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Research Paper

The Nordic/Baltic spot electric power system price: univariate nonlinear impulse-response analysis

Per B. Solibakke

Faculty of Economics, Norwegian University of Science and Technology, 7491 Trondheim, Norway; email: per.b.solibakke@ntnu.no

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ABSTRACT

This paper revisits the conditional mean and volatility density characteristics of the system price settled by the Nordic/Baltic spot electric power market (1993–2017). The main aim of this paper is an analysis of the nonlinear impulse-response features (shocks) in the nonstorable commodity market. Initially, we extract all deterministic seasonality and nonstationary trend and scale features from the series. A strictly stationary model reports serial correlation for the mean, and clustering, asymmetry and level effects for the volatility. For the mean, the impulse-response analysis reports linear and symmetric mean reversion for any price movements. For the volatility, small price movements show symmetric and decreasing volatility. In contrast, for larger absolute price movements, the volatility shows a nonlinear increase as well as fast-growing negative asymmetries. The impulse persistence is therefore relatively short. With the entrance of renewables into the energy market, the subperiod 2008–17 reports major systematic changes in the mean, volatility, asymmetry and persistence. In fact, the renewables era has changed the fundamental features of the Nordic/Baltic electricity market.

Keywords: Nordic/Baltic Spot electricity market; seasonality; conditional heteroscedasticity; asymmetry; renewables; impulse-response functions.

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1 INTRODUCTION

This paper studies the characteristics of the conditional mean and volatility of daily price movements of the system price for the Nordic/Baltic one-day-ahead spot electric power market. The spot market is a Nordic contract market where electric power trades in a daily auction-like market with physical delivery the following day, with full obligation to pay. The price settlement is based on all participants' collected purchase and sale requests. The "system" price is the balance price for the aggregated supply and demand graphs, ie, the market equilibrium.[1] Factors such as the weather or power plants not producing to their full capacity (eg, under maintenance) may affect prices. This study elaborates and extends earlier studies of the dynamics of the Nordic/Baltic electricity market prices (Solibakke 2002) to nonlinear dynamic structures.

The nonstorable energy market is highly volatile and is sensitive to several covariates (wind, consumption, perturbation, power plant maintenance, etc). This analysis focuses on price movements and impulses from the covariates. It therefore has three objectives. First, the univariate price movements are modeled using a semi-nonparametric methodology. For stationarity, we extract deterministic seasonality, trend and scale from the series. From the univariate model, we report fit characteristics and mean and volatility features. Second, we perform an impulse-response analysis, with particular focus on multi-step-ahead mean and volatility characteristics. The main objective here is potential opportunities for market participants from changing market features and dynamic strategic market positions. Third, we highlight the subperiod 2008–17 for analysis of market influence from renewables in the Nordic/Baltic electricity market. Here, the main questions are the effect of the introduction of renewables and potential new dynamic opportunities for market participants.

The methodology is the semi-nonparametric (SNP) time series analysis introduced by Gallant and Nychka (1987) and Gallant and Tauchen (1992, 2014). The method employs an expansion in Hermite functions to approximate the conditional density of the time series processes. The leading term of the model expansion process is therefore an established parametric model known to give a reasonable approximation of the process; higher-order terms (Hermite functions) capture departures from the model (Robinson 1983). The SNP model is fitted using conventional maximum likelihood together with a model selection strategy (the Bayesian information criterion (BIC))

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[1] The Nordic/Baltic auction market establishes prices from supply and demand for twenty-four hours. This analysis studies the average twenty-four-hour prices, and therefore the daily prices. Alternatively, the analysis could focus on one hour (eg, 08:00) or a block of hours (eg, 16:00–20:00) (Nord Pool 1998).

that determines the appropriate order of expansion (Schwarz 1978). The model is well designed for the computation of the nonlinear functionals of the densities.[2]

The remainder of this paper is organized as follows. In Section 2, we describe the impulse-response functionals and establish a bootstrapping technique for statistical significance. Section 3 gives a literature review of the Nordic/Baltic electricity price series (ELSPOT) and the SNP methodology. In Section 4, we extract seasonality, and scale and trend effects from the price series to obtain an ergodic and stationarity time series analysis. From the adjusted price, the SNP specification can be used to find consistent mean and volatility equation specifications. The Hermite function expansion extends the model approximation for the conditional density, which completely summarizes the probability distribution and completely characterizes the price movement process.[3] We report specifications and key properties. Section 5 applies the impulse-response methodology put forward in Sims (1980) and refined by Doan *et al* (1984) and others.[4] The dynamic properties of the nonlinear model are elicited in Section 5.2 by perturbing the vector of conditioning arguments in the conditional density function (Gallant *et al* 1993; Gallant and Tauchen 2010, 2014). To study the effects of introducing renewables into energy markets, in Section 5.3 we perform an analysis on the subperiod 2008–17. Section 6 summarizes the paper and presents our conclusions.

2 IMPULSE-RESPONSE FUNCTIONALS

We apply the methodologies outlined by Gallant *et al* (1993, 2014) to define step-ahead forecasts for the mean conditioned on the history as $g(y_{t-\infty+1}, \dots, y_t) = \mathbb{E}(y_{t+1} \mid (y_{t-k})_{k=0}^{\infty})$ in general and $g(y_{t-L+1}, \dots, y_t) = \mathbb{E}(y_{t+1} \mid (y_{t-k})_{k=0}^{L-1})$ for a Markovian process, where L is the number of lags. We set

Gallant *et al* (2014) does not correspond to any bibliography entry: do you mean Gallant and Tauchen (2014)?

$$
\begin{aligned}
\hat{y}_j(x) &= \mathbb{E}(g(y_{t-L+j}, \dots, y_{t+j}) \mid x_t = x) \\
&= \mathbb{E}(\mathbb{E}(y_{t+j} \mid y_{t-L+j}, \dots, y_{t+j}) \mid x_t = x),
\end{aligned}
$$

and therefore $\hat{y}_j^i$, for impulse ranges $i = -60\%, \dots, 60\%$ and for five steps (days) ahead $j = 0, \dots, 5$, where $x = (y_{-L+1}, \dots, y_0)$ and, as above, L represents the number of lags in the Markovian process. The conditional mean profiles $\{\hat{y}_j^i\}_{j=1}^{\infty}$ for $i = -60\%, \dots, 60\%$ are the conditional expectations of the trajectories of the one-step conditional mean. Note that $\{\hat{y}_j^{-10\%}\}_{j=1}^{\infty}$ therefore represents the mean response to a negative 10% impulse (error shock). The responses depend on the initial x,

[2] The computer cluster at Norwegian University of Science and Technology (NTNU) was used for estimation/implementation.

[3] The conditional density is a complicated nonlinear function of a large number of arguments.

[4] The impulse-response methodology is also known as error shock methodology.

which reflects the nonlinearity. Moreover, the law of iterated expectations implies that $\hat{y}_j(x) = \mathbb{E}(y_{t+j} \mid x_t = x)$. The sequences $\{\hat{y}_j^i - \hat{y}_j^0\}_{j=1}^{\infty}$ for $i = -60\%, \dots, 60\%$ represent the effects of the shocks on the trajectories of the process itself. A conditional moment profile can now be defined as $\mathbb{E}[g(y_{t+j-J}, \dots, y_{t+j}) \mid \{y_{t-k}\}_{k=0}^{L=1}]$, $j = 0, \dots, 5$, where "moment" refers to the time-invariant function $g(y_{-J}, \dots, y_0)$.

Similarly, the one-step-ahead variance, also called the volatility, is the one-step-ahead forecast of the variance conditional on the history becoming

$$\begin{aligned}&\operatorname{var}(y_{t+1} \mid (y_{t-k})_{k=0}^{\infty}) \\ &\quad = \mathbb{E}\{[y_{t+1} - \mathbb{E}(y_{t+1}\{y_{t-k}\}_{k=0}^{\infty})]x[y_{t+1} - \mathbb{E}(y_{t+1} \mid \{y_{t-k}\}_{k=0}^{\infty})]' \mid \{y_{t-k}\}_{k=0}^{\infty}\}\end{aligned}$$

or $\operatorname{var}(y_{t+1} \mid (y_{t-k})_{k=0}^{L-1})$ for a Markovian process ($L \ll \infty$). By suitably defining the function $g(\cdot)$, we can measure the effect of impulses on volatility. Now, we can write

$$\begin{aligned}\hat{\Psi}_j(x) &= \mathbb{E}(g(y_{t-L+j}, \dots, y_{t+j}) \mid x_t = x) \\ &= \mathbb{E}(\operatorname{var}(y_{t+j} \mid x_{t+j-1}) \mid x_t = x)\end{aligned}$$

for $j = 0, \dots, 5$, where $x = (y_{-L+1}, \dots, y_0)$. $\hat{\Psi}_j(x)$ is the conditional expectation of the trajectories of the step-ahead conditional variance matrix j, conditional on $x_t = x$. Therefore, as for the conditional mean, the $\{\hat{\Psi}_j^{-10\%}\}_{j=1}^{\infty}$ represents the volatility response from a negative 10% impulse (shock). The net effects of perturbations ∂y^i on volatility are assessed by plotting the profiles compared with the baseline $\{\hat{\Psi}_j^i - \hat{\Psi}_j^0\}_{j=1}^{\infty}$ for $i = -60\%, \dots, 60\%$. Note that the conditional volatility profile defined above is different from the path described by the j-step-ahead square-error process. Analytical evaluation of the integrals in the definition of the conditional moment profiles is difficult. However, it is well suited to Monte Carlo integration. Let $\{y_j^r\}_{j=1}^{\infty}$, $r = 1, \dots, R$, be R simulated realizations of the process starting from $x_0 = x$. That is, y_1^r is a random draw from $f(y \mid x)$ with $x = (y'_{-L+1}, \dots, y'_{-1}, y'_0)'$, y_2^r is a random draw from $f(y \mid x)$ with $x = (y'_{-L+2}, \dots, y'_0, y'_1)'$, and so on. Now, applying the invariant function of a stretch of $\{y_j\}$ with length j, we obtain

$$\begin{aligned}\hat{g}_j(x) &= \int \cdots \int g(y_{j-J}, \dots, y_j)\left[\prod_{i=0}^{j-1} f(y_{i+1} \mid y_{y-L+1}, \dots, y_i)\right] \mathrm{d}y_1 \cdots \mathrm{d}y_j \\ &\doteq \left(\frac{1}{R}\right) \sum_{r=1}^{R} g(y_{j-J}^r, \dots, y_j^r),\end{aligned}$$

with the approximation error tending to zero almost surely as $R \to \infty$, under mild regulatory conditions on f and g. For statistical inference, sup–norm bands are constructed by bootstrapping, using simulations to consider the sampling variation in the estimation of $\hat{f}(y \mid x)$, ie, changing the seed that generates densities and the basis for impulse-response analyses. The paper applies 500 samples and 95% confidence intervals. A 95% sup–norm confidence band is an ε-band around the profile $\hat{f}(y \mid x)$ that is just wide enough to contain 95% of the simulated profiles.

3 LITERATURE REVIEW

3.1 The electricity system price for the Nordic spot electricity market

Several international studies (see, for example, Kristiansen 2014; Goto and Karolyi 2004; Bystrøm 2003; Solibakke 2002) have explored the characteristics and dynamics of Nordic/Baltic spot electricity price (auction market) series. Financial models use historical price series, and assuming stationarity, we can extract reliable characteristics for both the mean and volatility. Spot electricity prices exhibit high volatility, strong mean reversion (see, for example, Lucia and Schwartz 2002; Geman and Roncoroni 2006), frequent spikes and seasonal patterns (see Higgs and Worthington 2008; Huisman and Mahieu 2003; Thomas *et al* 2011) and differ from region to region (Li and Flynn 2004). Moreover, Goto and Karolyi (2004) find a mean-reversion effect with seasonal changes in volatilities as well as volatility clustering for electricity trading hubs in the United States, Australia and the Nordic/Baltic market. Chan and Gray (2006) find serial correlation in both the mean and volatility for several electricity markets. Theodorou and Karyampas (2008) study the less developed and illiquid Greek electricity market and find mean reversion and the presence of serial correlation in both the mean and the volatility.

A considerable number of models have been proposed in the literature to attempt to capture the dynamics of electricity prices. One class of models includes stochastic models, regime-switching models, cointegration analysis, mean-reverting models and other empirical models (see De Vany and Walls 1999; Higgs and Worthington 2008; Huisman and Mahieu 2003; Huisman and Kilic 2013; Haldrup and Nilsen 2006; Knittel and Roberts 2005; Li and Flynn 2004; Lindstrøm and Regland 2012; Mount *et al* 2006; Robinson 2000; Robinson and Baniak 2002; Rubin and Babcock 2011; Tashpulatov 2013; Weron 2006, 2008). These models fail to capture the full volatility dynamics of electricity prices as well as the price and volatility interrelationships. Another class of models introduces univariate generalized autoregressive conditional heteroscedasticity (GARCH) conditional volatility models, as well as other variations of GARCH modeling, such as exponential (EGARCH) and threshold (TGARCH) (see Chan and Gray 2006; Escribano *et al* 2011; Habell *et al* 2004; Higgs and Worthington 2005; Koopman *et al* 2007; Solibakke 2002). These models capture the price and volatility dynamics of electricity prices as well as price shock transmissions. Finally, Knittel and Roberts (2005) find an inverse leverage effect for electricity prices in the United States. Other studies have found similar results (see, for example, Weron 2006, 2008; Harris 2006; Geman and Roncoroni 2006; Koopman *et al* 2007; Pilipović 2007; Sotiriadis *et al* 2016).

