A Hierarchical View on Evolution-In-Materio Computations

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***Abstract*—Evolution-In-Materio, an unconventional computing paradigm exploiting physical properties of materials for achieving computations, is addressed here as a system which exhibits dynamical hierarchies. A description of computations is provided to show that computations within Evolution-In-Materio systems arise from the dynamics at different hierarchical levels. An information theoretic approach to formalising the notion of dynamical hierarchies is used. The approach is based on the descriptions of the system at different hierarchical levels. The concrete material addressed in this paper is a carbon nanotube / polymer nanocomposite. The choice of material is based on previous work motivated by a number of experiments conducted on such material samples. Presented findings are valuable for several reasons: better understanding of computations within Evolution-In-Materio systems, useful hints to modelling this kind of unconventional computations, useful ideas for further development of similar unconventional computing systems such as using quantum properties of charge carriers within the material and the magnetic field for guiding the search for the solution of the computational problem at hand.**

I. Introduction

The unconventional computing paradigm Evolution-In-Materio (EIM) [1], [2] is based on several earlier attempts to use physical properties of materials for computations [3], [4]. Recently, it has been a subject of investigation under the NASCENCE project (NAnoSCale Engineering for Novel Computation using Evolution) [5], [6]. A number of computational tasks have been successfully solved [7]–[12], however, more as a proof of principle. A wider use of EIM as a potential replacement of classical digital computers based on semiconductor electronics is still out of sight. This is mainly due to the issues which are still to be addressed within the field, such as repeatability and stability of solutions, the way of interpretation of the material response, as well as the very choice of the material which is used as a computing substrate. We believe that better understanding of computations within an EIM framework can lead not only to the solutions to the existing challenges, it can also point towards further areas of investigation which may bring in novel ways of using physical properties of materials for useful computations.

This article is an endeavour in such direction. Based on the ideas presented in [13], it further describes computations within an EIM scenario. This is achieved by addressing computations at different hierarchical levels which may be recognised within the system. More precisely, the principles of dynamical hierarchies [14] are addressed within the sougth system. At this place, we provide a description of an EIM system showing that it can be said to exhibit dynamical hierarchies.

Dynamical hierarchies are one of the basic principles of complex living matter [15]. Units at lower levels interact and undergo some dynamics. As a result of the performed dynamics, *novelty* can be recognised as arising at higher levels, be it in a form of a novel structural entity, novel pattern or novel behavioural trait observable at that level. Novelty accounts for the fact that *the whole is larger than the sum of its parts*, a well-known principle of emergence [16]. Another property of this type of hierarchies is a *loss of information* due to an abstraction of the lower level properties. A number of approaches have been made to formalise this notion, in particular within the ALife community [17]–[24]. However, most of them show weaknesses and incompleteness at capturing important (defining) aspects of dynamical hierarchies. An information theoretic approach has been suggested as a way to overcome some of the shortcomings met. Such an approach, based on [24], is adopted and used here toward EIM systems.

Different levels of an EIM system need to be addressed within such framework. Since the physics of the material used for computations is the basis for achieving computations, our analysis begins with it. In particular, we address the material which has been used in the experiments within the NASCENCE project – a nanocomposite made of Single-Walled Carbon Nanotubes (SWCNT) and polymer molecules, produced especially for EIM experiments within the project [25]. Moreover, for the ease of understanding, we use a concrete computational task – a Time Series Prediction (TSP) of the financial data.

The organisation of the paper is as follows. Section II presents principles of EIM systems and the recognised conceptual domains related to hierarchical levels within such systems. Section III summarises principles of dynamical hierarchies as expressed via entropies within an information theoretic approach given in [24]. Section IV provides the description of the system from the level of the material physics. Sections V and VI provide descriptions of the system at the level of measurements and in the domain of interpretations respectively. They also relate provided descriptions to the lower level description through entropic relations. Finally, Section VII

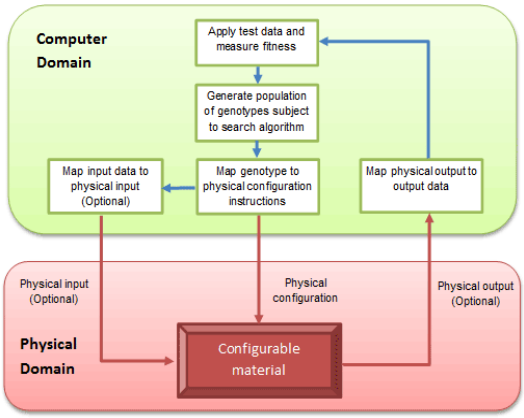


Fig. 1. Schematic view of the Evolution-in-Materio, adapted from [26].

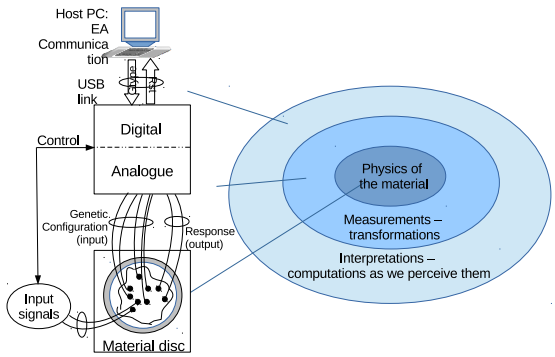


Fig. 2. Conceptual domains of the EIM computing system, from [13].

provides some discussion on the description of computations as presented in this article, the benefits it may have on further research areas within EIM and wider unconventional computing and, finally, concludes the paper with some ideas for future research directions.

