Framework for Combined Diagnostics, Prognostics and Optimal Operation of a Subsea Gas Compression System *

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Abstract: The efficient and safe operation of subsea gas and oil production systems sets strict requirements to equipment reliability to avoid unplanned breakdowns and costly maintenance interventions. Because of this, condition monitoring is employed to assess the status of the system in real-time. However, the condition of the system is usually not considered explicitly when finding the optimal operation strategy. Instead, operational constraints on flow rates, pressures etc., based on worst-case scenarios, are imposed. This can lead to unnecessarily restrained operation and significant economic losses. To avoid sub-optimal operation, we propose to integrate diagnostics and prognostics with the optimal decision making process for operation to obtain an operational strategy which is optimal subject to the expected system degradation. This allows us to proactively steer the system degradation, rather than simply reacting to it. We use the operation of a subsea gas compressor subject to bearing degradation as a case example.

Keywords: Model predictive control, robust control, diagnostics, prognostics

1. INTRODUCTION

Subsea processing is an enabling technology for fields that were previously deemed too remote, too deep or far away from existing infrastructure. However, several industrial challenges arise when moving topside equipment to the seabed. One of the potentially most prohibitive challenges is the inaccessibility of the plant for large parts of the year, and the need for specialized intervention ships. Consequently, unplanned shut-downs can be very costly and must be avoided as far as possible. In order to achieve this, strict reliability constraints are imposed on design and operation of the plant. While these safety margins provide a method to ensure reliable operation, they might be overly restrictive. One reason for this is because the information from the health monitoring system is often not utilized directly in the decision making process. Instead, a "worst-case" approach is often used to determine production set-points.

In this paper we propose a method for integrating health monitoring, prognostics and control to obtain an operational strategy that ensures maximum economic profit without jeopardizing the plant reliability. In particular, we include a health degradation model in our optimization routine, resulting in a model-predictive control (MPC)-like framework where we impose constraints on the remaining useful life (RUL) of the equipment.

MPC has gained increasing popularity in industry in recent years due to its ability to deal with constrained,

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multivariate, and nonlinear control problems, is based on the repeated optimization of the objective function, subject to constraints (Morari and Lee, 1999). The first input of the optimized input trajectory is implemented in the plant, before new measurements are taken and the model is re-optimized.

The concept of health-aware control has been investigated by a few authors in recent years. The term "healthaware control" itself was first used by Escobet et al. (2012) to describe a control structure which through the combination of prognostics and health monitoring (PHM) and feedback control simultaneously can fulfill the control objectives and extend the component RUL. The method was applied to a conveyor belt system, and later to wind turbines (Sanchez et al., 2015). Similar ideas of combining PHM and MPC were previously discussed by Pereira et al. (2010) and Salazar et al. (2016), with application to control effort distribution, and pumps in drinking water networks, respectively.

1.1 General description of framework

In this paper we propose an integrated framework for combining diagnostics, prognostics, and optimal operation using MPC. Our framework contains the following steps:

- **Step 1 (Data acquisition):** Collect measurements from the plant, including measurements of equipment health indicators
- Step 2 (Diagnostics): Estimate states and system health
- Step 3 (Prognostics): Estimate prediction model parameters

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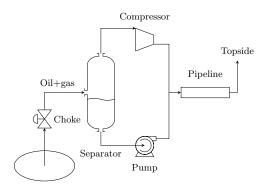


Fig. 1. Subsea gas compression station

Step 4 (Optimal operation strategy): Use

current health estimates and prediction model in addition to system model to find operation that maximizes the profit without violating safety constraints

Step 5: Implement first control input

Step 6: Repeat: go to step 1

In the following sections, we will show how these steps can be applied to the case of a subsea compression station. Following a description of the process in Section 2, we cover step 1 and 2 in Section 3.1, step 3 in Section 3.2, and step 4 and 5 in Section 4. The results from the case study are presented in Section 5.

2. PROCESS DESCRIPTION

Our case study is a subsea gas compression station, similar to installations on the Åsgard field and the Ormen Lange pilot. The purpose of the gas compression station is to boost the pressure of the stream so that it is sufficiently high to overcome the pressure drop in the transportation pipeline and arrive at the receiving facility topside with the desired outlet pressure. A multiphase boosting pump could be used for this purpose, but since the maturity level of the technology is limited, it is chosen to split the well stream into its gas and liquid parts before increasing the pressure of each individual stream. An illustration of the process is shown in Figure 1.