For the purpose of this paper, we build on and extend the work of Solibakke (2002) and follow the methodology of Gallant and Tauchen (1992, 2014). Seasonalities and trends are extracted, and a strictly stationary time series SNP model (Gallant and Tauchen 2010) is estimated. We perform a postestimation analysis for the nonlinear impulse-response methodology.

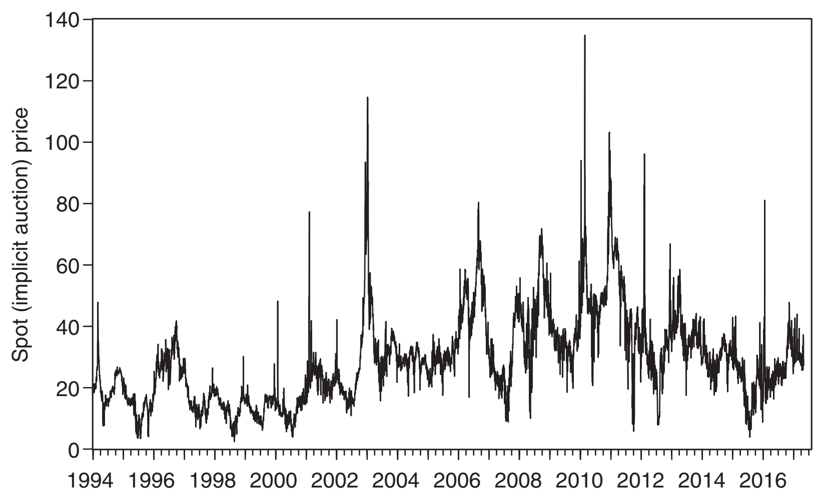
3.2 The semi-nonparametric model

Nonlinear stochastic models will, in our study, imply conditional models. Autoregressive moving average (ARMA) denotes terms applied to the structure of the conditional mean, while GARCH denotes terms applied to the structure of the conditional volatility. ARMA models can be studied in detail in, for example, Mills (1990), while ARCH specifications were first studied by Engle (1982), and extended by Bollerslev (1986), who specified the generalized ARCH (GARCH), primarily owing to the number of lags in the ARCH specification (Gallant and Tauchen (1998) found eighteen (!) ARCH lags for time series retrieved from the US financial market). ARCH/GARCH specifies the volatility as a function of historic price movements and volatility. In the international finance literature, several studies have demonstrated the use of results from these works (see, for example, Bollerslev *et al* 1992; Bollerslev 1987; Engle and Bollerslev 1986; Engle and Ng 1993; Nelson 1991; de Lima 1995a,b). For a comprehensive introduction to ARCH models and applications in finance see Gouriéroux (1997). Ding *et al* (1993) extend the symmetric GARCH model into asymmetric GARCH, and the truncated (GJR) GARCH is described by Glosten *et al* (1993).

The term semi-nonparametric was coined by Gallant and Nychka (1987) (see also Gallant and Tauchen 1992, 1998, 2014) to suggest that it lies halfway between parametric and nonparametric procedures.[5] The leading term of the series expansion is an established parametric model known to give a reasonable approximation of the process; higher-order terms capture departures from that model. With this structure, the SNP approach does not suffer from the curse of dimensionality to the same extent as kernels and splines. In regions where data is sparse, the leading term helps to fill in smoothly between data points. Where data is plentiful, the higher-order terms accommodate deviations from the leading term and fits are comparable with the kernel estimates proposed by Robinson (1983). The theoretical foundation of the method

[5] The SNP code and user guide are available at www.aronaldg.org/. The software is free and can be redistributed and/or modified under the terms of the GNU General Public License as published by the Free Software Foundation (either version 2 of the License or (optionally) any later version).

FIGURE 1 Nordic/Baltic system price series for the period December 1993–August 2017.



is the Hermite series expansion, which, for time series data, is particularly attractive based on both modeling and computational considerations. In terms of modeling, the Gaussian component of the Hermite expansion makes it easy to subsume into the leading term familiar time series models, including vector autoregression, ARCH and GARCH models (Engle 1982; Bollerslev 1986). These models are generally considered to give excellent first approximations in a wide variety of applications. In terms of computation, a Hermite density is easy to evaluate and differentiate. Also, its moments are easy to evaluate because they correspond to higher moments of the normal, which can be computed using standard recursions. Finally, a Hermite density turns out to be very practical to sample from, which facilitates simulation.

4 ENERGY MARKET DATA AND ADJUSTMENTS FOR STATIONARITY

The study uses daily prices from the Nordic/Baltic spot market (system price) for electric power spanning the period from December 1993 to August 2017 (twenty-four years, 8616 observations). The daily system prices are the average prices for twenty-four hours' auction one-day-ahead prices settled daily at 12:45 by the aggregate demand and supply interactions. Hence, there are twenty-four daily one-day-ahead

TABLE 1 Statistics for system price Nord Pool spot auction market 1993–2017.

(a) Unadjusted price series: returns

Mean (all)/ M (– drop)	Median SD	Maximum/ minimum	Moment kurt/skew	Quantile kurt/skew	Quantile normal	Cramer–von Mises	Serial dependence $Q(12)$	Serial dependence $Q^2(12)$	VaR (%) (1%; 2.5%)
0.00470	–0.56171	118.8913	15.09867	0.56667	118.9174	66.9275	2576.1	2012.80	–28.144
0.00359	10.57146	–104.3984	0.86285	0.05043	{0.0000}	{0.0000}	{0.0000}	{0.0000}	–42.414

BDS Z-statistic ($e = 1$) $m = 2$	BDS Z-statistic ($e = 1$) $m = 3$	BDS Z-statistic ($e = 1$) $m = 4$	BDS Z-statistic ($e = 1$) $m = 5$	KPSS (stationary) intercept	KPSS (stationary) I + trend	ADF test	ARCH (12)	RESET (12;6)	CVaR (1%; 2.5%)
28.0034	36.1362	39.8564	42.6974	0.01440	0.01219	–18.3889	1237.57	342.0436	–42.414
{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.6259}	{0.5219}	{0.0000}	{0.0000}	{0.0000}	–31.030

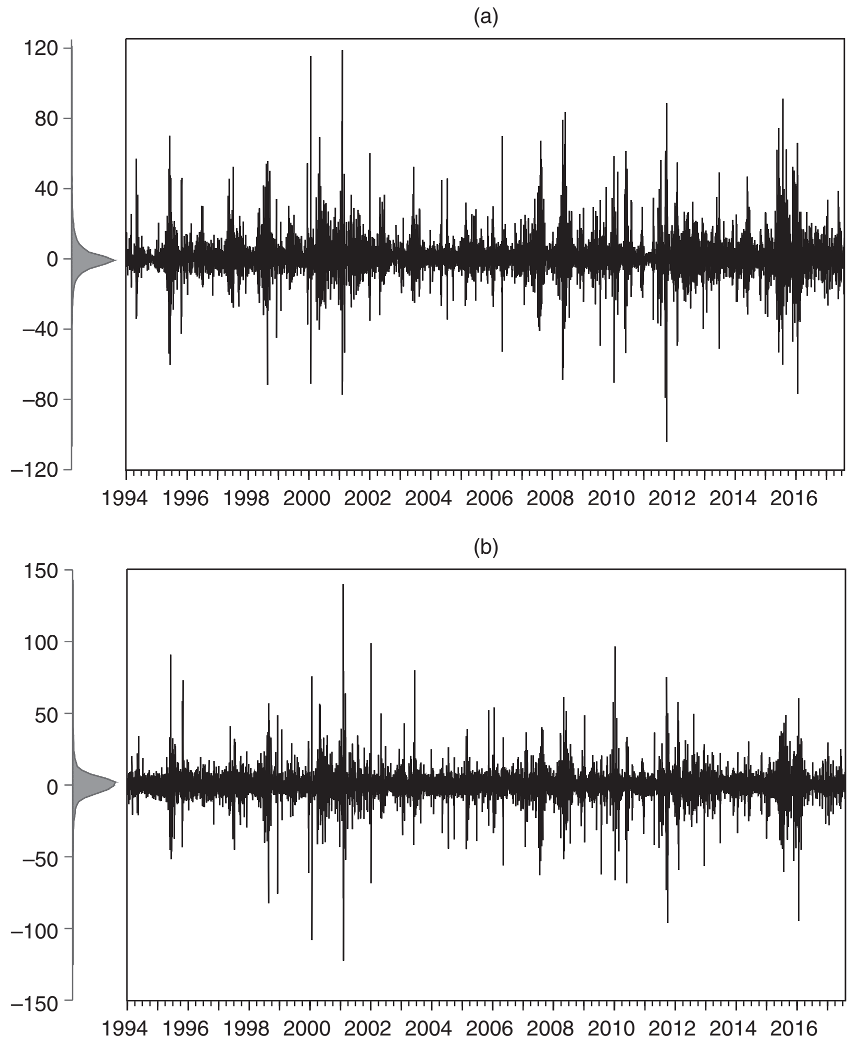
(b) Adjusted price series: returns

Mean (all)/ M (– drop)	Median SD	Maximum/ minimum	Moment kurt/skew	Quantile kurt/skew	Quantile normal	Cramer–von Mises	Serial dependence $Q(12)$	Serial dependence $Q^2(12)$	VaR (%) (1%; 2.5%)
0.00470	0.31526	140.2612	19.10913	0.11576	9.2619	49.1697	242.780	2132.50	–32.949
0.00313	10.57146	–122.5952	–0.23815	–0.05568	{0.0097}	{0.0000}	{0.0000}	{0.0000}	–21.428

BDS Z-statistic ($e = 1$) $m = 2$	BDS Z-statistic ($e = 1$) $m = 3$	BDS Z-statistic ($e = 1$) $m = 4$	BDS Z-statistic ($e = 1$) $m = 5$	KPSS (stationary) intercept	KPSS (stationary) I + trend	ADF test	ARCH (12)	RESET (12;6)	CVaR (1%; 2.5%)
34.2538	40.1002	43.1724	45.4459	0.15165	0.02995	–26.9585	139.041	42.32337	–49.343
{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.0846}	{0.1402}	{0.0000}	{0.0000}	{0.0000}	–35.134

SD denotes standard deviation. ADF denotes augmented Dickey–Fuller.

FIGURE 2 (a) Unadjusted and (b) seasonal adjusted price movement series for 1993–2017.



alternative time series candidates for the analysis.[6] Figure 1 plots the twenty-four-year daily average price series from the Nord Pool spot market. The log difference

[6] The data set is constructed by Nord Pool. Nord Pool provides liquid, efficient and secure day-ahead and intraday markets. It is the counterparty for all trades, guaranteeing settlement and delivery. See www.nordpoolspot.com.

of the unadjusted price series, P_i, is taken to create the price movement series, $100(\log P_i - \log P_{i-1})$. Many authors (see, for example, Solibakke 2002; Higgs and Worthington 2005) have noted systematic calendar (weekends, (moving) holidays and summer/winter), trend and scale effects in both the mean and variance of the system price movements. To adjust for these documented shifts in both the mean and volatility of the raw electricity price series, a two-stage adjustment procedure is carried out, in which systematic and deterministic effects are removed first from the mean and then from the variance (Gallant *et al* 1992). Let ϖ denote the raw electricity price movements to be adjusted by the procedure. Initially, the mean-regression equation $\varpi = x\beta + u$ is fitted, where x denotes the systematic (and deterministic) calendar variables that are most convenient for the time series, calendar day separation variables or other subperiods (moving holidays) and contains parameters for linear and squared trends. For the estimated least square residuals, $\hat{u}$, the variance equation model $\log(\hat{u}^2) = x\gamma + \varepsilon$ is estimated. The expression $z = \hat{u}/\mathrm{e}^{x\hat{\gamma}/2}$ standardizes the residuals for the mean, leaving a series with mean zero and (approximately) unit variance. Finally, the series $\hat{\varpi} = a + b(\hat{u}/\mathrm{e}^{x\hat{\gamma}/2})$ is taken as the adjusted series, where a and b are chosen so that

$$\frac{1}{T}\sum_{i=1}^{T}\hat{\varpi}_i = \frac{1}{T}\sum_{i=1}^{T}\varpi_i \quad \text{and} \quad \frac{1}{T-1}\sum_{i=1}^{T}(\hat{\varpi}_i - \bar{\varpi})^2 = \frac{1}{T-1}\sum_{i=1}^{T}(\hat{u}_i - \bar{u})^2.$$

The purpose of the above location and scale transformation is to aid interpretation (via the same unit measurement). This first-round analysis reports significant seasonal price patterns,[7] weekly price change patterns and negative patterns for the joint summer holidays.[8] For the second-round adjustments, the volatility analysis finds significant day and week effects as well as linear and squared trends. The greatest increase in volatility is on Mondays, followed by Saturdays, while from Tuesday to Friday volatility seems to calm down. The negative linear trend function from the analysis suggests a decreasing volatility, implying a maturing electricity market. However, the positive squared trend suggests an increasing volatility effect. This nonlinearity is more difficult to interpret, but a potential explanation could be the introduction of renewables and a higher variance in the overall electricity production after 2007/8. Based on the adjusted price movements, we propose that some random process can describe an observed realization of the random variables. A relatively simple model can establish the properties of these stochastic processes. For us, the relationship between observations corresponding to different periods is important, so that we can exploit the

[7] For example, day-of-week price patterns (Saturday, Sunday and Monday); price changes are strongly positive on Mondays and become strongly negative on Fridays, Saturdays and Sundays.
[8] For this paper, GAUSS (www.aptech.com) programs implement the adjustment procedures. Results are not reported due to space restrictions.

dynamic properties of the series to generate predictions for future periods. We thus impose weak stationarity, and the means, variances and covariances are independent of times (rather than the entire distribution). That is, a process $\{y_t\}$ is weakly stationary if, for all t, it holds that $\mathbb{E}\{y_t\} = \mu \leqslant \infty$, $V\{y_t\} = \mathbb{E}\{(y_t - \mu)^2\} = \gamma_0 < \infty$ and $\text{cov}\{y_t, y_{t-k}\} = \mathbb{E}\{(y_t - \mu)(y_{t-k} - \mu)\} = \gamma_k$, $k = 1, 2, 3, \dots$. A shock to a stationary autoregressive process of order 1 (AR(1)) affects all future observations, with diminishing impact.