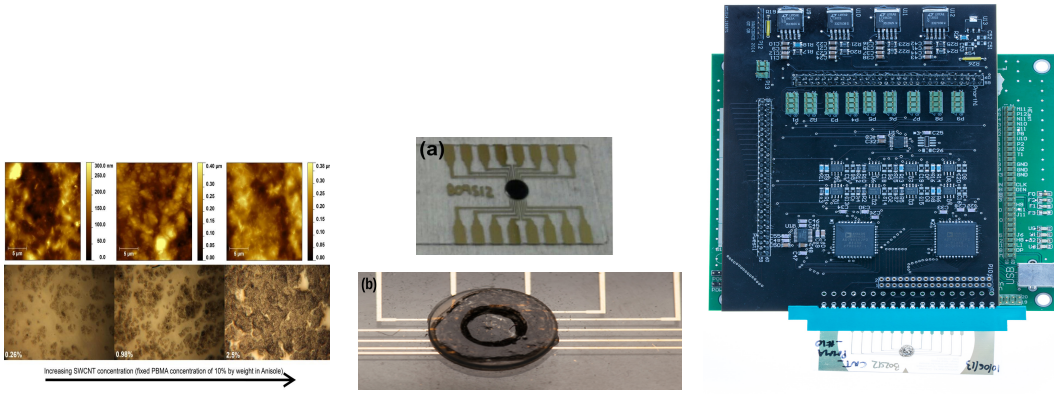
II. Evolution-In-Materio

EIM is an unconventional computing paradigm whereby physical properties of materials are used for computations [1], [2]. Material undergoes the changes, or reconfiguration, guided by an evolutionary algorithm (EA), e.g., a genetic algorithm (GA) [27] until desired computational task is solved. As a GA is performed by a classical digital computer, we speak of a computer controlled evolution (CCE) [28]. Namely, a digital computer is in the loop where it assesses the response of the material against some fitness function and, accordingly, sends configuration signals to the material which configure it so as to perform the computational task. This is schematically shown in Figure 1. The material physics and the changes it undergoes are of analogue nature. However, the GA is performed by a digital computer so that a computational problem at hand is translated and expressed in the digital world, i.e., the genome, the response of the material and the fitness function are all in the digital domain.

In [13] we have recognised different conceptual domains within which EIM computations can be described. Figure 2 shows these domains alongside a schematic view of the EIM scenario. It was postulated that computations emerge at higher levels, which correspond to domains of measurements and interpretations, as a result of the dynamics at the level of the material physics. Therefore, the system may exhibit dynamical hierarchies.

The conceptual domain of the material physics depends on the concrete material which is used for computations. In the experiments conducted within the NASCENCE, the physics of the blobs, i.e., unstructured matter, of CNT composites were used. The blob of material is treated as a black box which means that the manipulation of its properties is not performed in a top-down approach, it rather happens at the observation level, domain of measurements, and is done in a bottom-up fashion where evolutionary changes guide the transformations at low level towards useful computations observed at higher levels. The property used for computation is the conductivity of the blob or, more precisely, the change of the material conductivity due to the changes of the electric field in which the material is placed. The changes of the field are achieved by changing the voltages on the configuration (input) electrodes. The conductivity of the blob is based on the conductivity of SWCNTs which form a percolating network of conductive paths dispersed among polymer molecules, the latter electrically behaving as isolators. However, since CNTs naturally form ropes due to the attractive Van der Vaals forces and since they are randomly dispersed among polymer molecules thereby stretching in all directions, it is not an easy task to determine exact paths of current flow. It has been shown [30] that the conductivity in CNT-polymer nanocomposites depends on a number of factors - concentration of CNTs, fabrication method, presence of surfactants etc. The production of the samples used in our experiments is described in [25]. The samples show dependence of conductivity mainly based on the concentration of CNTs since other parameters of the production process remain the same. Figure 3(a) shows micrographs of the samples for different CNT concentrations. Figure 3(b) shows the material blob on the glass slide which is used for connecting the material to the rest of the system. This part belongs to the domain of material physics as illustrated in Figure 2.

The slides carrying samples are attached to the edge connector on the board which serves as an interface between the material slide and the digital computer running the GA. Figure 3(c) shows the Mecobo board [29] specially developed for this purpose within the NASCENCE project. The board receives configuration and input signals from the digital computer, performs D/A conversion and sends input signals further to the material via input electrodes. It also reads the response of the material through a set of output electrodes, performs A/D conversion and further sends the material response in



(a) AFM (top) and optical microscope (bottom) images of the SWCNT/PBMA films.

(b) Glass slide with the material dispersed over gold electrodes (bottom: close view).

(c) Mecobo and material slide [29].

Fig. 3. SWCNT-poly nanocomposites used in NASCENCE experiments, from NASCENCE documentation.

digital form to the computer to be processed by the GA. Which electrodes are input / configuration or output is determined by the genome and can be changed during an evolutionary run. In this way its operation belongs to the domain of measurements as shown in Figure 2.

