Regional Frequency Analysis of Extreme Precipitation with Consideration of Uncertainties to Update IDF Curves for the City of Trondheim

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Summary Regional frequency analysis based on the method of *L*-moments is performed from annual maximum series of extreme precipitation intensity to update Intensity-Duration-Frequency (IDF) curves for the city of Trondheim. The main problems addressed are (1) reduction of uncertainties of different sources for reliable estimation of quantiles: (i) testing of trend patterns and stationarity of the data series from the target site and demonstrating the dependency of results on the data used; (ii) testing regional homogeneity of extreme precipitation events for the climate regime in the study area and "pooling" of regional data for data augmentation and reduction of uncertainty due to short length of data series; and (iii) selection of distributions for extreme precipitation events of different durations to reduce the uncertainty due to choice of distributions; and (2) assessment and quantification of sampling uncertainty in terms of interval estimates (confidence bounds) of quantiles. Trend patterns and check for stationarity have been demonstrated for a data from a target site based on both non-parametric Mann-Kendall and parametric regression tests. Selection of distributions has been done based on Z-statistics and *L*-moment ratio diagrams. Non-parametric balanced bootstrap resampling has been used to quantify the sampling uncertainty. For extreme precipitation events of shorter durations (5 min. to 30 min.) there are statistically significant increasing trend patterns for the data series with start years of 1992 to 1998 while there are no significant trend patterns for recent extremes and there are no statistically significant trend patterns for longer durations (45 min. to 180 min.). The results of the analyses indicate that: (1) significance tests for trend patterns and stationarity are dependent on the data series used but the stationarity assumption is valid for the data series used from the target site. (2) the extreme precipitation events from four sites in Trondheim are homogeneous and can be "pooled" for regional analysis; (3) different types of distributions fit to extreme precipitation events of different durations which shows that thorough selection of distributions is indispensable rather than fitting a single distribution for the whole durations; (4) interval estimates from balanced bootstrap resampling indicated that there is huge sampling uncertainty in quantile estimation that needs to be addressed in any frequency analysis; and (5) large differences are observed between the IDF curves from this study and the existing IDF curves (i.e. Imetno). The IDF

curves from this study are from data augmented through regional analysis, based on thorough procedures for selection of distributions and also include uncertainty bounds and hence are more reliable than the existing one. Hence, the methods and procedures followed in this study are expected to contribute to endeavors for estimating reliable IDF curves.

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1. Introduction

Frequency analysis of extreme precipitation events of different durations have long been used for the estimation of extreme quantiles corresponding to return periods of interest. Estimated quantiles are summarized in the form of IDF curves from which design storm hyetographs can be derived. The information is then useful for the design and management of urban drainage infrastructure, bridges, spillways, risk analysis for landslide hazards, etc.

However, due to the prevalence of extreme precipitation events and vulnerability of urban environments, urban floods have resulted in catastrophic damages in the recent years for instance flooding in the city of Trondheim in August, 2007 (Thorolfsson et al., 2008). There is growing interest from different stakeholders such as municipalities, companies, engineers, etc. for reliable analysis of extreme precipitation events with uncertainty bounds and procedures for routine updating. Therefore, reliable estimation of quantiles and derivation of design storm hyetograph are required to reduce prediction uncertainty and hence to reduce the costs associated with either spillover or over design.

For sites with sufficient record length as compared to the return period of the extreme precipitation quantile of interest, at-site frequency analysis can be employed. But some sites are not gaged at all or long historical records are not usually available to be able to make reliable prediction of extreme quantiles for larger return periods. Hence data augmentation from the regional observations is performed by utilizing extreme precipitation intensity records in a region. This regional frequency analysis is based on delineation of hydrologically homogeneous sites in the region and can also be useful to characterize the spatial relationships of extreme precipitation events and to study the regional patterns of climatic variability or change besides its main purpose of data augmentation.

