

# Optimal Hedging Strategies for Salmon Producers

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## Abstract

We study the optimal hedging decisions for a risk-averse salmon producer. The hedging decisions are determined using a multistage stochastic programming model. The objective is to maximize the weighted sum of expected revenues from selling salmon either in the spot market or in futures contracts and Conditional Value-at-Risk (CVaR) of the revenues over the planning horizon. The scenario tree for the multistage stochastic programming model is generated based on a procedure that combines Principal Component Analysis and state space modelling. We present results for 3 different CVaR percentiles and different degrees of risk-aversion. The results indicate that salmon producers should use futures contracts to hedge price risk already at fairly low degrees of risk-aversion. The methods described in this paper will be useful as a decision support tool for determining fish companies' risk management and hedging strategies.

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## 1. Introduction

Global production of farmed Atlantic salmon has increased considerably over the past decades, from approximately 5300 tonnes in 1980 to almost 2.4 million tonnes in 2015. Throughout this period Norway has been the world's largest producer, with a production share of 54.7% in 2015 (FAO, 2017). The increase in production has been mainly driven by innovations leading to productivity growth and lower production costs, such as feeding technology or disease management. These innovations combined with a process of industrialization have made the salmon farming industry more similar to other food-producing industries (Kumar and Engle, 2016; Kvaløy and Tveterås, 2008; Asche, 2008).

Salmon producers face many types of risk. Among Norwegian producers, future salmon prices, diseases, as well as various regulatory issues have been identified as the most important sources of risk (Bergfjord, 2009). The regulatory framework is defined through a political process that cannot be controlled by an individual producer. Diseases are part of the production risk that is to

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a large degree due to the biological nature of the production process. Abolofia et al. (2017) for example show the considerable economic impact of sea lice infestation on the Norwegian salmon farming industry. Different approaches to studying and reducing production risk in salmon farming have been developed: Asche and Tveterås (1999) model production risk using a two-step procedure and show that this approach is valid for Norwegian salmon farming. Kumbhakar and Tveterås (2003) use panel data from Norwegian salmon farms to derive the risk preference function in the presence of production risk. They find that capital and labour have a risk-reducing effect, while feed and fish input increase risk. They also find that all salmon farmers in the study are risk-averse. Kvaløy and Tveterås (2008) show that vertical integration can reduce the production risk with the Norwegian salmon farming industry. Spatial diversification can also be used to mitigate production risk (Oglend and Tveterås, 2009).

Price risk can also be mitigated to a certain extent: Financial futures contracts (as well as other instruments such as swaps and options) have a long history of use as a means by which producers of a commodity – e. g. farmed salmon – can insure, or hedge, against unfavourable variations in the sale prices of that commodity (see Bernstein, 1996, for an overview). The benefit of forward selling for a commodity producing company is that it gets a predetermined price for its production and avoids the risk of making a loss should the commodity price fall. This way, the company avoids the expected costs associated with financial distress (Tufano, 1996; Stulz, 1984). This comes at a “cost” of losing the opportunity to sell their fish at potential higher prices in the future. The trade-off between reward (e. g. measured as expected revenue) and risk (measured by the risk of losing money in some form) is a key element in all decisions under uncertainty. Within the seafood market, Martínez-Garmendia and Anderson (1999) study the hedging performance of shrimp futures at the Minneapolis Grain Exchange. Numerous other applications of futures contracts are documented for a wide range of commodity producing companies, such as gold miners (Betts and Mamik, 2011), oil and energy producers (Wallace and Fleten, 2003; Jin and Jorion, 2006), agricultural companies (Tomek and Peterson, 2001), and power companies (Fleten et al., 2002; Dupuis et al., 2016).

Prior to the establishment of Fish Pool as a regulated marketplace for buying and selling financial salmon contracts in 2005, most of the salmon production was sold in the spot market, resulting in highly volatile prices for producers (see Guttormsen, 1999, for a more detailed discussion). Since then, there has been a considerable interest from researchers in the characteristics of the salmon futures market and the properties of the salmon futures prices. Some recent examples are presented below. Larsen and Asche (2011) study the use of contracts in Norwegian salmon industry. Asche et al. (2016) study whether the Fish Pool futures prices can serve as an unbiased estimator of the future spot price and therefore provide a price discovery function. Based on data from 2006 to 2014, they do not find support for their hypothesis that forward prices provide a price discovery function and conclude that the salmon futures market has not yet matured enough to predict future spot prices. Fischer and Lai (2016) assess the efficiency of the futures market for salmon and come to a somewhat conflicting

result based on data from June 2006 to June 2016, concluding that the salmon futures market does indeed provide a price discovery function and can therefore be used for risk management purposes. Misund and Asche (2016) examine the hedging efficiency of futures contracts on Atlantic salmon. They find that hedging by means of futures contracts is able to reduce the risk by 30-40%. While this level is higher than for other seafood markets, it is lower than for agricultural commodities. According to the authors, this indicates that the salmon futures market is still less mature than other well-established commodity markets. Ewald and Ouyang (2017) use an extended two-factor model to analyse the Fish Pool futures market and confirm seasonality in salmon spot and futures prices.