Table 1 reports the characteristics of the unadjusted and adjusted price movement series. By construction, the means and standard deviations are equal for both series. The mean is positive and close to zero. The daily standard deviation of $\sigma = 10.6\%$ is higher than for other commodity markets. The nonstorable features of electricity, together with the increasing influence of renewables, may partly explain this high volatility. The daily maximum and minimum for the adjusted series (+140 and −123, respectively) are close to but slightly more widely spread than the unadjusted series. The kurtosis increased (19.1), while the skewness became negative (−0.24). The adjusted series is far from normally distributed, confirmed by a quantile normal of 9.3 and a Cramer–von Mises normal test statistic of 49.2.[9] The adjusted series show a strong decline in the twelfth lag autocorrelation function for both the ordinary series ($Q(12)$) and the squared series ($Q^2(12)$) (see Box and Jenkins 1976; Ljung and Box 1978), but both are still highly significant. Similarly, the twelfth lag ARCH test statistic (Engle 1982) of 1238 suggests highly significant conditional heteroscedasticity. Finally, for the adjusted series, the augmented Dickey–Fuller (ADF) and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) statistics (see Dickey and Fuller 1979; Kwiatkowski *et al* 1992) confirm stationarity, and the Brock–Dechert–Scheinkman (BDS) test statistic (Brock and Deckert 1988; Scheinkman 1990; Brock *et al* 1996) reports general nonlinear data dependence. Figure 2 shows the path and distribution for both the unadjusted price movements (part (a)) and the seasonal and trend-adjusted (part (b)) price movements. The general appearance of the adjusted series is typical for market data, indicating that the data is not overprocessed by the adjustments. We also experimented with breaking trends in the adjustment equations, but our results suggested little evidence for trend breaks. For electricity producers and retailers in the Nordic/Baltic electricity market, the value-at-risk (VaR) is a well-known concept, and this market position will thus be of interest. Table 1 therefore includes the 2.5% and 1% VaR values.

I think our previous query about the font used for ‘e’ in the tables was confusing – sorry about that. The font used currently is the correct one for a variable e in a table in this journal, so I hope it’s OK as it is throughout? Or perhaps you meant that every instance of ‘$e = 1$’ in a table should be ‘$\varepsilon = 1$’? The notes in your PDF markup were unclear.

[9] The Cramer–von Mises test statistic is a procedure to test the null hypothesis that a sample comes from a population in which the variable is distributed according to a normal distribution.

5 EMPIRICAL ANALYSIS AND FINDINGS

5.1 The SNP density projection

Since the conditional density completely characterizes the price movement process, the density is naturally viewed as the fundamental statistical object of interest. The SNP model is fitted using conventional maximum likelihood together with a model selection strategy that determines the appropriate order of expansion (BIC). The Schwarz BIC (Schwarz 1978) is computed as

$$\text{BIC} = s_n(\hat{\theta}) + \frac{1}{2}\left(\frac{p_p}{n}\right)\log(n),$$

with small values of the criterion preferred. Table 2 reports the maximum likelihood (ML) estimates[10] of the parameters of the BIC-optimal SNP model.[11] First, for the mean, the intercept is insignificant and the serial correlations ($B[1, x]$) are significant, implying dependence up to fourteen (days) lags (η_{13}–η_{26}). We find that, in particular, lags 7 (η_{19}) and 14 (η_{26}) (the number of days in one week and two weeks) show strong positive autocorrelation.[12] The negative correlation at lags 2–5 (η_{14}–η_{17}) may suggest mean reversion. The week structure in the serial correlation is therefore a natural candidate for the structure of the mean. Second, the conditional variance coefficients (η_{27}–η_{31}) are all strongly significant. Conditional heteroscedasticity is therefore clearly present (η_{27}–η_{29}). Moreover, asymmetry (η_{30}) and level effects (η_{31}) are present.

I'm afraid I didn't really understand your note about the 's' that appears in this table and Table 5. If it is a matrix, then journal style would be to have it as either sloping s or even bold sloping $\boldsymbol{s}$. Would one of those options be acceptable?

Please forgive our earlier mistake with these symbols: the conversion from your Word file to LaTeX did not work nearly as well as it usually does. Please check carefully that all necessary changes have been applied in this second proof.

Moreover, for the volatility, the R_0 coefficient is 0.23. This coefficient is quite high and suggests a constant and long lasting conditional volatility for the market. The P coefficient is 0.42 and reports the shock effects from the previous period (ARCH). The Q coefficient is 0.91 and reports the serial correlation in the conditional volatility (GARCH). The model (Baba–Engle–Kraft–Kroner formulation) reports a significant negative asymmetry (V) of -0.33 and a level effect (W) of 0.72. Together, the three parameters R, P and Q imply that the volatility process follows both a high-valued and very erratic path. The largest eigenvalue of the conditional variance function P and Q companion matrix is 1.004. However, due to the use of an additional transformation

[10] Based on likelihood ratio test (LRT) statistics, the Student t loglikelihood function is strongly preferred to a normal likelihood function.
[11] The BIC optimal SNP model is the $L_u = 14$, $L_g = 1$, $L_r = 1$, $L_v = 1$, $L_\omega = 1$, $L_p = 1$, $K_z = 12$, $K_x = 0$ specification.
[12] The serial correlation may stem from the strong day effects found in the adjustment procedure. Our procedure seems not to remove all systematic seasonal effects.

TABLE 2 Nord Pool spot electricity system price. [Table continues on next page.]

(a) Hermite polynomials

Mean equation				
Var	**SNP coefficient**	**Mode**	**Standard error**	***t*-statistics**
η_1	$a_0[1]$	−0.00169	0.00569	−0.2979
η_2	$a_0[2]$	−0.22774	0.00906	−25.14244
η_3	$a_0[3]$	−0.01378	0.00588	−2.3447
η_4	$a_0[4]$	0.17467	0.00717	24.36335
η_5	$a_0[5]$	0.00788	0.00709	1.11145
η_6	$a_0[6]$	−0.09744	0.00945	−10.31244
η_7	$a_0[7]$	−0.01437	0.0077	−1.86754
η_8	$a_0[8]$	0.03859	0.00794	4.85979
η_9	$a_0[9]$	0.02689	0.00895	3.00521
η_{10}	$a_0[10]$	−0.04425	0.00865	−5.11661
η_{11}	$a_0[11]$	0.01564	0.01041	1.50289
η_{12}	$a_0[12]$	0.05425	0.00909	5.96919

(b) Mean correlation

Mean equation				
Var	**SNP coefficient**	**Mode**	**Standard error**	***t*-statistics**
η_{13}	$B(1,1)$	0.02384	0.01223	1.94893
η_{14}	$B(1,2)$	−0.06324	0.01063	−5.95071
η_{15}	$B(1,3)$	−0.03656	0.01042	−3.50856
η_{16}	$B(1,4)$	−0.02595	0.01039	−2.49822
η_{17}	$B(1,5)$	−0.02262	0.01013	−2.23383
η_{18}	$B(1,6)$	0.02606	0.00996	2.61741
η_{19}	$B(1,7)$	0.12074	0.00950	12.70797
η_{20}	$B(1,8)$	0.011	0.00980	1.12274
η_{21}	$B(1,9)$	−0.02232	0.00943	−2.36758
η_{22}	$B(1,10)$	−0.01414	0.00949	−1.49101
η_{23}	$B(1,11)$	−0.01798	0.00921	−1.95234
η_{24}	$B(1,12)$	−0.03037	0.00902	−3.36581
η_{25}	$B(1,13)$	0.01196	0.00872	1.37125
η_{26}	$B(1,14)$	0.08075	0.00831	9.71789

TABLE 2 Continued.

(c) Variance equation

Mean equation: Var	Mean equation: SNP coefficient	Mode	Standard error	t-statistics
η_{27}	$R_0[1]$	0.23008	0.0138	16.67838
η_{28}	$P[1,1]$s	0.42443	0.02862	14.82863
η_{29}	$Q[1,1]$s	0.90751	0.00639	142.09674
η_{30}	$V[1,1]$s	−0.32635	0.04579	−7.12717
η_{31}	$W[1,1]$s	0.71942	0.04002	17.97437
Observations (inc. drops)		8615	s_n	1.0658296
Loglikelihood		−4182.122379	AIC	1.0694280
			BIC	1.0821326
Largest eigenvalue of mean function companion matrix:				0.893251
Largest eigenvalue of variance function P and Q companion m:				1.003710

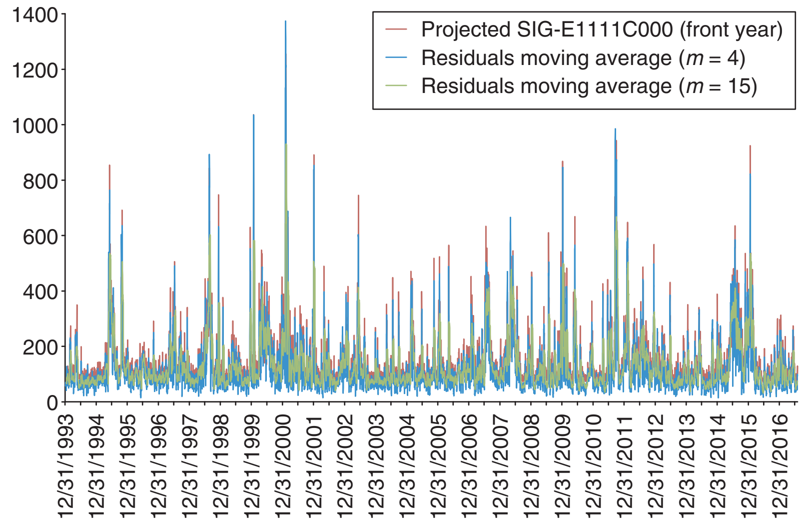
Statistical model SNP-14,111,12,000 -fit; semi-parametric-GARCH model.

(trigonometric spline),

$$\hat{x}_i = \begin{cases} \dfrac{1}{2}\left\{x_i + \dfrac{4}{\pi}\arctan\left[\dfrac{\pi}{4}(x_i + \sigma_{\text{tr}})\right] - \sigma_{\text{tr}}\right\}, & -\infty < x_i < -\sigma_{\text{tr}}, \\ x_i, & -\sigma_{\text{tr}} < x_i < \sigma_{\text{tr}}, \\ \dfrac{1}{2}\left\{x_i + \dfrac{4}{\pi}\arctan\left[\dfrac{\pi}{4}(x_i + \sigma_{\text{tr}})\right] - \sigma_{\text{tr}}\right\}, & \sigma_{\text{tr}} < x_i < \infty, \end{cases}$$

where x_i denotes an element of x_{t-1}, the dictum that the sum of the squared coefficients (squares) must be less than 1 (under the spline transform it suffices that the sum of the squares of the coefficients is less than 2) no longer holds. Finally, the Hermite functions coefficients (η_1–η_{12}), which capture parametric model departures, are BIC preferred up to the twelfth polynomial lag expansions. Hence, the Hermite result clearly suggests departures from the classical normally distributed and parametric conditional model. From these specifications, we show in Figure 3 a graphical representation of the conditional variance together with a calculation of moving averages with lags (days) of 4 and 15 ($m = 4$ and $m = 15$). Figure 4 shows the asymmetric volatility represented by the conditional variance function and the quadrature density distributions. The reaction to large negative price movements is only marginally higher than that of large positive movements. The density shows marginal higher densities for positive price movements. Therefore, from these plots, asymmetry is

FIGURE 3 Conditional variance and moving average for lags 4 and 15.