The system consists of a well choke with which the flow of the hydrocarbons from the reservoir can be controlled. From the reservoir, the stream enters a gas-liquid separator, whose purpose it is to separate the gas from the oil and water. Due to imperfect separation, liquid droplets can be carried over to the gas outlet of the separator. The separator efficiency is modeled as a function of the gas velocity and the average fluid density (Austrheim, 2006). The pressure of the liquid outlet is boosted by a pump before being recombined with the gas. Meanwhile, the pressure of the gas outlet is increased in a compressor. The compressor is modeled as a wet-gas compressor which can handle moderate amounts of liquid carry-over (Aguilera, 2013). Suction gas-volume-fractions of 0.95 to 1.0 can be tolerated at the compressor inlet.

3. DIAGNOSTIC AND PROGNOSTIC MODELING

Diagnostics and prognostics form the backbone of any PHM system (Heng et al., 2009). In order to make mean-

ingful decisions about future production, it is not only necessary to know what the health state of the equipment is at the current time, but one must also be able to predict how the condition of the equipment will develop in the future. Diagnostics is about the detection and monitoring of faults, whereas prognostics is about the prediction of health evolution and estimation of equipment RUL.

Prognostics and diagnostics of a large system such as a gas compression station is a challenging task, due to the high complexity and large number of components. Condition monitoring systems should be able to detect a wide range of faults, including everything from signal failure to external impact of foreign objects. A variety of methods are used to monitor subsea production systems in industry. For example, sand erosion and corrosion rates are monitored in vulnerable parts of the pipeline, such as in bends. Erosion and corrosion rates are estimated through measurements of electrical resistance or by periodic inspection of coupons. Detection of leaks is also an important topic. Leaks are usually monitored through a combination of visual surveillance, electrical resistance measurements of the seawater, and temperature/pressure measurements of seals.

In order to limit the scope of the remainder of the paper, we make the simplifying assumption that only the most crucial faults of the system need to be considered. It is known that rotating machinery such as compressors and pumps are prone to faults due to their many moving parts and mechanical complexity (Heng et al., 2009). This means that for the studied process, the compressor, the pump and the well choke need to be monitored closely due to the relatively high likelihood of critical faults occurring here.

Furthermore, in this paper, we exclude failures which cannot be influenced directly by manipulation of the inputs. This excludes a large number of important fault modes. Since the purpose of this paper is to combine condition monitoring and control, we chose to neglect faults which are independent of operational decisions for now. These kinds of faults will have to be addressed in future work.

3.1 Diagnostics

Vibration monitoring of rotating machinery is commonly used to assess their health. Imbalance caused by the onset of a fault will result in a periodic force with a characteristic periodicity and magnitude, which can be detected as vibrations. This technique can be used to detect defects on the shaft, bearings and impeller blades. Current subsea gas compression stations use magnetic bearings to stabilize the impeller, but since this technology is relatively new, not many degradation models are available in the open literature. Ball bearings, on the other hand, are widely used for a multitude of applications, including on-shore gas compressors. We will therefore use the case of a subsea compressor with ball bearings in this paper to demonstrate our framework.

Ball bearings, which are commonly found in pumps and compressors, are subject to large stresses due to their constant load and high rotational speed. At the same time, their survival is crucial for the operation of the machine.

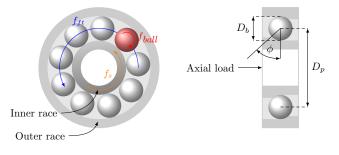


Fig. 2. Illustration of a ball bearing

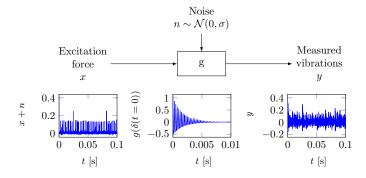


Fig. 3. Signal path from excitation force x to measured vibrations y, via impulse response model g.

Therefore, condition monitoring of bearings is important to ensure high availability of the pump or compressor. The inner workings of a ball bearing are shown in Figure 2.

