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Design of plantwide control systems with focus on maximizing throughput

NTNU

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Abstract

This thesis discusses plantwide control configuration with focus on maximizing throughput. The most important plantwide control issue is to maintain the mass balances in the plant. The inventory control system must be consistent, which means that the mass balances are satisfied. Self-consistency is usually required, meaning that the steady-state balances are maintained with the local inventory loops only. We propose the *self-consistency rule* to evaluate consistency of an inventory control system.

In many cases, economic optimal operation is the same as maximum plant throughput, which corresponds to maximum flow through the bottleneck(s). This insight may greatly simplify implementation of optimal operation, without the need for dynamic optimization based on a detailed model of the entire plant.

Throughput maximization requires tight bottleneck control. In the simplest case when the bottleneck is fixed to one unit, maximum throughput can be realized with single-loop control. The throughput manipulator should then be located at the bottleneck unit. This gives a short effective delay in the control loop. Effective delay determines the necessary back off from constraints to ensure feasible operation. Back off implies a reduction in throughput and an unrecoverable economic loss and should therefore be minimized. We obtain a rough estimate of the necessary back off based on controllability analysis.

In some cases it is not desirable to locate the throughput manipulator at the bottleneck. To reduce the effective time delay in the control loop from the throughput manipulator to the bottleneck unit, dynamic degrees of freedom, like most inventories, can be used to reduce the effective time delay.

In larger plants there may be several independent feeds, crossovers and splits that should all be utilized to obtain maximum throughput. The proposed *coordinator MPC* both identifies the bottlenecks and implements the optimal policy. A key idea in the coordinator MPC is to decompose the plantwide control problem by estimating the remaining capacity for each unit using models and constraint in the local MPC applications. The coordinator MPC is demonstrated by dynamic simulation and by implementation on a large-scale gas processing plant.

Acknowledgements

Contents

Chapter 1

Introduction

The purpose of this chapter is to motivate the research, to define the scope and place it in a wider perspective. The contributions and publications arising from this thesis are listed.

1.1   Motivation and focus

Optimal economic operation of processes is important, especially in mature industries where it is difficult to maintain competitive advantages. In some cases, steady-state considerations may be sufficient to track the economic operation point. In other cases, where the important economic disturbances are frequent compared to the plant response time, dynamic considerations to track the optimum is preferable. Some dynamic economic disturbances that most likely call for dynamic optimization are feed flow, feed quality, energy supplies and product specifications (Strand, 1991). To decide whether a dynamic or steady-state process model should be used, the dynamics of the plant and the disturbances must be considered.

   In practice, the control and optimization is organized in a hierarchical structure (or layer) (e.g. Findeisen *et al.* 1980; Skogestad and Postlethwaite 2005). Each layer acts at different time intervals (time scale separation) and a typical control hierarchy is displayed in Figure 1.1.

   This thesis discusses the control layer, that is, the regulatory control and supervisory control. In addition, implementation of maximum throughput (local optimization) in the control layer is discussed. The stabilizing regulatory control typically includes single-loop PID controllers. Supervisory control (or advanced control) should keep the plant at its target values and model predictive control (MPC) has become the unifying tool with many applications (Qin and Badgwell,
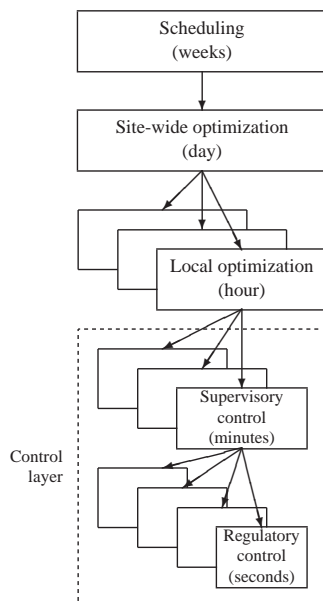
Figure 1.1: Typical control system hierarchy in chemical plants (Skogestad and Postlethwaite, 2005, p.387).

2003) and has replaced previous complex systems with selectors, decouplers, feed-forward control and logic.

Engell (2007) gives a review of how to realize optimal process operation by feedback control with direct optimization control, that is, optimization of a online economic cost criterion over finite horizon. Optimal operation can be implemented by conventional feedback control if a self-optimizing control structure is found. This is called *self-optimizing control* where acceptable operation is achieved under all conditions with constant set points for the controlled variables (Skogestad, 2000a; Morari *et al.*, 1980). Today, model based economic optimization has become common, and several real-time optimization (RTO) applications based on detailed nonlinear steady-state models are reported (Marlin and Hrymak, 1997). However, there are several challenges regarding (steady-state) RTO. To mention some of these challenges, an RTO requires highly predictive and robust models. Steady-state detection and data reconciliation are necessary to detect current operation point and to update models and this is not a straight forward task (Forbes *et al.*, 2006; Marlin and Hrymak, 1997).

In particular, for plants that are seldom in steady-state, dynamic optimization is more suitable, which may be realized using dynamic RTO (DRTO) or nonlinear model predictive controller (MPC) with an economic objective, e.g. Kadam *et al.*

(2007); Engell (2007); BenAmor *et al.* (2004); Tosukhowong *et al.* (2004); Diehl *et al.* (2002).

In many cases, we can assume that optimal economic operation is the same as maximizing plant throughput, subject to achieving feasible operation (satisfying operational constraints in all units) with the available feeds. This corresponds to a constrained operation mode (Maarleveld and Rijnsdorp, 1970) with maximum flow through the bottleneck(s). Note that the overall feed rate (or more generally the throughput) affects all units in the plant. For this reason, the throughput is usually not used as a degree of freedom for control of any individual unit, but must be set at the plant-wide level. The throughput manipulators are decided at the design stage and cannot easily be moved later because this requires reconfiguration of the inventory loops to ensure self-consistency (Chapter 2). Plant operation depends on its control structure design and plantwide control related to that design for complete chemical plants (Skogestad, 2004). The focus in this thesis is the control configuration design for throughput maximization.

The economic importance of throughput and the resulting earnings from improved control is stated by Bauer and Craig (2008). They performed a web-based survey by over 60 industrial experts in advanced process control (APC) on the economic assessment of process control. From the survey they found that in particular *throughput* and quality were the important profit factors: *"Both suppliers and users regard an increase in throughput and therefore production as the main profit contributor of process control. Several respondents estimate that the throughput increase lies between 5% and 10%."*.

In this thesis, dynamic optimization is approached by using linear MPC under the assumption of the economic optimum is at maximum throughput (Chapter 5 and 6). Since the objective function is simplified to a linear and constrained function, approaching dynamic optimization by linear MPC is suitable. In the simplest cases, the regulatory control layer can realize throughput maximization (Chapter 3 and 4).

1.2   Thesis overview

The thesis is composed of six independent articles, five of them in the main part of the thesis as chapters and one already published conference paper in the appendix. Some of the chapters have their own appendices. The thesis has a common bibliography. The chapters are written as independent articles, so background material is in some cases repeated. At the end of the thesis, there is a concluding chapter.

The starting point for this research was that the optimum operating policy in many cases is the same as maximum throughput that can be realized with a coordinator MPC (Chapter 5). The location of the throughput manipulator is crucial

when it comes to the required back off in the maximum throughput case. The effect the throughput manipulator location has on the required back off and its effect on the bottleneck unit was studied next (Chapter 3). The inventory control configuration is (partly) derived from the placement of the throughput manipulator, and a clear rule for a self-consistent inventory control structure was developed as it was not reported in the open literature (Chapter 2). Another path that arose from tight bottleneck control was the idea to include dynamic degrees of freedom (hold-up volumes) to obtain tighter bottleneck control (Chapter 4). Finally, through my employer, StatoilHydro, I got the possibility to implement the coordinator MPC in practice at a gas processing plant (Chapter 6). A short summary of the contents of the thesis is given next.

In **Chapter 2: Self-consistent inventory control**, we define consistency and self-consistency for an inventory control system. Consistency means that the (steady-state) mass balances are fulfilled and self-consistency means that the mass balances in the individual units are satisfied by the local inventory loops. This leads to the proposed self-consistency rule. The proposed rule is demonstrated on several examples, including units in series, recycle systems and closed systems. Specific rules that deal with the inventory control system are developed from the self-consistency rule.

In **Chapter 3: Throughput maximization requires tight bottleneck control**, we derive under which conditions maximum throughput is an optimal economic operation policy. We discuss back off in a general setting and for the special case for maximum throughput. We consider the case with a fixed bottleneck where a single-loop controller can realize maximum throughput. Further, the location of the throughput manipulator is discussed, where the effective time delay from the throughput manipulator to the bottleneck is important. The location of throughput manipulators is illustrated through examples. Possible improvements to reduce back off and hence increase the throughput are listed.

Chapter 4: Dynamic degrees of freedom for tighter bottleneck control, extend the ideas from Chapter 3 to include dynamic degrees of freedom to reduce the effective delay from the throughput manipulator to the bottleneck. The control structure single-loop with ratio control is proposed to include dynamic degrees of freedom for cases with fixed bottleneck. A multivariable controller like MPC that uses inventory set points as manipulated variables can also be used. Both control structures are demonstrated with an example. The required inventory size is estimated for the case with single-loop with ratio control structure.

In **Chapter 5: Coordinator MPC for maximizing plant throughput**, we consider the case where the bottlenecks may move, with parallel flows that give rise to multiple bottlenecks and with crossover flows as extra degrees of freedom. We present a coordinator MPC that solves the maximum throughput problem dynamically. The plantwide control problem is decomposed by estimating the capacity to each unit, that is, the feed rate each unit is able to receive within feasible operation. The coordinator MPC is demonstrated with a case study.

In **Chapter 6: Implementation of a coordinator MPC for maximizing throughput at a large-scale gas plant**, the industrial implementation of a coordinator MPC (Chapter 5) at the Kårstø gas plant is described. This includes design, modelling and tuning of the coordinator MPC, in addition to the plantwide decomposition by the remaining capacity estimate. Experiences from implementation and test runs are reported.

Chapter 7: Conclusions and directions for further work sums up and concludes the thesis, together with proposals for further work.

Appendix A: Implementation of MPC on a deethanizer at Kårstø gas plant discusses implementation of MPC on a deethanizer column located at the Kårstø gas plant. The appendix contains basic information about MPC design, dynamic modelling and tuning. The MPC software, SEPTIC*, is described briefly. The SEPTIC MPC tool is used in other parts of the thesis (Chapter 5 and 6) and the Appendix is therefore included for completeness.

1.3   Main contributions

The main contributions of the thesis are:

- Plantwide decomposition by estimating the remaining capacity in each unit. An important parameter for the maximum throughput case is the maximum flow for the individual (local) units. This can be obtained by using the models and constraint in the local MPC applications. This decomposes the plant significantly, leading to a much smaller plantwide control problem.

- The idea of using a "decentralized" coordinator MPC to maximize throughput. Throughput manipulators strongly affect several units and are therefore left as "unused" degree of freedom to be set at the plant-wide level. The coordinator manipulates on feed rates, splits and crossover (throughput manipulators) to maximize the plant throughput subject to feasible operation.

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*Statoil Estimation and Prediction Tool for Identification and Control

The remaining capacity estimate for each unit is constraints in the coordinator MPC.

- The self-consistency rule and the explanation of a self-consistent inventory control system. Consistency is a very important property of inventory control that must be fulfilled. An experienced engineer can usually immediately say if a proposed inventory control system is workable. However, for a student or newcomer to the field it is not obvious, and even for an experienced engineer there may be cases where the experience and intuition fails. Therefore, we find the self-consistency rule useful together with the illustrative examples.

- Single-loop with ratio control as an alternative structure to obtain tight bottleneck control. With a fixed bottleneck and with a long effective delay from the throughput manipulator to the bottleneck, tight bottleneck control can still be obtained by using dynamic degrees of freedom. Single-loop with ratio control use inventories upstream the bottleneck by adding bias to the inventory controller outputs, whereas the throughput manipulator (e.g. feed rate) controls the bottleneck flow rate. This structure makes it possible to obtain tight bottleneck control without moving the throughput manipulator or reconfiguring the inventory loops.

1.4   Publications

The following is a complete list of the publications written during the work contained in this thesis. This includes submitted, accepted and published work.

Chapter 2

Aske, E.M.B. and Skogestad, S. Self-consistent inventory control. *Ind. Eng. Chem. Res.*, Submitted.

Chapter 3

Aske, E.M.B, Skogestad,S. and Strand, S. Throughput maximization by improved bottleneck control. *8th International Symposium on Dynamics and Control of Process Systems (DYCOPS)*. Vol. 1, June 6-8 2007, Cancun, Mexico. pp 63-68.

Chapter 4

Aske, E.M.B. and Skogestad, S. Dynamic degrees of freedom for tighter bottleneck control. *Comput. Chem. Eng.*, Submitted.

Aske, E.M.B. and Skogestad, S. Dynamic degrees of freedom for tighter bottle-neck control. *10th International Symposium on Process Systems Engineering*, August 16-20, 2009, Salvador-Bahia, Brazil. Submitted.

Chapter 5

Aske, E.M.B., Strand S. and Skogestad, S. Coordinator MPC with focus on maximizing throughput, In: *Proc. PSE-ESCAPE Symposium*, (W. Marquardt and C. Pantelides, Eds.), July 10-13 2006, Garmisch-Partenkirchen, Germany. Published by Elsevier, ISBN 0-444-52969-1 978-0-444-52969-5, Vol. 21B, pp. 1203-1208.

Aske, E.M.B., Strand, S. and Skogestad, S. Coordinator MPC for maximization of plant throughput. *AIChE Annual Meeting*, San Francisco, USA, Nov. 2006, Abstract and Presentation 330b.

Aske, E.M.B., Strand, S. and Skogestad,S. Coordinator MPC for maximizing plant throughput. *Comput. Chem. Eng.* **32**, 195-204 (2008).

Chapter 6

Aske, E.M.B., Strand, S. and Skogestad, S. Implementation of Coordinator MPC on a Large-Scale Gas Plant. *AIChE Annual Meeting*, Philadelphia, USA, Nov. 2008, Abstract and Presentation 409g.

Aske, E.M.B., Strand, S. and Skogestad, S. Industrial implementation of a co-ordinator MPC for maximizing throughput at a large-scale gas plant. *International Symposium on Advanced Control of Chemical Processes*, July 12-15, 2009, Istanbul, Turkey. Submitted.

Appendix A

Aske, E.M.B., Strand, S. and Skogestad, S. Implementation of MPC on a deethanizer at Kårstø gas plant. In: *16th IFAC World Congress*, Prague, Czech Republic, July 2005, paper We-M06-TO/2. CD-rom published by International Federation of Automatic Control.

Chapter 2
Self-consistent inventory control

Is not included due to copyright

Chapter 3

Throughput maximization requires tight bottleneck control

With sufficiently high product prices and the feed is available, it is shown that maximum throughput is an optimal economic operation policy. This paper discusses the maximum throughput case, which is characterized by the existence of a *bottleneck* and the need for *back off* from active constraints to ensure feasibility. To implement maximum throughput, maximum flow in the bottleneck(s) must be realized. Obtaining tight bottleneck control in practice requires that the throughput manipulator is located close to the bottleneck (short effective delay). If the throughput manipulator is located close enough compared to the disturbance time constant, automatic control can reduce the back off significantly. Poor control of the bottleneck, including any deviation or back off, implies a reduction in throughput and an unrecoverable economic loss.

3.1 Introduction

In general, real-time optimization (RTO) based on a detailed process model may be used to find the optimal operation conditions of a plant, including identifying the optimal active constraints and computing the optimal set point for the unconstrained variables. However, in many cases, prices and market conditions are such that optimal operation is the same as maximizing plant throughput. Hence, the problem formulation can be simplified, and RTO based on a detailed nonlinear process model is not needed.

Maximum throughput in a network is a common problem in several settings (e.g. Phillips *et al.*, 1976; Ahuja *et al.*, 1993). From network theory, the *max-flow min-cut* theorem states that the maximum throughput in a plant (network) is limited by the "bottleneck" of the network. In order to maximize the throughput, the flow through the bottleneck should be at its maximum flow. In particular, if the actual flow at the bottleneck is not at its maximum at any given time, then this gives a loss in production which can never be recovered (sometimes referred to as a "lost opportunity").

To implement maximum throughput there are three important issues: 1) locate the bottleneck unit(s), 2) implement maximum throughput in the bottleneck unit and 3) minimize the back off from active constraints in the bottleneck unit. To locate the bottleneck in the first place, there are several opportunities. The most common is simply to increase the flow rate during operation (online) until feasible operation is no longer possible. Alternatively, the location can be estimated using a commercial flowsheet simulator or plant data. Litzen and Bravo (1999) discuss how to estimate the capacity for process units and find the bottleneck(s) for debottlenecking (design) purposes (steady-state). A third approach is to use the models that are implemented in the model predictive controllers (MPC) to estimate the available capacity for each unit on-line (dynamically) (Aske *et al.*, 2008).

Maximizing throughput requires manipulation of the *throughput manipulator* (TPM). This is usually the feed rate (Price *et al.*, 1994), but it can more generally be anywhere in the plant. Usually the location of the TPM is determined by the original design of the control system for the plant, and cannot be easily changed because it requires reconfiguration of the inventory loops to ensure a self-consistent inventory control system (Chapter 2). If one is free to place the TPM(s), then two considerations may come into account. First, one must consider its effect on the inventory control structure, including propagation of disturbances, dynamic lags, process time constants and interactions (Luyben, 1999). A second consideration, which is based on economics, is to locate the TPM such that tight control of the bottleneck unit is possible. Skogestad (2004) propose to set the production rate at the bottleneck.

Price and coauthors (Price and Georgakis, 1993; Price *et al.*, 1994) propose a plantwide design structure using a tiered framework with throughput, inventory and product quality controls. They discuss the importance of proper selection of the TPM and their general recommendation is to select an internal process flow as the TPM because: 1) "*they impede the propagation of disturbances through the system*" and 2) "*internal flows have a substantial chance of more rapidly affecting a throughput change*". On the other hand, Cheng *et al.* (2002) claim the opposite; the TPM should be a feed or product flow, and internal flows should be avoided from a dynamic interaction point of view. Price *et al.* (1994) also mentioned on

TPM location that "*some plants have a single processing unit which is markedly more difficult to control than the others. Selecting a flow very close to that unit as the throughput manipulator will help minimize or control the variation affecting the unit and so should make it easier to control*." Moore and Percell (1995) evaluated control alternatives by simulation on a three-unit module and concluded that "*the plant is capable of the highest production rate with the widest variation in feed composition when the production rate is set at the column feed, which is immediately before the process bottleneck*". However, there are no attempts trying to explain the results from the simulation study. Luyben *et al.* (1997) propose a heuristic design procedure for plantwide control. In the procedure, the authors recommend locating the TPM so it provides a smooth and stable production rate transitions and reject disturbances. However, all these approaches lack an economic evaluation of the TPM selection; whereas Larsson and Skogestad (2000) point out that the economics is a key factor for the placement of the TPM. They suggest that for a plant running at maximum capacity, the production rate should be set at the bottleneck, which is usually inside the plant.

From a literature search and based on our own industrial experience, it seems like the feed valves (or more general the throughput manipulator) is very rarely used in practice for closed-loop control, in spite of its great importance on the plant economics in cases where maximum throughput is optimal. The reason is probably the large effect the feed rate has on the operation of the entire plant, but the result may be a loss in economic performance. The main goal of this paper is to discuss the importance of using the throughput (often the feed rate) for closed-loop control.

When operating at maximum throughput, the plant is at the limit to infeasibility. For this reason, a "safety factor" or "back off" is required to achieve feasible operation under presence of disturbances, uncertainties, measurement error and other sources for imperfect control (Narraway and Perkins, 1993; Govatsmark and Skogestad, 2005). More precisely, the back off is the distance between the active constraint and the actual average value (set point). The necessary back off can generally be reduced by improving the control of the bottleneck unit, for example, by retuning the control system to reduce the dynamic variation. The idea is that improved control requires a smaller back off or, in short, "squeeze and shift" (squeeze the variance - and shift the set point closer to the constraints) (e.g. Richalet *et al.*, 1978; Richalet, 2007).

This paper addresses the maximized throughput case, and starts by considering the case under which considerations this is optimal (Section 3.2). In Section 3.3, back off is defined and reasons for why back off is needed together with its influence on the economics is discussed. The location of the throughput manipulator is discussed in Section 3.4, whereas in Section 3.5 the characteristics of maximum

throughput are treated. By using controllability analysis, an estimate of minimum back off is given in Section 3.6 with a more detail description is given in Appendix 3.A. In Section 3.7 we discuss actions to reduce back off, followed by a discussion in Section 3.8 before we conclude in Section 3.9.

3.2 Optimal operation (steady-state)

In this section, we discuss under which considerations, maximum throughput is economically optimal.

3.2.1 Modes of optimal operation

Mathematically, steady-state optimal operation is to minimize the cost J (or maximize the profit $-J$), subject to satisfying given specifications and model equations ($f = 0$) and given operational constraints ($g \leq 0$):

$$
\begin{aligned}
&\min_{u} \; J(x,u,d) \\
&\text{s. t. } f(x,u,d) = 0 \\
&\qquad g(x,u,d) \leq 0
\end{aligned}
\tag{3.1}
$$

Here are u the degrees of freedom (manipulated variables including the feed rates F_i), d the disturbances and x the (dependent) state variables.

A typical profit function is

$$
-J = \sum_{j} p_{P_j} \cdot Pj - \sum_{i} p_{F_i} \cdot F_i - \sum_{k} p_{Q_k} \cdot Q_k
\tag{3.2}
$$

where P_j are product flows, F_i the feed flows, Q_k are utility duties (heating, cooling, power), and p (with subscript) denote the prices of the corresponding flow and utility. Let F be a measure of the throughput in the plant. Depending on market conditions, a process has two main modes in terms of optimal operation:

Mode 1. *Given throughput (F given). The economic optimum is then usually the same as optimal efficiency, that is, to minimize utility (energy) consumption for the given throughput.*

This mode of operation typically occurs when the feed rate is given (or limited) or the product rate is given (or limited, for example, by market conditions), and the optimization problem (3.1) is modified by adding a set of constraints on the feed rate, $F_i = F_{i0}$.

Mode 2. *Feed is available and the throughput F is a degree of freedom. We here have two cases:*

(a) **Maximum throughput**. This mode of operation, which is the main focus of this paper, occurs when product prizes are sufficiently high and feed is available. We then have that the cost can be written $J = -pF$ where $p > 0$ (see (3.6) below). Optimal economic operation then corresponds to *maximizing* the throughput F, subject to achieving feasible operation and this does not depend on cost data. The optimum is *constrained* with respect to the throughput, and we have $dJ/dF_i < 0$ where the feed rates F_i are degrees of freedom.

(b) **Optimized throughput**. This mode of operation occurs when feed is available, but it is not optimal to go all the way to maximum throughput because the efficiency drops as the throughput increases. For example, increased throughput may be possible by increasing the purge rate, but this result in less efficient operation because of loss of valuable components. The optimum is *unconstrained* with respect to the feed rates F_i and we have $dJ/dF_i = 0$. Thus, increasing F_i above its optimal value is feasible, but gives a higher cost J.

3.2.2 Maximum throughput (Mode 2a)

We here want to show that when product prices are high compared to feed and utility costs, optimal operation of the plant is the same as maximizing throughput (Mode 2a). Let F be a measure of the throughput in the plant, and assume that all feed flows are set in proportion to F,

$$F_i = k_{F,i}F \tag{3.3}$$

Then, under the assumption of constant efficiency in all units (independent of throughput) and assuming that all intensive (property) variables are constant, all extensive variables (flows and heat duties) in the plant will scale with the throughput F (e.g. Skogestad, 1991). In particular, we have that

$$P_j = k_{P,j}F \qquad Q_k = k_{Q,k}F \tag{3.4}$$

where the gains $k_{P,j}$ and $k_{Q,k}$ are constants. Note from (3.4) that the gains may be obtained from nominal (denoted 0) mass balance data:

$$k_{P,j} = P_{j0}/F_0 \qquad k_{F,i} = F_{i0}/F_0 \qquad k_{Q,k} = Q_{k0}/F_0 \tag{3.5}$$

Substituting (3.3) and (3.4) into (3.2) gives

$$(\text{-}J) = \left(\sum_j p_{P_j} \cdot k_{P,j} - \sum_i p_{F_i} \cdot k_{F,i} - \sum_k p_{Q_k} \cdot k_{Q,k} \right) F = pF \tag{3.6}$$

where p is the operational profit per unit of feed F processed. From the above derivation, p is a constant for the case with constant efficiencies. We assume $p > 0$ such that we have a meaningful case where the products are worth more than the feed stocks and utilities. Then, from (3.6) it is clear that maximizing the profit $(-J)$ is equivalent to maximizing the (plant) throughput F. However, F cannot go to infinity, because the operational constraints ($g \le 0$) related to achieving feasible operation (indirectly) impose a maximum value for F.

In practice, the gains $k_{P,j}$, $k_{F,i}$ and $k_{Q,k}$ are not constant, because the efficiency of the plant changes. Usually, operation becomes less efficient and p in (3.6) decreases when F increases. Nevertheless, as long as p remains positive, we have that $d(-J)/dF = p > 0$ is nonzero, and we have a constrained optimum with respect to the throughput F. From (3.6) we see that p will remain positive if the product prices $p_{P,j}$ are sufficiently high compared to the prices of feeds and utilities.

If the efficiency drops, for example because $k_{Q,k}$ increases and $k_{P,j}$ decreases when the feed rate is increased, then p in (3.6) may become negative. Then there is no bottleneck and Mode 2b (optimized throughput) is optimal. This mode of operation is common for recycle systems. For example, this applies to the ammonia synthesis problem (Araújo and Skogestad, 2008).

3.3   Back off

Back off is a general concept that applies to operation close to any "hard" output constraint (not only to bottleneck operation). In this section we present a general discussion of back off.

Arkun and Stephanopoulos (1980) discussed moving away from the nominal optimal operation point to ensure feasible operation when there are disturbances. Narraway and Perkins (1993) discussed this in more detail and introduced the term "back off" to describe the distance from the active constraint that is required to accommodate the effects of disturbances.

3.3.1   Definition of back off

We use the following definition of back off (also see Figure 3.1):

Definition 3.1. *Back off*. *The (chosen) back off is the distance between the (optimal) active constraint value ($y_{constraint}$) and its set point (y_s) (actual steady-state operation point),*

$$Back\ off = b = |y_{constraint} - y_s|, \tag{3.7}$$

which is needed to obtain feasible operation in spite of:
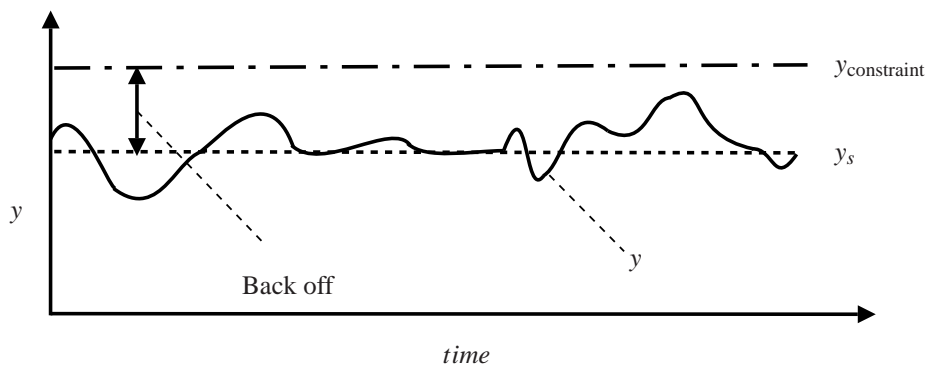
Figure 3.1: Illustration of back off, $b = |y_{\text{constraint}} - y_s|$

1. *Dynamic variations in the variable y caused by imperfect control (due to disturbances, model errors, effective delays and other sources of imperfect control).*

2. *Measurement errors.*

Remark 1 Here we assume integral action, such that on average $y_s = \bar{y}$ where

$$\bar{y} = \lim_{T \to \infty} \frac{1}{T} \int_0^T y(t)dt$$

In this case, only the steady-state measurement error (bias) is of importance, and not its dynamic variation (noise).

Remark 2 Back off was defined by Govatsmark and Skogestad (2005, eq. 20) as the difference between the actual set points and some reference values for the set points:

$$b = c_s - c_{s,ref}$$

where c_s is the actual set point and $c_{s,ref}$ is some reference value for the set point which depends on the method for set point computation (e.g. nominal, robust, on-line feasibility correction). Definition 3.1 coincides with their definition.

3.3.2 Required back off

Back off is needed to avoid constraints violation, and the required back off b depends on whether the active constrained variable y is an input or an output.

Output constraints

Generally, back off is *always* required for output constraints. Let us first distinguish between two constraint types:

- *Hard constraint:* Constraint cannot be violated at any time.

- *Steady-state (average) constraint:* Constraint must be satisfied at steady-state average, but dynamic violation is acceptable.

Safety constraints, like pressure and temperature limitations, are usually hard constraints. An example of a steady-state constraint is the composition of the overhead product from a distillation column which goes to a storage tank where mixing takes place. Another example may be emissions from a plant which often are in terms of hourly or daily averages.

