one can be at the lower bound. The result conclude that the same input cannot be at the upper and lower constraint at the same time.

Case II: Active state constraints

If the state, x_1, are at the upper constraint will $S_{1,1} = 0$ and

$$S_{1,1}^{(1)} = a_{1,1}x_1 + a_{1,2}x_2 + b_{1,1}u_1 = 0 \tag{4.1.13}$$

This implies that the Hamiltonian can be written as

$$\min_{u_1 \in \mathcal{U}} H_1 = p_1 u_1 \tag{4.1.14}$$

Looking closer to the case where the states are at the upper bounds it can be seen that

$$S_{1,1} = x_1 - x_{ub,1} = 0 \Longrightarrow x_1 = x_{ub,1} \tag{4.1.15}$$

Implementing this result in f(x,y,t) will the following result for the inputs be obtained

$$a_{1,1}x_{ub,1} + a_{1,2}x_2 + b_{1,1}u_1 = 0 \Rightarrow u_1 = -\frac{1}{b_{1,1}}(a_{1,1}x_{ub,1} + a_{1,2}x_2) \tag{4.1.16}$$

A similar expression is obtained for u_2. The state bounds and the first derivative of S is

$$S_{2,1} = x_2 - x_{ub,2} = 0 \Rightarrow x_2 = x_{ub,2} \tag{4.1.17}$$

$$S_{2,1}^{(1)} = a_{2,1}x_1 + a_{2,2}x_2 + b_{2,2}u_2 + k_2 = 0 \tag{4.1.18}$$

respectively. By implementing equation (4.1.17) in (4.1.18), and rearranging, the following expression for u_2 is obtained

$$u_2 = -\frac{1}{b_{2,2}}(a_{2,1}x_1 + a_{2,2}x_{ub,2} + k_2) \tag{4.1.19}$$

A similar expression for u will be obtained at the lower bounds:

$$S_{3,1}^{(1)} = -(a_{1,1}x_1 + a_{1,2}x_2 + b_{1,1}u_1) = 0 \tag{4.1.20}$$

$$S_{3,1} = x_1 - x_{lb,1} = 0 \tag{4.1.21}$$

The expression for u will thus be

$$u_1 = -\frac{1}{b_{1,1}}(a_{1,1}x_{lb,1} + a_{1,2}x_2) \tag{4.1.22}$$

In the same was as already explained, will the expression for u_2 at the lower bound be

$$u_2 = -\frac{1}{b_{2,2}}(a_{2,1}x_1 + a_{2,2}x_{ub,2} + k_2) \tag{4.1.23}$$

From the above result one can conclude that when the states are at the constraint will the input be equal to the expression in equation (4.1.16), (4.1.19), (4.1.22) and (4.1.23) respectively, depending on if one are at the upper or lower bound.

Case III: Unconstrained input

This section will give an expression for the unconstrained input when the other one have active state constraints. Assuming that x_1 are at the lower bound, the expression for u_1 will thus be as given in equation (4.1.22). The Hamiltonian can be written as

$$H = p^T u + \lambda \begin{pmatrix} 0 \\ \dot{x}_2 \end{pmatrix} = p^T u + \lambda_2 \dot{x}_2 \tag{4.1.24}$$

By implementing the expression for $\dot{x}_2$, will the Hamiltonian be given as

$$H = pu_1 + pu_2 + \lambda_2(a_{2,1}x_{lb,1} + a_{2,2}x_2 + b_{2,2}u_2 + k_2) \tag{4.1.25}$$

The Hamiltonian is to be minimized with respect to u_2, and since u_1 is given, this implies that one has to consider the sign of

$$p_2 + \lambda_2 b_{2,2} \tag{4.1.26}$$

If

$$p_2 + \lambda_2 b_{2,2} > 0 \tag{4.1.27}$$

then $u_2 = u_{lb,2}$, and similar, if

$$p_2 + \lambda_2 b_{2,2} < 0 \tag{4.1.28}$$

is the optimal input, $u_2 = u_{ub,2}$. This result correspond to the obtained solution in section 4.1 (no state constraint). The input that are unconstrained will thus be at the upper or lower bound. A similar expression will be obtained if u_1 is unconstrained.

$$p_1 + \lambda_1 b_{1,1} \tag{4.1.29}$$

4.2 Disturbance modelling

The main disturbances are the outdoor temperature, the mass of air in to the room and the energy price.

The value of the air change rate depends on the house and on wind and temperature changes [Murphy]. If anything else is not specified is the amount of air through the ventilation system and the outdoor temperature assumed a constant value of $0.06\frac{kg}{s}$ and $0\,^{\circ}\mathrm{C}$ respectively.

The power consumption varies throughout the day, and as a result so will the energy price [U.S. Energy Information Administration, 2011]. The energy price will be lower when the demand is low, and higher when the demand increases. The demand is normally higher in the morning before people go to work and in the afternoon when one come home from work. This implies that the energy price will be higher in these peak periods compared to the rest of the day. For simplicity, the energy price is in this project assumed to vary between a high and low value as given in Figure 4.2.1 [Huseiernes Landsforbund, 2013]. The Norwegian currency, NOK, will be used throughout this project.

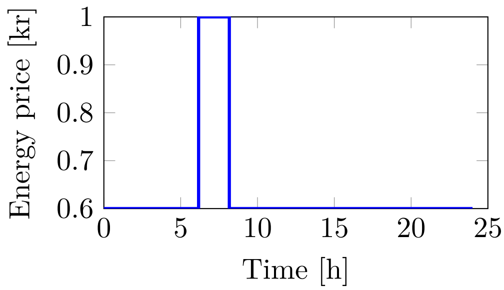

Figure 4.2.1: Energy price as a function of time

5 Shape of optimal solution

A description and illustration of how the solution of the linear optimization problem looks like will be proposed in this chapter.

From the analysis in chapter 4.1 it can be seen that the system has two operational modes:

1. Minimum energy consumption ($u = u_{lb}$ or $x = x_{lb}$)
2. Store energy ($u = u_{ub}$ or $x = x_{ub}$)

By implementing this simple structure, can one reformulate the optimization problem such that the decision variables becomes when to be in operational mode 1 or in operational mode 2. The optimal solution is thought to have a behaviour as illustrated in Figure 5.0.2.

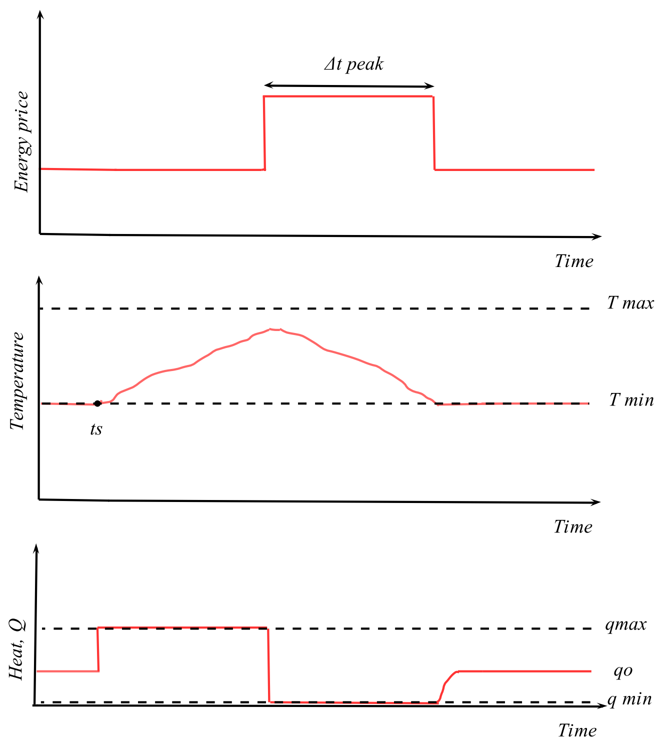

Figure 5.0.2: Shape of temperature and heat in the optimal solution

The state is assumed to be at the lower bound in the beginning of the simulation, as shown in Figure 5.0.2. During this period will the input be at the value that maintain a constant temperature. The input will switch to the upper bound when the system is to be heated up, labelled ts on Figure 5.0.2, and the state will increase as a result of that. However, the heat will be turned off when the energy price increases. The temperature will thus decrease, and when it reaches the lower bound will the input return to the value that keep the state at the constant temperature.

In the above solution it is important to know the time where the input is switched between the upper and lower bound. Since the future energy price is assumed known will the time where the input is switched off be known (which is when the energy price increases). When the state reaches its minimum constraint and during the periods when it is kept at that lower bound, will the heat be at a value that maintain a constant temperature (given in equation (4.1.16), (4.1.19), (4.1.22) or (4.1.23)). This means that the only parameter left to specify is the switching time, ts. At this time will the input be switched to the upper bound.

6 Reformulation of optimal control problem

This section reformulate the approach to solve the optimization problem.

Assuming that one know where the inputs switch from the lower to the upper bound, as described in chapter 5, will the optimization problem be reformulated to when one needs to switch from operational mode 1 to operational mode 2. This time, the switching time, will thus be the only DOF left to specify.

The new optimization problem can be stated as

$$\min_{t_{switch}} J \int_0^t P(t) \times u \; \mathrm{d}t \tag{6.0.1}$$

subject to

$$\dot{\mathbf{x}} = A(t)\mathbf{x}(t) + B\mathbf{u}(t) + K(t) \tag{6.0.2}$$

$$\mathbf{x}(t_o) = x_o \tag{6.0.3}$$

$$u = f(t_{switch}) \tag{6.0.4}$$

The problem was initially solved by single shooting optimization with the matlab function *fmincon*, but it turned out to be difficult to solve with this method. A reason for this may be that the initial guess was not good. However, a lot of different initial guess were tried, so the main reason why *fmincon* could not solve the problem was that the function most likely had problems handling the large variation in the inputs. It should be pointed out that it was the optimization problem described in chapter 4.1 that was tried solved with this method. The matlab script is given in Appendix F.

The optimization problem described in this chapter was solved by using the function *fminsearch* in Matlab, which finds the minimum of a scalar function of several variables [MathWorks]. One only need to specify an initial estimate to be able to solve the problem. The Matlab code was written by Chriss Grimholt, and it is found in Appendix E.1.

The Nelder-Mead simplex algorithm is used by *fminsearch* to solve the problem [MathWorks]. This is a gradient free method that makes a simplex (a n-dimensional version of a triangle) around the initial guess. At each step in the simplex will a solution be calculated. At a certain point in the simplex, called the reflected point, will a solution, f(r), be calculated. This solution, f(r), will be compared to the other solutions obtained in the simplex, and according to the value of f(r), will one of the sides in the simplex be reduced, expanded or reflected [MathWorks]. The same procedure will continue for the new simplex, and so on until a stopping criterion is met [MathWorks].

7 Optimal control

In this chapter will the result of the dynamic optimization problem be presented. Two possible ways to store the heat will be analysed; 1) using only one heating device and 2) using both heaters in the floor and room. The disturbances, m_{in} and T_o, will be kept constant during simulations.

7.1 Where to store the heat

To find the solution of the dynamic optimization problem it is of value to know where it is most optimal to store the heat. The two possible solutions to store the heat are in the floor and in the room. The maximum heat stored is calculated by the following formula:

$$q = mCp(T_{max} - T_{min}) \tag{7.1.1}$$

The calculated values for the room and floor are $560kJ$ and $32000kJ$ respectively. Considering that $(T_{max} - T_{min})$ is almost the same for both the floor and room, and that mCp is much larger for the floor than the room, it can be shown that the heat should be stored in the floor. This result is verified by the calculated values. This implies that storage of heat in the room can thus be neglected since the amount of heat stored in the floor is much bigger than the amount that can be stored in the room.

This report will consider two scenarios where 1) the heat is stored in the floor and 2) where both heaters in the floor and room are used to heat up the room. In each scenario will the length and starting point of the peak period, Δt_{peak}, be varied. The three different cases that are analysed are given in Table 7.1.1.

Table 7.1.1: High price intervals for the optimal case, case with broader peak and the scenario where the system does not have time to heat up

Scenario	**High price interval**	$\boldsymbol{\Delta t_{\mathrm{peak}}}$	**Unit**
Optimal case	$6-8$	2	$[h]$
Broad case	$5-12$	7	$[h]$
Not enough time to heat	$3-10$	7	$[h]$

7.2 Storing of heat in the floor

The system was simulated over a finite horizon, and the value of the initial guess and disturbances used throughout this chapter are given in Table

7.2.1. The results from simulations are given below. The headline on each

Table 7.2.1: Value of the initial guess and disturbances used during simulation

Parameter	Value	Unit
Disturbances		
m_{in}	0.06	$[kg]$
T_o	273	$[K]$
Initial values		
$T_{F,initial}$	293	$[K]$
$T_{R,initial}$	292	$[K]$

chapter refer to the scenarios described in Table 7.1.1.

Optimal case

The optimal solution of the problem is to keep the temperature in the floor and room at the lower bound for as long as possible. However, one should store enough heat to avoid using energy when the energy price is high. Figure 7.2.1 gives the results from the optimization. The red and blue line represents the result for the room and floor respectively.

The heating in the floor starts after 2 hours and 48 minutes. The optimal solution is, according to Figure 7.2.1, that T_F should not reach the maximum constraint before the energy price increases. The temperature in the floor decreases slowly, and will reach the lower bound at 11.33 a.m.

Similar, it can be seen that the room temperature is kept constant at the minimum constraint throughout the simulation time, apart from between 5 and 7. The temperature in the room increase due to an increases in T_F. The room heat will decrease before the specified interval and be turned off in it in order to keep T_R constant. On the same time will the floor heat be at the maximum limit. Since the floor heat is transferred to the room, will this cause an increase in T_R. At 6 o'clock will the floor heat be turned off and thus cause a decrease in the room temperature.

Figure 7.2.1 prove that the solution of the optimization problem always are on one of the constraints, i.e. the constraints are never broken. The floor temperature is at the minimum constraint in the beginning. At 3 o'clock, when T_F increases, will the floor heat be switched to the upper bound, and during the peak period will the heat be at the minimum constraint. At the

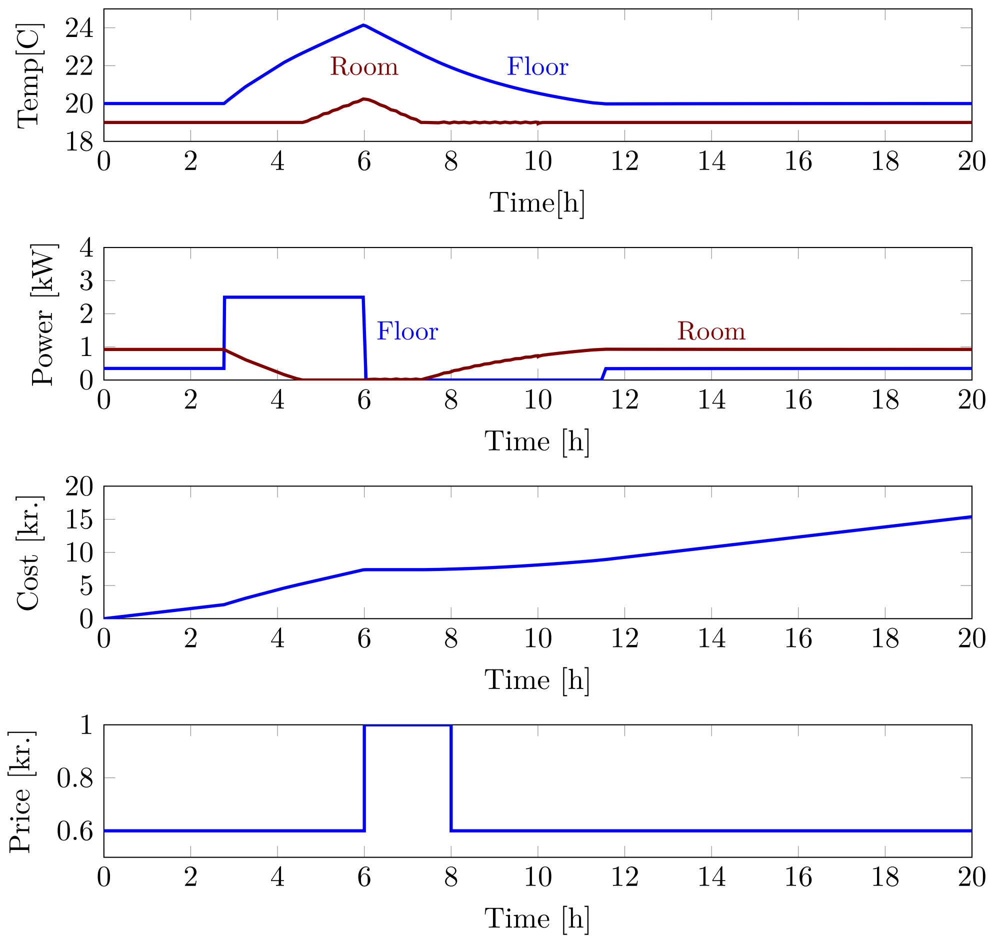

Figure 7.2.1: Results from optimization: optimal case. Temperature and heat versus time for the heat and floor, in addition to the cost function and energy price as a function of time

time when T_F reach its minimum constraint will the floor heat increase to the value that maintain a constant T_F. The room temperature and heat will have the same behaviour. At the time when T_R is not on the lower bound, will $q_{H/R}$ be turned off.

From Figure 7.2.1 it can be seen that the cost increases monotonically, but is constant in most of the the interval with high energy price. This means that no heat is purchased when the energy price is at the highest. The total cost after 20 hours are 15.4 kr.

Figure 7.2.1 shows that the cost function increases at 7.42 a.m even if none of the temperatures increases. The heat in the floor is turned off, but the heat in the room start to increase around 7/8 o'clock. The reason for this behaviour is that the room heat must be turned on in order to keep T_R at 19 °C.

Broad case

The peak period will begin one hour earlier and be five hours longer in this scenario compared to in the above case. The optimal solution with these conditions are given in Figure 7.2.2. The red and blue line represents the result for the room and floor respectively.

Figure 7.2.2 show that the heating of the system will start after just under one hour (54 minutes), and the temperature in the floor will reach the maximum constraint just before 5 o'clock. The temperature in the floor will then decrease and reach the lower bound around 11 o'clock. A loss of one hour is thus obtained, i.e. one uses heat in one hour when the energy price is at the highest. This is illustrated in Figure 7.2.2, where the heat in the floor increases at 11 a.m.

For the same reasons as described in the above section, the temperature in the room will be constant during most of the simulation time, apart from between 3 and 7.

From Figure 7.2.2 it can be seen that no heat is purchased between 5 and 8, which is proved by a constant cost function in that time interval. The total cost after 20 hours is 16.8 kr. In addition, the same figure proves that the temperature and heat in the floor and room will never break its constraints.

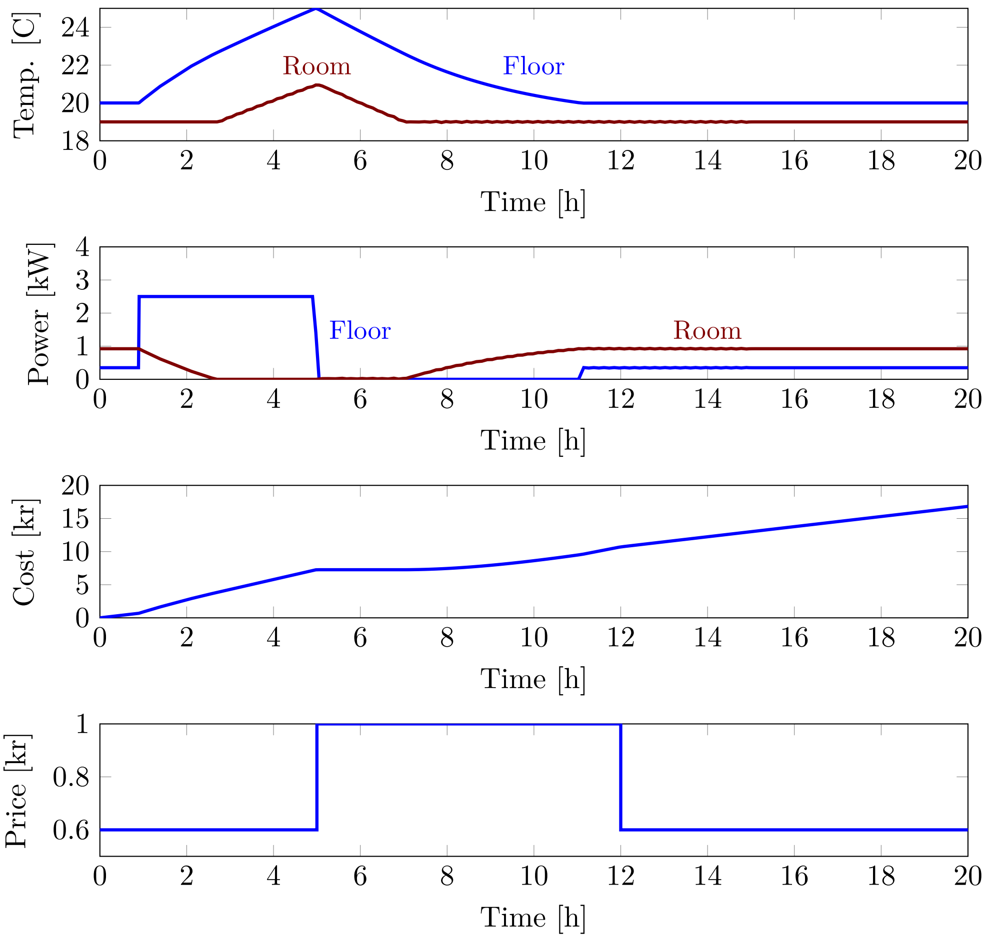

Figure 7.2.2: Results from optimization: broad case. Temperature and heat versus time for the heat and floor, in addition to the cost function and energy price as a function of time

Not enough time to heat

The third scenario considers the case where the heating device does not have time to heat the system before the peak period begins. The interval where the energy price is high will be seven hours long, the same as in the broad case, but it will begin earlier. Figure 7.2.3 gives the optimal solution from simulations. As in the previous cases, the red and blue line represents the result for the room and floor respectively.

Figure 7.2.3 shows that the optimal scenario is to start the heating of the system immediately in order to store as much heat as possible before the peak period. It can be seen that the floor temperature does not reach the upper bound before the energy price increases. At 3 o'clock will T_F decrease and 13 minutes past 8 will the temperature in the floor have reached the lower constraint. This means that the system will have a loss of approximately 2 hours, since the energy price is high until 10 o'clock. Figure 7.2.3 illustrates this, where the floor heat is turned on just after 8 o'clock.

As in the previous cases, the room temperature is kept constant throughout the simulation time, except from between 2 and 4 o'clock.

The total cost after 20 hours is 17.2 kr. Figure 7.2.3 show that the cost function is constant between 3 and 4, when no heat is used in the system, while it increases the rest of the simulation time.