For a full overview of bearing fault diagnostics, see e.g. Wang and Kootsookos (1998). To make this paper selfcontained, we give a short summary below. A surface defect on a bearing results in a periodic excitation force with characteristic frequency f_{fault} . The excitation force can be described as an impulse train, and the severity of the fault can be estimated by looking at the magnitudes of the impulses. In addition to the periodic impulses, random vibrations (noise) act on the bearing. The sum of these periodic impulses and the random noise is shown on the left in Figure 3. The resulting force is modulated by the impulse response function of the equipment to create the measured vibrations. This impulse response can be seen as the modulation of the original signal from the fault location to the vibration sensor, due to e.g. the resonance vibrations of the bearing housing. The impulse response model q is a damped harmonic oscillator, as illustrated in the middle plot in Figure 3. Finally, the measured vibrations, i.e. the modulated signal, are shown on the right in Figure 3.

The fault frequency f_{fault} is dependent on the location of the fault and the specific geometry of the bearing, but is ultimately a function of the shaft frequency f_s . Let us first define the fundamental train frequency f_{ft} as

$$f_{ft} = \frac{f_s}{2} \left[1 - \left(\frac{D_p}{D_b}\right)^{-1} \cos(\phi) \right]. \tag{1}$$

In the above expression, D_p is the pitch diameter, D_b is the ball diameter, and ϕ is the contact angle. ϕ is the angle between the raceway and the ball, which is larger than 0 for bearings with axial loads. See Wang and Kootsookos (1998) for details. For an inner race (IR) fault, the fault frequency is then

$$f_{IR \ fault} = n_b \cdot (f_s - f_{ft}) \,. \tag{2}$$

Similarly, for outer race (OR) and rolling element (RE) failures, the fault frequencies are

$$f_{OR \ fault} = n_b \cdot f_{ft} \tag{3}$$

and

$$f_{RE\ fault} = \frac{f_s}{2} \left(\frac{D_p}{D_b}\right) \left[1 - \left(\frac{D_p}{D_b}\right)^{-2} \cos(\phi)\right], \quad (4)$$

respectively, where n_b is the number of balls.

Furthermore, the amplitude of the excitation force is modulated by a sine wave with characteristic periodicity depending on the transmission path and the loading conditions of the bearing. For instance, under stationary loading an OR fault will be without periodicity, while the amplitudes of the impulse train in an IR fault will have periodicity f_s due to the varying distance to the vibration sensor. See e.g. Wang and Kootsookos (1998) for a full overview of the periodic characteristics of the faults.

Knowing how the vibration signal is created, we can now take the reverse path to recover the original fault-induced impulse train x from the vibration measurements y by demodulating the signal. Assuming an estimate of g can be found experimentally, the demodulation is performed by solving

$$\bar{x} = G^{-1}y,\tag{5}$$

where G is the Toeplitz convolution matrix of g.

From the estimated excitation force \bar{x} , the original faultinduced impulse train can be recovered by removing the additive noise. A Wiener filter can be used for this purpose if the signal-to-noise ratio is known from experiments. Alternatively, the properties of the Wiener filter can be identified blindly by maximizing the spectral kurtosis (fourth moment) of the output of the filter (Antoni and Randall, 2006). In this work, we use a standard Wiener filter the signal-to-noise ratio assumed to be known.

3.2 Prognostics

A widely applied prognostic model for surface defects is Paris' crack propagation model (Paris and Erdogan, 1963), which states that the crack length a will develop according to

$$\frac{da}{dn_{cycles}} = D \cdot \left(\Delta K\right)^n,\tag{6}$$

where n_{cycles} is the number of cycles, D is a material constant, ΔK is the range of strain and n is an exponent. In the case of bearing faults, Paris' law can be reformulated as

$$\frac{da}{dt} = c_{Paris} \cdot \left(T^2 \cdot f_s\right) = c_{Paris} \cdot \left(\frac{P^2}{f_s}\right), \qquad (7)$$

by assuming that the motor torque can be used as a health indicator for gross strain (Bechhoefer et al., 2008). In the above equation, c_{Paris} is a lumped parameter, T is the motor torque and P is the motor power.

The true value of c_{Paris} is not known exactly, so c_{Paris} must be estimated from past measurements. A moving horizon estimator is used for this purpose. The "measurements" utilized in this case are the estimated crack lengths based on the past vibrational data.

