Stochastic modeling of imperfect markets

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1 Abstract

Stochastic optimization approaches ignore that the decisions of different actors in markets typically do not lead to a system-wide optimal solution. Suppliers in markets with entrance barriers or other aspects that hinder competition, can use their dominant positions to exert market power and drive up market prices. To represent such *gaming behavior*, a different modeling approach is needed. Equilibrium models can represent varying market structures, including perfect competition and oligopoly. This chapter presents a multi-stage stochastic equilibrium model for a general commodity market wherein suppliers, transportation agents and storage agents make capacity investment decisions while facing uncertainty in future market circumstances and production and sales decisions in later stages when the uncertainty has been revealed. An illustrative example for the natural gas market is used to show how market power may affect decisions and expected profits, and discuss the value of the stochastic solution for the different agents in our gaming setting.

Keywords: stochastic equilibrium model, resource markets, Karush-Kuhn-Tucker conditions, investment under uncertainty, value of stochastic solution

2 Introduction

Many resource markets are characterized by uneven resource distribution, highly expensive up-front investments for exploration and production, as well as by high transportation and storage costs due to physical and regulatory constraints. As a consequence, barriers to enter such markets are high, which reduces competition and allows market participants to exert market power over consumers.

Moreover, the future development of demand is uncertain and driven by factors largely external to the commodity market in question. Suppliers have to make their decisions on infrastructure investments ahead of time, and thereby risk creating significant overcapacities, or missing out on business opportunities. These factors pose great risks for the firms and consumers involved, and increase price volatility. Examples of unforeseen dramatic market changes in recent years include the rapid expansion of US natural gas and oil production due to technological advancements in shale-bed exploration and exploitation (Stevens 2010), the significant drop in oil prices since June 2014 due to overcapacity and low demand (Bowler 2015), and the boom and bust in the Australian mining business (Yeomans, 2016).

Hence, on the one hand, consumer prices are affected by the technological and general economic development; while on the other hand, they are prone to manipulation by suppliers. As a consequence, market prices can significantly differ from the fundamental supply cost of goods. When analyzing commodity markets, these two major factors, market power and uncertainty, should be represented in the model. In the following, we briefly touch on the basic concepts of mathematical models which can account for market power exertion and exogenous uncertainty.

Markets are usually represented by the game-theoretical concept of the Nash-equilibrium (Nash 1950). In a commodity market, Nash equilibrium is reached if all suppliers produce at an output level that leads to a market price that does not incentivize any suppliers to change his output. Mathematically, the behavior of each supplier is modeled by an optimization problem. Consequently, the Nash-equilibrium is equivalent to a vector of outputs fulfilling the Karush-Kuhn-Tucker conditions of all optimization problems *simultaneously*. Such problems are referred to as equilibrium problems (Gabriel et al. 2013).

For the special case of a perfectly competitive market, the corresponding market equilibrium coincides with the one resulting from social welfare maximization under the assumption that the optimization problems of the individual suppliers are convex (Bertocchi et al. 2011). Hence, in practice, for competitive markets rather than an equilibrium problem, the equivalent optimization problem is usually solved.

Uncertainty (for instance in demand) can be incorporated into a model by defining a scenario tree representing different outcomes of random events (e.g., different inverse demand functions). Subsequently, the suppliers optimize the value of their objective function, given all possible scenarios, and the respective probabilities that the scenarios will play out. Unfortunately, as the number of variables scales with the number of scenarios and time steps (stages), stochastic models can quickly become intractable. One common approach to solving stochastic problems with large data instances is to decompose the problem into subproblems, and iteratively solve these to provide a solution for the original problem. One such method is Benders decomposition (see for instance Gabriel & Fuller (2010), and Egging (2013) for development and application of Benders decomposition variants to energy market models). Another approach recently proposed by Devine et al. (2016) is to limit the time steps considered for investment decisions to a subset of the entire planning horizon, and to gradually roll the optimization horizon forward in time; this technique is similar to "receding horizon control", which is frequently applied in process control. The obtained results are suboptimal, but may approximate the planning process in practice, particularly if scenarios with a low probability materialize.

Several approaches to model imperfectly competitive markets exist. In contrast to including uncertainty, which mainly increases the *size* of the problem, the incorporation of market power increases the *mathematical complexity* of the formulation, and hence, a previously tractable large-scale problem can become intractable. To avoid this, the key is to preserve the convexity properties of the original formulation. We can distinguish between single-level and multi-level formulations: specifically, single-level models preserve convexity, while multi-level models typically do not.

Markets with competition à la Bertrand, Cournot, Cournot with competitive fringe, and conjectural variations[1] fall into the category of single-level formulations. A model with agents competing à la Bertrand has a solution equivalent to the perfectly competitive market and can be represented as a welfare maximization problem. The remaining single-level problems can be represented by mixed complementarity problems (MCP). Examples include Boots et al. (2004), Egging & Gabriel (2006), Egging et al. (2008), Holz et al. (2008), and Lise et al. (2008) for the natural gas market, Haftendorn & Holz (2008) for the coal market, and Huppmann & Holz (2009) for the oil market. Besides energy and commodity markets, other industries are also affected by market power exertion by dominant players and are studied with similar model types. See, for instance, Alves & Forte (2015) who study the effects of an open sky agreement in the airline industry, and Hu et al. (2014) who study vegetable supply chains in Taiwan.

Multi-level approaches, such as the Stackelberg leader-follower game, can be modeled as Mathematical Problems with Equilibrium Constraints (MPEC), and can require even more involved formulations. Examples include Chen et al. (2006), who model emission allowance markets with a Stackelberg game, and Huppmann (2013) who investigates the development of oil prices via a Stackelberg game. As MPEC representations are non-convex, and therefore not suitable for modeling large-scale markets, we refrain from going into more detail here.

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[1] Conjectural variations concern supplier perception of how competitors adjust their supply in response to a price change. A value of 1 implies that competitors will adjust so much that total supply in the market stays the same; this is equivalent to a perfectly competitive market. A value of 0 implies that competitors will not respond at all; this is equivalent to a Cournot oligopoly. Conjectural variations have been used by many researchers, especially in energy markets. See, e.g., Day et al. (2002), Hobbs et al. (2004); Egging & Gabriel (2006), Lise et al. (2006), and Holz et al. (2008). See also footnote 7.

In the remainder of this chapter, we present a stochastic model for commodity and resource markets with market power exertion using conjectural variations. This problem can be formulated as an MCP and is thus suitable to represent large-scale markets – a pre-requisite when assessing resource / energy / commodity markets on a global scale. In the next section, we introduce the notation and formulate the model. In Section 4, we demonstrate the capabilities of the model with a small, insightful example, and discuss the effects of market power and consideration of uncertainty by the suppliers. Section 5 summarizes and concludes this chapter. Appendices at the end of the Chapter as well as online provide additional details and a model implementation.

3 A stochastic equilibrium model for resource markets

Suppliers produce resources in one or more nodes (countries) and sell the resources to consumers in various countries, including domestically. Suppliers can invest in production capacity expansions to allow for higher future production levels. A supplier can export resources through a transportation network and can use storage to arbitrage between seasons in a year, and thereby, benefit from differences in consumers' willingness to pay between seasons. Infrastructure service operators manage and operate the available transport and storage capacities. To make use of infrastructure services, the suppliers must pay a cost, consisting of a base tariff plus a market-determined congestion rate, for each unit of capacity used. Suppliers and service operators maximize their expected discounted profit. Consumers are represented by downward-sloping affine inverse demand curves, which represent consumer surplus maximization. The information structure is a stochastic scenario tree, which is common to all agents in the market (see **Figure 1**). **Figure 2** shows a stylized example of a resource market with the notation used in the model formulation below.

