Game-Theoretical Analysis of PLC System Performance in the Presence of Jamming Attacks

Yun Ai[1], Manav R. Bhatnagar[2], Michael Cheffena[1], Aashish Mathur[3], and Artem Sedakov[4]

[1] Norwegian University of Science and Technology (NTNU), 2815 Gjøvik, Norway,
{yun.ai; michael.cheffena}@ntnu.no
[2] Indian Institute of Technology Delhi, Hauz Khas, Delhi 110016, India,
manav@ee.iiid.ac.in
[3] Indian Institute of Technology (BHU) Varanasi, Uttar Pradesh 221005, India,
amathur.ece@iitbhu.ac.in
[4] Saint Petersburg State University, Saint Petersburg 199034, Russia,
a.sedakov@spbu.ru
(The authors are listed in the alphabetical order.)

Abstract. In this paper, we investigate the performance of power line communication (PLC) network in the presence of jamming attacks. The legitimate nodes of the PLC network try to communicate with the anchor node of the network while the jamming node attempts to degrade the system performance. The fading, attenuation and colored noise of the PLC channel with dependence on the frequency and transmission distance are taken into account. To investigate the jamming problem, we frame the adversarial interaction into a Bayesian game, where the PLC network tries to maximize the overall expected network capacity and the jammer node has the opposite goal. In the Bayesian game, both players have imperfect knowledge of their opponents. We study effects of total power available to the players on the equilibrium of the game by formulating it into zero-sum and non-zero-sum games, respectively. It is found that under some network setup, there exists a threshold power for which the actual gameplay of the legitimate nodes does not depend upon the actions of the jamming node, and vice versa. This allows us to choose the appropriate power allocation schemes given the total power and the action of the jamming node in some cases.

Keywords: security, jamming attack, game theory, zero-sum game, non-zero-sum game, Bayesian Nash equilibrium, power line communication

1 Introduction

In recent years, power line communication (PLC) has gained increasing interests from both the industry and academia due to the vision of widespread information transmission through power lines. With the advantages of omnipresence of power line and no need to invest in new infrastructure, PLC is set to be a promising technology with wide applications in smart grid, home automation and networking, etc. [1–3].

As in the case of wireless communications, PLC system is inherently based on broadcast transmission. This open and shared nature of the PLC transmission medium poses significant challenges for the communication secrecy and privacy in the presence of potential malicious attacks [4]. The nature of the malicious attacks generally indicates conflict and cooperation between the participants in the communication system. These kind of problems can be often addressed with the game theory approach, which has been widely used by the communication and networking research community to tackle various problems [5–7]. The anti-eavesdropping problem in the presence of selfish jamming is studied as a Bertrand game by assuming the single-channel multi-jammer and multi-channel single-jammer models in [5]. In [6], the authors consider a scenario where a jammer attacks one sub-band of a multi-channel wireless communication system. The strategies for both players are about the sub-channels to transmit or attack. The dependence of the equilibria of the formulated game on the relative position of the jammer is investigated. A reactive jamming scenario where the jammer may not always be able to accurately detect the legitimate transmissions is considered in [7]. Overall, depending on the specific scenario and the proposed strategy, different games and solutions can be formulated.

In this paper, we consider the jamming problem of PLC network. The PLC channel tremendously differs from the wireless channel in terms of the attenuation characteristics, fading distributions, and noise characteristics; the nature of wire transmission also makes the scenario of jamming different from the wireless case [8–10]. All these differences make the vast number of analysis and solutions for the wireless communication systems under malicious attacks inapplicable for the PLC systems. More specifically, we investigate the PLC system in the presence of jamming attack, where a malicious node attempts to degrade the network performance by contrasting the transmission at the physical layer. We interpret the legitimate nodes of the PLC network as one player (denoted as player L) with the aim of maximizing the system performance in terms of capacity while the malicious jamming node is considered as another player (denoted as player J) with the goal of minimizing the overall system performance. Therefore, the considered jamming problem can be well framed as a zero-sum game and analyzed with the game theory approach. Additionally, we consider a setup where the jammer has a goal of minimizing its losses assuming it can be tracked and then fined, thus the game becomes non-zero-sum.

The overall capacity of the PLC network, depends on the received signal-to-noise ratio (SNR) or signal-to-noise-plus-interference ratio (SINR) in case of jamming attack of each subchannel. The SNR or SINR highly depends upon the transmission power and the used frequency since the distances from the legitimate nodes to the anchor node in the PLC network are generally fixed. We assume that the legitimate nodes can allocate their spectrum depending on its power situation and their distances to the anchor node. Any feasible allocation of the spectrum by the legitimate nodes, we call type of player L. Meanwhile, the position of the jamming node is determined by its distance to the anchor node, which is supposed to be the type of the player J. We additionally assume that (i) the jammer has an imperfect knowledge on a particular spectrum allocation of the legitimate nodes but it knows all feasible allocations, and (ii) the legitimate nodes have imperfect information on a particular distance of the jammer to the anchor node but they have a knowledge on all feasible distances. Under these assumptions, the investigated game becomes a Bayesian game [11]. In our analysis, our objective is to investigate the role of the power allocation for both players and understand the corresponding effects on the resulting Bayesian equilibrium.

The remainder of the paper is organized as follows. In Section 2, we describe the considered system and PLC channel models. In Section 3, the investigated problem is formulated as Bayesian games (zero-sum and non-zero-sum); the Bayesian Nash equilibria (BNE) and the equilibrium payoffs to the formulated games are presented. The numerical results are presented in Section 4; and the impact of the number of sub-bands on the system performance is discussed. Section 5 concludes the paper.

2 System and Channel Model

PLC channel is tremendously different from the wireless channel. Attenuation in PLC systems depends on the characteristics of the power cables, length of transmission, and the operating frequency. The wireless channel noise stems from the thermal noise, which is modeled as additive white Gaussian noise (AWGN) [12]. However, the background noise in the PLC channel is not white but colored. The amplitude fading statistics in PLC environments are not well established compared to wireless communications. A vast number of measurement results show that distributions such as Rayleigh, Rician, and lognormal are recommended for defining the path amplitudes in PLC channels [13]. In our analysis, we will assume the amplitude following Rayleigh distribution, which was found to be the best fit for a wealth of PLC field measurements [14–18].