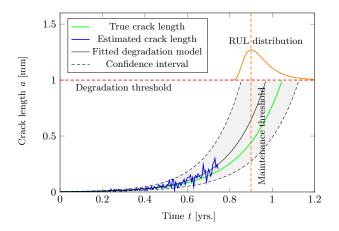


Fig. 4. Illustration of estimated degradation model based on "measurements" of the crack length (which themselves are estimated from vibrational data)

Confidence limits for the parameters are obtained from the covariance matrix of the parameter estimation (Lopez-Negrete and Biegler, 2012), which in turn can be used to predict the RUL distribution with Monte Carlo sampling. An illustration can be seen in Figure 4.

4. OPTIMIZING ECONOMIC PERFORMANCE SUBJECT TO HEALTH CONSTRAINTS

The estimated system health and the health degradation model can now be integrated in the decision making process by imposing constraints relating to the maximum allowable degradation. The optimal control problem (OCP) can then be solved with state-of-the art nonlinear programming (NLP) solvers such as IPOPT (Wächter and Biegler, 2006). Since information about the probability distribution of the parameter estimates is available, this should be embedded in the optimization problem to obtain a better solution. This gives rise to a stochastic NLP due to uncertainty in the parameter values, which can be written as

$$\min_{\mathbf{x}_k, \mathbf{u}_k} \sum_{k=1}^{N} \phi\left(\mathbf{x}_k, \mathbf{u}_k, \pi\right)$$
(8a)

s.t.
$$f(\mathbf{x}_k, \mathbf{u}_k, \pi) \le 0 \quad \forall k = 1...N$$
 (8b)

$$g(\mathbf{x}_k, \mathbf{u}_k, \pi) = 0 \quad \forall k = 1...N \tag{8c}$$

where **x** are the states, **u** are the inputs, π are the stochastic parameters, N is the horizon length, ϕ is the objective function, f and g are the inequality- and equality constraints, respectively. Since π is continuously distributed, finding the analytic solution of the resulting infinite-dimensional optimization problem maybe impossible. Listed below are three methods to deal with this.

S

- (1) The most naive approach for solving the stochastic problem is to substitute all uncertain parameters by their expected values. The solution obtained through this approach is likely to be sub-optimal, or even infeasible in the case where some constraints are active.
- (2) Another approach is to substitute the uncertain parameters by their worst-case realizations. The rationale is that if the solution holds for the worstcase scenario, it should hold for any scenario. This

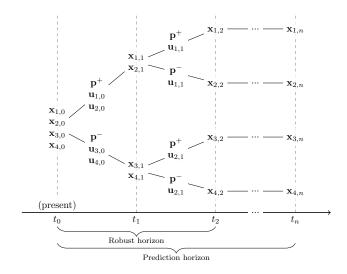


Fig. 5. Scenario tree with robust horizon N = n and prediction horizon S = 4.

approach is also known as the min-max approach in literature. While it works for many practical applications, the min-max approach usually results in a very conservative, possibly even infeasible, solution.

(3) A third approach is to employ a scenario-based method to explicitly deal with the parametric uncertainty. The idea stems from multistage stochastic programming, in which the uncertainty is discretized into a finite number of possible realizations, subject to which the optimization must be performed. Since the possibility of future recourse is taken explicitly into consideration, this approach is usually less conservative than the min-max approach. Due to the repeated measurement updates and input adjustments, stochastic optimal control problems are well suited to be solved with a scenario-based approach.

In this paper, we will use the scenario-based approach to solve the stochastic problem. The scenario-based deterministic equivalent of the stochastic OCP reads as

$$\min_{\mathbf{x}_{i,k},\mathbf{u}_{i,k}} \sum_{i=1}^{S} p_i \sum_{k=1}^{N} \phi\left(\mathbf{x}_{i,k}, \mathbf{u}_{i,k}\right)$$
(9a)

s.t.
$$f(\mathbf{x}_{i,k}, \mathbf{u}_{i,k}) \le 0 \quad \forall i = 1...S, \ k = 1...N \quad (9b)$$

$$g(\mathbf{x}_{i,k}, \mathbf{u}_{i,k}) = 0 \quad \forall i = 1...S, \ k = 1...N \quad (9c)$$

$$\sum_{i=1}^{\infty} \mathbf{A}_{i,k} \mathbf{u}_{i,k} = 0 \quad \forall k = 1...N$$
 (9d)

where S is the number of scenarios p_i is the probability associated with scenario *i* and **A** are the non-anticipativity constraints. Figure 5 shows a scenario tree corresponding to N = n and S = 4.