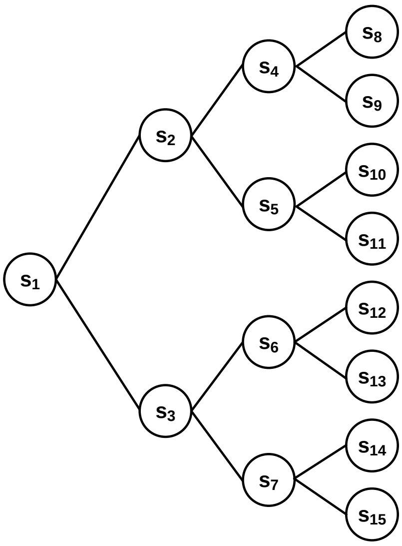

Figure 1. Scenario tree illustration

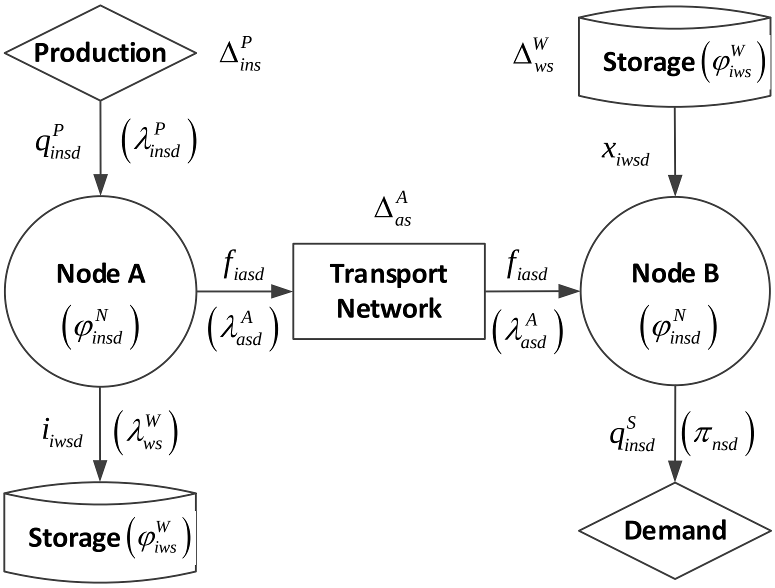

Figure 2. Value chain illustration with variable names (dual variables in parentheses)

3.1 Model formulation

Each market agent is represented by an expected profit maximization problem subject to operational and engineering constraints. The interaction between agents is governed by market clearing conditions. To allow determination of market equilibria, Karush-Kuhn-Tucker (KKT) conditions of all the suppliers and service operators are derived and combined with the market clearing conditions. This leads to an instance of an MCP.

Table 1 introduces the notation used in the model formulation below. For a natural gas market model, typical units of measurements for volumes (*rates*) and capacities are millions of cubic meters per day (Mcm/d) or billion cubic feet per day (bcf/d). Operational costs and prices can be in USD per thousand cubic meters ($/kcm) or USD per 1000 cubic feet $/mcf). Expansion costs may be denoted in $/mcm/d or $/bcf/d.

Table 1: Notation used to formulate the resource model.

Indices and Sets

$a \in A$	Transportation arcs
$A_n^+,\ A_n^-$	Inward resp. outward arcs from node *n*
$d \in D$	Demand seasons
$i, j \in I$	Suppliers
$n \in N$	Geographical location nodes
$s \in S$	Scenario tree nodes
$pred(s)$	Predecessors of scenario tree node[2]
$succ(s)$	Successors of scenario tree node[3]
$w \in W$	Seasonal storages
W_n	Storages at node *n*

Parameters

A_{nsd}	Intercept of inverse demand curve at location *n* for scenario node *s* in season *d*
B_{nsd}	Slope of inverse demand curve
c_{as}^{A}	Base cost for transport, linear
$c_{ins}^{P}(\ )$	Production cost, convex
c_{ws}^{W}	Base cost for usage of seasonal storage, linear
$c_{as}^{\Delta A}$	Arc expansions costs, linear
$c_{ins}^{\Delta P}$	Production capacity expansion cost, linear
$c_{ws}^{\Delta W}$	Storage expansions costs, linear
δ_{insd}^{CV}	Conjectural variation value[4]
$\overline{\Delta}_{as}^{A}$	Bound on arc expansion
$\overline{\Delta}_{ins}^{P}$	Bound on production capacity expansion
$\overline{\Delta}_{ws}^{W}$	Bound on storage capacity expansion
γ_s	Discount rate for scenario node
l_a^A	Loss rate for transport
l_w^W	Loss rate for storage injections
L_d	(Relative) length of season
n_a^+	End node of arc
n_a^-	Start node of arc
n_w	Node where storage is located
$\overline{Q}_{as}^{A}$	Initial arc capacity[5]
$\overline{Q}_{ins}^{P}$	Initial production capacity[Error! Bookmark not defined.]
$\overline{Q}_{ws}^{W}$	Initial capacity of seasonal storage[Error! Bookmark not defined.]
$\Pi_{insd}(\ )$	Supplier-specific price function
$\Pi_{nsd}(\ )$	Inverse demand curve
θ_s	Probability of scenario tree node

Variables (all nonnegative)

Δ_{as}^{A}	Arc capacity expansion
Δ_{ins}^{P}	Production capacity expansion
Δ_{ws}^{W}	Storage capacity expansion
f_{asd}^{A}	Total allocated transport capacity, auxiliary[6]
f_{iasd}	Supplier transport capacity usage
i_{wsd}^{W}	Total alloc. storage injection capacity, aux.
i_{iwsd}	Supplier storage injection rate
q_{insd}^{P}	Supplier production rate
q_{insd}^{S}	Supplier sales rate
x_{wsd}^{W}	Total alloc. storage extraction capacity, aux.
x_{iwsd}	Supplier storage extraction rate

Duals – nonnegative, unless indicated

λ_{asd}^{A}	Dual to arc capacity
λ_{insd}^{P}	Dual to production capacity
λ_{ws}^{W}	Dual to capacity of seasonal storage
$\varphi_{insd}^{N} \in \mathbb{R}$	Dual to nodal mass balance
$\varphi_{iws}^{W} \in \mathbb{R}$	Dual to supplier storage cycle
$\pi_{nsd} \in \mathbb{R}$	Wholesale market price, auxiliary
$\rho_{as}^{\Delta A}$	Dual to arc expansion limit
$\rho_{ins}^{\Delta P}$	Dual to production capacity expansion limit
$\rho_{ws}^{\Delta W}$	Dual to expansion limit of seasonal storage
τ_{asd}^{A}	Transport arc usage charge, auxiliary

[2] E.g., in Figure 1: $s' \in pred(s_{13}) = \{s_1, s_3, s_6\}$

[3] E.g., in Figure 1: $s' \in succ(s_3) = \{s_6, s_{13}\}$

[4] Value in the range 0 to 1; closer to 1 represents a higher level of market power.

[5] Subscript *s* allows including exogenous capacity expansions, e.g., known to come on stream in the future.

[6] Auxiliary variables are used when setting up the model, but will be substituted out before implementation.

3.1.1 Optimization problem of the supplier

A supplier produces, transports, stores, and sells goods. Transportation and storage services are obtained from external providers. These service providers charge a minimum base fee equal to their marginal cost (all agents are *price takers* in the markets for infrastructure services), and an additional congestion charge whenever capacity is restrictive. All storage costs (and losses, if applicable) are accounted for at the moment the goods are added to storage (*injections*). Each supplier i maximizes expected discounted profits Eq. (3.2.1) and is subject to production capacity constraints (3.2.2), production capacity expansion constraints (3.2.3), nodal mass balance constraints (3.2.4), and storage cycle constraints (3.2.5). Decision variables are production q_{insd}^{P}, production capacity expansions Δ_{insd}^{P}, transport shipments (*flows*) f_{iasd}^{A}, storage injection i_{iwsd} and extraction rate x_{iwsd} and sales levels q_{insd}^{S} in all stages and seasons. Objective function (3.2.1) is a probability weighted (θ_s) discounted (γ_s) sum of revenues $L_d \pi_{insd}(\ ) q_{insd}^{S}$, production costs $L_d c_{ndm}^{P}\left(q_{insd}^{P}\right)$, transport costs $\tau_{asd}^{A} f_{iasd}^{A}$, storage costs $\tau_{wsd}^{W} i_{iwsd}$, and capacity expansion costs $c_{ins}^{\Delta}\Delta_{ins}^{P}$.[7] The nodal mass balance constraint (3.2.4) states that production, storage extractions and inward flows must meet sales, storage injections, and outward flows. Storage cycle constraints (3.2.5) state that the sum of loss-corrected storage injections must cover extractions. This assumes that injections occur before extractions, but leaves flexibility in which season injections and extractions occur.

$$\max_{\substack{q_{insd}^{P}, i_{iwsd}, x_{iwsd}, \\ f_{iasd}, q_{insd}^{S}, \Delta_{ins}^{P}}} \sum_{n,s} \theta_s \gamma_s \left\{ \sum_d L_d \begin{pmatrix} \Pi_{insd}(\ ) q_{insd}^{S} - c_{ins}^{P}\left(q_{insd}^{P}\right) \\ -\sum_{a \in A_n^{+}} \tau_{asd}^{A} f_{iasd} - \sum_{w \in W_n} \tau_{wsd}^{W} i_{iwsd} \end{pmatrix} - c_{ins}^{\Delta P}\Delta_{ins}^{P} \right\} \tag{3.2.1}$$

s.t.

$$q_{insd}^{P} \le \overline{Q}_{ins}^{P} + \sum_{s' \in pred(s)} \Delta_{ins'}^{P} \qquad \left(\lambda_{insd}^{P}\right) \qquad \forall n,s,d \tag{3.2.2}$$

$$\Delta_{ins}^{P} \le \overline{\Delta}_{ins}^{P} \qquad \left(\rho_{ins}^{\Delta P}\right) \qquad \forall n,s \tag{3.2.3}$$

$$q_{insd}^{P} + \sum_{w \in W_n} x_{iwsd} + \sum_{a \in A_n^{+}} \left(1 - l_a^{A}\right) f_{iasd} = q_{insd}^{S} + \sum_{w \in W_n} i_{iwsd} + \sum_{a \in A_n^{-}} f_{iasd} \qquad \left(\varphi_{insd}^{N}\right) \qquad \forall n,s,d \tag{3.2.4}$$