For a steady-state (average) constraint, integral action is sufficient to ensure that $\bar{y} = y_{\text{constraint}} = y_s$ (on average) and no back off is required for dynamic variations caused by imperfect control. However, back off is required to account for possible steady-state measurement errors (bias).

In summary, we have:

- **Hard output constraints:** Required back off is sum of expected dynamic variation and steady-state measurement error (bias).

- **Steady-state (average) output constraint:** Required back off is equal to the steady-state measurement error (bias).

Note that there in addition may be maximum limits (hard constraints) on the allowed dynamic variation even for steady-state (average) constraints.

If no constraint violation is allowed, then the worst-case variation gives the required back off b together with the measurement error. However, in many cases a small constraint violation for a short-time is acceptable and therefore the worst-case variation may be too strict to determine the required back off. In practice, for stochastic signals, one needs to specify an acceptable likelihood for constraint violations. For example, the likelihood is 99.7% that the signal variation remains within ± 3 times standard deviation (σ), or 95% that the variations are within $\pm 2\sigma$ (for normal distribution). In this paper, we consider the worst-case variation and do not include probability for constraint violation.

Input constraints

Inputs have no associated control error. However, for cases where the input constraint does not correspond to a physical (hard) constraint, we must introduce back

off to guard against steady-state measurements errors. For example, there may be a constraint on the allowed flow that goes to the effluent.

For *hard* input constraint, there is normally no need to introduce back off, because we may simply set the input at its constrained value (it cannot be violated even if we want to). There is one exception and this is when the input variable is optimally saturated and is used for (dynamic) control. For example, the cooling rate to a reactor, which optimally should be at maximum, may be needed to stabilize the reactor if the desired operating point is unstable. In other cases, the input may be needed for dynamic control to obtain tight control of an important output variable.

In summary, we have:

- **Hard input constraint:** No back off is normally required.

- **Steady-state (average) input constraint:** Required back off is equal to the steady-state measurement error (bias).

3.3.3 Reducing effect of back off on economics

Any back off from an active constraint will results in an economic loss and should be as small as possible. There are in principle two ways of reducing the economic penalty caused by back off:

1. "Squeeze and shift" (e.g. Richalet, 2007): By improved control one can reduce ("squeeze") the variation and "shift" the set point towards the constraint to reduce back off. Also improved measurements that reduces the measurements variation will reduce the required back off.

2. "Move variation to variables where the economic loss is small": In many cases one can reconfigure the control system (single-loop control) or change the control weights (multivariable control) to obtain tighter control of economically important variables. In practice, this means:

   (a) Move variation to variables without hard constraints

   (b) Move variation to variables where a back off has a small economic effect. For example, this may be quantified by the Lagrange multiplier (shadow prices) (e.g. Edgar *et al.*, 2001).

Mathematically, for a constrained optimization problem, the economic loss caused by back off from an active constraint is represented by the Lagrange multiplier λ

$$\text{Loss} = \frac{\partial(\text{-}J^*)}{\partial c} \cdot \Delta c = \lambda \cdot b \qquad (3.8)$$

where $-J^*$ is the optimal value of the profit, c is the active constraint variable with back off $b = \Delta c$, and λ is the Lagrange multiplier.

At the end, selecting the back off is a trade-off between the improved profit resulting from a small back off and the cost of reducing the back off (e.g. by improved measurements or improved control).

3.4   Throughput manipulator

In this section, we discuss and define the term throughput manipulator. The structure of the inventory control system depends mainly on where in the process the *throughput manipulator*, see Figure 3.2 (Buckley, 1964; Price and Georgakis, 1993):

1. **Feed as TPM (given feed)**: inventory control system in the direction of flow (conventional approach).

2. **Product as TPM ("on-demand")**: inventory control system opposite to flow.

3. **TPM inside plant (general case)**: radiating inventory control.

These rules follow from the requirement of a self-consistent inventory control system, as discussed in detail in Chapter 2.

In terms of location of the TPM, Scheme 1 (Figure 3.2(a)) is the natural choice for Mode 1 with given feed rate, Scheme 2 (Figure 3.2(b)) is the natural choice for Mode 1 with given product rate, whereas Scheme 3 (Figure 3.2(c)) is usually the best choice for Modes 2a and 2b (feed rate is degree of freedom) where the optimal throughput is determined by some conditions internally in the plant.

In the above discussion, we have used the term "throughput manipulator" (TPM) without defining it. The term was introduced by Price and Georgakis (1993), but they did not give a clear definition. From the discussions of Price and coauthors (Price and Georgakis, 1993; Price *et al.*, 1994) on throughput manipulator, it is implicitly understood that a plant has only one throughput manipulator, which is related to the main feed stream. This is reasonable in most cases, because if a plant has several feeds, then these are usually set in proportion to each other, for example, based on the reaction stoichiometric. This was also used in (3.3) and (3.4), were we assumed that all flows and utilities are set in proportion to the throughput F.

However, there are cases that are not quite as simple. First, some plants may have several similar or alternative feeds that do not need to be set in proportion to each other. Thus, fixing one feed rate does not indirectly determine the value of

(a) Scheme 1: Throughput manipulator at feed, inventory control in the direction of flow

(b) Scheme 2: Throughput manipulator at product, inventory control in the direction opposite to flow

(c) Scheme 3: Throughput manipulator inside plant, radiating inventory control
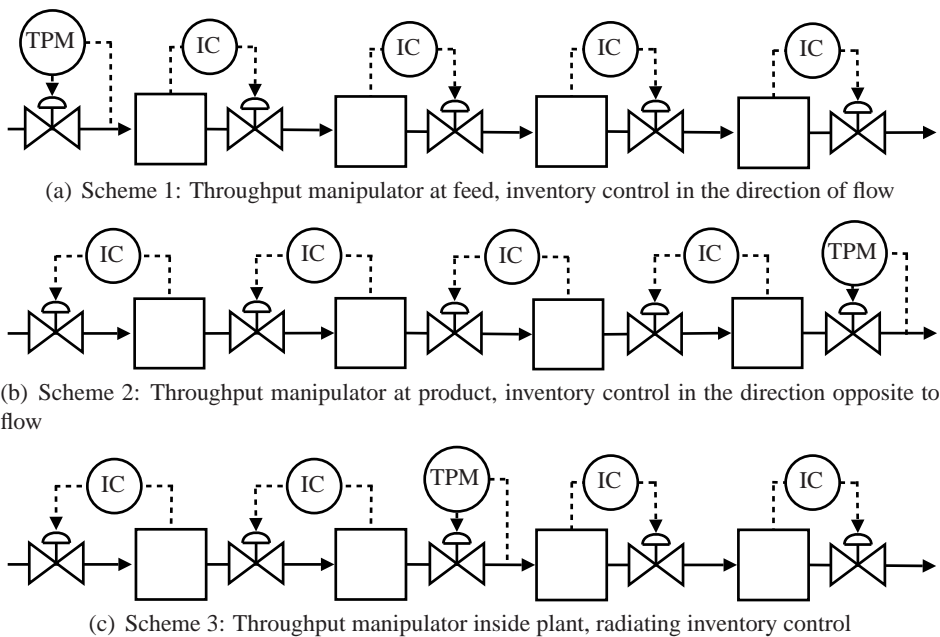
Figure 3.2: Basic schemes for inventory control. IC stands for inventory control and are typically a level controller (liquid) or a pressure controller (gas).

the others. Second, plants with parallel trains must have at least one TPM for each train. There may also be parallel trains inside the process, and the corresponding split may be viewed as a throughput manipulator. In addition, plants with parallel trains may have crossover flows, which also affect the throughput and may be viewed as throughput manipulators. To account for this, we propose the following general definition:

Definition 3.2. *Throughput manipulator (TPM). A throughput manipulator is a degree of freedom that affects the network flows (normally including feed and product flows), and which is not indirectly determined by other process requirements.*

Thus, a TPM is an "extra" degree of freedom, which is not needed for the control of individual units, but that can be used to set or optimize the network flows. Splits and crossovers can be viewed as throughput manipulators but they do not necessarily affect both the feed and the product flows. For example, if there is a split and the parallel processes are combined further downstream, the split factor will affect neither the feed nor the product flow. In Definition 3.2, "other process requirements" are often related to satisfying the component material balances, as discussed in the following examples.

Example 3.1. *Consider a process with two feeds, F_A of pure component A and F_B of pure component B, where the reaction $A + B \rightarrow P$ (product) takes place. Normally, in order to avoid losses, the feeds should be stoichiometric. Thus, we need $F_A = F_B$ at steady-state, which indirectly removes one degree of freedom, so the process has only one TPM.*

Example 3.2. *Consider the same process as in Example 3.1 with three feeds F_A, F_B and F_{AB}, where the latter consist of a mixture of A and B. The stoichiometry imposes one constraint, but otherwise the optimal ratio between these feeds is determined by plantwide economic arguments, and not by process requirements. Thus, according to Definition 3.2, this process has two TPMs. For example, the TPMs could be F_A and F_{AB}, with F_B adjusted to satisfy the stoichiometry.*

Example 3.3. *Consider a process with two feeds, F_A with pure component A and F_{AI} with A plus some inert I. The reaction $A \rightarrow P$ (product) takes place. This process has two TPMs because the (optimal) amount of the two feeds is determined by plantwide consideration.*

Example 3.4. *Consider a process with two feeds; F_A contains pure A and F_B contains pure B. The reactions $A \rightarrow P + X$ and $B \rightarrow P + Y$ take place, where P is the main product, and X and Y are byproducts. This process has two TPMs, because the ratio F_A/F_B is not given by other process requirements.*

In summary, we see from these examples that even quite simple processes can have more than one TPM. In addition to these examples, we have the more obvious cases of multiple TPMs, such as a process with parallel trains and crossovers.

3.5 Characteristics of the maximum throughput case

We have shown that maximum throughput is often the economically optimal mode of operation. In this section, we want to identify the main characteristics of the maximum throughput case.

3.5.1 Bottleneck

The *max-flow min-cut* theorem (Ford and Fulkerson, 1962, p.11) from linear network theory states that: "for any network the maximal flow value from source to sink is equal to the minimal cut capacity of all cuts separating source and sink". In simple terms, the theorem states that the maximum flow in a network is dictated by its bottleneck. To study bottlenecks in more detail, we need to define some terms.

Definition 3.3. *Maximum flow (capacity) of a unit. The maximum flow (capacity) of a unit is the maximum feed rate the unit can accept subject to achieving feasible operation.*

Mathematically, this corresponds to solving the maximum flow problem (3.1) with $(-J) = F_{\max,i}$, where $F_{\max,i}$ is the maximum feed for the unit i and u_i are the degrees of freedom for unit i. This means to find the maximum value of $F_{\max,i}$ that satisfies the constraints $f_i = 0$ and $g_i \leq 0$ for the unit.

Definition 3.4. *Maximum throughput of a plant. Let the throughput F be the (weighted) sum of all the feed flows. The maximum throughput $F_{\max}$ of a plant is the maximum network flow that a plant accept subject to achieving feasible operation.*

In the optimization problem, implied by Definition 3.4, all degrees of freedom (all F_i's) should be used to maximize the throughput, subject to achieving feasible operation (satisfying the constraints).

Definition 3.5. *Bottleneck. A unit is a bottleneck if maximum throughput (maximum network flow for the system) is obtained by operating this unit at maximum flow (see Definition 3.3).*

Definition 3.6. *Bottleneck constraints. The active constraints in the bottleneck unit are called the bottleneck constraints.*

The term "unit" in Definitions 3.5 and 3.6 needs some attention. For a simple process, where the process units are in series, a "unit" is the same as a single process unit. However, for integrated processes, one may need to consider a combined system of integrated units as a "unit". For example, for a chemical reactor with recycle, the combined "unit" may be the system of units consisting of the reactor, separator and recycle unit (e.g. compressor or pump). This is because the maximum flow to the combined system is not necessarily determined by the maximum flow in an individual unit. For example, if the chemical reactor is too small such that the conversion is too small (and thus in practice is a bottleneck); then this will result in increased recycle of unconverted reactant (also known as the "snowball effect"), which eventually will overload the separator, the compressor or pump. Thus, it will appear that one of these units is the bottleneck, whereas it is really the entire reactor system, and the reactor in particular, which is the problem in terms of capacity.

In Definition 3.5, note that if a flow inside a unit is at its maximum, this does not necessarily mean that the unit is a bottleneck. The unit is only a bottleneck if it operates at maximum feed rate according to Definition 3.3. For example, the heat flow in a distillation column (the unit) may optimally be at its maximum, because

overpurification of the "cheap" product is optimal in order to recover more of the valuable product. This does not mean that the column is a bottleneck, because it is possible, by reducing the overpurification, to increase the feed rate to the column. Only when *all* degrees of freedom are used to satisfy active constraints, do we have a bottleneck.

Note that in Definition 3.6, the active constraints in a bottleneck unit do not need to be flows or even extensive variables. For example, for the distillation column just mentioned, as the feed rate is increased, one will eventually reach the purity constraint on the "cheap" product, and if there are no remaining unconstrained degrees of freedom, the distillation column becomes the bottleneck unit. The active purity constraints on the products together with the maximum heat flow constraint then comprise the "bottleneck constraints".

3.5.2 Back off

Back off is generally required to guarantee feasibility when operating at active constraints (except for hard input constraints), as discussed in Section 3.3. We here discuss the implication of this. As we reach the bottleneck (and encounter a new active constraint), the throughput manipulator (e.g. feed rate) is the only remaining unconstrained input. To operate at the bottleneck, the throughput manipulator must be used as a degree of freedom to control this new active constraint. Based on the discussion in Section 3.3, we have the following cases:

1. The new bottleneck constraint is an output variable. The result in terms of control is "obvious": the TPM controls this output at the active constraint (with back off included).

2. The new bottleneck constraint is an input constraint. Here we have two cases:

   (a) The input variable is not used for control. Then the input is simply set at its constraint (no back off for hard input constraints).

   (b) The input variable is already used for control of a constrained output variable. There are two possibilities, depending on which back off is most costly:

       i. The TPM takes over the lost task. However, we usually have to increase the back off on this output, because of poorer dynamic control, since the TPM is generally located farther away from the output constraint than the saturated input.

       ii. Alternatively, we can let the original loop be unchanged, but we must then introduce an additional a back off on the input to en-

counter for dynamic variations. The TPM is then used to keep the input in desired operation range.

3.5.3   Summary of characteristics of maximum throughput case

From the discussion above we derive the following useful insights (rules) for the TPM in the maximum throughput case (Mode 2a):

Rule 3.1. *All plants have at least one throughput manipulator and at maximum throughput the network must have at least one bottleneck unit.*

Rule 3.2. *Additional independent feeds and flows splits may give additional TPMs (see Definition 3.2) and additional bottlenecks. The idea of "minimal cut" from network theory may be used to identify the location of the corresponding bottleneck units.*

Further, for tight control of the bottleneck unit and to minimize loss the following insights (rules) are stated for the maximum throughput case:

Rule 3.3. *The throughput manipulator(s) (TPM) is the steady-state degree of freedom for control of the bottleneck unit(s). Typically, the TPM is used to control one of the bottleneck constraints (Definition 3.6). The TPM should therefore be located so that controllability of the bottleneck unit is good (Skogestad, 2004).*

Rule 3.4. *Bottleneck unit: focus on tight control on the bottleneck constraint with the most costly back off in terms of loss in throughput.*

The last rule follows because any deviation from optimal operation in the bottleneck unit due to poor control (including any deviation or back off from the bottleneck constraints) implies a loss in throughput which can never be recovered (Section 3.3.3).

3.5.4   Moving bottlenecks

In the simplest maximum throughput case, the bottleneck is fixed and known and we can use single-loop control (Skogestad, 2004), where the TPM controls the constraint variable in the bottleneck unit.

If the bottleneck moves in the plant, then single-loop control requires reassignment of loops. Reassignment will involve the loop from TPM to the bottleneck (Rule 3.3), as well as the inventory loops needed to ensure self-consistency in the plant (Chapter 2). In addition, the moving bottleneck(s) itself needs to be identified.

For moving bottlenecks, a better approach in most cases is to use multivariable control were also input and output constraints can be included directly in the problem formulation (e.g. MPC). A case study using MPC for maximizing throughput with moving bottlenecks is described in Aske *et al.* (2008). In this case study, the capacity of the individual units is obtained using the models in the local (units) MPC. The main TPMs are located at the feed (conventional inventory control, Figure 3.2(a)), but there are additional degrees of freedom (splits and crossovers) to manipulate the throughput.

3.6 Obtaining (estimate) the back off

If we have a maximum throughput situation (Mode 2a) and the bottleneck has been correctly identified, then operation is optimal, except for the economic loss associated with the back off from active constraints. Back off is usually most costly in the bottleneck unit. It is important to know (or estimate) the expected back off in order to quantify the possible benefits of moving the TPM (changing the inventory control system), adding dynamic degrees of freedom (Chapter 4), changing or retuning the supervisory control system etc.

In the following we consider the case with a single input (TPM) that controls an active output constraint (y) in the bottleneck unit. A back off is then required to account for dynamic variations caused by imperfect control.

The magnitude of the back off for the dynamic control error should be obtained based on information about the disturbances and the expected control performance. Mathematically, this is given by the worst-case control error (variation) in terms of the "∞-norm" (maximum deviation). In the time domain the dynamic control error (and hence the minimum back off) is given by:

$$b_{\min} = \max_{d,\Delta} \|y(t) - y_s\|_\infty \tag{3.9}$$

where d and Δ denotes disturbance and uncertainty, respectively. The optimal (minimal) back off b is equal to the expected dynamic variation in the controlled variable y. In practice, determining the expected dynamic variation is difficult. However, the point here is not to estimate the minimum back off exactly, but to obtain a rough estimate. The simple method is based on controllability analysis.

3.6.1 Model-based approach (controllability analysis)

Without control, we assume here that the effect of the disturbance on the output (in this case a bottleneck constraint variable) is given by a first-order response with steady-state gain k_d ($= |\Delta y|/|\Delta d|$) and the time constant τ_d. Without control, the

(a) Step response in d at $t = 0$

(b) Corresponding frequency response of Sg_d where $S = (1 - gc)^{-1}$
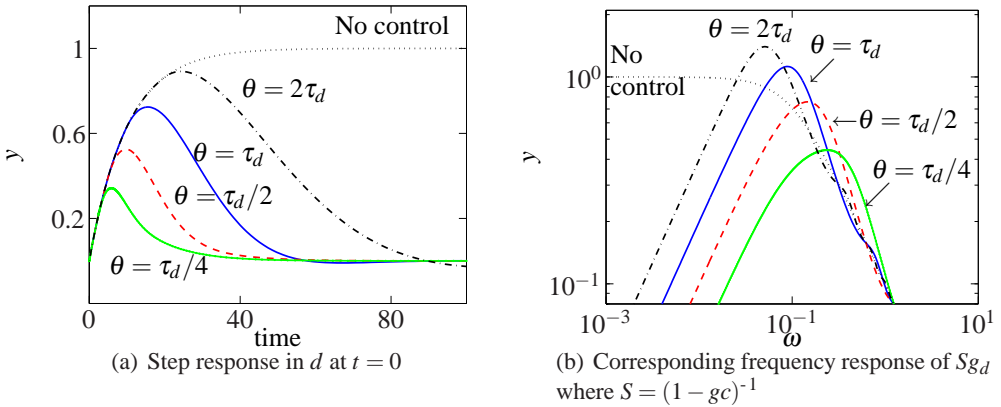
Figure 3.3: PI-control of first-order disturbance: Effect of effective delay θ. $y = g(s)u + g_d(s)d$. Process: $g(s) = k \frac{e^{-\theta s}}{\tau_1 s + 1}$, $\tau_1 = 10$. Disturbance: $g_d = \frac{1}{\tau_d s + 1}$, $\tau_d = 10$. Controller: $c(s) = K_c \frac{\tau_I s + 1}{\tau_I s}$ where $K_c = \frac{1}{k} \frac{\tau_1}{\tau_c + \theta}$ and $\tau_c = \theta$.

required minimum back off is then $b_{\min} = k_d |d_0|$, where $|d_0|$ is the magnitude of the disturbance. To counteract the effect of the disturbance using feedback control, and thus be able to reduce the back off, the control system needs response with a closed-loop time constant τ_c less than about τ_d. The main "enemy" of feedback control, which limits the achievable τ_c, is the time delay θ. In practice, most processes do not have a "pure" time delay, but they have an "effective" time delay θ_{eff}, which can be estimated from the dynamic model, for example, using the "half rule" of Skogestad (2003).

A simple example of a PI-controlled process with a first-order disturbance is illustrated in Figure 3.3: We see from Figure 3.3(a) that when the delay θ is equal to about τ_d or larger, then there is no significant improvement for a step disturbance. In fact, if we look at sinusoidal disturbances (Figure 3.3(b)), significant improvement in the maximum peak (which determines the necessary back off) is obtained by requiring $\theta \le \tau_d/4$. A more realistic process with five units is given in Example 3.5.

Example 3.5. *Minimum back off for different TPM locations. Consider a process with 5 units in series and a fixed bottleneck which is located at the outlet of the last unit (Figure 3.4). The objective is to maximize the throughput using single-loop control in spite of disturbances d_1 to d_5. The disturbances are of equal magnitude, but d_1 is located closest to the bottleneck and has therefore the major effect on the bottleneck. Consider three locations of the TPM:*

- *A: the conventional approach where the TPM is located at the feed,*
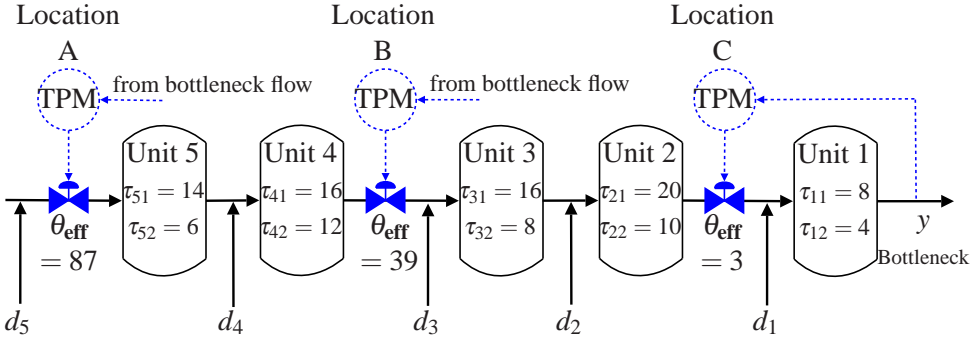
Figure 3.4: The process example with different placements of the TPM with reconfiguration of inventory loops. Inventory control is in direction of flow downstream TPM and in direction opposite to flow upstream TPM. The time constants for each unit is displayed together with the effective dead time (θ_{eff}) for each location for the throughput manipulator.

- **B:** *the TPM is located inside the process,*

- **C:** *the recommended approach in this paper where the TPM is located at the bottleneck.*

Each unit is represented by a second order model where the time constants (τ_1, τ_2) are stated in Figure 3.4. In addition unit 1 has a delay $\theta_1 = 1$. The disturbances d_1 to d_5 enter between the units. This gives the following disturbance transfer functions (G_{d_i}) from the disturbances (d_1, d_2, d_3, d_4, d_5) to the bottleneck flow (y):

$$G_{d_j} = k_d \cdot \prod_{i=1}^{j} \frac{e^{-\theta_1 s}}{(\tau_{i1}s+1)(\tau_{i2}s+1)}$$

The disturbance gain is given by k_d and is here selected to $k_d = 1$. The process transfer functions G_A, G_B and G_C from the input (TPM at location A, B, or C) are the same as for the disturbances, except that the process gain is given by k and here selected to $k = 2$.

The TPM (u) is adjusted using a PI feedback controller ($y = Ku$, $K = K_c(1 + \frac{1}{\tau_I s})$) that controls the bottleneck flow (y) and tuned using the SIMC tuning rules with $\tau_c = 3\theta_{eff}$. The resulting sensitivity function $S = (I + GK)^{-1}$ for the three alternatives is showed in Figure 3.5. Note that the response is much faster with the TPM located close to the bottleneck (location C).

The minimum back off b_{min} for each disturbance $|Sg_d|$ is displayed as a function of frequency for the TPM located at feed (A), in the middle (B) and at the bottleneck (C) in Figure 3.6(a), 3.6(b) and 3.6(c), respectively. Note that a linear scale on back off b is used since the cost is linear in back off (Equation (3.8)).
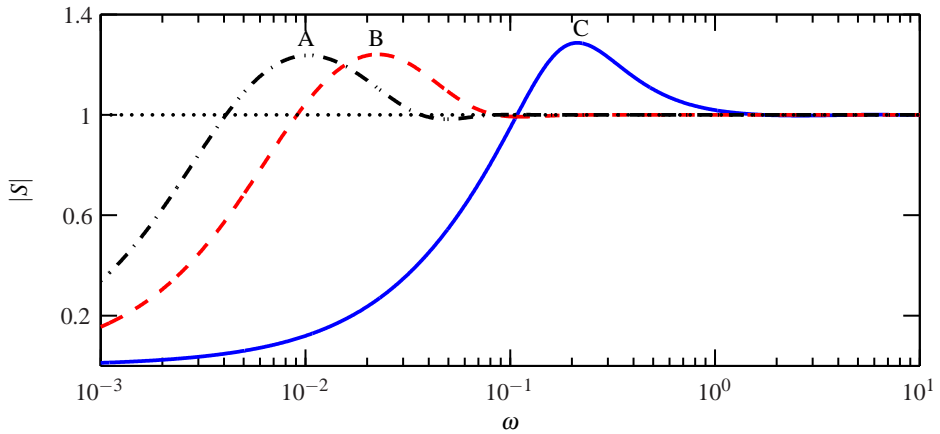
Figure 3.5: Sensitivity $|S|$ as a function of frequency for different placement of the TPM (location A,B and C) in Example 3.5. $S = (I + GK)^{-1}$ and K is a PI-controller.

With the TPM located at the bottleneck (Figure 3.6(c)), the peak of $|Sg_d|$ is reduced significantly, and especially disturbances d_2 to d_5 (upstream the TPM) have a very small effect on the bottleneck flow. With the TPM placed at the feed (Figure 3.6(a)), all the disturbances have almost the same effect on the bottleneck. At the worst-case frequency, the peak of $|Sg_d|$ is about 1.25 which is higher than the value of 1 (because the peak of $|S|$ is $M_s = 1.25$). Of course, we need to apply control to avoid steady-state drift, but this indicates that further detuning of the controller should be considered (the larger τ_c will reduce M_S), but this will lead to poorer set point tracking. For the TPM located inside the process string (Figure 3.6(b)), the peak of $|Sg_d|$ for d_1 (the most important disturbance) has almost the same magnitude as for TPM located at the feed, but the effect of the disturbances d_2 to d_5 is reduced.

The peak of $|Sg_d|$ with TPM located at the bottleneck is reduced from 0.7 to 0.3 by using a PID-controller instead of a PI. For the two other locations there is only a very small difference in the peak of $|Sg_d|$ between PI- and PID-controllers. In practice, PI-controllers are more common to use than PID since the latter is sensitive to noise and therefore a PI-controller is used here.

From the more detailed derivations of estimating minimum back off (Appendix 3.A.1) we have:

- An "easy" (slow) disturbance has a time constant $\tau_d > 4\theta_{\text{eff}}$. In this case tight bottleneck control (tight control of y) is helpful for rejecting the disturbance. The worst-case frequency is $\omega_{wc} \approx \frac{1}{\tau_d}$ and the resulting minimum back off assuming PI-control with "tight" control is given by $b_{\min} \approx \frac{2\theta_{\text{eff}}}{\tau_d} \cdot k_d |d_0| \leq$

(a) TPM at feed (location A) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 87$

(b) TPM in middle (location B) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 39$

(c) TPM at bottleneck (location C) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 3$

Figure 3.6: Minimum back off ($|Sg_d|$) as a function of frequency for the disturbances d_1 to d_5 on the bottleneck flow, for the three different locations of TPM (A, B, C) in Figure 3.4.

$k_d|d_0|$ (assuming a SIMC PI-controller with $\tau_c = \theta$). This shows that the back off can be significantly reduced if θ_{eff} is small compared to τ_d.