In order to reduce the number of scenarios, and thus the size of the OCP, it is common to define a robust-horizon $N_{robust} < N$ (Lucia et al., 2013a). The robust horizon is defined as the stage up until which branching occurs in the scenario tree. Since branching represents the availability of new information in the future, shortening the robust horizon means disregarding future state information. The justification for doing so is that additional branching at

later stages results in a much larger dimensionality of the NLP, with little improvement in the objective function.

Creating a scenario tree which captures the true nature of the uncertainty is a difficult task in and of itself, but is out of the scope of this paper. On one hand, the scenario tree should as detailed as possible to be a good approximation of the probability distribution. On the other hand, the scenario tree should be as small as possible due to the curse of dimensionality. We refer the interested reader to Dupačová et al. (2000). In the current work, we do as proposed by Lucia et al. (2013b), which is to generate the scenario tree by using combinations of the maximum, minimum, and the nominal uncertain parameters, as identified in the parameter estimation step.

5. RESULTS

The proposed framework for integrating diagnostics, prognostics and control was applied to the subsea compression system. The goal is to optimize production while making sure that the wet-gas compressor remains operational until the time scheduled for maintenance. In our case study, a maintenance stop is planned after 5 years after initial startup. An outer race bearing fault was simulated under stationary compressor loading. The fault was initiated at time t = 0, with an initial crack length of 0.01 mm. The degradation threshold is defined as the time when the crack length exceeds 1mm. The ball bearing consists of 10 rolling elements, and has a $\frac{D_p}{D_b}$ -ratio of 5.45. The compressor runs at a nominal speed of 60 Hz, with an operating window from 45-63 Hz. The operational objective is to maximize the net present value (NPV) of the gas production. Additionally, excessive control movement is penalized to avoid oscillatory or jumping solutions. The health-aware control problem reads

$$\min_{\mathbf{c}_{i,k},\mathbf{u}_{i,k}} \sum_{i=1}^{S=3} p_i \sum_{k=1}^{N=20} \left(-\frac{\dot{m}_{gas_{i,k}}}{(1+r)^{t_k}} + w\Delta \mathbf{u}_{i,k}^2 \right)$$
(10a)

where the discount factor r = 0.015, and the control movement penalty $w \ge 0$. We chose w = 100, resulting in approximately twice as much weight on the gas production term as on the control penalty term. The constraints are

$$f(\mathbf{x}_{i,k}, \mathbf{u}_{i,k}) \le 0 \quad \forall i = 1...3, \ k = 1...20$$
(10b)

$$g(\mathbf{x}_{i,k}, \mathbf{u}_{i,k}) = 0 \quad \forall i = 1...3, \, k = 1...20$$
(10c)

$$\mathbf{u}_{i=1,k=1} = \mathbf{u}_{i=2,k=1} = \mathbf{u}_{i=3,k=1}$$
 (10d)

The constraints f contain upper and lower bounds on the inputs \mathbf{u} (choke opening $0 < \mathbf{u}_{choke} < 1$ and compressor speed $45 < \mathbf{u}_{compressor} < 63$ Hz), as well as constraints relating to the allowable operating region, i.e. to prevent compressor surge and compressor choke. A minimum discharge pressure of $P_{discharge} = 150$ bar is imposed after the compressor to ensure flow through the long pipeline to the topside, as well.

The uncertainty in the parameter c_{Paris} in the crack propagation model, Equation 7, is included in the problem formulation through the three scenarios. Each scenario represents a discrete realization of c_{Paris} , namely the 5% percentile, the 95% percentile and the expected value. A robust horizon of length $N_{robust} = 1$ is used, making the problem effectively a two-stage problem from a stochastic programming perspective.

The OCP described in Equation 10 is solved repeatedly in a shrinking horizon fashion. That is, the OCP is solved with the initial values for the states being the latest state estimates from the plant. After a solution is obtained, only the inputs corresponding to the current time step are implemented. This is also illustrated in the flow diagram in Section 1.1.