[7] Note that the definition of the sales price function $\Pi_{insd}(\ )$ implements the competitive behavior of the supplier. Having $\Pi_{insd}(\ ) = \pi_{nsd}$ implies perfectly competitive behavior, i.e., taking the market price as given. Incorporating the inverse demand curve $\Pi_{insd}(\ ) = \Pi_{nsd}\left(\sum_i q_{insd}^{S}\right) = A_{nsd} - B_{nsd}\sum_i q_{insd}^{S}$ represents oligopolistic behavior à la Cournot. Intermediate market power levels in hybrid market structures between perfectly competitive and oligopoly can be represented using conjectural variation values δ_{ins}^{CV} in the following way: $\Pi_{insd}(\ ) = \delta_{insd}^{CV}\Pi_{nsd}\left(\sum_i q_{insd}^{S}\right) + \left(1 - \delta_{insd}^{CV}\right)\pi_{nsd}$.

$$\left(1-l_w^W\right)\sum_d L_d i_{iwsd} = \sum_d L_d x_{iwsd} \qquad \left(\varphi_{iws}^W\right) \qquad \forall w,s \qquad \textbf{(3.2.5)}$$

3.1.2 Optimization problem of the transport system operator

The transport system operator (TSO) manages and operates the network of transportation arcs by allocating transport capacity f_{asd}^A and expanding the network Δ_{as}^A. The TSO's objective is to maximize the expected value of the network, which is reflected by the probability-weighted, discounted congestion revenues minus operational and investment costs (3.2.6), subject to are capacity constraints (3.2.7) and restrictions to allowable expansions (3.2.8).

$$\max_{f_{asd}^A,\Delta_{as}^A} \sum_{a,s}\theta_s\gamma_s\left(\sum_d L_d\left(\tau_{asd}^A - c_{as}^A\right)f_{asd}^A - c_{as}^{\Delta A}\Delta_{as}^A\right) \qquad \textbf{(3.2.6)}$$

$$\text{s.t.} \quad f_{asd}^A \le \overline{Q}_{as}^A + \sum_{s'\in pred(s)}\Delta_{as'}^A \qquad \left(\lambda_{asd}^A\right) \qquad \forall a,s,d \qquad \textbf{(3.2.7)}$$

$$\Delta_{as}^A \le \overline{\Delta}_{as}^A \qquad \left(\rho_{as}^{\Delta A}\right) \qquad \forall a,s \qquad \textbf{(3.2.8)}$$

3.1.3 Optimization problem of the storage system operator

The Storage System Operator (SSO) manages and operates seasonal storage.[8] The SSO rents out storage injection capacity i_{wds}^W to suppliers and expands storage capacity by Δ_{ws}^W. The SSO's objective is to maximize the discounted congestion revenues minus investment costs, subject to storage capacity constraints.[9] All storage costs and losses are accounted for when goods are injected (added) to storage. Note that suppliers are responsible for the storage cycle balance, see Eq. (3.2.5) above.

$$\max_{i_{wsd}^W,\Delta_{ws}^W} \sum_{w,s}\theta_s\gamma_s\left(\sum_d L_d\left(\tau_{wsd}^W - c_{ws}^W\right)i_{wsd}^W - c_{ws}^{\Delta W}\Delta_{ws}^W\right) \qquad \textbf{(3.2.9)}$$

$$\text{s.t.} \quad \left(1-l_w^W\right)\sum_d L_d i_{wsd}^W \le \overline{Q}_{ws}^W + \sum_{s'\in pred(s)}\Delta_{ws'}^W \qquad \left(\lambda_{ws}^W\right) \qquad \forall w,s \qquad \textbf{(3.2.10)}$$

$$\Delta_{ws}^W \le \overline{\Delta}_{ws}^W \qquad \left(\rho_{ws}^{\Delta W}\right) \qquad \forall w,s \qquad \textbf{(3.2.11)}$$

3.2 Market clearing conditions

Eq. (3.3.1) equilibrates allocated and used transport capacity between the TSO and the suppliers.

$$f_{asd}^A = \sum_j f_{jasd} \qquad \left(\tau_{asd}^A\right) \qquad \forall a,s,d \qquad \textbf{(3.3.1)}$$

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[8] The formulation given in this section can be too restrictive for other storage types, in particular for storages with more than one injection/extraction cycle per stage.

[9] We refrain from adding constraints on injection/extraction in this basic formulation.

Eq. (3.3.2) balances allocated and used storage injection capacity between the SSO and the suppliers.

$$i^W_{wsd} = \sum_j i_{jwsd} \qquad (\tau^W_{wsd}) \qquad \forall w,s,d \qquad \textbf{(3.3.2)}$$

The inverse demand curve, Eq. (3.3.3), is the relationship between market price and supplied quantities.

$$\pi_{nsd} = A_{nsd} - B_{nsd} \sum_j q^S_{jnsd} \qquad (\pi_{nsd}) \qquad \forall n,s,d \qquad \textbf{(3.3.3)}$$

Note that all market clearing conditions are substituted out of the formulation in a later section, based on a result by Baltensperger et al. (2016). Details for this are provided in the Appendix.

3.3 Formulation as a mixed complementarity problem

MCPs can be used to determine equilibria in imperfect markets (Ferris & Pang (1997), Facchinei & Pang (2003)). Formally, for a given function $F:\mathbb{R}^{\bar{n}} \to \mathbb{R}^{\bar{n}}$, the MCP is to find a vector $x \in \mathbb{R}^{\bar{n}}$ such that:

$$\begin{cases} x_i = b_i & \Rightarrow F_i(x) \le 0 \\ a_i < x_i < b_i & \Rightarrow F_i(x) = 0 \\ x_i = a_i & \Rightarrow F_i(x) \ge 0 \end{cases}$$

where a and b are lower and upper bound vectors, respectively, and $a_i, b_i \in \mathbb{R} \cup \{-\infty, +\infty\}, a_i < b_i, \forall i = 1,2,\ldots,\bar{n}$ (Facchinei & Pang 2003). For the application at hand, the value of $a = 0$ and of $b = \infty$; the MCP reduces to $0 \le x_i \perp F_i(x) \ge 0, \forall i$, where the *complementarity* operator $\perp$ indicates that the product $x_i F_i(x) = 0, \forall i$.

To transform a collection of optimization problems and market clearing conditions into an MCP, Karush-Kuhn-Tucker (KKT) optimality conditions are derived for all the optimization problems. Each optimization problem can be brought into the form:

$$\begin{aligned} \min \quad & f(z) \\ s.t. \quad & g(z) \le 0 \\ & h(z) = 0 \end{aligned}$$

with $f:\mathbb{R}^n \to \mathbb{R}, g:\mathbb{R}^n \to \mathbb{R}^m, h:\mathbb{R}^n \to \mathbb{R}^l$, and 0 a zero-vector with appropriate dimensions. The corresponding KKT conditions are as follows.

$$\begin{aligned} & \nabla f(z^*) + \sum_{j=1}^m \mu_j \nabla g_j(z^*) + \sum_{k=1}^l \nu_k \nabla h_k(z^*) = 0 \\ & g_j(z^*) \le 0,\ j \in \{1,\ldots,\mathrm{m}\} \\ & h_k(z^*) = 0, k \in \{1,\ldots,\mathrm{l}\} \\ & \mu_j \ge 0,\ j \in \{1,\ldots,\mathrm{m}\} \\ & \mu_j g_j(\mathrm{z}^*) = 0,\ j \in \{1,\ldots,\mathrm{m}\} \end{aligned}$$

By combining primal variables z, and dual variables μ and ν of all optimization problems, and all variables from the market clearing conditions in x, and by combining all KKT and market clearing conditions in $F(x)$, the transformation to an MCP is completed.