The input/output model of a PLC system over Rayleigh fading channel can be expressed as

$$y = h \cdot x + w, \tag{1}$$

where x is the channel input with unit energy, i.e., $\mathrm{E}[|x|^2] = 1$, w represents the PLC background noise modeled as colored Gaussian distributed additive noise, and y is the channel output. The envelope of the channel gain, i.e., $|h|$, is Rayleigh distributed with PDF given by

$$f_{|h|}(z) = \frac{z}{\sigma^2} \cdot \exp\Big(-\frac{z^2}{2\sigma^2}\Big), \quad z \geqslant 0. \tag{2}$$

where $\sigma > 0$ is the scale parameter of the distribution, which determines the statistical average and the variance of the random variable as $\mathrm{E}[|h|] = \sigma\sqrt{\pi/2}$

and $\mathrm{Var}[|h|] = (2 - 0.5\pi)\sigma^2$, respectively. In model (1), the average power of $h \cdot x$ depends on the transmit power P_L and the power attenuation $a(D_L, f)$ over transmission distance D_L at operating frequency f[5], i.e.,

$$\mathrm{E}[|h|^2 \cdot |x|^2] = \mathrm{E}[|h|^2] = P_L \cdot a(D_L, f). \tag{3}$$

Due to the nature of the cable propagation environment, the PLC attenuation model is significantly different from that of wireless channel and the attenuation $a(D, f)$ can be modeled by [19]

$$a(D_L, f) = e^{-2(\alpha_1 + \alpha_2 \cdot f^k) \cdot D_L}, \tag{4}$$

where α_1 and α_2 are constants with dependence on the system configurations; the exponent k is the attenuation factor with typical values between 0.5 and 1. It is obvious from (4) that the attenuation increases dramatically with higher frequency and larger transmission distance.

The widely used assumption of white noise for wireless channel does not hold for PLC channel. Instead, the background noise is colored and the average power per unit bandwidth, namely, the power spectral density (PSD), can be written as [19]

$$N(f) = \mathrm{E}[|w|^2] = 10^{0.1 \cdot (\beta_1 + \beta_2 \cdot e^{-f/\beta_3})} \qquad [\mathrm{mW/Hz}], \tag{5}$$

where β_1, β_2, and β_3 are some constants.

With the aforementioned system, in case of no jamming, the received average SNR $\overline{\gamma}$ at the transmission distance D_L and frequency f can be expressed as

$$\overline{\gamma}(D_L, f) = \frac{P_L \cdot a(D_L, f)}{N(f)}. \tag{6}$$

A jammer J is located D_J away from the receiver and is transmitting noise-like power P_J over the concerned channel. To simplify our analysis, we can approximate the average SINR expression by using the variance of the Jammer's channel. This practice leads to an approximation, which is found reasonable in practice. Then, the corresponding average SINR can be simply expressed as

$$\overline{\gamma}(D_L, D_J, f) \approx \frac{P_L \cdot a(D_L, f)}{P_J \cdot a(D_J, f) + N(f)}. \tag{7}$$

It is well-known that under the Rayleigh fading channel, the instantaneous SNR or SINR γ is distributed according to an exponential distribution given by

$$f_\gamma(z) = \frac{1}{\overline{\gamma}} \cdot \exp\Big(-\frac{z}{\overline{\gamma}}\Big), \quad z \geqslant 0, \tag{8}$$

where the parameter $\overline{\gamma}$ is expressed in (6) or (7).

[5] The frequency f is in MHz throughout the paper.

The ergodic capacity (a.k.a. Shannon capacity) is defined as the expectation of the information rate over all states of the fading channel. The ergodic capacity of the PLC channel pertaining to the frequency f is expressed as [20]

$$C_f(P_L, D_L; P_J, D_J) = \int_0^{\infty} \log_2(1+z) f_\gamma(z)\, dz = \log_2(e) \cdot e^{\frac{1}{\overline{\gamma}}} \cdot E_1\Big(\frac{1}{\overline{\gamma}}\Big),$$

where the function $E_1(\cdot)$ is the exponential integral of first order given by $E_1(x) = \int_1^{\infty} \frac{e^{-xt}}{t}\, dt$ [21].

For transmission over a frequency band B, the corresponding ergodic capacity per bandwidth becomes

$$C_B(P_L, D_L; P_J, D_J) = \frac{1}{|B|} \cdot \int_B \log_2(e) \cdot e^{\frac{1}{\overline{\gamma}}} \cdot E_1\Big(\frac{1}{\overline{\gamma}}\Big)\, df, \tag{9}$$

integrating over all frequencies f within frequency band B, where the average SNR or SINR $\overline{\gamma}$ is expressed in (6) or (7) depending on the presence of the jammer [22]. It is not possible to obtain closed-form expressions for the integral in (9), but it is simple and straightforward to evaluate it numerically using mathematical softwares such as Matlab and Mathematica.

3 Game-Theoretical Approach of the Jamming Attacks in PLC Network

3.1 Case of a Zero-Sum Game

We assume that the considered PLC network is represented by a finite set of legitimate nodes $\{1, \ldots, m\}$, all of which transmit information to an anchor node. The PLC system operates in the frequency division multiple access (FDMA) mode by using $n \geqslant m$ equal subchannels $B_1, \ldots, B_n$ within the available frequency band B. Denote by $B(\ell) \in \{B_1, \ldots, B_n\}$ the subchannel assigned to legitimate node ℓ. The legitimate nodes may transmit at different power levels depending upon its available power and distance. One jammer node exists in the PLC network which launches "brute-force" hostile attacks at the physical layer by raising the interference level on the transmitting frequency band. The jamming node is also intelligent enough to attack different subchannels with different powers. This considered scenario is quite practical as the PLC network can be readily extended into a core network in future smart grid network (e.g., in Fig. 1, the anchor node is the router connecting all devices over power line and a jamming node can potentially attack the PLC network). As a more practical illustration of usage, the legitimate node might be a wifi access point within a room where there exists no fiber or a sensor node which collects data on the surrounding environment, etc. The anchor node might be a router, which transfers the accumulated information within the PLC network to the data center or the Internet [23].