Several studies (Adamowski et al., 1996; Gellens et al., 2002; Nguyen et al., 2002; Fowler et al., 2003; Lee et al, 2003; Trefry et al., 2005; Wallis et al, 2007 and Norbiato et al., 2007) have been done on regional frequency analysis of extreme precipitation or rainfall events based on *L*-moments or updating of IDF curves for different parts of the world. Gaál et al. (2008) applied region of influence (ROI) approach and *L*-moment based "index storm" procedure for frequency analysis of heavy precipitation in Slovakia. Kyselý et al. (2007) have derived the regional growth curves from regional frequency analysis based on *L*-moments for improved estimates of design values and they have concluded that the regional approach is most advantageous for variables such as precipitation that exhibit high random spatial variability. Yang et al (2010) have analyzed rainfall extremes in the Pearl River basin in China using L-moments augmented by tests for stationarity and correlation. Data pooling and regionalization procedures which are based on the method of L-moments is widely employed due to its rigorous statistical tests rather than simple approaches such as based on averaging (for instance, Bengtsson and Milloti, 2010) of storm depths from at-site quantile estimations from the stations in the region.

However, regional frequency analysis of extreme precipitation events and hence derivation of IDF curves is subject to the major uncertainties of different sources which are not addressed in the previous studies (see also Hailegeorgis and Burn, 2009):

a. Data series used: data quality, which is related to questions like is the data series stationary and independent; and sampling of data, which are related to the time period and length of data series and the sampling type (i.e. annual maximum series or partial duration series which is peaks over thresholds);
b. Selection of frequency distribution;
c. Parameter estimation; and
d. Regionalization and quantile estimation

One of the main assumptions in the statistical frequency analysis from historical (observed) data series is the stationarity assumption. However, there may be observed trends in extreme precipitation events mainly due to anticipated climate change and hence there is uncertainty involved with the stationarity assumption. The presence of significant non-stationarity in hydrologic time series cannot be ignored when estimating design values for future time horizons (Cunderlik and Burn, 2003). Bradley (1998) found that there is strong evidence for climate-related non-randomness in extreme precipitation in the Southern plains of the United States. Adamowski et al. (2003) detected significant trends in annual maxima rainfall data for durations ranging from 5 minute to 12 hour for Ontario (Canada) using the regional average Mann-Kendall. Crisci et al. (2002) have studied the uncertainties due to trends connected with the estimation of the design storms for Tuscany (Italy) by Pearson linear correlation coefficient and the Mann-Kendall tests. They have demonstrated that the hydrological consequences of this kind of climate variability have a major impact on the design of hydraulic works in the basin. However, there are still limitations in the commonly used trend test procedures due to the dependency of their results on the data series used.

Long time series data is required for reliable analysis of trend and to substantiate whether there is really a change or not for the long-term planning purposes. Therefore, analysis of such type of trends is not an objective in here. But, an insight in to the patterns of trend and check for stationarity of the historical data can also be pursued from the available relatively short records in this type of analysis. Zhang et al. (2010) analyzed the pattern of trends of streamflow based on different start and end years with a length of records from 10 years to 80 years. Bengtsson and Milloti (2010) have analyzed trends in hourly and sub-hourly annual maximum precipitation of events of 25 years to 27 years long. Being data dependent analysis, estimation of extreme events need to be updated regularly when new extremes data are recorded in the region as regular update is indispensable for management and evaluation of the performances of water infrastructure, for vulnerability and risk analysis, etc.

Annual maximum series rather than partial duration series (peaks over thresholds) method of sampling of extreme precipitation have been used in this study. This avoids the uncertainty related to subjective choice of the threshold values above which the extreme events are included in the analysis. One may opt for comparing the results of regional frequency analysis based on the *L*-moment from sampling based on annual maximum series vs. the peaks over threshold type of sampling. But this task is not an objective of the present study.

Independence (i.e. no correlation) in the data series is also a main assumption in frequency analysis. Correlation can be spatial correlation or serial correlation. Hosking and Wallis (1997) noted that a small amount of serial dependence in annual data series has little effect on the quality of quantile estimates. Data sampling based on the annual maximum series which provides an additional advantage of avoiding the problem of serial correlation in the data. Spatial correlation in data series as demonstrated by Hosking and Wallis (1988; 1997), Mikkelsen et al. (1996), Martins and Stedinger (2002), Madsen et al. (2002), Bayazit et al. (2004), and Castellarin et al. (2008) can have an effect on the homogeneity test statistics in regional frequency analysis. The effect of intersite dependence on the regional *L*-moment algorithm is to increase the variability of the regional averages and this increases the variability of estimated growth curve (Hosking and Wallis, 1997). Madsen et al. (2002), based on partial duration series (PDS) of extreme rainfall analysis for Denmark, found that in general the correlation is a decreasing function of distance and the correlation being larger for larger durations. Also higher intersite correlation may be expected for low intensity (longer duration) frontal storms which covers large areas than high intensity (shorter duration) localized convective storms. The data used in this study from different sites in the region have short concurrent records. Hence, intersite correlation in the data series is not a focus in this study.