Rather than studying properties of spot and futures prices for salmon or characteristics of the futures market, this paper examines how the information provided by Fish Pool can be used by a salmon producer to reduce the exposure to price risk. For studies regarding the price volatility in aquaculture markets, the interested reader is referred to Dahl (2017) or Dahl and Oglend (2014). Focus of this paper is the question how the producer's risk preferences influence the decision on whether to sell the salmon using a futures contract in order to secure the price or to sell the fish in the spot market. We formulate a multistage stochastic programming problem to model the producer's sales decisions with time consistent risk constraints (see e. g. Shapiro, 2009). The objective is to maximize the weighted sum of expected revenue on one hand and the conditional value-at-risk (CVaR) of the future revenue on the other hand. We use the weights associated with expected revenue and the CVaR as a measure of the producer's risk preference and varying the weight allows us to examine how the producer's decisions change as the degree of risk aversion increases. This paper is the first to use multistage stochastic programming with a time-consistent risk measure in the objective function to study how the hedging decisions of a commodity producer depend on the producer's risk preferences. The scenario tree needed for the optimization model is generated using a novel method based on the combination of Principal Component Analysis and state space modelling.

The remainder of this paper is structured as follows: in Section 2 we provide a verbal description of the salmon producer's problem of deciding upon a hedging strategy. The optimization model is given in Section 3. The scenario generation method is presented in Section 4. The results from our computational study based on real-world data from Fish Pool are presented and discussed in Section 5. We conclude with Section 6.

## 2. Problem Description

Norwegian salmon producers perceive themselves to be moderately risk-averse and consider price risk to be one of the most important sources of risk (Bergfjord, 2009). The price risk can be mitigated by selling salmon through a futures contract, guaranteeing a deterministic revenue at the cost of foregoing the chance of selling the fish later at a higher price. The producer therefore faces the decision of selecting a set of contracts to enter into with different maturities

and prices that will sell all the fish and – depending on the producer’s risk preference – maximize expected revenue, minimize price risk or seek to optimize a combination of these two objectives. Note that we consider selling in the spot market as entering into a futures contract with maturity 0.

We use Conditional Value-at-Risk (CVaR) as risk metric for the producer. CVaR is closely related to Value-at-Risk (VaR). In a financial context, VaR for a given portfolio and time period is defined as the loss that will not be exceeded with a given probability  $\alpha$ . Typical values for  $\alpha$  are 0.9, 0.95 and 0.99. For the same portfolio, time period and probability  $\alpha$ , CVaR represents the expectation of the losses under the condition that they will exceed VaR (see e. g. Zenios, 2008, for more detailed information). In terms of revenues of the salmon producer, CVaR represents the expected value of the  $(1 - \alpha) \cdot 100\%$  worst revenues over all scenarios. A risk-averse salmon producer will therefore try to maximize CVaR. We use a weight,  $\lambda \in [0, 1]$ , to describe the producer’s risk preference, with a risk-neutral producer being defined by  $\lambda = 0$ .

We assume that salmon producers are mainly concerned with hedging price risk connected with their own production, rather than speculating in the salmon market. This implies here that a futures contract will not be closed out before it expires. We also assume that short sales of salmon are not possible.

Production risk is not taken into account here, instead we use deterministic forecasts for the production volumes that need to be sold in the different time periods. This approach does not necessarily reflect biomass development in the real world, however by using a sensible lower bound on the production volumes our model might still provide valuable insight on the properties of the optimal hedging strategy. Any volumes above the chosen lower bound can simply be assumed sold in the spot market.

### 3. Optimization Model

We provide a multistage stochastic programming formulation for the hedging problem. The objective to maximize the weighted sum of expected revenues and CVaR with respect to revenues over the planning horizon. The main focus in terms of decisions is, given the producer’s risk preference, on how much of the production should be sold in the spot market and at which point in time a futures contract should be chosen.

Time consistency is a requirement for ensuring optimal decisions in risk-averse multistage stochastic programming problems (see Rudloff et al., 2014, for more details). According to Shapiro (2009), time consistency is characterized by the requirement that “optimal decisions should not depend on scenarios we already know cannot happen in the future.” Following Rudloff et al. (2014) and Pisciella et al. (2016), we therefore use a nested CVaR-implementation to ensure time consistent optimal decisions.

Let us first introduce the following notation for our problem formulation:

- Sets

$\mathcal{M}$	Set of maturities for contracts. Spot sales have a maturity of 0.
$\mathcal{N}$	Set of event nodes in the scenario tree.
$\mathcal{N}_i$	Set of event nodes at stage $i$ in the scenario tree.
$\mathcal{C}(n)$	Set of children nodes (successors) of node $n$ , $n \in \mathcal{N}$ .
$\mathcal{S}$	Set of scenarios.
$\mathcal{S}(n)$	Set of scenarios passing through event node $n$ of the scenario tree, $\mathcal{S}(n) \subseteq \mathcal{S}$ .
$\mathcal{T}$	Set of time periods.
$\mathcal{T}_i$	Set of time periods belonging to stage $i$ , $\mathcal{T}_i \subseteq \mathcal{T}$ .