For player L, let D_ℓ, $\ell = 1, \ldots, m$, represent the distance between the anchor node and the ℓth legitimate node. The total available power for player L is

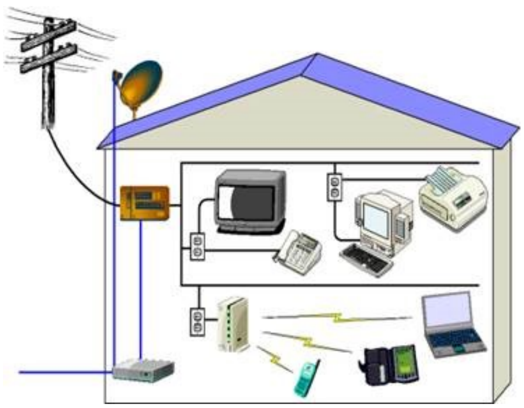

Fig. 1: A typical PLC network where jamming security can be an issue. [24]

denoted as P_L. As explained in Section 1, the uncertainty of the allocation scheme of the legitimate nodes on B is modeled by a set $\mathcal{T}_L$ of different types of allocation schemes of the spectrum, where $|\mathcal{T}_L|$ is finite. A type $t_L \in \mathcal{T}_L$ is an assignment profile $(B(1), \ldots, B(m))$ whose components are subchannels assigned to legitimate nodes. As the available frequency band B is equally divided into n subchannels, it is straightforward to see that $|\mathcal{T}_L| = n(n-1)\cdots(n-m+2)$. The corresponding prior probability for the type t_L is denoted as $p_L(t_L)$, and $\sum_{t_L \in \mathcal{T}_L} p_L(t_L) = 1$. For player J, the uncertainty of the distance from the jamming node to the anchor node is also simulated by a set $\mathcal{T}_J$ of different types of the distance, where $|\mathcal{T}_J|$ is also assumed to be finite. A type $t_J \in \mathcal{T}_J$ describes the distance D_J between the jamming node and the anchor node. Similarly, the corresponding prior probability for the type t_J in the finite set $\mathcal{T}_J$ is written as $p_J(t_J)$, and $\sum_{t_J \in \mathcal{T}_J} p_J(t_J) = 1$. The total available power for player J is denoted as P_J. The set of types of allocation schemes of the spectrum $\mathcal{T}_L$ (for player L), the set of types of distance $\mathcal{T}_J$ (for player J), available powers P_L and P_J, distances between legitimate nodes and the anchor node D_ℓ, $\ell = 1, \ldots, m$, and probabilities $p_L(t_L)$, $t_L \in \mathcal{T}_L$, and $p_J(t_J)$, $t_J \in \mathcal{T}_J$, are common knowledge. We suppose that the types of players are selected by Nature, the terminology in game theory standing for a fictitious player which introduces randomness to the game, according to the commonly known prior probability distributions [25].

Once the type of the allocation scheme for player L has been assigned by Nature according to the probability distribution, player L chooses its action $A_L = (P_{L,1}, \ldots, P_{L,m})$ from a finite set of actions $\mathcal{A}_L$ to its advantage subject to the constraint $\sum_{\ell=1}^{m} P_{L,\ell} = P_L$. Similarly for player J, the type of the distance from the jamming node to the anchor node is first assigned by Nature according to the probability distribution, player J takes its action $A_J = (P_{J,1}, \ldots, P_{L,n})$ from a finite set of actions $\mathcal{A}_J$ to its advantage subject to the constraint $\sum_{j=1}^{n} P_{J,j} = P_J$. It should be noted that the strategies, i.e., different power levels allocated to different nodes for player L or subchannels for player

J, should take discrete values. This assumption is reasonable since in practical communication systems, the power generally takes discrete values.

Since the set of power allocation schemes for player L is finite, its strategy set denoted by $\mathcal{X}_L$ will consist of $|\mathcal{X}_L| = |\mathcal{A}_L|^{|\mathcal{T}_L|}$ pure strategies whose entries $X_L \in \mathcal{X}_L$ are $|\mathcal{T}_L|$-tuples assigning a power allocation for L for any possible type. As an illustration of the action set, we can think of the simplest case where there are only a single node for player L and $|\mathcal{A}_L| = 2$ actions, then there are only two pure strategies for player L. Similarly, for player J, its strategy is to allocate different powers to n subchannels. Further, since the set of power allocation schemes for player J is also finite then it is straightforward to see that its strategy set denoted by $\mathcal{X}_J$ consists of $|\mathcal{X}_J| = |\mathcal{A}_J|^{|\mathcal{T}_J|}$ pure strategies whose entries $X_J \in \mathcal{X}_J$ are $|\mathcal{T}_J|$-tuples assigning a power allocation for J for any possible type. With the above knowledge, given two power allocation schemes A_L and A_J for players L and J respectively, we can then represent the expected ex ante payoff of player L (with its goal to maximize the overall expected network capacity) as follows

$$\begin{aligned} \mathrm{E}[U_L(X_L, X_J)] = &\sum_{t_L \in \mathcal{T}_L} \sum_{t_J \in \mathcal{T}_J} p_L(t_L) \cdot p_J(t_J) \\ &\times \left(\sum_{\ell=1}^{m} C_{B(\ell)}(P_{L,\ell}, D_\ell; P_{J,B(\ell)}, D_J) \right). \end{aligned} \tag{10}$$

The capacity for each subchannel used by each legitimate node can be readily obtained by substituting (6) or (7) into (9). Since player J has the opposite goal (it aims at minimizing the overall network capacity), its expected payoff can be expressed as $\mathrm{E}[U_J(X_L, X_J)] = -\mathrm{E}[U_L(X_L, X_J)]$ for all $X_L \in \mathcal{X}_L$ and $X_J \in \mathcal{X}_J$. Thus the problem under consideration can be modeled by means of a zero-sum game.