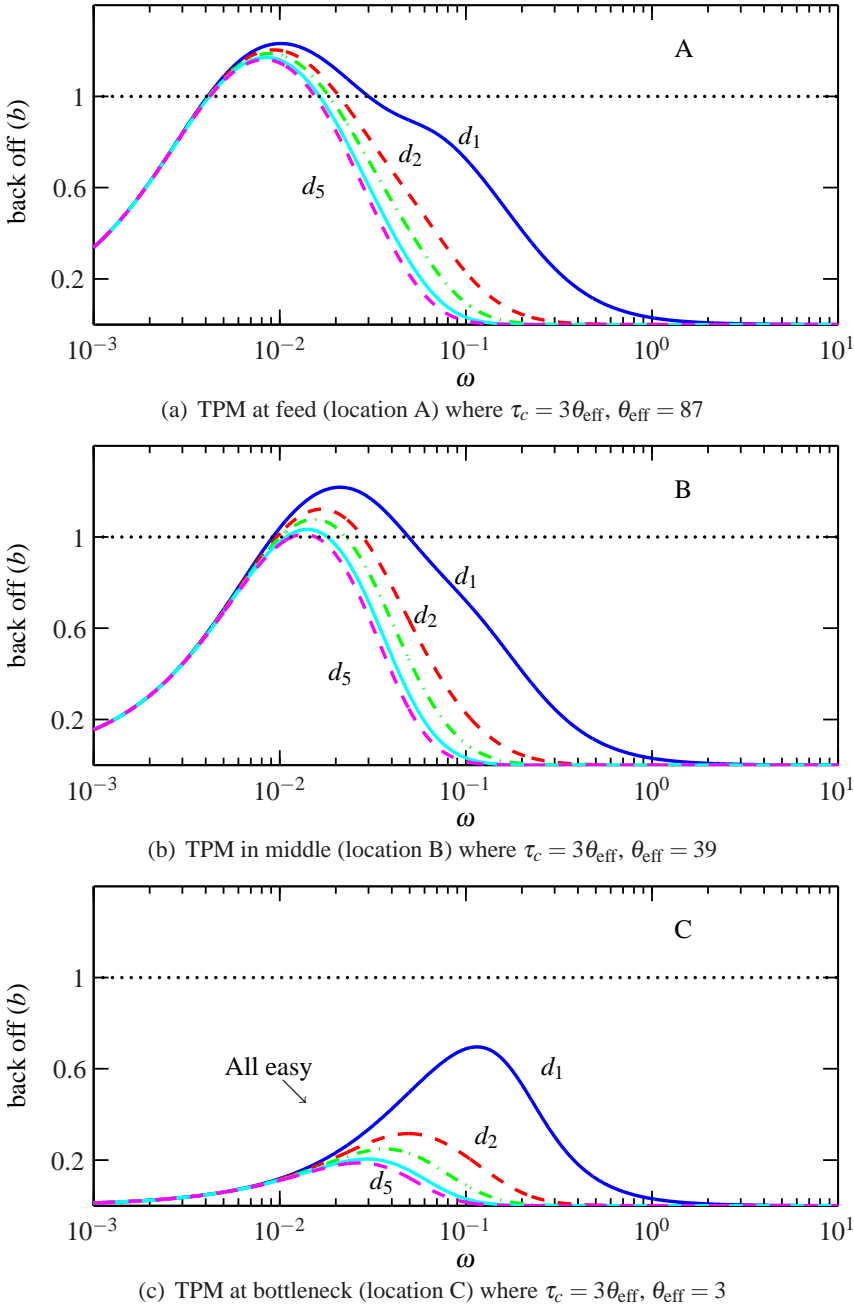- A "difficult" (fast) disturbance has a time constant $\tau_d < 2\theta_{\text{eff}}$. In this case, control actually gives a larger back off than no control. However, control is necessary for set point tracking. The worst-case frequency is $\omega_{wc} \approx \omega_{peakS}$ where ω_{peakS} is the peak frequency of $|S|$ defined as $|S(j\omega_{peakS})| = \max_\omega |S(j\omega)| = M_S$. To reduce the peak M_S, it is recommended to used "smooth" control (with $\tau_c \geq 2\theta$), that is , for following slow changes in the bottleneck constraints. The minimum back off is given by $b_{\min} \approx M_S \cdot k_d|d_0|$.

In summary, the requirement $\theta_{\text{eff}} < \frac{\tau_d}{4}$ to have benefit of control implies that the TPM must be located very close to the bottleneck to have any benefit of improved control and reducing back off. This also explains in most cases why the loss with manual control, where the operator adjusts the TPM, is usually small.

A more detail mathematical model-based approach for estimating the minimum back off is discussed by Narraway *et al.* (1991); Heath *et al.* (1996) and Loeblein and Perkins (1999) (see Appendix 3.A.2 for more details). The approach requires a nonlinear dynamic model of the process and optimizes simultaneously the control structure and controller parameters in order to find the minimum back off required accommodating the effects of disturbances. However, this approach is too rigorous to be useful as a practical engineering tool.

3.6.2   On-line identification

On-line identification or simply manual adjustment based on experience is the most common approach to determine the back off. In practice, instead of identifying the disturbances itself, it is easier to identify from plant data the output variance. The back off must be set larger than the observed variations to ensure feasible operation even with worst-case disturbances. The back off may be successively reduced from the initial value with increasing disturbance experience. On-line identification is the simplest method, but may be time consuming and requires extensive monitoring of the plant.

3.7   Reducing the back off

Reducing the back off may possibly increase the throughput and give large improvements in profit. To reduce the back off, the first step is to reduce the dynamic variation (squeeze) in the variables with the most costly back off. In the following, suggestions to obtain less dynamic variation are listed.

Improvement 1: Retune the control loops, especially those associated with the bottleneck unit in order to reduce dynamic variations, primarily in the active "hard" constraints variables.

Improvement 2: Move, add or make use of additional degrees of freedom, that influence the flow through the bottleneck (e.g. throughput manipulator, crossovers, splits, extra feeds, inventories) to obtain tighter dynamic control of the bottleneck unit.

Improvement 3: Introduce feedforward control from measured disturbances to obtain tighter control.

Improvement 4: Introduce feedforward control from expected changes in the active constraint variable ($y_{\text{constraint}}$) to the set point (y_s) to keep the back off b unchanged.

Improvement 5: Adjust the back off b depending on expected disturbance level. Importantly, the back off b can be reduced (move y_s closer to $y_{\text{constraint}}$) when the expected disturbance level is low ("calm periods").

Improvement 6: Exploit the hold-up volume in buffer volumes as a dynamic degree of freedom to obtain tighter bottleneck control.

Improvement 7: Add buffer tank to dampen disturbances that affect the active constraints.

A more detailed discussion of each Improvement is given below.

Improvement 1: Retune control loops

As shown in Section 3.6, the possibility to reduce the back off by achieving tight control of the bottleneck unit itself is limited in most cases, unless the TPM is located close to the bottleneck. However, this does not mean that retuning is not important, because retuning the control loop may avoid *unnecessary* variations in variables that may propagate dynamic variations to the bottleneck unit. An example is a poorly tuned temperature controller in a distillation column upstream the bottleneck unit. The temperature controller performance can be acceptable for composition control in the distillation column itself, but it may lead to unnecessary flow variations that disturb the downstream (bottleneck) unit(s).

Improvement 2: Move, add or use additional degrees of freedom

As mentioned in Section 3.5.3, the TPM should be moved close to the bottleneck unit in order to reduce the effective time delay from the TPM to the bottleneck.

However, other alternatives should be considered because moving the TPM requires reconfiguration of the inventory loops to obtain a self-consistent inventory control system (see Section 3.4). Note that it is possible to move the TPMs without reconfiguration, but then the inventory control system will only be consistent and may consist of "long loops". Such a "long loop" requires larger hold-up volume because of longer physical distance and hence longer effective time delay. Other ways to shorten the possible "long loop" from the TPM to the bottleneck unit is to use other variables that affect the throughput, like crossovers between parallel units and feed splits (see Rule 3.2). The key point for using additional degrees of freedom is to reduce the effective time delay from the manipulated variable to the active constraint in the bottleneck unit.

Improvement 3: Feedforward control from measured disturbances

Feedforward control from (important) disturbances can reduce the dynamic variation in the controlled variable (bottleneck constraint) y. This leads to tighter control and the back off can be reduced.

Improvement 4: Follow changes in $y_{constraint}$ (feedforward action)

From (3.7), the back off is $b = |y_{constraint} - y_s|$, so the actual set point y_s is set by $y_{constraint}$ and the back off b. The "hard" constraint $y_{constraint}$ may change due to disturbances and we want y_s to follow these variations (at least to some extent) to avoid an unnecessary change in back off (b). For example, consider a distillation column operating at maximum throughput. The maximum feed rate to the column depends on the feed composition, and a change in the feed composition may increase the maximum feed rate, hence an increase in $y_{constraint}$ occurs. By increasing y_s correspondingly to $y_{constraint}$, the back off b will remain constant. With available disturbance measurements, feedforward can be applied to adjust y_s.

Improvement 5: Adjust back off depending on disturbance level (feedforward action)

Compared to Improvement 4, where y_s is adjusted to keep a *constant* back off, we want here to *adjust the back off b* itself depending on the expected disturbance level. The idea is that the back off can be reduced in (expected) "calm periods". For example, consider a plant that receives feed gas at high pressure through a long pipeline, where the feed composition is monitored at the pipe inlet. The feed composition is an important disturbance, and by monitoring the feed composition in the pipeline, one will know in advance when the changes will occur. In periods with no feed composition changes, the back off b can be reduced. It is important

that the monitoring of disturbance level is reliable, so that the back off can be increased again during periods with larger disturbances.

Improvement 6: Buffer volume as dynamic degree of freedom

The hold-up volume in a process can be exploited as *dynamic degree of freedom* to obtain faster (short-term) corrections of the flow to the downstream unit. When using inventories, the hold-up volume must be refilled from upstream source to avoid emptying, so this requires acceptable speed of the inventory control systems. The hold-up volume should be large enough to change the throughput in the downstream unit for the period it takes to refill it. Implementing hold-up volumes can be done by by using ratio control (single-loop) or a multivariable dynamic controller (e.g. MPC) that manipulate on the buffer volume (level). These issues are discussed in more detail in Chapter 4.

Improvement 7: Add buffer volume

The buffer volume can dampen the variations (or the disturbances) by exploiting its hold-up volume. This requires smooth tuning of the buffer volume, otherwise inflow $\approx$ outflow and no smoothing will be obtained. Buffer volumes that is added to smooth out disturbances that affect the bottleneck must be placed upstream the bottleneck. Buffer volumes downstream the bottleneck has no effect on the bottleneck (the active constraint) and no reduction in back off will be obtained. However, note that hold-up volumes placed between the throughput manipulator and the bottleneck increases the effective time delay for flow rate changes, and tight control of the bottleneck unit becomes more difficult if the buffer volume is not exploited.

Example 3.6. *Using buffer volumes as dynamic degrees of freedom to obtain tighter bottleneck control. This example illustrates tighter bottleneck control by using hold-up volumes as dynamic degrees of freedom. Consider three units, each followed by a buffer (hold-up) volume, as displayed in Figure 3.7. Maximum capacity for each unit changes due to disturbances and the bottleneck moves. The objective is maximum throughput and the throughput manipulator is located at the feed but the hold-up volumes are exploited for tighter control of the bottleneck.*
Three different control structures are studied:

1. *Manual control where the TPM is set at a rate that ensures feasibility in spite of the predefined disturbances.*

2. *An MPC controller that uses only the TPM as manipulator to maximize throughput and consider the constraints in each unit.*
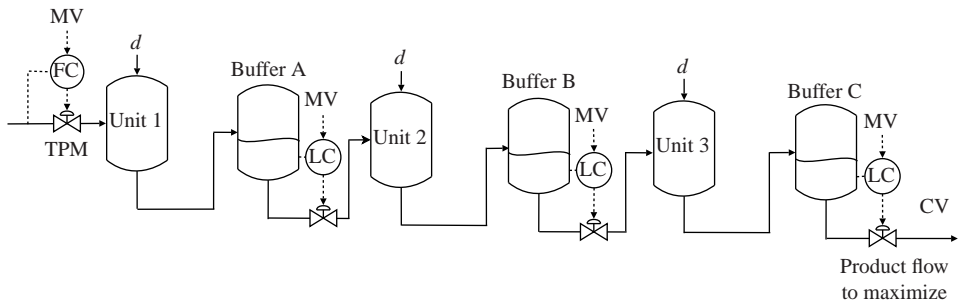
Figure 3.7: Diagram of the simulated process in Example 3.6 with MPC that uses the feed rate and the buffer volumes to maximize throughput (control structure 3).

> 3. An MPC controller that uses the TPM in addition to the set point to the level controller in the three buffer volumes as manipulated variables to maximize throughput and consider the constraints in each unit.

The predefined step-disturbances are regarded as unmeasured and not included as disturbance variables (DVs) in the MPC controller. The necessary back off from the hard capacity constraints in the units are found by trial-and error. With the predefined step-disturbances present, accumulated production for the three control structures is given in Figure 3.8. Using the hold-up volumes (structure 3) tighter bottleneck control is obtained and the accumulated production is increased. Using only the feed rate is only marginally better than manual control. This is due to the long effective delay ("long loop") from the feed valve to the constraint and hence a large back off is needed.

3.8   Discussion

3.8.1   Network theory

The maximum throughput case in production systems is closely related to the maximum flow problem in networks considered in operations research. Such a network consists of sources (feeds), arcs, nodes and sinks (products) (e.g. Phillips *et al.*, 1976). An arc is like a pipeline or unit with a given (maximum) capacity and the nodes may be used to add or split streams. We assume that the network is linear, which requires that the splits are either free variables ("actual" splits or crossovers in process networks) or constant (typically, internal splits in the units in process networks, for example, a distillation column that splits into two products). We then have a linear programming problem, and the trivial but important conclusion is that the maximum flow is dictated by the network bottleneck. To see this, one introduces "cuts" through the network, and the capacity of a cut is the sum of the
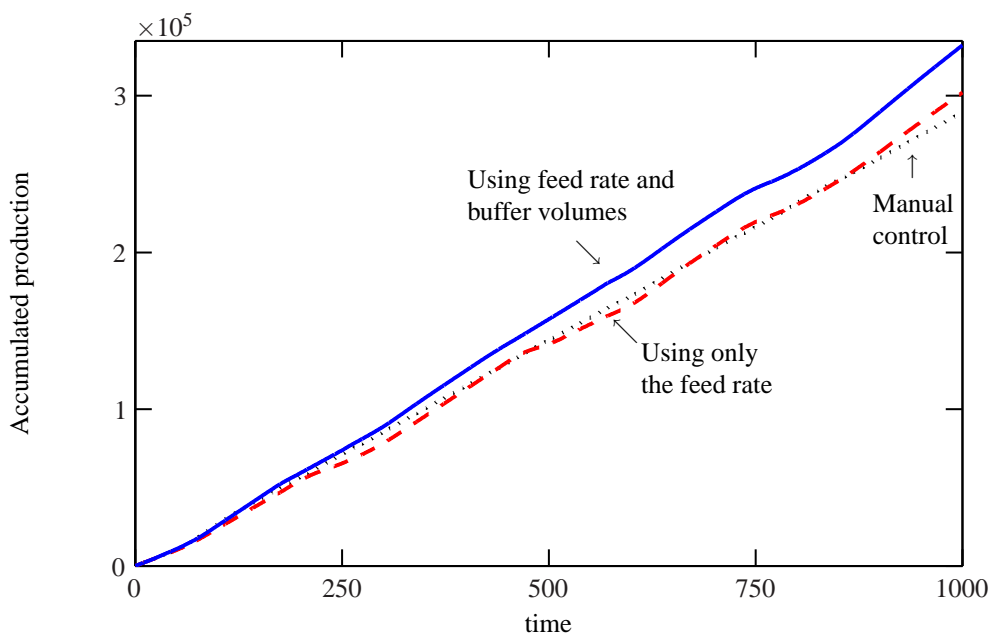
Figure 3.8: Accumulated product rate manual control (TPM constant, dotted line), TPM in closed-loop (dashed) and both using TPM and hold-up volumes (solid).

capacity of the forward arcs (arcs that is leaving the node) that it cuts through. The *max-flow min-cut theorem* (Ford and Fulkerson, 1962) says that the maximum flow through the network is equal to the minimum capacity of all cuts (the minimal cut). We then reach the important insight that maximum network flow (maximum throughput) requires that all arcs in some cut have maximum flow, that is, they must all be bottlenecks (with no available capacity left). Figure 3.9 illustrates parts of a chemical plant with sources ($s1 - s3$), arcs, nodes (units $u1 - u11$ and junctions $m1 - m3$ in our terms) and sinks ($n1 - n12$) and a possible location of the minimal cut. The location of the minimum cut shows that the units $u1$ and $u11$ are bottlenecks units. Note that a cut separating the source and the sink is a partition of the nodes into two subset S and $\bar{S}$ where the source nodes are in S and the sink nodes are in $\bar{S}$ (e.g. Phillips *et al.*, 1976). The arc denoted c (crossover) is not included in the summation of the capacity in the minimal cut since it is directed from a node in $\bar{S}$ to a node in S. A network like the one displayed in Figure 3.9 with multiple sources and sinks can be converted to a single-source single-sink by creating an imaginary super source and an imaginary super sink (Phillips *et al.*, 1976), but this is not included here. Therefore it does not seem like all the sink nodes are located in the subset $\bar{S}$ in Figure 3.9.
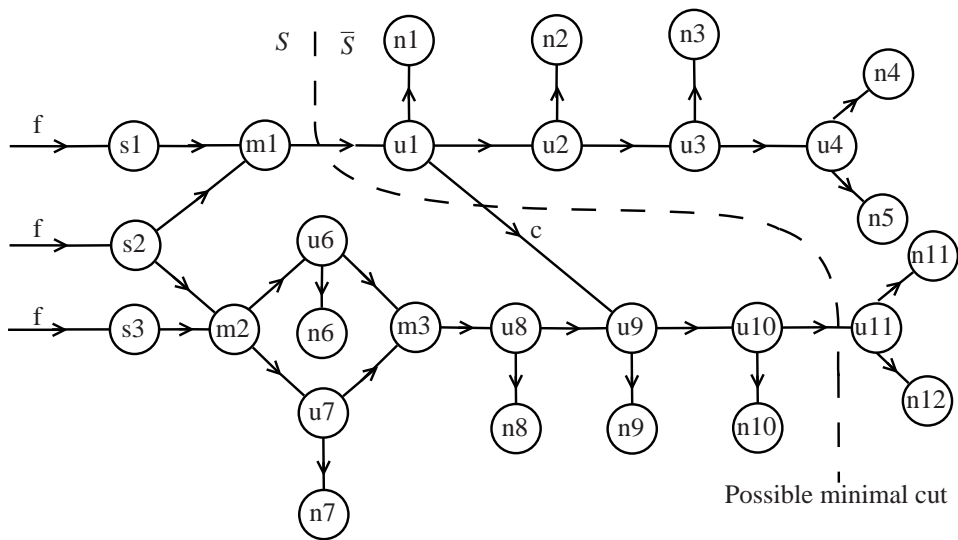
Figure 3.9: A part of a gas plant illustrated as a flow network, with a possible minimal cut. The corresponding flow sheet of the gas processing plant is displayed in Figure 5.3.

To apply network theory to production systems, we first need to obtain the capacity (maximum flow) of each unit (arc). This is quite straightforward, and involves solving a (nonlinear) feasibility problem for each unit (see Definition 3.3). The capacity may also be computed on-line, for example, by using local MPC implementations as proposed by Aske *et al.* (2008).

The main assumption for applying network theory is that the mass flow through the network is represented by linear flow connections. Note that the nonlinearity of the equations within a unit is not a problem, but rather the possible nonlinearity in terms of flows between units. The main problem of applying linear network theory to production systems is therefore that the flow split in a unit, e.g. a distillation column, is not constant, but depends on the state of its feed, and, in particular, of its feed composition. The main process unit to change composition is a reactor, so decisions in the reactor may strongly influence the flow in downstream units and recycles. Another important decision that affects composition, and thus flows, is the amount of recycle. One solution to avoid these sources of nonlinearity is to treat certain combinations of units, like a reactor-recycle system, as a single combined unit as seen from maximum throughput (bottleneck) point of view.

Although the linearity assumptions will not hold exactly in most of "our" systems, the bottleneck result is nevertheless likely to be optimal in most cases. The reason is that the location of active constraints (bottleneck) is a structural issue.

3.8.2   Issues on estimation of back off

Estimating the dynamic variation in a controlled variable y by using controllability analysis has some limitations. The back off estimation is only valid for single-loop control where the controller is tuned by using the SIMC-tuning rules. The tuning rules are not really a limitation, since the speed of the closed-loop response is a degree of freedom. However, the simplified analytic estimation needs a model of the disturbance and assumes that the shape of G_d is flat up to the break frequency where the disturbance rolls off. The asymptotic consideration of the disturbance will be wrong, especially for higher order. For a higher order disturbance, the assumption that G_d is "flat" up to ω_{bd} will not be correct, since the disturbance starts to roll of at a lower frequency.

With our experience from industry today, on-line identification is by far the most used. A model is not required in this case, only plant data. For a new plant, estimating necessary back off has minor importance; because during a plant start up, optimal production is not the issue, but rather to obtain stabilized production. After reaching nominal production, reducing back off and optimal production becomes an operating issue, but at that time plant data is available. Operating margins is typically reduced gradually. With close follow-up from personnel, the time spent to move the plant from nominal to optimal production can be reduced.

Back off is based on experience and therefore the importance of the manual control should not be underestimated. However, a new regime of closed-loop control of the throughput can be fulfilled, but now with the back off as the available manipulator for the operators instead of the throughput. This makes the back off (and also the loss) more visual instead of being "baked into" the throughput set point.

3.9   Conclusion

In this paper, we have shown that "maximum throughput" is an optimal economic operation policy in many cases. To implement maximum throughput, the key is to achieve maximum flow through the bottleneck unit(s). However, to achieve feasible operation (no constraint violation), is usually necessary to "back off" from the optimally active constraints. Back off leads to a lower flow through the bottleneck and an unrecoverable economic loss. This leads to the obvious but important conclusion that "throughput maximization requires tight bottleneck control". However, achieving tight bottleneck control in practice is not so simple because the throughput manipulator is often located too far away from the bottleneck unit (with a large effective delay θ_{eff}) to be effective for reducing the effect of disturbances on the key bottleneck variables. For example, to significantly reduce the

effect of a first-order disturbance (and be able to reduce the back off), we must require $\theta_{\text{eff}} < \tau_d/4$ where τ_d is the first-order response times for the disturbance. In practice, the requirement $\theta_{\text{eff}} < \tau_d/4$ is unlikely to be satisfied unless the TPM is located at the bottleneck unit. Thus, "tight bottleneck control" (and reducing the back off) in practice requires that the TPM is located close to the bottleneck unit. This can either be achieved by moving TPM (which requires reconfiguration of the inventory control system) or for some plants, to utilize "extra" TPMs such as crossovers and splits (Chapter 5). Another alternative is to make use of dynamic degrees of freedom (variations in the inventories) as is further discussed in Chapter 4. Increased throughput can also be achieved by strategies where the back off is reduced in "calm" periods where there are less disturbances. Possible improvements to reduce back off are listed in Section 3.7.

3.A   Estimation of minimum back off

We here use a controllability analysis for identifying the dynamic control varia-
tions. This requires a model of the process together with assumption of the ex-
pected frequency and amplitude of the disturbances. Controllability is a property
that is independent of the detailed controller tuning, but here we assume that IMC-
tuning are used. The issue here is to estimate the minimum required back off from
a model without designing a controller.

3.A.1   Simplified analytic estimation for single-loop control

Let y denote the controlled active constraint in the bottleneck unit, for which we
want to estimate the expected dynamic variation which is equal to the minimum
back off. Let u denote the manipulated variable (e.g. TPM or a dynamic variable
that affects y) and d the disturbance. For the linearized system $y = Gu + G_d d$,
the closed-loop transfer function from a disturbance d to y is (e.g. Skogestad and
Postlethwaite, 2005)

$$y = (I + GK)^{-1} \cdot G_d d = SG_d d \qquad (3.10)$$

where G is the process model, K is the feedback controller, $S = (I + GK)^{-1}$ is the
sensitivity function and G_d is the disturbance model. Assume that the disturbances
are sinusoidal, $d(t) = d_0 \sin(\omega t)$, and that $|d_0|$ is bounded. We consider only scalar
disturbances (i.e. one disturbance at a time). The worst-case amplification (peak
output variation as a function of disturbance frequency) from d to y then gives the
optimal (minimum) back off, thus

$$b \geq b_{\min} = \max_{\omega, d} |y| = \max_d \|SG_d\|_\infty \cdot |d_0| \qquad (3.11)$$

where $\max_{\omega, d} |y|$ represents the effect of the worst-case disturbance over all fre-
quencies and directions and therefore represents the minimum back off. Note that

$$\|SG_d\|_\infty \triangleq \max_\omega |SG_d(j\omega)| = |SG_d(j\omega_{wc})| \qquad (3.12)$$

where ω_{wc} is the worst-case frequency where $|SG_d|$ has its peak.

Worst-case frequency

The minimum back off for a given disturbance is given by $\|SG_d\|_\infty = SG_d(j\omega_{wc})$,
but what is the worst-case frequency (peak frequency) ω_{wc}? It is difficult to know
ω_{wc} beforehand, but typically the peak frequency for $|SG_d|$ is located around the
closed-loop bandwidth frequency. Thus, two interesting frequencies are the peak

$\frac{\tau_c}{\theta}$	0	1	2	3	4	5
$\omega_S \cdot \theta$	0.741	0.511	0.414	0.357	0.319	0.291
$\omega_{peakS} \cdot \theta$	1.38	1.14	1.02	0.947	0.891	0.849
M_s	3.13	1.59	1.35	1.25	1.19	1.16

Table 3.1: Frequencies for sensitivity function (S) and robustness margins for different τ_c using SIMC-settings ($K_c = \frac{1}{k}\frac{\tau_1}{\theta+\tau_c}$, $\tau_I = \tau_1$, $\tau_D = \tau_2$) in the PID-controller.

frequency for $|S|$ defined as $|S(j\omega_{peakS})| = \max_\omega |S(j\omega)| = M_S$, and the frequency ω_S defined as $|S(j\omega_S)| = 1$. Using these two specific frequencies we have

$$b_{\min} \geq |Sg_d(j\omega_{wc})| \cdot |d_0| \geq \left\{ \begin{array}{l} |Sg_d(j\omega_s)| \cdot |d_0| = |g_d(j\omega_s)| \cdot |d_0| \\ |Sg_d(j\omega_{peakS})| \cdot |d_0| = M_S \cdot |g_d(j\omega_{peakS})| \cdot |d_0| \end{array} \right.$$

(3.13)

These two lower bounds on the minimum back off are fairly tight for a first-order model of g_d. For a disturbance model g_d of higher order, general rules for estimating the minimum back off $b_{\min} = \max_\omega ||SG_d||_\infty$ is difficult to state. For example, a g_d of high order will roll off quickly at higher frequencies and ω_S and ω_{peakS} may not represent the worst-case frequencies.

Nevertheless, the two frequencies will always provide a lower bound, so it is interesting to estimate ω_S and ω_{peakS}. Table 3.1 gives the peak of $|S|$ ($= M_s$) and the frequencies ω_s and ω_{peakS} for a first-order process with time delay, $G_1 = ke^{-\theta s}/(\tau_1 s + 1)$, controlled with a PI-controller using the SIMC-tunings rules ($K_c = \frac{1}{k}\frac{\tau_1}{\theta+\tau_c}$, $\tau_I = \tau_1$) as a function of the tuning parameter τ_c (the closed-loop time constant). The same values apply to a second order with time delay process delay ($G_2 = e^{-\theta s}/((\tau_1 s + 1)(\tau_2 s + 1))$) controlled with a PID-controller if we select the derivative time $\tau_D = \tau_2$. In both cases the closed-loop transfer function becomes $L = GC = \frac{e^{-\theta s}}{(\tau_c+\theta)s}$.

Selection of the tuning variable τ_c

The sensitivity function depends on the controller tuning K, that is, the closed-loop time constant τ_c. Here we want to state some recommendations for selection of τ_c in our further development of an assumption of minimum back off.

1. We want to minimize $||SG_d||_\infty$ to minimize the back off. This leads to selecting a small τ_c to reject "easy" disturbances upstream the input u (tight control) and a large τ_c to reject "difficult" disturbances after the input u (leads to M_s small).

2. For robustness we want $\|S\|_\infty = M_s \le 1.6$, which implies $\tau_c \ge \theta$ approximately, see Table 3.1.

3. We want to minimize τ_c to have fast set point tracking.

To make some more specific recommendations of what τ_c should be, consider the disturbance break frequency ω_{bd} defined as

$$\omega_{bd} = \frac{1}{\tau_d} \tag{3.14}$$

where τ_d is the largest disturbance time constant in g_d. In other words, ω_{bd} is the frequency where the disturbance gain starts dropping. Consider two cases:

Case 1: "Difficult" ("fast") disturbances with $\omega_{bd} > \omega_S$. Here, $|g_d|$ is "flat" at the frequency ω_S (and approximately "flat" at ω_{peakS}), so the use of feedback will give worse response than with no control at some frequencies because $|S|$ has an unavoidable peak at the resonance frequency ω_{peakS}. This leads to the worst-case frequency $\omega_{wc} \approx \omega_{peakS}$, and we have $\|Sg_d\|_\infty \approx M_S|g_d(j\omega_{peakS})| \cdot |d_0| \approx M_S \cdot k_d|d_0|$. To reduce M_S we want τ_c large (but on the other hand we want τ_c small for set point tracking (y_s)). In summary, a steady-state analysis is sufficient for back off estimation and we have $b_{min} \approx M_S \cdot k_d|d_0|$ where $k_d = g_d(0)$ is the steady-state disturbance gain. To minimize M_S we want τ_c large.

Case 2: "Easy" ("slow") disturbance with $\omega_{bd} < \omega_S$. In this case ω_{bd} is approximately the worst-case frequency because $|S| \approx \frac{\omega}{\omega_S}$ increases linearly with ω in a log-log plot in the frequency region up to ω_S (Skogestad and Postlethwaite, 2005) and $|g_d| \approx k_d$ up to ω_{bd}. In summary, $b_{min} \approx |Sg_d(j\omega_{bd})| \approx k_d\frac{\omega_{bd}}{\omega_S}$ and we want ω_S as large as possible for disturbance rejection, which corresponds to τ_c small.