- Indices and superscripts

$i$	Stage of the scenario tree, $i = 1 \dots I$ .
$m$	Maturity (in months) of the contract, $m \in \mathcal{M}$ .
$n$	Event node index for the scenario tree, $n \in \mathcal{N}$ .
$s$	Scenario superscript, $s \in \mathcal{S}$ .
$t$	Time period index, $t \in \mathcal{T}$ .

- Parameters, constants, and coefficients

$P_{t,m}^s$	Price per kg of salmon at time $t$ in a contract with maturity $m$ in scenario $s$ . Note that the salmon is actually delivered at time $t + m$ .
$Q_t$	Production quantity of salmon to be delivered at time period $t$ .
$U_{t,m}$	Upper bound on the share of production at time $t$ that can be sold in a futures contract with maturity $m$ , $U_{t,m} \in [0, 1]$ .
$X_t$	Share of production quantity in period $t$ sold through futures contracts before the start of the planning period.
$p^n$	Conditional probability of reaching node $n$ from its predecessor.
$\alpha$	Confidence (percentile) level for VaR and CVaR.
$\lambda$	Weight for the salmon producer's risk preference, $\lambda \in [0, 1]$ .

- Decision variables

$k_{i,n}$	Objective function value at stage $i$ and node $n$ of the scenario tree.
$x_{t,m}^s$	Share of production quantity at time $t + m$ sold in a contract with maturity $m$ in scenario $s$ . The contract is entered into at time $t$ .
$y_{i,n}$	Shortfall of revenues with respect to conditional value-at-risk at stage $i$ and node $n$ of the scenario tree.
$z_{i,n}$	Auxiliary variable for modelling CVaR.

We now provide the mathematical formulation of the optimal hedging strategy problem for a salmon producer:

$$\begin{aligned} \max \frac{1}{|\mathcal{S}|} \cdot \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}_1} \sum_{\{m \in \mathcal{M} | m+t \leq |\mathcal{T}|\}} x_{t,m}^s \cdot Q_{t+m} \cdot P_{t,m}^s + \\ (1 - \lambda) \cdot \left( \sum_{n \in \mathcal{N}_2} p^n \cdot k_{2,n} \right) + \lambda \cdot \left( z_{2,1} - \frac{1}{1 - \alpha} \sum_{n \in \mathcal{N}_2} p^n \cdot y_{2,n} \right) \end{aligned} \quad (1)$$

subject to

$$z_{i+1,n} - k_{i+1,n'} \leq y_{i+1,n'} \quad i = 1 \dots I - 1, n \in \mathcal{N}_i, n' \in \mathcal{C}(n), \quad (2)$$

$$\begin{aligned} \frac{1}{|\mathcal{S}(n)|} \cdot \sum_{s \in \mathcal{S}(n)} \sum_{t \in \mathcal{T}_i} \sum_{\{m \in \mathcal{M} | m+t \leq |\mathcal{T}|\}} x_{t,m}^s \cdot Q_{t+m} \cdot P_{t,m}^s + \\ (1 - \lambda) \cdot \left( \sum_{n' \in \mathcal{C}(n)} p^{n'} \cdot k_{i+1,n'} \right) + \\ \lambda \cdot \left( z_{i+1,n} - \frac{1}{1 - \alpha} \sum_{n' \in \mathcal{C}(n)} p^{n'} \cdot y_{i+1,n'} \right) = k_{i,n} \end{aligned} \quad (3)$$

$$i = 2 \dots I - 1, n \in \mathcal{N}_i, \quad (3)$$

$$\sum_{s \in \mathcal{S}(n)} \sum_{t \in \mathcal{T}_I} \sum_{\{m \in \mathcal{M} | m+t \leq |\mathcal{T}|\}} x_{t,m}^s \cdot Q_{t+m} \cdot P_{t,m}^s = k_{I,n} \quad n \in \mathcal{N}_I, \quad (4)$$

$$\sum_{\{m \in \mathcal{M} | m+t \leq |\mathcal{T}|\}} x_{t,m}^s + X_{t+m} = 1 \quad t \in \mathcal{T}, s \in \mathcal{S}, \quad (5)$$

$$x_{t,m}^s \leq U_{t,m} \quad t \in \mathcal{T}, m \in \mathcal{M}, s \in \mathcal{S}, \quad (6)$$

$$\frac{1}{|\mathcal{S}(n)|} \sum_{s' \in \mathcal{S}(n)} x_{t,m}^{s'} = x_{t,m}^s \quad t \in \mathcal{T}, m \in \mathcal{M}, n \in \mathcal{N}, s \in \mathcal{S}(n), \quad (7)$$

$$x_{t,m}^s \geq 0 \quad t \in \mathcal{T}, m \in \mathcal{M}, s \in \mathcal{S}, \quad (8)$$

$$y_{i+1,n} \geq 0 \quad i = 1 \dots I - 1, n \in \mathcal{N}_{i+1}, \quad (9)$$

$$k_{i+1,n} \geq 0 \quad i = 1 \dots I - 1, n \in \mathcal{N}_{i+1}, \quad (10)$$