An additional challenge is that the results of frequency analysis from historical data are dependent on the data series used. The length of the sample data may not be sufficient to represent the underlying population especially for longer return periods, and there is no general consensus on the guidelines regarding the required length of data series. For instance, Jacob et al. (1999) suggested a 5T guideline that states “pooling” group should contain at least 5T station-years of data so as to obtain reasonably accurate estimates of the T-year quantile while Mamen and Iden (2009), stated that one needs a series of at least 25 years to calculate values for return period of 100 years. Hence, in general there is always uncertainty due to sampling as different data series may result in different quantile estimates.

Selection of frequency distribution is also a major source of uncertainty in the estimation of extreme quantiles as the sample data may reasonably fit to two or more distributions but with significant differences in quantile values.

There are different methods of parameter estimation in frequency analysis which results in different quantile estimates. The method of *L*-moments is used for parameter estimation in this study due to its advantages mentioned by Hosking, 1990 such as *L*-moments being linear functions of the data are less sensitive than are conventional moments to sampling variability or measurement errors in the extreme data values and *L*-moment ratio estimators have small bias and variance in comparison with the conventional moments. Hence uncertainty due to parameter estimation is not dealt with in this study.
Also there is always uncertainty pertinent to delineation of homogeneous regions. Different homogeneous regions can be delineated based on the criteria presented by Hoskings and Wallis (1997) but may result in different estimated quantiles. Uncertainty due to regionalization is not addressed in this study since it is not possible to form a big region for the target site due to the availability of extreme precipitation data only at municipality level which are many hundreds of kilometers apart and hence with a potentially heterogeneous climate regime.

Furthermore, there is also uncertainty related to quantile estimation based on the “index storm” procedure as use of different index values for instance the mean vs. the median values may result in different quantile values. A middle-sized storm such as the mean or the median can be used as an “index storm”. The difference between the mean and the median depends on the skewness of the data fitted to a particular distribution. For instance, for a normal distribution (i.e. zero skewness) the mean is equal to the median and both corresponds to the 50% probability of exceedence. Grover et al. (2002) have tested median flood as “index flood". Also someone may be interested to test percentiles other than the mean and the median. Nevertheless, investigation of the effect of choosing different "index storms" and its pertinent uncertainty in the regional frequency analysis of extreme precipitation is not the scope (objective) of this work. Plots of the "index storms" which are the mean of the annual maximum series used in the present study are given in Fig. 5 while plots of the annual maximum series are given in Fig. 6 to Fig. 9 for different durations of extreme precipitation events for different sites considered in this study.

Therefore, the existing wide practice of frequency analysis and derivation of IDF curves entails the following major limitations:

i. Only at-site frequency analysis based on short record length is widely applied which makes quantile estimates of large return values less reliable;

ii. A single statistical distribution is fitted to extreme precipitations of different durations without any thorough choice of the "best-fit" distribution which increases the uncertainty due to the choice of distributions;

iii. There is no improved uncertainty bounds associated with the estimated quantiles hence the end users are not able to propagate the uncertainty due to the IDF curves to the derivation of IDF based design storm hyetographs and in the simulation of urban runoff (floods); and

iv. Lack of tests for trend patterns and stationarity in data series and lack of comprehensive procedures which helps routine updating of the IDF curves.

1.1. Objectives of the study

The limitations which are stated above need to be addressed for improved predictions to minimize the risks pertinent to the uncertainty in predictions. Hence the main objectives of this study geared towards:

i. Application of procedures for trend patterns and stationarity tests in extreme precipitation events of different durations for a target site to demonstrate the limitations in the existing trend and stationarity test procedures due to their dependency on the data series used and hence to assess the uncertainty pertinent to stationarity assumption;

ii. Detailed review of the derivations and procedures of regional frequency analysis of extreme precipitation events based on the method of *L*-moments for better understanding of the method. Estimation of *L*-moments directly from ordered observations and their corresponding weights have been presented as a rather handy approach for implementation of the method of *L*-moments and extension of the procedures and tools presented by Hosking and Wallis (1997);

iii. Fitting the “best-fit” statistical distributions for each duration of extreme precipitation events to reduce the uncertainty due to the choice of statistical distributions;

iv. Quantification of uncertainty in quantile estimation due to sampling of data series; and

v. Application of the methods to the climate regime in central Norway (i.e. city of Trondheim) and updating of the IDF curves based on the regional analysis for the city.