In the above case definitions, ω_S is used to determine the disturbance case and hence decide the tuning parameter τ_c. However, ω_S depends on the selection of τ_c. From Table 3.1 a relation between ω_S, θ and τ_c are given, and we can state ω_S approximately

$$\omega_S \approx \frac{1}{\tau_c + \theta} \tag{3.15}$$

From the arguments above, we can suggest a "rule of thumb" for selection of τ_c:

$$\tau_c = \begin{cases} 3\theta, & \text{for } \omega_{bd} > \frac{1}{2\theta} \text{ or } \tau_d < 2\theta \\ \theta, & \text{for } \omega_{bd} < \frac{1}{4\theta} \text{ or } \tau_d > 4\theta \end{cases} \tag{3.16}$$

The choice of $\tau_c = 3\theta$ is a trade-off between disturbance rejection and set point trajectory: we want to minimize τ_c to track set points, but at the same time we want

to maximize τ_c to reduce M_S. The choice $\tau_c = 3\theta$ gives $M_S \approx 1.25$ (see Table 3.1), so the use of feedback gives 25% extra back off.

The recommendations (3.16) do not state a selection of τ_c in the intermediate range $2\theta < \tau_d < 4\theta$. The disturbances with $\tau_d > 4\theta$ are "slow" disturbances and the control system are able to reject them fairly good. For $\tau_d < 2\theta$ the disturbances are fast and here the control is poorer for disturbance rejection than no control because of the peak of $|S|$. In the intermediate range τ_c should be increased from θ up to 3θ.

Summary of simplified analytic estimation of back off

The minimum back off $b_{\min}$ is given by (3.11). The frequencies ω_S and ω_{peakS} are expressed by θ and τ_c in Table 3.1, and the recommendations for τ_c are given in (3.16). In the idealized case we assume that $\frac{1}{\tau_d} = \omega_{bd}$ and that g_d is approximately "flat" at frequencies below ω_{bd}. In addition, we assume that $|S| \approx \frac{\omega}{\omega_S}$ between ω_S and ω_{peakS}, in other words, the slope of $|S|$ is approximately $+1$ in the given range. Then the location of the peak frequency and the magnitude of the necessary back off can be summarized as:

For "difficult" disturbance with $\tau_d < 2\theta$: $\quad \omega_{wc} \approx \omega_{peakS}$
$$b_{\min} \approx M_S \cdot k_d |d_0| \qquad (3.17)$$

For "easy" disturbance with $\tau_d > 4\theta$: $\quad \omega_{wc} \approx \dfrac{1}{\tau_d}$
$$b_{\min} \approx \dfrac{2\theta}{\tau_d} \cdot k_d |d_0| \leq k_d |d_0| \quad (3.18)$$

To conclude the estimation of back off, we see from (3.17) and (3.18) that control is helpful for $\tau_d > 4\theta_{\text{eff}}$. Otherwise the back off is given by steady-state disturbance effect.

To illustrate the estimation of back off, consider the introductory example.

Example 3.5 (continued). *Minimum back off for different TPM locations. The necessary back off for the "difficult" disturbance d_1 (difficult because it is located close to the bottleneck) is calculated using Table 3.1 and Equations (3.17)-(3.18). The tuning variable is selected to $\tau_c = 3\theta$ for all three TPM locations. The disturbance time constant for d_1 is $\tau_d = 8$ or equivalent $\omega_{bd} = 0.125$. The calculated frequencies and minimum back off are compared with the observed ones in Table 3.2. Note that location C with $\theta_{eff} = 3$ is in the intermediate range $2\theta < \tau_d < 4\theta$ and it is not clear if (3.17) or (3.18) should be used. Here, (3.18) is selected since the disturbances have started to roll off and a stationary analysis will be less correct.*

Location	A ($\theta_{\text{eff}} = 87$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.17)	0.004	0.011	0.011	1.25
Observed	Fig. 3.6(a)	0.004	0.010	0.010	1.23
Location	B ($\theta_{\text{eff}} = 39$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.17)	0.009	0.024	0.024	1.25
Observed	Figure 3.6(b)	0.009	0.022	0.021	1.22
Location	C ($\theta_{\text{eff}} = 3$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.18)	0.12	0.32	0.13	0.75
Observed	Figure 3.6(c)	0.11	0.22	0.12	0.70

Table 3.2: Estimated and observed frequencies (ω_s, ω_{peakS} and ω_{wc}) and minimum back off ($b_{\min}$) to account for disturbance d_1 (with $\tau_d = 8$) from Example 3.5. The frequencies and back off are estimated by using Table 3.1, Equation (3.17) and (3.18). The observations are from Figure 3.6.

We see that ω_{peakS} provides a good estimate of the worst-case frequency for processes with long effective time delay θ (location A and B) whereas ω_{bd} provides a good estimate for the worst-case frequency for processes with a short effective time delay θ (location C). For the back off calculation, $M_s \cdot k_d |d_0|$ gives a good estimate for long effective time delay. For a short effective time delay θ the back off estimate is also good. However, by using the estimated frequency of ω_S instead of the approximation of $\omega_S \approx \frac{1}{2\theta}$, the estimated minimum back off becomes larger than the observed minimum back off, since the disturbance has started to roll off (it is not really a "fast" disturbance but an "intermediate"). Note that the location of the peak to $|Sg_d|$ moves from ω_{peakS} towards ω_{bd} with smaller effective time delay between TPM and bottleneck. To move the TPM from location A to location B has very little effect in terms of reducing minimum back off. The disturbances are still fast compared to the closed-loop response and control is not helpful for rejecting the major disturbance.

Assume that it is possible (and preferable in terms of costs) to increase the hold-up between the inlet of the plant and the middle of the plant (refer to location A and B in Example 3.5). To evaluate the effect of larger holdups between location A and B in terms of minimum back off, consider a new example.

Example 3.7. *Minimum back off in a process with large hold-up volumes. Consider the same process string as in Example 3.5, but now with significantly larger hold-up volumes in unit 1 and 2. The bottleneck flow (y) is considered fixed at the outlet of the last unit. The time constants for each unit are displayed in Table 3.3.*

The minimum back off b_{min} for each disturbance $|Sg_d|$ is displayed as a function of frequency for the TPM located at feed (A), in the middle (B) and at the bottleneck (C) in Figure 3.10., With the TPM located at the bottleneck (Figure 3.10(c)), the

Unit	τ_1	τ_2
1	200	100
2	50	50
3	16	8
4	20	10
5	8	1

Table 3.3: Time constants τ_1 and τ_2 for the units in Example 3.7.

peak of $|Sg_d|$ is reduced significantly compare to when the TPM is located in A and B. For TPM located in A an B there is almost no difference for the worst disturbance d_1, but the effect of the disturbances d_2 to d_5 is reduced when TPM is moved from location A to B.

By using Table 3.1 together with (3.17) and (3.18), the frequencies ω_S, ω_{peakS} and ω_{wc} are estimated together with minimum back off. The observed and the estimated frequencies and back off are compared in Table 3.4. Here location A and B is in the area for steady-state analysis ($\tau_d < 2\theta$). For location C the worst disturbance d_1 is fast compared to the closed-loop response ($\tau_d > 4\theta$).

Location	A ($\theta_{\text{eff}} = 214$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.17)	0.0017	0.0044	0.0044	1.25
Observed	Fig. 3.6(a)	0.0017	0.0040	0.0040	1.26
Location	B ($\theta_{\text{eff}} = 36$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.17)	0.010	0.026	0.026	1.25
Observed	Figure 3.6(b)	0.010	0.024	0.023	1.22
Location	C ($\theta_{\text{eff}} = 1.5$)	ω_S	ω_{peakS}	ω_{wc}	$b_{\min}$
Estimated	Tab. 3.1, Eq. (3.18)	0.24	0.62	0.13	0.38
Observed	Figure 3.6(c)	0.22	0.48	0.18	0.49

Table 3.4: Estimated and observed frequencies (ω_s, ω_{peakS} and ω_{wc}) and the minimum back off ($b_{\min}$) to account for disturbance d_1 (with $\tau_d = 8$) from Example 3.7. The frequencies and back off are estimated by using Table 3.1, Equation (3.17) and (3.18). The observations are from Figure 3.10.

We see that ω_{peakS} provides a good estimate of the worst-case frequency for processes with long effective time delay θ (location A and B) whereas ω_{bd} provides a good estimate for the worst-case frequency for processes with a short effective time delay θ (location C). For the back off calculation, $M_s \cdot k_d |d_0|$ gives a good estimate for long effective time delay. For location C the worst-case disturbance is categorized as "easy" and here the estimate is lower than the observed minimum back off. However, by using the estimated frequency of ω_S instead of the approxi-

(a) TPM at feed (location A) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 214$

(b) TPM in middle (location B) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 36$

(c) TPM at bottleneck (location C) where $\tau_c = 3\theta_{\text{eff}}$, $\theta_{\text{eff}} = 1.5$
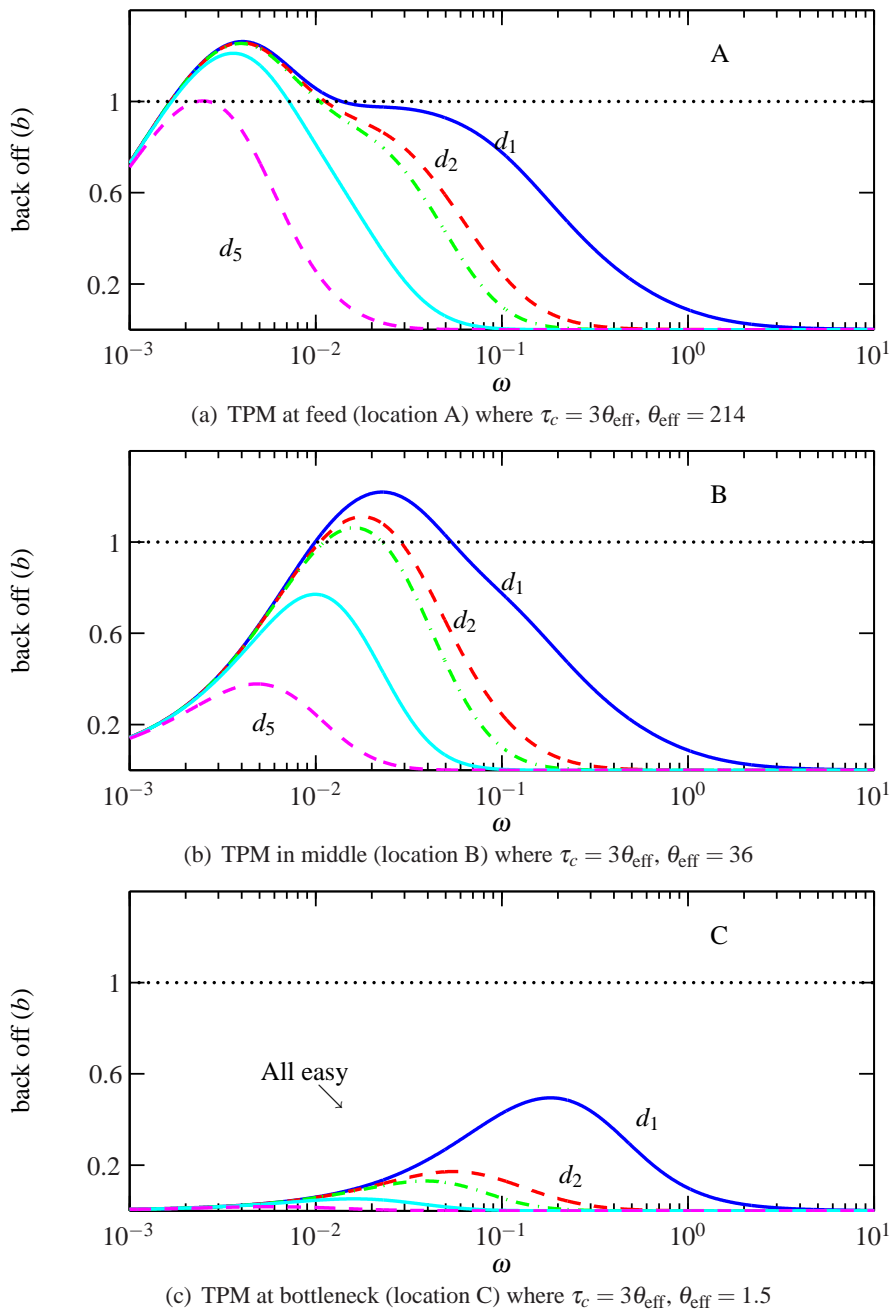
Figure 3.10: $|Sg_d|$ as a function of frequency; effect of the disturbances d_1 to d_5 on the bottleneck flow, for the three different locations of TPM given in Example 3.7.

mation of $\omega_S \approx \frac{1}{2\theta}$, the estimated minimum back off becomes slightly larger than the observed back off. Note that even though the difference in effective time delay between location A and B is now much larger than in Example 3.5, the minimum back off is almost the same. The effective time delay with TPM at location B is still large compared to the most important disturbance time constants, so a stationary analysis is still valid.

3.A.2 Comments on mathematical approach

A mathematical approach to estimate the necessary back off is treated by e.g. Perkins and coauthors (Narraway *et al.*, 1991; Narraway and Perkins, 1993, 1994; Heath *et al.*, 1996; Loeblein and Perkins, 1998, 1999) and Romagnoli and coauthors (Bandoni *et al.*, 1994; Bahri *et al.*, 1996; Figueroa *et al.*, 1996).

Narraway *et al.* (1991) present a method to assess the impact of disturbances on plant economics. Their approach is to perform an economic evaluation of the necessary back off (dynamic economics) to select the control structure (pairing) that minimize the economic impact of disturbances on the process economics. They consider so-called stationary disturbances that are fast disturbances which do not change the steady-state optimum but requires back off since they affect the size of the dynamic operating region. The analysis is performed to a linearized plant dynamic model with assumption of perfect control to the chosen control objectives.

Narraway and Perkins (1993) presents a modification of the method proposed in Narraway *et al.* (1991) for the *a priori* assessment of the effect of disturbances on the economics, in addition to a branch and bound algorithm for the choice of control structure based on the economic criteria. Further, Heath *et al.* (1996) modifies the method by using multiloop PI structures tuned by Ziegler Nichols gains/resets instead of the assumption of perfect control in the control structure selection algorithm.

Loeblein and Perkins (1999) integrate dynamic economics and average deviation from optimum in order to obtain a unified measure for the economic performance by adding the back off from the dynamic economics and from average deviation from optimum. Regulatory back off is evaluated using the unconstrained MPC law with QP algorithm for a stochastic description of disturbances. This leads to a quadratic program which can be solved analytically since the inequality constraints on the input variables are neglected during the back off calculation. The statistical variation of the variables to which constraint are to be applied is described by a density function of a Gaussian distribution with zero mean and known covariance. The regulatory back off is described with a probability that is specified a priori.

To find the necessary back off by using a detailed model-based approach is

unrealistic to solve exact for real systems. It requires a dynamic model of the plant together with disturbance characteristics, where the information is limited, especially prior to plant operation. In addition, the variations in the controlled variables are dependent on the regulatory control structure and its parameters and the use of advanced process control (e.g. MPC).

Chapter 4

Dynamic degrees of freedom for tighter bottleneck control

In many cases, optimal plant operation is the same as maximum throughput. To realize maximum throughput, tight control of the bottleneck unit(s) is necessary. Dynamic degrees of freedom can be used to obtain tighter bottleneck control. Here, "dynamic" means that the degree of freedom has no steady-state effect on plant operation. For example, most inventories (levels) have no steady-state effect. Nevertheless, temporary changes of inventories can allow for dynamic changes in the flow through the bottleneck that keeps the process closer to its bottleneck constraint and increase the throughput. A simple structure is to use a single-loop bottleneck controller that adjusts the feed flow, combined with a simple ratio control scheme that adjusts the dynamic degrees of freedom. The idea is to change all the flows upstream of the bottleneck simultaneously, instead of waiting for inventory loops to move the feed rate change through the units. The required buffer volume for plant design is analyzed for upstream disturbances and bottleneck set point changes.

4.1   Introduction

In many cases, prices and market conditions are such that optimal operation is the same as maximizing plant throughput. In this case, the optimum lies at constraints, and in order to maximize throughput, the flow through the bottleneck(s) should be at its maximum at all times (Chapter 3). If the actual flow through the bottleneck is not at its maximum at any given time, then this gives a loss in production that can never be recovered (sometimes referred to as a "lost opportunity"). Tight

bottleneck control is therefore important for maximizing throughput and avoiding losses.

In existing plants, the most common approach for controlling the throughput is to set the feed flow at the inlet of the plant and use inventory control in the direction of flow (Price *et al.*, 1994). One reason for this is that most of the control structure decisions are done at the design stage (before the plant is built), where one usually assumes a fixed feed rate. However, tight bottleneck control requires that the throughput manipulator (TPM) is located close to the bottleneck (Skogestad, 2004). The term "close to the bottleneck" means that there is a short effective delay from the input (TPM) to the output (bottleneck flow).

Ideally the TPM should be located at the bottleneck, but this may not be desirable (or even possible) for other reasons. First, if the TPM is moved, the inventory loops must be reconfigured to ensure self-consistency (Chapter 2). Second, there may be dynamical reasons for avoiding a so-called on-demand control structure with inventory control opposite the direction of flow, which is required upstream of the TPM to ensure self-consistency. Luyben (1999) points out several inherent dynamic disadvantages with the on-demand structure, including propagation of disturbances, dynamic lags, process time constants and interactions. Third, if a bottleneck(s) moves in the plant due to disturbances, then single-loop control requires relocation of TPM and reconfiguration of inventory loops. Thus, in practice one is often left with a fixed throughput manipulator, usually the feed rate. This usually leads to a large effective delay ("long loop") because the bottleneck is usually located inside the plant. This leads to an economic loss because of a large required back off from the bottleneck constraints.

Instead, with the TPM fixed, for example at the feed, one may introduce additional degrees of freedom to reduce the back off:

1. For plants with parallel trains one may use crossover and splits (Aske *et al.*, 2008). This are "extra" degrees of freedom that usually cannot be used by a single unit.

2. More generally, one may use "dynamic" degrees of freedom. This is the topic of the present paper. By "dynamic" degrees of freedom we mean manipulated variables with no steady-state effect. The most common examples are liquid inventories (levels) and buffer tank inventories.

The idea is to change the inventory to make temporary flow rate changes in the units between the TPM (feed) ant the bottleneck. This may give tighter bottleneck control, but the cost is that the inventory itself will be less tightly controlled. However, in many cases, inventories need only to be kept within a given range and tight set point control is not needed.

Faanes and Skogestad (2003) defined a buffer tank (surge tank) as a unit where the holdup (volume) is exploited to provide improved operation. They applied control theory to the design of buffer tanks, including deciding on the number of tanks and tank volumes required to dampen the fast (i.e., high-frequency) disturbances, which cannot be handled by the feedback control system. In this paper, the issue is to use the buffer volume to introduce dynamic flow rate changes.

There are also related issues in business systems. Supply chains are sometimes modelled as continuous processes and Schwartz *et al.* (2006) used simulation to study decision policies for inventory management. To improve the financial benefits, they use the inventory set points for intermediate storage subject to maintain acceptable performance in the presence of significant supply and demand variability and forecast error as well as constraints on production, inventory levels, and shipping capacity.

The organization is as follows. Section 4.2 explains how to include dynamic degrees of freedom using either single-loop with ratio control or using a multivariable controller. The use of dynamic degrees of freedom for tighter bottleneck control is demonstrated by an example in Section 4.3. Transfer functions are developed for the single-loop with ratio control structure in Section 4.4 and these functions are further analyzed to estimate the required inventory for disturbances (Section 4.5). A discussion follows in Section 4.6. A summary of the implications for design of inventory tanks is given in Section 4.7 before the paper is concluded in Section 4.8.

4.2 Alternative strategies for bottleneck control

Assume that the objective is to maximize the flow through the bottleneck and that the feed rate is available as a degree for freedom (throughput manipulator, TPM). Figure 4.1 shows four ways of achieving this using simple single-loop control structures.

In the traditional configuration in Figure 4.1(a), the feed rate is the degree of freedom for manipulating throughput (TPM), and inventory control is in the direction of flow. To maximize the flow through the bottleneck, the operators change the feed valve manually based on information about the plant operation and experience. However, careful attention by the operators is required in order to keep the bottleneck flow close to its maximum at all times, so we want to use automatic control.

Alternative 1: Single-loop control of bottleneck flow using the feed rate. (Figure 4.1(b))
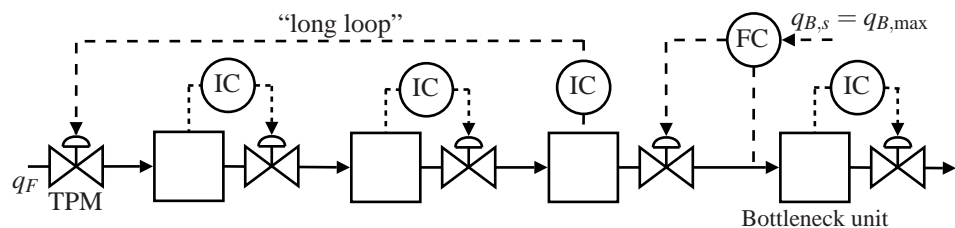The simplest is to use single-loop feedback control where the feed rate (TPM) is
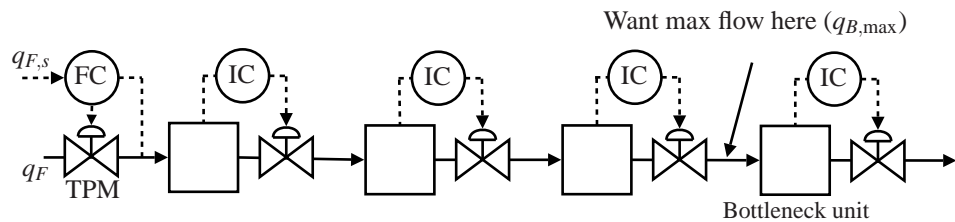
(a) Traditional configuration (manual control of feed rate)
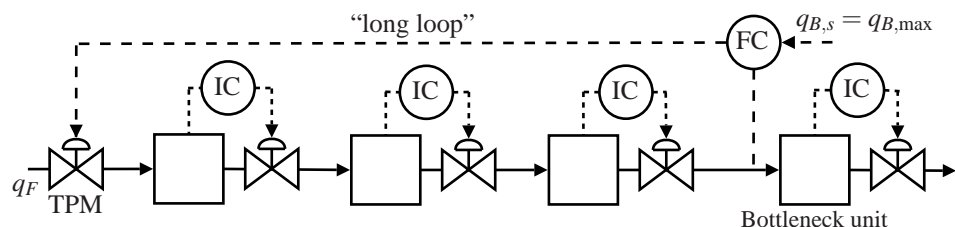
(b) Alternative 1: Single-loop control where the feed rate controls the bottleneck flow (Problem: "long loop" with large effective delay).

(c) Alternative 2: Throughput manipulator moved to bottleneck without reconfiguration of the inventory loops in the other units. Feed rate controls the "lost task", in this case the upstream inventory (Problem: "long loop" with large effective delay).
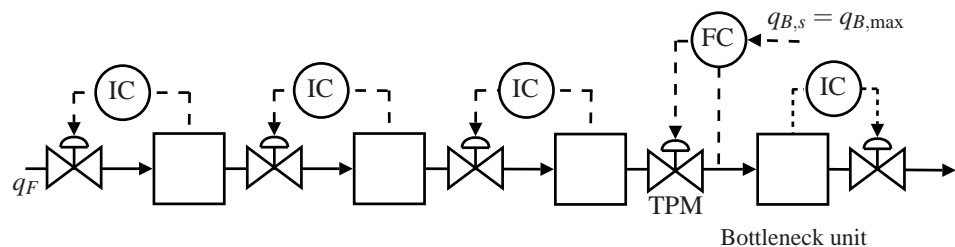
(d) Alternative 3: Throughput manipulator moved to bottleneck with reconfiguration of inventory loops upstream of bottleneck (Problem: reconfiguration).

Figure 4.1: Simple single-loop control structures for maximizing bottleneck flow in serial process. IC stands for inventory controller (e.g. level controller).

manipulated to keep the bottleneck flow close to its maximum. However, there is often a large effective delay from the feed flow (input) to the bottleneck flow (output), so tight control of the bottleneck flow is not possible because of disturbances.

Alternative 2: Move TPM from feed to bottleneck and let feed control "lost task". (Figure 4.1(c))
The bottleneck flow is set directly at its maximum, which corresponds to moving the throughput manipulator to the bottleneck. The inventory loops are not reconfigured, so the feed rate now needs to take over the "lost task" which in this case is control of the inventory upstream of the bottleneck. In this case, tight bottleneck control is achieved, but inventory control may be poor, leading to possibly emptying or overflowing the upstream tank because of a large effective delay from the feed flow (input) to the tank (output).

Alternative 3: Reconfigure inventory control. (Figure 4.1(d))
The TPM is moved to the bottleneck and all the upstream inventory loops are reconfigured to be in the opposite direction of flow upstream the bottleneck. In this case, both tight bottleneck control and good inventory control may be achieved. However, the reconfiguration of inventory loops is usually very undesirable from a practical point of view.

In summary, none of these alternatives are desirable. To improve control and keep the flow through the bottleneck closer to its maximum at all times, we would like to have additional degrees of freedom, and the only ones that are normally available are the inventories (holdups) in the buffer tanks, which can be used to make dynamic flow changes. The word "dynamic" is used because most inventories have no steady-state effect on plant operation.

The main idea is as follows: To change the flow through the bottleneck, for example, to increase it, we temporarily reduce the inventory in the upstream holdup volume. However, this inventory needs to be kept within bounds, so if we want to increase the bottleneck flow permanently, we need to increase the flow into this part of the process and so on, all the way back to the feed (throughput manipulator). The simplest (but not generally optimal) approach is to use a "ratio" control system where all flows upstream the bottleneck are increased simultaneously by the same relative amount. The idea is illustrated in Figure 4.2.

Alternative 1D: Single-loop plus ratio control. (Figure 4.2(a))
The idea is to control the bottleneck flow by simultaneously changing all the flows upstream of the bottleneck by the same relative amount. The advantage is that the effective delay from the feed to the bottleneck may be significantly reduced and even eliminated in some cases. However, the dynamic flow changes are counteracted by the inventory controllers. In particular, note that the feed flow is the only

degree of freedom that has a steady-state effect on the bottleneck flow. The strategy may also be viewed as a "ratio feedforward controller" from the feed flow to the downstream flows.

Alternative 2D: Move TPM to bottleneck and add ratio control to "lost task".
(Figure 4.2(b))
The TPM is moved to the bottleneck and the "lost task" (inventory upstream the bottleneck) is controlled by the feed rate. The use of ratio control is the same as for Alternative 1D. The effective delay from the feed rate to the lost task is reduced by using ratio control.

Alternative 4: Multivariable controller. (Figure 4.2(c))
A multivariable controller (e.g. MPC) uses the feed rate and the inventories as manipulated variables (MVs). The controlled variables (CVs) are the bottleneck flow and inventory constraints.

In this paper we focus on Alternative 1D. One reason is that the analytic treatment is quite simple. To understand how the "ratio control" works, consider first inventory control of an individual buffer tank. The "normal" feedback inventory controller (IC) can be written

$$q = K(s)(I - I_s) + q_0 \tag{4.1}$$

where I is the inventory (e.g. level), I_s is its set point, q is the flow in our out of the tank (output from controller) and q_0 is the flow bias term of the controller. The feedback controller $K(s)$ has a negative sign if q is an inflow and a positive sign if q is an outflow. Now, to introduce the inventory as a degree of freedom one can either adjust the inventory set point (I_s) or adjust the bias (q_0). The most obvious is to adjust the inventory set point I_s, but it is more direct in terms of flow changes to adjust the bias. Actually, the two approaches are not very different, because a change in q_0 can equivalently be implemented as a set point change by choosing $I_s = -q_0/K(s)$. In this paper, we choose to use the bias q_0 as the dynamic degree of freedom for ratio control.