$$z_{i+1,n} \geq 0 \quad i = 1 \dots I - 1, n \in \mathcal{N}_i. \quad (11)$$

The first term of the objective function (1) is the deterministic revenue from sales made in the sport market or through futures contracts entered into in the periods of the first stage. The second and third term are part of the nested CVaR implementation (see e. g. Pisciella et al., 2016, for more details): the second term in (1) represents the expected objective function value whereas the third term is the CVaR with respect to the objective function value from the second stage. Constraints (2) are the necessary nested CVaR constraints. See Uryasev (2000), Rockafellar and Uryasev (2000) and Kaut et al. (2007) for details on the linear programming formulation of CVaR. The recursive formulation for calculating the objective function value at a given node in the scenario tree is given by equations (3). Equations (4) calculate the income in the last

stage for each scenario. Constraints (5) ensure that the entire production for a given time period is sold either in the spot market or through futures contracts. Restrictions (6) limit the share of the production volume at time  $t$  that can be sold using a contract with maturity  $m$ . Equations (7) are the non-anticipativity constraints enforcing the relationship between stages, periods and scenarios. In short, they force decisions that are based on the same information to be equal across the scenario tree. Finally, we have non-negativity constraints (8)-(11) for the decision variables.

#### 4. Price Forecasts and Scenario Generation

In order to analyse the optimal hedging policies for salmon producers, we need to generate scenarios for spot and futures contract prices. A general description of the methodology for forecasting prices and generating the scenario tree required by the multistage stochastic programming model is presented in this section.

##### 4.1. Preprocessing of Data

Starting point for the scenario generation procedure are the historic spot and forward prices provided by Fish Pool (fishpool.eu) for the period June 2006 to December 2016. The Fish Pool Index, FPI<sup>TM</sup>, serves as the basis for the spot price. The index is a reference price intended to facilitate the settling of futures contracts. It is a weighted average of three different index elements and calculated for each week (see <http://fishpool.eu/price-information/spot-prices/fishpool-index/> for more information). Please note that Fish Pool's terminology is imprecise as their forward contracts actually are futures contracts.

The futures price curve is derived on a daily based on closing prices. The curve provides today's prices for salmon deliveries in the months ahead. In our analysis, we include futures prices with maturities from 1 month ahead to 12 months ahead. Note that the futures contracts only specify the delivery month, rather than an exact date. As a consequence of the different time resolutions, spot and futures prices cannot be compared directly. We therefore aggregate all prices into monthly averages such that spot prices, futures prices as well as maturities of futures contracts are available on a monthly basis.

##### 4.2. Principal Component Analysis and State Space Modelling

The 13 different time series for both spot and futures prices are first converted into log-prices. In an attempt to reduce the dimensionality of the multivariate time series, we perform a Principal Component Analysis on the covariance matrix of the log-prices (see e.g. Tsay, 2010, for an introduction to Principal Component Analysis). The analysis of the eigenvalues reveals that the first principal component explains more than 90% of the variance and the remaining components are therefore discarded in the subsequent analysis. This reduces the 13-variate time series to a univariate time series. Figure 1 illustrates the loss of information when representing the spot and futures price time

series by a single principal component: the overall pattern in price development is well captured, but the intra-period variations between the different time series are lost.

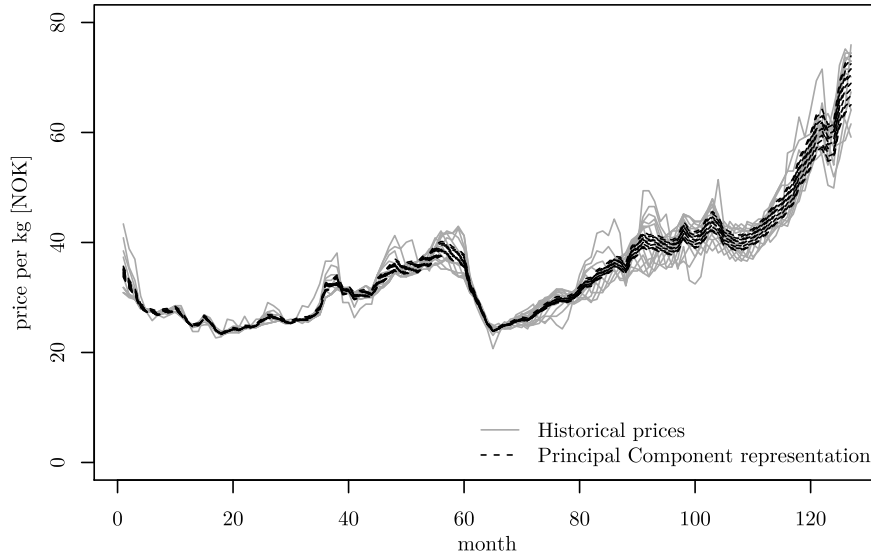


Figure 1: Historical spot and futures prices and their representation based on the first principal component

We use a state space model to describe the univariate time series for the first principal component. State space models are very flexible and can incorporate non-stationarities in time series, such as trends and cycles, directly. State space models have been used for generating price forecasts for quite some time. For example, Vukina and Anderson (1993) use state space forecasting to study intertemporal cross-commodity hedging between fishmeal and soybean meal. The same authors also use a state space model to create short term price forecasts for the Japanese salmon wholesale market (Vukina and Anderson, 1994). Gu and Anderson (1995) apply a multivariate state space model on deseasonalized data to produce short term forecasts for the US salmon market. Kåresen and Husby (2000) use a state space model to create joint forecasts for both spot and futures prices in the Nordic electricity market. Myklebust et al. (2010) implement a multivariate state space model to forecast the prices of different natural gas components. For an introduction to time series analysis using state space models, see e.g. Harvey (1990); Commandeur and Koopman (2007) or Petris et al. (2009).