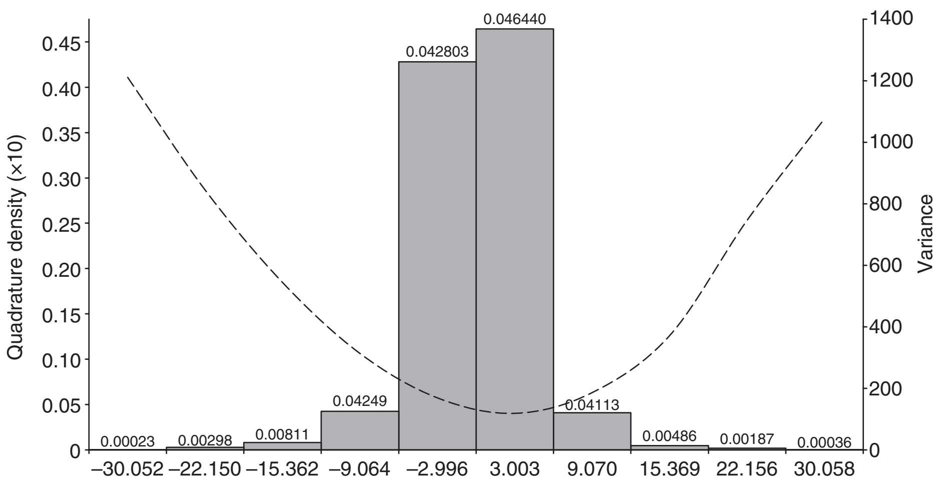


difficult to verify for the Nordic/Baltic electricity market. From the semiparametric SNP model's (standardized) residuals, we can establish a basis for model specification tests. Table 3 shows specification test statistics together with residual distributional properties. The mean is close to zero and the standard deviation is close to 1 ($N(0, 1)$).

I have changed the final two column headings to (2.5%; 1%) as I think you requested, but please check (here and in Table 6) as they were originally 5%/1% not 2.5%/1%.

However, a maximum of 12.2, a minimum of −7.9, a kurtosis of 11.6 and a skewness of 0.24 suggest deviations from a standard normal distribution. The Cramer–von Mises test statistic (12.3) suggests deviations from a normal distribution of the standardized residuals. As a first specification test of the model, we calculate the twelfth-order Ljung–Box statistic (Ljung and Box 1978) for the standardized residuals (Q) and squared standardized residuals (Q^2). We find no significant evidence of serial correlation for the residuals ($Q(12)$) and the squared residuals ($Q^2(12)$). The twelfth lag ARCH test statistic (8.1) for the standardized residuals indicates conditional homoscedasticity and the RESET (12;6) (Ramsey 1969) test statistic (28) cannot reject nonlinearity in the mean at the 1% level. The BDS (Brock *et al* 1996) test statistic for standardized residuals cannot reject independent and identical distribution. The joint bias test (Engle and Ng 1993) reports no significant biases in standardized residuals. The specification tests therefore report an appropriate model specification. The SNP projection gives access to one-step-ahead densities $f_K(\tilde{y}_t \mid x_{t-1}, \hat{\theta})$, conditional on the values for $x_{t-1} = (\tilde{y}_{t-1}, \tilde{y}_{t-2}, \dots, \tilde{y}_{t-L})$. Hence, we plot densities conditioned on several values of x_{t-1}. Plots of the conditional one-step-ahead densities are

FIGURE 4 The conditional variance function for the system price.



shown in Figure 5, where all lags are set to the unconditional mean of the data (0.003), and Figure 6, where lags are set explicitly (−40%; +40%) without reference to the mean. Figure 5 plots the density together with a normal distribution, while Figure 6 is conditional on seventeen values of x ($x_{t-1} = -60\%, \dots, 60\%$). Both figures show densities typically shaped for data from other commodity (and financial) markets: peaked with fatter tails than the normal with a bit of asymmetry. Moreover, the larger the x_{t-1} variable becomes in absolute terms, the wider the distribution and therefore the greater the uncertainty of the next day's possible price movement intervals. These two plots indicate simply that market participants experience much greater uncertainty when daily price movements, and therefore daily volatility, are high.

5.2 The impulse-response functionals for the period 1993–2017

Figure 7 reports the Nordic/Baltic spot system price mean impulse response functions defined in Section 2. The plot contains the conditional mean profiles $\{\hat{y}_j^i - \hat{y}_j^0\}_{j=1}^5$ for seventeen impulses $i = -60, \dots, +60\%$ and for steps ahead $j = 1, \dots, 5$. The impulse response functions for the conditional mean show the well-known characteristics of mean reversion. The baseline mean profile is $\hat{y}_j^0$, and negative (positive) response lines are continuous (dotted). The plot shows that for day 0 the model is given impulses for price movements between −60% and 60%. For multi-steps ahead 1–5, the plot shows that the price movements revert immediately to zero. Moreover, the mean effects are symmetric and totally dissipated within one step ahead of the impulse,

TABLE 3 Residual statistics for the seasonal adjusted Nordic/Balkan electricity system.

Mean/ mode	Median SD	Maximum/ minimum	Moment kurt/skew	Quantile kurt/skew	Quantile normal	Cramer–von Mises	Serial dependence $Q(12)$	Serial dependence $Q^2(12)$
0.00042	0.02507	12.23843	11.56939	0.05327	1.40263	12.32548	6.4938	8.3971
	0.99999	−7.93856	0.23749	−0.01644	{0.4959}	{0.0000}	{0.7950}	{0.7755}

BDS Z-statistic ($e = 1$) $m = 2$	$m = 3$	$m = 4$	$m = 5$	ARCH (12)	RESET (12;6)	Joint bias	VaR (%) (2.5%; 1%)	CVaR (%) (2.5%; 1%)
1.55505	2.75108	2.08730	1.57246	8.054767	28.02767	5.45309	−1.5104	−2.3183
{0.1191}	{0.0091}	{0.0452}	{0.1159}	{9.7808}	{0.0055}	{0.0676}	−2.7234	−3.8218

FIGURE 5 One-step-ahead conditional returns density.

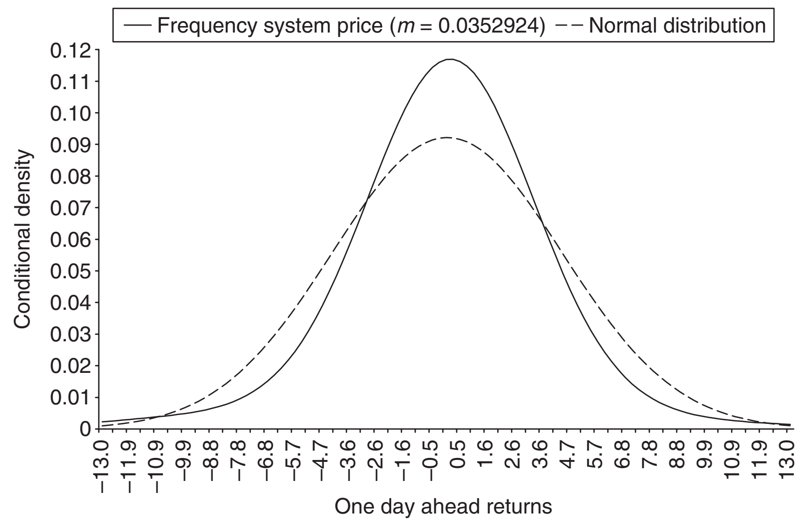


FIGURE 6 One-step-ahead conditional returns density for $f_K(y_t \mid x_{t-1}) = -40, -20, -10, -5, -3, -1, 0, 1, 3, 5, 10, 20, 40\%$.

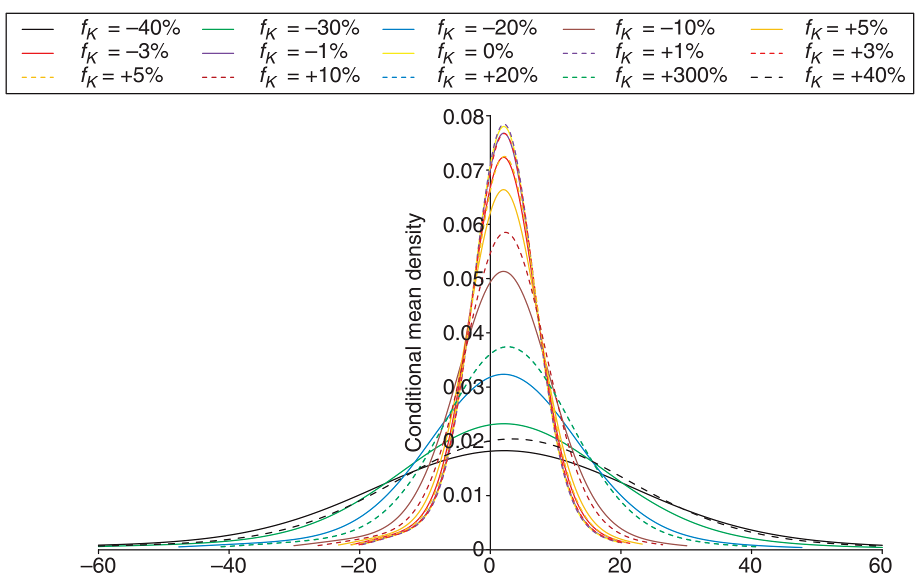
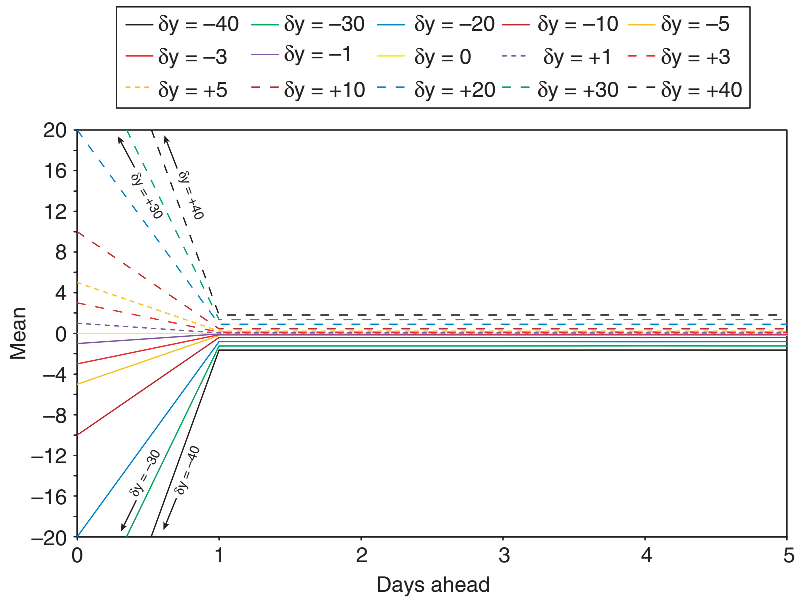


FIGURE 7 The location and scale of the mean impulse-response functions.

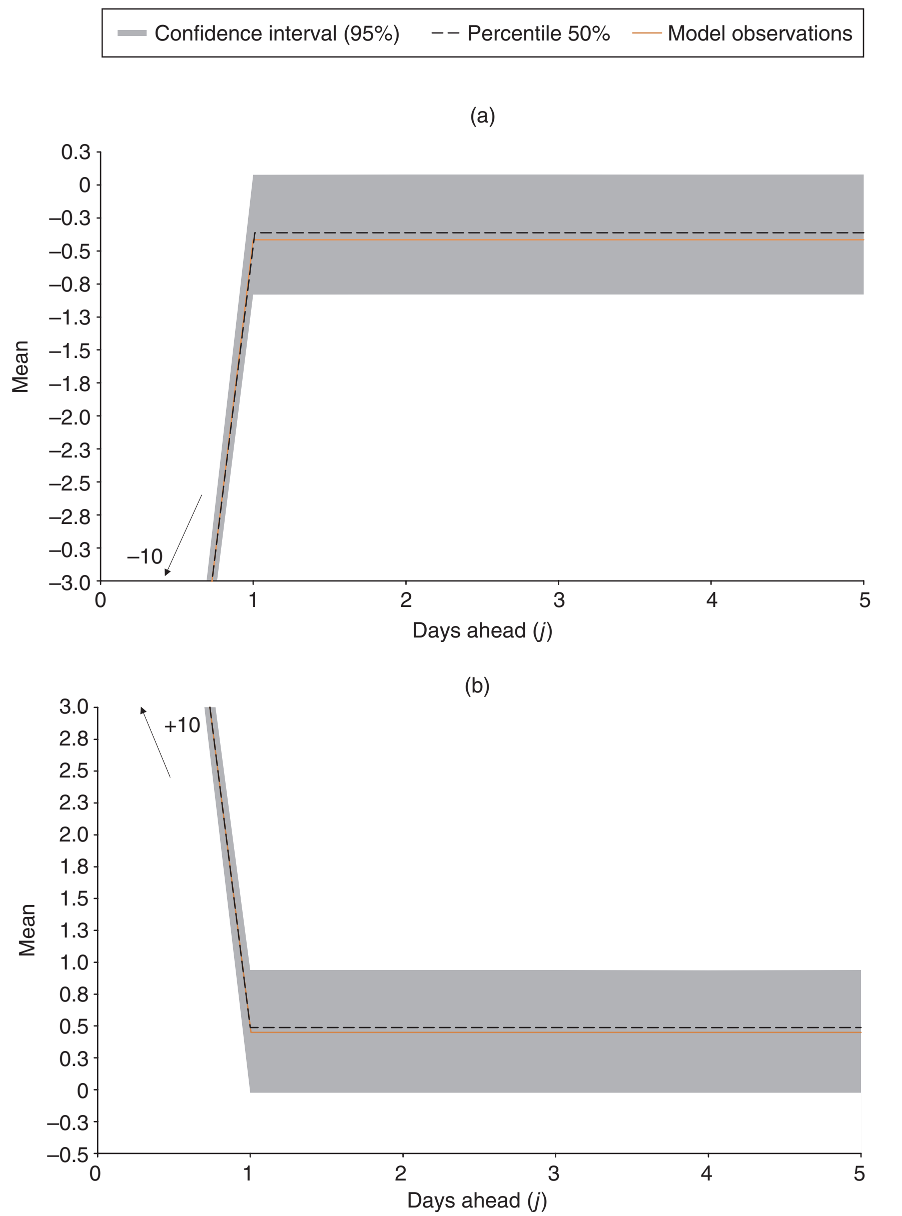


Mean $\mathbb{E}[\mu(y_{k,j}) \mid x-1]$ versus days ahead for $\delta y = -40, -30, -20, -10, -5, -3, -1, 0, 1, 3, 5, 10, 20, 30, 40$ steps.