In summary, the investigated Bayesian zero-sum game $\mathcal{G}$ can be characterized as

$$\mathcal{G} = \{\mathcal{P}, \mathcal{T}, \theta, \mathcal{A}, \mathcal{U}\}, \tag{11}$$

where the parameters are elaborated as follows:

- Player set $\mathcal{P} = \{L, J\}$ consists of two players, namely player L: the all legitimate nodes and player J: the jamming node;
- Type sets $\mathcal{T} = \{\mathcal{T}_L, \mathcal{T}_J\}$, where the type of player L is determined by the frequency band allocation scheme, and the type of player J is determined by the distance from the jamming node to the anchor node;
- Probability set $\theta = \{\theta_L, \theta_J\}$, where θ_L and θ_J are the prior probability distributions of the types on $\mathcal{T}_L$ and $\mathcal{T}_J$ assigning probabilities $p_L(t_L)$, $t_L \in \mathcal{T}_L$, and $p_J(t_J)$, $t_J \in \mathcal{T}_J$ for players L and J, respectively;
- Action sets $\mathcal{A} = \{\mathcal{A}_L, \mathcal{A}_J\}$, where $\mathcal{A}_L$ and $\mathcal{A}_J$ being the transmitting power allocations of the available frequency band B of players L and J, respectively;
- Utility functions $\mathcal{U} = \{\mathrm{E}[U_L], \mathrm{E}[U_J]\}$ where $\mathrm{E}[U_L]$ is determined by (10) and $\mathrm{E}[U_J] = -\mathrm{E}[U_L]$.

3.2 Bayesian Nash Equilibrium

With a goal to find a BNE, which is a saddle point in a zero-sum game, we note that the equilibrium may not exist in pure strategies. For this reason, we introduce a mixed strategy ξ_L of player L as a probability distribution over set $\mathcal{X}_L$ of its pure strategies, where $\xi_L(X_L)$ denotes the probability of choosing pure strategy $X_L \in \mathcal{X}_L$ with $\sum_{X_L \in \mathcal{X}_L} \xi_L(X_L) = 1$. Similarly, a mixed strategy ξ_J of player J is a probability distribution over set $\mathcal{X}_J$, where $\xi_J(X_J)$ stands for the probability of choosing pure strategy $X_J \in \mathcal{X}_J$ with $\sum_{X_J \in \mathcal{X}_J} \xi_J(X_J) = 1$. Let Ξ_L and Ξ_J denote the sets of mixed strategies of players L and J, respectively. Given two mixed strategies ξ_L and ξ_J of players L and J, the expected payoff of player L (with its goal to maximize the overall expected network capacity) is given by

$$\mathrm{E}[U_L(\xi_L, \xi_J)] = \sum_{X_L \in \mathcal{X}_L} \sum_{X_J \in \mathcal{X}_J} \xi_L(X_L)\xi_J(X_J)\mathrm{E}[U_L(X_L, X_J)], \tag{12}$$

and $\mathrm{E}[U_J(\xi_L, \xi_J)] = -\mathrm{E}[U_L(\xi_L, \xi_J)]$ for all $\xi_L \in \Xi_L$ and $\xi_J \in \Xi_J$. We call a pair (ξ_L^*, ξ_J^*) BNE, if $\mathrm{E}[U_L(\xi_L, \xi_J^*)] \leqslant \mathrm{E}[U_L(\xi_L^*, \xi_J^*)] \leqslant \mathrm{E}[U_L(\xi_L^*, \xi_J)]$ for any $\xi_L \in \Xi_L$ and $\xi_J \in \Xi_J$. The expected payoff $\mathrm{E}[U_L(\xi_L^*, \xi_J^*)]$ for BNE (ξ_L^*, ξ_J^*) is called the value of the game, which we denote by v.

BNE of the zero-sum game can be found with Minmax Theorem, which is closely related to the linear programming. According to Minmax Theorem, there exists at least one Nash equilibrium and all equilibria yield the same payoff for each player [25]. The mixed strategy under BNE ensures that the value v is maximized in the worst case due to the strategy played by the opponent [26]. This is mathematically expressed as

$$\begin{aligned} v &= \max_{\xi_L \in \Xi_L} \underbrace{\min_{X_J \in \mathcal{X}_J} \sum_{X_L \in \mathcal{X}_L} \xi_L(X_L)\mathrm{E}[U_L(X_L, X_J)]}_{v_L} \\ &= \min_{\xi_J \in \Xi_J} \underbrace{\max_{X_L \in \mathcal{X}_L} \sum_{X_J \in \mathcal{X}_J} \xi_J(X_J)\mathrm{E}[U_L(X_L, X_J)]}_{v_J}. \end{aligned} \tag{13}$$

The above optimization can be further reformulated as the following dual linear programs:

$$\max_{\xi_L \in \Xi_L,\, v_L} \quad v_L \tag{14}$$

$$\text{subject to} \begin{cases} v_L \leqslant \sum\limits_{X_L \in \mathcal{X}_L} \xi_L(X_L)\mathrm{E}[U_L(X_L, X_J)], \ \ \forall X_J \in \mathcal{X}_J, \\ \sum\limits_{X_L \in \mathcal{X}_L} \xi_L(X_L) = 1, \\ \xi_L(X_L) \geqslant 0, \ \ \forall X_L \in \mathcal{X}_L, \end{cases}$$

and

$$\min_{\xi_J \in \Xi_J,\, v_J} \quad v_J \tag{15}$$

$$\text{subject to} \begin{cases} v_J \geqslant \sum\limits_{X_J \in \mathcal{X}_J} \xi_J(X_J) \mathrm{E}[U_L(X_L, X_J)], \ \ \forall X_L \in \mathcal{X}_L, \\ \sum\limits_{X_J \in \mathcal{X}_J} \xi_J(X_J) = 1, \\ \xi_J(X_J) \geqslant 0, \ \ \forall X_J \in \mathcal{X}_J. \end{cases}$$

Let (ξ_L^*, v_L^*) and (ξ_J^*, v_J^*) represent the optimal solutions to the above linear programs (14)-(15). From Minmax Theorem it follows that pair (ξ_L^*, ξ_J^*) is a BNE in mixed strategies and $v = v_L^* = v_J^*$ is the value of the game. The optimal solutions of the linear programs can be readily obtained using Matlab command '`linprog()`' [27].