The third domain, domain of interpretations, refers to the problem definition regarding available resources, primarily number of electrodes and types of signals used for computations. This domain is case-specific. For example, in [26], a Travelling Salesman Problem is solved in-materio for a 9 city problem. The sample was used on a slide with 12 electrodes out of which 3 were used as configuration inputs. Remaining 9 electrodes were assigned to the 9 cities, an electrode per city, and a simple ordering of output voltages by their values represents the result of the computation for the given configuration. It is obvious then that further scaling of the problem is limited by the number of available electrodes. Further, when addressing this domain we shall use an example of a Time Series Prediction (TSP) [1] solved in materio.

III. Information Theoretic Approach to Dynamical Hierarchies

The most complex systems created by nature exhibit hierarchical organisation [15]. The hierarchies met in living systems are of a specific kind - they arise from the constant flow of the matter: molecules of cells are incessantly being replaced as nutrients enter and the waste leaves them. The dynamics supported by such a flow gives rise to interactions between the cells which then give rise to formation of higher levels of organisation, i.e., tissue, organs, organism as a whole. Therefore, it is the dynamics undergone by the constituent parts that builds the hierarchies and we rather speak of the organisation of the matter in living systems than of some fixed and a once-for-all built hierarchical structure. Such form of hierarchies is termed "dynamical hierarchies" [14], [21], [31].

Dynamical hierarchies have been studied within ALife with the intention of providing formalism for their description and definition. A number of approaches have been adopted from simulation frameworks [17]–[19] to theoretical definitions of related notions [20]–[24]. We focus on the information theoretic approach as presented in [24]. Such approach is not tied to any specific framework, like in the example of [17], nor does it lack specification of novelty at a higher level which arises from the interactions at lower levels as in the example of [20]. For the sake of completeness, in this section we summarise the main contributions made in [24] in order to make the rest of the paper easier to follow.

For such hierarchical system it is needed that there exist descriptions of the system at diferent hierarchical levels and that the system so described is entropic. The descriptions are given in a form of a state space description, i.e., as a set of discrete states $\{s_i\}$ and the transition function tf which determines the transitions between the states. This can be written as:

$$S = \{s_i(t) : s_i(t) \xrightarrow{tf} s_i(t+1); \{p\}\} \tag{1}$$

In Equation 1, t denotes a time instant in which the system is in a given state s_i and p stands for a set of parameters since the description of the state space is in general parameterised. For example, for a physical system the parameters may be some environmental influences like temperature or magnetic field. Further, each state is assigned certain probability with which the system is to be found in that particular state. Therefore, it is possible to define entropy in the information theoretic sense as [32]:

$$H[S] = -\sum_{s} P(S=s)\log(P(S=s)) \tag{2}$$

[1]Note that further references to TSP are for the Time Series Prediction, not the Travelling Salesman Problem

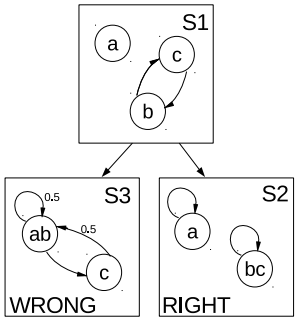


Fig. 4. Two hyperdescriptions, $S2$ and $S3$ of the system $S1$, adapted from [24].

and its conditional equivalent:

$$H[S|Q] = -\sum_{q}\sum_{s} P(S=s|Q=q)\log(P(S=s|Q=q)) \tag{3}$$

where S and Q stand for the state space descriptions of the system which are of a general form as given by Equation 1.

Entropy can also be understood as a measure of uncertainty as when applied to communication channels within the Shannon's information theory [32]. In that sense, the larger the entropy, the larger the uncertainty about the future of the system. So, for a system with some units at the lowest level of our observation, there exists a set of states, $S = \{s_a\}$ which have associated probabilities $P(s_a)$ and the transition function tf_a for the transitions between the states. An example of one such system description, $S1$, is given in Figure 4.

Further, let us assume that there exists another description of the same system at a higher level, $S_b = \{\{s_b\}, tf_b\}$, such that there exists a description function d_{ba} from the states $\{s_a\}$ to the states $\{s_b\}$. Then, we say that S_b hyperdescribes S_a see [24]. What is important to note is that such description has to be chosen carefully so as to capture the behaviour of the system well and also well describe the dynamics of the system. In the example given in [24] for the concrete system shown in Figure 4, two higher level descriptions are given, one which is not good at capturing the system description, $S3$, and another which is, $S2$. For such hyperdescriptions, the following values are defined:

$$r(S,t) = 1 - \frac{H[s(t+1)|s(t)]}{H[s(t+1)]} \tag{4}$$

$$q(S_b, S_a, t) = 1 - \frac{H[s_b(t+1)|s_b(t)]}{H[s_a(t+1)|s_b(t)]} \tag{5}$$

which are termed *state dependence* and *distinctness* respectively. The closer to the value of 1 they are, the more prominent the hierarchies are.