Our state space model for the first principal component is given by the set of equations (12)-(18). The formulation includes a trend, seasonality as well as first order autoregressive effects:

$$y_t = \mu_t + \gamma_{1,t} + \psi_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (12)$$



$$\mu_t = \mu_{t-1} + \beta_t + \eta_t \quad \eta_t \sim N(0, \sigma_\eta^2) \quad (13)$$

$$\beta_t = \beta_{t-1} + \nu_t \quad \nu_t \sim N(0, \sigma_\nu^2) \quad (14)$$

$$\psi_t = \Phi\psi_{t-1} + \kappa_t \quad \kappa_t \sim N(0, \sigma_\kappa^2) \quad (15)$$

$$\gamma_{1,t+1} = -\gamma_{1,t} - \dots - \gamma_{11,t} + \omega_t \quad \omega_t \sim N(0, \sigma_\omega^2) \quad (16)$$

$$\gamma_{2,t+1} = \gamma_{1,t} \quad (17)$$

⋮

$$\gamma_{11,t+1} = \gamma_{10,t} \quad (18)$$

Equation (12) is the observation equation. The value at time  $t$  of the first principal component  $y_t$  depends on the realization of the unobservable state at the same time. The components explaining  $y_t$  are a level  $\mu_t$ , monthly growth  $\beta_t$ , autoregressive effects  $\psi_t$  and a monthly seasonal factor,  $\gamma_{1,t}$ . Equations (13) to (18) constitute the state equation. Equations (13) and (14) represent a local level and a local trend, respectively. The autoregressive effects are introduced through equation (15). The seasonal component is modelled by means of equations (16) to (18). Modelling seasonality requires  $s - 1$  state equations, where  $s$  is the periodicity of the seasonality (see Commandeur and Koopman, 2007, for more details on modelling seasonality). For the monthly seasonal component in the model above,  $s = 12$ .

We assume that the disturbances  $\varepsilon_t$ ,  $\eta_t$ ,  $\nu_t$ , and  $\kappa_t$  follow a normal distribution and are mutually uncorrelated. The model is implemented and solved using the `d1m`-package for R, (see e.g. Petris, 2010; Petris and Petrone, 2011).

#### 4.3. Generating the Scenario Tree

The scenario tree used in the computational study has 4 stages and covers a planning horizon of 6 months. The structure of the tree is illustrated in Figure 2. The total number of scenarios in the tree will depend on the number of scenarios that are generated in each stage (indicated by the dotted lines in Figure 2).

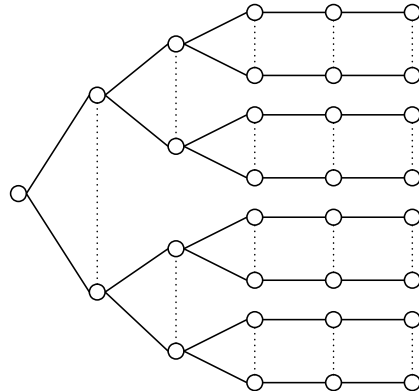


Figure 2: Structure of the multistage scenario tree used in the optimization model

The scenario tree for the multistage stochastic programming problem (1)-(11) is generated in R, using the functionality provided by the `d1m`-package. The function `d1mForecast` evaluates means and variances of the distributions of both observations and states and can be used to predict future states and observations. It can also draw samples from these distributions for future states and observations (Petris et al., 2009).

The scenario generation procedure is described by the following steps:

1. The last observation of the time series is used as first month (and first stage) of the scenario tree.
2. We use `d1mForecast` to generate a sample of a predefined size for the second month.
3. For each of the scenarios generated in the previous step, the time series for the first principal component is extended with the sampled observation. We then use `d1mFilter` to apply the Kalman filter in order to update the filtered values of the state variables. After that we generate another sample of a predefined size for the third month of the scenario tree.
4. Step 3 is repeated for generating the scenarios for the first period (and fourth month of the planning horizon) of the fourth and last stage of the tree.
5. The time series for each scenario are extended and updated with the new scenarios. For this last stage of the tree, we generate scenarios as a deterministic 3-month forecast.
6. The generated scenario tree for the first principal component is then transformed back into a scenario tree for spot and futures prices.

In this paper, we use a sample size of 50 scenarios in each stage of the tree. The resulting scenario tree therefore has 125,000 scenarios. Figure 3 shows the expected future spot and futures prices in the generated scenario tree as well as the minimum and maximum prices across all time series and scenarios.

## 5. Computational Results

The optimization model is implemented in Mosel and solved using FICO Xpress. All calculations have been carried out on a laptop, running Windows 10, with an Intel® Core™ i7-6500U processor and 8GB RAM.