Let now q_F be the feed flow computed by the flow controller (FC) in Figure 4.2(a). Then, the bias adjustment in all the inventory controllers (IC) in the figure is
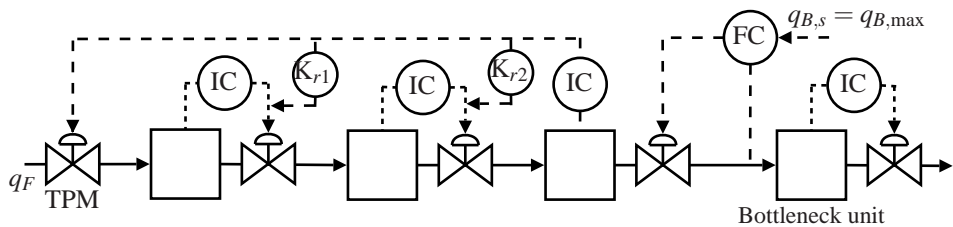
$$\Delta q_0 = K_r \Delta q_F \tag{4.2}$$

where K_r is the steady-state gain for the effect of q_F on q_0. The overall IC then becomes

$$\Delta q = K(s)(I - I_s) + \underbrace{K_r \Delta q_F}_{\Delta q_0} \tag{4.3}$$
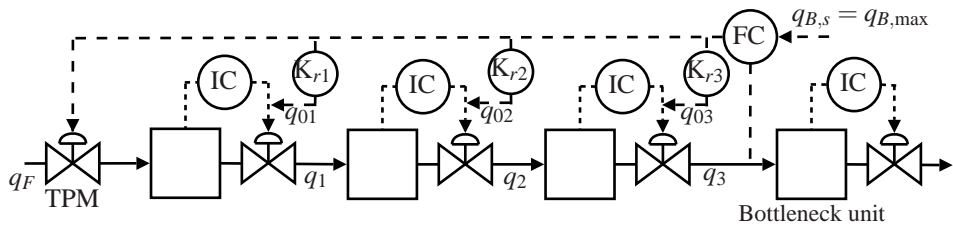
(a) Alternative 1D: The feed rate (TPM) controls the bottleneck flow with use of inventories as additional dynamic degrees of freedom (here shown using a "bias" adjustment of the flow from each unit).

(b) Alternative 2D: The TPM is moved without reconfiguration of inventory loops. The feed rate controls the lost task, in this case the inventory upstream the bottleneck (large effective delay) and inventories are used as dynamic degrees of freedom.
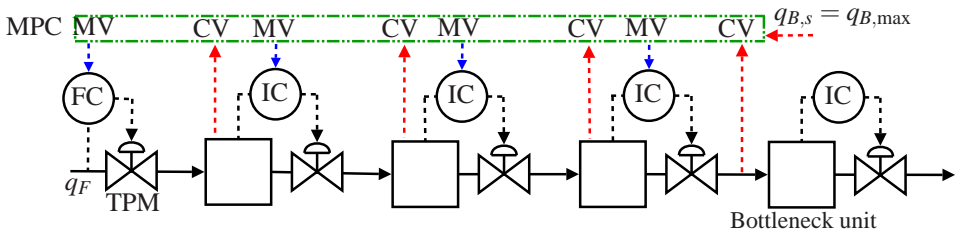
(c) Alternative 4: Multivariable control structure (e.g. MPC) where the feed rate and the inventory controller set points are MVs.

Figure 4.2: Structures for controlling bottleneck flows that use inventories as dynamic degrees of freedom (with no reconfiguration of the inventory loops). Alternative 1D is studied in this paper. IC stands for inventory controller (e.g. level controller) and K_i is a constant gain (ratio controller).

	De-ethanizer	De-propanizer	De-butanizer	Butane splitter
Number of trays	32	48	40	92
Feed tray	20	20	19	45
Nominal feed [*kmol/min*]	75.6	29.4	11.6	8.1
Holdup sump (M_B) [*Kmol*]	121	38	18	66
Holdup condenser (M_D) [*Kmol*]	226	227	62	88
Nominal boil up [*Kmol/min*]	88.6	32.3	9.9	34.5
Nominal reflux [*Kmol/min*]	71.5	27.1	8.6	25.9

Table 4.1: Sizes and nominal flows for the distillation columns in Example 4.1.

The important point to note is that there are no dynamics in K_r. This means that all the flows q are changed simultaneously when q_F changes. This is not generally optimal, but it is the simplest and is used in this paper.

4.3 Introductory example

The example given below illustrates how tight bottleneck control can be obtained by use of dynamic degrees of freedom.

Example 4.1. *Four distillation columns in series. Consider four distillation columns in series, as shown in Figure 4.3. The four columns represent the liquid upgrading part of a gas processing plant and consist of a deethanizer, a depropanizer, a debutanizer and a butane splitter. Assume that the butane splitter (the last unit) has the lowest processing capacity and is therefore the bottleneck unit. The throughput is manipulated at the feed to the first column. The idea is to use the column inventories (sump or condenser drum holdup) as dynamic degrees of freedom to obtain tighter bottleneck control.*

The distillation column models are implemented in Matlab/Simulink. Each of the four columns is modelled as multicomponent distillation with one feed and two products, constant relative volatilities, no vapor hold-up, constant molar flows, total condenser and liquid flow dynamics represented by the Francis weir formula. All columns use the "LV-configuration" where distillate (D) and bottoms flow (B) are used for inventory control (M_D and M_B). To stabilize the column composition profile, all columns have temperature control in the bottom section by manipulating the boilup. Some relevant sizes and flows for the columns are given in Table 4.1. Note that there is a crossover flow from the bottoms of the deethanizer where $q_{cross} = 15.8$ kmol/min, as displayed in Figure 4.3.

Four different control structures for maximizing throughput are tested:

1. ***Manual**. Traditional (manual) control of the throughput.*

Figure 4.3: Distillation process: Four columns in series, here shown with throughput controlled by using single-loop with ratio control (Alternative 1D).

2. **Single-loop**. *Single-loop control where the bottleneck flow is controlled using the feed rate (Alternative 1 in Section 4.2).*

3. **Single-loop with ratio**. *Use of the inventories as dynamic degrees of freedom by adding a bias (q_0) to the inventory controller outputs as in Figure 4.3. (Alternative 1D in Section 4.2).*

4. **Multivariable**. *MPC with the feed rate and the inventory set points as MVs and the bottleneck flow and level constraints as CVs (Alternative 4 in Section 4.2).*

The column inventories M_D and M_B are controlled with P-controllers with gain $K_c = 1/\tau_V$. Here we use "smooth" level control where we set $\tau_V = V_{tank}/q_{out}$ (Skogestad, 2006) where q_{out} is the flow out of the volume (D or B). With a nominal half-full tank we can then handle a 50% change in the product flow (D and B) without emptying or overfilling. Actually, the flow into the inventory is considerably larger, but disturbances in boilup (or reflux) are counteracted by the temperature controller (Skogestad, 2007). The temperature controllers (TC) are tuned with SIMC PI-tuning (Skogestad, 2003) with $\tau_c = 0.5$ min. The TCs and ICs tunings are identical in all four columns and in the four tested control structures.

Two disturbances are considered. First, at $t = 10$ min, we make a set point change in the bottleneck flow, for example, caused by a disturbance in the bottleneck unit (the butane splitter). Second, at $t = 210$ min, there is an unknown change in the feed rate.

1. *For manual control, we assume that a skilled operator can immediately change the feed rate to the value corresponding to the new bottleneck flow set point. However, we assume that the operator does not notice the unmeasured feed flow disturbance, so no adjustment is therefore done for the feed rate disturbance.*

2. *For the single-loop control structure we want smooth tuning to avoid overshoot and "aggressive" use of the feed valve. Therefore, the bottleneck flow controller (FC) is tuned with SIMC tunings with $\tau_c = 3\theta$ for smooth tuning (Skogestad, 2006). This gives a PI-controller with $K_c = 3.0$ and $\tau_I = 14$ min.*

3. *For the single-loop control with ratio (bias) adjustment (Alternative 1D), there is no effective delay and the bottleneck flow controller (FC) is tightly tuned with a short integral time ($K_c = 0.5$ and $\tau_I = 0.3$ min), which are typical FC tuning parameters.*

4. *In the multivariable structure the FC at the feed is omitted and the MPC manipulates directly the feed valve. The built-in MPC toolbox in Matlab is*

used and tuned with a low penalty on the use of inventories (MV moves) and a high penalty on the deviation from the bottleneck flow set point (CV set point).

The four control structures are evaluated in terms of how tightly the bottleneck flow (q_B) is controlled in spite of disturbances. As mentioned, two disturbances are considered:

- *At $t = 10$ min: 5% increase in bottleneck flow set point ($q_{B,s}$).*

- *At $t = 210$ min: 8% decrease in the feed rate to the deethanizer (q_F). The net feed flow is $q_F = q_{F,u} + q_{F,d}$, where $q_{F,u}$ is the flow contribution from the controller (initially $q_{F,d} = 0$ and $q_F = q_{F,u} = 100$, but then $q_{F,d} = -8$ at $t = 210$).*

The resulting bottleneck flow (q_B), the net feed flow (q_F) and the inventories used as dynamic degrees of freedom (deethanizer M_B, depropanizer M_B and debutanizer M_D) for the four different control structures are displayed in Figure 4.4. The first observation is that we have significantly tighter bottleneck control with ratio control and MPC (Alternative 3 and 4) where inventories are used as dynamic degrees of freedom (Figure 4.4(a)). The inventories (levels) are quite tightly controlled with surprisingly small variations as shown in Figure 4.4. There is some steady-state offset because we use P-control (no integral action).

In summary, we can operate closer to the capacity constraint of the butane splitter (reduce the back off) and hence increase the throughput when dynamic degrees of freedom (inventories) are used.

4.4   Analysis of use of dynamic degrees of freedom

In this section, the single-loop with ratio control scheme (Alt. 1D in Section 4.2) is analyzed in more detail. The main reason is to later use the results to estimate the required buffer volume for dynamic degrees of freedom (Section 4.5). The dynamic degrees of freedom are either the inventory set point (V_s) or the bias adjustment (q_0), but here we only consider q_0.

To make the control structure in Figure 4.2(a) clearer, consider a similar structure, which consists of only one unit, or more precisely, a process unit (G) followed by an inventory (G_V), as displayed in Figure 4.4. The outflow q_B from the inventory is assumed to be the bottleneck flow that should be tightly controlled. However, q_B cannot be set freely because it is already used for level control. Thus, to improve the dynamic response, we add a bias term q_0 which is set in proportion to the net feed flow q_F, computed by the bottleneck controller. This single-loop
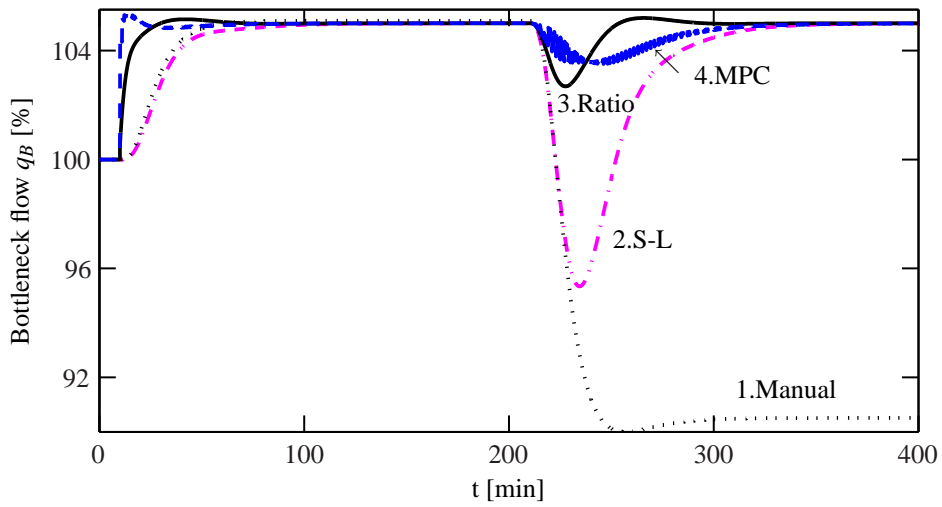
(a) Response in Bottleneck flow q_B (CV)
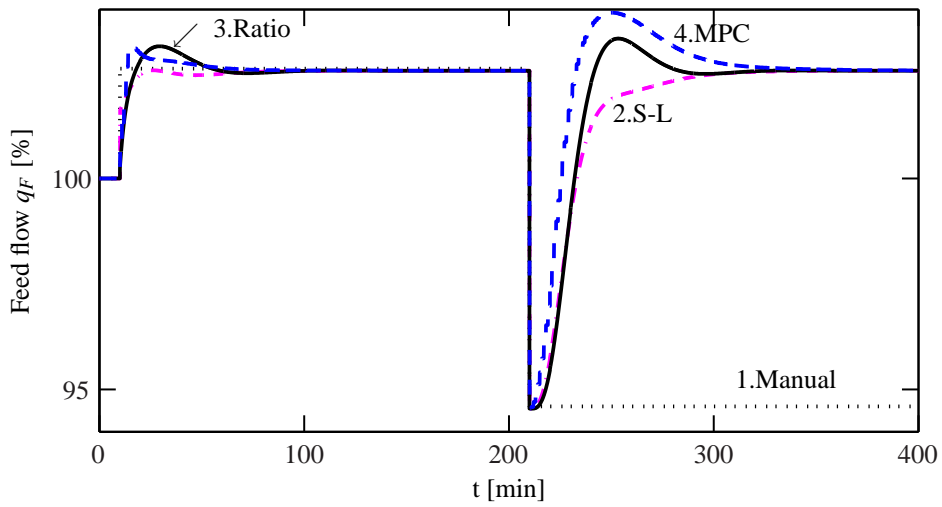
(b) Responses in Feed flow q_F (MV)

Figure 4.4: Continued on next page.
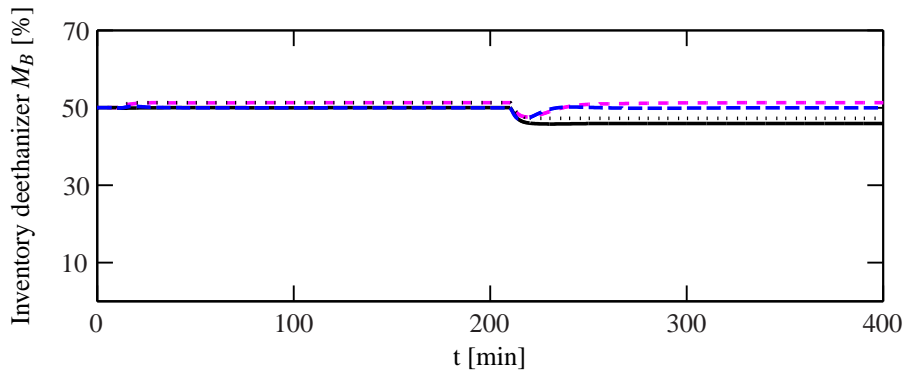
(c) Responses in deetanizer bottoms level M_B (dynamic degree of freedom no. 1)
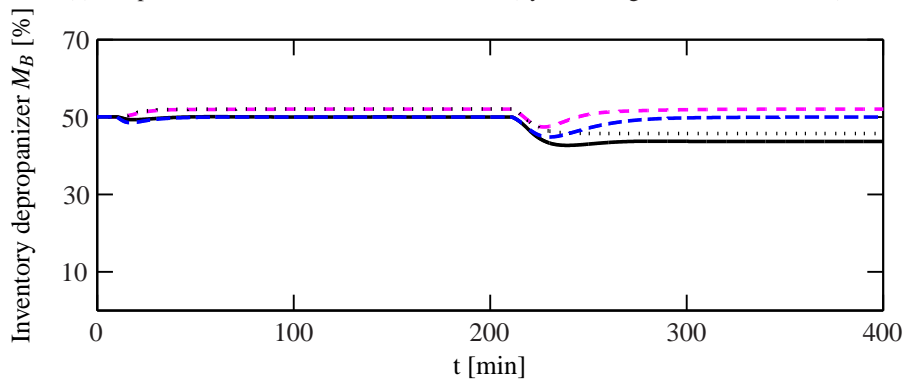
(d) Responses in depropanizer bottoms level M_B (dynamic degree of freedom no. 2)
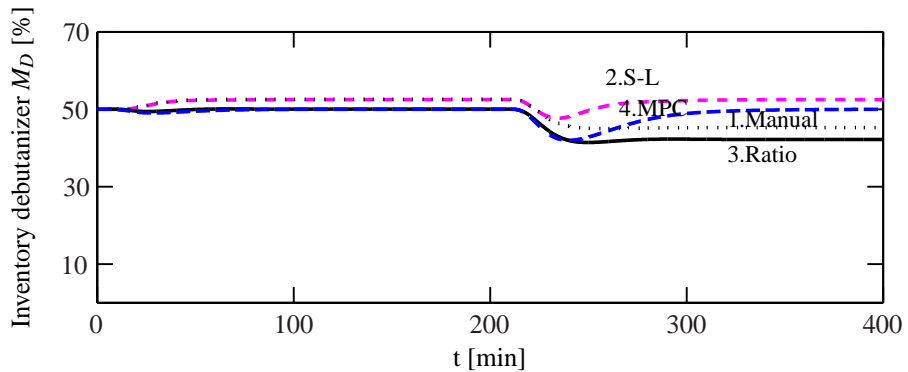
(e) Responses in debutanizer distillate level M_D (dynamic degree of freedom no. 3)

Figure 4.4: Bottleneck control of the distillation process for four different control structures. 1) Manual control (dotted), 2) Single-loop control (dash-dotted), 3) Single-loop with ratio (bias adjustment on inventory flows, solid), 4) MPC using both feed rate and inventories as MVs (dashed). Disturbances: 5% increase in bottleneck flow set point $q_{B,s}$ at $t = 10$ and 8% unknown decrease in feed rate q_F at $t = 210$.
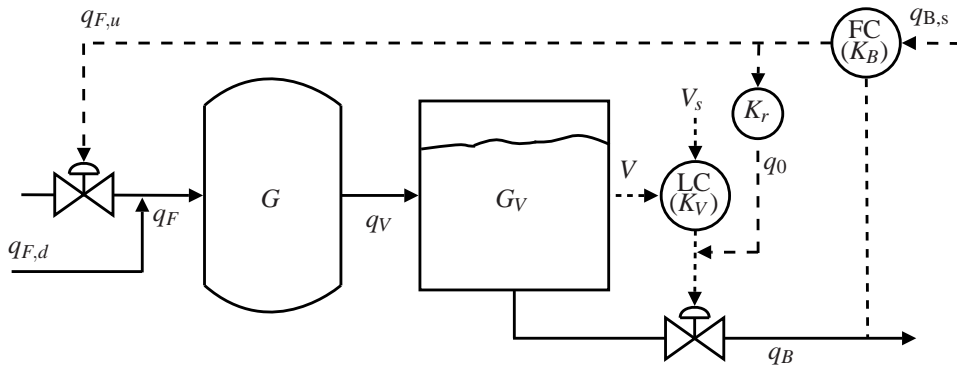
Figure 4.4: Example of single-loop control with a linear bias adjustment added to the level controller output.

with static ratio control structure can be viewed as feedforward control combined with feedback, where the flows in downstream units are increased proportionally to the feed rate q_F. This idea is also used sometimes by skilled operators, e.g. during start-up of a plant. We will now analyze this system in more detail.

The mass balance for the holdup volume V, assuming constant density, is

$$\frac{dV}{dt} = q_V - q_B \tag{4.4}$$

where q_V is the inflow and q_B is the outflow (see Figure 4.4). Upon taking the Laplace transform and introducing deviation variables, we get

$$V(s) = \frac{1}{s}(q_V - q_B) \tag{4.5}$$

Thus, the transfer function for the inventory is $G_V(s) = \frac{1}{s}$. Next, assume that the inlet flow to the buffer volume q_V is given by

$$q_V = G(s) \cdot q_F \tag{4.6}$$

where G is the process transfer function for the upstream process between the feed and the buffer volume. The net feed flow q_F is defined as

$$q_F = q_{F,u} + q_{F,d} \tag{4.7}$$

where $q_{F,u}$ is the flow contribution from the bottleneck (flow) controller and $q_{F,d}$ is an unmeasured disturbance in the flow. The bottleneck flow q_B is given by the level controller with transfer function $K_V(s)$ plus the ratio (bias) contribution q_0,

$$q_B = K_V(s)(V - V_s) + q_0 \tag{4.8}$$
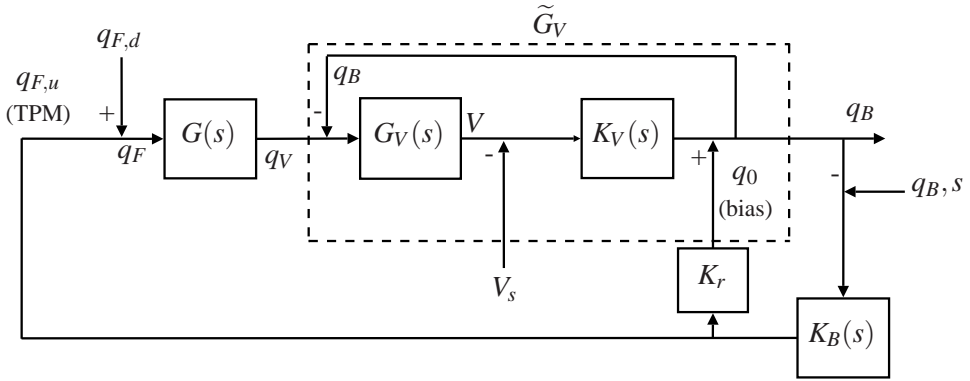
Figure 4.5: Corresponding block diagram of Figure 4.4 in the Laplace domain. q_0 (bias) and V_s (inventory set point) are the dynamic degrees of freedom for control of the bottleneck flow q_B.

where V_s is the set point for the inventory volume. Note that we want the level controller to be a "slow" (averaging) level controller, because otherwise no exploitation of the holdup volume can be obtained. In most cases, we use a proportional-only controller, where $K_V(s) = 1/\tau_V$ (a constant). Typically, to be able to exploit all the volume, τ_V is chosen equal to the nominal residence time (V/q) of a half-full tank (Skogestad, 2006).

The corresponding block diagram of the control structure in Figure 4.4 is given in Figure 4.5. The block K_B is the bottleneck flow controller (FC in Figure 4.4), K_V is the level controller (LC in Figure 4.4) and K_r is the ratio (bias) controller. The block $\widetilde{G}_V$ gives the closed-loop transfer function from the flow into the inventory q_V to the bottleneck flow q_B and consists of the buffer volume plus the level controller. This block also has the two dynamic degrees for bottleneck control as inputs, namely V_s and q_0.

Without active bottleneck control

With only the inventory controller (i.e., without the bottleneck control active, $K_B = 0$) we get from the block diagram (in deviation variables)

$$q_B = \frac{K_V G_V G}{1 + K_V G_V} \cdot q_F + \frac{1}{1 + K_V G_V} \cdot q_0 - \frac{K_V}{1 + K_V G_V} \cdot V_s \qquad (4.9)$$

$$V = \frac{G_V G}{1 + K_V G_V} \cdot q_F - \frac{G_V}{1 + K_V G_V} \cdot q_0 + \frac{K_V G_V}{1 + K_V G_V} \cdot V_s \qquad (4.10)$$

Introducing $G_V(s) = 1/s$ gives

$$q_B = \frac{K_V G}{s + K_V} \cdot q_F + \frac{s}{s + K_V} \cdot q_0 - \frac{K_V s}{s + K_V} \cdot V_s \tag{4.11}$$

$$V = \frac{G}{s + K_V} \cdot q_F - \frac{1}{s + K_V} \cdot q_0 + \frac{K_V}{s + K_V} \cdot V_s \tag{4.12}$$

The steady-state effect is obtained by setting $s = 0$. Thus, we note, as expected, that only q_F has a steady-state effect on the bottleneck flow q_B.

For the further analysis, we assume that the process $G(s)$ is first-order with gain K_r and time constant τ_G

$$G = \frac{K_r}{\tau_G s + 1} \tag{4.13}$$

We assume that the level controller is a proportional controller

$$K_V \triangleq \frac{1}{\tau_V} \tag{4.14}$$

Now, Equations (4.11) and (4.12) become:

$$q_B = \frac{K_r}{(\tau_G s + 1)(\tau_V s + 1)} \cdot q_F + \frac{\tau_V s}{\tau_V s + 1} \cdot q_0 - \frac{s}{\tau_V s + 1} \cdot V_s \tag{4.15}$$

$$V = \frac{K_r \tau_V}{(\tau_G s + 1)(\tau_V s + 1)} \cdot q_F - \frac{\tau_V}{\tau_V s + 1} \cdot q_0 + \frac{1}{\tau_V s + 1} \cdot V_s \tag{4.16}$$

The effective delay from q_F to q_B in this simple case with PI control is, using the half rule (Skogestad, 2003), $\theta_{\text{eff}} = \min(\frac{\tau_V}{2}, \frac{\tau_G}{2})$. From Equation (4.15) and (4.16), the block $\widetilde{G}_V$ in Figure 4.5 is summarized in Table 4.2. The transfer functions given in Table 4.2 are of interest also for MPC.

q_V	q_0	V_s	
$\downarrow$	$\downarrow$	$\downarrow$	
$\dfrac{1}{\tau_V s + 1}$	$\dfrac{\tau_V s}{\tau_V s + 1}$	$\dfrac{-s}{\tau_V s + 1}$	$\rightarrow q_B$
$\dfrac{\tau_V}{\tau_V s + 1}$	$\dfrac{-\tau_V}{\tau_V s + 1}$	$\dfrac{1}{\tau_V s + 1}$	$\rightarrow V$

Table 4.2: Block $\widetilde{G}_V$ in Figure 4.5 with $G_V(s) = 1/s$ and $K_V(s) = 1/\tau_V$ (P-control).

4.5 Analysis of single-loop with ratio control

In this section, the objective is to find the required buffer tank volume V_{min}. In principle, this can be done by either dynamic simulation or analytically. Here we

choose to use the single-loop with ratio control result from the previous section to derive an analytical expression for the required inventory to find an estimate. The most common control structure for dynamic degrees of freedom would probably be MPC, but as shown in the introductory example (Figure 4.4), the inventory volume variations in these two control structure were similar (see also Tables 4.4 and 4.5), although they will depend on the MPC tuning. Note that V denotes the volume of the liquid in the tank and V_{tank} is the actual tank volume.

4.5.1 Developing transfer functions for single-loop with ratio control

Consider Figure 4.4, with one unit followed by a volume where its inventory is exploited dynamically by single-loop with ratio control structure.

Response with "perfect" bias adjustment (ratio controller)

We assume "perfect" static bias adjustment where a feed change is accomplished by a corresponding relative change in downstream flows. This corresponds to the static bias adjustment

$$q_0 = K_r^* q_{F,u} \tag{4.17}$$

where K_r^* is the nominal steady-state ratio $\Delta q_B / \Delta q_{F,u}$. If there are no flow splits or junctions between the feed and the bottleneck unit, then $K_r = 1$. We now want to study the effect of adding the bias ratio adjustment. We assume that the inventory set point is constant ($V_s = 0$). Then, from Equation (4.15), the effect of $q_{F,u}$ and $q_{F,d}$ on the bottleneck flow q_B is

$$
\begin{aligned}
q_B &= \frac{1 + \tau_V s(\tau_G s + 1)}{(\tau_V s + 1)(\tau_G s + 1)} \cdot K_r \cdot q_{F,u} + \frac{1}{(\tau_V s + 1)(\tau_G s + 1)} \cdot K_r \cdot q_{F,d} \\
&= h_{q_B q_{F,u}}(s) \cdot q_{F,u} + h_{q_B q_{F,d}}(s) \cdot q_{F,d}
\end{aligned}
\tag{4.18}
$$

Note that there is a "direct effect" from $q_{F,u}$ to q_B, because of the bias from the static ratio controller. Thus, the effective delay from $q_{F,u}$ to q_B is zero and "perfect" control of q_B is in theory possible. However, one must take into account the variations in $q_{F,u}$ and the volume (level) constraints.

Similarly, from Equation (4.16), the effect of $q_{F,u}$ and $q_{F,d}$ on the volume (level) V is

$$
V = \frac{-\tau_G \tau_V s}{(\tau_V s + 1)(\tau_G s + 1)} \cdot K_r \cdot q_{F,u} + \frac{\tau_V}{(\tau_V s + 1)(\tau_G s + 1)} \cdot K_r \cdot q_{F,d}
\tag{4.19}
$$

Response with "perfect" bottleneck flow controller

To study the expected variations in volume (level), assume a "perfect" bottleneck flow controller K_B that gives $q_B = q_{B,s}$ at all times. This assumption requires the fastest variations in the manipulated input may be expected to lead to the worst-case variation in inventory (V) with the given inventory controller tuning.

Setting $q_B = q_{B,s}$ (perfect bottleneck control), the resulting change in the feed rate from (4.18) is:

$$
\begin{aligned}
q_{F,u} &= \frac{1}{h_{q_B q_{F,u}}} \cdot q_{B,s} - \frac{h_{q_B q_{F,d}}}{h_{q_B q_{F,u}}} \cdot q_{F,d} \\
&= \frac{(\tau_V s + 1)(\tau_G s + 1)}{1 + \tau_V s(\tau_G s + 1)} \frac{1}{K_r} \cdot q_{B,s} - \frac{1}{1 + \tau_V s(\tau_G s + 1)} \cdot q_{F,d}
\end{aligned}
\tag{4.20}
$$

and from Equations (4.19) and (4.20), the resulting change in the inventory with perfect bottleneck control is:

$$
\begin{aligned}
V &= \frac{-\tau_G \tau_V s}{1 + \tau_V s(\tau_G s + 1)} \cdot q_{B,s} + \frac{\tau_V}{1 + \tau_V s(\tau_G s + 1)} \cdot K_r \cdot q_{F,d} \\
&= h_{V q_{B,s}} \cdot q_{B,s} + h_{V q_{F,d}} \cdot q_{F,d}
\end{aligned}
\tag{4.21}
$$

We note that a feed disturbance $q_{F,d}$ has a steady-state effect on the volume (level) because we use a P-only level controller. However, these should be within the allowed bounds when we use an averaging (smooth) level controller when gain $K_V = 1/\tau_V = |\Delta q_0|/|\Delta V_{max}|$ (Skogestad, 2006, Eq.25). A bottleneck flow change $q_{B,s}$ has no steady-state effect of V, but there will be dynamic variations, as studied in more detail below.