The state dependence as defined in Equation 4 means that the higher level description contains some information about the future states of the system: if we know the description at this level, uncertainty about the future state of the system is less than if this description is not known. This statement captures the novelty brought in at a higher level: the higher level description captures the novelty in its formulation.

The distinctness defined by Equation 5 means that knowing the state of the system as described at a higher level, there is less uncertainty about the future state the system will be in as seen at this level than there is uncertainty about its state as described at a lower level. This entropic relation captures the loss of information when we move from the lower to the higher level. For example, we say that some moving body has certain acceleration without looking into forces which act on its constitutive molecules which result in the acceleration of the body as a whole. For the example given in Figures 4 it would be a loss of information about the transitions between the states **b** and **c** which pertain to the lower level description but are lost at the higher level description although the system behaviour is still correctly captured by this description.

Further, we provide descriptions of the system as given by Equation 1 for the conceptual domains recognised within EIM systems. In other words, for each of the domains, a description is provided in a form of a set of discrete states each of which with the assigned probability with which the system is to be found in it. The superscripts $1, 2$ and 3 denote descriptions for particular domains: domain of material physics, domain of measurements and domain of interpretation respectively. Since each state in the system description is assigned certain probability, the system is entropic and the entropies can be established for conceptual domains mentioned. Also the entropic relations given by Equations 4 and 5 are shown to hold for identified conceptual domains.

IV. The Domain of Material Physics

At this place we address the basis of the CNT conductivity, i.e., the electron transport in CNTs and the change of the electromagnetic field at the accordingly scaled level. Both electric and magnetic components of the field are addressed for the sake of generality. This is needed in the approach herein due to the voltage controlled current change in the CNT and the possible formation of small magnetic fields.

A. A Brief Look at the Electromagnetic Reality

Electromagnetic reality can be described in several ways. Since operations at the nanoscale level are addressed, the description which is adopted accounts for quantum properties of charge carriers and the relativistic nature of the phenomena which are manifested as electromagnetic field. It can be shown [33] that for such description these two equations suffice which can replace Maxwell's equations [34], i.e.:

$$\mathbf{k} = \frac{q_0}{\hbar}\mathbf{A} \tag{6}$$

$$\Box^2\mathbf{A} = -\mu_0\mathbf{J} \tag{7}$$

where $\mathbf{k}$ is a propagation vector of a charge carrier in a four-vector notation, q_0 elemental charge, $\hbar$ reduced Planck constant, $\mathbf{A} = [\overrightarrow{\mathbf{A}}, \frac{V}{c}]$ electromagnetic four-potential which

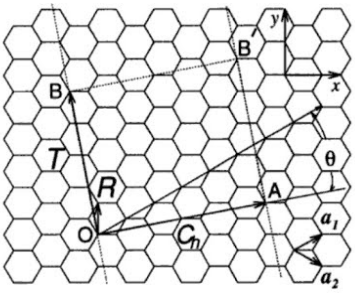


Fig. 5. Unrolled honeycomb lattice of CNT, taken from [35], p.38.

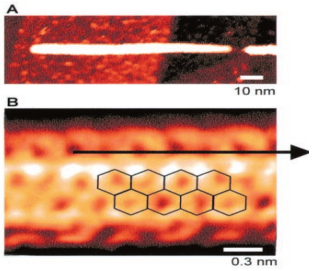


Fig. 6. Scanning Tunneling Microscopy (STM) atomically resolved image of CNT - electron waves visible, taken from [42]

accounts for both scalar (electrostatic) and vector potential, **J** four-vector current density, μ_0 the magnetic constant and $\Box = [\nabla, -\frac{1}{c}\frac{\partial}{\partial t}]$ four-gradient operator. The changes observed in the electromagnetic reality are the manifestation of the dynamics of the collective electron system [33] so at the very bottom of the observed phenomenon lies the dynamics of electrons.

B. Electron transport in CNTs

Electronic structure and the properties of electron transport in CNTs are determined by the CNT geometry [35]–[37]. On one side, the fact that the length of a CNT is orders of magnitude larger than its diameter - μm vs nm - makes them behave electronically like quantum wires at low temperatures [38]. On the other side, the geometry of the carbon atom lattice which makes a CNT, primarily its chirality, further determines electrical properties of CNTs. CNTs are obtained from the graphene sheet by cutting and rolling it into a tube. Figure 5 shows geometrical properties of the carbon lattice which are determined by the graphene sheet. Chirality is defined by the chiral vector C_h see Figure 5 which determines the direction along which a graphene sheet is cut. Dependent on the chirality, CNTs can be electrically semiconducting or metallic. Further, the energy gap in semiconducting CNTs is inversely proportional to the tube diameter.

Apart from the tube geometry, its conductance is also dependent on the electromagnetic field in which the tube is placed – the phenomenon exploited in the NASCENCE experiments. Moreover, conductance fluctuations due to the changes in the magnetic field, known as *universal conductance fluctuations*, can change the character of the CNT. For example, the magnetic field which is parallel to the CNT axis can change the semiconducting character of the tube to metallic [39]. The change happens due to the lowering of the gap between valence and conduction energy bands. Such remarkable properties are akin to the effects of the magnetic field on the electrons which are also subject to the periodic potential caused by the atoms in the crystal lattice as in [40].