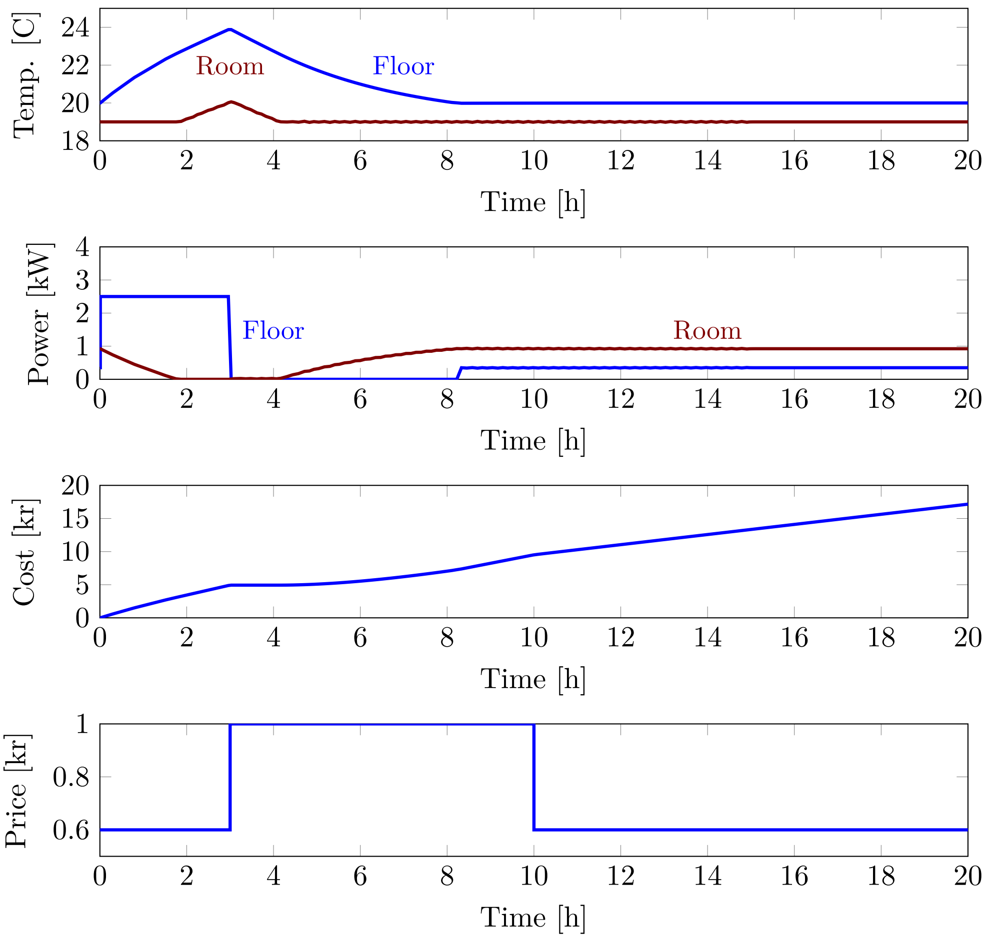

Figure 7.2.3: Results from optimization: case where one does not have time to heat. Temperature and heat versus time for the heat and floor, in addition to the cost function and energy price as a function of time

7.3 Storing of heat in the floor and room

This section will analyse the scenario when both heaters in the floor and room are used to heat the system. The value of the initial guess and disturbances used during simulation are given in Table 7.2.1. The system was simulated over a finite horizon, and the results are given below.

Optimal case

The optimal solution with a peak period that starts at 6 o'clock and is 2 hours long, are given in Figure 7.3.1. As before, the red and blue line represents the result for the room and floor respectively. The headline of each chapter refer to the scenarios described in Table 7.1.1.

Figure 7.3.1 shows that the optimal solution is to keep T_R constant during most of the simulation time. This means that storage of heat in the room will not be necessary in this case. Similar to the optimal case in section 7.2, will the room temperature increases between 5 and 7, due to an increase in T_F when the room heat is off.

The heating of the floor starts 3 hours before the peak period. The temperature in the floor does not reach the maximum constraint before the peak period, it will then decrease and reach the minimum at 11.16 a.m. This will give a loss of approximately 1 hour.

The cost function is constant between 6 and 8, and the total cost after 20 hours is 15.4 kr.

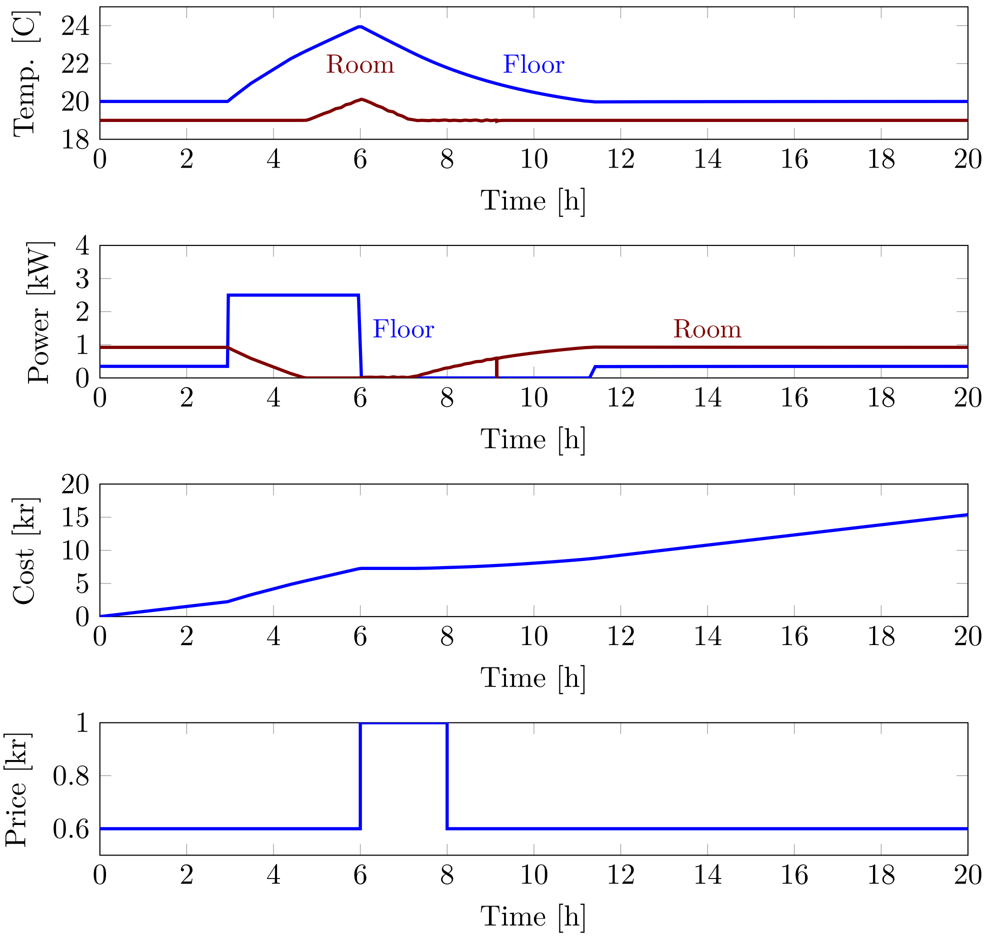

Figure 7.3.1: Results from optimization: optimal case. Both room and floor heat are implemented in the initial guess to find the solution. Temperature and heat versus time for the heat and floor, in addition to the cost function and energy price as a function of time

Broad case

The optimal solution of the optimization problem with a peak interval that starts at 5 a.m and is 7 hours long are given in Figure 7.3.2. The red and blue lines represent the result for the room and floor respectively.

Figure 7.3.2 illustrates that the optimal solution is to use only the heat in the floor for storage, since the room temperature is to be kept constant at the minimum constraint. The floor should be heated up after 54 minutes.

The floor temperature reaches the upper bound just before 5 o'clock and decreases when the peak period begins. At 11 a.m will T_F be at the minimum constraint, which gives a loss of 1 hour.

The temperature in the room is kept constant during the simulations, apart from between 3 and 7. The heat in the room will be at the minimum limit in this interval, but the floor heat is turned on from 1 to 5 a.m.

The cost function is constant between 5 and 7, when no heat is used. The total cost of the optimal solution is 16.8 kr.

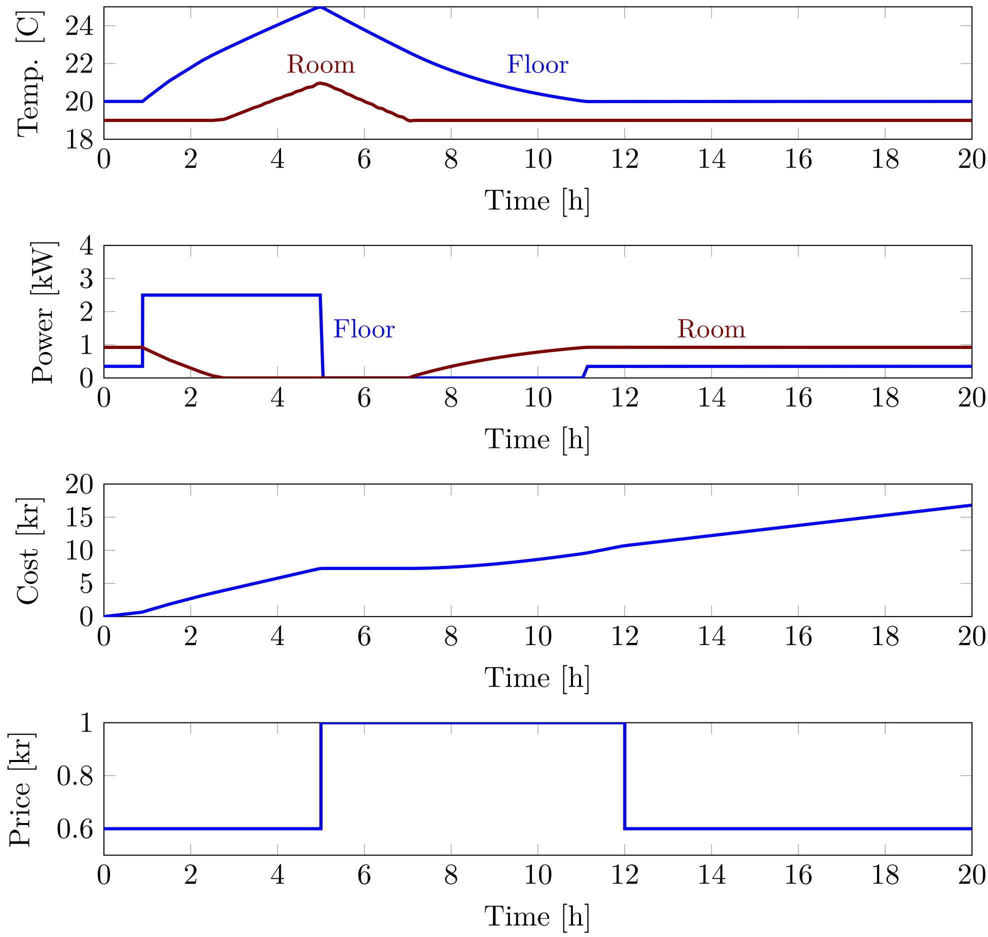

Figure 7.3.2: Results from optimization: broad case. Both room and floor heat are implemented in the initial guess to find the solution. Temperature and heat versus time for the heat and floor, in addition to the cost function and energy price as a function of time

Not enough time to heat

The results from simulations where Δt_{peak} begins at 3 a.m and are seven hours long are given in Figure 7.3.3. The red and blue lines in the figure represent the result for the room and floor respectively.

As in the same case but with storing of heat in only the floor, is the optimal solution to start the heating of the floor immediately. In contrast to the optimal and broad case described above, should the room heat not be kept constant, but be turned to the maximum value 12 minutes past 2.

The floor heat will be kept at the upper bound during storing, and be switched off when the peak period begins. At 9 a.m, when T_F reaches the lower bound, will the floor heat increase again. Figure 7.3.3 shows that the system will have a loss of one hour. From Figure 7.3.3 it can be seen that T_R decreases more rapidly than T_F.

From Figure 7.3.3 it can be seen that the room heat will decrease in the beginning. The reason for this behaviour is that the aim of the system is to keep T_R constant while T_F increases, at least until the heating of the room begins. The room heat is switched to the maximum value during the period of storage. The room temperature will reach the maximum constraint before it decreases and at approximately 5 a.m will T_R reach the lower constraint again.

The cost function is constant between 3 and 5, and the total cost of the optimal solution is 16.9 kr.

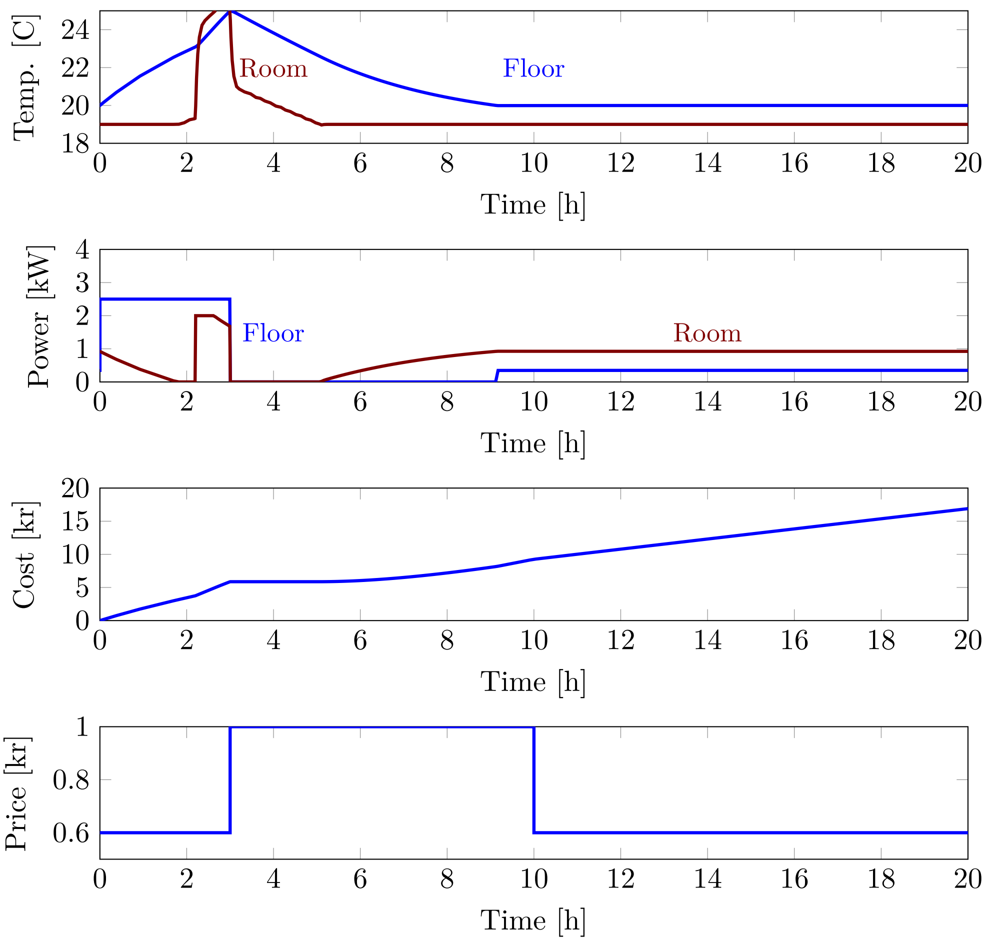

Figure 7.3.3: Results from optimization where the there is no time to heat. Both room and floor heat are implemented in the initial guess to find the solution. Temperature and heat versus time for the heat and floor, in addition to the cost function and energy price as a function of time

7.4 Summary of the results when storing heat in the floor and when using both heaters for storage

A summary of the obtained values for the switching time, loss and total cost for the scenarios described in this chapter are given in Table 7.4.1. The first column give the results from optimization when only the floor heat is used for storage, while the second scenario contain the results from when both heaters are used to heat the system.

Table 7.4.1: Summary of the obtained values of the switching time, the floor and room, the loss and total cost for the scenarios with heating i the floor and where both heaters are used for storage

	Storage: floor		**Storage: floor and room**	
Optimal case	ts_F	2.48 a.m.	ts_F	2.57 ≈ 3 a.m.
	ts_R	–	ts_R	–
	J_{total}	15.4 kr	J_{total}	15.4 kr
Broad case	ts_F	54 minutes	ts_F	53.24 ≈ 54 minutes
	ts_R	–	ts_R	–
	Loss	1 hour	Loss	1 hour
	J_{total}	16.8 kr	J_{total}	16.8 kr
No time to heat	ts_F	0 hours	ts_F	0 hours
	ts_R	–	ts_R	2.12 a.m.
	Loss	2 hour	Loss	1 hour
	J_{total}	17.2 kr	J_{total}	16.9 kr

The floor heat will be the only heating device necessary for storage in the optimal case. The room heat is to be kept constant in order to save energy costs. From the two columns in Table 7.4.1 it can be shown that the system is heated up around the same time in both scenarios. The difference between the time to start heating the system is 12 minutes. The same scenario (where only the floor heat is used for storage) is analysed in both cases, and the switching time should thus be the same. The reason for the difference in ts is that the optimum is flat, and it is thus difficult to obtain the same values. However, this difference is not of significance for the total cost since it is the same for both scenarios.

The result for the broad case is similar to the optimal case, only the floor heat is needed when storing energy. The system is heated up at the same time in both scenarios, and the total cost and loss are the same.

In the third case, when the peak period starts too early for the temperature to have time to reach the upper bound, will the result differ from each other in the two scenarios. In both scenarios will the floor heat be turned on immediately in order to save energy cost. But the temperature in the floor will not reach the upper constraint when using only the floor heat for storage. When using both the floor and room heat will T_F and T_R reach the upper constraint. More energy will be stored in this case, which means that less heat will be used when the energy price is high. Table 7.4.1 illustrates this; the system will have a loss of 2 hours when using only the floor heat, while a loss of 1 hour is obtained when using both heaters. The total cost will be higher for the first scenario compared to the scenario where both heaters are used for storage.

From Table 7.4.1 it can be seen that the longer the peak period, the more energy needs to be stored which results in a higher total cost. The broad and third case have the same length of Δt_{peak}, but the energy price starts to increase earlier in the latter case. Naturally, if the peak period starts too early will the temperature in the system not have time to reach the maximum value, even if the system starts to heat immediately. Using both heaters for storage may prevent this, as shown in Table 7.4.1. The system will have time to heat up when using both heaters and the total cost will be reduced. The total cost in the third case, when both heaters are used for storage is similar to the total cost in the broad case.

From this results one can conclude that it is better to use both heaters to heat the system in the case where one does not have time to heat. On the other hand, if the length and starting point of Δt_{peak} is the same as in the optimal and broad case should only the floor heat be used.

8 Development of switching rules

The objective of the optimization is to minimize the energy costs. Assuming that the future energy cost is known. The optimal scenario is to store heat just before the peak period, when the energy price is low, and the idea is to use this stored energy in the peak period. On the other hand, the storing of heat should not begin too early because excess heat will be transferred from the floor to the room, and from the air to the outside.

This chapter will derive rules to find the optimal time to start heating the system. A switching rule for the optimal and broad case described in section 7.1 will be given together with the result from simulations. The headline of each section represents the scenarios described in Table 7.1.1 in section 7.1

8.1 Switching rule for optimal case

The time to start heating the room should not be too early nor too late in order to save energy costs. The following formula, that finds the length of the interval where one should heat, Δts, was derived:

$$\Delta t_s^q = \frac{\Delta t_{peak}}{\frac{q_{max}}{q_o} - 1} \tag{8.1.1}$$

The different terms in equation (8.1.1) are explained in Figure 8.1.1. The amount of heat stored should be equal to the amount of heat used in the peak period:

$$\Delta t_s \times (q_{max} - q_o) = \Delta t_{peak} \times q_o \tag{8.1.2}$$

Rearranging this equation gives

$$\Delta t_s = \frac{\Delta t_{peak} q_o}{q_{max} - q_o} \tag{8.1.3}$$

which is the expression given in equation (8.1.1).

Results from simulation when using only the floor for storage of heat

The switching time was found from the calculated value of Δts^q and used as the optimal value in the simulations. The calculations of Δts and the appurtenant ts are given in Appendix B. The same constant disturbances as given in section 7.2 was used, and the result are given in Figure 8.1.2.

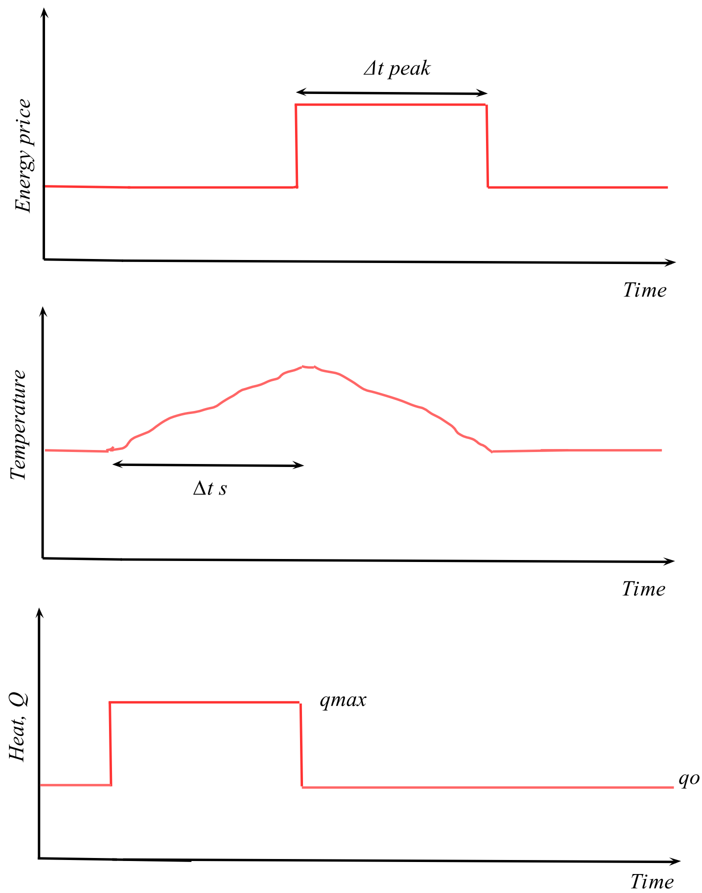

Figure 8.1.1: Heat, temperature and energy price as a function of time. Illustration of what the parameters given in the switching rule represent

According to the calculations from the switching rule, will the floor heat be turned on after 3 hours and 41 minutes. From Figure 8.1.2 it can be seen that the floor temperature does not reach the upper bound before the energy price increases. The minimum constraint will be reached 37 minutes past 10.

The room temperature is kept constant, but it will increase by 0.5°C between 5.30 and 6. When the floor temperature increases will the room heat start to decrease in order to maintain a constant room temperature. In the specified interval will the room heat has reached the minimum constraint, but the floor heat will still be at the maximum. Since heat is transferred

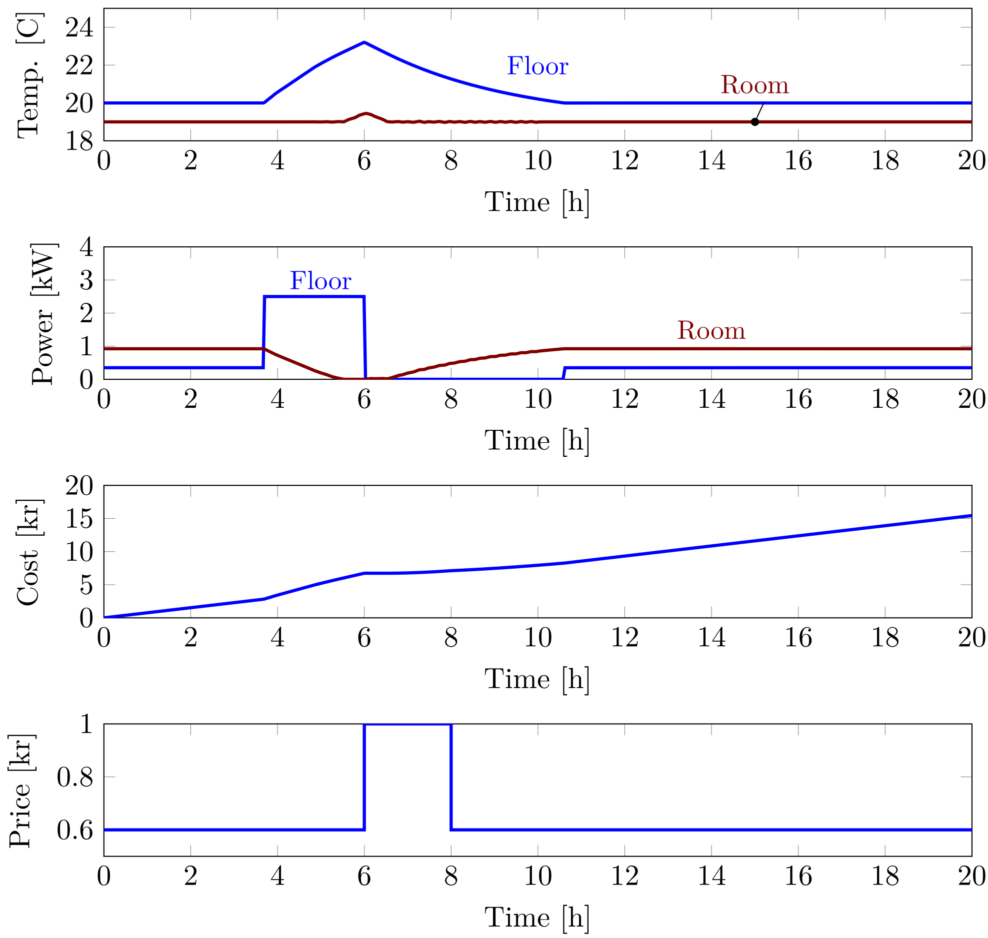

Figure 8.1.2: Results from optimization when using the switching rule for the optimal case. Temperature and heat versus time for the heat and floor, in addition to the cost function and energy price as a function of time

from the floor to the room will this cause an increase in T_R. The floor heat will be turned off when the energy price increases, and this will result in a decrease in the room temperature. Similar to when T_F increases, will $q_{H/R}$ increase when T_F decreases, and when the floor temperature reaches the minimum constraint will the room heat be kept constant.