4.5.2 Required inventory volume for single unit

The following results are for a single unit (Figure 4.4).

Requirements for bottleneck flow $q_{B,s}$

From (4.21), the transfer function from bottleneck flow changes ($q_{B,s}$) to volume changes (V) with "perfect" bottleneck control is

$$
h_{V q_{B,s}} = \frac{-\tau^2 s}{\tau^2 s^2 + 2\tau \zeta s + 1} \quad \text{where } \tau = \sqrt{\tau_G \tau_V} \; ; \; \zeta = \frac{1}{2}\sqrt{\frac{\tau_V}{\tau_G}}
\tag{4.22}
$$

The peak magnitude for $h_{Vq_{B,s}}$ occurs at frequency $\omega_{peak} = \frac{1}{\tau} = 1/\sqrt{\tau_G \tau_V}$ (see Appendix 4.A for details) and we get

$$|\Delta V_{peak,B}| = \frac{-\tau^2 \omega_{peak}}{\sqrt{(1 - \omega_{peak}^2 \tau^2)^2 + (2\omega_{peak}\tau\zeta)^2}} \cdot |\Delta q_{B,s}| = \frac{\tau}{2\zeta} \cdot |\Delta q_{B,s}| = \tau_G \cdot |\Delta q_{B,s}|$$

(4.23)

This means that the peak of $|V|$ is equal to $\tau_G \cdot |\Delta q_{B,s}|$ and is independent of the level tuning τ_V. This somewhat surprising result follows because of the assumption of perfect bottleneck control, which means that the bottleneck flow controller will counteract the level controller actions.

Requirements for upstream disturbances $q_{F,d}$

Consider next unmeasured disturbances in the feed rate. From (4.21), assuming no overshoot (i.e. $\zeta \geq 1$ or $\tau_V \geq 4\tau_G$), the largest volume change is found at steady-state and is given by

$$|\Delta V_{peak,d}| = \left. \frac{K_r \tau_V}{1 + \tau_V s(\tau_G s + 1)} \right|_{s=0} \cdot |\Delta q_{F,d}| = K_r \tau_V \cdot |\Delta q_{F,d}|$$

(4.24)

Note here that the volume variation depends directly on the level tuning τ_V, so we may use (4.24) to derive the slowest allowed level tuning.

Acceptable variations in feed rate $q_{F,u}$

We want to avoid too large variations in the feed rate caused by bottleneck set point changes. The transfer function from $q_{B,s}$ to $q_{F,u}$ is given by $1/h_{q_B q_{F,u}}(s)$ (Equation (4.20)). Let us assume $q_{B,s}$ can vary sinusoidally and that we do not want more than 50% overshoot in the manipulated feed rate, that is, $|q_{F,u}/q_{F,ss}| \leq M = 1.5$ at all frequencies, where the steady-state change is $q_{F,ss} = q_{B,s}/K_r$. To achieve this we must require

$$\tau_V \geq \frac{\tau_G}{M - 1} = 2\tau_G$$

(4.25)

as derived in Appendix 4.B.

4.5.3   Required inventory volume for units in series

We here consider units in series, for example, as shown for the distillation columns in Figure 4.3. In this case, the above expressions do not strictly hold, even for the case when we can approximate the flow dynamics in each part of the process by a first-order response with time constant τ_G. Consider three units in series, where

1 is the first unit, 2 is the intermediate unit, and 3 is the last unit upstream of the bottleneck.

The above expressions do not hold because the counteracting effect of the level control in upstream units is neglected. Nevertheless, let as assume that with perfect bottleneck control the resulting feed rate change is given by (4.20), except that we must use the dynamics for the last unit (unit 3). We then have for the effect of $q_{B,s}$ on $q_{F,u}$:

$$\frac{q_{F,u}}{q_{B,s}} = \frac{(\tau_{V,3}s+1)(\tau_{G,3}s+1)}{1+\tau_{V,3}s(\tau_{G,3}s+1)} \frac{1}{K_{r,3}} \tag{4.26}$$

This is the flow rate change into the first unit (unit 1). Note that if we assume that $\tau_{V,3} \gg \tau_{G,3}$ then this transfer function approaches $(1/K_{r,3})$, which means that $q_{F,u}$ changes to its steady-state value (which is $q_{B,s}/K_{r,3}$) and stays there (with no overshoot). We assume in the following that this holds, that is

$$q_{F1,u} = q_{F,u} = (1/K_{r,3})q_{B,s} \tag{4.27}$$

For the other units we similarly get if we neglect the counteracting effect of the upstream level controller.

$$q_{F2,u} = q_{0,1} = (K_{r,1}/K_{r,3})q_{B,s} \tag{4.28}$$

$$q_{F3,u} = q_{0,2} = (K_{r,2}/K_{r,3})q_{B,s} \tag{4.29}$$

$$q_{F4,u} = q_{0,3} = q_{B,s} \tag{4.30}$$

Requirements for bottleneck flow $q_{B,s}$

In (4.23), $\Delta q_{B,s}$ is the flow into the next bottleneck unit (unit 4 in our case). With our assumptions of immediate flow changes, the same expression applies also to the other units and we have that the expected maximum change in inventory volume is

$$|\Delta V_{peak,B,i}| = \tau_{G,i} \cdot \Delta q_{B,i} \tag{4.31}$$

where $\Delta q_{B,i} = (K_{r,i}/K_{r,3}) \cdot \Delta q_{B,s}$ is the steady-state flow change in tank i resulting from a change in the bottleneck flow. We note from the derivation that this formula is only approximate, but nevertheless we find by comparing with simulations that it holds quite well (see below).

Requirements for upstream disturbances

The maximum volume change for disturbances occurs at steady state, which means that (4.24) will hold well also for units in series. The general expression for tank i becomes

$$|\Delta V_{peak,d,i}| = \tau_{V,i} \cdot \Delta q_{d,i} \tag{4.32}$$

where $\Delta q_{d,i}$ is the effect of a disturbance on the flow in tank i. For a feed flow disturbance we have $\Delta q_{d,i} = K_{r,i} \cdot \Delta q_{F,d}$.

Acceptable variation in feed rate $q_{F,u}$

The feed rate change is primarily determined by the dynamics in the last unit, see (4.26). Equation (4.25) therefore applies to the last unit only, that is, for the last unit (here denoted 3) we must require to have an overshoot in the feed rate of less than a factor M for sinusoidal variations in $q_{B,s}$:

$$\tau_{V,3} \geq \frac{1}{M-1} \tau_{G,3} \tag{4.33}$$

which is equal to $2\tau_{G,3}$ when $M = 1.5$ (50% overshoot).

4.5.4 Example of required inventory size using single-loop with ratio control

To check the required inventory, we compare for the introductory example the observed volume variations with the estimated volume variation derived in (4.31) and (4.32).

Example 4.1 (continued). *Required buffer volume for four distillation columns in series. The relevant flow dynamics for each column is approximated by a first-order transfer function $K_r/(\tau_G s + 1)$ where τ_G is found from simulations. The time constant τ_G was found as the time for the flow rate into the inventory to reach 63% of its steady-state change following a step change in column feed rate (outflow of previous inventory). The time constants and gains are summarized in Table 4.3. For example, following a step change in the deethanizer column feed rate, it takes $\tau_G = 0.85$ min before the liquid flow into the column reboiler has changed 63%.*

Inventory	τ_G [min]	τ_V [min]	K_r
1. Deethanizer sump (M_B)	0.85	2.7	0.602
2. Depropanizer sump (M_B)	3.9	3.3	0.254
3. Debutanizer condenser (M_D)	1.2	7.7	0.209

Table 4.3: Time constant flow change (τ_G, approximated), inventory (τ_V) and the static ratio gain (K_r) for the distillation columns in Example 4.1.

The observed variations in the volumes (deethanizer M_B, depropanizer M_B and debutanizer M_D) are normalized to find $\Delta V/\Delta q_{F,d}$ and $\Delta V/\Delta q_{B,s}$ and are compared with the estimated volume variations given by (4.31) and (4.32). For example, for the deethanizer the estimate from (4.31) is $|\Delta V|/|\Delta q_{B,s}| = \tau_{G,1} \cdot K_{r,1}/K_{r,3} =$

0.85 $min \cdot 0.602/0.209 = 2.4$ min, and from (4.32) the estimate is $|\Delta V|/|\Delta q_{F,d}| = \tau_{V,1} K_{r,1} = 2.7$ $min \cdot 0.602 = 1.6$ min. From Table 4.4 we see that the estimated volume variations compare well with the observed variations. There is some difference for the bottleneck set point change, but this is expected since the time constant τ_G is only an approximation. For the feed rate disturbance the steady-state volume is the same as estimated, but there are slight overshoots in the volume. This is caused by the overshoot in the manipulated feed rate $q_{F,u}$ (see Figure 4.4(b)).

		1. Deethanizer M_B [min]	2. Depropanizer M_B [min]	3. Debutanizer M_D [min]
$\frac{|\Delta V|}{|\Delta q_{B,s}|}$	Observed at $t = \infty$	0	0	0.026
	Observed max	0.69	1.4	1.8
	Estimated max (4.31)	2.4	4.7	1.2
$\frac{|\Delta V|}{|\Delta q_{F,d}|}$	Observed at $t = \infty$	1.6	0.83	1.6
	Observed max	1.7	0.97	1.8
	Estimated max (4.32)	1.6	0.84	1.6

Table 4.4: Calculated and observed volumes variations in Example 4.1 for single-loop with static bias adjustment (Alternative 1D in Section 4.2).

The corresponding volume variations with MPC are given in Table 4.5. The inventory usage is about the same initially for the two control alternatives, but the MPC has integral action so the inventories return to their set points. However, note that the variations depend on the specific set points weights and penalty on MV moves used in MPC.

		1. Deethanizer M_B [min]	2. Depropanizer M_B[min]	3. Debutanizer M_D[min]
$\frac{|\Delta V|}{|\Delta q_{B,s}|}$	Observed at $t = \infty$	0	0	0
	Observed max	2.2	3.7	3.0
$\frac{|\Delta V|}{|\Delta q_{F,d}|}$	Observed at $t = \infty$	0	0	0
	Observed max	1.2	0.86	1.6

Table 4.5: Observed volumes variations in Example 4.1 with MPC (Alternative 4 in Section 4.2).

The advantages of including dynamic degrees of freedom in throughput maximization are clear. Including buffer volumes leads to tighter control at the bottleneck unit and less back off is required under presence of disturbances, leading to improvement of the plant throughput. The simple formulas developed here can be used to determine the buffer tank volume in plant design. For upstream disturbances the required buffer volume is given by (4.32), and for bottleneck set point changes the required buffer volume is given by (4.31); see also the discussion.

4.6 Discussion

Effect of level control tuning

In the above simulations, the level controllers were actually quite tightly tuned (Figure 4.4). A tight inventory controller counteracts the bias added to the inventory output (q_0) and this leads to poorer bottleneck control. It may also lead to some overshoot in q_F, because the flow controller must generate a larger signal to q_0. On the other hand, with a smoother tuning there is a risk for overfilling or emptying the tank. Thus, tuning of the level controller is a trade-off. This is illustrated by simulation in Figure 4.6 where smoother level tunings are used (τ_c about 7 times larger). The results are summarized in Table 4.6. We see, as expected, that the volume variations are significantly larger, but the control of the bottleneck is better. There is now no overshoot in q_F for the ratio structure. Again, the observed and estimated volume variations are close (Table 4.6).

		1. Deethanizer M_B [min]	2. Depropanizer M_B [min]	3. Debutanizer M_D [min]
$\frac{|\Delta V|}{|\Delta q_{B,s}|}$	Observed at $t = \infty$	0	0.02	0.05
	Observed max	1.42	3.2	2.5
	Estimated max (4.31)	2.4	4.7	1.2
$\frac{|\Delta V|}{|\Delta q_{F,d}|}$	Observed at $t = \infty$	12	5.1	10
	Observed max	12	5.1	10
	Estimated max (4.32)	12	5.1	10

Table 4.6: Calculated and observed volumes variations for the introductory example with smooth inventory tunings. The control structure is single-loop with static bias adjustment.

Finally, note that with smoother level tunings, manual or single-loop bottleneck control is poorer, because it then takes longer time for the flow rate change to move through the system. An important conclusion is that for manual or single-loop bottleneck control we should have tight the level control on the path from the feed (TPM). However, the conclusion is opposite of we make use of the levels as dynamic degrees of freedom. In practice, this may imply that we may need to detune the level loops if we want to use the levels as dynamic degrees of freedom.

Bias or set point adjustment?

Use of the inventories as dynamic degrees of freedom can be realized with either bias adjustment (used here for the ratio scheme) or with set point changes (used here in MPC). Use of bias adjustment does not affect the control system directly, and the inventory set point is still available to operators. However, it may not be
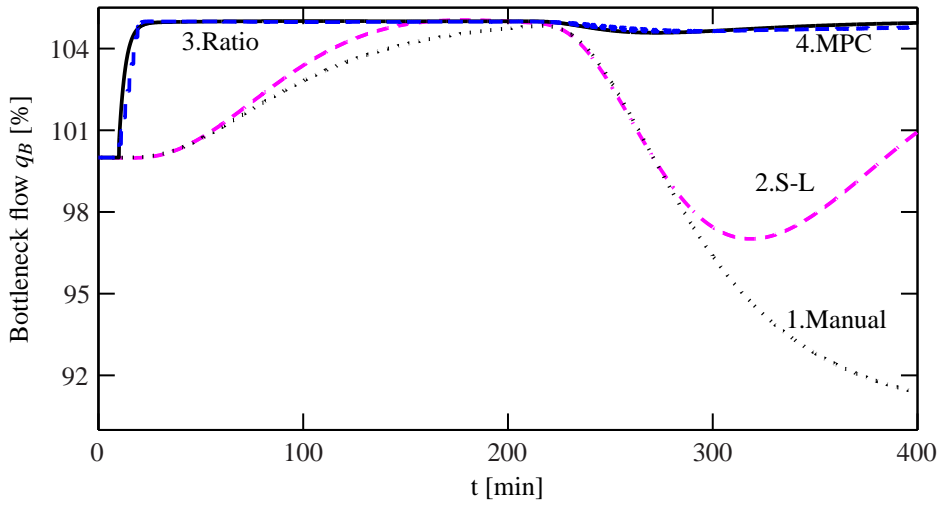
(a) Response in Bottleneck flow q_B (CV)
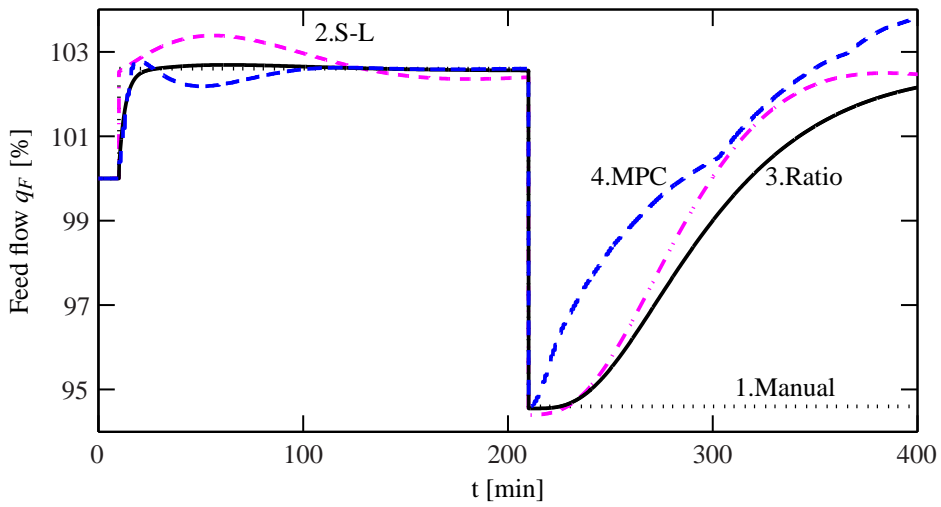
(b) Responses in Feed flow q_F (MV)

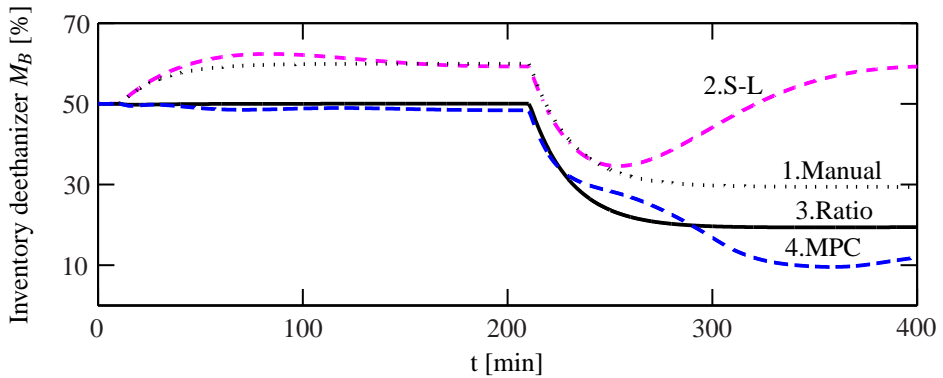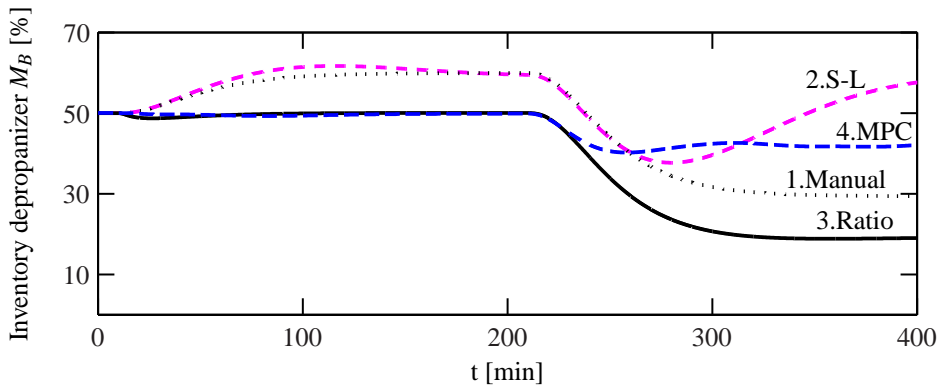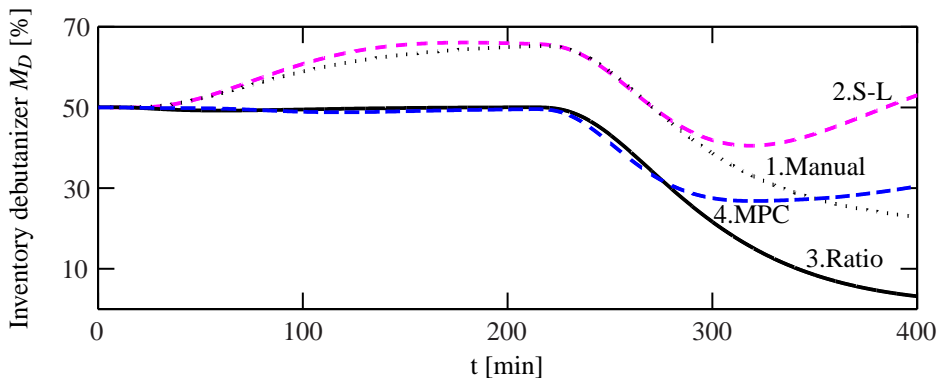Figure 4.6: Continued on next page.

(c) Responses in deetanizer bottoms level M_B (dynamic degree of freedom no. 1)

(d) Responses in depropanizer bottoms level M_B (dynamic degree of freedom no. 2)

(e) Responses in debutanizer distillate level M_D (dynamic degree of freedom no. 3)

Figure 4.6: Same as in Figure 4.4, but with slower (smoother) inventory control.

possible in practice to include bias adjustment because it is not available in the digital control system (DCS). On the other hand, with use of set point adjustment, the use of inventories is very dependent on the inventory tuning.

Placement of the buffer volume

When the feed is the throughput manipulator, the inventory must be placed (and exploited) upstream the bottleneck on the path from the throughput manipulator. Alternatively, they may be placed at the path from important disturbances. If the bottleneck is fixed, then all inventories should be upstream the bottleneck. If the bottleneck is moving, then inventories should be distributed in the plant.

Variations in static ratio gain

The single-loop with ratio control scheme is sensitive to errors in the static ratio gains. This follows because the static ratio gain gives a feedforward control action and feedforward is in general sensitive to modelling errors. In particular, with a too small value of K_r, one will get an overshoot in the feed rate (q_F).

4.7   Summary: Implications for design of inventory tanks

We have derived two formulas, (4.31) and (4.32), for the expected volume variations when inventories are used as dynamic degrees of freedom to achieve bottleneck control. The validity of (4.32) and to some extent (4.31) have been confirmed by simulations. We here summarize the practical use of these formulas for design of inventory tanks.

Tank size

A desired change in tank throughput Δq_B results in a volume variation ΔV and from (4.31) we have

$$|\Delta V| = \tau_G \cdot |\Delta q_B| \tag{4.34}$$

where τ_G is the time constant for "refilling" the tank. In practice, τ_G is the time for the flow rate into V to reach 63% of its steady-state change following a step in flow rate out of the (closest) upstream inventory. This is for the normal case when the TPM is upstream the bottleneck; the same formula applies also when it is downstream. For design purposes, the flow change $|\Delta q_B|$ is the (steady-state) flow change through tank resulting from the largest expected throughput (bottleneck flow) change. (Here, "largest change" should be evaluated over a time period

shorter than τ_G, approximately, because slower changes do not pose a problem in terms of dynamic changes in tank volume).

Equation (4.34) is useful for sizing the tank (inventory volume). In words, (4.34) says that the expected volume variation for an inventory used for bottleneck control (V [m^3]) is approximately the expected variation in flow through the unit (Δq_B [m^3/min]) multiplied by the time constant (τ_G [min]) for the flow dynamics for "refilling" V from the upstream inventory. As expected, a large tank is required if τ_G is large.

For our distillation columns process, we get from (4.34) the following minimum inventories if we assume a 5% desired change in the throughput (bottleneck flow). Note that we here give the inventory in *kmol* (M) rather than in m^3 (V).

$$\text{Deethanizer: } M_B = 0.85 \ min \cdot 29.4 \ kmol/min \cdot 0.05 = 1.2 \ kmol$$
$$\text{Depropanizer: } M_B = 3.9 \ min \cdot 11.6 \ kmol/min \cdot 0.05 = 2.3 \ kmol$$
$$\text{Debutanizer: } M_D = 1.2 \ min \cdot 8.1 \ kmol/min \cdot 0.05 = 0.49 \ kmol$$

For comparison, the actual holdups are 121 *kmol*, 38 *kmol* and 62 *kmol*, respectively, which is from about 40 to 200 times larger than the minimum. This explains why the variations in the inventories for the first 200 *min* in the simulations (Figure 4.4 and Figure 4.6) are so small for the cases 3 and 4 where the inventories are used as degrees of freedom for bottleneck control.

Level control tuning

Next consider (4.32), which involves the closed-loop time constant (τ_V) for the level control loop in the inventory tank. We get

$$|\Delta V_{peak}| = \tau_V \cdot |\Delta q_d| \tag{4.35}$$

where Δq_d is the flow rate change through the tank in question. Equation (4.35) can be used to tune the level controller, and then gives the well-known formula for smooth (averaging) level control. To see this, note that for a nominally half-full tank we must require $|\Delta V_{peak}| < 0.5 \ V_{\text{tank}}$ to avoid overfilling or emptying. If we furthermore assume that the maximum expected change in flow through the tank is 50% of the nominal flow, then $q_d = 0.5 \ q$. Inserting into (4.35) then gives

$$\tau_V < \frac{V_{\text{tank}}}{q} \tag{4.36}$$

where τ_V is the closed-loop time constant for the level control loop. Thus, selecting $\tau_V = V_{\text{tank}}/q$ (the well-known value for smooth level control, (e.g. Skogestad, 2006), gives the slowest possible controller tuning subject to not overfilling or emptying the tank for 50% flow rate changes.

Applying the formula $\tau_V = V_{tank}/q$ to our distillation column example gives (the factor 2 is because we assume that the tank is nominally half full).

Deethanizer: $\tau_V = 2 \cdot 121 \; kmol/(29.4 + 15.8) \; kmol/min = 5.4 \; min$

Depropanizer: $\tau_V = 2 \cdot 38 \; kmol/11.6 \; kmol/min = 6.6 \; min$

Debutanizer: $\tau_V = 2 \cdot 62 \; kmol/8.1 \; kmol/min = 15.3 \; min$

The actual values used in the simulations were 2.7 *min*, 3.3 *min* and 7.7 *min*, respectively, which is half of the values given above and results in smaller variations in the volumes. In addition, the flow rate disturbance was only 8%, and this is why the variations in the inventories for the last 200 *min* in the simulations were so small. In the later simulations (Figure 4.6), τ_V was increased by about a factor 7 in all three level loops. As expected, this resulted in much larger variations in the inventories (about 7 times larger for the last 200 *min* of simulations), but it also resulted in better bottleneck control (for ratio control and MPC where the inventories are used as dynamic degrees of freedom).

We have also derived a formula (4.33) which applies for the level tuning in the last tank upstream of the bottleneck. It says that we should have τ_V for the last tank significantly larger than τ_G. In our case we have $\tau_G = 1.2 \; min$ for the last unit upstream of the bottleneck (debutanizer), whereas $\tau_V = 7.7 \; min$ for the last tank, so this is satisfied.

By comparing Figure 4.4(a) and 4.6(a) we note that bottleneck control is only weakly dependent on the inventory control tuning (value of τ_c) for cases 3 and 4 where the inventories are used as degrees of freedom for bottleneck control (bottleneck control is slightly better in Figure 4.6(a) with smoother inventory control). This is good, because it means that the inventory controllers (value of τ_c) can be tuned independently of the plantwide issue of throughput control.

On the other hand, for cases 1 and 2 where we only use the feed rate as a degree of freedom, bottleneck control is much better with tight inventory control (Figure 4.4(a)) because the effective deadtime from the feed flow to the bottleneck is then reduced. On the other hand, tight inventory control results in little damping of flow disturbances. Thus, there will be a trade-off between wanting tight inventory control (for good bottleneck control) and slow inventory control (to dampen flow disturbances).

4.8   Conclusion

Tight bottleneck control is important for maximizing throughput and avoiding economic losses. However, achieving tight bottleneck control in practice is not so simple because the throughput manipulator is often located away from the bottleneck

unit (with a large effective delay θ_{eff}). In this paper we propose to reduce the effective delay by using dynamic degrees of freedom. The main idea is as follows: To change the flow through the bottleneck, for example, to increase it, we temporarily reduce the inventory in the upstream holdup volume. However, this inventory needs to be kept within bounds, so if we want to increase the bottleneck flow permanently, we need to increase the flow into this part of the process and so on, all the way back to the feed (throughput manipulator). The simplest approach is to make a control system where all flows upstream the bottleneck are increased simultaneously by the same relative amount, like a single-loop bottleneck controller that adjusts the feed flow, combined with ratio controllers that adjust the dynamic degrees of freedom. In this paper a static bias adjustment is studied. Two formulas (4.31) and (4.32) are derived for the expected volume variations when inventories are used as dynamic degrees of freedom to achieve bottleneck control. These two formulas can be used for inventory design purposes.

4.9 Acknowledgments

4.A   Derivation of the peak frequency for second order transfer function

4.A.1   Peak frequency for a second order system

The transfer function $h_{V_{qB,s}}$ is of second order. To analyze the transfer function, consider first a general second order system

$$G(s) = \frac{K}{\tau^2 s^2 + 2\tau\zeta s + 1} \tag{4.37}$$

where K is gain of the second order model, τ is the system time constant and ζ is the damping factor. The magnitude $|G|$ as a function of frequency ω is given by (e.g., Seborg *et al.* (1989, eq. 14-35a))

$$|G| = \frac{K}{\sqrt{(1 - \omega^2\tau^2)^2 + (2\omega\tau\zeta)^2}} \tag{4.38}$$

The transfer function $h_{V_{qB,s}} = (-\tau_G \tau_V s)/(1 + \tau_V s(\tau_G s + 1))$ has a differentiation (s) in the numerator and a second order system in the denominator. The differentiation has a slope of $+1$ in the whole frequency range. The peak frequencies of $h_{vB,s}$ is where the derivative with respect to frequency are zero, thus the denominator should have slope -1 in this point, since the integrator in the numerator always has the slope $+1$.

The phase to a second order system is always $-90°$ at $\omega = \frac{1}{\tau}$, see Seborg *et al.* (1989, Figure 14.3). For stable minimum-phase systems the slope is approximately -1 at $\phi = -90°$ (Skogestad and Postlethwaite, 2005, Eq. 2.12), and this is a commonly used approximation. Thus, the peak frequency of $h_{V_{qB,s}}$ is located at the break frequency, $\omega_{peak} = \frac{1}{\tau}$. The peak frequency can also be found analytically by differentiating (4.22) with respect to ω and let the derivative be zero, as shown in Appendix 4.A.2. Note that in this case the peak frequency is independent of the damping factor ζ.