2. Study region and data

The study site is the city of Trondheim, Norway. The city of Trondheim is chosen for the study due to recent prevalence of extreme precipitation events (Thorolfsson et al., 2008), growing interest by different stakeholders for better analysis of extreme precipitation events, and relatively good records of regional data. Moreover, the availability of urban storm runoff research catchment at Risvollan in the city gives the opportunity for further research related to propagation of the uncertainties due to the IDF curves to design flood values. There is also a plan to expand the regional methodology pertinent to data augmentation and prediction for ungaged sites for other regions in Norway. But the importance of this work is not site and problem specific that the methodologies and procedures developed or followed in this study can be utilized elsewhere for similar objectives of analyzing extreme hydro-meteorological events such as storms, floods, lowflows, wind speed, etc.

Extreme precipitation data is available for the period 1967 to 2009. Extreme precipitation intensity data from four stations are “pooled” for regional analysis for this study (Table 1). The mean annual precipitation from the existing metrological stations in the city ranges from 740 mm to 900 mm. Trondheim experiences extreme rainfalls during summer. It also experiences precipitation in the form of snowfall during winter (from November to March). The target site of Risvollan is located about 4 km southeast of the center of city of Trondheim and have been an active urban research catchment since 1987 with separate storm sewer networks of about 20 ha residential area. The site is equipped with instruments for measuring precipitation, temperature, short wave solar radiation, wind velocity, relative humidity, snow melt and storm water runoff (Matheussen, 2004). Owing to the availability of several measurements, it is possible to execute further research for instance propagation of the uncertainty in IDF curves to urban runoff simulation and analysis of flooding risks.

3. Methodology

3.1. Trend pattern and stationarity

Significance tests for trends are commonly used to detect the steady change (a trend) in hydrologic time series before use for statistical analysis. Both non-parametric and parametric methods are used to detect the significance of trends. The non-parametric test has made no assumption about the statistical distributions of the data and hence they are not subject to the uncertainty in the assumptions of the types of distributions. The parametric tests assume that the time series data follows some particular distribution.

Non-parametric test: Mann-Kendall test

The non-parametric Mann-Kendall test (Mann, 1945; Kendall, 1975) is commonly used for detection of direction of trend patterns in hydrological variables. The test procedures for Mann-Kendall test have been described by many researchers for instance by Adamowski et al. (2003) and McBean et al. (2008). For a time series of n data points where X_i and X_j are a member of the data series where i = 1,2,….,n-1 and j = i+1, i+2, i+3,……,n; each data point X_i is compared with all corresponding X_j data points to compute the sign (i.e. direction of trends). The Kendall's S-statistics is computed from the sum of the signs and the variance of the S-statistics is computed. The null hypothesis to test (H_0) is there is no monotonic trend in the data and the alternative hypothesis (H_1) is there is monotonic trend in the data. The test is based on the Z-test. If $|Z_s| > (Z_{obs} = Z_{\alpha/2})$, we have an evidence to reject the null hypothesis and hence that there is significant trend in the data where α is significance level. A significance level of 5% i.e. a confidence level of 95% is used in this study.

Parametric test: linear regression test

In order to detect the trend, linear regression can be fitted between a response variable which is the annual maximum series of precipitation intensity with the independent variable which is the time (i.e. year) for different durations. The significance test is done for the slope parameter of the linear regression model. Then from the statistical significance of the slope parameter it can be inferred that there are trends in the annual time series data. The Null hypothesis for trend test (H_0) is there is no significant trend and the alternative hypothesis (H_1) is there is significant trend. The test is based on the t-test (see Rawlings et al, page 122).

The critical t-value is $t_{crit} = t_{\alpha/2, n-p}$. If | t_{obs} | < t_{crit}, we fail to reject the null hypothesis (i.e. no significant trend).

3.2. Regional frequency analysis based on *L*-moments

Frequency analysis of extreme precipitation events requires the availability of sufficient extreme precipitation data especially for reliable estimation of rare events (i.e. quantiles with large return periods). In regional frequency analysis, additional information from homogeneous sites within the region is utilized to improve the at-site estimates. Hosking and Wallis (1990; 1993; 1997), Burn (1988; 1990; 2003) and Martins and Stedinger (2002) demonstrated the importance of using regional information for frequency analysis of extreme hydrological events.