3.3.1 Complementarity conditions related to supplier decisions

Appendix A to this Chapter contains the derivation steps, including details for the derivation of KKT conditions, and the substitution of variables and equations. Here, we present the resulting, final model.

$$\forall i,n,s,d: \quad 0 \le q^P_{insd} \perp \theta_s \gamma_s L_d \frac{dc^P_{ins}\left(q^P_{insd}\right)}{q^P_{insd}} + \lambda^P_{insd} - \varphi^N_{insd} \ge 0 \tag{3.5.1}$$

$$\forall i,n,s: \quad 0 \le \Delta^P_{ins} \perp \theta_s \gamma_s c^{\Delta P}_{ins} - \sum_{s' \in succ(s),d} \lambda^P_{ins'd} + \rho^{\Delta P}_{ins} \ge 0 \tag{3.5.2}$$

$$\forall i,a,s,d: \quad 0 \le f_{iasd} \perp \theta_s \gamma_s L_d c^A_{as} + \lambda^A_{asd} + \varphi_{in^-_a sd} - \left(1 - l^A_a\right) \varphi_{in^+_a sd} \ge 0. \tag{3.5.3}$$

$$\forall i,n,s,d: \quad 0 \le q^S_{insd} \perp -\theta_s \gamma_s L_d \left(A_{nsd} - B_{nsd} \left(\sum_j q^S_{jnsd} + \delta^{CV}_{insd} q^S_{insd} \right) \right) + \varphi^N_{insd} \ge 0 \tag{3.5.4}$$

$$\forall i,w,s,d: \quad 0 \le i_{iwsd} \perp \theta_s \gamma_s L_d c^W_{ws} + \left(1 - l^W_w\right) L_d \lambda^W_{ws} + \varphi^N_{insd} - \left(1 - l^W_w\right) L_d \varphi^W_{ins} \ge 0 \tag{3.5.5}$$

$$\forall i,w,s,d: \quad 0 \le x_{iwsd} \perp -\varphi^N_{insd} + L_d \varphi^W_{ins} \ge 0 \tag{3.5.6}$$

$$\forall i,n,s,d: \quad 0 \le \lambda^P_{insd} \perp q^P_{insd} - \overline{Q}^P_{ins} - \sum_{s' \in pred(s)} \Delta^P_{ins'} \ge 0 \tag{3.5.7}$$

$$\forall i,n,s: \quad 0 \le \rho^{\Delta P}_{ins} \perp \overline{\Delta}^P_{ins} - \Delta^P_{ins} \ge 0 \tag{3.5.8}$$

$$\forall i,n,s,d: \quad \varphi^N_{insd} \quad fis, \; q^P_{insd} + \sum_{w \in W_n} x_{iwsd} + \sum_{a \in A^+_n} \left(1 - l^A_a\right) f^A_{iasd} - q^S_{insd} - \sum_{w \in W_n} i_{iwsd} - \sum_{a \in A^-_n} f^A_{iasd} = 0 \tag{3.5.9}$$

$$\forall i,w,s: \quad \varphi^W_{iws} \quad fis, \left(1 - l^W_w\right) \sum_d L_d i_{iwsd} = \sum_d L_d x_{iwsd} \tag{3.5.10}$$

3.3.2 Complementarity conditions related to TSO decisions

$$\forall a,s,d: \quad 0 \le \Delta^A_{as} \perp \theta_s \gamma_s c^{\Delta A}_{as} - \sum_{s' \in succ(s),d} \lambda^A_{as',d} + \rho^{\Delta A}_{as} \ge 0 \tag{3.5.11}$$

$$\forall a,s,d: \quad 0 \le \lambda^A_{asd} \perp \overline{Q}^A_{as} + \sum_{s' \in pred(s)} \Delta^A_{as'} - \sum_j f_{jasd} \ge 0 \tag{3.5.12}$$

$$\forall a,s,d: \quad 0 \le \rho^A_{as} \perp \overline{\Delta}^a_{as} - \Delta^a_{as} \ge 0 \tag{3.5.13}$$

3.3.3 Complementarity conditions related to SSO decisions

$$\forall w,s: \qquad 0 \le \Delta_{ws}^{W} \perp \theta_s \gamma_s c_{ws}^{\Delta W} - \sum_{s' \in succ(s)} \lambda_{ws'}^{W} + \rho_{ws}^{\Delta W} \ge 0 \qquad \textbf{(3.5.14)}$$

$$\forall w,s: \qquad 0 \le \lambda_{ws}^{W} \perp \overline{Q}_{ws}^{W} + \sum_{s' \in pred(s)} \Delta_{ws'}^{W} - \left(1 - l_w^W\right) \sum_{j,d} L_d i_{jwsd}^{W} \ge 0 \qquad \textbf{(3.5.15)}$$

$$\forall w,s: \qquad 0 \le \rho_{ws}^{\Delta W} \perp \overline{\Delta}_{ws}^{W} - \Delta_{ws}^{W} \ge 0 \qquad \textbf{(3.5.16)}$$

These sixteen KKT conditions are implemented in GAMS (Brooke et al. 1998), parameterized to represent a stylized natural gas market (details in Appendix C), and solved for equilibrium using the PATH solver (Dirkse & Ferris (1995), Ferris & Munson (2000)).

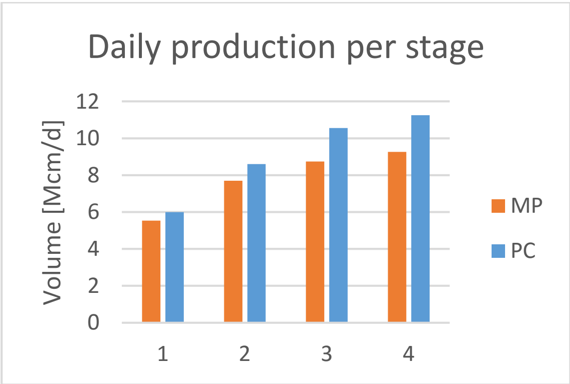
4 An imperfect natural gas market

In this section, we first demonstrate the effects of market power exertion by suppliers on the market outcomes, and next, we look into the impact of uncertainty on the decisions of the suppliers and service operators, and discuss the value of stochastic solution (VSS) to the various agents.

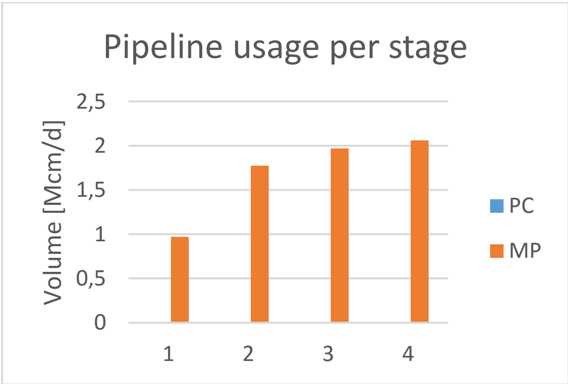
4.1 Effects of market power

To demonstrate the effects of market power, we introduce a model with three nodes, where each node is connected to both of the other nodes. This setting is symmetrical for all suppliers and consumers, and leads to symmetrical supply, consumption, and trade / pipeline usage. The symmetry allows a succinct presentation and discussion of the results because production and consumption in all three nodes are the same in an equilibrium, as are the flows in all six pipelines.

We compare two market settings: in the first case, all suppliers behave in a perfectly competitive (PC) manner, while in the second case, they exert market power à la Cournot (MP). Here, we exclude uncertainty and seasonality and assume perfect foresight over all four stages of the simulations, such that the effects of market power can be assessed independently from the effects of uncertainty.



a)



b)

Figure 3: a) Daily production in each node, b) daily pipeline usage. Consumption levels are not shown as they correspond to production minus losses from transportation and storage, and hence, are similar to production levels.

Market-power exerting agents withhold supplies to drive up market prices. Figure 3a shows that production is significantly lower in the MP case in all stages. Compared to competitive suppliers, market-power exerting suppliers have an incentive to supply lower amounts domestically (to drive up domestic prices), while diversifying supplies to other markets (see also Egging & Gabriel (2006) or Baltensperger et al. (2016)). This diversification is illustrated clearly in Figure 3b by the high pipeline usage in MP. In fact, in our symmetrical network, in the PC case suppliers only supply at their own node causing a welfare maximizing and cost-minimizing equilibrium outcome. Finally, in MP lower supply implies lower consumption, and thus higher prices (c.f., the inverse demand function, Footnote 7).

4.2 Impact of uncertainty on market outcomes

We extend the model with three interconnected nodes from above by allowing investment in storage capacity at node two and imposing uncertainty using the scenario tree in **Figure 1**. We assume all suppliers to behave à la Cournot. We represent uncertainty by varying the intercept of the inverse demand curve among the scenarios in future stages (see Table 3 in Appendix C). This adjusted setting is symmetrical for suppliers and consumers in nodes one and three, as well as the pipelines to and from nodes one and three to node two, which allows a succinct analysis and presentation of the results.

We carry out two simulations: a deterministic benchmark problem considering the storage expansion option, and the full stochastic problem. In the deterministic problem all agents base their choice of actions on the assumption that the average scenario will happen with 100% certainty: the *Expected Value* problem (EV). In the stochastic problem, all agents consider all scenarios individually and maximize expected profits under the assumption that all scenarios are equally likely: the recourse problem (RP) (cf., (Birge & Louveaux 2011)).

First, we analyze the impact of uncertainty on investment decisions. By assumption, production capacity expansions in each stage cannot be higher than four Mcm/d, and storage and pipeline expansions cannot be higher than two Mcm/d.