4.A.2   Analytic derivation of peak frequency

Here the peak frequency for Equation (4.22) is derived analytically and we confirm the arguments in Section 4.A.1. To evaluate the magnitude of $h_{V_{qB,s}}$, replace s with $j\omega$ in (4.22)

$$h_{V_{qB,s}} = \frac{-\tau^2 j\omega}{\tau^2 (j\omega)^2 + 2\zeta\tau\omega + 1} \tag{4.39}$$

The magnitude is given by

$$|h_{V_{qB,s}}| = \frac{\tau^2 \omega}{\sqrt{(1 - \tau^2\omega^2)^2 + (2\zeta\tau\omega)^2}} \tag{4.40}$$

Differentiation with respect to ω

$$\frac{d|h_{Vq_{B,s}}|}{d\omega} = \left(\frac{u}{v}\right)' = \frac{u' \cdot v - v' \cdot u}{v^2} \quad \text{where} \tag{4.41}$$

$$u = \tau^2 \omega$$

$$v = [(1 - \tau^2 \omega^2)^2 + (2\tau\zeta\omega)^2]^{\frac{1}{2}} = n^{\frac{1}{2}}$$

$$\frac{du}{d\omega} = \tau^2$$

$$\frac{dv}{d\omega} = \frac{1}{2} n^{-\frac{1}{2}} \frac{dn}{d\omega}$$

$$\frac{dn}{d\omega} = [(1 - \tau^2 \omega^2)^2 + (2\omega\tau\zeta)^2]'$$

$$= -4\tau^2 \omega + 4\tau^4 \omega^3 + 8\tau^2 \zeta^2 \omega$$

$$\frac{dv}{d\omega} = \frac{1}{2} n^{-\frac{1}{2}} \cdot (-4\tau^2 \omega + 4\tau^4 \omega^3 + 8\tau^2 \zeta^2 \omega)$$

$$= (-2\tau^2 \omega + 2\tau^4 \omega^3 + 4\tau^2 \zeta^2 \omega) \cdot n^{-\frac{1}{2}}$$

Inserting for u and v in (4.41) gives

$$\frac{d|h_{Vq_{B,s}}|}{d\omega} = \frac{\tau^2 n^{\frac{1}{2}} - (-2\tau^2 \omega + 2\tau^4 \omega^3 + 4\tau^2 \zeta^2 \omega) \cdot n^{-\frac{1}{2}} \cdot \tau^2 \omega}{n} \tag{4.42}$$

Multiply numerator and denominator with $n^{\frac{1}{2}}$ gives

$$\frac{d|h_{Vq_{B,s}}|}{d\omega} = \frac{\tau^2 n - (-2\tau^2 \omega + 2\tau^4 \omega^3 + 4\tau^2 \zeta^2 \omega) \cdot \tau^2 \omega}{n^{\frac{3}{2}}} \tag{4.43}$$

We want to find the peak frequency, which corresponds to setting the derivative to zero. Here it is sufficient to evaluate the numerator in Equation (4.43). This yield

$$\tau^2 n - (-2\tau^2 \omega + 2\tau^4 \omega^3 + 4\tau^2 \zeta^2 \omega) \cdot \tau^2 \omega = 0$$

$$1 - 2\tau^2 \omega^2 + \tau^4 \omega^4 + 4\tau^2 \zeta^2 \omega^2 + 2\tau^2 \omega^2 - 2\tau^4 \omega^4 - 4\tau^2 \zeta^2 \omega^2 = 0$$

$$1 - \tau^4 \omega^4 = 0$$

$$\omega^4 = \frac{1}{\tau^4}$$

$$\omega = \frac{1}{\tau}$$

Hence, the peak frequency for $h_{Vq_{B,s}}$ is derived analytically to be $\omega = \frac{1}{\tau} = \frac{1}{\sqrt{\tau_G \tau_V}}$.

4.B   Analytic derivation of acceptable variations in feed rate

The variations in feed rate caused by bottleneck set point changes is given by (4.20) and we have $|q_{F,u}/q_{F,ss}| = |q_{F,u}K_r/q_{B,s}|$ where

$$\frac{q_{F,u}K_r}{q_{B,s}} = \frac{\tau_V\tau_G s^2 + (\tau_V + \tau_G)s + 1}{\tau_V\tau_G s^2 + \tau_V s + 1} \tag{4.44}$$

which can be written as a second order system

$$\frac{q_{F,u}K_r}{q_{B,s}} = \frac{\tau s^2 + 2\tau\zeta_n s + 1}{\tau s^2 + 2\tau\zeta_d s + 1} \quad \text{with}$$

$$\tau = \sqrt{\tau_G\tau_V}$$

$$\zeta_n = \frac{\tau_V + \tau_G}{2\sqrt{\tau_G\tau_V}} = \frac{1}{2}\sqrt{\frac{\tau_V}{\tau_G}}\frac{\tau_V + \tau_G}{\tau_G} \geq \zeta_d \tag{4.45}$$

$$\zeta_d = \frac{1}{2}\sqrt{\frac{\tau_V}{\tau_G}}$$

The magnitude of a second-order system is given in Equation (4.38).

$$\left|\frac{q_{F,u}\cdot K_r}{q_{B,s}}\right| = \frac{\sqrt{(1-\omega^2\tau^2)^2 + (2\omega\tau\zeta_n)^2}}{\sqrt{(1-\omega^2\tau^2)^2 + (2\omega\tau\zeta_d)^2}} \tag{4.46}$$

From Section 4.A.1, a stable minimum-phase, second-order system has its magnitude peak at frequency $\omega = 1/\tau = 1/\sqrt{\tau_G\tau_V}$ and inserting this gives:

$$\left|\frac{q_{F,u}\cdot K_r}{q_{B,s}}\right|_{\max} = \frac{\frac{\tau_V + \tau_G}{\sqrt{\tau_G\tau_V}}}{\sqrt{\frac{\tau_V}{\tau_G}}} = 1 + \frac{\tau_G}{\tau_V} \tag{4.47}$$

Let M denote the allowed overshoot (e.g. $M = 1.5$ if us allow 50% overshoot). Then we must require

$$\left|\frac{q_{F,u}\cdot K_r}{q_{B,s}}\right| \leq M \tag{4.48}$$

and from (4.47) we get

$$1 + \frac{\tau_G}{\tau_V} \leq M \tag{4.49}$$

$$\tau_V \geq \frac{\tau_G}{M-1}$$

For example, with $M = 1.5$ we get $\tau_V \geq 2\tau_G$.

Chapter 5

Coordinator MPC for maximizing plant throughput

In many cases economic optimal operation is the same as maximum plant throughput, which is the same as maximum flow through the bottleneck(s). This insight may greatly simplify implementation. In this paper, we consider the case where the bottlenecks may move, with parallel flows that give rise to multiple bottlenecks and with crossover flows as extra degrees of freedom. With the assumption that the flow through the network is represented by a set of units with linear flow connections, the maximum throughput problem is then a linear programming (LP) problem. We propose to implement maximum throughput by using a coordinator model predictive controller (MPC). Use of MPC to solve the LP has the benefit of allowing for a coordinated dynamic implementation. The constraints for the coordinator MPC are the maximum flows through the individual units. These may change with time and a key idea is that they can be obtained with almost no extra effort using the models in the existing local MPCs. The coordinator MPC has been tested on a dynamic simulator for parts of the Kårstø gas plant and performs well for the simulated challenges.

5.1 Introduction

Real-time optimization (RTO) offers a direct method of maximizing an economic objective function. Most RTO systems are based on detailed nonlinear steady-state models of the entire plant, combined with data reconciliation to update key parameters, such as feed compositions and efficiency factors in units, see for example Marlin and Hrymak (1997). Typically, the RTO application reoptimizes and up-

dates on an hourly basis the set points for the lower-layer control system, which may consists of set points of local MPCs based on simple linear dynamic models. A steady-state RTO is not sufficient if there are frequent changes in active constraints of large economic importance. For example, this could be the case if the throughput bottleneck in a plant moves frequently, which is the case for the application studied in this paper. At least in theory, it is then more suitable to use dynamic optimization with a nonlinear model, which may be realized using dynamic RTO (DRTO) or nonlinear MPC with an economic objective (Tosukhowong *et al.*, 2004; Kadam *et al.*, 2003; Strand, 1991). However, a centralized dynamic optimization of the entire plant is undesirable (Lu, 2003). An alternative is to use local unit-based MPCs, but the resulting steady-state target calculation may be far from optimal (Havlena and Lu, 2005). Coordination of multiple local MPCs has been studied by several authors. Cheng *et al.* (2004, 2006, 2007) have suggested to approach this "coordination" problem by identifying appropriate interactions for linking constraints to find the steady-state targets for the local MPCs. Rawlings and Stewart (2007) discuss a cooperative distributed MPC framework, where the local MPC objective functions are modified to achieve systemwide control objectives. Ying and Joseph (1999) propose a two-stage MPC complement that track changes in the optimum caused by disturbances. The approach permits dynamic tracking of the optimum which is not achievable with a steady-state RTO used in conjunction with a single-stage MPC.

In this paper, we present a different and simpler solution that achieves economic optimal operation without any of these complexities. This solution applies to the common case where prices and market conditions are such that economic optimal operation of the plant is the same as maximizing plant throughput. The main objective is then to maximize the feed to the plant, subject to achieving feasible operation (satisfying operational constraints in all units). This insight may be used to implement optimal operation, without the need for dynamic optimization based on a detailed model of the entire plant.

The *max-flow min-cut* theorem (Ford and Fulkerson, 1962) from linear network theory states that the maximum throughput in a linear network is limited by the "bottleneck(s)" of the network (Aske *et al.*, 2007). In order to maximize the throughput, the flow at the bottlenecks should always be at their maximum. In particular, if the actual flow at the bottleneck is not at its maximum at any given time, then this gives a loss in production that can never be recovered (sometimes referred to as a "lost opportunity").

The throughput manipulators (TPMs) are the degrees of freedom available for implementing maximum throughput. They affect the flow through the entire plant (or at least in more than one unit), and therefore cannot be used to control an individual unit or objective. Ideally, in terms of maximizing plant production and
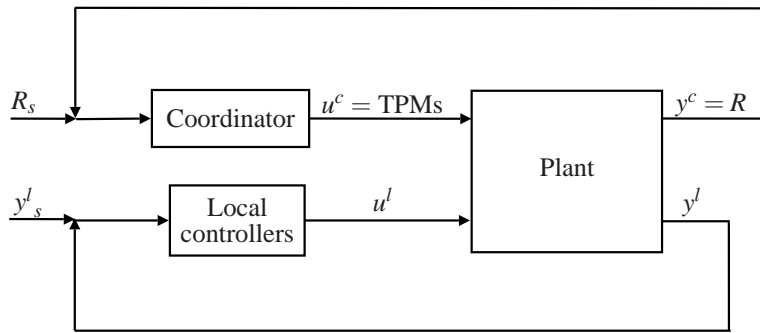
Figure 5.1: The coordinator uses the throughput manipulators (u^c = TPMs) to control the remaining capacity ($y^c = R$) in the units.

minimizing the back off, the TPM should be located at the bottleneck (Aske *et al.*, 2007). However, the bottleneck may move depending on plant operating conditions (e.g. feed composition), and it is generally very difficult to change the TPM, once a decision on its location has been made. The reason is that the location of the TPM affects the degrees of freedom available for local control, and thus strongly affects the structure of the local control systems and in particular the structure of the inventory control system (Buckley, 1964; Price and Georgakis, 1993). The TPM will therefore generally be located away from the bottleneck, for example at the feed. For dynamic reasons it will then not be possible to achieve maximum flow through the bottleneck at all times, and a loss in production is inevitable.

The use of a coordinator controller that uses the throughput manipulators ($u^c =$ TPMs) to control the remaining local capacity ($y^c = R = F^l_{max} - F^l$) in the units as illustrated in Figure 5.1. In the simplest case with a fixed bottleneck and feed rate as the TPM, the coordinator may be a single-loop PI-controller with the feed rate as the manipulated variable (u^c) and the bottleneck flow as the controlled variable (y^c) (Skogestad, 2004). However, more generally the coordinator must be a multivariable controller. Note from Figure 5.1 that the "coordinator" and the "local" controllers for the individual units are actually on the same level in the control hierarchy, like in decentralized control. Nevertheless, the term coordinator is used because the TPMs strongly affect all the units and because in general the coordinator controller must be designed based on a flow network model of the entire plant. An alternative to the decentralized structure is to combine all the local MPCs into a large combined MPC application that include the throughput manipulators as degrees of freedom.

Optimal operation corresponds to $R = 0$ in the bottleneck, but if the maximum flow through the bottleneck is a hard constraint, then to avoid infeasibility ($R < 0$)
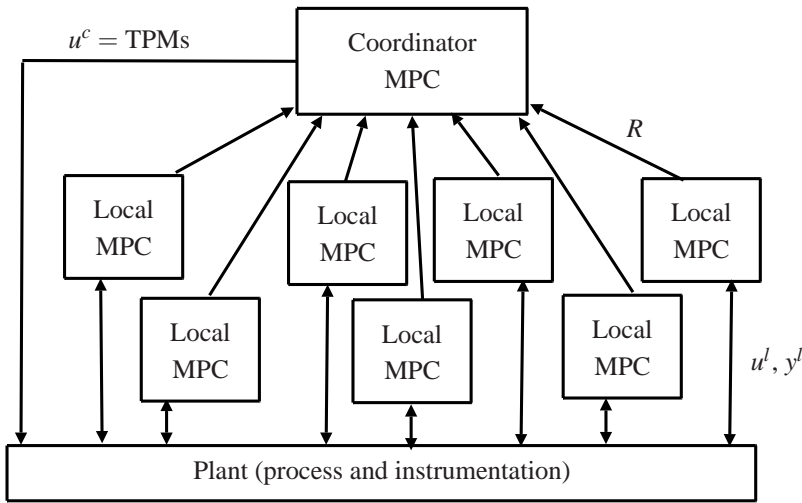
Figure 5.2: Proposed control structure where the coordinator MPC receives information from the local MPC about the remaining capacity (R) in the units.

dynamically, we need to "back off" from the optimal point

$$\text{Back off } (b) = R_s = F^l_{max} - F^l_s \tag{5.1}$$

More generally, the back off is the distance to the active constraint needed to avoid dynamic infeasibility in the presence of disturbances, model errors, delay and other sources for imperfect control (Narraway and Perkins, 1993, Govatsmark and Skogestad, 2005). The back off is a "safety factor" and should be obtained based on information about the disturbances and the expected control performance.

In this paper, we consider cases where the bottlenecks may move and with parallel trains that give rise to multiple bottlenecks and multiple throughput manipulators. This requires multivariable control and the proposed coordinator MPC both identifies the bottlenecks and implements the optimal policy. The constraints for the coordinator MPC are non-negative remaining capacities ($R \geq b \geq 0$) in all units. The values of R may change with time and a key idea is that they can be obtained with almost no extra effort using the existing local MPCs, as illustrated in Figure 5.2.

The paper is organized as follows. Economic optimal operation and the special case of maximum throughput is discussed in Section 5.2. Section 5.3 describes the coordinator MPC in addition to the capacity calculations in the local MPCs. Section 5.4 describes a dynamic simulation case study for a gas plant. A discussion follows in Section 5.5 before the paper is concluded in Section 5.6.

5.2   Maximum throughput as a special case of optimal operation

Mathematically, the optimum is found by minimizing the cost J (i.e., maximize the profit $(-J)$), subject to satisfying given specifications and model equations ($f = 0$) and operational constraints ($g \leq 0$). At steady-state:

$$\min_{u} J(x, u, d) \tag{5.2}$$
$$\text{s. t. } f(x, u, d) = 0$$
$$g(x, u, d) \leq 0$$

Here u are the degrees of freedom (or manipulated variables, MVs), d the disturbances and x the (dependent) state variables. The degrees of freedom are split into those used for local control (u^l) and the TPMs used for throughput coordinator (u^c),

$$u = \begin{bmatrix} u^l \\ u^c \end{bmatrix} \tag{5.3}$$

A typical profit function is

$$(-J) = \sum_{j} p_{P_j} \cdot P_j - \sum_{i} p_{F_i} \cdot F_i - \sum_{k} p_{Q_k} \cdot Q_k \tag{5.4}$$

where P_j are the product flows, F_i the feed flows, Q_k the utility duties (heating, cooling, power), and p denote the prices.

In many cases, and especially when the product prices are high, optimal operation of the plant (maximize $-J$) is the same as maximizing throughput. To understand this, let F denote the overall throughput in the plant, and assume that all feed flows are set in proportion to F,

$$F_i = k_{F,i} F \tag{5.5}$$

Then, under the assumption of constant efficiency in the units (independent of throughput) and assuming that all intensive (property) variables are constant, all extensive variables (flows and heat duties) in the plant will scale with the throughput F e.g, Skogestad (1991). In particular, we have that

$$P_j = k_{P,j} F; \; Q_k = k_{Q,k} F \tag{5.6}$$

where the gains $k_{P,j}$ and $k_{Q,k}$ and are constants. Note from (5.6) that the gains may be obtained from nominal (denoted 0) mass balance data:

$$k_{P,j} = P_{j0} / F_0; \; k_{F,i} = F_{i0} / F_0; \; k_{Q,k} = Q_{k0} / F_0 \tag{5.7}$$

Substituting (5.5) and (5.6) into (5.4) gives

$$(-J) = \left(\sum_j p_{P_j} \cdot k_{P,j} - \sum_i p_{F_i} \cdot k_{F,i} - \sum_k p_{Q_k} \cdot k_{Q,k} \right) F = pF \qquad (5.8)$$

where p is the operational profit per unit of feed F processed. From the above derivation, p is a constant for the case with constant efficiencies. We assume $p > 0$ such that we have a meaningful case where the products are worth more than the feedstocks and utilities. Then, from (5.8) it is clear that maximizing the profit $(-J)$ is equivalent to maximizing the throughput F. However, F cannot go to infinity, because the operational constraints ($g \leq 0$) related to achieving feasible operation (indirectly) impose a maximum value for F.

In practice, the gains $k_{P,j}$ and $k_{Q,k}$ and are not constant, because the efficiency of the plant changes. Usually, operation becomes less efficient and p decreases when F increases. Nevertheless, as long as p remains positive, $d(-J)/dF = p > 0$ is nonzero, and we have a constrained optimum with respect to the throughput F. From (5.8) we see that p will remain positive and optimal operation is the same as maximum throughput if the feed is available and product prices $p_{P,j}$ are sufficiently high compared to the prices of feeds and utilities.

5.3 Coordinator MPC for maximizing throughput

The overall feed rate (or more generally the throughput) affects all units in the plant. For this reason, the throughput is usually not used as a degree of freedom for control of any individual unit, but is instead left as an "unused" degree of freedom to be set at the plant-wide level. Most commonly, the throughput manipulators (u^c) are set manually by the operator, but the objective here is to coordinate them to achieve economic optimal operation.

It is assumed that the local controllers (e.g. local MPCs) are implemented on the individual units. These adjust the local degrees of freedom u^l such that the operation is feasible. However, local feasibility requires that the feed rate to the unit F_k^l is below its maximum capacity, $F_{k,max}^l$, and one of the tasks of the plant-wide coordinator is to make sure that this is satisfied. $F_{k,max}^l$ may change depending on disturbances (e.g. feed composition) and needs to be updated continuously. One method is to use the already existing models in the local MPCs, as discussed in Section 5.3.2.

5.3.1 The coordinator MPC

The steady-state optimization problem (5.2) can be simplified when the optimal solution corresponds to maximizing plant throughput. Consider the steady-state

optimization problem

$$\max_{u^c}(\text{-}J) \quad \text{s. t.} \tag{5.9}$$

$$F^l = Gu^c \tag{5.10}$$

$$R = F^l_{max} - F^l \geq b \geq 0 \tag{5.11}$$

$$u^c_{min} \leq u^c \leq u^c_{max} \tag{5.12}$$

Here F^l is a vector of local feeds to the units and R is a vector of remaining capacities in the units. If the objective is to maximize throughput with a single feed, then $(\text{-}J) = F$. More generally, with different values of the feedstocks and products, the profit function in (5.4) is used. G is a linear steady-state network model from the throughput manipulators u^c (independent feed and crossover flows) to all the local flows F^l. In order to achieve feasible flow through the network, it is necessary that $R \geq 0$ in all units. However, to guarantee dynamic feasibility, an additional back off from the capacity constraint may be required, which is represented by the vector b in (5.11). The main difference from the original optimization problem (5.2) is that only u^c (TPMs) are considered as degrees of freedom for the optimization in (5.9)-(5.12) and that the original constraints for the units ($f = 0, g \leq 0$) are replaced by a linear flow network and flow constraints ($R \geq b$).

It is assumed that the local controllers generate close-to optimal values for the remaining degrees of freedom u^l, while satisfying the original equality ($f = 0$) and inequality constraints ($g \leq 0$). This implies that no coordination of the local controllers is required, or more specifically that constant set points for the local controllers give close to optimal operation. In other words, it is assumed that we for the local units can identify "self-optimizing" controlled variables Skogestad (2000b). If this is not possible then centralized optimization (RTO or maybe even DRTO) is required.

With the linear profit function $(\text{-}J)$ in (5.4), the optimization problem in (5.9)-(5.12) is an LP problem. The optimal solution to an LP problem is always at constraints. This means that the number of active constraints in (5.11) and (5.12) must be equal to the number of throughput manipulators, u^c. Note that an active constraint in (5.11) corresponds to having $R_k = F^l_{max,k} - F^l_k = b_k$, that is, unit k is a bottleneck. This agrees with the max-flow min-cut theorem of linear network theory. However, to solve the LP problem, we will not make use of the max-flow min-cut theorem.

The steady-state optimization problem in (5.9)-(5.12) can be extended to the

dynamic optimization problem:

$$\min_{u^c} (J - J_s)^2 + \Delta u^{cT} Q_u \Delta u^c \quad \text{s. t.} \tag{5.13}$$

$$F^l = G_{dyn} u^c \tag{5.14}$$

$$R = F^l_{max} - F^l \geq b \geq 0 \tag{5.15}$$

$$u^c_{min} \leq u^c \leq u^c_{max} \tag{5.16}$$

$$\Delta u^c_{min} \leq \Delta u^c \leq \Delta u^c_{max} \tag{5.17}$$

Maximum throughput under the presence of disturbances is dynamic in nature, and here, G_{dyn} is a linear dynamic model from u^c (manipulated variables, MVs) to the remaining capacity in each unit, R_k. Obtaining the dynamic models may be time consuming. However, it is possible to use simple mass balances to calculate the steady-state gains of G_{dyn}, see (5.7).

The dynamic cost function (5.13) includes penalty on the MV moves to ensure robustness and acceptable dynamic performance. The constraints are: back off on capacity to each unit (5.15), MV high and low limits (5.16) and MV rate of change limits (5.17). MV rate of change is mainly a safeguard for errors and is normally not used for tuning.

The term $\Delta u^{cT} Q_u \Delta u^c$ makes the objective function quadratic, whereas the objective function in the original problem (5.9) is linear. To obtain a quadratic objective function that fits directly into the MPC software used here, we have used a common trick of introducing a quadratic term $(J - J_s)^2$. The profit set point J_s is high and unreachable with a lower priority than the capacity constraints. An alternative approach would be to include a linear term in J in (5.13).

Standard MPC implementations perform at each time step two calculations (Qin and Badgwell, 2003). First, the steady-state optimization problem with all the constraints is solved to obtain a feasible steady-state solution. Second, the dynamic problem is solved using the feasible targets obtained from the steady-state calculation. In our case, the steady-state part gives a feasible set point for the profit (or total flow) that replaces J_s in the subsequent solution of the dynamic problem. The dynamic terms involving Δu^c do not matter in the steady-state part, so the steady-state solution is identical to the LP problem in (5.9)-(5.12).

It is assumed that the local controllers (including local MPCs) are closed before obtaining the dynamic flow model G_{dyn}. To ensure good performance, it is then advisable that the coordinator operates with a longer time horizon than the local MPCs.

5.3.2 Capacity calculations using local MPCs

An important parameter for the coordinator is the maximum flow for the individual (local) units, F_{max}^l. A key idea in the present work is to obtain updated values using on-line information (feedback) from the plant. Note that it is not critical that the estimate of the maximum capacity is correct, except when the unit is actually approaching its maximum capacity and the corresponding capacity constraint $R = F_{max}^l - F^l \geq b$ becomes active. The use of on-line information from the actual plant will ensure that this is satisfied.

In simple cases, one may update the maximum capacity using the distance (Δconstraint ≥ 0) to a critical constraint in the unit,

$$F_{max}^l = F^l + c \cdot \Delta\text{constraint}$$

where c is a constant and F^l is the present flow through the unit. For example, for a distillation column Δconstraint $= \Delta p_{max} - \Delta p$ could be difference between the pressure drop corresponding to flooding and the actual pressure drop.

In more complex cases, there may be more than one constraint that limits the operation of the unit and thus its maximum capacity. MPC is often implemented on the local units to improve dynamic performance and avoid complex logic. The maximum feed for each unit k can then be easily estimated using the already existing models and constraints in the local MPC applications. The only exception may be that the model must be updated to include the feed to the unit, F_k^l, as an independent variable. The maximum feed to the unit k is then obtained by solving the additional steady-state problem:

$$F_{k,max}^l = \max_{u_k^l, F_k^l} F_k^l \tag{5.18}$$

subject to the linear model equations and constraints of the local MPC, which is a LP problem. Here u_k^l is the vector of manipulated variables in the local MPC, and the optimization is subject to satisfying the linear constraints for the unit. To include past MV moves and disturbances, the end predictions of the variables should be used instead of the present values.

5.4 Kårstø gas processing case study

The Kårstø plant treats gas and condensate from central parts of the Norwegian continental shelf. The products are dry gas, which is exported through pipelines, and natural gas liquids (NGL) and condensate, which are exported by ships. The Kårstø plant plays a key role in the pipeline structure in the Norwegian Sea and
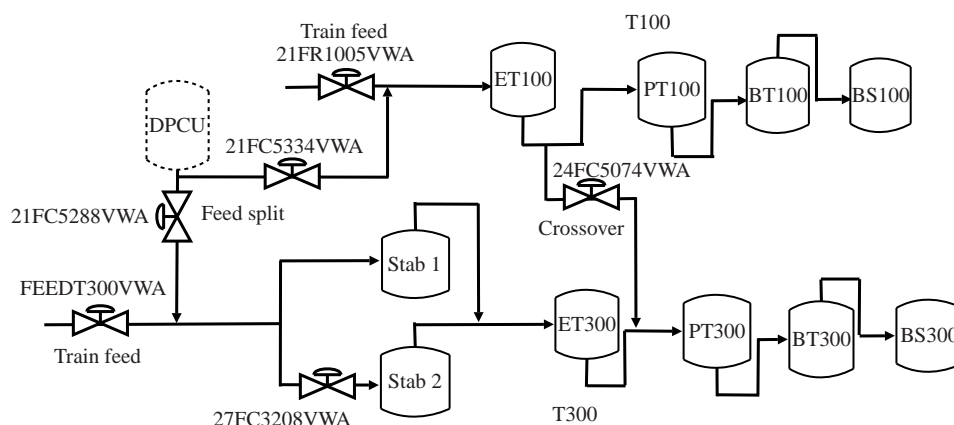
Figure 5.3: The simulated parts of the Kårstø plant

therefore is maximum throughput usually the main objective. Also, from an isolated Kårstø point of view, the plant has relative low feed and energy costs and high product prices that favor high throughputs. There are no recycles in the plant. Usually, feed is available and can be manipulated within given limits.

The feed enters the plant from three different pipelines and the feed composition may change frequently in all three lines. Changes in feed compositions can move the main bottleneck from one unit to another and affect the plant throughput. The coordinator MPC approach has been tested with good results using the Kårstø Whole Plant simulator. This is a dynamic simulator built in the software D-SPICE®.

5.4.1 The case

To demonstrate the applicability of the coordinator MPC, we use a detailed simulator model of parts of the Kårstø plant. To avoid the need for large computer resources to run the process simulator, only parts of the whole plant are used in the case study, see Figure 5.3. The selected parts include two fractionation trains, T100 and T300. Both trains have a deethanizer, depropanizer, debutanizer and a butane splitter. In addition T300 has two stabilizers in parallel. There are six throughput manipulators (u^c) as indicated by valves in Figure 5.3: two main train feeds, two liquid streams to the trains from the dew point control unit (DPCU), a crossover from train T100 to T300, and a flow split for the parallel stabilizers in train T300.

The local MPCs and the coordinator are implemented in Statoils SEPTIC[*]

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[*]Statoil Estimation and Prediction Tool for Identification and Control

MPC software (Strand and Sagli, 2003). Data exchange between the simulator and the MPC applications is done by the built-in D-SPICE® OPC server. The detailed dynamic simulator was used to obtain "experimental" step response models (G_{dyn}) in the coordinator MPC. This approach has been found to work well in practice (Strand and Sagli, 2003).

5.4.2 Implementation of the local MPCs

The main control objective for each column is to control the quality in the top and bottom streams, by manipulating boil-up (V) and reflux flow (L). In addition the column must be kept under surveillance to avoid overloading, which is an important issue when maximizing throughput. Column differential pressure (Δp) is used as an indicator of flooding (Kister, 1990). The remaining feed capacity for each column (R_k) is calculated in the local MPC.