No heat will be used between 6 and 6.30, which result in a constant cost function in the specified interval. The total cost after 20 hours are 15.4 kr.

8.2 Switching rule for broad case

The interval with high energy price will, in contrast to the optimal case, begin earlier and be longer in this scenario. The floor temperature will reach the upper constraint before the peak period begins. Taking this into account a different switching rule was derived. By starting with equation (3.2.3)

$$\dot{T}_F = \frac{-(UA)_{F/R}}{(mCp)_F}(T_F - T_R) + \frac{q_{H/F}}{(mCp)_F}$$

and assuming that the transfer of energy from the floor to the room, $(UA)_F(T_F - T_R)$, is small compared the heat in the floor and therefore neglected, the following expression of T_F is obtained

$$\frac{\mathrm{d}T_F}{\mathrm{d}t} = \frac{q_{H/F}}{(mCp)_F} \tag{8.2.1}$$

Rearranging this equation gives the time to start heating the system from the starting value, T_o, to T_{max}. It is assumed that T_o always equals T_{min}. The heat in the floor, $q_{H/F}$, is replaced by $\Delta q = (q_{max} - q_o)$, which is the amount of heat necessary to heat the floor from T_o to T_{max}.

$$\int_{T_{min}}^{T_{max}} \mathrm{d}T = \frac{q_{max} - q_o}{mCp} \int_{t_1}^{t_2} \mathrm{d}t \tag{8.2.2}$$

Integrating and rearranging gives the new switching rule:

$$\Delta t_s^T = \frac{T_{max} - T_{min}}{q_{max} - q_o} mCp \tag{8.2.3}$$

Results from simulation when using only the floor for storage of heat

The switching time was found from the calculated value of Δts^T and used as the optimal value in the simulations. The calculations of Δts and ts are

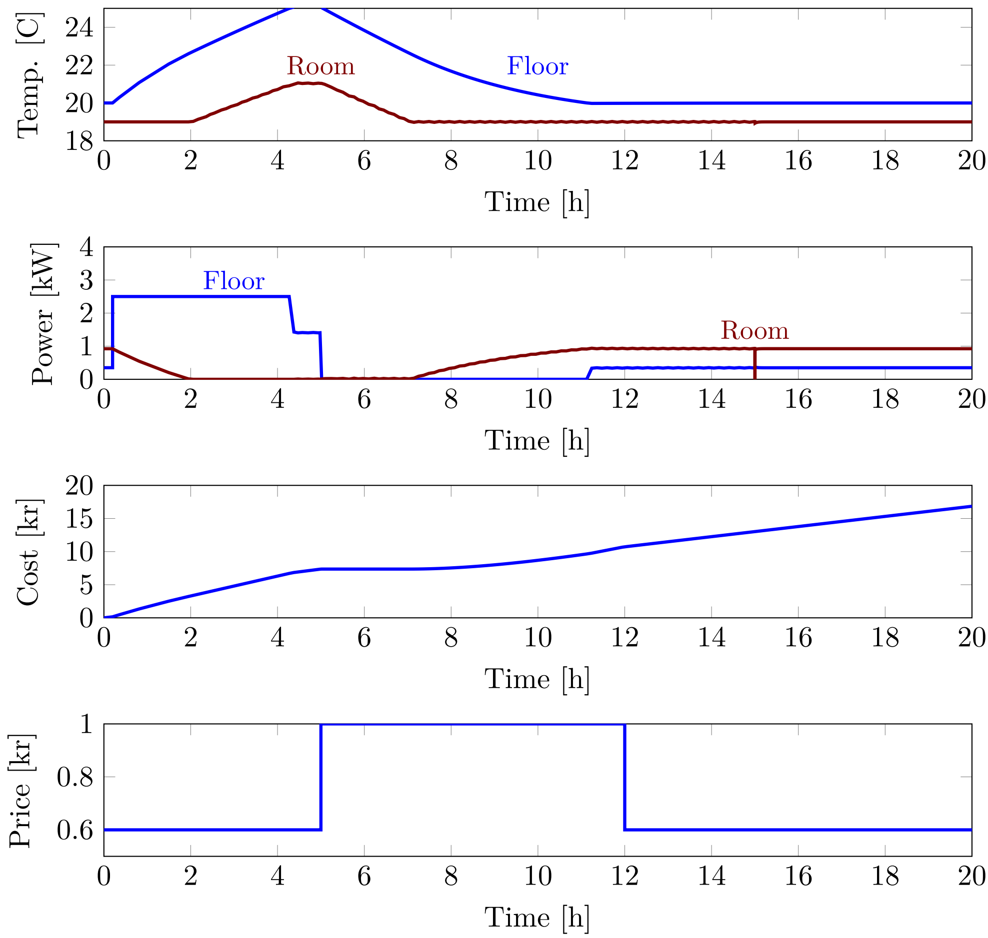

Figure 8.2.1: Results from optimization when using the switching rule for the broad case. Temperature and heat versus time for the heat and floor, in addition to the cost function and energy price as a function of time

given in Appendix B. The same constant disturbances as given in section 7.2 was used, and the result are given in Figure 8.2.1.

The length of the interval where one should heat was calculated to 4.48 h. This means that the heat in the floor will be switched to the maximum value after 12 minutes, since the peak period begins at 5 a.m. The floor heat will increase and reach the upper bound around 4, and be kept there until the peak period begins. The floor temperature will decrease when the floor heat is turned off, and it will reach the minimum constraint 7 minutes after 11. This will give a loss of approximately 1 hour.

The room temperature will be constant most of the time, apart from between 2 and 7 where it will increase. As described in the previous chapter, the temperature will increase because the floor heat is turned on, and the room heat is turned off.

From Figure 8.2.1 it can be seen that the cost function is monotonically increasing, and the total cost after 20 hours is 16.8 kr.

8.3 Switching rule for the case where there is no time to heat

The same switching rule as obtained in section 8.1 was used in this case. A detailed description of the calculations of Δts is given in Appendix B.

The calculated value of Δts is 8.09 h. This means that the switching time is 5 hours before the simulation time, and the solution is thus infeasible.

8.4 Summary

Table 8.4.1 gives a summary of the calculated values of Δts, ts, the total cost and loss obtained in the cases described above.

The switching rules give a good estimate for the optimal and broad case, while the result is poor in the third case.

From Table 8.4.1 it can be seen that the longer the peak period, the higher energy costs, since more energy needs to be stored in advance. In addition, it can be seen that the constraints will never be broken in any of the cases.

Table 8.4.1: Summary of the obtained values of the switching time, Δts, loss and total cost for the scenario with heating i the floor. The values of Δts was calculated from the switching rules

	Storage of heat: floor	
Optimal case	Δts^q	2.31 h
	ts	10 h and 37 min
	J_{total}	15.4 kr
Broad case	Δts^T	4.8 h
	ts	1h and 7 min
	Loss	5 h and 30 min
	J_{total}	16.8 kr
No time to heat	Δts^q	8.09 h
	ts	Infeasible

8.5 Comparison of optimal solution with the switching rules

Table 8.5.1 compares the obtained values of the loss, total cost and ts from the optimal control problem and switching rules. For sake of comparison, only the scenario where the system use the floor heat for storage is analysed.

Table 8.5.1: Comparison of the switching time, loss and total cost obtained from the optimal solution and switching rules

	Switching rule		Optimal control	
Optimal case	ts	3h and 41 min	ts	2h and 48 min
	J_{total}	15.4 kr	J_{total}	15.4 kr
Broad case	ts	12 min	ts	54 min
	Loss	1 hour	Loss	1 hour
	J_{total}	16.8 kr	J_{total}	16.8 kr
No time to heat	ts	Infeasible	ts	0 hours
	Loss	–	Loss	2 hours
	J_{total}	–	J_{total}	17.2 kr

The switching rule gives a good estimate of the switching time in the optimal and broad case. Even if the system is heated up for a longer period of time (56 min before the optimal solution) in the case where the switching rule is used, will the total cost be the same in both scenarios. The same result is obtained for the broad case. The switching time is estimated to begin 42 minutes before when using the switching rule. In spite of this will the total cost for the optimal solution and switching rule be the same. The reason for the difference in switching time may be due to the flat optimum.

One ought to think that if the system is heated up for a longer period of time, should the total cost also be higher. Especially in the broad case, when T_F is kept at the maximum for about an hour when using the switching rule, compared to just a few minutes in the solution of the optimal control problem. The total cost will be the same in both scenarios because the room heat will start to decrease and eventually be turned off when the T_F increases. This means that if the system is heated up earlier, will the room heat be off for a longer period of time, compared to the case where the heating starts later.

From Table 8.5.1 it can bee seen that the loss obtained from the optimal control problem and switching rule is the same in the broad case. The floor temperature will decrease at the same rate in both scenarios, and it will start to drop when the energy price increases. The floor temperature will reach the maximum constraint before the peak period in both scenarios. This means that T_F will be at the same value in both scenarios when it start to decrease, and T_F will thus reach the minimum constraint at the same. The loss is found by subtracting the time where the peak period ends with the time T_F reach the minimum constraint, and since the peak period is equal in both cases will the loss be similar to one another in the two scenarios.

The same switching rule as in the optimal case is used in the scenario where the system does not have time to heat up. The estimation of ts will in this case be no good, since a value before the starting point of the simulation is estimated.

This comparison demonstrates that the switching rule give good results for the optimal and broad case, while the estimate is poor for the last case.

9 Optimal control with varying disturbances

This chapter gives the result of the optimal control problem when varying the outdoor temperature. The first part will give the result from the optimal control problem described in chapter 6 and 7, while the last part illustrates how the switching rules respond to disturbances.

In the previous chapters has the outdoor temperature been kept constant at 0 °C. This chapter will analyse the behaviour of the system when the outdoor temperature is changed to another constant value or when a step is implemented. The optimal case, where the peak period is 2 hours long and starts at 6 o'clock, will be used as reference case. An illustration of the energy price in the optimal case is shown in Figure 9.0.1. In addition, only the floor heat is used for storage of heat in this chapter.

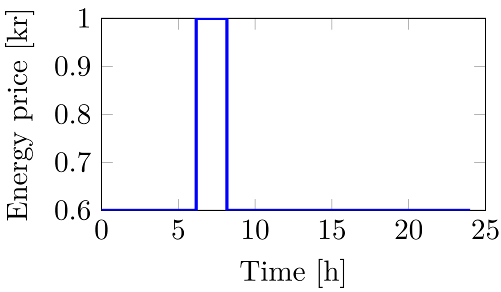

Figure 9.0.1: Illustration of the peak period in the base case

A variation in the outdoor temperature before and after the peak period will be implemented.

9.1 Variation in the outdoor temperature before the peak period

In this case will the outdoor temperature still be constant, but it will vary between +\- 5 and +\- 9 degrees from the base case. Figure 9.1.1 and 9.1.2 gives the result from simulation. The first figure compares the temperature and power in the floor and room for the various changes in T_o, while the latter figure shows the cost function and outdoor temperature as a function of time. The blue line represents the reference case, where the outdoor temperature is 0 °C. The red, green, yellow and purple line represent a outdoor temperature of 9, 5, -5 and $-9\,°C$, as illustrated in Figure 9.1.1.

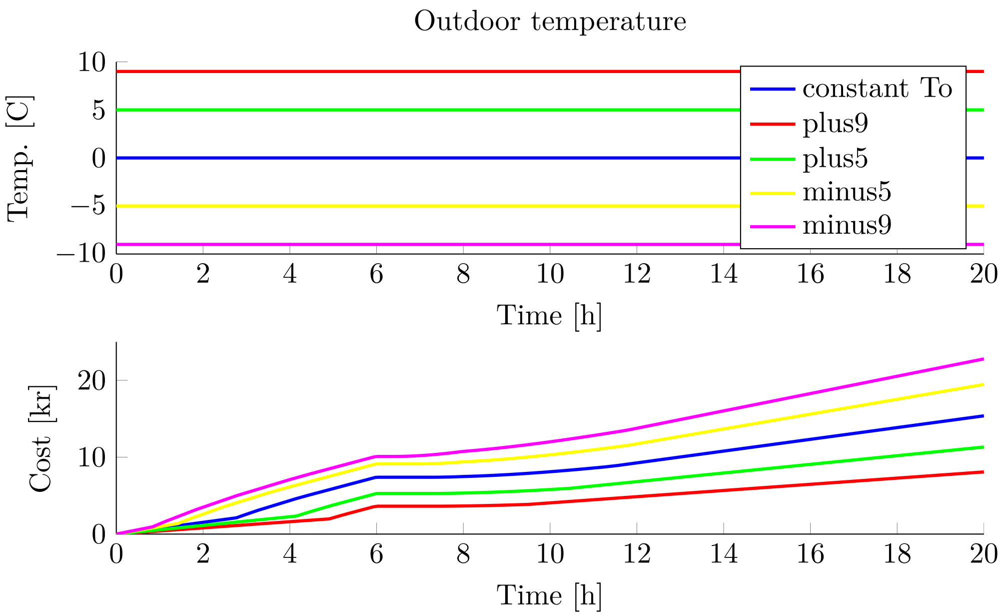

Figure 9.1.1: Variation in the outdoor temperature before the peak period. Cost function and outdoor temperature as a function of time

According to Figure 9.1.2 it can be seen that the switching time for the floor changes when the outdoor temperature varies. The optimal time to start heating the system for the case where the outdoor temperature is 9, 5, 0, -5 and $-9\,^{\circ}$C are 4 h 56 min, 4 h and 10 min, 2 h and 48 min, 1 h and 26 min and lastly 50 min respectively.

The floor temperature will reach the maximum constraint when the outdoor temperature is -5 and $-9\,^{\circ}$C. For higher outdoor temperature will the maximum constraint not be attained. Similar, the temperature will reach its lower bound after 11 hours and 50 minutes when the outdoor temperature is -5 and $-9\,^{\circ}$C. For an outdoor temperature of 0, 5 and $9\,^{\circ}$C will the minimum limit be reached at 11.28, 10.28 and 9.30 a.m respectively.

The room temperature will be constant most of the time, but it will have a peak around 6 in all the cases. The warmer it is outside, the later does the system need to be heated up. The room heat is starting to decrease at the switching time, when $q_{H/F}$ is turned to max. At the same time, the floor heat will be turned off at 6 a.m in all the cases. At this time will T_F

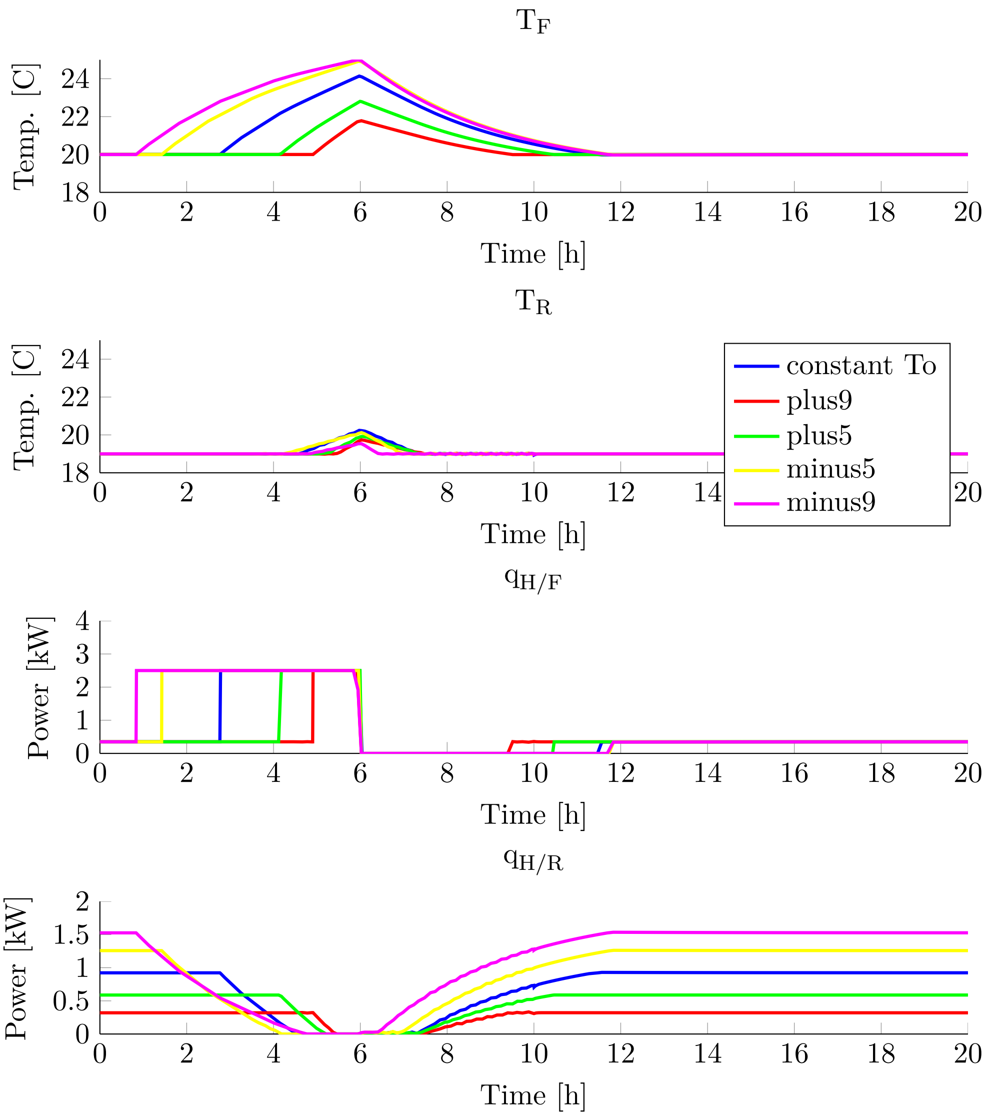

Figure 9.1.2: Variation in the outdoor temperature before the peak period. Input and states as a function of time

decrease, and the room heat will start to increase. From Figure 9.1.2 it can bee seen that the earlier the switching time, the earlier will $q_{H/R}$ be turned off. This means that T_R will increase for a longer period of time the colder the outdoor temperature is. However, at an outdoor temperature of $-9\,°C$ will the room temperature increases least of all. At such a low outdoor temperature will the room heat need to use a lot of power to keep the room temperature at the minimum constraint. It will thus take a while for the room heat to decrease to the minimum constraint, and from Figure 9.1.2 it can be seen that the interval where $q_{H/F}$ is off is smaller than the same interval when the outdoor temperature is $-5\,°C$.

The total cost increases with decreasing outdoor temperature. The total cost after 20 hours when the outdoor temperature is 9, 5, 0, -5 and $-9\,°C$ is 8.1, 11.3, 15.4, 19.5 and 22.8 kr.

From Figure 9.1.2 it can be seen that the temperature and heat will always be on the constraints or between, they are never broken.

9.2 Variation in the outdoor temperature after the peak period

A step at 9 o'clock, where T_o will change from $0\,°C$ to +\- 5 and +\- 9 degrees, will be implemented in this case. Figure 9.2.1 and 9.2.2 give the results from simulation. The first figure shows the input and states as a function of time for the different outdoor temperatures, while the latter figure compares the cost function and outdoor temperature in each case. The red, green, yellow and purple line represent a outdoor temperature of 9, 5, -5 and $-9\,°C$. The blue line gives the result from the reference case.

Figure 9.2.1 shows that the switching time will be approximately the same in all the cases (around 3 o'clock) when a change in the outdoor temperature happens after the peak period. The room and floor heat will therefore have similar behaviour in all the cases until 9 o'clock. At 9 a.m when a step in the outdoor temperature is implemented, as illustrated in Figure 9.2.2, will the amount room heat used vary in each case. Naturally, the colder the outdoor the outdoor temperature the more heat needs to be used in order to keep the room temperature at the minimum constraint. Similar, the warmer the weather is outside, less heat will be needed to maintain a constant temperature.

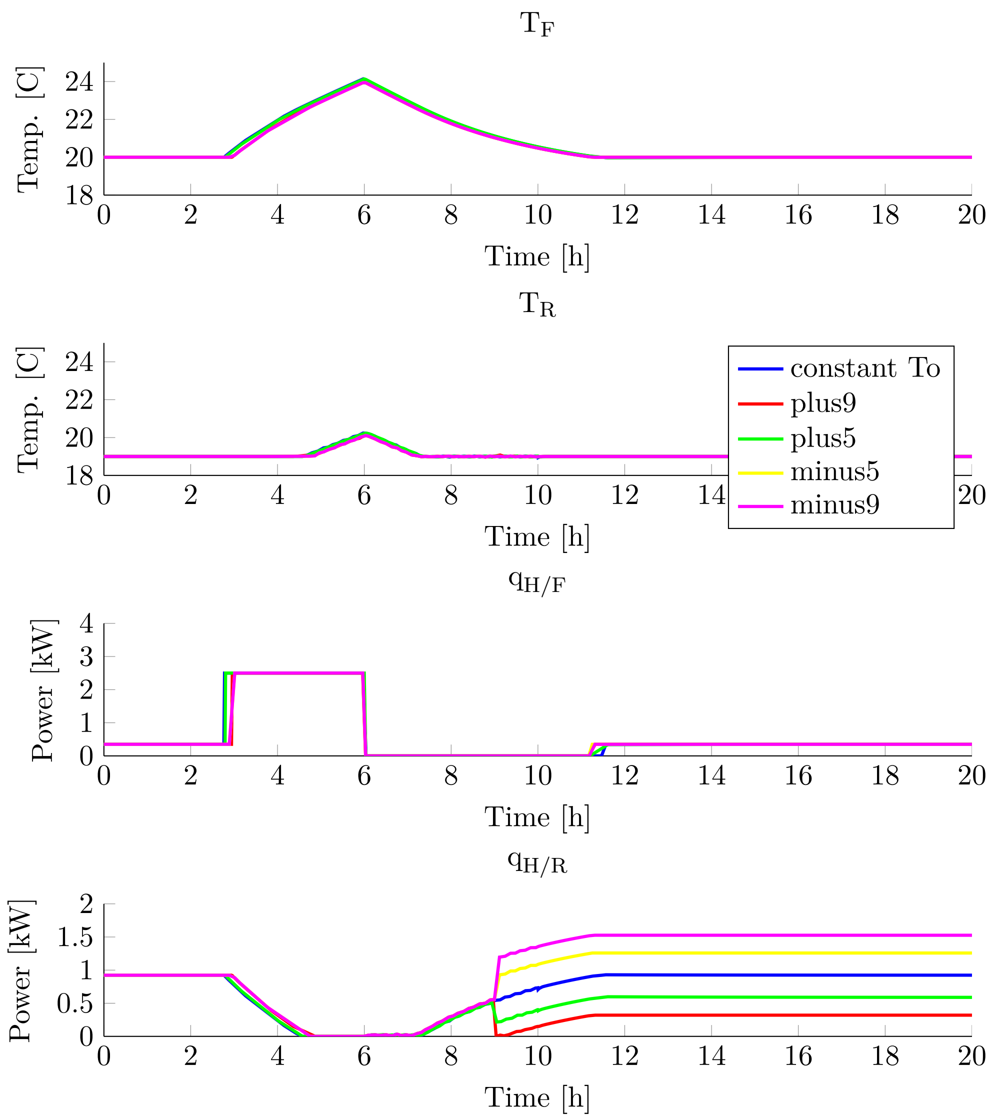

Figure 9.2.1: Step in the outdoor temperature after the peak period. Input and states as a function of time

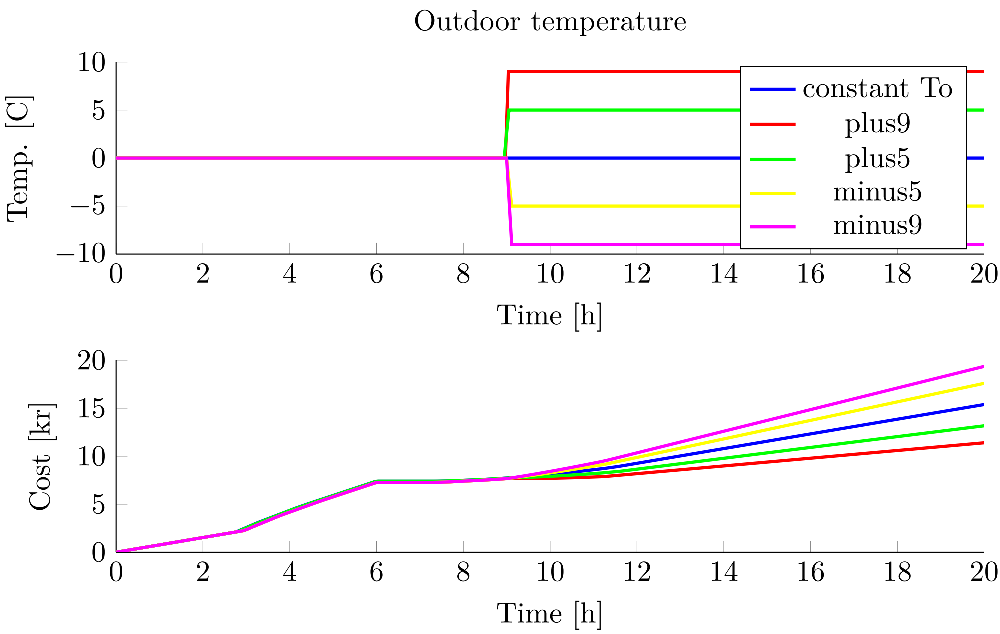

Figure 9.2.2: Step in the outdoor temperature after the peak period. Cost function and outdoor temperature as a function of time

According to Figure 9.2.2 it can be seen that the cost function is more or less the same until 9 a.m for all the different outdoor temperatures. After 9 a.m will the increase in the cost function be higher the colder it is outside. The total cost after 20 hours when T_o changes till 9, 5 , 0 , -5 and $-9\,^{\circ}$C is 11.4, 13.2, 15.4, 17.6 and 19.4 kr.