### 5.1. Problem Instances

Examining the historical prices from Fish Pool during the period June 2006 to December 2016 (see e. g. Figure 3), two characteristic periods stand out: There is a short, but sharp drop in prices starting in Spring 2011 and lasting for about 9 months. Since then, the prices have exhibited an increasing trend, starting early 2012 and lasting all through 2016. We therefore choose two different points in time to study the salmon producer’s optimal hedging strategy: the first one is towards the end of the drop in salmon prices, starting with the

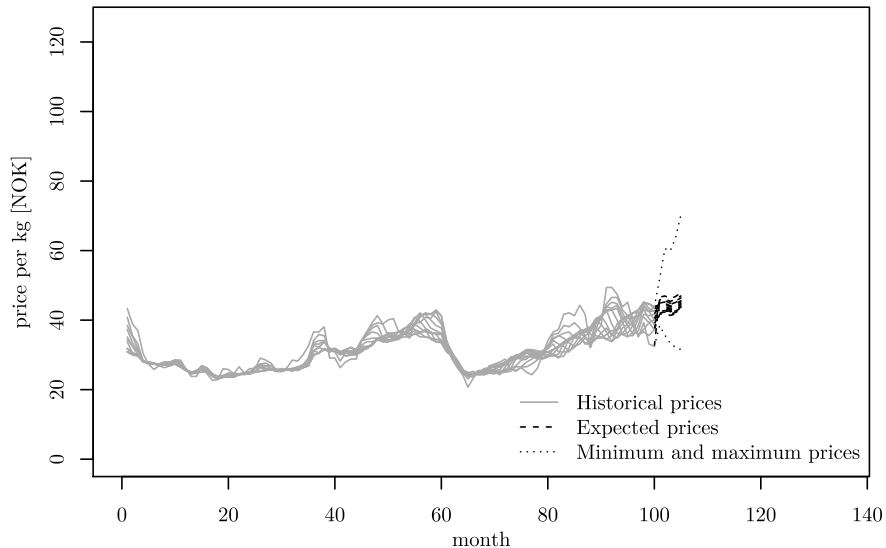


Figure 3: Historic and generated expected prices in the scenario tree

first stage in September 2011. The second problem instance starts after experiencing a long period of increasing prices, with January 2016 as the start of the planning horizon. Figures 4 and 5 illustrate the scenario trees for the problem instances starting in September 2011 and January 2016, respectively. An interesting difference between these two trees is that the expected prices in 2011 are moderately decreasing, while they are slightly increasing in 2016. Also, there seems to be a larger downside potential for the prices in the scenario tree for 2011.

We use the weekly export statistics for fresh salmon from Statistics Norway as starting point for determining the monthly production quantities (Statistics Norway, 2017). The weekly export volumes are aggregated into monthly production volumes and 10% of this aggregated volume is used as production quantity in the optimization model. The analysis thus reflects a salmon producer with a Norwegian market share (in production) of approximately 9-10%. The period for the production volumes matches the planning horizon as given by the two different scenario trees.

The weight  $\lambda$  specifying the producer's risk preference is increased in steps of 0.01 in the interval  $[0, 1]$ . The problem is solved for three different CVaR percentiles, i. e. values of  $\alpha$ : 0.9, 0.95 and 0.99. Sales in futures contracts do not have an upper bound, i. e.  $U_{t,m}$  is set to 1.

The resulting optimization problem with 125,000 scenarios has about 1,010,000 variables and 628,000 constraints.

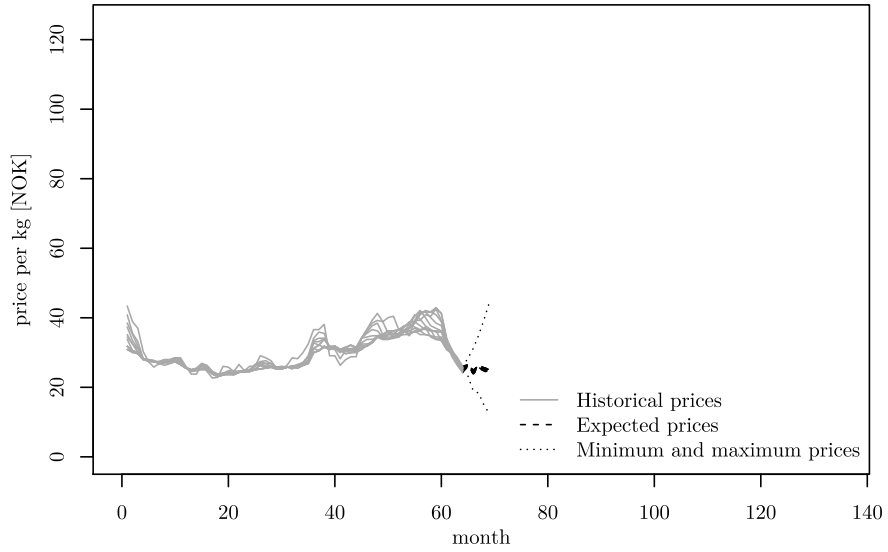


Figure 4: Historic and generated prices in the scenario tree starting in September 2011