3.3 Case of a Non-Zero-Sum Game

In the previous subsection we formulated and examined the problem of jamming attacks in the PLC network when legitimate nodes and the jamming node have opposite goals. However in some cases this approach seems less practical: for example, players do not necessarily aim at maximizing (minimizing) the overall network capacity. Below we propose an extension of the formulated problem to a case of a non-zero-sum game. Let as previously player L transmit the signal over the distance D_L at the frequency band B with the transmit power P_L whereas player J being at the distance D_J away from the receiver transmit noise with the transmit power P_J over the concerned frequency band. To be as close to the previous model as possible and at the same time extending it in line with [28], we define the payoffs of players L and J as $C_B(P_L, D_L; P_J, D_J)$ and $-(1-\varrho) \cdot C_B(P_L, D_L; P_J, D_J) - \varrho \cdot F$, respectively, where the newly introduced parameters will be described followingly.

From the definitions of players' payoffs, we observe that player L still aims at maximizing its network capacity when transmitting the signal over the distance D_L at the frequency band B with power level P_L under the presence of the jammer (alternatively, player L maximizes its profit from the transmission of a signal receiving one unit of utility for providing one unit of capacity). On the other hand, player J minimizes his expected losses assuming he can be tracked when transmitting the noise signal over the frequency band B with a given constant probability ϱ and then fined a constant penalty $F > 0$. In practice, the penalty might be a fine to the jammer by the utility company after finding the jamming actions (with probability ϱ). Thus the goal of player J is to minimize the expected losses when transmitting the noise at power level P_J being at the distance D_J away from player L. Note that players L and J have completely opposite goals when the jammer can never be tracked, i.e., when the probability $\varrho = 0$. In this case the game becomes zero sum.

It is worth mentioning that players' behavior patterns remain unchanged: a strategy of player L, X_L, is a power allocation A_L among selected subchannels based on an assignment profile t_L realized with probability $p_L(t_L)$, while a strategy of player J, X_J, is a power allocation A_J among all subchannels within the available frequency band based on a distance to the anchor node t_J selected with probability $p_J(t_J)$. Given a strategy profile (X_L, X_J), we represent the expected payoffs of players. Since the goal of player L is still in the maximization of the expected network capacity, the expected payoff of L will have the form of (10), but the expected payoff of player J is given by

$$\begin{aligned} \mathrm{E}[U_J(X_L, X_J)] = &- \sum_{t_L \in \mathcal{T}_L} \sum_{t_J \in \mathcal{T}_J} p_L(t_L) \cdot p_J(t_J) \\ &\times \left((1-\varrho) \sum_{\ell=1}^{m} C_{B(\ell)}(P_{L,\ell}, D_\ell; P_{J,B(\ell)}, D_J) + \varrho m F \right). \end{aligned} \tag{16}$$

The formulated game is not zero sum and it can be characterized by the same components as in (11) with the only difference that players' utility functions $\mathcal{U} = \{\mathrm{E}[U_L], \mathrm{E}[U_J]\}$ represented by their expected payoffs in the PLC network are determined by (10) and (16), respectively. Similarly, introducing mixed strategies ξ_L for player L and ξ_J for player J, we can write the expected payoffs of players as follows

$$\mathrm{E}[U_L(\xi_L, \xi_J)] = \sum_{X_L \in \mathcal{X}_L} \sum_{X_J \in \mathcal{X}_J} \xi_L(X_L) \xi_J(X_J) \mathrm{E}[U_L(X_L, X_J)], \tag{17}$$

$$\mathrm{E}[U_J(\xi_L, \xi_J)] = \sum_{X_L \in \mathcal{X}_L} \sum_{X_J \in \mathcal{X}_J} \xi_L(X_L) \xi_J(X_J) \mathrm{E}[U_J(X_L, X_J)], \tag{18}$$

where $\xi_L(X_L)$ and $\xi_J(X_J)$ stand for the probabilities of choosing pure strategies $X_L \in \mathcal{X}_L$ and $X_J \in \mathcal{X}_J$, respectively, with $\sum_{X_L \in \mathcal{X}_L} \xi_L(X_L) = 1$ and $\sum_{X_J \in \mathcal{X}_J} \xi_J(X_J) = 1$, whereas $\mathrm{E}[U_L(X_L, X_J)]$ and $\mathrm{E}[U_J(X_L, X_J)]$ are defined by (10) and (16). We call a pair (ξ_L^*, ξ_J^*) BNE in the non-zero-sum game if $\mathrm{E}[U_L(\xi_L, \xi_J^*)] \leqslant \mathrm{E}[U_L(\xi_L^*, \xi_J^*)]$ for any $\xi_L \in \Xi_L$ and at the same time the relationship $\mathrm{E}[U_J(\xi_L^*, \xi_J) \leqslant \mathrm{E}[U_J(\xi_L^*, \xi_J^*)]$ holds for any $\xi_J \in \Xi_J$. We denote the expected equilibrium payoffs $\mathrm{E}[U_L(\xi_L^*, \xi_J^*)]$ and $\mathrm{E}[U_J(\xi_L^*, \xi_J^*)]$ for BNE (ξ_L^*, ξ_J^*) by v_L^* and v_J^*, respectively.

It is well-known that the Nash theorem guarantees the existence of at least one BNE in the game [25]. Moreover from the theory of non-zero-sum games we conclude that BNE satisfies the conditions:

$$v_L^* = \max_{X_L \in \mathcal{X}_L} \sum_{X_J \in \mathcal{X}_J} \xi_J^*(X_J) \mathrm{E}[U_L(X_L, X_J)],$$

$$v_J^* = \max_{X_J \in \mathcal{X}_J} \sum_{X_L \in \mathcal{X}_L} \xi_L^*(X_L) \mathrm{E}[U_J(X_L, X_J)].$$

For a two-person games with finite sets of strategies, there has been developed a combinatorial algorithm for finding an equilibrium (so-called the Lemke–Howson algorithm [29]). The mixed BNE can be obtained using Matlab function '`LemkeHowson()`' [30].