suggesting there is very little evidence of nonlinearity in the conditional mean of the price movement processes. From the −60% and 60% extreme price impulses, the step-ahead responses are very close to zero. In fact, all the impulse-response profiles, even for extreme profiles, consistently show dissipated responses. The response differences between positive and negative impulses are negligible. Implementing bootstrapping, we can report 95% sup–norm bands. Each band is computed using the bootstrap procedure described in Section 2 with 500 refittings of the SNP model. The band computed for the case where the mean is −10% and 10% is reported in Figure 8. The ε-band is located around zero, suggesting no obvious advantageous positions for market participants. For the −10% shock in part (a) the 95% ε-band is between 0.07 and −0.83 with an expectation of −0.37, and for the +10% shock in part (b) the ε-band is between 0.94 and −0.02 with an expectation of 0.49. In fact, all mean ε-bands in the price movements ranges between −60% and 60% include zeros. Hence, all mean impulses suggest immediate market mean reversions.[13]

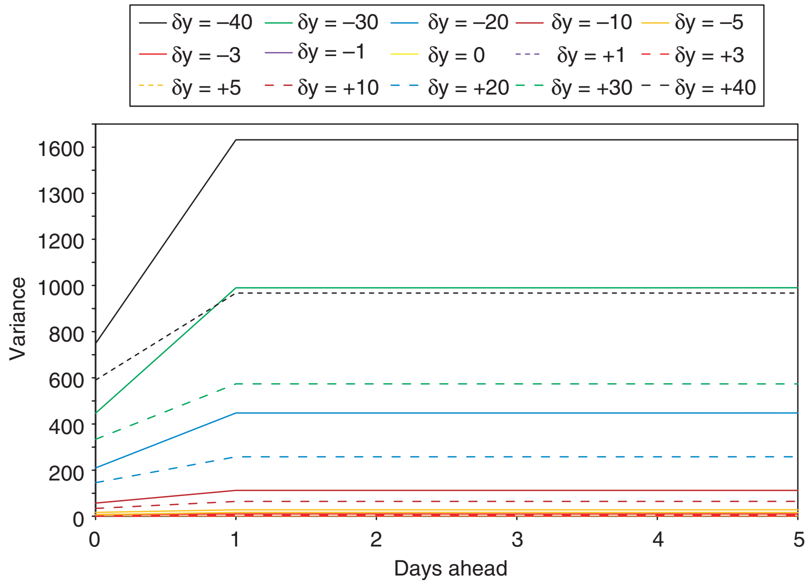
[13] Mean tables are not reported due to space restrictions. All tables are available from the author upon request.

FIGURE 8 Mean and mean-difference characteristics and confidence intervals.



(a) Confidence intervals for multiple days ahead system price mean function responses $\delta y = -10\%$. (b) Confidence intervals for multiple days ahead system price mean function responses $\delta y = +10\%$.

FIGURE 9 The location and scale of the variance impulse-response functions.

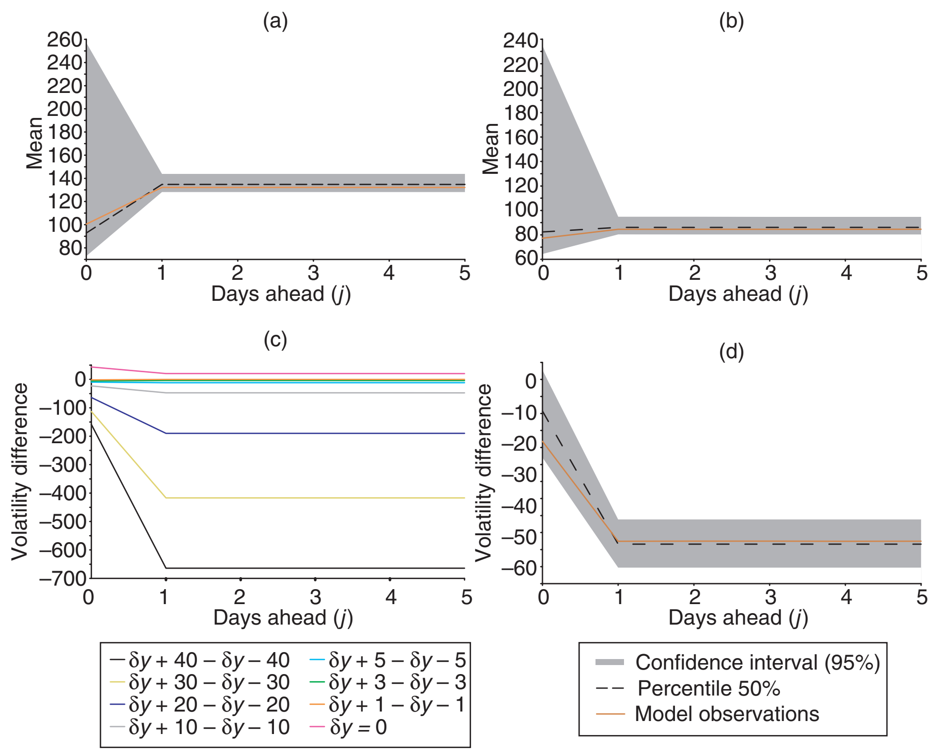


Variance $\mathbb{E}[\text{var}(y_{k,j}) \mid x-1]$ versus days ahead for $\delta y = -40, -30, -20, -10, -5, -3, -1, 0, 1, 3, 5, 10, 20, 30, 40$ steps.

Figure 9 reports the impulse-response variance functions (conditional variance profiles) $\{\hat{\Psi}_j^i - \hat{\Psi}_j^0\}_{j=1}^5$ for impulses $i = -60\%, \dots, 60\%$ and multi-steps ahead $j = 1, \dots, 5$ as described in Section 2. The baseline variance profile is $\hat{\Psi}_j^0$.

For absolute price movement impulses below 10%, the variance responses are both small and symmetric. In contrast, large absolute movement impulses ($\geqslant$ 10%) report quickly increasing variance responses together with a fast-growing negative asymmetry. For all absolute price movements between 1% and 60%, we find higher volatility for negative price movements than for similar positive price movements. Moreover, for all impulses, the differences are negative and, for absolute impulses greater than 5%, the differences increase rapidly. Hence, the results clearly indicate negative asymmetry for the electricity market. For statistical significance, Figure 10 reports the 95% confidence intervals (sup–norm ε-bands) for -10% impulses (part (a)) and $+10\%$ impulses (part (b)) to access responses to the volatility functionals. The ε-bands from bootstrapping here are clearly visible and do not include zeros for either the -10% or the 10% impulses. The ε-band responses for the negative 10% impulses relative to positive 10% impulses are multi-step-ahead shifted somewhat higher. The ε-bands for day 0 responses are naturally wider for positive impulses.

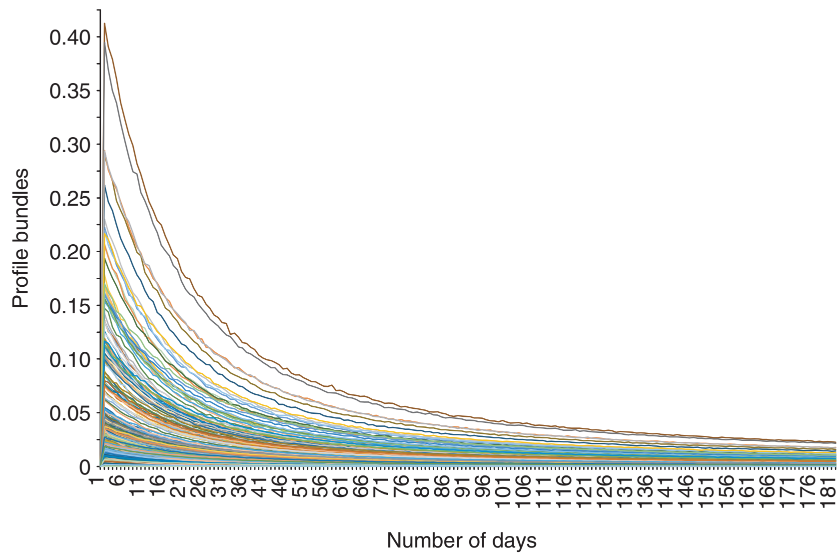
FIGURE 10 Mean and volatility response confidence intervals (95%) for −10% and +10% price impulses.



(a) Confidence intervals for multiple days ahead system price volatility function responses $\delta y = -10\%$. (b) Confidence intervals for multiple days ahead system price volatility function responses $\delta y = +10\%$. (c) Volatility asymmetry for log spot system price $(+\delta y\% - (-\delta y\%))$. (d) Confidence intervals for multiple days ahead system price volatility function responses $\delta y = (+10\% - (-10\%))$.

The 95% confidence intervals clearly indicate significant volatility increases for both −10% and 10% price impulses. For asymmetry, Figure 10(c) reports volatility price sign response differences for −10% and +10% impulses. The volatility response differences seem negligible for small price movements. Larger price movements show differences that grow relatively fast. Figure 10(d) reports the 95% ε-band for the response differences between −10% and 10% impulses. As this response ε-band difference does not include zero for positive steps ahead, we are able to reject at 5% statistical significance the null hypothesis of symmetry. For day zero, the ε-band includes zero, which suggest that the asymmetry must be rejected at 5% statistical significance. Since our nonlinear impulse response analysis traces out the dynamic

FIGURE 11 Conditional volatility persistence.



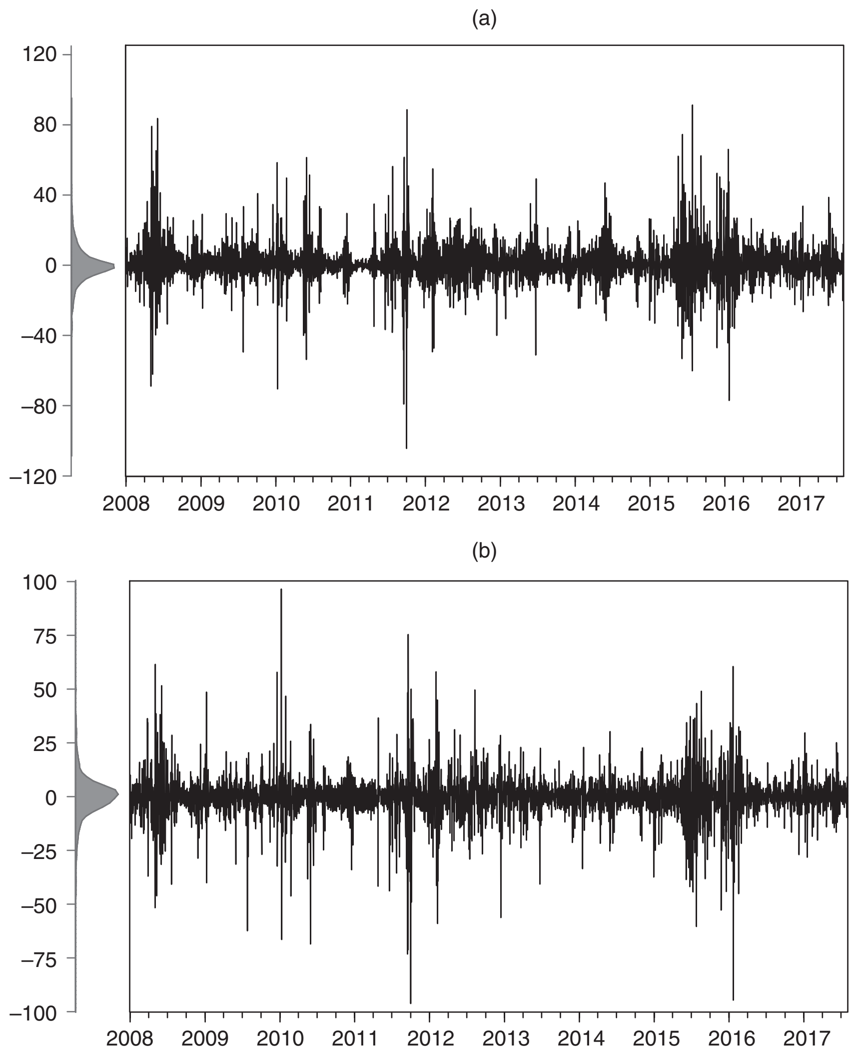
Average half-life (days): 22.4582. Standard deviation: 7.9718.

effects of shocks, it is very well suited for assessing the empirical importance of the asymmetry (also called the leverage effect).[14] Finally, Figure 11 shows the persistence based on the SNP specification ($\hat{f}(y \mid x)$). Each profile uses data up to date $t-1$. At date t, the profile shows mean reversion typically for GARCH(1, 1) processes. The measure of the persistence in a volatility model is the "half-life" of volatility. This is defined as the time taken for the volatility to move halfway back toward its unconditional mean following a deviation from it. The half-life definition is given as (Engle and Patton 2001)

$$\tau = k : |h_{t+k|t} - \sigma^2| = \tfrac{1}{2}|h_{t+1|t} - \sigma^2|.$$

The volatility from approximately the 1000 latest observations (2013–17) in the data set is defined in the plot to be 22.5 days with a standard deviation of 7.97 days.