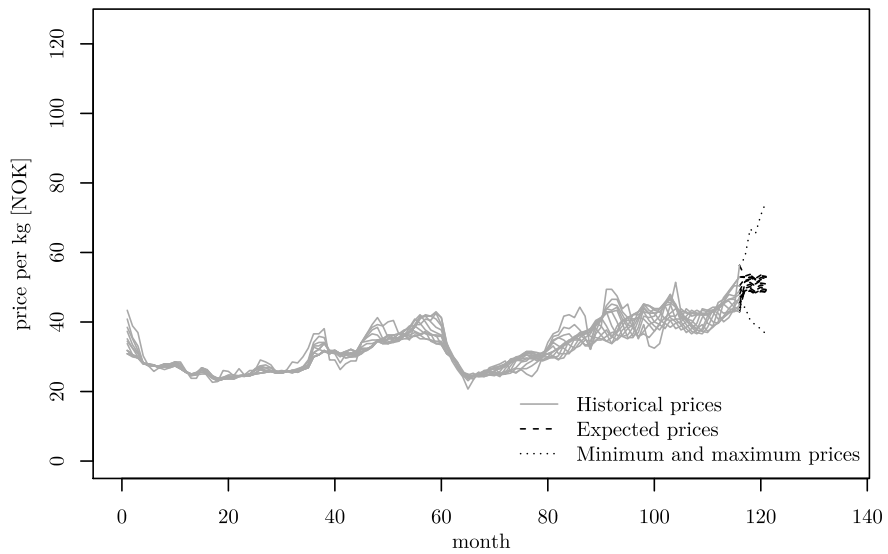


Figure 5: Historic and generated prices in the scenario tree starting in January 2016

## 5.2. Results and Discussion

The solution to the first problem instance – the scenario tree starting in September 2011 – is trivial. Irrespective of the salmon producer’s risk preference or the chosen CVaR, the model recommends to sell the entire production during the planning horizon in the first period, using the available futures contracts. This generates a deterministic profit that is equal across all scenarios, thus eliminating all risk. The reason for this result lies in the scenario tree: The scenario generation and price forecasting procedure has picked up on the steep drop in prices, almost 40% in the three months prior to the start of the planning horizon, and generates scenarios where the deterministic futures price in the first stage exceeds the expected spot prices in the corresponding latter periods. It is therefore optimal to always sell the planning horizon’s entire production in the first period.

The second problem instance starting in January 2016 provides different results. When the salmon producer is risk-averse, i. e.  $\lambda$  close to 1, the model suggests to sell all fish in the first period, generating a deterministic, risk-free profit. When the producer is risk-neutral, i. e.  $\lambda = 0$ , the production volume is to a large degree sold in the spot market. This applies to all three different CVaR percentiles.

We see from the results that the salmon producer starts reducing the amount of salmon sold in the spot market as the degree of risk-aversion increases. While there might be price scenarios in which the salmon producer does not reduce the amount of spot trades, the average volumes are clearly decreasing. Consider for example the sales of salmon produced in period 4 given the 90% CVaR requirement as shown in Figure 6: The risk-neutral producer sells on average about 40% of the salmon in the spot market. The remaining 60% are sold using a futures contract with maturity 1 in period 3. Note that none of the salmon is sold in either period 1 or period 2. As the degree of risk-aversion increases, the salmon producer chooses to increase the volume of salmon sold through contracts in period 3 while the volume sold on the spot market decreases. It is first from values  $\lambda > 0.05$  that contracts with maturities longer than 1 month are chosen. Eventually contracts with longer maturities start to replace the 1-month contracts until the entire production is sold in the first period.

The results show a gradual shift towards futures contracts with longer maturities. While only the results for period 4 are shown here, this gradual shift is observed for the other periods of the planning horizon as well. The same behaviour is also observed for the other two CVaR criteria, see Figures 7 and 8.

When comparing the graphs for the three different CVaR percentiles (see Figures 6 to 8), it is obvious that the chosen CVaR criterion also influences the hedging decisions. In general, the higher the CVaR percentile, the less risk-averse the salmon producer needs to be before moving sales volume from the spot market to the futures market. While the risk preference parameter of the salmon producer,  $\lambda$ , has to be larger than 0.4 in the example above, to sell the entire production in period 4 through contracts in the first period, we observe the same behaviour for values less than 0.4 in case of both the 95% CVaR and the 99% CVaR criterion.

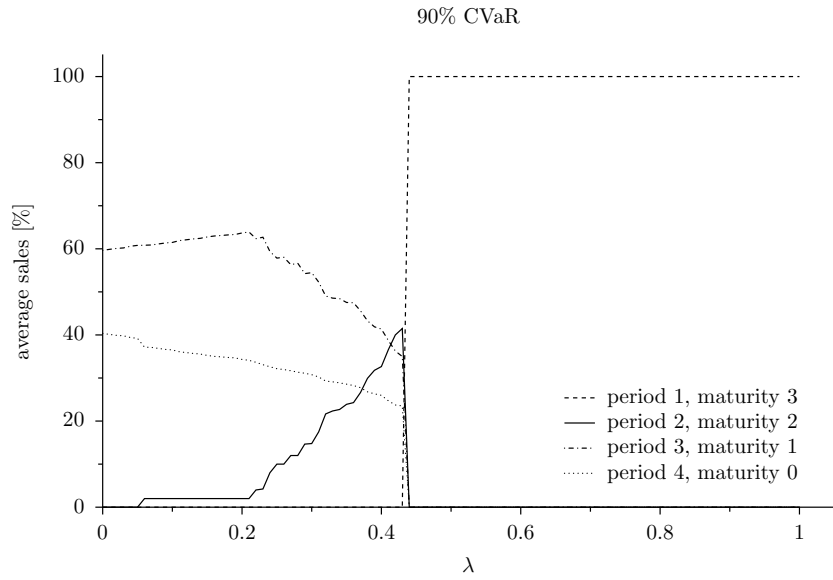


Figure 6: Average sales for period 4 using different futures contracts for 90% CVaR