For the descriptions of electrons at the quantum level, the Schrödinger equation contains the needed information which is, in general, of the form:

$$H\psi = i\hbar\frac{\partial\psi}{\partial t} \tag{8}$$

where ψ is the electron wave function and H the Hamiltonian, i.e., the operator corresponding to the total (kinetic and potential) energy of the system. For the case when an electron in a periodic potential $V(\mathbf{r})$ of a crystal lattice is also exposed to perturbing magnetic and electric fields, as is the case in our experiments, Hamiltonian is of the form [40]:

$$H = \frac{(\mathbf{p} - \frac{e\vec{\mathbf{A}}}{c})^2}{2m} + V(\mathbf{r}) + e\phi(\mathbf{r}) \tag{9}$$

where $\mathbf{p}$ stands for an electron momentum, e elemental charge, $\vec{\mathbf{A}}$ vector potential, i.e., magnetic field, and $\phi(\mathbf{r})$ a perturbing electric field in general taken to be slowly varying along the CNT in comparison with the periodic potential $V(\mathbf{r})$ of the lattice. All values are given for the position $\mathbf{r}$ within the lattice. Electronic bands, i.e., allowed electronic states can then be found as a solution of the equation of the form:

$$H\psi = E\psi \tag{10}$$

in accordance with the so-called tight binding method [41]. Since electron transport occurs along the CNT where the periodic potential $V(\mathbf{r})$ caused by its lattice atoms is exhibited, the wave function of the electron is of the following form, in accordance with the Bloch theorem [2]:

$$\psi_{\mathbf{k}} = u_{\mathbf{k}}(\mathbf{r})exp(i\mathbf{k}\cdot\mathbf{r}) \tag{11}$$

where $u_{\mathbf{k}}(\mathbf{r})$ has the period of the crystal lattice. It can be shown that a wave vector is quantised in circumferential direction, while it can be quantised for the wave vectors corresponding to the direction of the CNT axis if the length of the CNT is of the order of μm [42]. Figure 6 shows an STM image of electron wave functions which clearly shows quantised states of the electron waves in CNTs.

The character of electron transport in CNTs is dependent on the conditions which make it exhibit its wave or particle nature. Different transport regimes exist: ballistic, localised weak / strong or classic [35]. They are dependent on the relations

[2]Bloch theorem states the solution of the Schrödinger equation for a periodic potential

between three kinds of characteristic lengths attributed to electrons at the mesoscopic level: the momentum length (or mean free path), L_m, the Fermi wavelength, λ_F, and the phase-relaxation length, L_ϕ [35]. Further, we shall assume ballistic transport at low temperatures so that scattering effects can be neglected and the quantised electronic states assumed.

C. State Space Description for the Domain of Material Physics

The wave function, such as the one given in Equation 11, contains enough information about any measurable property of the electron. The probability of the electron occupying a concrete position in time-space, is then:

$$p_s = \psi^* \psi \tag{12}$$

Also, according to the Pauli exclusion principle, maximum two electrons can occupy the same energy state provided they are of opposite spins. From what has been said so far, the description at the level of physics, S^1 can be given by the state $s^1(t)$ defined by the propagation vectors of all the electrons which are moving along CNTs in the system and the transfer function tf^1 by the four-potential $\mathbf{A}$ since the changes of the electron propagation vector happen due to the changes of the electromagnetic field:

$$s(t) = s^1(t) = \{\mathbf{k}_i(t)\}_{i=1}^{N_e} \tag{13}$$

where N_e is the total number of electrons in the system. The transfer function defined by the four-potential:

$$tf^1 = f(\mathbf{A}; \{p\}) \tag{14}$$

where $f(\cdot)$ denotes some functional dependence on the variables in parentheses. Parameter p refers to parameters like temperature, irregularity of the carbon lattice, presence of impurities etc.

V. Domain of Measurements

Values measured in the EIM system are voltages on electrodes which interface the material. Here the distinction must be made between the actual analogue voltages which are on the electrodes and their corresponding digital values which are the result of the A/D conversion by the interface between the analogue and digital domain (here the Mecobo interface platform). The latter are further transmitted to the computer to be processed by the EA. In providing the description of the system for the domain of measurements, the analogue voltages on the electrodes will be assumed. So, the description at this level is somewhat straightforward - it is given by the set of voltages on the electrodes. For the description in the form given by Equation 1, it can be written:

$$s(t) = s^2(t) = \{V_i\}_{i=1}^{N} \tag{15}$$

where N is the number of electrodes used to interface the material, and the transition function:

$$tf^2 = f(\{V_j\}; \{p\}) \tag{16}$$

which consists of the changes of voltages on configuration electrodes, i.e., the index j goes over the electrodes which are used as configuration electrodes. Parameters are those which may affect measured values of voltages, such as temperature, measuring equipment etc.

In order to show that such description hyperdescribes the description at the level of material physics, we need to show that there exists a description function $d_{21}(s^1(t))$ which results in discrete states $s^2(t)$ and that the *state dependence* and *distinctness* hold for descriptions S^1 and S^2.