The LV-configuration with a temperature loop is used for regulatory control of the columns (Skogestad, 2007), and the local MPCs are configured as follows:

- CV (set point + constraint): Impurity of heavy key component

- CV (set point + constraint): Impurity of light key component

- CV (constraint): Column differential pressure

- MV: Reflux flow rate set point

- MV: Tray temperature set point in lower section

- DV: Column feed flow

These MVs correspond to u^l (local degrees of freedom), and CVs are the same as y^l. The feed rate is a disturbance variable (DV) for the local MPC, and is used as a degree of freedom when solving the extra LP problem to obtain the remaining capacity (R) to be used by the coordinator. Some of the columns have additional limitations that are included as CVs in the local MPC. The product qualities are described as impurity of the key component and a logarithmic transformation is used to linearize over the operating region (Skogestad, 1997). The high limits on the product qualities are given by the maximum levels of impurity in the sales specifications and the differential pressure high limit is placed just below the flooding point.

The control specification priorities for solving the steady-state feasibility problem for the local MPC are as follows:

1. High limit differential pressure

2. Impurity limits

3. Impurity set points

where 1 has the highest priority. The priority list is used in the steady state part in the MPC solver and leads to relaxation of the impurity set points (and in worst case limits) to avoid exceeding the differential pressure high limit (Strand and Sagli, 2003). By quality relaxation the column can handle the given feed rate without flooding the column. The low-priority quality set points are not used when solving the extra steady-state LP problem to obtain the remaining capacity R, because set point deviations are acceptable if the alternative is feed reduction. In the dynamic optimization part the constraints violations are handled by adding penalty terms to the objective function.

The local MPC applications are built with experimental step response models as described in Aske *et al.* (2005). The prediction horizon is 3 to 6 hours, which is significantly longer than the closed-loop response time. The sample time in the local MPC is set to 1 minute. From experience this is sufficiently fast for the distillation column applications and is the actual sample time used in the plant today.

5.4.3　The design and implementation of the coordinator MPC

The objective function for the coordinator is to maximize the total plant feed, $-J = F = \sum F_i$, which is the sum of the train feeds and the flows from the DPCU (FEEDT300VWA + 21FC5288VWA + 21FC5334VWA + 21FR1005VWA). The CVs and MVs for the coordinator MPC are:

- CV (high set point): Total feed flow F to the plant (PLANT FEED).

- CVs (constraints): Remaining feed capacity R_k in columns, 10 in total (R-ET100, R-PT100, R-BT100, R-BS100, R-STAB1, R-STAB2, R-ET300, R-PT300, R-BT300, R-BS300)

- CV (constraint): T100 deethanizer sump level controller output (LC OUTLET)

- MV: Feed train 100 (21FR1005VWA)

- MV: Feed train 300 (FEEDT300VWA)

- MV: Feed from DPCU to train 100 (21FC5334VWA)

- MV: Feed from DPCU to train 300 (21FC5288VWA)

- MV: Crossover flow from T100 to T300 (24FC5074VWA)

- MV: Stabilizers feed split (27FC3208VWA)

These MVs correspond to u^c (coordinator degrees of freedom). The deethanizer sump level controller output CV (gives the feed to PT100) is used to avoid emptying or overfilling up the sump level in ET100 when manipulating the crossover. The total plant feed has a high unreachable set point with low priority. The remaining feed capacity low limits, and high and low limits of the level controller output have high priority.

Note that each train has two feeds; one train feed and one from the dew point control unit (DPCU). The two feeds have different compositions, and this makes it possible for the coordinator to adjust the feed composition, and thus adjust the load to specific units. The two stabilizers are identical in the simulator, so the stabilizer split (27FC3208VWA) will ensure equal load to the stabilizers. The coordinator uses experimental step response models, obtained in the same way as for the local MPCs. The models were obtained at 80-95% of the maximum throughput, which is typical for the current plant operation. The coordinator execution rate is slower than in the local MPCs to ensure robustness and is here chosen to be 3 minutes. The prediction horizon is set to 20 hours.

The coordinator attempts to maximize the total feed rate while satisfying the capacity constraints for the units. Since the capacity constraints are "hard", it is necessary to introduce at steady-state a back off b to ensure $R \geq 0$ also dynamically. Tuning of the coordinator MPC is a trade-off between robustness and MV (feed) variation on the one side and keeping the flows through the bottlenecks close their maximum on the other side. The required back off b needs to be obtained after observing over some time the performance of coordinator MPC. In the case study, the value of b is about 1-2% of the feed to the unit.

5.4.4 Results from the simulator case study

The coordinator MPC performance is illustrated with three different cases:

1. Take the plant from unconstrained operation (with given feed rate) to maximum throughput (at $t = 0$ min)

2. Change in feed composition (at $t = 360$ min)

3. Change in a CV limit in a local MPC (at $t = 600$ min)

All three cases are common events at the Kårstø plant. Feed composition changes are the most frequent ones. The coordinator should also be able to handle operator changes in the local MPCs as illustrated by changing a local CV limit.

The most important CVs in the coordinator MPC are displayed in Figure 5.4 and the corresponding coordinator MVs are shown in Figure 5.5. CVs far from their constraints are omitted. The vertical lines in the Figures indicate the time where disturbances are introduced (Cases 2 and 3). The back off from the capacity constraints is indicated by dashed horizontal lines in Figure 5.4. Figure 5.6 shows the response of a local MPC application (BS100).

Case 1: Take the plant to maximum throughput

Initially, the plant is not operating at maximum throughput, and Figure 5.5 shows that all four feed rates are ramped up over the first hour. The crossover (named 24FC5074VWA in Figure 5.5) is reduced to unload train 300 where BS300 is close to its capacity limit even initially (the plant is not steady state at $t = 0$ min). From Figure 5.4, ET100 and the T300 stabilizers (Stab1 and Stab2) impose a bottleneck upstream of the crossover, whereas BS300 is a bottleneck downstream the crossover, at least for some period. The remaining capacity in BS300 violates its lower limit of $b = 1.6$ t/h, and is actually just below zero for some time. Hence the back off b is not sufficiently large to keep the remaining capacity just above zero in this case. From Figure 5.6, we see that the local MPC application for BS100 relaxes the quality set points because the column reaches the differential pressure high limit.

Case 2: Change in feed composition

A feed composition step change is introduced to the train 100 feed (which is sum of 21FR1005VWA and 21FC5335VWA). The composition change is given in Table 5.1 and occurs at time $t = 360$ minutes, at the first vertical line in Figures 5.4, 5.5 and 5.6. The reduction in ethane content leads to an increase in the remaining feed capacity in ET100, which is a bottleneck at that time, and the coordinator can increase the train feed. However, the increase in iso-butane content reduces the remaining feed capacity in the further downstream butane splitter (BS100), which becomes a new bottleneck. The coordinator increases the crossover to make use of some remaining capacity in train 300.

Case 3: Change in a CV limit in a local MPC

The bottom quality high limit in BS100 is reduced at a time where BS100 is already operating at its capacity limit, as can be seen at $t = 600$ minutes in Figure 5.6. This leads to a reduction in the remaining feed capacity in BS100 of about 2 t/h. The coordinator MPC responds by increasing the crossover flow from T100 to T300 in addition to T100 feed reduction. The two butane splitters (BS100 and BS300) are
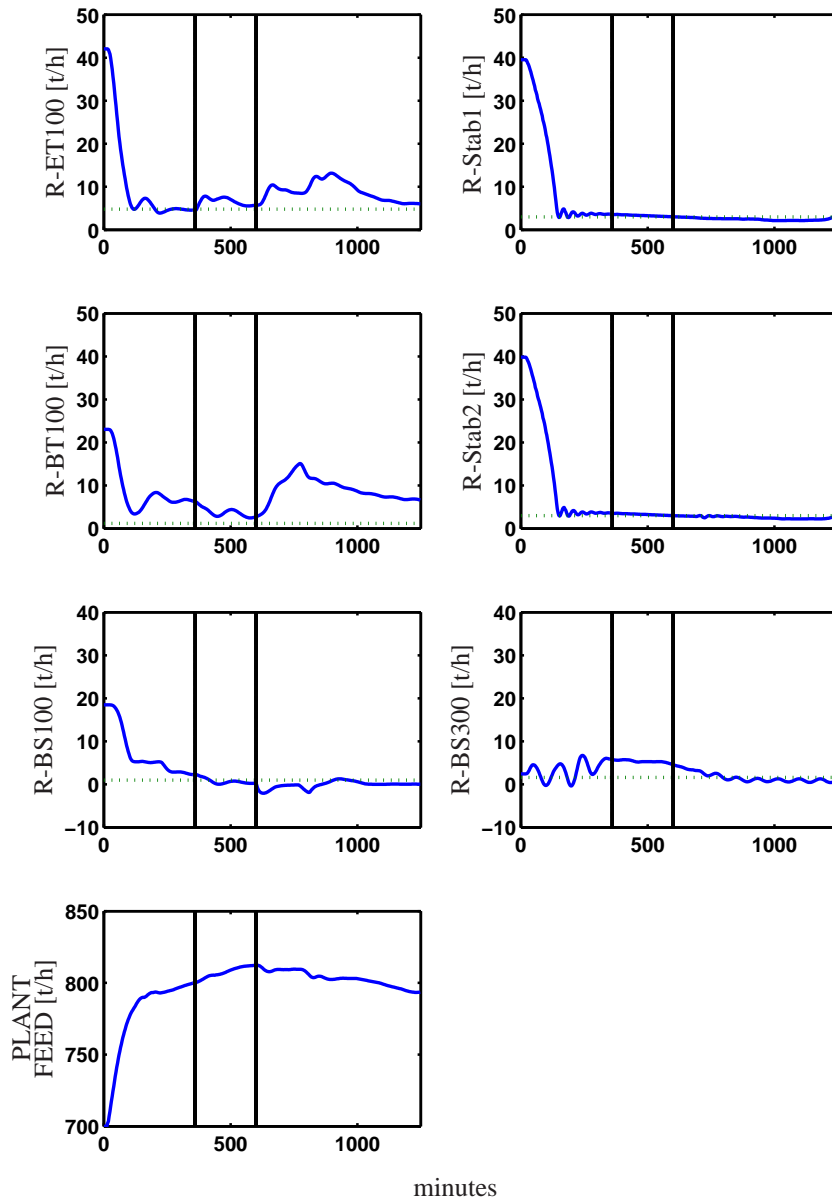
Figure 5.4: The most important CVs in the coordinator MPC (solid) with CV limits (dotted)
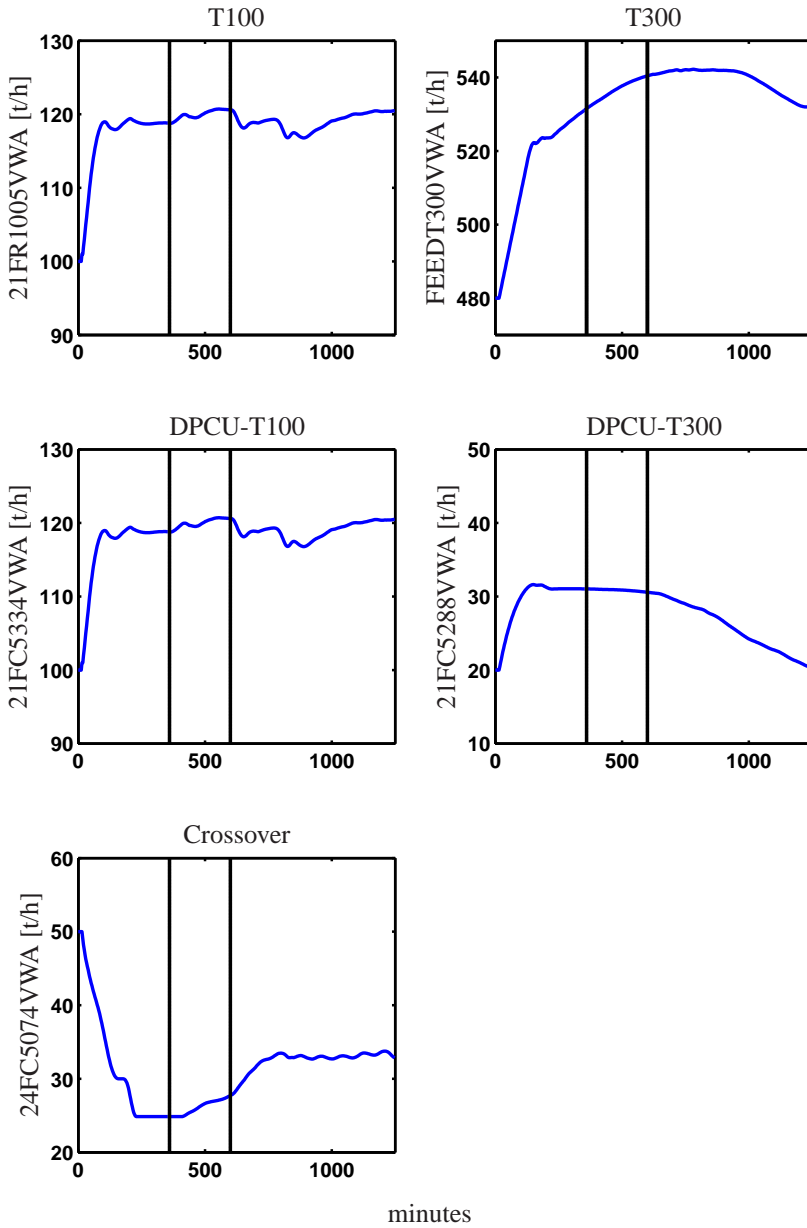
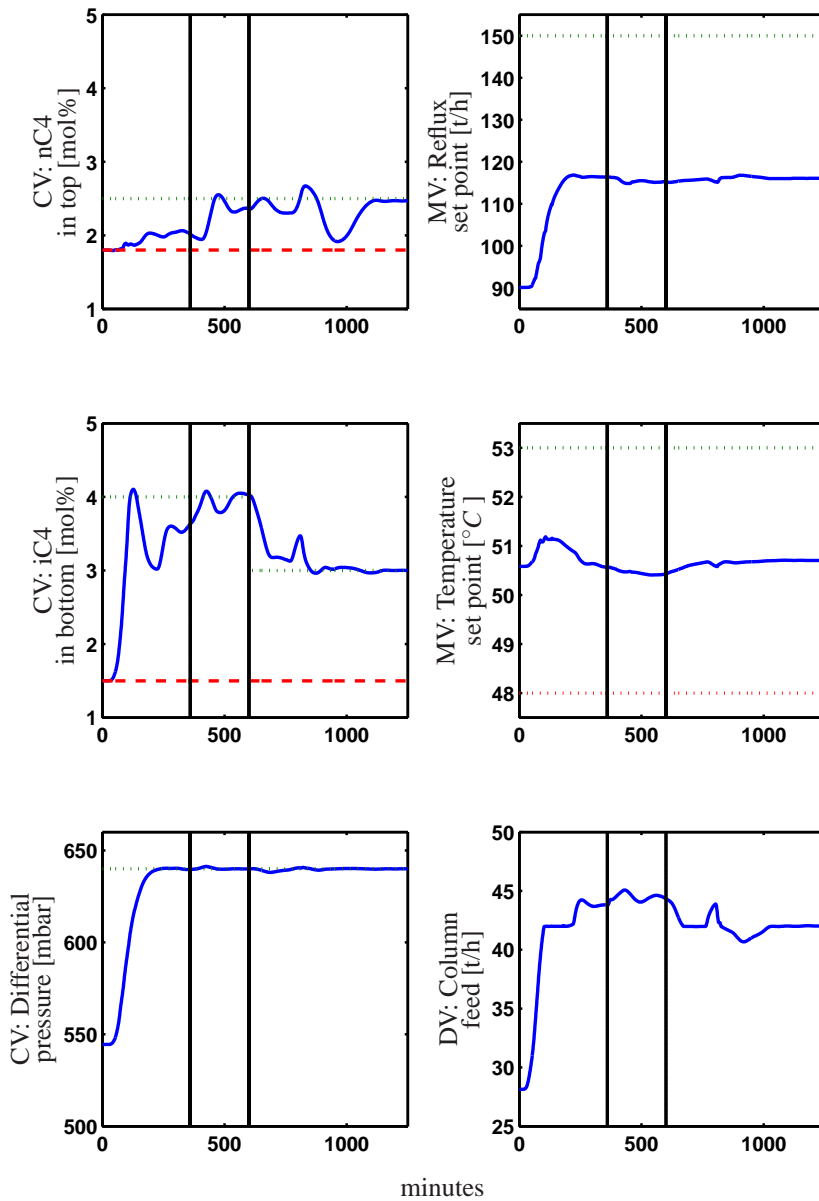Figure 5.5: MVs in the coordinator MPC. Vertical lines indicate new case.

Figure 5.6: CVs, MVs and DV in BS100 MPC. Horizontal lines are set points (dashed) and limits (dotted).

Component	Nominal [mol%]	Points change [%]
Ethane	37.3	-1.1
Propane	35.4	0.71
Iso-butane	5.64	5.6
N-butane	11.3	-0.34
Iso-pentane	1.79	0.09
N-pentane	1.79	0.10

Table 5.1: The feed composition change in the T100 feed at $t = 360$ minutes

now the bottlenecks together with the stabilizers. As expected, the overall effect of the stricter quality limit is reduction in the total plant feed. The reduction takes a long time, however, because the bottleneck in the butane splitters is quite far from the plant feeds.

5.5   Discussion

The main assumption behind the proposed coordinator MPC (see (5.13)-(5.17)), is that optimal operation corresponds to maximum throughput. This will always be the case if the flow network (G_{dyn}) is linear because we then have a LP problem. However, as discussed in Section 5.2, even a nonlinear network will have maximum throughput as the optimal solution provided the product prices are sufficiently high. Thus, the use of a linear flow network model (G_{dyn}) in the coordinator MPC is not a critical assumption. The coordinator identifies the maximum throughput solution based on feedback about the remaining capacity in the individual units, and the main assumption for the network model is that the gains (from feed rates to remaining capacities) have the right sign. Nevertheless, a good network model, both static and dynamic, is desired because it improves the dynamic performance of the coordinator MPC.

In this application, the remaining capacity is obtained for individual units. However, in some cases, for example, reactor-recycle systems, it may be better to consider *system bottleneck* caused by the combination of several units (Aske *et al.*, 2007).

By using a decoupled strategy based on the remaining feed capacity in each unit, the coordinator MPC exploits the already existing models in the local MPCs. This leads to a much smaller modelling effort compared to alternative approaches, like RTO based on a detailed nonlinear model of the entire plant. The computation time in the coordinator MPC is small, and facilitates fast corrections of disturbances, model errors and transient dynamics. The coordinator MPC effectively solves the DRTO problem with acceptable accuracy and execution frequency.

An alternative coordinator MPC strategy would be to combine all the local MPCs into one large combined MPC application including the throughput manipulators. However, for a complete plant the application will be over-complex leading to challenging modelling and maintenance. The improvement by using a combined approach compared to our simple coordinator MPC is expected to be minor since the set points to the MPC are not coordinated. Set point coordination would require a nonlinear model for the entire plant, for example, RTO.

A back off from the maximum throughput in the units is necessary due to unmeasured disturbances and long process response times. The back off should be selected according to the control performance and acceptable constraint violations. In general, the back off can be reduced by improving the dynamic network model and including more plant information to allow for feed-forward control. For example, feed composition changes could be included in the coordinator MPC to improve performance. Due to the lack of fast and explicit feed composition measurements in the plant, feed composition changes are treated as unmeasured disturbances in the simulations in the current concept. However, the concept can be extended by using intermediate flow measurements as indicator for feed composition changes. Therefore, the use of alternative model structures that will simplify and propagate model corrections from intermediate flow measurements should be evaluated.

The most effective way of reducing the back off is to introduce throughput manipulators that are located closer to the bottlenecks. This reduces the dynamic response time and gives tighter control of the flow through the bottleneck. In the case study, the crossover flow introduces a throughput manipulator in the middle of the plant, which improves the throughput control of the units downstream the crossover. It is also possible to include additional dynamic throughput manipulators that make use of the dynamic buffer capacity in the various units and intermediate tanks in the network.

The coordinator requires that the local MPC are well tuned and work well. If the local MPC is not well tuned, a larger back off is needed to avoid constraint violation in the coordinator MPC. In the case study, the BS300 MPC should be retuned to give less oscillation at high throughputs.

The term "coordinator" is used by authors (Venkat *et al.* and Cheng *et al.*) to describe coordination of multiple MPCs where the coordinator is at the level above and generates set points to the local MPCs. In this work the term "coordinator" is used in the meaning of coordinating the flow through the plant, and the coordinator at the same level in the control hierarchy as the local MPCs (see Figure 5.1). However, the tuning is assumed to be done sequentially, with the local MPCs being closed before obtaining the flow network model and tuning the coordinator MPC.

5.6   Conclusion

In many cases, optimal operation is the same as maximum throughput. In terms of realizing maximum throughput there are two issues, first identifying bottleneck(s) and second, implementing maximum flow at the bottleneck(s). The first issue is solved by using the models and constraints from the local unit MPC applications to obtain an estimate of the remaining feed capacity of each unit. The second issue is solved using a standard MPC framework with a simple linear flow network model. The overall solution is a coordinator MPC that manipulates on plant feeds and crossovers to maximize throughput. The coordinator MPC has been tested on a dynamic simulator for parts of the Kårstø gas plant, and it performs well for the simulated challenges.

Chapter 6
Industrial implementation of a coordinator MPC for maximizing throughput at a large-scale gas plant

Is not included due to copyright

Chapter 7

Conclusions and directions for further work

7.1 Conclusions

This thesis has discussed plantwide control configuration with focus on the maximum throughput case. In the general case, an important task for the plantwide control system, if not the most important, is to maintain the plant mass balances. The proposed *self-consistency rule* in **Chapter 2** fills this lack of a general rule that applies to all cases. It may be regarded as an obvious rule, but is often forgotten in a plantwide perspective.We believe the self-consistency rule states the mass balances in a clear manner and will be very useful for students and newcomers in the field.

In **Chapter 3** we have shown that "maximum throughput" is an optimal economic operation policy in many cases. This occurs when product prices are sufficiently high and feed is available and the throughput F is a degree of freedom. Optimal economic operation then corresponds to maximizing the throughput F subject to achieving feasible operation.

From a literature search and based on our own industrial experience, it seems like the feed valve (or more general the throughput manipulator) is very rarely used in practice for closed-loop control, in spite of its great importance on the plant economics in cases where maximum throughput is optimal. The reason is probably the large effect of feed rate on the operation of the entire plant, but the result may be a loss in economic performance.

This thesis discussed several methods for implementing maximum throughput in the control layer. The nature of maximum throughput simplifies the implementation because the optimum is constrained and corresponds to maximum throughput in the bottlenecks(s). Maximum throughput can then be implemented in the

control layer and the approaches discussed in this thesis are:

Chapter 3: To obtain tight bottleneck control, move the throughput manipulator
to the bottleneck unit and control the bottleneck flow with single-loop con-
trol. The approach requires the bottleneck to be fixed in one unit. The disad-
vantage is that the inventory loops upstream the bottleneck must be recon-
figured when moving the throughput manipulator to ensure self-consistency.

Chapter 4: In cases where it is not desired to move the throughput manipulator,
dynamic degrees of freedom can be included to shorten the effective time
delay from the throughput manipulator to the bottleneck. With dynamic
degrees of freedom, we mean manipulated variables with no steady-state ef-
fect. The most common examples are liquid levels and buffer tank levels.
To include dynamic degrees of freedom in single-loop control, the struc-
ture *single-loop with ratio control* is proposed. This control structure uses
the original location of the throughput manipulator (usually the feed rate)
and use inventories dynamically by adding bias to the inventory controller
outputs. The structure can be used for cases with fixed bottleneck. The
single-loop with ratio control structure has no need for reconfiguration of the
inventory loops, even the control parameter tunings can remain unchanged
(except if the inventories are poorly tuned). An multivariable controller (e.g.
MPC) can also be used to include dynamic degrees of freedom with through-
put manipulator (feed rate) and inventories (inventory controller set point or
directly manipulating the valve) as manipulated variables.

Chapters 5 and 6: In larger plants, there are often independent feeds and parallel
trains with crossovers and splits between them that give rise to multiple bot-
tlenecks and multiple throughput manipulators. This requires multivariable
control and the proposed coordinator MPC both identifies the bottlenecks
and implements the optimal policy. The coordinator uses the remaining de-
grees of freedom (u^c) to maximize the flow through the network subject to
given constraints. The remaining degrees of freedom (u^c) include feed rates,
splits and crossovers and the local MPCs provide estimates of the available
capacity constraints ($R_k > 0$) in each node for the network. The constraints
for the coordinator MPC are non-negative remaining capacities (R_k) for each
unit k, that is, how much more the unit is able to receive within feasible op-
eration. The values of R_k may change with time and a key idea is that they
can be obtained with almost no extra effort using the existing local MPCs.

In the latter approach, coordinator MPC for maximizing throughput, the plant-
wide control problem is decomposed by estimating the remaining capacity of each
unit in the local MPC applications. The remaining capacity (R_k) is estimated from

the present initial state, linear model equations and constraints used in the local MPC. To calculate the current maximum feed for each unit, the end predictions (steady-state gain) for the variables are used. In this thesis, the estimate is based on experimental models, most of them linear (some are gain scheduled). However, rigorous models for local units can also be used to predict the remaining capacity and makes decomposition flexible where the best available model can be used to predict the remaining capacity. The major advantage of decomposition is that the overall plant application becomes smaller in size and hence easier to understand and maintain. The coordinator MPC can also easily be built in steps with successive local MPC applications included in the coordinator.

The coordinator MPC is an effective tool for plantwide dynamic optimization. It uses simple models and by estimating remaining capacity of each unit, the plant is decomposed in an effective way. Dynamic optimization with simple models and decomposition of the plantwide control problem is satisfactorily in many cases compared to traditional (steady-state) RTO. This thesis discusses an objective function equal to maximum throughput and dynamic optimization using linear models. However, the coordinator MPC is not imitated to this. The objective function can be economic, for example with a price weighting between the feeds. The coordinator can also use non-linear, rigorous models when it is necessary.

To implement maximum throughput, the key is to achieve maximum flow through the bottleneck unit(s). However, to achieve feasible operation it is usually necessary to "back off" from the optimally active constraints. Back off leads to a lower flow through the bottleneck and an unrecoverable economic loss. This leads to the obvious conclusion that "throughput maximization requires tight bottleneck control". It is important to know (or estimate) the expected back off in order to quantify the possible benefits of moving the throughput manipulator (changing the inventory control system), adding dynamic degrees of freedom, changing or retuning the supervisory control system etc. The magnitude of the back off should be obtained based on information about the disturbances and the expected control performance. In practice, determining the expected dynamic variation is difficult. In this thesis, we obtain a rough estimate of the necessary back off based on controllability analysis. In summary, the requirement that that the effective time delay in the bottleneck controller loop should be less than 1/4 of the disturbance time constant to have benefit of control. This implies that the throughput manipulator must be located very close to the bottleneck to have any benefit of improved control and reducing back off.

7.2 Directions for further work

Within the scope of this thesis, some issues for further work are listed below.

Uncertainty in the static ratio gain

In the single-loop with ratio control, the bias adjustment is considered constant (static). However, this gain may change, for example due to feed composition changes. The performance of the control structure is not considered if the static ratio changes significantly. An alternative implementation can be a nonlinear bias adjustment to account for significant gain changes, but this structure is not studied in detail.

Information loss in plantwide control decomposition

In the estimate of the remaining capacity of a unit, only a single unit is considered in the local MPC application. Thus, some information between the units is therefore lost in the decomposition. For example, the capacity of one unit may depended on how an another unit is operated. Are there any effective ways to add cross-information between the units but still be able to decompose the plant and not include all variables? How large is this loss in cross-information in terms of economics? How much more effort must be added to avoid this loss?

Further implementation of the coordinator MPC

The coordinator MPC is implemented at the Kårstø gas plant, covering about half of the processing units. This should be extended to cover the whole plant and include export gas quality to achieve the real maximum plant throughput. In the estimation of remaining capacity, an LP solver that includes relaxation of the constraints should be implemented. It is preferable that the estimate returns the best possible solution instead of "giving up" and this improves the robustness of the coordinator MPC.

Throughput maximization in recycle systems

The maximum throughput case in production systems is closely related to the maximum flow problem in networks considered in operations research. The main assumption for applying network theory is that the mass flow through the network is represented by linear flow connections. The main process unit that creates nonlinearity in terms of flows between the units is a reactor. Another important decision that affects composition, and thus flows, is the amount of recycle. In this thesis, these sources of nonlinearity are viewed as a single combined unit as seen from maximum throughput (bottleneck) point of view. Combined units are not treated in detail and should be understand better in terms of maximum throughput. However, such systems with reactors will often be in Mode 2b, optimized throughput,

with an unconstrained optimum with no bottlenecks, but there might be cases when such plants are in Mode 2a, maximum throughput.

Obtain an back-off estimate on more realistic example

In Chapter 3, controllability analysis is used to obtain necessary back off to ensure feasibility in spite of disturbances. The controllability analysis should be performed on more realistic example.

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Appendix A
Implementation of MPC on a deethanizer at Kårstø gas plant

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