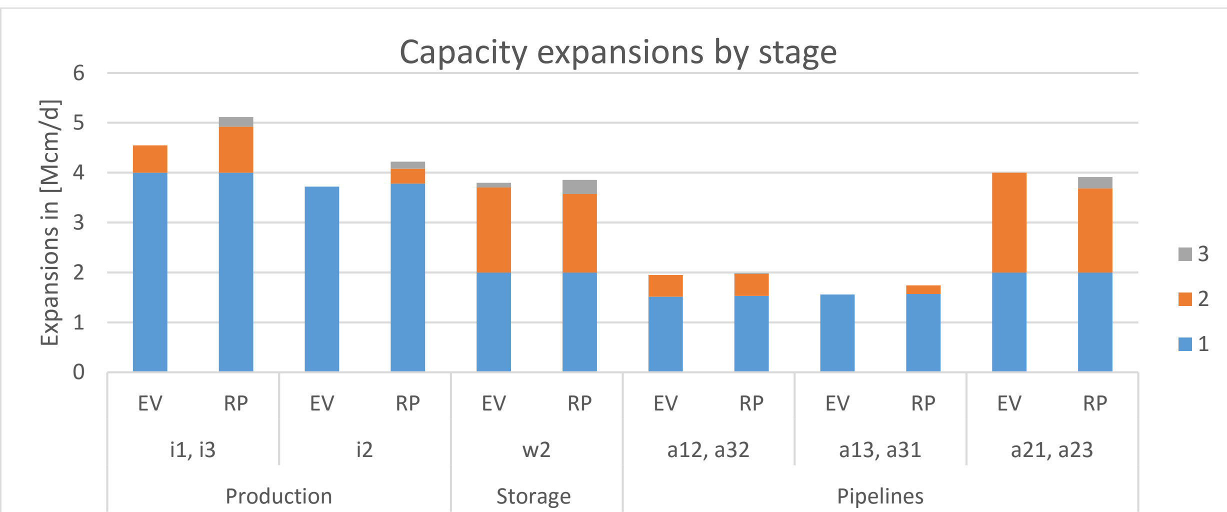


Figure 4: Average capacity expansions by stage in production, storage, and pipeline capacities in EV and RP. Due to symmetry in the problem characteristics, several bars represent results for multiple indices.

Figure 4 shows that the infrastructure expansions are generally higher in RP, most notably in production capacity. At nodes one and three, production capacity expansions (i1, i3) in the first stage are restricted to four by the upper bound. Similarly, the first-stage expansions of storage w2 at node two, and of arcs from node two to nodes one (a21) and three (a23) are at the upper bound. Where upper bounds are not restrictive, the first-stage expansions in RP are slightly higher, indicating that the upward profitability potential, due to the uncertainty, more than outweighs the downward potential and that expected profitability warrants higher investment early on. That expansions of a21 and a23 are much higher than of the other arcs, indicates that the storage facility to be constructed at node two also affects the supply at the other nodes. In the second stage, the picture is mixed. Storage investment is lower in RP than in EV, whereas pipeline expansions vary. Furthermore, we observe that some storage and pipeline investments are delayed. The possibility for *recourse*, corrective decisions in later stages after some of the uncertainty has been resolved, results in third-stage investment in capacity in the stochastic problem in all capacity types. In contrast, only very small investment in storage will happen in EV. Here, we see a culmination of two effects. One the one hand, only infrastructures investments that are profitable in expectation are made. On the other hand, in later stages, the capacities are optimized considering fewer scenarios, which boosts the weight of profitable investments in the investment decision.

Naturally, we should not draw hard conclusions based on such a small example. It illustrates, however, the combined impact of 1. Market power, which results in more trade and pipeline capacity expansions compared to perfect competition, and 2. The impact of uncertainty, which tends to favor higher capacity expansion in general to benefit from profitability opportunities while hedging upward and downward potential (the latter might be different for risk-averse agents).

4.3 Value of stochastic solution

The value of the stochastic solution (VSS) is a concept that shows the added value of considering uncertainty explicitly in optimization models (Birge & Louveaux 2011). To determine the VSS, we carry out additional simulations: we take the capacity expansions in each stage from the EV solution and fix the expansion values, and thereby, all capacities in the RP. The resulting model will not determine any new investment decisions, but only the optimal operational volumes (production, flow, and storage injection and extraction) in the stochastic setting given the now exogenous capacities. The result is referred to as the Expected outcome of the Expected Value (EV) problem: the EEV.

Because we don’t have an optimization problem, but an equilibrium problem, the VSS definition does not translate directly to the multi-agent settings. Still, to give some insight in the impact of including uncertainty, we use the concept. Formally, the VSS is defined as the difference between the maximized objective value of the RP and the EEV (Birge and Louveaux, 1997):

$$VSS = \max_{x,y} \mathrm{E}_{\upsilon}\left(a^T x + z\left(y\left(x,\bar{\upsilon}\right)\right)\right) - \mathrm{E}_{\upsilon} \max_{x,y}\left(a^T x + z\left(y\left(x,\bar{\upsilon}\right)\right)\right) \geq 0\,.$$

Figure 5 shows that the VSS is positive for most agents, but not for one supplier and the storage operator, and is also positive when considering the total social welfare (SW, the aggregate of agent profits and consumer surpluses). As it turns out, the on average slightly lower storage investment in EV vs RP causes the "EV" storage capacity to be more restrictive when evaluated in the stochastic setting: the EEV results. This drives up congestion rents and hence profitability of storages. A result by (Birge & Louveaux 2011) shows that VSS has to be non-negative for optimization models. However, several authors have found negative VSS for some of the agents in *multi-agent* game theoretic problems such as the one discussed above (e.g., Zhuang & Gabriel (2008), Genc et al. (2007), and Egging (2010)).

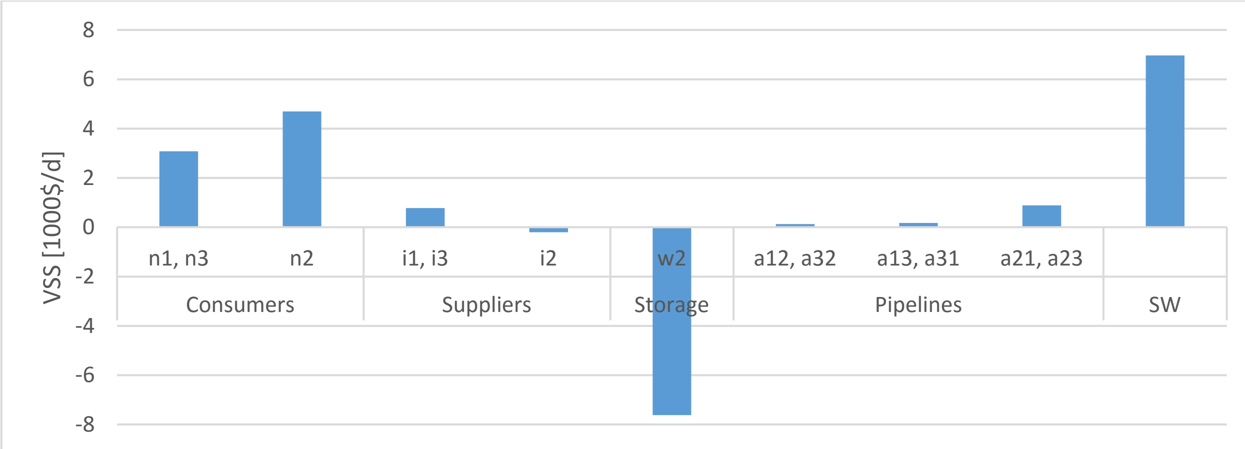


Figure 5: VVS by agent and on aggregate.

5 Conclusion

This chapter introduces a stochastic model to analyze resource markets under uncertainty where suppliers may exert market powers towards consumers. We consider capacity investment in production, storage, and the transportation network in a multi-stage stochastic model. A stylized case study for a four-stage problem considering a natural gas market on a three-node network shows the combined impact of market power and uncertainty. In this stylized network, we observe how market power exertion leads to higher investment in the transportation network, and how uncertainty leads to higher capacity investment in general. Additionally, we show that in this multi-agent gaming setting, the value of the stochastic solution is negative for some of the agents.

6 Model extensions

The stochastic MCP introduced in this chapter contains the elements necessary to illustrate the merits of this modeling approach. Several functional extensions have been introduced in the literature, although most of them ignore uncertainty. Table 2 gives an overview of extensions that have been included by researchers in MCPs, with references.

Table 2: Functional extensions to the introduced resource market model.