FIGURE 12 Seasonal (a) unadjusted and (b) adjusted log price movements for 2008–17.



5.3 The impulse-response functionals for the subperiod 2008–17

We study the subperiod 2008–17 separately to see if the massive subsidizing of renewables through green certificates and direct investment support influenced the

[14] The leverage effect is the tendency for a price decline to lead to a subsequent volatility increase that is larger than that in volatility associated with a price rise of the same magnitude. For further details see Black (1976), Christie (1982), Nelson (1991) and Campbell *et al* (1993).

TABLE 4 Returns statistics for system price Nord Pool spot auction market.

(a) Unadjusted series

Mean (all)/ (ex drop)	Median SD	Maximum/ minimum	Moment kurt/skew	Quantile kurt/skew	Quantile normal	Cramer–von Mises	Serial dependence $Q(12)$	Serial dependence $Q^2(12)$	VaR (%) (1%; 2.5%)
−0.01282	−0.59032	91.2158	11.25465	0.58256	50.7990	23.6019	1105.0	941.84	−34.8769
−0.01570	11.79544	−104.398	0.47028	0.04681	{0.0000}	{0.0000}	{0.0000}	{0.0000}	−22.7377

BDS Z-statistic ($e = 1$) $m = 2$	$m = 3$	$m = 4$	$m = 5$	KPSS (stationary) intercept	I + trend	ADF test	ARCH (12)	RESET (12;6)	CVaR (1%; 2.5%)
19.2824	23.9124	26.1585	27.8458	0.01193	0.01194	−14.7431	617.49	246.5713	−48.1537
{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.4630}	{0.1460}	{0.0000}	{0.0000}	{0.0000}	−35.281

(b) Adjusted series

Mean (all)/ (ex drop)	Median SD	Maximum/ minimum	Moment kurt/skew	Quantile kurt/skew	Quantile normal	Cramer–von Mises	Serial dependence $Q(12)$	Serial dependence $Q^2(12)$	VaR (%) (1%; 2.5%)
−0.01282	0.28700	78.2743	10.28586	0.14518	4.1374	20.6714	111.190	890.34	−37.1136
−0.00754	11.79544	−116.636	−0.54752	−0.04266	{0.1264}	{0.0000}	{0.0000}	{0.0000}	−24.3012

BDS Z-statistic ($e = 1$) $m = 2$	$m = 3$	$m = 4$	$m = 5$	KPSS (stationary) intercept	I + trend	ADF test	ARCH (12)	RESET (12;6)	CVaR (1%; 2.5%)
22.8589	27.3007	29.8005	32.1585	0.02247	0.01926	−48.2467	468.888	93.8097	−53.8098
{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.4630}	{0.1460}	{0.0000}	{0.0000}	{0.0000}	−39.4372

SD denotes standard deviation. ADF denotes augmented Dickey–Fuller.

spot price market auction dynamics. The 2008–17 log price series is reported in Figure 1. We apply the same adjustments procedures for this ten-year subperiod as for the entire twenty-four-year period. For the mean, the adjustments show only minor differences (joint holidays) from the full-length period. The volatility adjustment includes a smaller number of weeks; the linear and squared trends are not significant, the midweek days effects are not significant and the weeks report changing volatility patterns.

Figure 12 shows the unadjusted and the specifically adjusted time series for the subperiod 2008–17, and the characteristics for the subperiod are reported in Table 4. These do not diverge dramatically from the full period series. The SNP model specification is repeated for the ten-year subperiod 2008–17. The BIC optimal nonlinear semiparametric model is nearly unchanged, but the BIC optimal model shows fewer Hermite functions for the normal model exceptions.[15] The model is reported in Table 5. The SNP model's subperiod coefficient values are clearly different from the full period model. For the mean correlation, we pinpoint the change in coefficient signs. For the variance equation, the ARCH term (lagged errors) is increased quite strongly, while the GARCH term (history) is reduced slightly. Asymmetry and level effects are still strongly significant. The subperiod seems more sensitive to the daily price movements, as the ARCH component for the conditional volatility moves from 0.42 to 0.44. The relevance of historic volatility shows a similar influence when the GARCH component of the conditional volatility falls to 0.894 from 0.907. Table 6 reports specification test statistics for the subperiod, together with residual distributional properties. The Cramer–von Mises test statistic (4.4) suggests deviations from the normal distribution of the standardized residuals. As for the full-length period, we calculate the twelfth-order Ljung–Box statistic for the standardized residuals (Q), squared standardized residuals (Q^2), the twelfth lag ARCH test statistic for the standardized residuals, the RESET (12;6) (Ramsey 1969) test statistic, the BDS (Brock *et al* 1996) test statistic for standardized residuals and the joint bias test (Engle and Ng 1993). All the test statistics are nonsignificant for almost all lags. The specification tests therefore suggest an appropriate subperiod model specification. Figure 13(a) shows graphically the conditional variance together with a calculation of moving averages with lags of four and fifteen (days) ($m = 4$ and $m = 15$). In Figure 15(b), we report the asymmetric volatility represented by the conditional variance function and the quadrature density distribution. The volatility reaction from large negative price movements is only marginally higher than from large positive price movements. The quadrature density shows high densities for small absolute price movements and lower densities for large absolute price movements. Parts (a) and (b) of Figure 14

Apologies: we included the wrong figure for figure 12 in the previous proof. All OK now? Please check throughout.

[15] The BIC optimal SNP model is the $L_u = 14$, $L_g = 1$, $L_r = 1$, $L_v = 1$, $L_\omega = 1$, $L_p = 1$, $K_z = 10$, $K_x = 0$ specification.

TABLE 5 Nord Pool spot electricity system price. [Table continues on next page.]

(a) Hermite polynomials

Mean equation				
Var	SNP coefficient	Mode	Standard error	t-statistics
η_1	$a_0[1]$	0.00898	0.00884	1.01579
η_2	$a_0[2]$	−0.21791	0.01388	−15.70438
η_3	$a_0[3]$	0.00410	0.00887	0.46187
η_4	$a_0[4]$	0.09148	0.00887	10.31142
η_5	$a_0[5]$	0.02037	0.00990	2.05752
η_6	$a_0[6]$	−0.10808	0.01033	−10.46163
η_7	$a_0[7]$	−0.03809	0.01085	−3.51201
η_8	$a_0[8]$	0.10614	0.01291	8.22129

(b) Mean correlation

Mean equation				
Var	SNP coefficient	Mode	Standard error	t-statistics
η_9	$B(1,1)$	−0.02728	0.01687	−1.61671
η_{10}	$B(1,2)$	0.02136	0.01499	−9.34976
η_{11}	$B(1,3)$	0.11752	0.01443	−4.7198
η_{12}	$B(1,4)$	0.00965	0.01513	−2.15493
η_{13}	$B(1,5)$	0.00102	0.01464	−2.31574
η_{14}	$B(1,6)$	−0.00492	0.01505	1.4245
η_{15}	$B(1,7)$	−0.02944	0.01419	8.14486
η_{16}	$B(1,8)$	−0.02767	0.0136	0.63783
η_{17}	$B(1,9)$	−0.03045	0.01267	0.06953
η_{18}	$B(1,10)$	0.074	0.01177	−0.32683
η_{19}	$B(1,11)$	0.22402	0.01309	−2.07443
η_{20}	$B(1,12)$	0.43753	0.03824	−2.03561
η_{21}	$B(1,13)$	0.89433	0.00819	−2.40367
η_{22}	$B(1,14)$	−0.42789	0.03641	6.28876

repeat the plots for the unconditional and conditional mean densities, respectively. As reported for the full period, the larger the absolute terms for the x_{t-1} variable, the wider the distribution, and therefore the greater the uncertainty. Moreover, the distributions have all shifted somewhat to the left. The conditional mean is 0.029 for the period 2008–17 and 0.035 for the period 1993–2017. Hence, the prices for

TABLE 5 Continued.

(c) Variance equation

Mean equation: Var	Mean equation: SNP coefficient	Mode	Standard error	t-statistics
η_{27}	$R_0[1]$	0.22402	0.01309	17.11931
η_{28}	$P(1,1)$s	0.43753	0.03824	11.44194
η_{29}	$Q(1,1)$s	0.89433	0.00819	109.1575
η_{30}	$V(1,1)$s	−0.42789	0.03641	−11.75271
η_{31}	$W(1,1)$s	0.49455	0.0576	8.58557
Observations (inc. drops)		3411	s_n	1.1098592
Loglikelihood		−3886.726765	AIC	1.1175690
			BIC	1.1413197
Largest eigenvalue of mean function companion matrix:				0.884274
Largest eigenvalue of variance function P and Q companion m:				0.991253

Statistical model SNP-14,111,8, 000 -fit; semi-parametric-GARCH model.

the subperiod 2008–17 show lower but positive drift. The quadrature distribution in Figure 13(b) confirms this drift, suggesting that the density mass has moved to the left. For the mean impulse-response functions for the subperiod 2008–17, we find the same symmetry and total dissipation within a day of the impulses, but with a clear sign of overreaction not visible for the period 1993–2017.

In fact, in Figure 15(a), the mean shows a tendency toward a significant negative serial correlation (overreaction/correction).[16] The plot shows that negative (continuous lines) and positive (dotted lines) impulses (conditioning using x_{t-1} values) report a negative (positive) overreaction followed by a positive (negative) one-step-ahead correction (negative serial correlation). For an impulse of −20% (20%) the response is 4.2% (−3.8%), and for an impulse of −60% (60%) the mean response is in fact as high as 12.5% (−11.6%). The mean response is therefore relatively symmetric, but the mean difference between −20% and −60% (20% and 60%) price movements (impulses) shows a response difference of close to 8.3% (7.8%). Large price movements seem therefore to surprise market participants, adjusting supply/demand behavior showing both mean reversion and forms of market overreactions/corrections. From the bootstrap analysis, Figure 16 reports the confidence intervals (95%) for −10% and 10% impulses. The analysis suggests clearly significant negative correlation response coefficients for the subperiod 2008–17. For an impulse of −10%

[16] See also Table 5 and, in particular, the structure of the mean correlation.

TABLE 6 Residual statistics for the seasonal adjusted Nordic/Balkan Electricity System.

Mean/ mode	Median SD	Maximum/ minimum	Moment kurt/skew	Quantile kurt/skew	Quantile normal	Cramer–von Mises	Serial dependence $Q(12)$	Serial dependence $Q^2(12)$
−0.00292	0.01341	8.33060	6.71178	0.02253	0.10360	4.39454	3.3338	10.852
	1.00008	−8.24064	−0.37840	0.00717	{0.9495}	{0.0000}	{0.9930}	{0.5420}

BDS Z-statistic ($e = 1$) $m = 2$	$m = 3$	$m = 4$	$m = 5$	ARCH (12)	RESET (12;6)	Joint bias	VaR (%) (2.5%; 1%)	CVaR (%) (2.5%; 1%)
1.52009	2.86097	2.17858	1.30630	10.8614	8.46616	4.41270	−1.5499	−2.3934
{0.1285}	{0.0042}	{0.0294}	{0.1915}	{0.5408}	{0.7477}	{0.1145}	−2.8409	−3.9497

FIGURE 13 (a) Conditional volatility and (b) quadrature plot for subperiod 2008–17.