4 Numerical Results

In this section, the analytical results derived in the previous sections are evaluated numerically with the use of Matlab. We adopt the PLC channel parameter values shown in Table 1, which are the experimental data from field measurements conducted in the industrial environments [31,32]. For simulation purpose, we investigate the simplest case where there are two nodes for player L and one jamming node for player J. Unless stated otherwise, the distances of the two nodes to the anchor node are $D_1 = 20\,\mathrm{m}, D_2 = 28\,\mathrm{m}$. The two frequencies used by PLC network are $B_1 = [10, 20]\,\mathrm{MHz}$ and $B_2 = [20, 30]\,\mathrm{MHz}$. The frequency bands are the types of player L, i.e., $\mathcal{T}_L = \{t_{L1}, t_{L2}\}$ where $t_{L1} = B_1$, $t_{L2} = B_2$, which are assigned with probabilities $p_L(t_{L1}) = 1/3$ and $p_L(t_{L2}) = 2/3$. There is no complete information on the position of the jammer except that it is located either 21 or 26 m away from the anchor node, thus these two distances are the types of player J and $\mathcal{T}_J = \{t_{J1}, t_{J2}\}$ where $t_{J1} = 21$ m, $t_{J2} = 26$ m. The probability distribution is $p_J(t_{J1}) = 3/7$ and $p_J(t_{J2}) = 4/7$.

It is known that $P_L = 16$ dBm/Hz and $P_J = 12$ dBm/Hz. The action spaces for players are as follows. For player L, $\mathcal{A}_L = \{A_{L1}, A_{L2}\}$ where $A_{L1} = (0.75P_L, 0.25P_L)$, $A_{L2} = (0.5P_L, 0.5P_L)$, and $\mathcal{A}_J = \{A_{J1}, A_{J2}\}$ where $A_{J1} = (0.25P_J, 0.75P_J)$, $A_{J2} = (0.75P_J, 0.25P_J)$, thus both players have two power allocation schemes. This implies both players' strategy sets consist of four pure strategies: $\mathcal{X}_L = \{X_{L1}, X_{L2}, X_{L3}, X_{L4}\}$ and $\mathcal{X}_J = \{X_{J1}, X_{J2}, X_{J3}, X_{J4}\}$. Players' strategies should be read as follows. The strategy X_{L1} dictates player L to choose A_{L1} if he is of type t_{L1} and A_{L1} if he is of type t_{L2}. The strategy X_{L2} prescribes him to choose A_{L1} if he is of type t_{L1} and A_{L2} if he is of type t_{L2}. When selecting X_{L3}, player L chooses A_{L2} if he is of type t_{L1} and A_{L1} if he is of type t_{L2}. And finally, when selecting X_{L4}, player L chooses A_{L2} if he is of type t_{L1} and A_{L2} if he is of type t_{L2}. Similarly for player J.

4.1 Results for the Case of Zero-Sum Game

By solving the linear programs (14) and (15), the zero-sum game admits the mixed Bayesian Nash equilibrium which is given by $\xi_L^* = (0.0038, 0.6224, 0, 0.3738)$, $\xi_J^* = (0.0424, 0.9576, 0, 0)$, that is, player L with probability 0.0038 plays X_{L1}, with probability 0.6224 plays X_{L2}, and with probability 0.3738 plays X_{L4} whereas player J with probability 0.0424 chooses X_{J1} and with probability 0.9576

Table 1: PLC Channel Parameters

attenuation model parameters		
$\alpha_1 = 9.33 \times 10^{-3}$ m^{-1}	$\alpha_2 = 5.1 \times 10^{-3}$ s/m	$k = 0.7$
noise model parameters (industrial environment)		
$\beta_1 = -123$	$\beta_2 = 40$	$\beta_3 = 8.6$

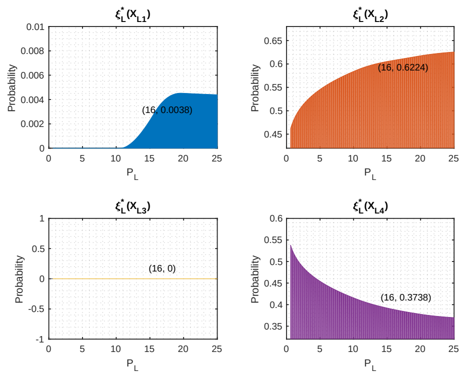

Fig. 2: Components of the equilibrium strategy ξ_L^* as a function of P_L with fixed value of $P_J = 12$ dBm/Hz.

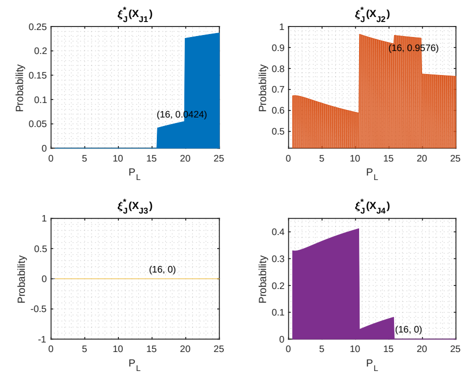

Fig. 3: Components of the equilibrium strategy ξ_J^* as a function of P_L with fixed value of $P_J = 12$ dBm/Hz.

chooses X_{J2}. The value of the game $v^* = 1.21912$. Figures 2 and 3 show the equilibrium mixed strategies ξ_L^* and ξ_J^* for both players as function of P_L with the fixed value of $P_J = 12$ dBm/Hz. We note that under the considered range of values of P_L, player L (J) never uses his strategy $X_{L3}(X_{J3})$ in any equilibrium, while he starts using strategy $X_{L1}(X_{J1})$ when P_L exceeds some threshold. At the same time, when P_L exceeds this threshold, player J stops using strategy X_{J4} in any equilibrium. Similar figures can be provided for ξ_L^* and ξ_J^* as functions of P_J with fixed value of P_L.