Challenge	**Addressed in**
Case study – global gas market	Egging (2010), Egging (2013), Egging & Holz (2016)
Daily variation	Huppmann & Egging (2014)
Decomposition – multiple fuels	Siggerud (2014)
Decomposition – single fuel	Egging (2010), Egging (2013)
Demand for energy services (rather than energy units)	Egging & Huppmann (2012), Huppmann & Egging (2014)
Investment budgets	Egging, Pichler, Kalvo & Walle-Hansen (2016)
Kirchhoff's laws in electricity markets	Leuthold et al. (2012)
Limitations in seasonal variation of infrastructure usage	Implemented in an industry project for a major French utility in the period 2009-2010 by coauthor Egging

Challenge	**Addressed in**
Multi-horizon uncertainty	Tomasgard et al. (2016), Egging Su & Tomasgard (2016)
Multi-level	Kalashnikov et al. (2010), Huppmann & Holz (2012), Neumann et al. (2011)
Multiple fuels	Egging & Huppmann (2012), Huppmann & Egging (2014), Abrell & Weigt (2012)
Resource / Reserve expansions	Huppmann & Egging (2014); Egging Su Tomasgard (2016)
Resource / Reserve limitations	Egging et al. (2008)
Risk aversion	Chin & Siddiqui (2014), Cabero et al. (2010), Luna et al. (2016), Egging, Pichler, Kalvo & Walle-Hansen (2016), Abada et al. (2014)
Supply contracts	Egging et al. (2010), Cabero et al. (2010), Chin & Siddiqui (2014)
Transformation	Egging & Huppmann (2012), Huppmann & Egging (2014)
Weymouth equations for pressure-flow in natural gas	Midthun (2007) (not an MCP but the discussed linearization of the Weymouth equations does allow for inclusion.)

7 Appendix A – Model derivation details

7.1 Supplier problem - standard form

To facilitate the derivation of Karush-Kuhn-Tucker (KKT) conditions, it is helpful to reformulate all maximization problems as minimization problems, and change all constraints to inequalities. In addition to the problem formulations presented in the main part of the chapter, inequality conditions and corresponding dual variables are added enforcing non-negativity for non-negative variables.

The intermediate derivation step is shown here for completeness, but is typically skipped by more experienced developers, since all newly introduced inequalities and dual variables are substituted out of the formulation again in a later step[10].

$$\min -Z_i = \sum_{n,s} \theta_s \gamma_s \left\{ \sum_d L_d \begin{pmatrix} c_{ins}^P \left(q_{insd}^P \right) + \sum_{a \in A_n^+} \left(c_{as}^A + \tau_{asd}^A \right) f_{iasd} \\ + \sum_{w \in W_n} \left(c_{ws}^W + \tau_{wsd}^W \right) i_{iwsd} - \Pi_{insd}\left(\quad \right) q_{insd}^S \end{pmatrix} + c_{ins}^{\Delta P} \Delta_{ins}^P \right\} \quad \textbf{(8.1.1)}$$

s.t.

$$q_{insd}^P - \overline{Q}_{ins}^P - \sum_{s' \in pred(s)} \Delta_{ins'}^P \leq 0 \quad \left(\lambda_{insd}^P \right) \quad \forall n,s,d \quad \textbf{(8.1.2)}$$

$$\Delta_{ins}^P - \overline{\Delta}_{ins}^P \leq 0 \quad \left(\rho_{ins}^{\Delta P} \right) \quad \forall n,s \quad \textbf{(8.1.3)}$$

$$-q_{insd}^P - \sum_{w \in W_n} x_{iwsd} - \sum_{a \in A_n^+} \left(1 - l_a^A \right) f_{iasd} + q_{insd}^S + \sum_{w \in W_n} i_{iwsd} + \sum_{a \in A_n^-} f_{iasd} = 0 \quad \left(\varphi_{insd}^N \right) \quad \forall n,s,d \quad \textbf{(8.1.4)}$$

[10] Concerning the supplier nodal mass balance equation Eq. (3.2.4) and storage cycle equation Eq. (3.2.5), there is no mathematical reason to change the '+' and '-' signs. However, starting from the perspective -SUPPLY + DEMAND ≤ 0 ensures that (i) the values of the free-in-sign duals will have a more intuitive interpretation, and (ii) when deriving the KKT-conditions in the following steps no adjustments are needed when implementing the model in GAMS (Brooke et al. 1998), and using the PATH solver (Dirkse & Ferris (1995), Ferris & Munson (2000)). The derivation, as presented here, ensures that the signs of the values in the Hessian matrix (of second order derivatives), which is used by the PATH solver, will be correct and consistent (www.gams.com/docs/pdf/path.pdf , http://pages.cs.wisc.edu/~ferris/path.html).

$$-\left(1-l_w^W\right)\sum_d L_d i_{iwsd} + \sum_d L_d x_{iwsd} = 0 \quad \left(\varphi_{iws}^W\right) \quad \forall w,s \tag{8.1.5}$$

$$-q_{insd}^P \le 0 \quad \left(\psi_{insd}^1\right) \quad \forall n,s,d \tag{8.1.6}$$

$$-\Delta_{ins}^P \le 0 \quad \left(\psi_{ins}^2\right) \quad \forall n,s \tag{8.1.7}$$

$$-f_{iasd} \le 0 \quad \left(\psi_{iasd}^3\right) \quad \forall a,s,d \tag{8.1.8}$$

$$-q_{insd}^S \le 0 \quad \left(\psi_{insd}^4\right) \quad \forall n,s,d \tag{8.1.9}$$

$$-i_{iwsd} \le 0 \quad \left(\psi_{iwsd}^5\right) \quad \forall w,s,d \tag{8.1.10}$$

$$-x_{iwsd} \le 0 \quad \left(\psi_{iwsd}^6\right) \quad \forall w,s,d \tag{8.1.11}$$

7.2 Lagrangian of the supplier problem

The next intermediate step is the formulation of the Lagrangian. (Like the previous step, this step is also typically skipped by more experienced developers, but is shown here for illustrative purposes.)

$$\begin{aligned}
L_i = \quad & \sum_{n,s}\theta_s\gamma_s\left\{\sum_d L_d\left(\begin{array}{l} c_{ins}^P\left(q_{insd}^P\right)+\sum_{a\in A_n^+}\tau_{asd}^A f_{iasd} \\ +\sum_{w\in W_n}\tau_{wsd}^W i_{iwsd} - \Pi_{insd}\left(\quad\right) q_{insd}^S \end{array}\right) + c_{ins}^{\Delta P}\Delta_{ins}^P\right\} \\
& +\sum_{n,s,d}\lambda_{insd}^P\left(q_{insd}^P - \overline{Q}_{ins}^P - \sum_{s'\in pred(s)}\Delta_{ins'}^P\right) + \sum_{n,s}\rho_{ins}^{\Delta P}\left(\Delta_{ins}^P - \overline{\Delta}_{ins}^P\right) \\
& +\sum_{n,s,d}\varphi_{insd}^N\left(-q_{insd}^P - \sum_{w\in W_n}x_{iwsd} - \sum_{a\in A_n^+}\left(1-l_a^A\right)f_{iasd} + q_{insd}^S + \sum_{w\in W_n} i_{iwsd} + \sum_{a\in A_n^-} f_{iasd}\right) \\
& +\sum_{w,s}\varphi_{iws}^W\left(-\left(1-l_w^W\right)\sum_d L_d i_{iwsd} + \sum_d L_d x_{iwsd}\right) \\
& +\sum_{n,s,d}\psi_{insd}^1\left(-q_{insd}^P\right) + \sum_{n,s}\psi_{ins}^2\left(-\Delta_{ins}^P\right) + \sum_{a,s,d}\psi_{iasd}^3\left(-f_{iasd}\right) \\
& +\sum_{n,s,d}\psi_{insd}^4\left(-q_{insd}^S\right) + \sum_{w,s,d}\psi_{iwsd}^5\left(-i_{iwsd}\right) + \sum_{w,s,d}\psi_{iwsd}^6\left(-x_{iwsd}\right)
\end{aligned} \tag{8.2.1}$$