L-moments and L-moment ratios

Let X be a real-valued random variable with cumulative distribution F(x), quantile function x(F) and probability distribution function f(x) or dF(x). For a set of ordered data by $x_{1:n} \leq x_{2:n} \leq, \ldots\ldots, \leq x_{n:n}$, certain linear combinations of the elements of an ordered sample contain information about the location, scale and shape of the distribution from which the sample is drawn hence *L*-moments are defined to be the expected values of these linear combinations, multiplied for numerical convenience by scalar constants (Hosking and Wallis, 1997). The *L*-moments of a probability distribution are defined by (Hosking, 1990; Hosking and Wallis, 1997; Serfling and Xiao 2006, 2007)

$$\lambda_k = n^{-1} \sum_{r=1}^{n} w_{r:n}^{(k)} E\left[X_{r:n}\right] \tag{1}$$

$$w_{r:n}^{(k)} = \sum_{j=0}^{\min\{r-1,k-1\}} (-1)^{k-1-j} \binom{k-1}{j} \times \binom{k-1+j}{j} \binom{n-1}{j}^{-1} \binom{r-1}{j} \tag{2}$$

Where, $w_{r:n}^{(k)}$ are the weights and r = 1,..., n are the ranks of the observations in ascending order. Hence the weights, which are the relative contributions of each observation to the first four *L*-moments for a sample size n are computed as:

$$w_{r:n}^{(1)} = 1$$

$$w_{r:n}^{(2)} = \frac{2r-n-1}{n-1}$$

$$w_{r:n}^{(3)} = \frac{(n-1)(n-2)+6(r-1)(r-2)-6(r-1)(n-2)}{(n-1)(n-2)}$$

$$w_{r:n}^{(4)} = \frac{1}{(n-1)(n-2)(n-3)} \times \begin{Bmatrix} -(n-1)(n-2)(n-3)+ \\ 20(r-1)(r-2)(r-3)+ \\ 12(r-1)(n-2)(n-3)- \\ 30(r-1)(r-2)(n-3) \end{Bmatrix} \tag{3}$$

L-moment ratios are independent of units of measurement and are given by Hosking and Wallis (1997) as follows:

$$\tau = \frac{\lambda_2}{\lambda_1},\ \tau_k = \frac{\lambda_k}{\lambda_2};\ k \geq 3 \tag{4}$$

Where, λ_1 is the *L*-location or the mean, λ_2 is the *L*-scale, τ is the *L*-CV, τ_3 is the *L*-skewness and τ_4 is the *L*-kurtosis.

Estimators of L-moments and L-moment ratios

Estimators of *L*-moments are obtained from finite sample. Hosking and Wallis (1997) (see formula 2.59 and Fig 2.6) have derived an expression for the sample *L*-moments (l_k) which are unbiased estimators of λ_k in terms of the ordered observations and their corresponding weights for the first four *L*-moments for a sample size of nineteen.

$$l_k = n^{-1} \sum_{r=1}^{n} w_{r:n}^{(k)} x_{r:n}\ ;\ k = 1,2,.. \tag{5}$$

Where $w_{r:n}^{(k)}$ are the weights as defined in eqns. (2 and 3), $x_{r:n}$ are the ordered observations and r = 1, 2, 3,…, n are the ranks of observations in ascending order. The first *L*-moment (λ_1) is the expectation or the mean of the distribution for a probability distribution and its estimator (l_1) is a sample mean and hence all the observations have equal weightages which are equal to one. Regional average *L*-moments are estimated from

$$l_k^{R} = \frac{\sum_{i=1}^{N} n_i \frac{l_k^{(i)}}{l_1^{(i)}}}{\sum_{i=1}^{N} n_i} \; ; \; k = 1,2,.. \quad (6)$$

Where, N is the total number of sites in the region, n_i is the number of records for each site and R denotes regional. Sample *L*-moment ratios t and t_k are natural estimators of τ and τ_k respectively and are not unbiased but their biases are very small in moderate or large samples (Hosking and Wallis, 1997) and are defined as $t = \frac{l_2}{l_1}, \; t_k = \frac{l_k}{l_2}; \; k \geq 3$ (7)

Implementation of eqns. (3 to 7) is not difficult. It can even be implemented as spreadsheet calculations so that it avoids relying mainly on previous work to apply the method of regional frequency analyses and also it encourages further extension or upgrading of the method with additional performances.