Description function

For each state from the description S^1, there is a set of electron propagation vectors $\{\mathbf{k}_i\}$ for which Equations 6 and 7 hold. Since the voltage measured on the electrode corresponds to the static potential of the material at the point in space where the electrode is placed, it can be concluded that the scalar part of the four vector potential $\mathbf{A}$ will suffice to provide the needed voltage values. Therefore, the description function $d_{21}(s^1(t))$ exists.

State dependence

For the state dependence as defined by Equation 4 to be less than 1, the following inequality needs to be satisfied:

$$H[s^2(t+1)|s^2(t)] < H[s^2(t+1)] \tag{17}$$

From the definitions of entropy and conditional entropy, Equations 2 and 3, and since the factors of summation for the entropy are of the form $y = x \log x, x = P(*)$ and factor x rises faster than $\log x$, it follows that this inequality is satisfied if:

$$\sum_{j=1}^{N_{s2}} P(s_j^2(t+1)) > \sum_{i=1}^{N_{s2}} \sum_{j=1}^{N_{s2}} P(s_j^2(t+1)|s_i^2(t)) \tag{18}$$

where N_{s2} is the total number of states pertaining to the description S^2. According to the Law of Total Probability for each state $s_j^2(t+1)$ in S^2:

$$P(s_j^2(t+1)) = \sum_{i=1}^{N_{s2}} (P(s_j^2(t+1)|s_i^2(t)) \cdot P(s_i^2(t))) \tag{19}$$

Therefore, it follows that the inequality 18 will hold if there exists at least one state in S^2 which can't be reached from all the other states, i.e., if the transfer function is such that this condition holds. This will be the case if the conduction paths in the material are not formed between all the electrodes for all configurations and if they are not of the same conductivity. Such physical conditions are provided for lower concentrations of CNTs.

Distinctness

The distinctness criterion as given by Equation 5 will be satisfied for the level of measurements if the following inequality holds:

$$H[s^2(t+1)|s^2(t)] < H[s^1(t+1)|s^2(t)] \tag{20}$$

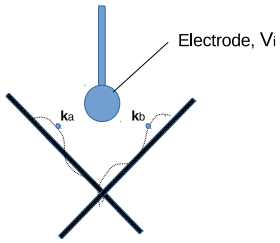


Fig. 7. Illustration of two propagation vectors resulting in the same electrode potential.

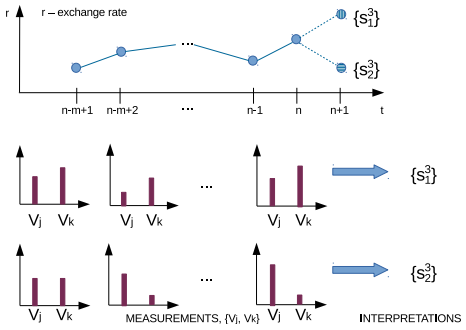


Fig. 8. Illustration of the state space description for the exchange rate prediction.

From Equation 3, it follows that this condition will be satisfied if:

$$\sum_{i=1}^{N_{s2}}\sum_{j=1}^{N_{s2}} P(s_j^2(t+1)|s_i^2(t)) > \sum_{i=1}^{N_{s2}}\sum_{k=1}^{N_{s1}} P(s_k^1(t+1)|s_i^2(t)) \quad (21)$$

To explain when such conditions will be met, let us refer to Figure 7. The Figure shows a simple system consisting of one electrode and two CNTs placed symmetrically to the electrode axis. If there is only one electron wave in the system which is described by the propagation vector $\mathbf{k}_a$ and if at some point in time t it corresponds to the dot marked as $\mathbf{k}_a$ in Figure 7, then the electrode will register some potential V_i. The same potential could be registered on the electrode had the electron wave been moving along the other CNT described by the propagation vector $\mathbf{k}_b$. In other words, for the state from the description S^2, there can exist more corresponding states from the description S^1 which are hyperdescribed by this state in S^2. Therefore, for the state $s^2(t)$ in S^2 there is more uncertainty about the next state $s^1(t+1)$ which belongs to the description S^1 than there is uncertainty about the next state $s^2(t+1)$ from the description S^2 from which Equation 20 immediately follows. An exact proof would contain the proof of Equation 21 which can be done easily by expressing the conditional probability on the left hand side of the inequality as a sum over the states in S^1 for which transitions exist to the specific state from S^2. Because of the reasoning illustrated in Figure 7, this sum is less or equal to the probability on the right hand side. Again, the equality holds for the case when all the states can be reached from one another and when the paths are of the same conductivity leading to the probabilities of the state transitions being the same.

VI. Domain of Interpretation

The description of the system at the level of the problem interpretation is tied to the particular problem at hand. It is based on the measurements provided by the level of measurements which are then interpreted for the target problem. As problems to be solved vary so does the interpretation of the voltages measured as the material response. Therefore, in order to provide a description of the system at this level, we choose one particular problem to solve in-materio: a Time Series Prediction, TSP.

TSP is a task where future values of some time series are predicted based on its values in the past [43]. Typically, there is a series in a form $\{x(t)\}$ where $x(t)$ denotes a value of the signal at time t. Usually, these values are provided for discrete time steps $n \cdot t$. As the values further in the past have less relevance, a time window is applied which selects only a finite number of the time steps back in the time series history for prediction of future value.