7.3 Supplier problem - partial derivatives

Next, the partial derivatives of the Lagrangian of supplier *i* with respect to the original, non-auxiliary primal and dual variables are shown:

$$\frac{\partial L_i}{\partial q^P_{insd}} = \theta_s \gamma_s L_d \frac{dc^P_{ins}\left(q^P_{insd}\right)}{q^P_{insd}} + \lambda^P_{insd} - \varphi^N_{insd} - \psi^1_{insd} \qquad \forall n,s,d \qquad \textbf{(8.3.1)}$$

$$\frac{\partial L_i}{\partial \Delta^P_{ins}} = \theta_s \gamma_s c^{\Delta P}_{ins} - \sum_{s' \in succ(s),d} \lambda^P_{ins'd} + \rho^{\Delta P}_{ins} - \psi^2_{ins} \qquad \forall n,s \qquad \textbf{(8.3.2)}$$

$$\frac{\partial L_i}{\partial f_{iasd}} = \theta_s \gamma_s L_d \tau^A_{asd} - \left(1 - l^A_a\right) \varphi^N_{in^-_a sd} + \varphi^N_{in^+_a sd} - \psi^3_{iasd} \qquad \forall a,s,d \qquad \textbf{(8.3.3)}$$

$$\frac{\partial L_i}{\partial q^S_{insd}} = -\theta_s \gamma_s L_d \frac{d\left(\Pi_{insd}\left(\ \right) q^S_{insd}\right)}{dq^S_{insd}} + \varphi^N_{insd} - \psi^4_{insd} \qquad \forall n,s,d \qquad \textbf{(8.3.4)}$$

$$\frac{\partial L_i}{\partial i_{iwsd}} = \theta_s \gamma_s L_d \tau^W_{wsd} + \varphi^N_{in_w sd} - \left(1 - l^W_w\right) L_d \varphi^W_{iws} - \psi^5_{iwsd} \qquad \forall w,s,d \qquad \textbf{(8.3.5)}$$

$$\frac{\partial L_i}{\partial x_{iwsd}} = -\varphi^N_{in_w sd} + L_d \varphi^W_{iws} - \psi^6_{iwsd} \qquad \forall w,s,d \qquad \textbf{(8.3.6)}$$

$$\frac{\partial L_i}{\partial \lambda^P_{insd}} = q^P_{insd} - \overline{Q}^P_{ins} - \sum_{pred(s)} \Delta^P_{inm} - \psi^5_{insd} \qquad \forall n,s,d \qquad \textbf{(8.3.7)}$$

$$\frac{\partial L_i}{\partial \rho^{\Delta P}_{ins}} = \Delta^P_{ins} - \overline{\Delta}^P_{ins} - \psi^2_{ins} \qquad \forall n,s \qquad \textbf{(8.3.8)}$$

$$\frac{\partial L_i}{\partial \varphi^N_{insd}} = -q^P_{insd} - \sum_{w \in W_n} x_{iwsd} - \sum_{a \in A^+_n} \left(1 - l^A_a\right) f^A_{iasd} + q^S_{insd} + \sum_{w \in W_n} i_{iwsd} + \sum_{a \in A^-_n} f^A_{iasd} \qquad \forall n,s,d \qquad \textbf{(8.3.9)}$$

$$\frac{\partial L_i}{\partial \varphi^W_{iwsd}} = -\left(1 - l^W_w\right) \sum_d L_d i_{iwsd} + \sum_d L_d x_{iwsd} \qquad \forall w,s,d \qquad \textbf{(8.3.10)}$$

7.4 Karush Kuhn Tucker conditions

Karush Kuhn Tucker conditions are derived by setting the partial derivatives Eqs. (8.3.1)-(8.3.10) equal to zero and eliminating the auxiliary variables $\psi^i, i = 1,2,..,6$. We first present all KKT-conditions that follow from the optimization problems and market clearing conditions, before we show how several conditions can be eliminated to find an equivalent however more compact formulation.

7.4.1 KKT conditions - suppliers

The partial derivative of Z_i with respect to q^P_{insd}, resulted in Eq. (8.3.1). We set Eq. (8.3.1) equal to zero and moved the auxiliary variable to the right-hand side to get the following equality:

$$\theta_s \gamma_s L_d \frac{dc_{ins}^P\left(q_{insd}^P\right)}{q_{insd}^P} + \lambda_{insd}^P - \varphi_{insd} = \psi_{insd}^1 \quad \textbf{(8.4.1)}$$

For a KKT point, we have q_{insd}^P and ψ_{insd}^1 complementarity to each other (ref. Eq.(8.1.6) and Section 3.3). We used this to eliminate ψ_{insd}^1 and obtained the following:

$$0 \le \theta_s \gamma_s L_d \frac{dc_{ins}^P\left(q_{insd}^P\right)}{q_{insd}^P} + \lambda_{insd}^P - \varphi_{insd} = \psi_{insd}^1 \perp -q_{insd}^P \le 0 \quad \textbf{(8.4.2)}$$

Next, rewrite and reorder this to the following KKT condition:

$$\forall n,s,d: \qquad 0 \le q_{insd}^P \perp \theta_s \gamma_s L_d \frac{dc_{ins}^P\left(q_{insd}^P\right)}{q_{insd}^P} + \lambda_{insd}^P - \varphi_{insd}^N \ge 0 \quad \textbf{(8.4.3)}$$

In a similar way, we can derive the remaining KKT conditions for supplier *i*:

$$\forall n,s: \qquad 0 \le \Delta_{ins}^P \perp \theta_s \gamma_s c_{ins}^{\Delta P} - \sum_{s' \in succ(s),d} \lambda_{ins'd}^P + \rho_{ins}^{\Delta P} \ge 0 \quad \textbf{(8.4.4)}$$

$$\forall a,s,d: \qquad 0 \le f_{iasd} \perp \theta_s \gamma_s L_d \tau_{asd}^A + \varphi_{in_a^- sd} - \left(1 - l_a^A\right)\varphi_{in_a^+ sd} \ge 0 \quad \textbf{(8.4.5)}$$

The KKT condition reflecting the stationarity condition for sales quantities reads:

$$\forall n,s,d: \qquad 0 \le q_{insd}^S \perp -\theta_s \gamma_s L_d \frac{d\Pi_{insd}(\;)q_{insd}^S}{dq_{insd}^S} + \varphi_{insd} \ge 0 \quad \textbf{(8.4.6)}$$

with $\Pi_{insd}(\;) = \delta_{insd}^{CV}\Pi_{nsd}\left(\sum_j q_{jnsd}^S\right) + \left(1-\delta_{insd}^{CV}\right)\pi_{nsd}$, and $\Pi_{nsd}\left(\sum_j q_{jnsd}^S\right) = A_{nsd} - B_{nsd}\sum_j q_{jnsd}^S$ (cf., Footnote 7), we have: $\frac{d\left(\Pi_{insd}(\;)q_{insd}^S\right)}{dq_{insd}^S} = \delta_{insd}^{CV}\left(A_{nsd} - B_{nsd}\left(\sum_j q_{jnsd}^S + q_{insd}^S\right)\right) + \left(1-\delta_{insd}^{CV}\right)\pi_{nsd}$. From Eq.**(**3.3.3**)**, we know that $\pi_{nsd} = A_{nsd} - B_{nsd}\left(\sum_j q_{jnsd}^S\right)$, hence:

$$\begin{aligned}\frac{d\left(\Pi_{nsd}(\;)q_{insd}^S\right)}{dq_{insd}^S} &= \delta_{insd}^{CV}\left(A_{nsd} - B_{nsd}\left(\sum_j q_{jnsd}^S + q_{insd}^S\right)\right) + \left(1-\delta_{insd}^{CV}\right)\left(A_{nsd} - B_{nsd}\left(\sum_j q_{jnsd}^S\right)\right) \\ &= A_{nsd} - B_{nsd}\left(\sum_j q_{jnsd}^S + \delta_{insd}^{CV} q_{insd}^S\right)\left(= \pi_{nsd} - \delta_{insd}^{CV} B_{nsd} q_{insd}^S\right)\end{aligned}$$

.

Finally, we can state the KKT conditions for the sales quantities q_{insd}^{S} as follows:[11]

$$\forall n,s,d: \qquad 0 \le q_{insd}^{S} \perp -\theta_s \gamma_s L_d \left(A_{nsd} - B_{nsd} \left(\sum_j q_{jnsd}^{S} + \delta_{insd}^{CV} q_{insd}^{S} \right) \right) + \varphi_{insd}^{N} \ge 0 \qquad \textbf{(8.4.7)}$$

The derivation of the remaining KKT conditions is straightforward:

$$\forall w,s,d: \qquad 0 \le i_{iwsd} \perp \theta_s \gamma_s L_d \tau_{wsd}^{W} + \varphi_{insd}^{N} - \left(1 - l_w^{W}\right) L_d \varphi_{ins}^{W} \ge 0 \qquad \textbf{(8.4.8)}$$

$$\forall w,s,d: \qquad 0 \le x_{iwsd} \perp -\varphi_{insd}^{N} + L_d \varphi_{ins}^{W} \ge 0 \qquad \textbf{(8.4.9)}$$

$$\forall n,s,d: \qquad 0 \le \lambda_{insd}^{P} \perp q_{insd}^{P} - \overline{Q}_{ins}^{P} - \sum_{s' \in pred(s)} \Delta_{ins'}^{P} \ge 0 \qquad \textbf{(8.4.10)}$$

$$\forall n,s: \qquad 0 \le \rho_{ins}^{\Delta P} \perp \overline{\Delta}_{ins}^{P} - \Delta_{ins}^{P} \ge 0 \qquad \textbf{(8.4.11)}$$

$$\forall n,s,d: \quad \varphi_{insd}^{N} \quad fis,\ q_{insd}^{P} + \sum_{w \in W_n} x_{iwsd} + \sum_{a \in A_n^{+}} \left(1 - l_a^{A}\right) f_{iasd}^{A} - q_{insd}^{S} - \sum_{w \in W_n} i_{iwsd} - \sum_{a \in A_n^{-}} f_{iasd}^{A} = 0 \qquad \textbf{(8.4.12)}$$

$$\forall w,s: \qquad \varphi_{iws}^{W} \quad fis, \left(1 - l_w^{W}\right) \sum_d L_d i_{iwsd} = \sum_d L_d x_{iwsd} \qquad \textbf{(8.4.13)}$$