Figure 4 shows the PLC system capacity at the Bayesian Nash equilibrium (the value of the zero-sum game v^*) as a function of the PSDs P_L and P_J. It is clear that in the equilibrium, the system capacity is proportional to the power of the legitimate nodes and inversely proportional to the power from the jamming node. In order to compare the system capacity resulting from different strategies for both players, we investigate the special case of two legitimate nodes for player L and one jamming node for player J. In this scenario, the three-dimensional Fig. 5 suffices to illustrate all the strategies of both players as well as the corresponding payoff. The PSDs for the legitimate nodes and the jamming node are set as $P_L = 16$ dBm/Hz and $P_J = 12$ dBm/Hz, respectively. It can be seen from Fig. 5 that the system capacity for a fixed strategy of player L is a convex function of $P_{J,1}$ while the capacity becomes a concave function of $P_{L,1}$ with fixed $P_{J,1}$. We can see that different strategies from both players lead to quite different system performances. However, there is a saddle point, which the legitimate nodes and the jammer both have no incentive to deviate. This saddle point or the Nash equilibrium is achieved while $P_{L,1} = 6$ dBm/Hz and $P_{J,1} = 12$ dBm/Hz .

Figure 6 demonstrates the relationship between the value of the game (Nash equilibrium payoff), the maxmin and minmax payoffs and the available power to player L, P_L. The maxmin payoff is simply the best payoff for player L when

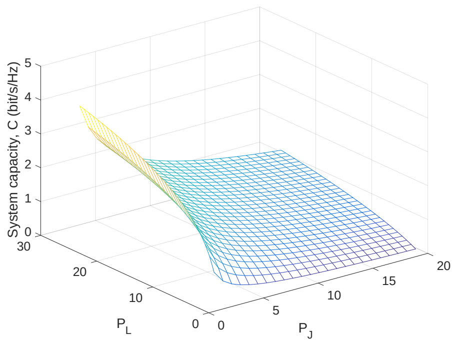

Fig. 4: The zero-sum game payoff at Bayesian Nash equilibrium (the value of the zero-sum game) as a function of the PSDs P_L and P_J.

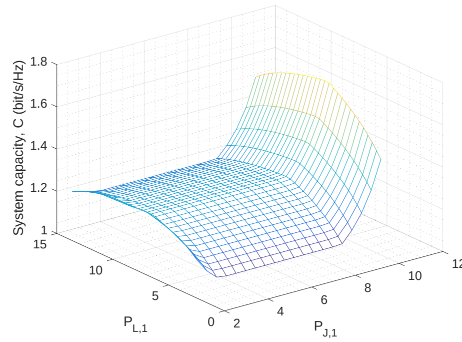

Fig. 5: The system capacity in an equilibrium in the zero-sum game for different strategies of both players as a function of $P_{L,1}$ and $P_{J,1}$ with fixed $P_L = 16$ dBm/Hz and $P_J = 12$ dBm/Hz.

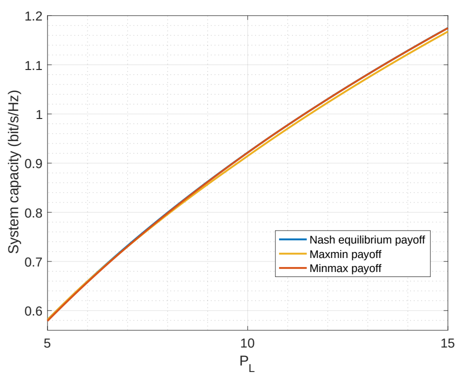

Fig. 6: The Bayesian Nash equilibrium payoff for player L as a function of P_L with the fixed value of $P_J = 12$ dBm/Hz.

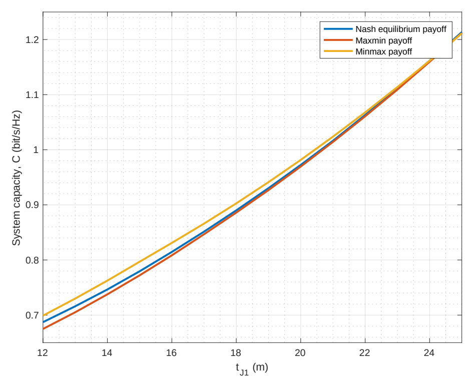

Fig. 7: The Bayesian Nash equilibrium payoff for player L as a function of the type t_{J1} with the other type t_{J2} fixed.

player J plays the most hostile strategy while the minmax payoff is player L's worst payoff when player J plays the least harmful strategy for player L. Clearly, the Nash equilibrium payoff is bounded by the maxmin and minmax payoffs. However, the three payoffs converge while the power available to L is larger than the threshold, which indicates that player L behaves, in this scenario, almost independently of player J's strategy, and vice versa. The similar pattern is presented in Fig. 7, where the relationship is shown between the value of the game (Nash equilibrium payoff), the maxmin and minmax payoffs and the one selected distance (type) of player J, t_{J1}, with the other distance t_{J2} being fixed.

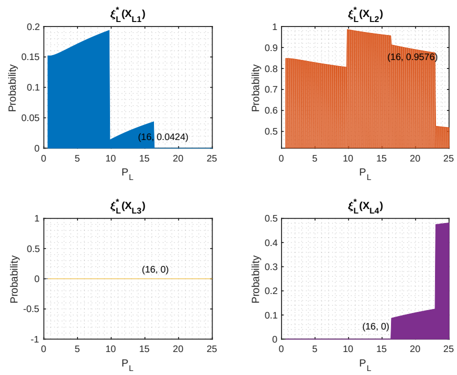

Fig. 8: Components of the equilibrium strategy ξ_L^* as a function of P_L with fixed value of $P_J = 12$ dBm/Hz.

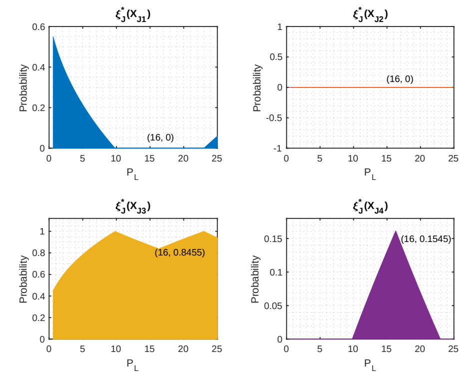

Fig. 9: Components of the equilibrium strategy ξ_J^* as a function of P_L with fixed value of $P_J = 12$ dBm/Hz.