Let us assume that a time series represents a currency exchange rate between two currencies and that the task to solve is the direction of this change in the future, i.e., whether the exchange rate will rise or fall. Further, let us assume that the electrode array over which the material is placed consists of altogether N electrodes where N is a finite integer number. Let us choose the interpretation of the problem in the following way. Two of the electrodes, out_j and out_k, $j \neq k, 1 \leq j, k \leq N$, will be used as output electrodes from which the material response, V_j and V_k respectively, will be read and interpreted as follows:

- if $V_j < V_k$, the exchange rate will rise
- if $V_j \geq V_k$, the exchange rate will fall

This is illustrated in Figure 8. The rest of the electrodes are used for configuration and for signal input where input refers to the time series past values.

For such interpretation of the problem, a state space description of the system can be made as following:

$$s(t) = s^3(t) = \{s_1^3(t), s_2^3(t)\} \quad (22)$$

where $s_1^3(t)$ denotes the state where the currency exchange rate will rise ($V_j < V_k$), $s_2^3(t)$ denotes the state that the rate will fall ($V_j \geq V_k$) and the transition function tf^3 is determined by the relation between the measured values V_j and V_k as stated by the interpretation of the problem.

Description function

The description function from the level of measurements, i.e., states defined by the set of voltages on all N electrodes

$\{V_i\}_{i=1}^{N}$ to the level of interpretation can be defined as:

$$d_{32}(s^2(t)) = \begin{cases} s_1^3(t), & \text{if } V_j < V_k \\ s_2^3(t), & \text{if } V_j \geq V_k \end{cases} \quad (23)$$

State dependence

With such description, the state dependence, Equation 4, will hold for:

$$H[s^3(t+1)|s^3(t)] < H[s^3(t+1)] \quad (24)$$

By definition:

$$H[s^3(t+1)] = -\sum_{i=1}^{2} P(s_i^3(t+1)) \log(P(s_i^3(t+1))) \quad (25)$$

and

$$H[s^3(t+1)|s^3(t)] = \\ -\sum_{l=1}^{2}\sum_{i=1}^{2} P(s_i^3(t+1)|s_l^3(t)) \log(P(s_i^3(t+1)|s_l^3(t))) \quad (26)$$

Further, from the Bayes' theorem it follows:

$$P(s_1^3(t)|s_1^3(t+1)) = \frac{P(s_1^3(t+1)|s_1^3(t))P(s_1^3(t))}{P(s^3(t+1))} \quad (27)$$

from which it follows that:

$$P(s_1^3(t+1)) > P(s_1^3(t+1)|s_1^3(t)) \quad (28)$$

since:

$$P(s_1^3(t)) > P(s_1^3(t)|s_1^3(t+1)) \quad (29)$$

Therefore, it can be written:

$$P(s_1^3(t+1)) > \\ \frac{1}{2}(P(s_1^3(t+1)|s_1^3(t)) + P(s_1^3(t+1)|s_2^3(t))) \quad (30)$$

Equivalent inequality holds for the state s_2^3 as well, or, in general, for other states from the description S^3. When this inequality is accounted for in equations 25 and 26, it follows that the state dependence criterion is satisfied as given in Equation 24.

Distinctness

The distinctness relation will hold for:

$$H[s^3(t+1)|s^3(t)] < H[s^2(t+1)|s^3(t)] \quad (31)$$

By definition of conditional entropy:

$$H[s^2(t+1)|s^3(t)] = \\ -\sum_{l=1}^{2}\sum_{i=1}^{2}(P(s_i^2(t+1)|s_l^3(t)) \cdot \log(P(s_i^2(t+1)|s_l^3(t))) \quad (32)$$

Let us refer to the description of the system at the level of interpretation given in the beginning of this section and illustrated in the top of the Figure 9 as $S3$. The description

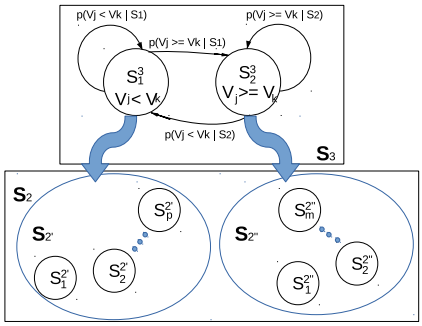


Fig. 9. Two disjoint subsets of S_2 which are hyperdescribed by the states of S_3.

is such that all the states from the description at the level of measurements can be partitioned into two disjoint subsets, $S^{2'}$ and $S^{2''}$ where $S^{2'}$ contains all the states $s^2 = s^{2'} = \{V_i\}$ from S^2 for which it holds that $V_j < V_k$ and $S^{2''}$ contains all the states $s^2 = s^{2''} = \{V_i\}$ from S^2 for which it holds that $V_j \geq V_k$ as illustrated in Figure 9. Then it can be written:

$$P(s_n^3(t+1)|s_1^3(t)) = \sum_{m \in S^{2'}} P(s_m^{2'}(t+1)|s_1^3(t)) \quad (33)$$

and

$$P(s_n^3(t+1)|s_2^3(t)) = \sum_{m \in S^{2''}} P(s_m^{2'}(t+1)|s_2^3(t)) \quad (34)$$

where n stands for index 1 or 2 for two possible states in S^3. From Equations 33 and 34, it follows that for a particular state s_m from the S^2 description it can be written:

$$P(s_n^3(t+1)|s_1^3(t)) < P(s_m^{2'}(t+1)|s_1^3(t)) \\ P(s_n^3(t+1)|s_2^3(t)) < P(s_m^{2''}(t+1)|s_2^3(t)) \quad (35)$$

From Equation 35 it follows that Equation 31 holds since, as mentioned, the factors of summation for the entropy are of the form $y = x \log x, x = P(*)$ and factor x rises faster than $\log x$. Therefore, it can be concluded that the description at the interpretation level for the problem of TSP as given at the beginning of this section, hyperdescribes the description of the system at the level of measurements.