9.3 Switching rules and how they respond to changes in the outdoor temperature

As in section 9.1 will the outdoor temperature be constant during the simulation time, but it will vary with +\-5 and +\-9 degree from the reference case. The switching rule derived in chapter 8.1 will be used to find the switching time. Table 9.3.1 gives the obtained switching times, while the calculations are given in Appendix C. The calculation of the switching time for the reference case is found in Appendix B.

Table 9.3.1: Calculated values of the switching time from the switching rule for the optimal case

Outdoor temperature	**Switching time**
9 °C	5 h and 16 min
5 °C	4 h and 48 min
0 °C	3 h and 41 min
−5 °C	2 h and 24 min
−9 °C	Infeasible

An infeasible switching time is obtained when the outdoor temperature is −9 °C, and this case will therefore be omitted in the simulations. Figure 9.3.1 and 9.3.2 give the result from simulation. The first figure shows the input and states as a function of time for the different outdoor temperatures, while the latter figure compares the cost function an outdoor temperature in each case. As previous, The red, green and yellow line represent an outdoor temperature of 9, 5, and −5 °C, while the blue line gives the base case.

Table 9.3.1 proves that the colder the outdoor temperature, the earlier will the switching time be. From Figure 9.1.2 it can be seen that the floor temperature never reaches the upper constraint in any of the cases, but the minimum constraint is reached at 11.15, 10.37, 9.30 and 8.48 a.m with an outdoor temperature of -5, 0, 5 and 9 °C respectively.

The room temperature will barely increase in any of the cases. The interval where the room heat increase the most is at a outdoor temperature of −5 °C, and opposite, T_R do barely increase when T_o equals 5 and 9 °C.

From Figure 9.3.2 it can be seen that the total cost after 20 hours when the outdoor temperature is -5, 0 ,5 and 9 °C is 9.5, 15.4, 11.4 and 8.1 kr

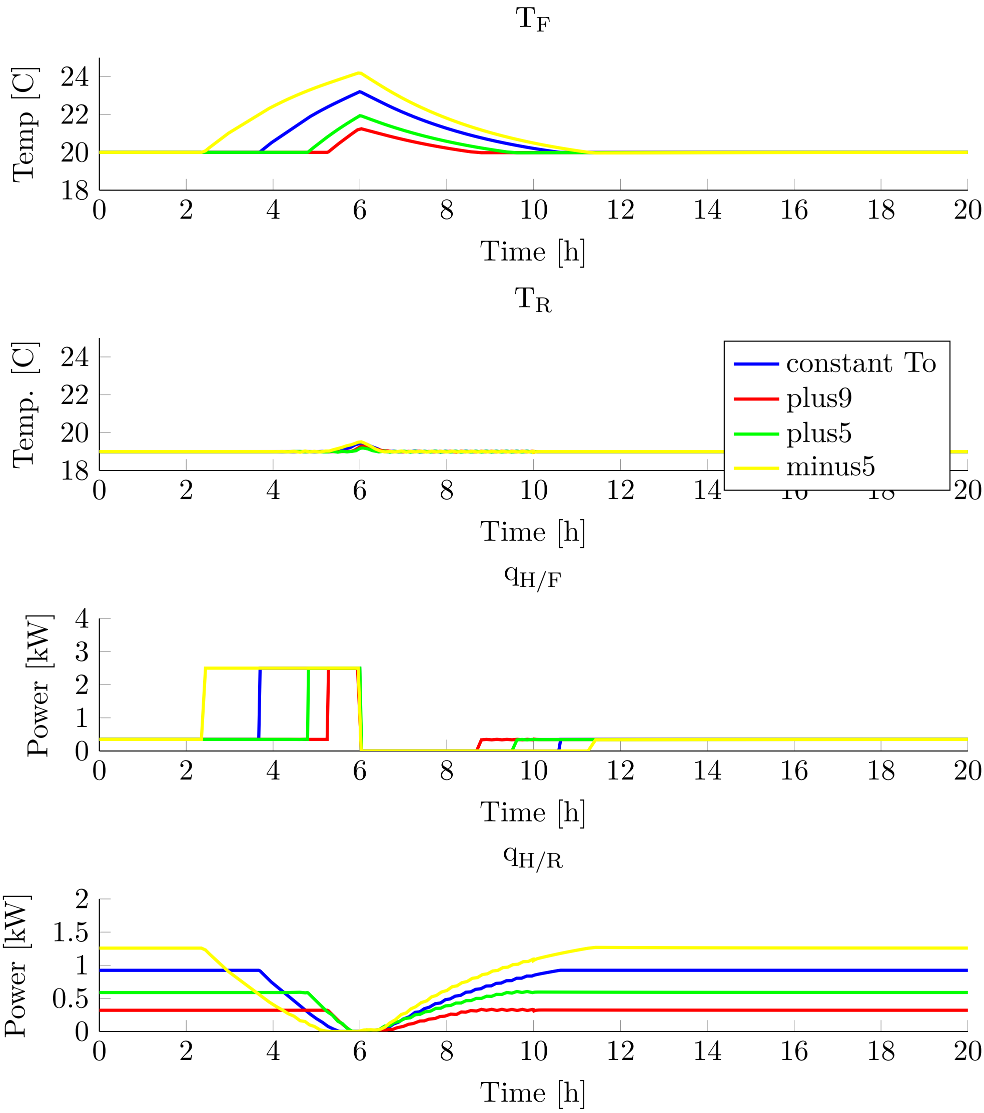

Figure 9.3.1: Step in the outdoor temperature before the peak period. Input and states as a function of time. The switching rule is used to find the optimal time to start heating the system

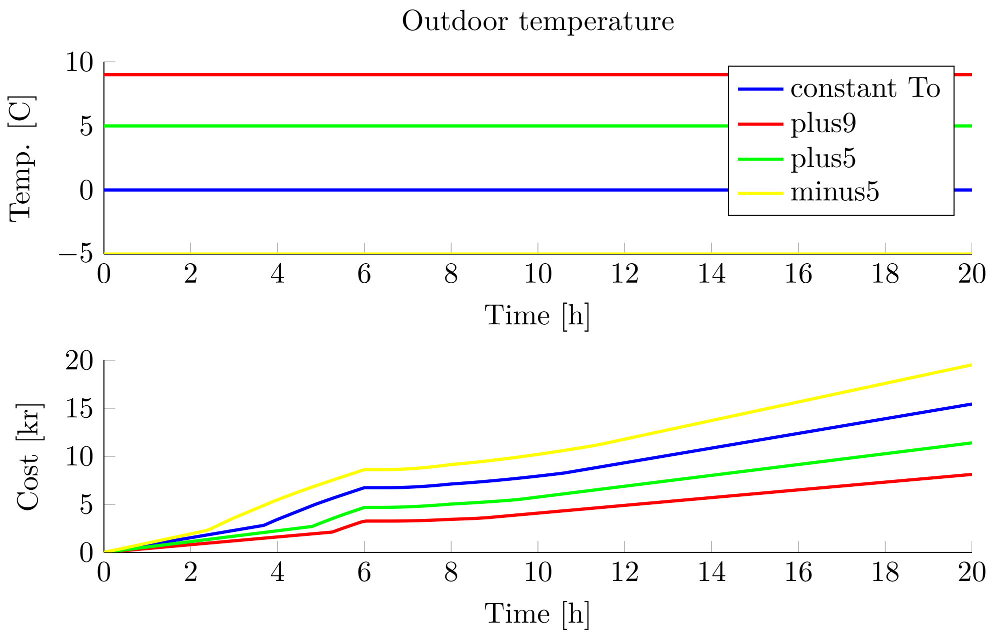

Figure 9.3.2: Step in the outdoor temperature before the peak period. Cost function and outdoor temperature as a function of time. The switching rule is used to find the optimal time to start heating the system

respectively.

9.4 Comparison

Table 9.4.1 gives a summary of the obtained switching time and total cost for the cases where the outdoor temperature is varied before and after the peak period. In addition, the results from the switching rules will also be included.

Before peak period

Table 9.4.1 shows that the obtained switching time from the switching rule and optimal case are similar. The difference between the optimal case and switching rule is 20, 38 and 58 minutes for an outdoor temperature of 9, 5 and $-5\,^{\circ}$C respectively. Although the optimal case starts to heat a bit earlier will the total cost be approximately the same for all the cases. For an outdoor temperature of $5\,^{\circ}$C will the total cost after 20 hours be 0.1 kr higher when using the switching rule.

Table 9.4.1: Summary of the obtained switching time and total cost for the cases where the outdoor temperature is changed before and after the peak period, in addition to the results from the switching rules

	Before peak period				**After peak period**	
	Optimal control		**Switching rule**		**Optimal control**	
T_o [°C]	ts [h]	J_{tot} [kr]	ts [h]	J_{tot}	ts [h]	J_{tot} [kr]
9	4.56	8.1	5.16	8.1	2.57	11.4
5	4.10	11.3	4.48	11.4	2.48	13.2
0	2.48	15.4	3.41	15.4	2.48	15.4
−5	1.26	19.5	2.24	19.5	2.58	17.6
−9	0.50	22.8	–	–	2.58	19.4

From Table 9.4.1 it can be seen that the switching time begins closer to the peak period when the outdoor temperature is warmer than the reference case, outdoor temperature of 0 °C. Similar, the system will be heated up earlier when the outdoor temperatures go below the base case. The cost function shows the same behaviour; the colder the outdoor temperature, the more heat is needed to keep the temperature at the desired value, and the higher the total cost.

Implementing a change in the outdoor temperature before the peak period result in a change in switching time. It can be seen that the colder the outdoor temperature, the earlier should the switching time be. Similar, when the outdoor temperature increases will the switching time be closer to the peak period. This prove that the system will take a change in disturbance before the peak period into consideration, and a new switching time that allow for the new outdoor temperature will be calculated.

After peak period

A variation in disturbance after the peak period will not give a change in switching time that are of any significance. The last but one column in Table 9.3.1 show that the switching time will be just before 3 o'clock in all the cases. This result indicates that the model does not allow for a variation in the outdoor temperature after the peak period. The switching time will barely vary from the reference case if the outdoor temperature increases or decreases.

Same as for a disturbance before the peak period, the cost function will be higher the colder the outdoor temperature is. However, comparing the cost function for the optimal case with a disturbance before and after the peak period it can be seen that:

1. $J_{\text{total, disturbance after peak}} < J_{\text{total, disturbance before peak}}$ when the outdoor temperature is below 0 °C

2. $J_{\text{total, disturbance after peak}} > J_{\text{total, disturbance before peak}}$ when the outdoor temperature is higher than the reference case

For temperatures above the reference case will less floor heat be used in the optimal case since the switch time is just before the peak period. This means that the energy consumption in the optimal case will be lower than for the switching rule, where the switching time will be around the same time as the base case. Similar, the interval where the heating of the system take place is longer than the base case when the outdoor temperature is below 0 °C in the optimal case. The energy consumption will thus increase, and it will be higher than the energy consumption when using the switching rule.

The above results conclude that the model, both the solution from the optimal control problem and switching rules, will take the disturbance into consideration if it is varied before the peak period. However, the model does not allow for a variation in the outdoor temperature after the peak period. The reason for is that the model cannot know what will happen in the future, and the solution obtained may not be the optimal one.

The switching rule handles a variation in the disturbance well, but when the outdoor temperature becomes too cold will the result be poor.

10 Optimal control using PI

This chapter presents the results of the dynamic optimization problem when using Simulink and PI controllers to solve the system. The chapter starts with a description on how the system was implemented in Simulink. How the optimization problem was solved and the results are given in the end.

10.1 Description of the Simulink model

The system described in chapter 3.2 was this time implemented in Matlab and Simulink, in addition to a simple feedback control structure. The control structure includes two PI controllers, where the two heat inputs are used to control the floor and room temperature respectively. The control structure is illustrated in Figure 10.1.1, the Matlab code and an illustration of the Simulink model are given in Appendix E, section E.2.

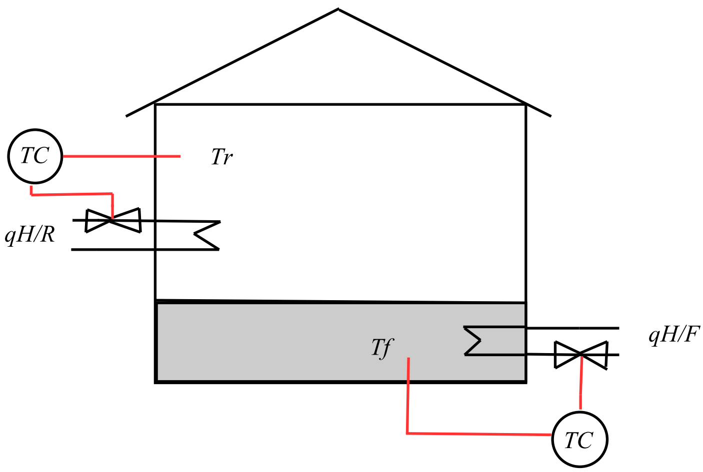

Figure 10.1.1: Overview of the control structure of the system

The parameters for the PI controllers were found using the tuning rules explained in chapter 2.7. The tuning parameters are given in Table 10.1.1, a more detailed description on how they were calculated are given in Appendix D.

Anti-windup is included in the PI controllers in Simulink. This means that the integral part in the PI controller is turned off when when the upper or lower limits of the saturation are reached.

Table 10.1.1: Tuning parameters for the two PI controllers

PI controller	$\mathbf{K_c}$	$\tau_\mathbf{I}$
$T_F - q_{H/F}$	127.7	0.04
$T_R - q_{H/R}$	249.7	0.04

10.2 Storing of heat in the floor

This chapter will use the result from chapter 7 to find the optimal solution of the problem. The obtained value of the switching time will be implemented as an initial guess in the Simulink model, and by the method trial and error will the optimal solution be obtained. Table 10.2.1 gives a summary of the obtained values of ts from the switching rule found in chapter 7.

Table 10.2.1: Obtained value of ts from the switching rules for the optimal case, broad case and the case where one do not have time to heat

Case	**ts**
Optimal case	3 h 41 min
Broad case	12 min
No time to heat	Infeasible

This section will analyse the case with storing of heat in only the floor. The temperature in the room is set to the minimum constraint, while the temperature in the floor will be kept at the minimum for as long as storage of heat is not necessary. During the period of storage, will the heater in the floor be kept at the maximum output.

For sake of comparison, the value of the inputs and disturbances during simulation are the same as in section 7.2. The same scenarios as explained in section 7.1 was analysed, and the headline of the below sections refer to the different cases with varying length and starting point of the peak period as described in Table 7.1.1.

Optimal solution

The optimization problem is solved by choosing an energy price somewhere between the high and low value, $P_{high/low}$. If the energy price is above this level will the temperature be kept at the minimum constraint, and if the price is below $P_{high/low}$ can the temperature be at the minimum or maximum depending on if storage is necessary or not. This means that if the time is within the interval Δts will the temperature be kept at the upper bound. Similar, the temperature should be at the lower bound if one is outside Δts. Appendix E, section E.3 gives a picture of the Simulink model.

The system was simulated over a finite horizon and the results are given below. Figure 10.2.1 compares the temperatures and the heat in the floor and room respectively for the optimal case and the solution from the switching rule, while Figure 10.2.2 compares the cost functions. The result from the switching rule given in equation (8.1.1) was used, while the optimal solution was found by the method trial and error. The red and blue line represent the solution from the switching rule and optimal case respectively.

With a Δt_{peak} of one hour it can be seen that the optimal solution is to start the heating at 2.45 a.m. The temperature in the floor will not reach the maximum value before the energy price increases. At 6 o'clock will T_F decrease and at 11.25 a.m will the temperature reach the lower constraint.

The room temperature is kept constant to minimize energy costs, but between 5 and 7 will it increase. When T_F increases will the heating in the room decrease in order to keep T_R constant. In the specified interval will the room heat has reached minimum constraint, while the floor heat is at the maximum. The floor heat is transferred to the room and thus cause the room temperature to increase. At 6 o'clock will the floor heat be turned off and T_R will start to decrease, as shown in Figure 10.2.1.

The time to start heating the system is estimated to a value too late by the switching rule, it will be heated up 53 minutes later than in the optimal case. This result is illustrated in Figure 10.2.1, where the floor heat is switched to the maximum value later than in the optimal case. The floor heat will be tuned on 35 minutes past 10 when T_F reach its minimum constraint.

As explained before, T_R will increase when $q_{H/R}$ is turned off and $q_{H/F}$ is still on. The interval where T_R increases is bigger in the optimal case than for the case where the switching rule is used, as shown in Figure 10.2.1. The room heat will start to fall when T_F increases. Similar, $q_{H/R}$ increases again when the T_F decreases. When T_F attains its minimum constraint again, will the room heat stabilize at a constant value that continue to maintain the constant room temperature. In the optimal case will the heat in the room start to fall earlier since the floor is heated up prior to the switching rule.

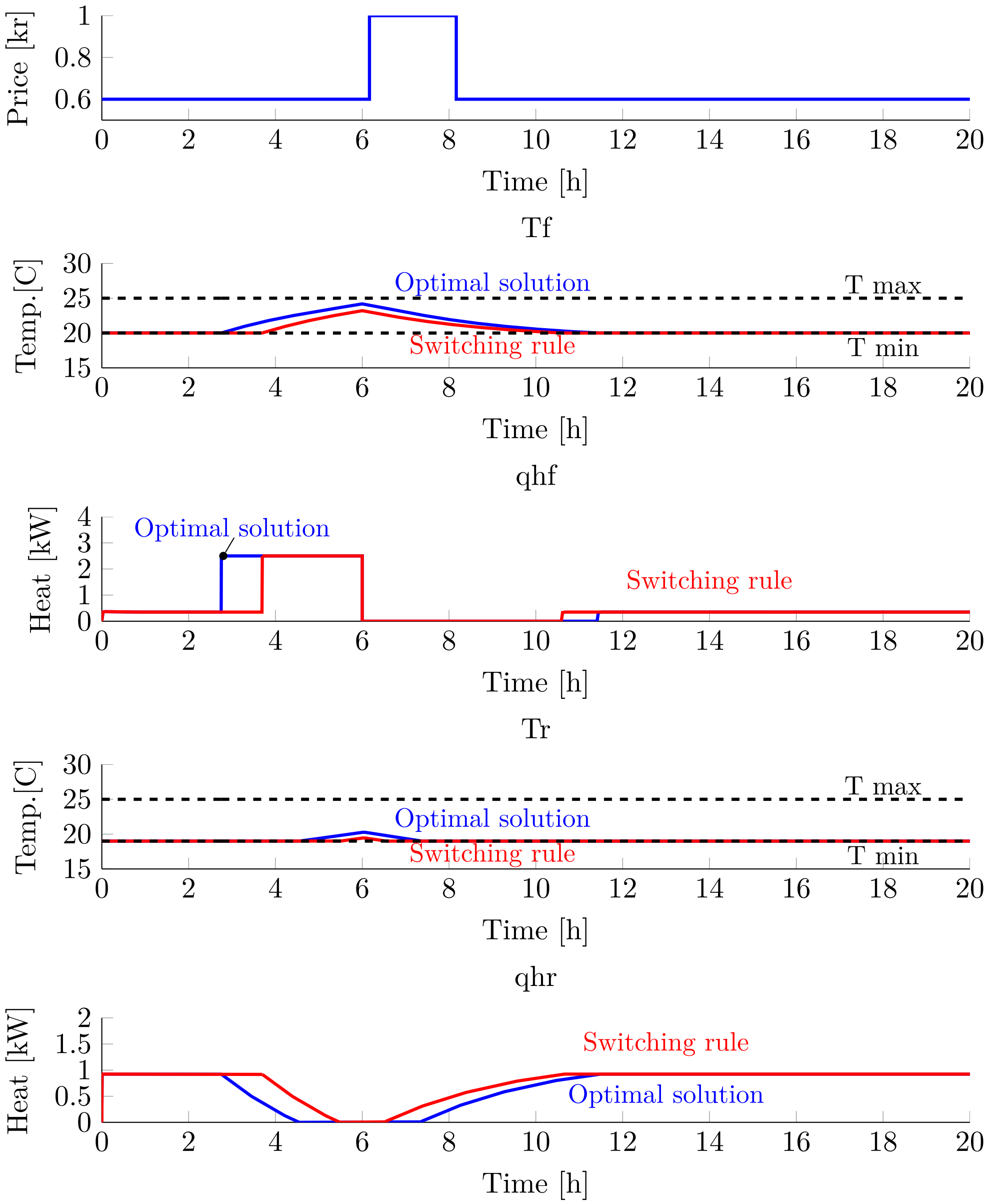

Figure 10.2.1: Temperature and heat in the floor and room versus time for the optimal case and the solution from the switching rule. The results are obtained by using PI controllers. The heat is stored in the floor

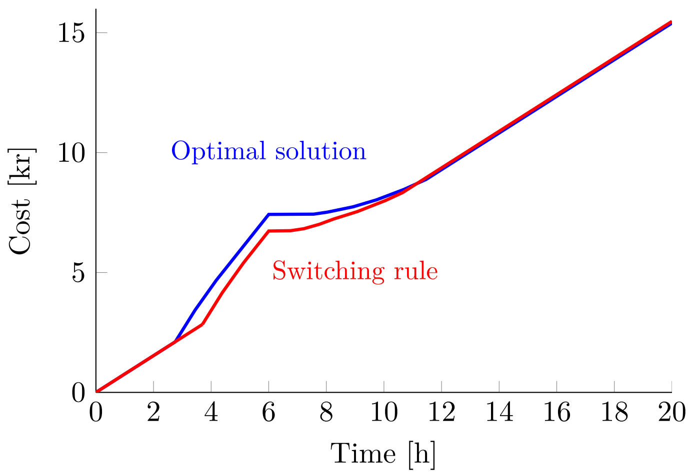

Figure 10.2.2: Cost function as a function of time for the optimal case and the case when using the switching rule. The results are obtained by using PI controllers. The heat is stored in the floor

In addition, T_F will reach its minimum constraint a 54 minutes later in the optimal case, and $q_{H/R}$ will thus be turned off for a greater amount of time in the optimal case.

Since the floor heat is turned to the maximum value for a longer period of time in the optimal case, will more heat be used in this case. Figure 10.2.2 proves this; the cost function for the optimal case will be higher than the cost function for the switching rule around 3 and 10. However, the room heat will be turned off for a longer period of time in the optimal case. The interval where no energy is purchased will therefore be longer in this case, as illustrated with a constant cost function in Figure 10.2.2.