7.4.2 KKT conditions – TSO

The stationarity conditions for the TSO are derived similarly as above for the supplier:

$$\forall a,s,d: \qquad 0 \le f_{asd}^{A} \perp -\theta_s \gamma_s L_d \left(\tau_{asd}^{A} - c_{as}^{A} \right) + \lambda_{asd}^{A} \ge 0 \qquad \textbf{(8.4.14)}$$

$$\forall a,s: \qquad 0 \le \Delta_{as}^{A} \perp \theta_s \gamma_s c_{as}^{\Delta A} - \sum_{s' \in succ(s),d} \lambda_{as',d}^{A} + \rho_{as}^{\Delta A} \ge 0 \qquad \textbf{(8.4.15)}$$

$$\forall a,s,d: \qquad 0 \le \lambda_{asd}^{A} \perp \overline{Q}_{as}^{A} + \sum_{s' \in pred(s)} \Delta_{as'}^{A} - f_{asd}^{A} \ge 0 \qquad \textbf{(8.4.16)}$$

$$\forall a,s: \qquad 0 \le \rho_{as}^{A} \perp \overline{\Delta}_{as}^{a} - \Delta_{as}^{a} \ge 0 \qquad \textbf{(8.4.17)}$$

7.4.3 KKT conditions – SSO

Except for the first equation below, Eq. (8.4.18) for storage capacity, the KKT conditions for the SSO are similar to the KKT conditions derived for the TSO.

$$\forall w,s,d: \qquad 0 \le i_{wsd}^{W} \perp -\theta_s \gamma_s L_d \left(\tau_{wsd}^{W} - c_{ws}^{W} \right) + \left(1 - l_w^{W}\right) L_d \lambda_{ws}^{W} \ge 0 \qquad \textbf{(8.4.18)}$$

---

[11] Note that for positive sales, we have: $\varphi_{insd} = \pi_{nsd} - \delta_{insd}^{CV} B_{nsd} q_{insd}^{S}$, which illustrates the effect of the conjectural variation.

$$\forall w,s: \quad 0 \le \Delta_{ws}^{W} \perp \theta_s \gamma_s c_{ws}^{\Delta W} - \sum_{s' \in succ(s)} \lambda_{ws'}^{W} + \rho_{ws}^{\Delta W} \ge 0 \tag{8.4.19}$$

$$\forall w,s: \quad 0 \le \lambda_{ws}^{W} \perp \overline{Q}_{ws}^{W} + \sum_{s' \in pred(s)} \Delta_{ws'}^{W} - \left(1 - l_{w}^{W}\right) \sum_{j,d} L_d i_{jwsd}^{W} \ge 0 \tag{8.4.20}$$

$$\forall w,s: \quad 0 \le \rho_{ws}^{\Delta W} \perp \overline{\Delta}_{ws}^{W} - \Delta_{ws}^{W} \ge 0 \tag{8.4.21}$$

7.5 Reduction of model equations and variables

We aim to reduce the number of variables and equations to simplify notation and maintenance of the model by applying the findings of Baltensperger et al. (2016). First, we substitute f_{asd}^{A} with $\sum_j f_{jasd}$, and i_{wsd}^{W} with $\sum_j i_{jwsd}$ in our formulation, since these terms are equivalent at all times – this follows directly from the market clearing conditions Eqs. (3.3.1) and (3.3.2). After this substitution, the market clearing conditions are redundant. The adjusted KKT conditions Eqs. (3.5.12) and (3.5.15) can be interpreted as a feasible region of one player which is affected by decisions of other players. This means that we now have a Generalized Nash Problem (GNP). However, because the model was set up as a Nash problem, and these substitutions are only made to the system of equations after deriving the KKT, the system still represents the original Nash problem. Additionally, Midthun (2007) shows that if a common constraint is valued the same by all agents, the solution of the GNP solved as an MCP is unique. Since the same dual variable / congestion price applies to all agents (namely λ_{asd}^{A} and λ_{ws}^{W}), we have the common valuation in place.

Second, we eliminate the conditions associated with f_{asd}^{A} and i_{wsd}^{W}: Eqs. (8.4.14) and (8.4.18). We can differentiate two cases for condition (8.4.14): if $f_{asd}^{A} > 0$, we obtain $\tau_{asd}^{T} = c_{as}^{A} + \frac{\lambda_{asd}^{A}}{\theta_s \gamma_s L_d}$; if $f_{asd}^{A} = 0$, then $\tau_{asd}^{T} \le c_{as}^{A} + \frac{\lambda_{asd}^{A}}{\theta_s \gamma_s L_d}$. Hence, by setting $\tau_{asd}^{T} = c_{as}^{A} + \frac{\lambda_{asd}^{A}}{\theta_s \gamma_s L_d}$, the model obtains an unambiguous solution for non-operating services ($f_{asd}^{A} = 0$). Eq. (8.4.5) now reads

$$0 \le f_{iasd} \perp \theta_s \gamma_s L_d \left(c_{as}^{A} + \frac{\lambda_{asd}^{A}}{\theta_s \gamma_s L_d} \right) + \varphi_{in_a^- sd} - \left(1 - l_a^{A}\right) \varphi_{in_a^+ sd} \ge 0, \tag{8.5.1}$$

which can be reformulated to

$$0 \le f_{iasd} \perp \theta_s \gamma_s L_d c_{as}^{A} + \lambda_{asd}^{A} + \varphi_{in_a^- sd} - \left(1 - l_a^{A}\right) \varphi_{in_a^+ sd} \ge 0. \tag{8.5.2}$$

Following the same argument, we set $\tau_{wsd}^{W} = c_{ws}^{W} + \frac{\left(1-l_w^{W}\right)\lambda_{ws}^{W}}{\theta_s \gamma_s}$, and reformulate Eq. (8.4.8) as follows:

$$\forall w,s,d: \quad 0 \le i_{iwsd} \perp \theta_s \gamma_s L_d \left(c_{ws}^{W} + \frac{\left(1-l_w^{W}\right)\lambda_{ws}^{W}}{\theta_s \gamma_s} \right) + \varphi_{insd}^{N} - \left(1 - l_w^{W}\right) L_d \varphi_{ins}^{W} \ge 0 \tag{8.5.3}$$

$$\forall w,s,d: \quad 0 \le i_{iwsd} \perp \theta_s \gamma_s L_d c_{ws}^W + \left(1-l_w^W\right) L_d \lambda_{ws}^W + \varphi_{insd}^N - \left(1-l_w^W\right) L_d \varphi_{ins}^W \ge 0 \tag{8.5.4}$$

This concludes the derivation of the KKT conditions, and the elimination of the redundant equations to arrive at a compact model formulation. This formulation is the basis for the implementation in GAMS.

8 Appendix B – GAMS code

We will make the GAMS code of the model available online. Add reference before final submission.

9 Appendix C – Input data

Values for the model parameterization are chosen for illustrative purposes. In the main text units of measurement have been added to improve readability.

The discount rate γ_s is 20% per stage for all agents.

Supplier data

Production and investment cost are the same for all suppliers (cf., Table 3).

Table 3: Supplier data.

	All stages All suppliers
Constant per unit cost	1.0
Linear per unit cost term	0.5
Initial capacity	6
Maximum expansion	4
Expansion cost per unit	2

Network data

All six transportation arcs have an initial capacity of 1 unit / day. The (regulated) basic fee for usage is \$ 1 per unit transported, and the loss rate is 10%. Expansion costs are 1 per unit for each arc, and the maximum expansion is 2 units per stage.

Demand data

The inverse demand curve varies by node in the scenario tree. Specifically, the value of the intercepts are chosen differently as illustrated in Table 4.

Table 4: Inverse demand curve data.

	Stage 1	stage 2		stage 3				stage 4							
	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10	s11	s12	s13	s14	s15
intercept	20	24	20	28	24	24	20	32	30	28	26	26	24	22	20
Slope	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Seasonality and storage

Three seasons exist: Low, High, and Peak demand. L, H, and P account for 50%, 30%, and 20% of the year and have relative to average loads of 70%, 110%, and 160%, respectively. (50%x70% + 30%x110%

+ 20% x 160% = 100%) The load values are used as multiplication factors of the intercepts presented in Table 4 to find the intercepts of the inverse demand curves in all seasons in all scenario nodes.)

Storage is present at node two but initially without capacity. Maximum expansion is 2 units per stage. Investment costs are 1.6 per unit. Injection costs are 2 per unit and the injection loss rate is 5%.

10 Appendix D – Detailed Results

This will automatically be part of the GAMS code that will be made available online.

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