Further it can be argued whether such statement holds for other problems which are solved by EIM. Of course, it is all dependent on the interpretation of the problem at hand and how the solution of the problem is formulated. However, all interpretations come down to a one-to-many mappings between the level of measurements and the level of interpretation so that the description at this level is a good one in a sense explained in Section III, i.e., it captures well the dynamics of the system.

Table I summarises hierarchies within an EIM system in a concise manner.

TABLE I
OVERVIEW OF DYNAMICAL HIERARCHICAL LEVELS IN AN EIM SYSTEM

Domain	State	Transfer function	Loss of information	Novelty
Material physics	$s^1 = \{\mathbf{k}_i\}$	change of four vector potential, $\mathbf{A}$	N/A	N/A
Measurements	$s^2 = \{V_i\}$	change of analogue voltages on input electrodes	number of states, information on individual $\mathbf{k}$	voltage, currents (new qualitative description)
Representation in a computer	$s^{2'} = \{V_i^{'}\}$	digitalised voltages on input electrodes	loss of information due to A/D conversion	new representation in a form of genome
Interpretation, case-specific	$s^3 = \{s_i\}$	GA outcome	loss of voltage values	meaning assigned within a problem domain (case-specific)

VII. DISCUSSION AND CONCLUSION

Hierarchies, somewhat intuitively understood within EIM systems, have been revisited in this paper. An EIM system is described from three hierarchical levels which correspond to the conceptual domains previously identified for the EIM computing scenario. Hierarchies are viewed as dynamical hierarchies, i.e., rather as organisations rising from the dynamics of units at lower levels than as firm structural hierarchies. In order to formally show that an EIM system exhibits dynamical hierarchies, an information theoretic approach was adopted. The conditions are based on the state space descriptions at different levels and entropic relations for such descriptions named *state dependence* and *distinctness*.

The EIM system addressed in this paper is as general as possible. Namely, some approximations were made and also some assumptions about physical properties of the system in order to make the presented reasoning easier to understand. The material used for computations was assumed to be a nanocomposite which consists of SWCNTs and polymer molecules as this material has been used within the group and the NASCENCE project in a number of experiments. Approximations were made as to the mechanisms of electron transport in CNTs so that mainly ballistic transport was considered. Other transport mechanisms were taken to be a consequence of various *parameters* which were included as such in the descriptions at different levels. Another approximation was that the electron transport was considered for a single CNT, while nanocomposites at hand contain bundled CNTs where conduction is of a more complex nature. Moreover, a concrete computational task was used, i.e., the TSP of the financial data.

Applied information theoretic formalism led us to conclusion that the conditions for the existence of dynamical hierarchies within EIM systems can exist. For the concrete example of the CNT-polymer nanocomposite, lower concentrations of CNTs are more likely to exhibit such behaviour since entropic relations between various level descriptions are more pronounced for such cases as explained within Section V. Findings regarding the state dependencies hold, however, for the general case of any material used for computations. The significance of these results lies in the fact that the properties of hierachical systems found in nature, primarily living systems, such as complexity of performed tasks and adaptability to environmental changes, can be expected to be possible in EIM systems as well. An open question is how to make use of the material properties for achieving them.

Further, an insight into the material physics at the lowest level has provided some guidelines for further research directions regarding the type of the material used for computations and the way of its manipulation with respect to the qualitative kind of signals and the method used for the search for the solution. Electron transport in CNTs and the possibility of using its quantum properties for computations is one of them. Further, so far it has been electrical signals, i.e., the changes of voltages which were used for manipulation of the material. Our deeper look into the material physics has shown that other kinds of material manipulation can also lead to the change of CNT conductivity and therefore be used for EIM computing systems. In the first place we think of the magnetic field which can change the conductivity of CNTs and therefore be used for the manipulation of a CNT-based material. Such novel ways of EIM manipulation also open new research directions towards search methods for the solution of computational problems, perhaps beyond EAs. Current work conducted within the group is directed towards attempts to demonstrate this in simulation.

Finally, the findings from this paper can be useful for modelling computations in EIM systems. So far, computations within EIM systems have been modelled without much consideration of the material physics although it is the physics of the material which is the basis of computations, i.e., it was modelled by artificial neural networks, cellular automata, resistor networks. Hierarchical approach with the material physics at its basis may prove a well fitting model which allows for the representation and simulation of the effects of different material properties on the results of computations.

In conclusion, we find this article valuable not only for describing EIM computations from the perspective of dynamical hierarchies but also for the number of new research directions it has pointed towards.

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