Around 10.30 will the cost function for the switching rule overtake the cost for the optimal case. As already explained, more room heat will be used in the case with the switching rule. In addition, the floor heat will be turned on earlier (around 10.30) in this case. By taking these facts into consideration, can one explain why the cost function for the switching rule will exceed the cost function for the optimal case.

The difference between the two cost functions are barely noticeable after 10.30, but Figure 10.2.2 proves that the cost function for the switching rule will be a bit higher. The total cost after 20 hours for the optimal case and the broad case is 15.4 and 15.5 kr respectively.

Broad case

The system was simulated over a finite horizon and the results are given below. Figure 10.2.3 compares the temperature and the heat in the floor and room respectively for the optimal case and the case using the switching rule given in equation (8.2.3). An example on how Δts was calculated are given in Appendix B. The optimal solution was found by the method trial and error. Figure 10.2.4 compares the cost function for both cases. The result from the switching rule is represented by a red line, while the optimal solution is illustrated with a blue line in both figures.

The optimal time to start the heating of the system is after 54 minutes, while the same value calculated from the switching rule is 12 minutes. In the optimal case will the temperature reach the upper bound a few minutes before the peak period, compared to the switching case, where T_F will be at the maximum constraint in 40 minutes before it decreases. This means that a greater amount of floor heat will be used in the case with the switching rule. Figure 10.2.3 illustrates this. It can be seen that the floor heat in the optimal case is switched to the lower bound later than in the case where the switching rule is used.

In both scenarios will the floor temperature reach the maximum constraint before the peak period. This means that T_F will decrease and reach its lower bound at the same time (4 minutes past 11) for the optimal case and when using the switching rule. A loss of approximately 1 hour is thus obtained for both cases.

From Figure 10.2.3 it can be seen that the room heat is turned off for a longer period of time when using the switching rule. The temperature in the room will start to fall 42 minutes later in the optimal case. However, the room heat will be turned on at the same time in both cases. As a result of this, will the room temperature when using the switching rule start to increase 42 minutes before the optimal solution, but T_R will attain its minimum limit at the same time in both cases.

The cost function for the switching rule is higher in the beginning. The reason for this is that more floor heat is used in this case. Although more room heat will be spend in the optimal case, will the total amount of heat (floor and room) be higher for the switching rule than in the optimal case.

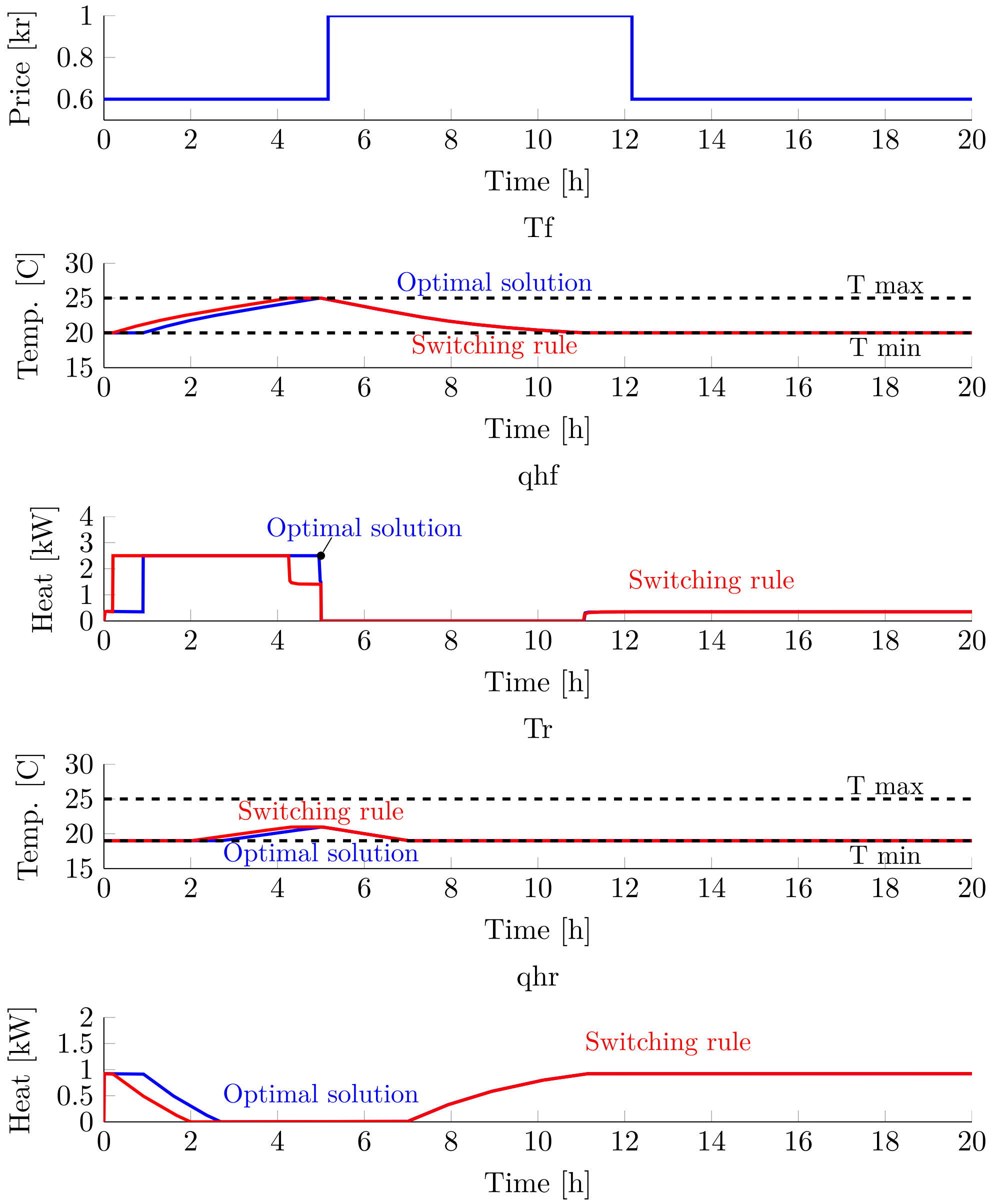

Figure 10.2.3: Temperature and heat in the floor and room versus time for the broad case and the solution from the switching rule. The results are obtained by using PI controllers. The heat is stored in the floor

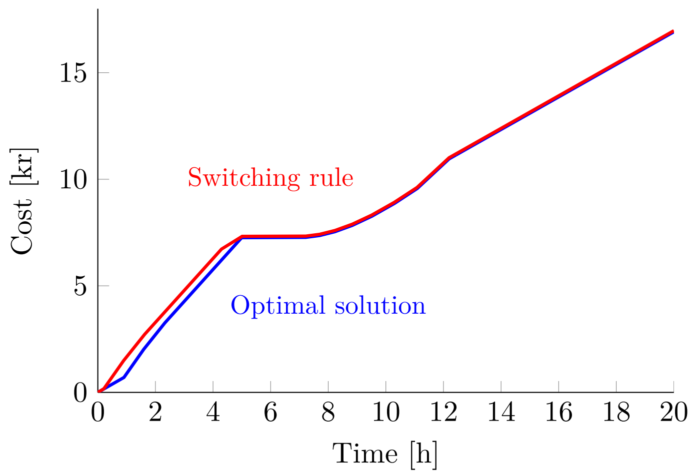

Figure 10.2.4: Cost function as a function of time for the broad case and the case when using the switching rule. The results are obtained by using PI controllers. The heat is stored in the floor

Around 7 a.m will the the energy consumption be similar to one another in both cases. The total cost after 20 hours i 16.9 and 17 kr for the optimal case and the case where using the switching rule.

No time to heat

In this case will the floor temperature not be able to reach the maximum constraint before the peak period begins, even if the floor heating is turned to max immediately. The reason for this is that the peak period begins too early.

From chapter 8 it was seen that the switching rule did not give any feasible solution, and it will therefore be omitted in this section. Figure 10.2.5 and 10.2.6 give the results from simulation. The method trial and error was used to obtain the optimal solution.

The optimal solution, according to Figure 10.2.5, is to start heating the system immediately. When starting to heat as early as possible will the amount of energy that needs to be used when the energy price is high be reduced. The floor temperature will start to decrease at 3 a.m and reach the lower bound at 8 o'clock. This will give a loss of 2 hours. The room temperature is kept constant during the simulations, apart from between 2

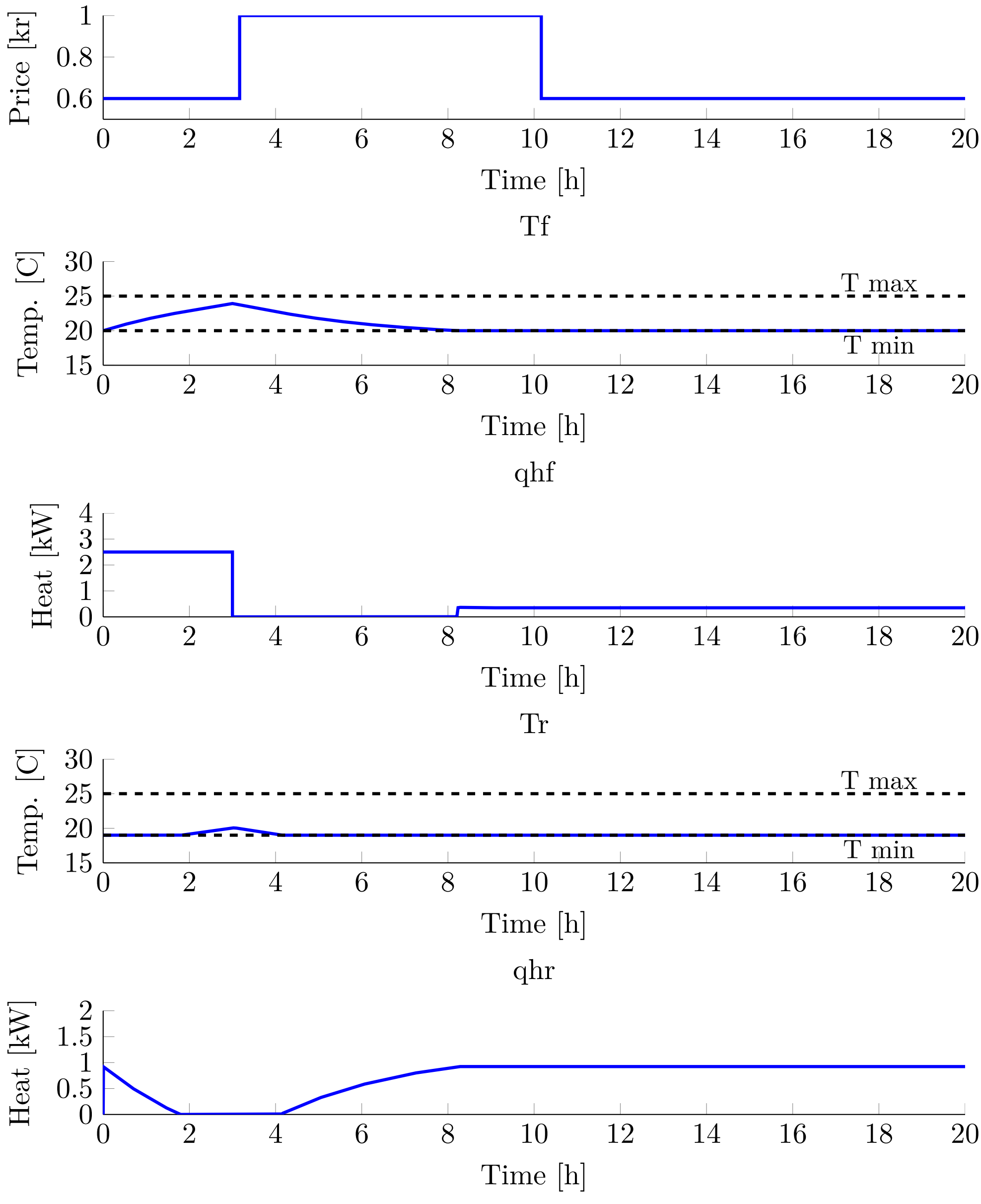

Figure 10.2.5: Temperature and heat in the floor and room versus time for the case where one does have time to heat. The results are obtained by using PI controllers. The heat is stored in the floor

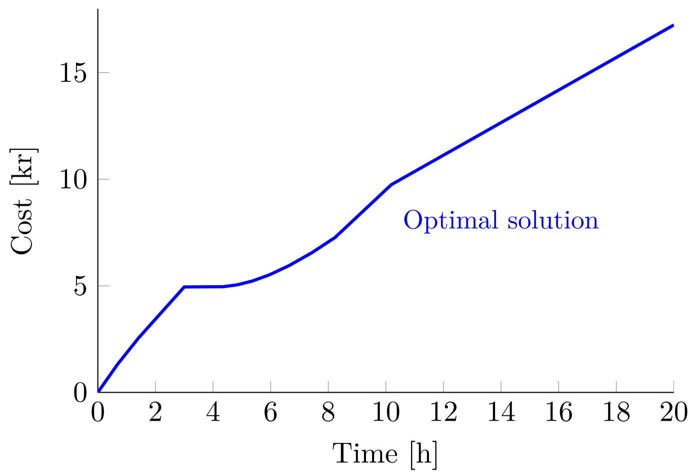

Figure 10.2.6: Cost function as a function of time for the case where there is no time to heat. The results are obtained by using PI controllers. The heat is stored in the floor

and 4 a.m.

From Figure 10.2.6 that the total cost after 20 hours is 17.2 kr.

10.3 Storing of heat in the floor and room

In this section are both heating elements in the floor and room used for storage of heat. Similar to the above case, the temperature in the floor and room will be kept at the lower bound as long as no heating take place, while the heaters will be kept at the maximum value during the period of storage.

From the results in section 7.3 it was shown that heating in the room will not be necessary in the optimal and broad case. Heating in the room will just be required if the system do not have time to heat up before the peak period.

No time to heat

Both the heaters in the floor and room will be used for storage. Figure 10.3.1 and 10.3.2 give the optimal result from simulations. The first figure compares the temperature and heat in the floor and room respectively, while the latter figure gives the cost function. The optimal solution was found by the method trial and error.

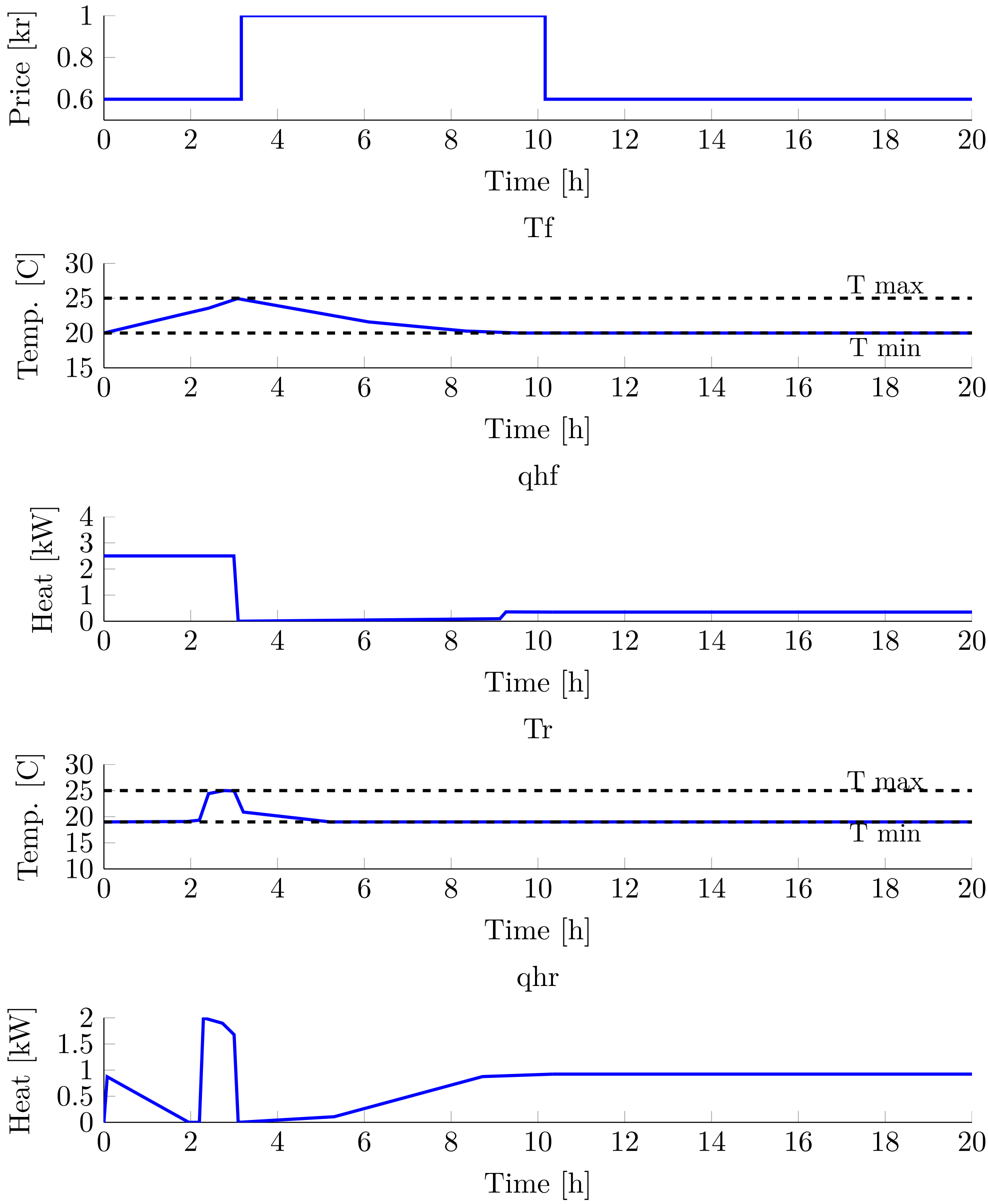

Figure 10.3.1: Temperature and heat in the floor and room versus time for the case where the system does not have time to heat up. The results are obtained by using PI controllers. Storage of heat in the floor and room

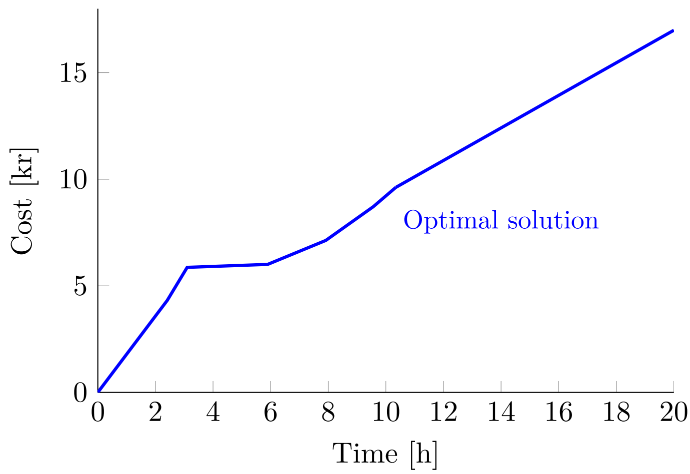

Figure 10.3.2: Cost function as a function of time for the time where the system does not have time to heat up. The results are obtained by using PI controllers. Storage of heat in the floor and room

The optimal time for the floor and room to start heating are immediately and at 2.12 a.m respectively. Both the floor and room temperature will reach the upper bound before they decrease again. From Figure 10.3.1 it can be seen that T_R decreases more rapidly than T_F. The floor temperature will reach it minimum constraint 7 minutes past 9, while the temperature in the room will reach its minimum 9 minutes past 5. This will give a loss of approximately 1 hour for the floor.

The floor and room heat will both be at the lower bound before the period of storage. After the switching time and until the peak period begins will the heaters be at the maximum value, before they are turned off at 3 o'clock.

Figure 10.3.2 shows that the total cot after 20 hours is 17 kr.

10.4 Summary of the obtained results when storing heat in the floor and when using both heaters for storage

The first part of this section will give a summary of the switching time, loss and total cost for the optimal solution, while the same results for the switching rule will be given in the end. A comparison of the optimal solution and switching rules will be given in the next section.

Summary of the optimal results

A summary of the obtained values for the switching time, loss and total cost for each case described in this chapter are given in Tale 10.4.1. The first column in the table gives the results from optimization when only the floor heat is used for storage, while the second scenario contains the results from when both heaters are used to heat the system.

Table 10.4.1: Summary of the optimal values of ts, the loss and total cost for the scenarios with heating in the floor and where both heaters are used for storage. The results are obtained by using PI controllers

	Storage: floor		**Storage: floor and room**	
Optimal case	ts_F	2h 45 min	–	—
	ts_R	–		
	J_{total}	15.4 kr		
Broad case	ts_F	54 minutes	—	—
	ts_R	–		
	Loss	1 hour		
	J_{total}	16.9 kr		
No time to heat	ts_F	Immediately	ts_F	Immediately
	ts_R	–	ts_R	2.12 a.m.
	$Loss_F$	2 hours	$Loss_F$	1 hours
	J_{total}	17.2 kr	J_{total}	17 kr

As mentioned in section 7.4, will the floor heat be the only heating device necessary for storage in the optimal and broad case. The room heat is to be kept constant for as long as possible in both scenarios.

In the third case will floor start the heating of the system immediately in order to store as much heat as possible before the peak period. Because the energy price starts to increase already at 3 a.m will T_F not be able to reach the upper constraint, and the system will have a loss of 2 hours. This result is valid if only the floor is used for storage of energy. In the scenario where both heaters are used for storage should the floor start the heating immediately, but in addition, the room will be heated up at 2.12 a.m. Both the floor and room temperature will reach the upper constraint.

More energy will be stored in the case where both heaters are used, and less amount of heat will thus be used when the energy price is high. The cost function for the case where both heaters are used for storage will be lower than the total cost for the case where the the floor heat is used for storage. The room heat will be on the maximum value for one hour when using both heaters for storage. The room heat will be turned off in the same interval when only the floor heat is used to heat the system. However, in the case where only the floor heat is used for storage will the floor heat be on the maximum constraint for a longer period of time (one hour) compared to the first case and the loss will also be bigger (one hour). The energy consumption in total will therefore be higher when using only the floor heat.

From the above result one can conclude that both heaters should be used for storage when one does not have time to heat. In addition, from Table 10.4.1 it can be seen that the total cost increases the longer the peak period is.

The heat and temperature for the floor and room will always be on or somewhere between the constraints, they are never broken. This result is valid for all the cases.

Summary of the results obtained from the switching rules

Table 10.4.2 gives a summary of the obtained values of the loss, total cost and ts from the switching rules for the scenarios with storing in the floor and when using both heaters for storage.

Table 10.4.2: Summary of the calculated values of ts obtained from the switching rules, in addition to the loss and total cost for the scenarios with heating in the floor. The results are obtained by using PI controllers

		Storage of heat: floor
Optimal case	ts_F^q	3 h 41 min.
	J_{total}	15.5 kr
Broad case	ts_F^T	12 min
	Loss	1 hour
	J_{total}	17 kr

The switching rule is able to calculate a switching time for the optimal and broad case. In the third case, where there is no time to heat, will the switching time obtained from the switching rule be infeasible.

10.5 Comparison of the optimal solution with the switching rules

From the results in Table 10.4.1 and 10.4.2 it can be seen that the switching rule gives a good estimate of the switching time in the optimal case. Even if the system is heated up for a longer period of time (around one hour) in the case where the switching rule is used, will the total cost be 0.1 kr higher than in the optimal case. The reason for the difference in switching time may be because of the flat optimum.

Similar to the optimal case, the switching rule derived for the broad case gives a good estimate of the switching time. It misses the optimal ts by 42 minutes, but the loss will be the same in both scenarios, and the total cost will be 0.1 kr higher when using the switching rule compared to the optimal solution.

The floor and room heat should be used for storage in the case where one does not have time to heat the system. The result from the switching rule is poor in this case, it estimates a switching time that is not feasible.

From this comparison one can conclude that the switching rule gives a good estimate of the switching time in the optimal and broad case, while the rules are no good in the last case.

11 Comparison of the solution from optimal control with the optimal control problem when using PI

This chapter compares the results from optimization when using the gradient free method (chapter 7) to solve the problem with the solutions obtained when using PI controllers (chapter 10).