Similarity measures and delineation of homogeneous regions

Similar and homogeneous regions are identified and delineated based on specific similarity measures and homogeneity criterion respectively as proper delineation of homogeneous region is crucial for reliable quantile estimation. The region of influence approach (Burn, 1990, Zrinji and Burn, 1994) is used to identify similar sites and rank them based on their proximity to the target site as shown in Table 1. The attributes used for the similarity distance metrics have equal weights and include

a. Altitude of the stations;

b. Locations (X and Y co-ordinates of the stations); and

c. Mean annual precipitation at the stations

Hosking and Wallis (1997) presented the regional homogeneity based on the theory of *L*-moments which compares the regional dispersion of *L*-moment ratios with the average dispersion of the *L*-moment ratios determined from NS simulations of homogeneous groups from a four parameter Kappa distribution influenced only by sampling variability. Three heterogeneity measures are used to test the variability of three different H-statistics namely H_1 for "coefficient of *L*-variation" (*L*-CV), H_2 for the combination of *L*-CV and *L*-skewness (*L*-SK) and H_3 for the combination of *L*-skewness (*L*-CS) and *L*-kurtosis (L-CK). Heterogeneity measures (H-statistics) are calculated as

$$H_i = \frac{V_{\text{observed}_i} - \mu_{\text{Vsimulated}_i}}{\sigma_{\text{Vsimulated}_i}}; i = 1,2,3 \tag{8}$$

Where $\mu_{\text{Vsimulated}_i}$ and $\sigma_{\text{Vsimulated}_i}$ are the means and standard deviations of the simulated values of dispersions (V_i) while V_{observed_i} are the regional dispersions calculated from the observations. The dispersions (V-statistics) are defined as

$$V_1 = \left\{ \frac{\sum_{i=1}^{N} n_i \left(t^{(i)} - t^R \right)^2}{\sum_{i=1}^{N} n_i} \right\}^{\frac{1}{2}}$$

$$V_2 = \frac{\sum_{i=1}^{N} n_i \left\{ \left(t^{(i)} - t^R \right)^2 + \left(t_3^{(i)} - t_3^R \right)^2 \right\}^{\frac{1}{2}}}{\sum_{i=1}^{N} n_i}$$

$$V_3 = \frac{\sum_{i=1}^{N} n_i \left\{ \left(t_3^{(i)} - t_3^R \right)^2 + \left(t_4^{(i)} - t_4^R \right)^2 \right\}^{\frac{1}{2}}}{\sum_{i=1}^{N} n_i} \tag{9}$$

$$t^R = \frac{\sum_{i=1}^{N} n_i t^{(i)}}{\sum_{i=1}^{N} n_i}, \quad t_k^R = \frac{\sum_{i=1}^{N} n_i t_k^{(i)}}{\sum_{i=1}^{N} n_i} \;; k \geq 3 \tag{10}$$

Where, V_1 is the standard deviation of the at-site sample *L*-CVs weighted based on record length. V_2 and V_3 are the weighted average distance from the site to the group weighted mean on graphs of t versus t_3 and of t_3 versus t_4 respectively, t^R, t_3^R and t_4^R are the regional average *L*-CV, *L*-SK, and *L*-CK respectively weighted proportionally to the sites' record length (n_i) and i represents the sites 1,2,…,N. Hosking and Wallis (1997) suggested that region can be regarded as "acceptably homogeneous" if $H < 1$, "possibly heterogeneous" if $1 \leq H < 2$, and "definitely heterogeneous" if $H \geq 2$.

Discordancy measure

A measure of discordancy between the *L*-moment ratios of a site and the average *L*-moment ratios of a group of similar sites identifies those sites that are discordant with the group as a whole and the procedures for discordancy measure as explained by Hosking and Wallis (1997) is as follows: Suppose there are N sites in the group, let $u_i = [t^{(i)}\ t_3^{(i)}\ t_4^{(i)}]^T$ be a vector containing the *L*-moment ratios t, t_3 and t_4 values for site i and the superscript T denotes transpose of a vector matrix, the group average ū and sample covariance matrix S are defined as

$$\bar{u} = \frac{1}{N}\sum_{i=1}^{N} u_i$$
$$S = \sum_{i=1}^{N}\left(u_i - \bar{u}\right)\left(u_i - \bar{u}\right)^T \tag{11}$$

Then the discordancy measure D_i for a site is given by equation

$$D_i = \frac{1}{3} N \left(u_i - \bar{u}\right)^T S^{-1} \left(u_i - \bar{u}\right) \tag{12}$$

A site should be declared discordant if $D_i \geq 3.0$.