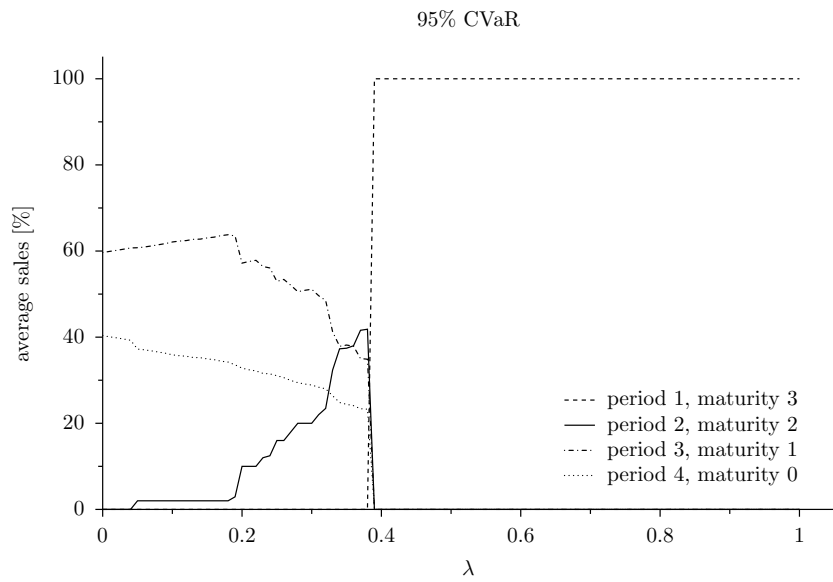


Figure 7: Average sales for period 4 using different futures contracts for 95% CVaR

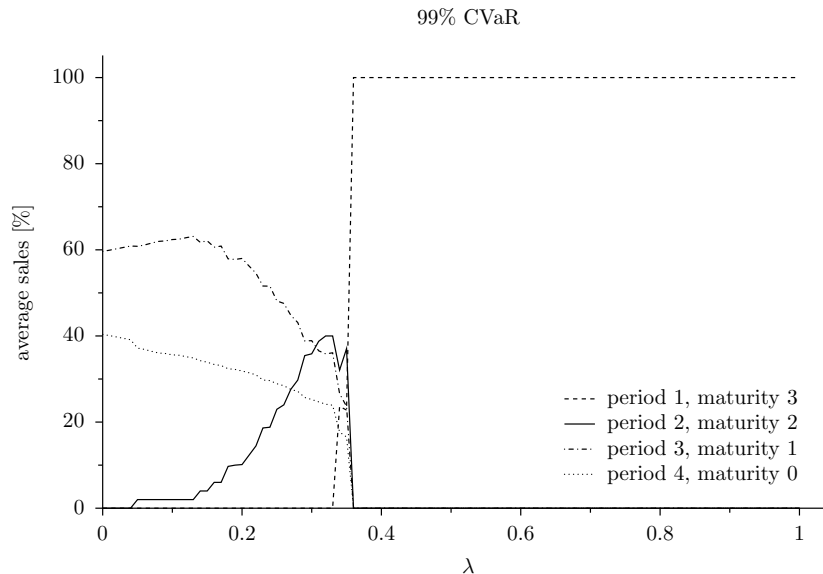


Figure 8: Average sales for period 4 using different futures contracts for 99% CVaR

A risk-neutral salmon producer was defined by  $\lambda = 0$ . The results above indicate that salmon producers should consider hedging strategies at already fairly low degrees of risk-aversion. This is also consistent with the first case, where the risk-neutral producer chooses to sell the planning horizon’s entire production through futures contracts in the first period.

## 6. Conclusions

We present in this paper a multistage stochastic programming model for determining the optimal hedging decision of a salmon producer. We see that the producer chooses to reduce exposure in the spot market and enters into futures contracts at quite low levels of risk-aversion and continues to do so as the degree of risk-aversion increases. The degree of risk-aversion influences also the maturity of the chosen futures contracts: the higher the degree of risk aversion, the longer the maturity of the futures contract.

Hedging decisions are complex, requiring a decision on both when to enter into a futures contract, for how long and the volume traded through the contract. We believe that the type of analysis in this paper can provide valuable decision support to a salmon producer regarding the choice of an optimal hedging strategy according to the company’s risk preferences. That is, how much to hedge and when. The model can also be used for sensitivity analysis and scenario analysis where consequences of changes in risk preferences and CVaR constraints are examined.

The presented analysis is static, whereas a salmon producer in the real world would make hedging decisions continuously. A natural extension to this work is

therefore to use the model in a more dynamic setting, e. g. in a rolling horizon, and study, how such a dynamic setting might affect the hedging strategies. Another path for future research is to introduce production risk into the analysis.

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