Storing of heat in the floor

Table 11.0.1 compares the obtained values of the total cost, loss and switching time for the optimal control problem when using the gradient free method and PI controllers. The system will store the heat in the floor and the three different scenarios with varying length and starting point of the peak interval will be given.

Table 11.0.1: Comparison of the obtained values of the switching time, loss and total cost for the optimal control problem and the control problem when using PI, and where the heat will be stored in the floor

		Storage of heat: floor	
		Optimal control	**Optimal control PI**
Optimal case	ts	2 h and 48 min	2 h and 45 min
	J_{total}	15.4 kr	15.4 kr
Broad case	ts	54 minutes	54 minutes
	Loss	1 hour	1 hour
	J_{total}	16.8 kr	16.9 kr
No time to heat	ts	Immediately	Immediately
	Loss	2 hours	2 hours
	J_{total}	17.2 kr	17.2 kr

The solution of the optimal control problem when using a PI controller and the gradient free method are similar to one another in the optimal case. The cost function will be the same in both cases, but the switching time is different. From Table 11.0.1 it can be seen that the system will be heated up 3 minutes earlier when using PI. However, the system has a flat optimum, which makes it difficult to find the exact minimum. This may be the reason for the difference in switching time for the two models.

The broad case have identical switching times and loss, but the total cost for the optimal control problem when using PI will be 0.1 kr higher than the solution of the optimal control problem. The reason for the higher total cost when using PI are illustrated in Figure 11.0.1. In the optimal case will

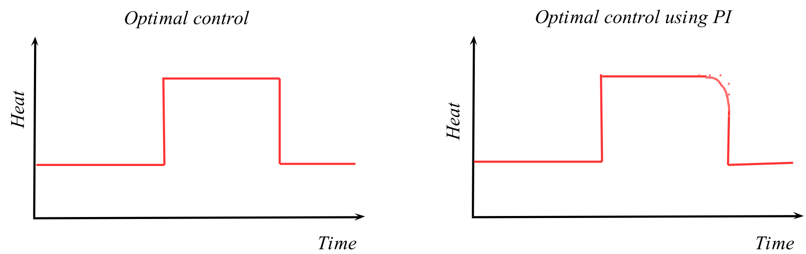

Figure 11.0.1: Behaviour of the heat as a function of time for the optimal control problem and the control problem when using PI

the heat be turned off immediately, but when using the PI controllers will the response be a bit slower. The heat will use a few minutes to be turned off and the energy consumption will increase as a result of that. If one zoom in on the figures for the broad case, this result will be verified. Nevertheless, the difference in the cost function is not significant, since the total cost after one year will be just 36 kr higher when using PI to solve the optimization problem.

In the third case, where there is no time to heat, the solution of the optimal control problem will be the same in both cases. The switching time, loss and total cost are identical to one another when using PI and when using fminsearch to solve the problem.

From the above result it can be concluded that the result from the optimal control problem when using PI will resemble the solution of the optimal control problem when using the gradient free method.

Storing of heat in the floor and room

From the result in chapter 7 it was found that storing of heat in the room and floor were only necessary in the case where there is no time to heat. Table 11.0.2 gives the result from the optimal control problem when both heaters are used for storage. The result from the optimal control problem from chapter 7 and 10 will be compared.

From the second last and last column in Table 11.0.2 it can be seen that the

Table 11.0.2: Comparison of the obtained values of the switching time, loss and total cost for the optimal control problem and the optimal control problem when using PI, and where the heat will be stored in the floor and room

		Storage of heat: floor and room	
		Optimal control	**Optimal control PI**
No time to heat	ts_F	Immediately	Immediately
	ts_R	2.12 a.m.	2.12 a.m
	Loss	1 hour	1 hour
	J_{total}	16.9 kr	17 kr

switching time for the floor and room, in addition to the loss will be similar to one another. However, the total cost for the control problem when using PI will be 0.1 kr higher than the optimal control problem. The reason for this behaviour might be because of a slower response time for the PI controller, as explained above.

One can conclude that the optimal control problem where one uses PI controllers to solve the problem will give similar results as the optimal control problem.

Comparison of the switching rules

The following Table compares the result from the switching rules for the optimal control problem and the control problem where PI controllers are used.

The two last columns in Table 11.0.3 shows that the switching time will be the same in the optimal case, but the total cost will be 0.1 kr higher when using PI controllers to solve the problem. Similar, the total cost for the broad case when using PI will be 0.2 kr higher than when solving the problem by fminsearch, but the switching time and loss will be identical in both cases. However, the difference in total cost between the two cases is not big. The energy costs for one year will be 36 kr and 73 kr higher for the optimal and broad case respectively when PI is used.

In the case where one does not have time to heat will the switching rule not give any feasible result. The obtained switching time will be before the simulation starts.

Table 11.0.3: Comparison of the obtained values of the switching time, loss and total cost obtained from the switching rule for the optimal control problem and the control problem when using PI

		Storage of heat: floor	
		Optimal control	**Optimal control PI**
Optimal case	ts	3 h and 41 min	3 h and 41 min
	J_{total}	15.4 kr	15.5 kr
Broad case	ts	12 min	12 min
	Loss	1 hour	1 hour
	J_{total}	16.8 kr	17 kr
No time to heat	ts	Infeasible	Infeasible
	Loss	–	–
	J_{total}	–	–

From the above result one can conclude that the result from the switching rules are good for the optimal and broad case, but not if the peak period starts too early. In addition, the switching rules give similar results for the optimal control problem when using the gradient free method and when using PI controllers.

12 Conclusion

12.1 Optimal control

For a system with constant outdoor temperature of 0 °C, it was found that the heat should be stored in the floor when the energy price has a peak interval of two and seven hours and where it start at 6 a.m, and 5 a.m respectively. On the other hand, if the system have no time to heat before the peak interval begins should both heaters in the floor and room be used for storage in order to minimize the energy costs.

As shown previously, the derived switching rules gives a good estimate of the switching time, apart from in the case where there is not time to heat. The obtained switching time are not far from the optimal solution, and the reason for the difference may be due to a flat optimum. Nevertheless, the total cost after 20 hours will be the same for the optimal solution as well as when using the switching rule.

Varying the outdoor temperature it was seen that both the solution from the optimal control problem and switching rules, will take the disturbance into consideration if it is varied before the peak period. On the other hand, the model do not allow for a variation in the outdoor temperature after the peak period. The reason for this is that model cannot know what will happen in the future, and the solution obtained may not be the optimal one.

It was found that the switching rule handle a variation in the disturbance before the peak period as good as the optimal control problem, but when the outdoor temperature becomes too cold will the result be poor.

12.2 Optimal control using PI

Similar to the above section, it was found that the heat should be stored in the floor when the energy price has a peak interval of two and seven hours when using PI to solve the optimization problem. If the there is no time to heat before the peak interval should both heaters in the floor and room be used for storage in order to minimize the energy costs. This result is valid if the outdoor temperature is constant at 0 °C.

It was found that the switching rules give a good estimation of the switching time. Similar to the optimal control problem will the switching rule give an infeasible switching time in the case where there is no time to heat.

However, It was found that the obtained switching time when using the switching rule will be similar to the optimal solution found when using PI.

12.3 Comparison of the two methods

As shown in the previous chapter, the result of the control problem when using PI will resemble the solution of the optimal control problem. This statement is correct both in the case when storage of heat are in the floor and in the floor and room. It was shown that the total cost and switching time will be similar for both methods and the input and states will always be on or somewhere between the constraints, they are never broken.

In addition, the solution when using the switching rules are similar for both methods. In some cases will the switching time differ from one another in the two methods, or the total cost when solving the problem using PI will be a bit higher. The reason for this difference may be due to the flat optimum and the slower response time for the PI controller respectively.

In general it can be seen that the longer the peak period the more energy needs to be stored, which in turn result in a higher total cost.

12.4 Further work

Further work in this thesis could include a further analysis on the effect the disturbances has on the system. The model where PI controllers is used to solve the problem could be tested with varying outdoor temperature as well, and the result from the two methods could be compared.

In addition, a variation in all the disturbances, not just the outdoor temperature, should be analysed.

Nomenclature

Parameter	Unit	Description
States		
x		State
T_F	°C	Temperature in the floor
T_R	°C	Temperature in the room
Inputs		
u		Input
$q_{H/F}$	kW	Floor heat
$q_{H/R}$	kW	Room heat
Disturbances		
T_o	°C	Outdoor temperature
$m_{in/out}$	kg	Mass of air in/out of the room
P	NOK	Energy price
Parameters		
$q_{F/R}$	kW	Heat flow from floor to the air in the room
$q_{R/O}$	kW	Heat flow from the room to the outside
U	$\frac{J}{sm^2K}$	Overall heat transfer coefficient
A	m^2	Heat transfer area
m	kg	Mass
V	m^3	Volume
Cp	$\frac{J}{kgK}$	Heat capacity
ρ	$\frac{kg}{m^3}$	Density
Simulation parameters		
x_{lb}	°C	Lower bound for the states
x_{ub}	°C	Upper bound for the states
u_{lb}	kW	Lower bound for the inputs
u_{ub}	kW	Upper bound for the inputs

Parameter	Unit	Description
Simulation parameters continued		
t	h	Time
ts	h	Switching time / time to start heating
Δt_s	h	Interval where the heating take place
Δt_{peak}	h	Length of interval with high energy price
J	NOK	Cost function
Parameters from background		
k_i	$[\frac{J}{smK}]$	Thermal conductivity
h_i	$[\frac{J}{sm^2K}]$	Convective coefficient
τ_I		Integral time
τ_D		Derivative time
τ_c		Tuning parameter
θ		Effective delay
K_c		Controller gain
k		Steady-state gain
N_F		Number of degrees of freedom
N_V		Number of process variables
N_E		Number of independent equations
H		Hamilton function
λ		Lagrange multiplier

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Appendices

A Calculation of process variables

Table A.0.1 and 3.1.1 give the values of the parameters necessary to calculate $UA_{RO/F}$, $mCP_{F/R}$ and m_R

Table A.0.1: Values of the parameters used in the system

Parameter	Unit	Value
$\rho_{concrete}$	$[\frac{kg}{m^3}]$	2400[ToolBox, c]
ρ_{air}	$[\frac{kg}{m^3}]$	1.17[ToolBox, b]
$Cp_{concrete}$	$[\frac{J}{kgK}]$	630[Geankoplis, 2003]
Cp_{air}	$[\frac{J}{kgK}]$	1005[ToolBox, b]
k_{wood}	$[\frac{J}{hmK}]$	748.80[Geankoplis, 2003]
$k_{\text{rock wool}}$l	$[\frac{J}{hmK}]$	140.76[Geankoplis, 2003]
h_{air}	$[\frac{J}{hm^2K}]$	54000[ToolBox, a]

Heat is transferred from the air in the room to the outside and from the floor to the air in the room, and the overall heat transfer coefficients are denoted $U_{R/O}$ and $U_{F/R}$ respectively. The heat from the room is descried with a combination of conductive and convective heat transfer, while $U_{F/R}$ is found by using the rues for conductive heat transfer. Equation (A.0.1) and (A.0.2) give the expression for $U_{R/O}$ and $U_{F/R}$ respectively.

$$U_{R/O} = \frac{1}{\frac{1}{h_{air}} + \frac{\Delta x_{wood}}{k_{wood}} + \frac{\Delta x_{\text{rock wool}}}{k_{\text{rock wool}}} + \frac{\Delta x_{wood}}{k_{wood}} + \frac{1}{h_{air}}} \tag{A.0.1}$$

$$U_{F/R} = \frac{1}{\frac{\Delta x_{wood}}{k_{wood}}} \tag{A.0.2}$$

The general formula to calculate the mass is a follows

$$m = \rho V \tag{A.0.3}$$

The room contains air, and the mass is calculated as described in equation (A.0.4).

$$m_{room} = \rho_{air}(A_{floor}h_{wall}) \tag{A.0.4}$$

The mass of the floor is found by using the density of concrete and volume of the floor.

$$m_{floor} = \rho_{concrete}(A_{floor}\Delta x_{concrete}) \tag{A.0.5}$$

In equation (A.0.4) and (A.0.5) are the height of the wall and the thickness of the concrete layer denoted h_{wall} and $\Delta x_{concrete}$ respectively.

The calculated values for $UA_{R/O}$,$UA_{F/R}$, mCp_R,mCp_F and m_R are given in Table A.0.2. To ease the calculations the values of the parameters are

Table A.0.2: Calculated value of the parameters

Parameter	**Dimension**	**Value**
$UA_{R/O}$	$[\frac{kW}{K}]$	0.0072
$UA_{F/R}$	$[\frac{kW}{K}]$	0.3467
mCp_R	$[\frac{kJ}{K}]$	70.3098
mCp_F	$[\frac{kJ}{K}]$	3780
m_R	$[kg]$	69.9600

round up/down to the values found in Table 3.2.1.

B Calculation of Δts

This appendix gives the calculations of Δt_s in the three cases with different length of the interval with high energy price. Table B.0.3 gives the value of the parameters necessary to calculated Δt_s, while the length of the peak periods in each case are illustrated in Table B.0.4.

Table B.0.3: Values of the parameters used to calculate Δt_s

Parameter	**Unit**	**Value**
$q_{max,F}$	2.5	$[kW]$
$q_{max,R}$	2.5	$[kW]$
$q_{o,F}$	0.34	$[kW]$
$q_{o,R}$	1	$[kW]$
$T_{max,F}$	298	$[K]$
$T_{max,R}$	298	$[K]$
$T_{min,F}$	293	$[K]$
$T_{min,R}$	292	$[K]$
mCp_F	4 000	$[\frac{kJ}{K}]$

Table B.0.4: Length of the peak period for the optimal case, broad case and the case where one does not have time to heat

Case	**Peak period**	**Unit**
Optimal case	$6-8$	$[h]$
Broad case	$5-12$	$[h]$
No time to heat	$3-10$	$[h]$

Different combinations of q_{max} and q_o was tried to find the value of Δt_s closest to the optimal solution. The combination that gave the best result was when using $q_{max,F}$, $q_{o,R}$ and $q_{o,F}$. The calculated values of Δt_s for the optimal scenario, the broad case and the case where the system does not have enough time to heat are given in equation (B.0.6), (B.0.7) and (B.0.8) respectively.

$$\Delta t_s^q = \frac{\Delta t_{peak}}{\frac{q_{max}}{q_o} - 1} = \frac{2}{\frac{2.5}{1+0.34} - 1} = 2.31 \text{ h} \tag{B.0.6}$$

$$\Delta t_s^T = \frac{T_{max} - T_{min}}{q_{max} - q_o} mCp = \frac{298 - 293}{2.5 - (1 + 0.34)} \times 4000 = 17241.37\text{s} = 4.8 \text{ h} \tag{B.0.7}$$

$$\Delta t_s^q = \frac{\Delta t_{peak}}{\frac{q_{max}}{q_o} - 1} = \frac{7}{\frac{2.5}{1+0.34} - 1} = 8.09 \text{ h} \qquad (B.0.8)$$

The switching time is found by subtracting the time where the peak period begin with Δts. Table B.0.4 gives the peak period in each case, and equation (B.0.9) and (B.0.10) gives the switching time for the optimal and broad case respectively.

$$ts_{\text{optimal case}} = 6 - 2.31 = 3.69 \text{ h} \qquad (B.0.9)$$

$$ts_{\text{broad case}} = 5 - 4.8 = 0.2 = 12 \text{ min} \qquad (B.0.10)$$

The calculated value of Δts in the third case, where one do not have time to heat, are 8.09. On the same time will the peak period start at 3 a.m. This means that the switching time will be 5 hours before the starting point for the simulations, and the solution is therefore infeasible.

Table B.0.5 gives a summary of the obtained switching time and appurtenant Δts for the optimal case, broad case and the case where one do not have time to heat.

Table B.0.5: Obtained value of Δts and ts from the switching rules for the optimal case, broad case and the case where one do not have time to heat

Case	**Δts**	**Unit**	**ts**
Optimal case	2.31	$[h]$	3 h 41 min
Broad case	4.8	$[h]$	12 min
No time to heat	8.09	$[h]$	Infeasible

C Calculation of Δts for different outdoor temperatures

The system will be implemented with a outdoor temperature of of 5, 9, -5 and $-9\,°C$. The switching rule derived in chapter 8.1

$$\Delta t_s^q = \frac{\Delta t_{peak}}{\frac{q_{max}}{q_o} - 1}$$

will be used to find the switching time when the outdoor temperature varies as described above. The value of the parameters $q_{o,F}$ and $q_{o,R}$ in each case are obtained from Figure 9.1.2 in chapter 9.1, and the values are given in Table C.0.6 below. The value of q_{max} is, as described in Appendix B, equal to 2.5 kW. The calculations of the Δts obtained when the outdoor

Table C.0.6: Value of $q_{o,F}$ and $q_{o,R}$ when the outdoor temperature are -5 ,-9, 5 and 9 °C

$\mathbf{T_o}$	$\mathbf{q_{o,F}}$	$\mathbf{q_{o,R}}$	**Unit**
9 °C	0.32	0.35	$[kW]$
5 °C	0.59	0.35	$[kW]$
−5 °C	1.26	0.35	$[kW]$
−9 °C	1.53	0.35	$[kW]$

temperature is 9, 5, -5 and $-9\,°C$ is given in equation (C.0.11), (C.0.12), (C.0.13) and (C.0.14).

$$\Delta t_{s,\text{plus 9}} = \frac{2}{\frac{2.5}{0.32+0.35} - 1} = 0.73\text{h} \tag{C.0.11}$$

$$\Delta t_{s,\text{plus 5}} = \frac{2}{\frac{2.5}{0.59+0.35} - 1} = 1.21\text{h} \tag{C.0.12}$$

$$\Delta t_{s,\text{minus 5}} = \frac{2}{\frac{2.5}{1.26+0.35} - 1} = 3.61\text{h} \tag{C.0.13}$$

$$\Delta t_{s,\text{minus 9}} = \frac{2}{\frac{2.5}{1.53+0.35} - 1} = 6.01\text{h} \tag{C.0.14}$$

The switching time, ts, is found be subtracting the time the peak period begins with Δts. The calculated values of ts are:

$$ts_{\text{plus 9}} = 6 - 0.73 = 5.628 = 5\text{ h and }16\text{ min} \tag{C.0.15}$$

$$ts_{\text{plus 5}} = 6 - 1.21 = 4.8 = 4 \text{ h and } 48 \text{ min} \tag{C.0.16}$$

$$ts_{\text{minuss 5}} = 6 - 3.61 = 2.4 = 2 \text{ h and } 24 \text{ min} \tag{C.0.17}$$

According to equation (C.0.14) will the length of the interval where the heating take place be 6 h and 6 minutes long when the temperature is $-9\,°C$. However, the peak period start at 6 a.m and the solution is therefore infeasible.

Table C.0.7 gives a summary of the obtained values of the switching time.

Table C.0.7: Calculated values of the switching time from the switching rule for the optimal case

Outdoor temperature	**Switching time**
9 °C	5 h and 16 min
5 °C	4 h and 48 min
−5 °C	2 h and 24 min
−9 °C	Infeasible

D Calculation of tuning parameters

The system was simulated with varying energy price and inputs (as illustrated in Figure D.0.1), but with constant outdoor temperature and m_{in}. The outdoor temperature and m_{in} equals 0 °C and $0.06kg$ respectively.

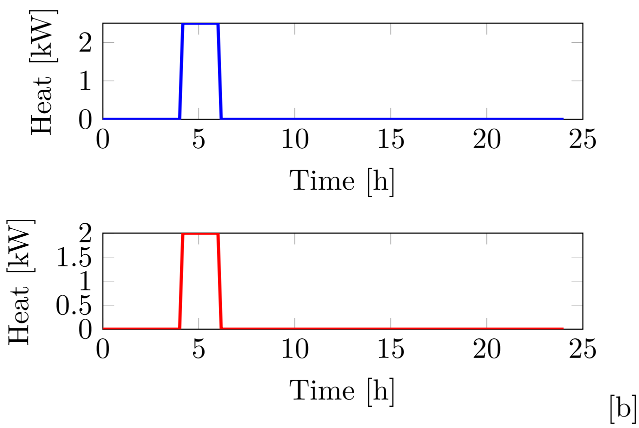


Figure D.0.1: Floor and room heat versus time

D.1 Tuning of T_F

To find the tuning parameters a step change that doubles the heat input was made. The plot of the open loop response can be seen in Figure D.1.1. The Figure D.1.1 it can be seen that this is an integrating process without any delay, $\theta = 0$. Assuming a small value for the tuning parameter, $\tau_c = 0.01$, the following tuning parameter was calculated.

$$k' = \frac{\Delta y}{\Delta t \Delta u} = \frac{37.44 - 17.98}{(16.41 - 10)5(5 - 0)} = 0.783 \qquad \text{(D.1.1)}$$

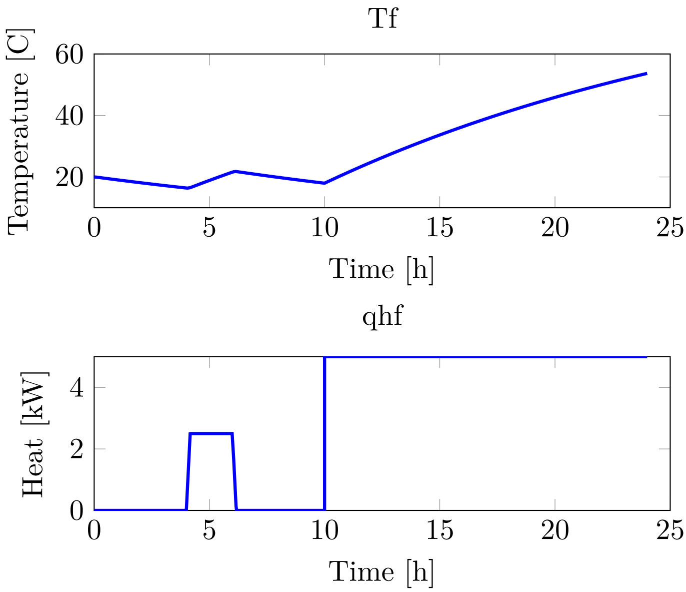
This gives a controller gain and integral time equal to:

$$K_c = \frac{1}{k'} \frac{1}{(\tau_c + \theta)} = \frac{1}{0.783} \frac{1}{0.01 + 0} = 127.7 \qquad \text{(D.1.2)}$$

$$\tau_I = 4 \times (\tau_c + \theta) = 4 \times (0.01) = 0.04 \qquad \text{(D.1.3)}$$

D.2 Tuning of T_R

Figure D.2.1 gives the open loop response of T_R with a 100% step change in $q_{H/R}$. This is a integrating process without any delay. Assuming $\tau_c = 0.01$,



[b]