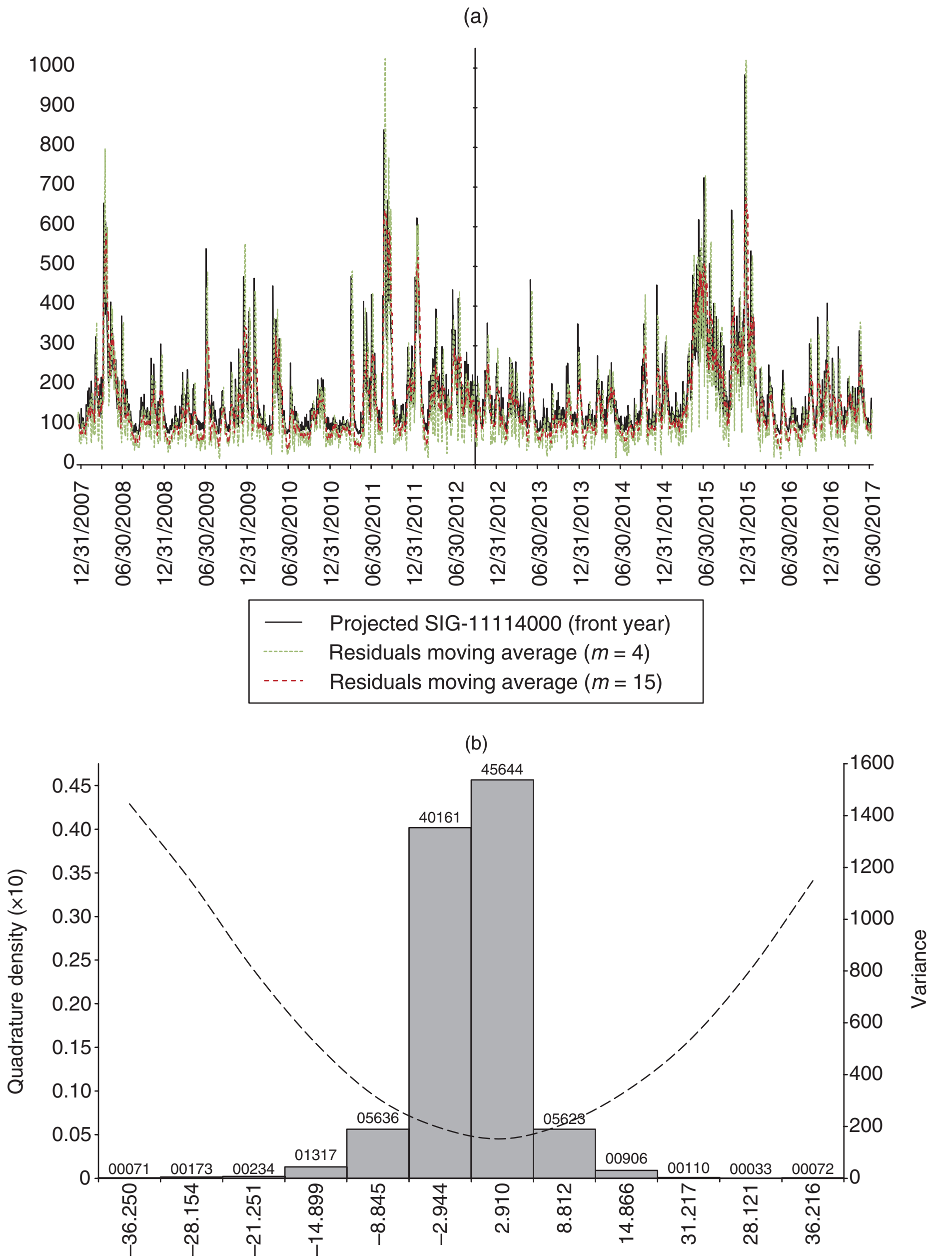
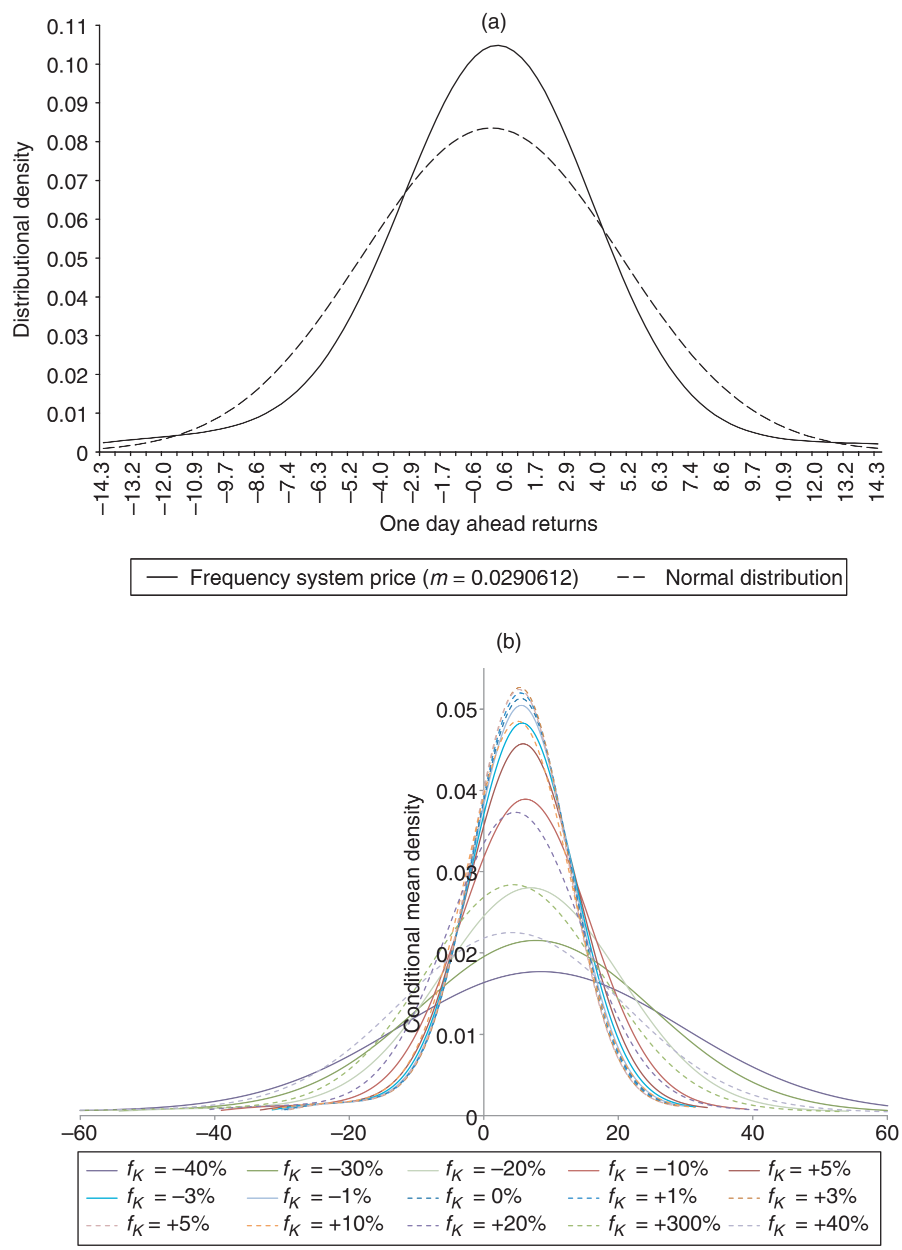


FIGURE 14 Characteristics for period densities for subperiod 2008–16.



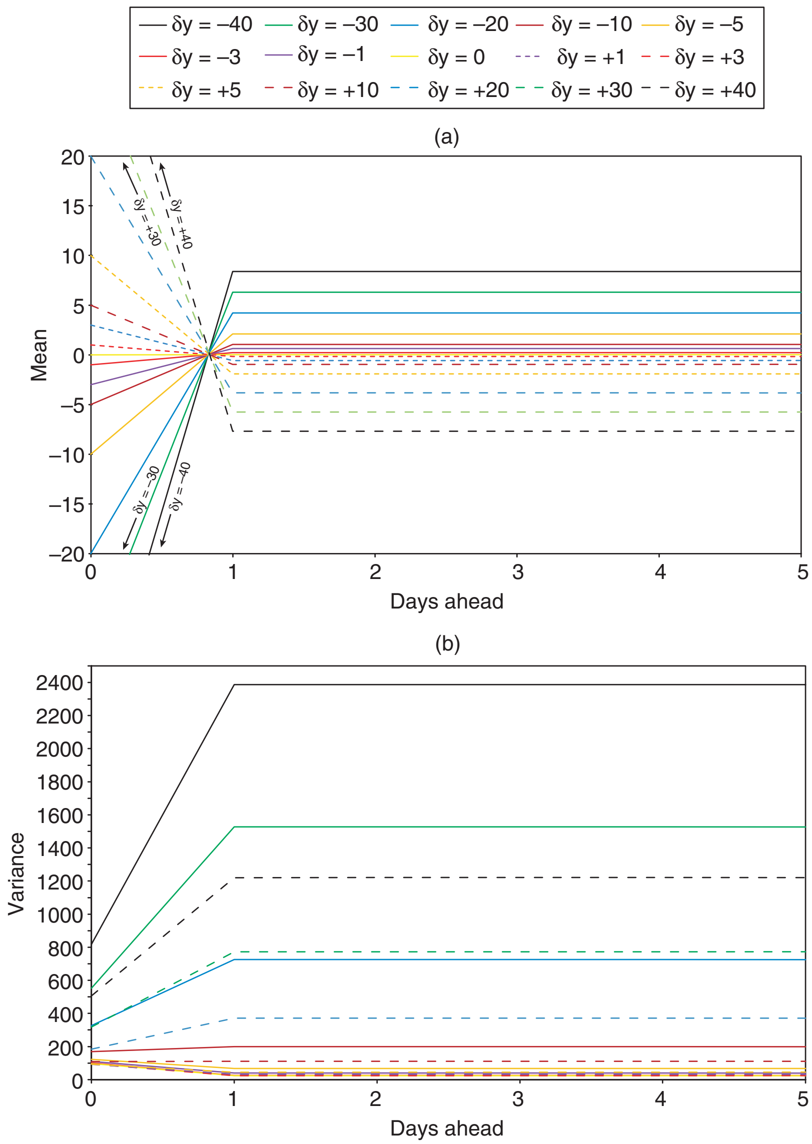
(a) One-day-ahead conditional returns density for unconditional mean $x_{t-1} = -0.00754$. (b) One-step-ahead conditional returns density for $f_K(y_t \mid x_{t-1}) = -40, -20, -10, -5, -3, -1, 0, 1, 3, 5, 10, 20, 40\%$.

(+10%), the response is +2.19% (−1.84%) with a 95% confidence interval between 1.54 and 2.83 (−1.19 and −2.46). The significant and quite symmetric mean difference for the 2008–17 period is clearly different from the close-to-nonexistent mean difference for the 1993–2017 period. The magnitude of the differences in mean, considering the size of the 95% confidence intervals within the two subperiods, suggests a significant change in mean dynamics between the periods. The subperiod 2008–17, which saw massive growth in renewables in the energy system, reports overreaction quite differently than the full period. The overreaction will most probably also induce higher volatility and asymmetry.

Figure 15(b) reports the volatility response functions for the subperiod 2008–17. For small price movements between −5% and +5%, the volatility seems to decrease. For larger absolute price movements, the step-ahead volatility increases. The increases are much larger for negative price movements than for positive price movements (asymmetry). Figure 17 reports confidence intervals for price impulses of −10% (panel (a)) and 10% (part (b)) and step-ahead volatility. The ε-band responses for negative −10% and positive 10% impulses and the step-ahead responses are shifted somewhat higher than for the positive 10% impulses. The ε-band for day 0 is naturally wider for positive step-ahead days. The 95% confidence intervals clearly indicate significant volatility increases for both −10% and 10% price impulses. Moreover, the volatility structure for the two periods seems similar. The general picture is that there exist small volatility differences for small price movements, but these grow quickly for large absolute price movements. From Figure 17(c), we see that the volatility differences are much larger for negative price impulses than for positive price impulses (increased asymmetry for subperiod 2008–17). For example, on day 1, for a price change impulse of (+20% − (−20%)), the volatility increases by 354. The same numbers for a price impulse of (+60% − (−60%)) give a volatility increase of 2034. Figure 18(b) shows that the 95% confidence intervals for the (+10 − (−10%)) impulse differences with mean 88 do not include zeros (72, 106), suggesting significant differences. Finally, the volatility from approximately 1000 of the latest observations (2013–17) in the data set for the subperiod 2008–17 is defined in Figure 18 to be 12.99 days, with a standard deviation of 2.98 days.