Selection of a regional frequency distribution (goodness-of-fit measure)

The choice of frequency distributions is determined based on the goodness-of-fit measures which indicate how much the considered distributions fit the available data. It entails hypothesis tests to reject the null hypothesis which says a certain distribution fits to the data better than the other candidate distributions. If we fail to provide evidence to reject the null hypothesis the distribution is said to be the “best-fit”. Hosking and Wallis (1997) tested several distributions for the regional analysis and found that the two parameter distributions are not robust and vulnerable to “misspecification” and suggested that they are not recommended for regional or at-site analyses. Therefore, in the present study we considered the three parameter distributions which have also the shape parameters in addition to the scale and location parameters for the regional analyses. The analysis in the present study is based on historical records for which the stationarity assumption is tested to be valid. So, the methodology in the present study is different from the non-stationary extreme value analysis (such as Hundecha et al., 2008; Mudersbach and Jensen, 2010, etc.) which considers an assumed time dependent patterns for some of the distribution parameters and also it is different from frequency analysis based on projected scenarios of extreme precipitation events (such as Monette et.al., 2012, etc.). Therefore, when new extreme events are added to the analysis, the “best-fit” distribution, distribution parameters and also the quantile estimates and recurrence intervals may change which is the main limitation of any data dependent or data driven models.

However, the ultimate objective is estimation of more reliable and robust quantile values with uncertainty bounds from historical records (observations) which is expected to be a more reliable approach than the analyses based on the projected scenarios and non-stationary analysis. Quantile estimates from distributions which have shape parameters are expected to be robust and not highly sensitive to some new extreme precipitation events which are not included in the regional analysis. Therefore, selection of distributions also comply with the main essence of the regional analysis which include as much as possible extreme records in the region in to the data by “trading space for time” for data length augmentation and robust/ reliable predictions at both gaged and ungaged sites. As it can be observed from Table 1, the regional extreme precipitation data is increased from 23 to 71 through pooling by the regional analysis based on the method of *L*-moments for the target site, Risvollan. When several distributions fit the data adequately, any of them is a reasonable choice for use in the final analysis, and the best choice from among them will be the distribution that is most robust (Hosking and Wallis, 1997). They proposed the five poarameter Wakeby distribution as a default regional distribution if none of the considered candidate distributions fulfills the requirements of goodness-of-fit statistics.

The goodness-of-fit criterions defined in terms of *L*-moments for each of various candidate distributions are the Z-statistics and *L*-moment ratio diagram:

a. The Z- statistic

Fit a four parameter Kappa distribution to the regional average *L*-moment ratios l_1^R, t^R, t_3^R, and t_4^R. Simulate a large number, N_{sim}, of realizations of a region with N sites, each from a four parameter Kappa distribution. For the mth simulated region, calculate the regional average *L*-kurtosis $t_4^{[m]}$, the bias and standard deviation of t_4^R

$$\beta_4 = \frac{1}{N_{sim}} \sum_{m=1}^{N_{sim}} \left(t_4^{[m]} - t_4^R \right)$$

$$\sigma_4 = \left\{ \frac{1}{N_{sim} - 1} \left[\sum_{m=1}^{N_{sim}} \left(t_4^{(m)} - t_4^R \right)^2 - N_{sim} \beta_4^2 \right] \right\}^{\frac{1}{2}} \tag{13}$$

And, for each candidate distribution, the goodness-of-fit measure is given by

$$Z^{DIST} = \frac{\tau_4^{DIST} - t_4^R + \beta_4}{\sigma_4} \quad (14)$$

Where, DIST refers to a particular distribution, β_4 and σ_4 are the bias and standard deviation of t_4^R respectively, N_{sim} is the number of simulated regional data sets in a similar way as for the heterogeneity statistics. The superscript m denotes the m^{th} simulated region. The fit is declared adequate if Z^{DIST} is sufficiently close to zero, a reasonable criterion being $|Z^{DIST}| \leq 1.64$.

b. *L*-moment ratio diagram

Selection of the "best-fit" regional distribution using *L*-moment ratio diagrams involves plotting of the regional sample *L*-moment ratios (*L*-skewness vs. *L*-kurtosis) as a scatter plot and comparing them with theoretical *L*-moment ratio curves, which are given by Hosking and Wallis, 1997, of the candidate distributions. The distribution to which the regional *L*-moment ratios computed from the sample are closer to the theoretical curve is selected as the "best-fit".