Between each optimization step, the vibration measurements are added by generating an impulse train for the excitation force and adding white noise. The force is modulated through the impulse response model of the compressor. We assume that g from Section 3.1 can be written as a damped sinusoid

$$g = \exp(-\lambda t) \cdot \cos(\omega t) \tag{11}$$

with decay factor $\lambda = 600 \text{ s}^{-1}$ and frequency $\omega = 4$ kHz, resulting in the impulse response model shown in the middle graph in Fig. 3. The magnitude of the original impulse is found by the method described in Section 3.1. Unfortunately, no real vibration data was available, so we used the same model for both generating the measurements and estimating the states. However, due to the added noise, the estimates were not perfect and the method can still be used to showcase the approach.

The closed-loop solution of the health-aware controller with a fixed maintenance horizon of 5 years is shown in Figure 6. The figure shows the evolution of the past inputs and the states and the optimal trajectories for the three scenarios at three different points in time, at t = 0, t = 2and t = 2.8 years.

6. DISCUSSION

The success of our proposed approach hinges on the quality of the degradation model and condition monitoring capabilities. Our approach relies on the equipment vendors to provide models and data for performing the diagnostics and prognostics. Since the objective of this paper is to demonstrate our framework, we have chosen to use a relative simple degradation and diagnostics model, that we adapted to our purposes.

From Figure 6, it can be seen that the predicted trajectories differ from the real trajectories. This is due to the NPV-term in the cost function and the presence of uncertainty. The closed-loop solution has two different operating regions, the first from t = 0 to t = 2.5 years where the compressor runs with maximum speed, and the second from t = 2.5 to t = 5 years where production has to be choked back in order to meet the required bearing health constraint. The abrupt change between operating regions occurs because of the NPV term. Since future production is valued less than present production, the optimizer will attempt to keep production at maximum as long as possible. In the second operating region, gas production is lower in order to meet the outlet pressure constraint and the minimum health requirement.

Overall, the results are as expected. It seems reasonable to require that the subsea installation is as profitable as possible while operating, i.e. that production is at

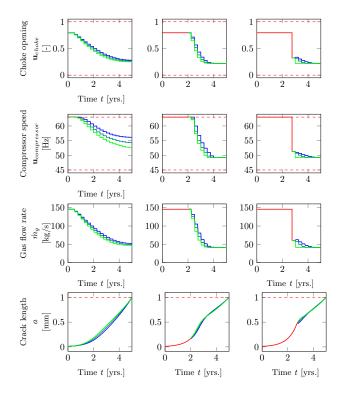


Fig. 6. Three snapshots of the closed-loop solution at t = 0, t = 2 and t = 2.8 yrs. The blue, turquoise and green scenarios are for the low, expected and high realizations of the stochastic parameter c_{Paris} , respectively. The maintenance horizon is fixed at 5 years.

maximum. However, the fact that the production has to be throttled down after a while indicates that the specified maintenance horizon of 5 years may have been too long. Preferably, maintenance intervention should have been scheduled earlier, so that maximum throughput could have been achieved the entire time. In future work, we will consider the possibility of adapting the maintenance time during operation to make sure that maximum throughput can be achieved. In this case maintenance is scheduled a year in advance, and will be decided by the optimizer. Nevertheless, the control structure successfully meets the constraints while maximizing the production.

7. CONCLUSION AND FUTURE WORK

We presented a framework to combine diagnostics, prognostics and control of a subsea gas compression plant subject to compressor bearing failure. By including measurements of fault indicators and fault prognostic models in the MPC framework, we can ensure that the operation is both economically optimal and safe. In the case of bearings, vibration measurements can be used to detectand estimate the severity of faults. Paris' law for crack propagation can be used to predict fault development.

In future work, we will consider multiple failure mechanisms, not only those which can only be influenced through input manipulation. For example, the production strategy will look different when a seal fault has been detected and failure is eminent. Careful operation in order to "save" the bearings will be suboptimal, since intervention is required in the near future to replace the faulty seal. In similar vein, we will look at the entire subsea plant as a whole, to ensure that operation is optimal not only for a single unit (e.g. the compressor), but for all units in the plant.

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