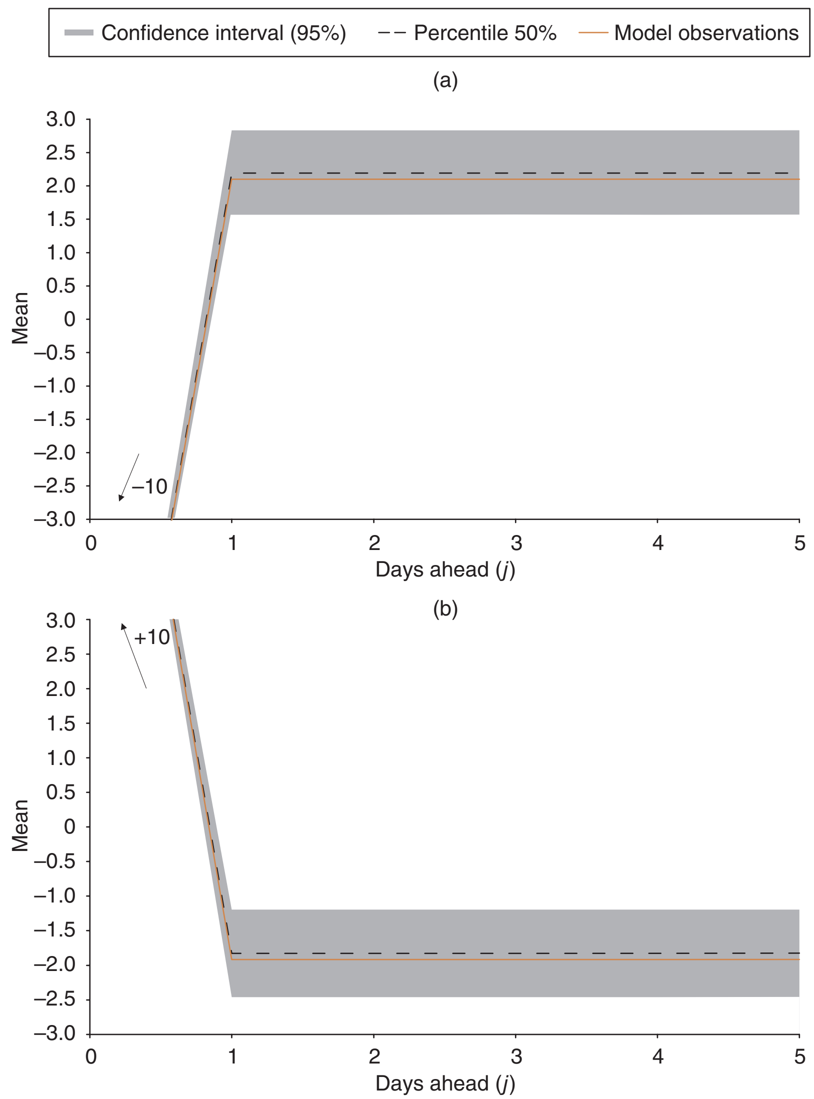
For comparison between the periods 1993–2017 and 2008–17, Figure 19(a) shows the mean differences and Figure 19(b) shows volatility differences. For the mean in part (a) the general picture is that negative impulses produce positive returns, while the positive impulses produce negative returns. The correction/overreaction result for the subperiod 2008–17 is therefore revealed. For the volatility in part (b), the general picture is of lower volatility for small price impulses and higher volatility produced by large price impulses. Moreover, the asymmetry for large price changes has increased. Figure 19(b) shows that the volatility differences are much greater for negative price impulses than for positive ones. In fact, for all price impulses, the negative asymmetry

FIGURE 15 The location and scale of the mean and variance of the impulse-response functions (subperiod 2008–17).



Mean $\mathbb{E}[\mu(y_{k,j}) \mid x-1]$ and variance $\mathbb{E}[\mathrm{var}(y_{k,j}) \mid x-1]$ versus days ahead for $\delta y = -40, -30, -20, -10, -5, -3, -1, 0, 1, 3, 5, 10, 20, 30, 40$ steps.

FIGURE 16 Mean response confidence intervals (95%) for (a) $+10\%$ and (b) -10% price impulses (subperiod 2008–17).



is much larger for 2008–17 than for 1993–2017. The negative asymmetries for impulse values of -20% and $+20\%$, for day zero and for one-step ahead, report responses that are 171 and 564 larger for the period 2008–17, respectively. The same negative

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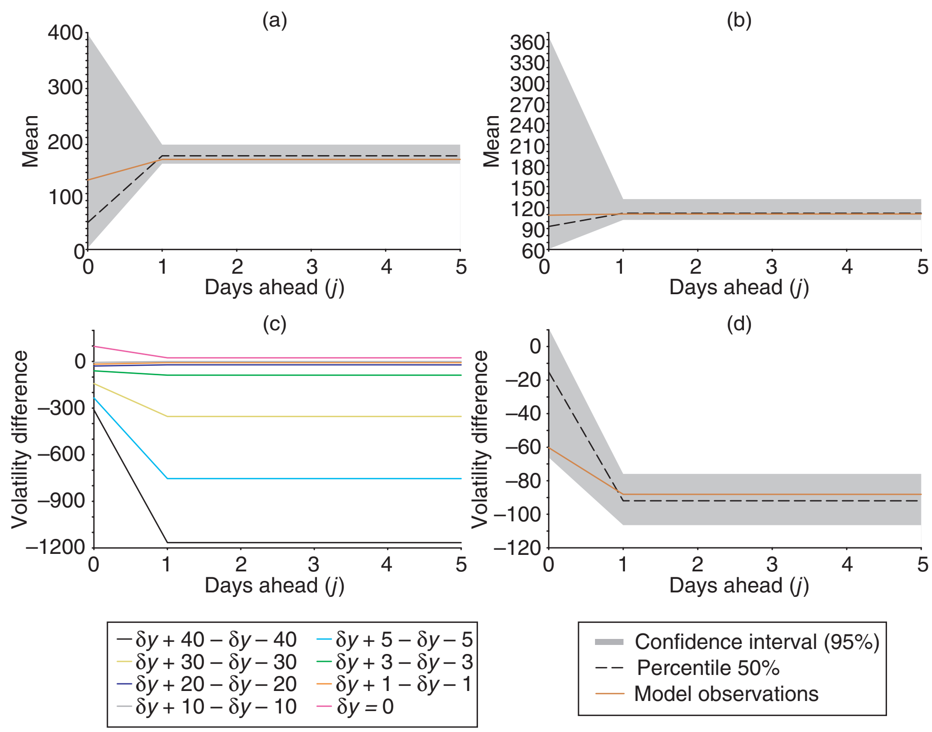
FIGURE 17 Volatility response confidence intervals (95%) for −10% and +10% price impulses (subperiod 2008–17).



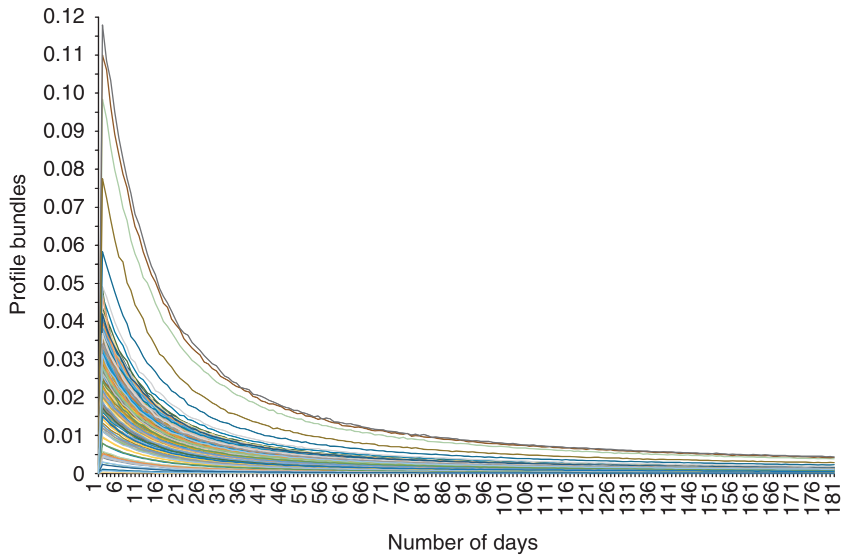
(a) Confidence intervals for multiple days ahead system price volatility function responses $\delta y = -10\%$. (b) Confidence intervals for multiple days ahead system price volatility function responses $\delta y = +10\%$. (c) Volatility asymmetry for log spot system price $(+\delta y\% - (-\delta y\%))$. (d) Confidence intervals for multiple days ahead system price volatility function responses $\delta y = (+10\% - (-10\%))$.

asymmetries for price impulses of −60% and 60% report responses that are 214 and 866 larger for the 2008–17 period, respectively.

6 SUMMARY AND CONCLUSIONS

We have modeled and estimated an ARMA–GARCH-in-mean model specification for the conditional mean and variance for the so-called system price in the Nordic electric power market for the period 1994–2017 (twenty-four years). The time series are adjusted for systematic seasonal, trend and scale effects, and all the estimated conditional specifications are BIC preferred. Our model captures the serial correlation structure in the return series, the effect of thick distribution tails (leptokurtosis)

FIGURE 18 Persistence characteristics (subperiod 2008–17).



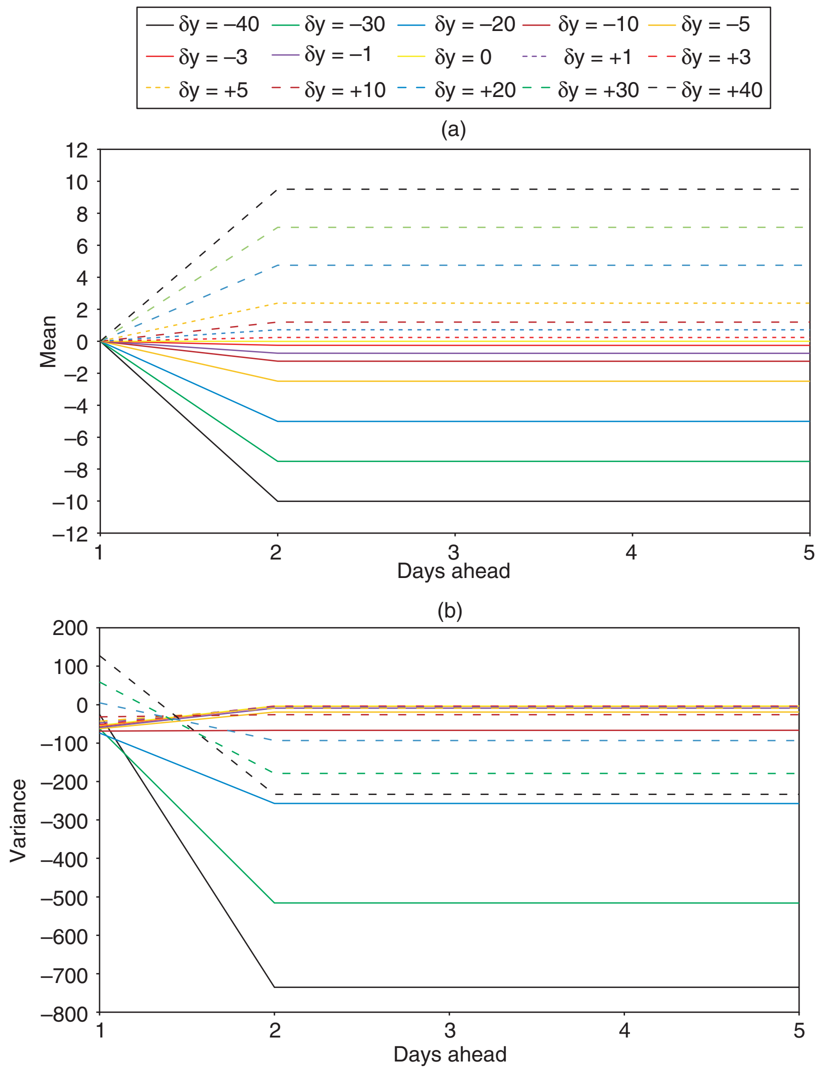
Average half-life (days): 12.9919. Standard deviation: 2.97868.

and residual risk in the conditional mean. The conditional variance equation captures shock, persistence and asymmetry and the two-equation specification control for conditional heteroscedasticity. A battery of statistical model specification tests cannot reject the BIC-optimal SNP specification. We summarize our results below.

The drift is close to zero. We find serial correlation structures up to fourteen days after an adjustment procedure that accounts for seasonal, trend and scale effects. Moreover, mean reversion in clearly visible in the time series. The volatility equation rejects conditional homoscedasticity. The empirical impulse-response analysis confirms immediate (one day) dissipation, suggesting linearity in the conditional mean equation. The impulse-response analysis reveals quite different conditional volatility responses from small to large impulses. For small price movements (impulses), the volatility shows modest increases relative to the period 1993–2017 (small responses). In contrast, for large price movements the volatility shows quite large responses.

As the impulse-response analysis is very well suited for assessing the empirical importance of asymmetry, our results show little negative asymmetry for small absolute price movements. However, the asymmetry becomes severe as price movements grow large. The persistence of shock for the period 1993–2017 is about 22.5 trading days with a standard deviation of eight days. For the subperiod 2008–17, the

FIGURE 19 (a) Mean and (b) volatility differences between subperiods 1993–2017 and 2008–17.



Mean $\mathbb{E}[\mu(y_{k,j}) \mid x-1]$ and variance $\mathbb{E}[\mathrm{var}(y_{k,j}) \mid x-1]$ versus days ahead for $\delta y = -40, -30, -20, -10, -5, -3, -1, 0, 1, 3, 5, 10, 20, 30, 40$ steps.

density mass around small negative and zero price movements for the conditional mean shows an increase relative to the 1993–2017 period. The impulse response functions for the mean suggest a change from positive serial correlation for the

Please advise which of the two original suggested changes is better here: "probability mass function" or "probability density function"? It wasn't clear in your corrections which version you preferred.

period 1993–2017 to negative serial correlation for the period 2008–17 (overreaction/correction). From price impulses for the period 2008–17, the nonlinear volatility shows both larger responses and stronger negative asymmetry. The persistence of shock for the period 2008–17 fell considerably, and was about 12.9 trading days with a standard deviation of three days.

Future research can extend these results to multivariate impulse-response analysis of contemporaneous spot prices and wind, consumption, production and perturbation forecasts. For example, as a starting point, a bivariate analysis of wind forecast and spot price movements may clarify strategic bidding behavior from existing flexible power producers (mainly hydro) in the Nordic/Baltic electricity market.

DECLARATION OF INTEREST

The author reports no conflicts of interest. The author alone is responsible for the content and writing of the paper.

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