Figure D.1.1: Open loop response of T_F with a step change in $q_{H/F}$

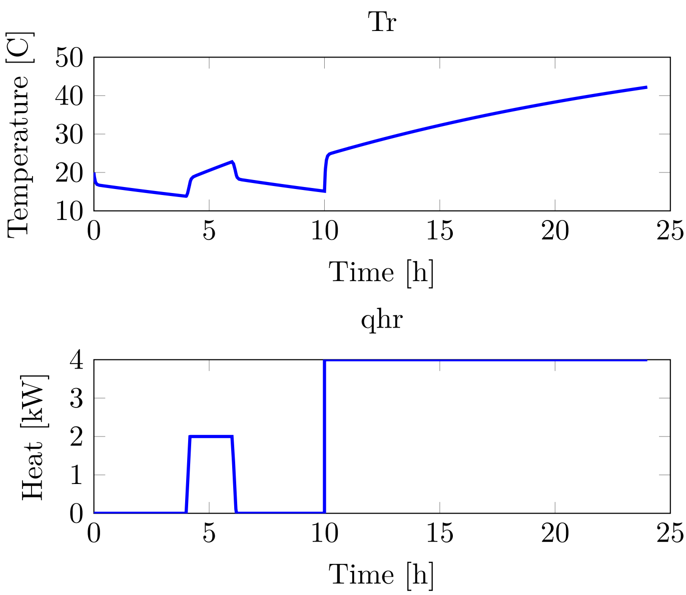


Figure D.2.1: Open loop response of T_R with a step change in $q_{H/R}$

the following tuning parameters was calculated:

$$k' = \frac{34.45 - 24.69}{(14.41 - 10.19)(4 - 0)} = 0.400 \tag{D.2.1}$$

$$K_c = \frac{1}{0.400}\frac{1}{0.01 + 0} = 249.7 \tag{D.2.2}$$

$$\tau_I = 4 \times (\tau_c + \theta) = 4 \times (0.01) = 0.04 \tag{D.2.3}$$

E Matlab code

E.1 Optimal control

```
%Optimal Temperature Control of rooms for Minimum Energy Cost

%Master thesis 2013
%Siri Hofstad Trapnes

close all
clear all
clc

% Solution of optimal control problem

% Function for finding the solution of the dynmaic optimization problem
% where the Matlab function fminsearch is used to find the solution
% Written by Chriss Grimholt, 2013


%% Parameters
par.UA_fr = 0.350; %[kJ/sK = kW/K]
par.UA_ro = 0.007; %[kJ/sK = kW/K]

par.mCp_f = 4000;  %[kJ/K]
par.mCp_r = 70;    %[kJ/K]
par.m_r   = 70;    %[kg]

% Simulation parameters
opt.sim_time = 20; %[h]

% Input Bounds
opt.lb.u.q_f = 0;   %[kW]
opt.ub.u.q_f = 2.5; %[kW]

opt.lb.u.q_r = 0;   %[kW]
opt.ub.u.q_r = 2;   %[kW]
```

```
% State Bounds
opt.lb.x.T_r = 19;  %[C]
opt.ub.x.T_r = 25;  %[C]

opt.lb.x.T_f = 20;  %[C]
opt.ub.x.T_f = 25;  %[C]

% Initial States
init.T_f   = 20;     %[C]
init.T_r   = 19;     %[C]
init.J     = 0;      %[kr]
init.vector = [init.T_f init.T_r init.J 0 0]';

% Disturbances
dist.price.value = [0.6 1 0.6];  %[kr/kWh]
dist.price.time  = [0 5 12];     %[h]

dist.m_in.value = [0.06];   %[kg]
dist.m_in.time  = [0];      %[h]

%Constant outdoor temperture
dist.T_o.value = 0;     %[C]
dist.T_o.time  = 0;     %[h]
%Varying outdoor temperature
% 1) step before the peak period:
dist.T_o.value = [0 -9];  %[C]
dist.T_o.time = [0 0];    %[h]
% 2) step after the peak period
dist.T_o.value = [0 -9];  %[C]
dist.T_o.time = [0 9];    %[h]

%% Optimization

%Storing in room and floor
% Initial guess:
%  qhf qhr
u = [2 2];
u = fminsearch(@(u) costfun(u,dist,par,init,opt), u)

```

```
% %Storing in floor
% Initial guess:
u = 2.91; %Time
u = fminsearch(@(u) costfun([u 10],dist,par,init,opt),
    u)
% %where 10 is a number after t(p_high)

%% Simulation
u = [u 10] %Optimal value
%u = [(6-2.31) 10] % Optimal value switching rule
option=odeset('AbsTol',1e-10,'RelTol',1e-3);
[t, x] = ode23(@(t,x) model(t,x,u,dist,par,opt),[0 opt
    .sim_time],init.vector(1:3),option);
cost = x(end,end) %The cost


%% Plotting

% Back calculating u in order to be able to plot the
    results
T_o = zeros(length(t),1);
m_in = zeros(length(t),1);
for i = 1:length(t)

    T_o_tmp = dist.T_o.value(dist.T_o.time <= t(i));
    T_o(i) = T_o_tmp(end);

    m_in_tmp = dist.m_in.value(dist.m_in.time <= t(i))
        ;
    m_in(i) = m_in_tmp(end);
end
u_f  = zeros(length(t),1);
u_r  = zeros(length(t),1);

t_int = find((t < u(1))>0);
u_f(t_int)  = max(par.UA_fr*(x(t_int,1)-x(t_int,2)),0)
    ;
t_int = find((t < u(2))>0);
u_r(t_int) = max(-par.UA_fr*(x(t_int,1)-x(t_int,2)) +
    (par.UA_ro + m_in(t_int)./par.m_r*par.mCp_r).*(x(
```

```
    t_int,2)-T_o(t_int)),0);

t_int = find((t > u(1)) .* (t <= dist.price.time(2))>0);
u_f(t_int) = opt.ub.u.q_f; %#ok<FNDSB>
t_int = find((t > u(2)) .* (t <= dist.price.time(2))>0);
u_r(t_int) = opt.ub.u.q_r;  %#ok<FNDSB>

t_int = find((t > u(1)) .* (t <= dist.price.time(2) .* (x(:,1) >= opt.ub.x.T_f))>0);
u_f(t_int)  = max(par.UA_fr*(x(t_int,1)-x(t_int,2)),0);
t_int = find((t > u(2)) .* (t <= dist.price.time(2) .* (x(:,2) >= opt.ub.x.T_r))>0);
u_r(t_int) = max(-par.UA_fr*(x(t_int,1)-x(t_int,2)) + (par.UA_ro + m_in(t_int)./par.m_r*par.mCp_r).*(x(t_int,2)-T_o(t_int)),0);


t_int = find(((t >dist.price.time(2)) .* (x(:,1) <= opt.lb.x.T_f))>0);
u_f(t_int) = max(par.UA_fr*(x(t_int,1)-x(t_int,2)),0);

t_int = find(((t > dist.price.time(2)) .* (x(:,2) <= opt.lb.x.T_r))>0);
u_r(t_int) =max( -par.UA_fr*(x(t_int,1)-x(t_int,2)) + (par.UA_ro + m_in(t_int)./par.m_r*par.mCp_r).*(x(t_int,2)-T_o(t_int)),0);
```

Cost function

```
%Cost function

function J = costfun(u,dist,par,init,opt)

%option=odeset('AbsTol',1e-10,'RelTol',1e-6);
[~, x] = ode23(@(t,x) model(t,x,u,dist,par,opt),[0 opt.sim_time],...
    init.vector(1:3));
J = x(end,end);
```

Model

```
%Function of the model

function dxdt = model(t,x,u,dist,par,opt)


%% States
T_f = x(1);
T_r = x(2);

%% Disturbances
price = dist.price.value(dist.price.time <= t);
price = price(end);

T_o = dist.T_o.value(dist.T_o.time <= t);
T_o = T_o(end);

m_in = dist.m_in.value(dist.m_in.time <= t);
m_in = m_in(end);


%% Inputs
switch 1
    case t < u(1)
        q_f = max(par.UA_fr*(opt.lb.x.T_f-T_r),0);
    case t > u(1) && t <= dist.price.time(2)
        if T_f >= opt.ub.x.T_f
            q_f = max(par.UA_fr*(opt.ub.x.T_f-T_r),0);
        else
            q_f = opt.ub.u.q_f;
        end
    case t > dist.price.time(2);
        if T_f <= opt.lb.x.T_f
            q_f = max(par.UA_fr*(opt.lb.x.T_f-T_r),0);
        else
            q_f = 0;
        end
end

```

```matlab
switch 1
    case t < u(2)
        q_r =max(-par.UA_fr*(T_f-opt.lb.x.T_r) + (par.UA_ro + m_in/par.m_r*par.mCp_r)*(opt.lb.x.T_r-T_o),0);
    case t > u(2) && t <= dist.price.time(2)
        if T_r >= opt.ub.x.T_r
            q_r =max(-par.UA_fr*(T_f-opt.ub.x.T_r) + (par.UA_ro + m_in/par.m_r*par.mCp_r)*(opt.ub.x.T_r-T_o),0);
        else
            q_r = opt.ub.u.q_r;
        end
    case t > dist.price.time(2)
        if T_r <= opt.lb.x.T_r
            q_r =max(-par.UA_fr*(T_f-opt.lb.x.T_r) + (par.UA_ro + m_in/par.m_r*par.mCp_r)*(opt.lb.x.T_r-T_o),0);
        else
            q_r = 0;
        end
end

%% Model
%Floor
dxdt(1) = (q_f/par.mCp_f)-(par.UA_fr/par.mCp_f)*(T_f-T_r);

%Room
dxdt(2) = (q_r/par.mCp_r)+(par.UA_fr/par.mCp_r)*(T_f-T_r)-(par.UA_ro/par.mCp_r)*(T_r-T_o)...
    +(m_in/par.m_r)*(T_o-T_r);

%Cost function
dxdt(3) = price*(q_f + q_r)/3600;

dxdt = dxdt'*3600;
```

E.2 Optimal control using PI: tuning parameters

```
%Optimal Temperature Control of rooms for Minimum Energy Cost

%Master thesis 2013

close all
clear all
clc

%Optimal control using PI
%Script that finds the tuning parameters of the system
%Written by Siri Hofstad Trapnes, 2013

%% Parameters

%UA = J/hK
%mCp = J/K

par.UAfr = 0.350; % [kJ/sK = kW/K]
par.UAro = 0.007; % [kJ/sK = kW/K]
par.mCpf = 4000; % [kJ/K]
par.mCpr = 70; % [kJ/K]
par.mr = 70; % [kg]

min = 0.06; % [kg]
mout = min;

%% Energy price

%Varying price
ts = 600; % 10min
tend = 24*60*60; % 24h
t = [0:ts:tend]'; % [s]
t_p = t/(60*60);

t1 = length(t(1):ts:t(37)); % start - 6h %
t2 = length(t(38):ts:t(49)); % 6 - 8 %
t3 = length(t(50):ts:t(end)); % 8 - end %
```

```

P1 = 1/(60*60);
P2 = 0.6/(60*60);

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2]; % 15h
P_plot = P;

figure(1)
stairs(t_p,(P*60*60),'b')
xlabel('Time [h]')
ylabel('Energy price [kr/kWh]')
title('Energy price')

%Saving in tikz
%matlab2tikz('Energy_price_plot.tikz')

var.time = t;
var.signals.values = P;
%var = [t; P]';
%var.signals.dimensions = dim;

%% Inputs and initial conditions

%Inputs
%qhr = 2; % No heat on: 0 %max: 2 [kW]
%qhf = 2.5; % No heat on: 0 %max: 2.5 [kW]
To = 273+0; % [K]

%Varying inputs
t1 = length(t(1):ts:t(25)); % start - 4h
t2 = length(t(26):ts:t(37)); % 4h - 6h
t3 = length(t(38):ts:t(end)); %15h

qhf_min = 0;
qhf_max = 2.5;
qhr_min = 0;
qhr_max = 2;

```

```
qhf = [ones(t1,1)*qhf_min; ones(t2,1)*qhf_max; ones(t3
    ,1)*qhf_min];
qhr = [ones(t1,1)*qhr_min; ones(t2,1)*qhr_max; ones(t3
    ,1)*qhr_min];

varqhf.time = t;
varqhf.signals.values = qhf;

varqhr.time = t;
varqhr.signals.values = qhr;

%Initial conditions
Tf = 293; % [K]
Tr = 293; % [K]
J = 0; % [NOK]
init = [Tf Tr J];

%Set point
Tf_set = 293; % [K]
Tr_set = 292; % [K]

%normal
Tfmax = 298; % [K]
Tfmin = 293; % [K] 293
Trmax = 298; % [K]
Trmin = 292; % [K] 293

%% Simulation without PI controller

sim('simulink_mastermdl')

%Outputs
Tf = simout(:,1)-273; % [C]
Tr = simout(:,2)-273; % [C]
J = simout(:,3);
Qhf = qhf_out;
Qhr = qhr_out;

%Simulation time
t = time/(60*60); %[h]
```

```
% Plotting
% figure(1)
% Subplot(4,1,1)
% plot(t,Tf,'b')
% ylabel('Temperature [C]')
% xlabel('Time [h]')
% title('Tf')
%
% subplot(4,1,2)
% plot(t,Qhf,'b')
% ylabel('Heat [kW]')
% xlabel('Time [h]')
% title('qhf')
%
% Subplot(4,1,3)
% plot(t,Tr,'r')
% ylabel('Temperature [C]')
% xlabel('Time [h]')
% title('Tr')
%
% subplot(4,1,4)
% plot(t,Qhr,'r')
% ylabel('Heat [kW]')
% xlabel('Time [h]')
% title('qhr')
%
% figure(2)
% plot(t,J,'g')
% ylabel('Cost [kr]')
% xlabel('Time [h]')
% title('Cost function, J')

%% Simulation with PI controller

%Simulations to find the tuning parameters

sim('simulink_master_PI')

%Outputs
```

```
Tf = simout(:,1)-273; % [C]
Tr = simout(:,2)-273; % [C]
J = simout(:,3);
Qhf = qhf_out;
Qhr = qhr_out;
t = time/(60*60); % [h]

%Energy consumption
Jend = J(end)

%Plot to find the tuning parameters
figure(3)
subplot(2,1,1)
plot(t,Tf,'b')
ylabel('Temperature [C]')
xlabel('Time [h]')
title('Tf')

subplot(2,1,2)
plot(t,Qhf,'b')
ylabel('Heat [kW]')
xlabel('Time [h]')
title('qhf')

%Saving in tikz
%matlab2tikz('open_loop_Tf_plot.tikz')

figure(4)
subplot(2,1,1)
plot(t,Tr,'b')
ylabel('Temperature [C]')
xlabel('Time [h]')
title('Tr')

subplot(2,1,2)
plot(t,Qhr,'b')
ylabel('Heat [kW]')
xlabel('Time [h]')
title('qhr')
```

```
%Saving in tikz
%matlab2tikz('open_loop_Tr_plot.tikz')

%% Simulation of control structure
sim('simulink_master_PI_control_structure')

%Outputs
Tf = simout(:,1)-273; % [C]
Tr = simout(:,2)-273; % [C]
J = simout(:,3);
Qhf = qhf_out;
Qhr = qhr_out;
t = time/(60*60); % [h]

%Energy consumption
Jend = J(end)
```

Representation of the simulink models

Figure E.2.1 , E.2.2 and E.2.3 give an overview of the Simulink models used in the matlab script above. The first, second and third figure represent the models called *simulink _ mastermdl*, *simulink _ master _ PI* and *simulink _ master _ PI _ control _ structure* respectively.

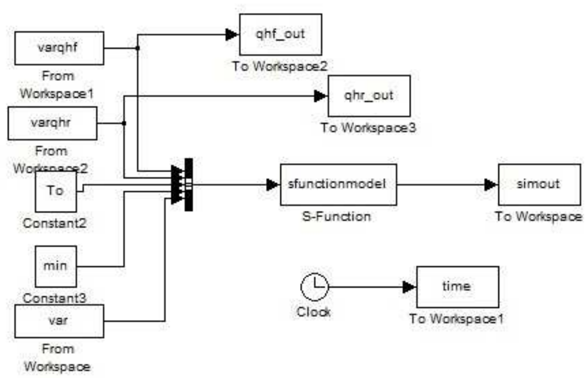


Figure E.2.1: Illustration of the Simulink model called *simulink _ master-mdl*

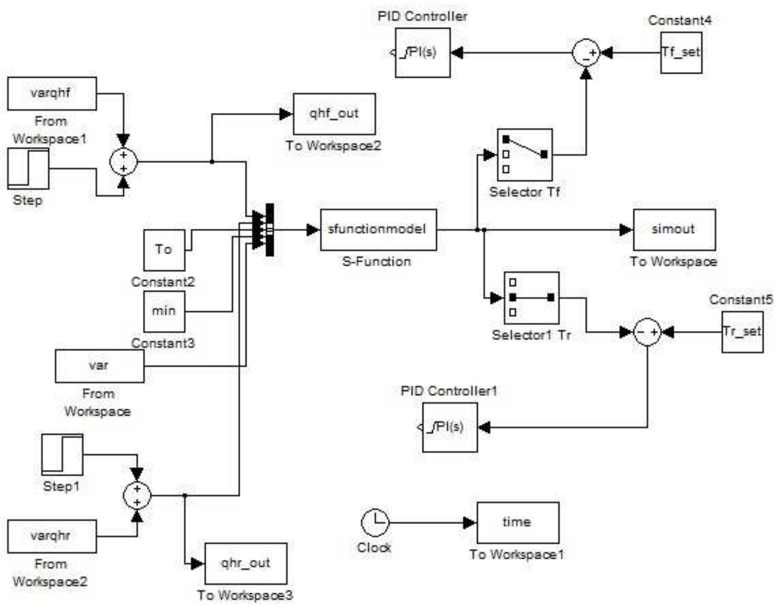


Figure E.2.2: Illustration of the Simulink model called *simulink _ master _ PI*

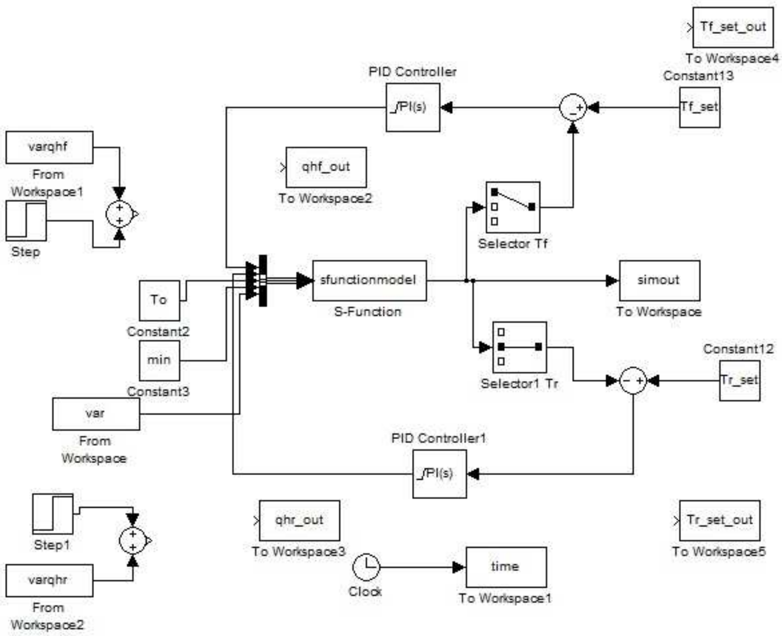


Figure E.2.3: Illustration of the Simulink model called *simulink _ master _ PI _ control _ structure*

E.3 Optimal control using PI: storage of heat in the floor

```
%Optimal Temperature Control of rooms for Minimum
    Energy Cost

%Master thesis 2013

close all
clear all
clc

% Solution of optimal control problem when using PI
    controllers

% Storage of heat in the floor
% Written by Siri Hofstad Trapnes, 2013

%% Parameters

%UA = J/hK
%mCp = J/K

par.UAfr = 0.350; % [kJ/sK = kW/K]
par.UAro = 0.007; % [kJ/sK = kW/K]
par.mCpf = 4000; % [kJ/K]
par.mCpr = 70; % [kJ/K]
par.mr = 70; % [kg]

min = 0.06; % [kg]
mout = min;

%% Energy price

%Varying price
ts = 600; % 10min
tend = 20*60*60; % 24h
t = [0:ts:tend]'; % [s]
t_p = t/(60*60);

t1 = length(t(1):ts:t(37)); % start - 6h %
```

```
t2 = length(t(38):ts:t(49)); % 6 - 8 %
t3 = length(t(50):ts:t(end)); % 8 - end %

P1 = 1/(60*60);
P2 = 0.6/(60*60);

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2]; %
    15h
P_plot = P;

var.time = t;
var.signals.values = P;

%% Inputs and initial conditions

%Inputs

%Constant inputs
%qhr = 2; % No heat on: 0 %max: 2 [kW]
%qhf = 2.5; % No heat on: 0 %max: 2.5 [kW]

%Disturbance
To = 273+0; % [K]

%Varying inputs
t1 = length(t(1):ts:t(25)); % start - 4h
t2 = length(t(26):ts:t(37)); % 4h - 6h
t3 = length(t(38):ts:t(end)); %15h

qhf_min = 0;
qhf_max = 2.5;
qhr_min = 0;
qhr_max = 2;

qhf = [ones(t1,1)*qhf_min; ones(t2,1)*qhf_max; ones(t3
    ,1)*qhf_min];
qhr = [ones(t1,1)*qhr_min; ones(t2,1)*qhr_max; ones(t3
    ,1)*qhr_min];

varqhf.time = t;
```

```
varqhf.signals.values = qhf;

varqhr.time = t;
varqhr.signals.values = qhr;

%Initial conditions
Tf = 293; % [K]
Tr = 292; % [K]
J = 0; % [NOK]
init = [Tf Tr J];

%Set point
Tf_set = 292; % [K]
Tr_set = 292; % [K]

%Upper and lower constraints
Tfmax = 298; % [K]
Tfmin = 293; % [K]
Trmax = 298; % [K]
Trmin = 292; % [K]

%% Optimizing, storing of heat in the floor

%Optimal solution
%High price interval between 6 and 8

sim('simulink_master_PI_2')

%Outputs
Tf = simout(:,1)-273; % [C]
Tr = simout(:,2)-273; % [C]
J = simout(:,3);
Qhf = qhf_out;
Qhr = qhr_out;
t = time/(60*60); % [h]

%Energy consumption
Jend = J(end)

%Solution from switching rule
```

```
ts = 600; % 10min
tend = 20*60*60; % 20h
t_switch = [0:ts:tend]'; % [s]
t_p_switch = t_switch/(60*60);

t1 = length(t_switch(1):ts:t_switch(37)); % start - 6h
t2 = length(t_switch(38):ts:t_switch(49)); % 6 - 8
t3 = length(t_switch(50):ts:t_switch(end)); % 8 - end

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2];
P_switch = P;
var.time = t_switch;
var.signals.values = P;

sim('simulink_master_PI_2_switch')

%Outputs
Tf_switch = simout(:,1)-273; % [C]
Tr_switch = simout(:,2)-273; % [C]
J_switch = simout(:,3);
Qhf_switch = qhf_out;
Qhr_switch = qhr_out;
t_switch = time/(60*60); % [h]
%Energy consumption
Jend_switch = J_switch(end)

% Broad peak
% High energy price from 5 h - 12 h

ts = 600; % 10min
tend = 20*60*60; % 20h
t_bp = [0:ts:tend]'; % [s]
t_p_broadpeak = t_bp/(60*60);

t1 = length(t_bp(1):ts:t_bp(31)); % start - 5h
t2 = length(t_bp(32):ts:t_bp(73)); % 5 - 12
t3 = length(t_bp(74):ts:t_bp(end)); % 12 - end

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2];
P_broadpeak = P;
```

```
var.time = t_bp;
var.signals.values = P;

sim('simulink_master_PI_broadpeak')

%Outputs
Tf_broadpeak = simout(:,1)-273; % [C]
Tr_broadpeak = simout(:,2)-273; % [C]
J_broadpeak = simout(:,3);
Qhf_broadpeak = qhf_out;
Qhr_broadpeak = qhr_out;
t_broadpeak = time/(60*60); % [h]

%Energy consumption
Jend_broadpeak = J_broadpeak(end)

%Solution from broad peak using switching rule

ts = 600; % 10min
tend = 20*60*60; % 24h
t_bp_s = [0:ts:tend]'; % [s]
t_p_broadpeak_s = t_bp_s/(60*60);

t1 = length(t_bp_s(1):ts:t_bp_s(31)); % start - 5h
t2 = length(t_bp_s(32):ts:t_bp_s(73)); % 5 - 12
t3 = length(t_bp_s(74):ts:t_bp_s(end)); % 12 - end

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2];
P_broadpeak_s = P;
var.time = t_bp_s;
var.signals.values = P;

sim('simulink_master_PI_broadpeak_switch')

%Outputs
Tf_broadpeak_s = simout(:,1)-273; % [C]
Tr_broadpeak_s = simout(:,2)-273; % [C]
J_broadpeak_s = simout(:,3);
Qhf_broadpeak_s = qhf_out;
Qhr_broadpeak_s = qhr_out;
```

```
t_broadpeak_s = time/(60*60); % [h]

%Energy consumption
Jend_broadpeak_s = J_broadpeak_s(end)

%Broad peak 2. No time to heat up the floor
%High energy price from 3 h - 10 h

ts = 600; % 10min
tend = 20*60*60; % 20h
t_bp2 = [0:ts:tend]'; % [s]
t_p_broadpeak2 = t_bp2/(60*60);

t1 = length(t_bp2(1):ts:t_bp2(19)); % start - 3h
t2 = length(t_bp2(20):ts:t_bp2(61)); % 3 - 10
t3 = length(t_bp2(62):ts:t_bp2(end)); % 10 - end

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2];
P_broadpeak2 = P;
var.signals.values = P;

sim('simulink_master_PI_broadpeak2')

%Outputs
Tf_broadpeak2 = simout(:,1)-273; % [C]
Tr_broadpeak2 = simout(:,2)-273; % [C]
J_broadpeak2 = simout(:,3);
Qhf_broadpeak2 = qhf_out;
Qhr_broadpeak2 = qhr_out;
t_broadpeak2 = time/(60*60); % [h]

%Energy consumption
Jend_broadpeak2 = J_broadpeak2(end)
```

Representation of the Simulink model

Figure E.3.1 gives an overview of one of the Simulink models used in the simulations. The model that is given is the first one in the above Matlab code, called *simulink_master_PI_2*.