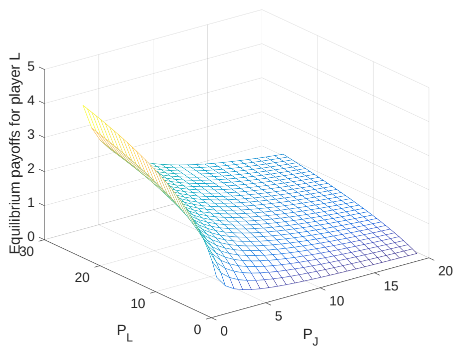

Fig. 10: The equilibrium payoff v_L^* as a function of the PSDs P_L and P_J.

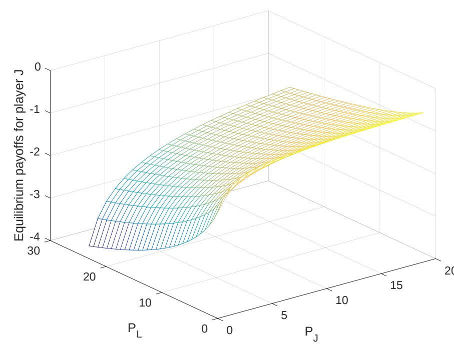

Fig. 11: The equilibrium payoff v_J^* as a function of the PSDs P_L and P_J.

4.2 Results for the Case of Non-Zero-Sum Game

In the case of the non-zero-sum game we additionally assume that the probability of tracking the jammer equals $\varrho = 0.2$ and the fine $F = 50$. In the non-zero-sum game the mixed Bayesian Nash equilibrium is given using the Lemke-Howson algorithm as follows: $\xi_L^* = (0.0424, 0.9576, 0, 0)$, $\xi_J^* = (0, 0, 0.8455, 0.1545)$. Under this equilibrium profile, player L with probability 0.0424 plays X_{L1} and with probability 0.9576 plays X_{L2} whereas player J with probability 0.8455 chooses X_{J3} and with probability 0.1545 chooses X_{J4}. The equilibrium payoffs are: $v_L^* = 1.21141$ and $v_J^* = -1.46913$. Figures 8 and 9 show the equilibrium mixed strategies for both players as a function of P_L with fixed value of P_J.

Figures 10 and 11 show the PLC system capacity v_L^* and jammer's losses v_J^* at the Bayesian Nash equilibrium as a functions of the PSDs P_L and P_J. Again in the equilibrium, the system capacity is proportional to the power of the legitimate nodes and inversely proportional to the power from the jamming

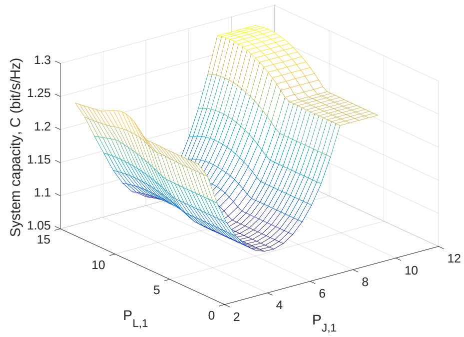

Fig. 12: The equilibrium payoff for player L as a function of $P_{L,1}$ and $P_{J,1}$ with fixed values of $P_L = 16$ dBm/Hz and $P_J = 12$ dBm/Hz.

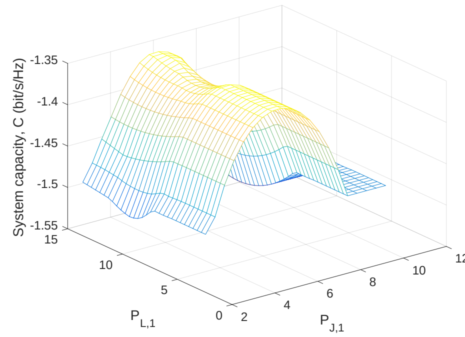

Fig. 13: The equilibrium payoff for player J as a function of $P_{L,1}$ and $P_{J,1}$ with fixed values of $P_L = 16$ dBm/Hz and $P_J = 12$ dBm/Hz.

node. For the jamming node, the higher the power of the legitimate nodes, the higher losses of J and the higher the power of J, the less losses it sustains in the system.

Figures 12 and 13 show the equilibrium payoffs as functions of $P_{L,1}$ and $P_{J,1}$ for players L and J, respectively. Here we observe a different pattern of the equilibrium payoff of L comparing with that in Fig. 5 (we recall that the PLC system capacity is the payoff of L). We see the intervals for $P_{L,1}$ where the equilibrium payoff of L can be a convex function for a fixed $P_{J,1}$, whereas in case of the zero-sum game it is a concave function of $P_{L,1}$. There are also intervals for $P_{J,1}$ where the equilibrium payoff of L is a concave function for a fixed $P_{L,1}$, whereas in Fig. 5 it is a convex function of $P_{J,1}$. Similar conclusion can be made also from Fig. 13 where the equilibrium payoff of player J is demonstrated, recalling that in the zero-sum game the payoff of J differs of L only in sign.

5 Conclusion

In this paper, we formulate the performance of a PLC network with the presence of jamming attacks into a Bayesian game. It was assumed that both players of the game have imperfect knowledge of the opponents, namely the spectrum allocation scheme for the legitimate nodes and the distance of the jamming node to the anchor node. Under some assumptions, we derived the Bayesian Nash equilibrium of the game. We further studied the effects of total power available to both players on the equilibrium. It is found that the equilibrium is unique in many setups, where the jamming node adopts a strategy following which it does not attack the subchannels used by legitimate nodes with specific power allocation. This allows the PLC network to choose the allocation schemes to its advantages in some cases.

It should be noted that the present model can be extended to the case when players have asymmetric information about types: when one player knows his own type but does not observe the type of his opponent what seems to be more practical in most cases. We leave this for future research.

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