The temperature in the room is set to a minimum value by the block *Trset* in Figure E.3.1. The switch called *P switch* represent the place where the energy price switches between the high and low value, $Price_{high/low}$, while the switching time is defined in *ts switch*. The last switch, *ts switch 1*, defines the time where the heating of the system should stop, i.e. the time where the peak period begins.

Only one Simulink model will be given in this report. The reason for this is that the models described in the above Matlab function will be similar to Figure E.3.1. The only difference between them are the values implemented in switch *ts switch* and *ts switch 1*. In the models where the name ends with *switch*, will the switching value be implemented in *ts switch*, while the optimal value will be given in *ts switch* in the rest of the models.

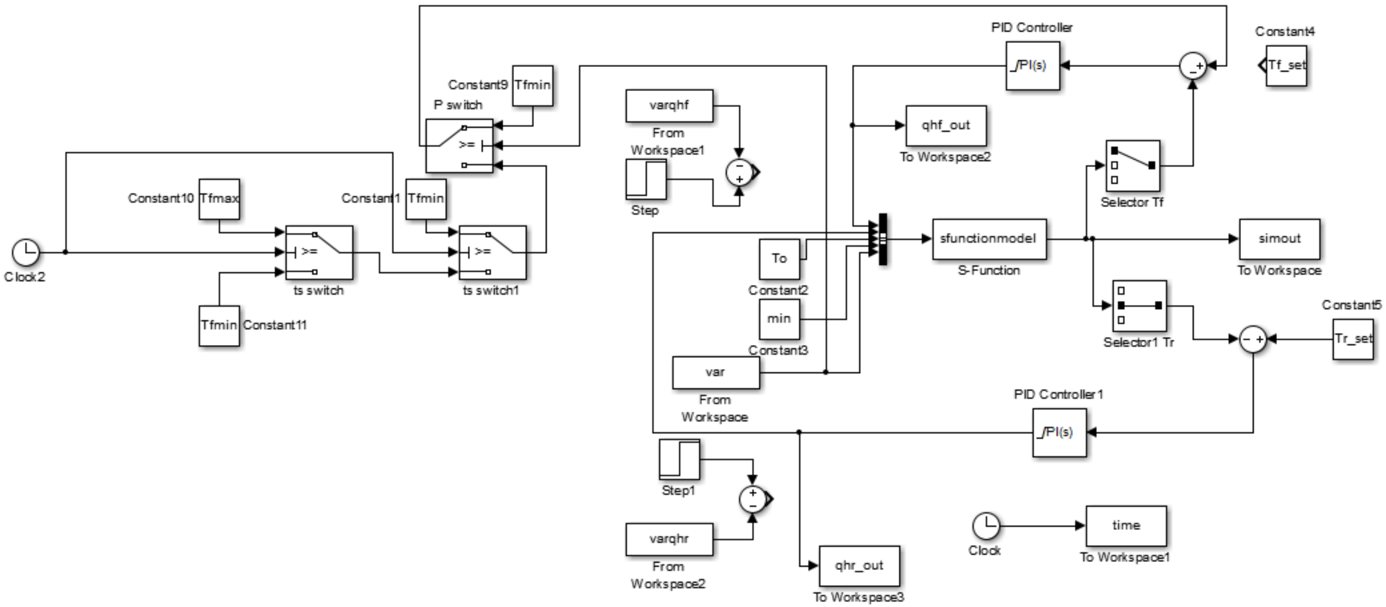


Figure E.3.1: Overview of Simulink model that solve the dynamic optimization problem

E.4 Optimal control using PI: storage of heat in the floor and room

```
%Optimal Temperature Control of rooms for Minimum Energy Cost

%Master thesis 2013

close all
clear all
clc

% Function for finding the solution of optimization problem when using
% PI controllers
% Storage of heat in the floor and room
% Written by Siri Hofstad Trapnes, 2013

%% Parameters

%UA = J/hK
%mCp = J/K

par.UAfr = 0.350; % [kJ/sK = kW/K]
par.UAro = 0.007; % [kJ/sK = kW/K]
par.mCpf = 4000; % [kJ/K]
par.mCpr = 70; % [kJ/K]
par.mr = 70; % [kg]

min = 0.06; % [kg]
mout = min;

%% Energy price

%Varying price
ts = 600; % 10min
tend = 20*60*60; % 20h
t = [0:ts:tend]'; % [s]
t_p = t/(60*60);

```

```
t1 = length(t(1):ts:t(37)); % start - 6h %
t2 = length(t(38):ts:t(49)); % 6 - 8 %
t3 = length(t(50):ts:t(end)); % 8 - end %

P1 = 1/(60*60);
P2 = 0.6/(60*60);

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2]; %
    20h
P_plot = P;

var.time = t;
var.signals.values = P;

%% Inputs and initial conditions

%Inputs
%qhr = 2; % No heat on: 0 %max: 2 [kW]
%qhf = 2.5; % No heat on: 0 %max: 2.5 [kW]
To = 273+0; % [K]

%Varying inputs
t1 = length(t(1):ts:t(25)); % start - 4h
t2 = length(t(26):ts:t(37)); % 4h - 6h
t3 = length(t(38):ts:t(end)); %15h

qhf_min = 0;
qhf_max = 2.5;
qhr_min = 0;
qhr_max = 2;

qhf = [ones(t1,1)*qhf_min; ones(t2,1)*qhf_max; ones(t3
    ,1)*qhf_min];
qhr = [ones(t1,1)*qhr_min; ones(t2,1)*qhr_max; ones(t3
    ,1)*qhr_min];

varqhf.time = t;
varqhf.signals.values = qhf;

varqhr.time = t;
```

```
varqhr.signals.values = qhr;

%Initial conditions
Tf = 293; % [K]
Tr = 292; % [K]
J = 0; % [NOK]
init = [Tf Tr J];

%Set point
Tf_set = 293; % [K]
Tr_set = 292; % [K]

%normal
Tfmax = 298; % [K]
Tfmin = 293; % [K]
Trmax = 298; % [K]
Trmin = 292; % [K]

%% Optimization, storing of heat in the floor and room

%Optimal solution
%High price interval between 6 and 8

sim('simulink_master_PI_2_storing_floor_room')

%Outputs
Tf = simout(:,1)-273; % [C]
Tr = simout(:,2)-273; % [C]
J = simout(:,3);
Qhf = qhf_out;
Qhr = qhr_out;
t = time/(60*60); % [h]

%Energy consumption
Jend = J(end)

%Switching rule
ts = 600; % 10min
tend = 20*60*60; % 20h
t_switch = [0:ts:tend]'; % [s]
```

```
t_p_switch = t_switch/(60*60);

t1 = length(t_switch(1):ts:t_switch(37)); % start - 6h
    %
t2 = length(t_switch(38):ts:t_switch(49)); % 6 - 8 %
t3 = length(t_switch(50):ts:t_switch(end)); % 8 - end
   %

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2];
P_switch = P;
var.time = t_switch;
var.signals.values = P;

sim('simulink_master_PI_2_storing_floor_room_switch')

%Outputs
Tf_switch = simout(:,1)-273; % [C]
Tr_switch = simout(:,2)-273; % [C]
J_switch = simout(:,3);
Qhf_switch = qhf_out;
Qhr_switch = qhr_out;
t_switch = time/(60*60); % [h]
%Energy consumption
Jend_switch = J_switch(end)

%Broad peak
%High energy price from 5 h - 12 h

ts = 600; % 10min
tend = 20*60*60; % 20h
t_bp = [0:ts:tend]'; % [s]
t_p_broadpeak = t_bp/(60*60);

t1 = length(t_bp(1):ts:t_bp(37)); % start - 5h
t2 = length(t_bp(38):ts:t_bp(73)); % 5 - 12
t3 = length(t_bp(74):ts:t_bp(end)); % 12 - end

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2];
P_broadpeak = P;
var.time = t_bp;
```

```
var.signals.values = P;

sim('simulink_master_broad_peak_floor_room')

%Outputs
Tf_broadpeak = simout(:,1)-273; % [C]
Tr_broadpeak = simout(:,2)-273; % [C]
J_broadpeak = simout(:,3);
Qhf_broadpeak = qhf_out;
Qhr_broadpeak = qhr_out;
t_broadpeak = time/(60*60); % [h]
%Energy consumption
Jend_broadpeak = J_broadpeak(end)

%Solution from broad peak using switching rule

ts = 600; % 10min
tend = 20*60*60; % 24h
t_bp_s = [0:ts:tend]'; % [s]
t_p_broadpeak_s = t_bp_s/(60*60);

t1 = length(t_bp_s(1):ts:t_bp_s(31)); % start - 5h
t2 = length(t_bp_s(32):ts:t_bp_s(73)); % 5 - 12
t3 = length(t_bp_s(74):ts:t_bp_s(end)); % 12 - end

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2];
P_broadpeak_s = P;
var.time = t_bp_s;
var.signals.values = P;

sim('simulink_master_broad_peak_floor_room_switch')

%Outputs
Tf_broadpeak_s = simout(:,1)-273; % [C]
Tr_broadpeak_s = simout(:,2)-273; % [C]
J_broadpeak_s = simout(:,3);
Qhf_broadpeak_s = qhf_out;
Qhr_broadpeak_s = qhr_out;
t_broadpeak_s = time/(60*60); % [h]
%Energy consumption
```

```
Jend_broadpeak_s = J_broadpeak_s(end)

%Broad peak 2. No time to heat up the floor
%High energy price from 3 h - 10 h

ts = 600; % 10min
tend = 20*60*60; % 20h
t_bp2 = [0:ts:tend]'; % [s]
t_p_broadpeak2 = t_bp2/(60*60);

t1 = length(t_bp2(1):ts:t_bp2(19)); % start - 3h
t2 = length(t_bp2(20):ts:t_bp2(61)); % 3 - 10
t3 = length(t_bp2(62):ts:t_bp2(end)); % 10 - end

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2];
P_broadpeak2 = P;
var.signals.values = P;

sim('simulink_master_case3_floor_room')

%Outputs
Tf_broadpeak2 = simout(:,1)-273; % [C]
Tr_broadpeak2 = simout(:,2)-273; % [C]
J_broadpeak2 = simout(:,3);
Qhf_broadpeak2 = qhf_out;
Qhr_broadpeak2 = qhr_out;
t_broadpeak2 = time/(60*60); % [h]

%Energy consumption
Jend_broadpeak2 = J_broadpeak2(end)
```

Representation of a Simulink model

Figure E.4.1 and E.4.2 give an overview of two of the Simulink models used in the simulations. The models that are given is the first and fifth one in the above Matlab code, called *simulink _ master _ PI _ 2 _ storing _ floor _ room* and *simulink _ master _ case3 _ floor _ room* respectively.

The function of each switch will be the same as described in section E.3: switch *P switch* give $Price_{high/low}$, Δts is implemented in *ts switch*, while *ts switch 1* define the time where the peak period begins.

Only these two models will be illustrated in this report. This because the only difference between the Simulink models are the values implemented in *ts switch* and *ts switch 1*. As in the above section, the switching value is implemented in the models where the name ends with *switch*, while the optimal value is used in *ts switch* in the rest of the models.

The models for the optimal and broad case will be similar to Figure E.4.1. The room temperature will be kept constant, and the set point of T_R is defined in the block $Trset$, as shown in Figure E.4.1. The case where the system does not have time to heat up before the peak period will be represented by the Simulink model given in Figure E.4.2. The room heat should increase at ts. This means that the set point of T_R should be defined in the same as the set point for T_F.

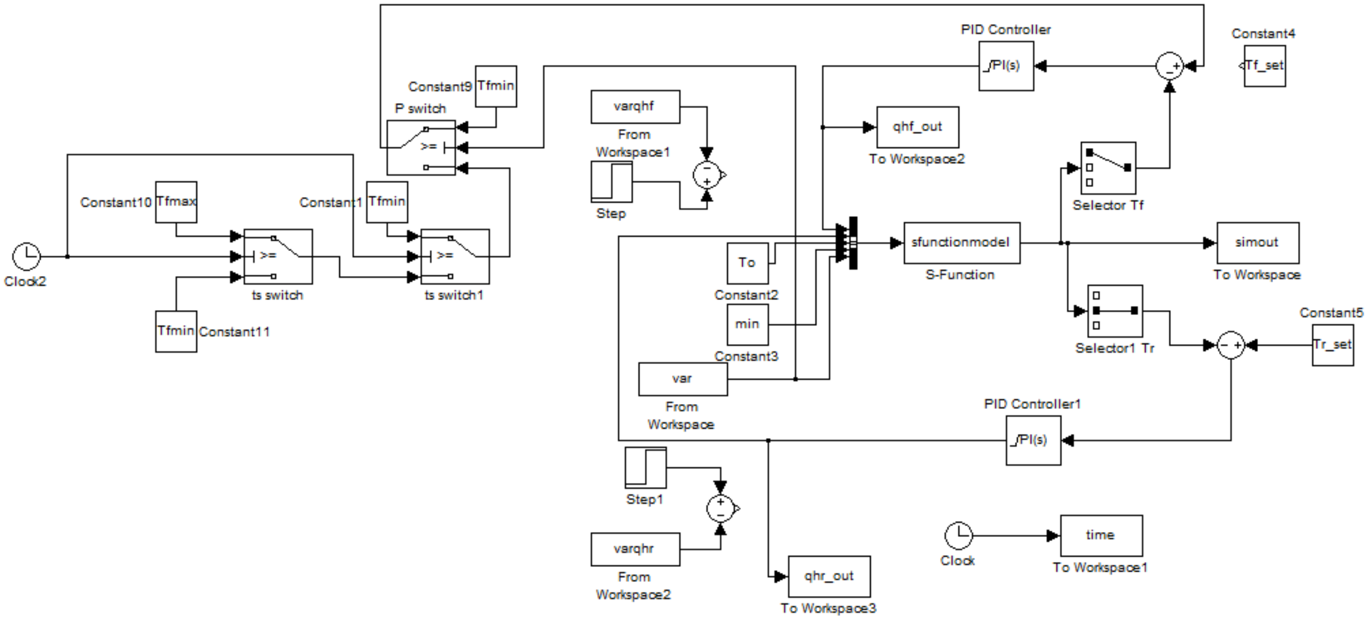


Figure E.4.1: Overview of Simulink model that solve the dynamic optimization problem

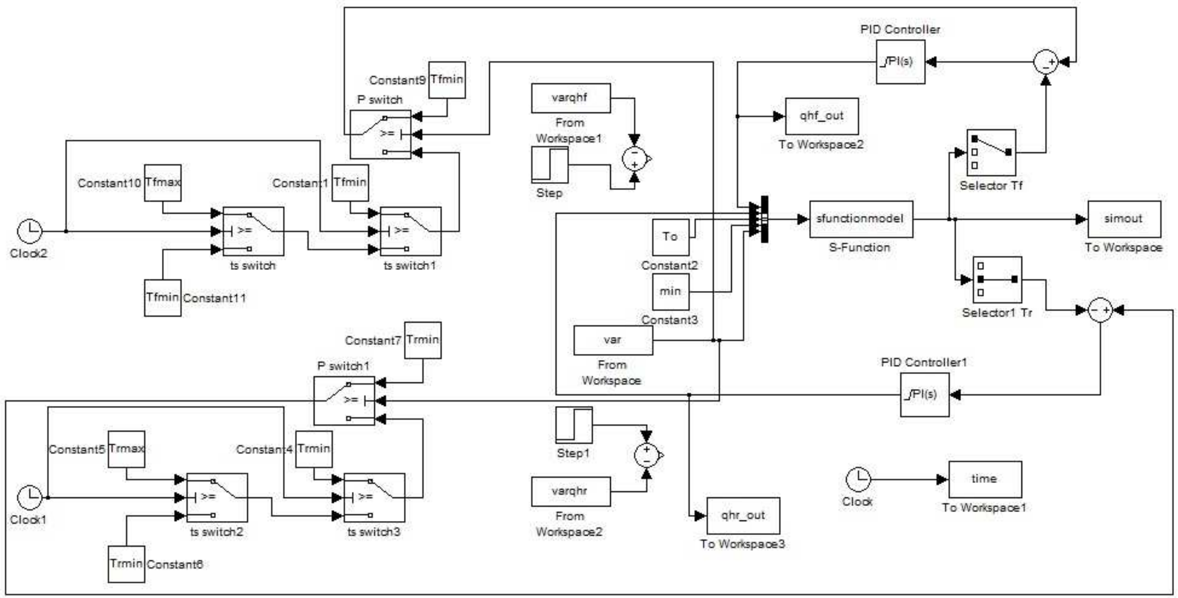


Figure E.4.2: Overview of Simulink model that solve the dynamic optimization problem. The case where the system do not have time to heat up before the peak period is implemented, and where the heat in the floor and room are used for storage

F Matlab code for the single shooting optimization problem

```
%Optimal Temperature Control of rooms for Minimum
    Energy Cost

%Master thesis 2013

close all
clear all
clc

% Single shooting optimization
% Written by Siri Hofstad Trapnes, 2013

%% Parameters

%UA = J/hK
%mCp = J/K

par.UAfr = 0.350; % [kJ/sK = kW/K]
par.UAro = 0.007; % [kJ/sK = kW/K]
par.mCpf = 4000; % [kJ/K]
par.mCpr = 70; % [kJ/K]
par.mr = 70; % [kg]

min = 0.06; % [kg]
%mout = min;

%% Inputs and initial conditions

%Inputs
qhr = 2; % No heat on: 0 %max: 2 [kW]
qhf = 2.5; % No heat on: 0 %max: 2.5 [kW]

%Disturbances
To = 273-0; % [K]

%Initial conditions
```

```
Tf = 293; % [K] 293
Tr = 293; % [K] 293
J = 0;
init = [Tf Tr J];

%% Energy price

%Varying price
ts = 600; % 10min
tend = 54000; % 15h
t = [0:ts:tend]'; % [s]
t_p = t/(60*60);

t1 = length(t(1):ts:t(37)); % start - 6h
t2 = length(t(38):ts:t(61)); % 6 - 10
t3 = length(t(62):ts:t(end)); % 10 - end

P1 = 1/(60*60);
P2 = 0.6/(60*60);

P = [ones(t1,1)*P2; ones(t2,1)*P1; ones(t3,1)*P2]; %
    15h

par.P = P;

%% Solving equations

%Constant parameters
% u = [qhf qhr]; %Constant u
d = [To min]; %Constant d


%Time varying parameters
% Input, u
qhf_min = 0; % [kW]
qhf_max = 2.5; % [kW]
qhr_min = 0; % [kW]
qhr_max = 2; %4.8; % [kW]

t1 = length(t(1):ts:t(25)); % 0:10min:4h
```

```
t2 = length(t(26):ts:t(37)); % 4:10min:6
t3 = length(t(38):ts:t(end)); % 6:10min:end

qhf1 = [ones(t1,1)*qhf_min; ones(t2,1)*qhf_max; ones(
    t3,1)*qhf_min]; % 15h
qhr1 = [ones(t1,1)*qhr_min; ones(t2,1)*qhr_max; ones(
    t3,1)*qhr_min]; % 15h

tintp = [0:ts:tend]';

u = [qhf1 ; qhr1];

[t x] = ode15s(@(t,x)equations(t,x,u,d,tintp,par),t,
    init);

%% Optimization

% Upper and lower constraints for the states
Tr_min = 293;  % [K]
Tr_max = 300;  % [K]
Tf_min = 292; % [K]
Tf_max = 300; % [K]

par1.Tr_min = Tr_min;
par1.Tr_max = Tr_max;
par1.Tf_min = Tf_min;
par1.Tf_max = Tf_max;

%Upper and lower bounds for the input
qhf_upper = 2.5; % [kW]
qhr_upper = 2; %4.8; % [kW]
qhf_lower = 0; % [kW]
qhr_lower = 0; % [kW]

LB = [ones(length(t),1)*qhf_lower ; ones(length(t),1)*
    qhr_lower];
UB = [ones(length(t),1)*qhf_upper ; ones(length(t),1)*
    qhr_upper];

%Initial values
```

```
% load heatfile.mat %Get the values from
    model_master_simulink
% qhf_g = interp1(time_sim,qhf_sim,t);
% qhr_g = interp1(time_sim,qhr_sim,t);
% u0 = [qhf_g ; qhr_g];
% u0 = [qhf_sim ; qhr_sim];
u0 = [2.5*ones(length(t),1); 2*ones(length(t),1)]; % [
    kW] qhf qhr

options = optimset('Display','Iter','Algorithm','
    active-set',...
     'TolCon',4*10^(-2),'TolX',0.01);
Z = fmincon(@(u)costfun(u,t,init,d,tintp,par),u0
    ,[],[],[],[],LB,UB,...
     @(u)nonlinconst(u,d,tintp,par1,par,t,init),options
        );


qhf_opt = Z(1:length(Z)/2,1);
qhr_opt = Z(length(Z)/2+1:end,1);
[t x] = ode15s(@(t,x)equations(t,x,Z,d,tintp,par),t,
    init);

% Cost function
J = P.*(Z(1:length(Z)/2,1)+Z(length(Z)/2+1:end,1));
```

Model

```
function dxdt = equations(t,x,u,d,tintp,par,var)

%Parameters
UAfr = par.UAfr;
UAro = par.UAro;
mCpf = par.mCpf;
mCpr = par.mCpr;
mr = par.mr;
%P = par.P;

%Inputs and disturbances
% qhf = u(1);
% qhr = u(2);
```

```
To = d(1);
min = d(2);

%To = 273

Tf = x(1);
Tr = x(2);
%J =x(3);

qhf1 = u(1:length(tintp),1);
qhr1 = u(length(tintp)+1:end,1);

qhf = interp1(tintp,qhf1,t');
qhr = interp1(tintp,qhr1,t');

% min1 = d(:,1);
% min = interp1(tintp,min1,t);
P1 = par.P;

P = interp1(tintp,P1,t');

%Floor
dxdt(1) = (qhf/mCpf)-(UAfr/mCpf)*(Tf-Tr);
%Room
dxdt(2) = (qhr/mCpr)+(UAfr/mCpr)*(Tf-Tr)-(UAro/mCpr)*(Tr-To)...
    +(min/mr)*(To-Tr);
%Cost function
dxdt(3) = P.*(qhf+qhr);
% dxdt(3) = P.*(qhf+qhr)^2;
% dxdt(3) = P.*(0.4(qhf+qhr)^+(qhf+qhr));
%           Floor       Room
dxdt = [dxdt(1) dxdt(2) dxdt(3)]';
```

Cost function

```
function J = costfun(u,t,init,d,tintp,par)

%qhf = u(1:length(u)/2,1);
%qhr = u(length(u)/2+1:end,1);

```

```
[t x] = ode15s(@(t,x)equations(t,x,u,d,tintp,par),t,
    init);

J = x(:,3);
J = J(end);

%J = trapz(tintp,P.*(qhf+qhr));
%J = trapz(tintp,P.*(qhf+qhr).^2);
%J = trapz(tintp,(P.*(0.4*(qhf+qhr)).^2+(qhf+qhr)));
```

Nonlinear inequality constraints

```
function [C Ceq] = nonlinconst(u,d,tintp,par1,par,t,
    init)
%Nonlinear inequality constraints

%Parameters
Tr_min = par1.Tr_min;
Tr_max = par1.Tr_max;
Tf_min = par1.Tf_min;
Tf_max = par1.Tf_max;

[t x] = ode15s(@(t,x)equations(t,x,u,d,tintp,par),t,
    init);

Tf = x(:,1);
Tr = x(:,2);

% C <= 0
C(1) = -(max(Tf) - Tf_max);  % [K]
C(2) = -(max(Tr) - Tr_max); % [K]
C(3) = -(Tf_min - min(Tf)); % [K]
C(4) = -(Tr_min - min(Tr)); % [K]

 C = [C(1) C(2) C(3) C(4)];

Ceq = [];
```