Estimation of parameters and quantiles

The main objective of frequency analysis is estimation of quantiles corresponding to a return period of interest. The parameters of distributions given in Appendix B are estimated from their relationship with *L*-moments and *L*-moment ratios as given by Hosking and Wallis (1997). Then the quantiles are estimated from quantile functions which are given in Appendix A. The "index storm" approach which is a similar approach to the index flood (Dalrymple, 1960) is used for quantile estimation of extreme precipitation events. The main assumption of an "index storm" procedure is that the sites forming a homogeneous region have identical frequency distribution called the regional growth curve but a site-specific scaling factor, the "index storm". Let x(F), 0 < F < 1, be the quantile function of the frequency distribution of extreme precipitation intensity at site i, for a homogeneous region

$$x_i(F) = \mu_i q(F) \quad (15)$$

Where i = 1, 2,…, N and μ_i is the site-dependent scale factor which is called the "index storm" and q(F) is the regional growth curve which is a dimensionless quantile function common to every site in a homogeneous region.

Following previous work (Hosking and Wallis, 1997, Nguyen et al., 2002, Gaál et al., 2008, etc.), the location estimator (i.e. the sample mean) of annual maximum series of extreme precipitation intensity is used as an "index storm" in this study. More detailed references on regional frequency analysis based on *L*-moments can be obtained from Hosking and Wallis (1997).

3.3. Balanced bootstrap resampling

Quantile estimate from a single data set in regional frequency analysis provides only a point estimate. Therefore, non-parametric balanced bootstrap resampling, which involves random sampling with replacement, is employed to quantify sampling uncertainty in terms of interval estimates (i.e. confidence intervals of quantile estimates). In bootstrap (Efron 1979; 1982), the samples are drawn with replacement from the original sample. Davison et al. (1986) presented balanced bootstrap resampling which reuses each of the observations equal number of times. In balanced bootstrap resampling, the total number of occurrences of each sample point in the whole resamples is the same and is equal to the number of resampling ($N_{resampling}$). Faulkner et al. (1999) derived confidence limits for growth curves of rainfall data by bootstrapping. Burn (2003) applied bootstrap resampling for flood frequency analysis and presented the main advantages of bootstrap resampling for constructing confidence intervals. Also the initial spatial correlation of the data from different sites is not affected in bootstrap resampling approach (Pujol et al., 2007).

In bootstrap, let the original sample data is $X = \{X_1, X_2, \ldots, X_n\}$ and the bootstrap resample of X is $X^* = \{X_1^*, X_2^*, \ldots, X_n^*\}$, the estimators such as confidence intervals can then be estimated from the resamples $(X^*)^{(1)}, (X^*)^{(2)}, \ldots, (X^*)^{(Nresampling)}$ of size $N_{resampling}$. The background and method of estimating the confidence intervals as presented by Faulkner and Jones (1999) and Carpenter (1999) is as follows: let Q_i is the estimate from the bootstrap sample i, Q_{sam} is the estimate from sample data and Q_{true} is the unknown true quantity, bootstrap residuals $e_i = Q_i - Q_{sam}$ and the actual unknown residual $e = Q_{sam} - Q_{true}$. Assuming that bootstrap residuals (e_i) to be representative of values drawn from the same distribution as the actual unknown residual (e), $Q_i - Q_{sam} \equiv Q_{sam} - Q_{true}$. If e_l and e_u are appropriate lower and upper percentage points of the unknown distribution of the residuals, such that the probability

$$\Pr\left(e_l \le e \le e_u\right) = 1-2\alpha \rightarrow$$
$$\Pr\left(e_l \le Q_{sam} - Q_{true} \le e_u\right) = 1-2\alpha \rightarrow$$
$$\Pr\left(Q_{sam} - e_u \le Q_{true} \le Q_{sam} - e_l\right) = 1-2\alpha$$

Then, $\left(LCL, UCL\right) = \left(Q_{sam} - e_u, Q_{sam} - e_l\right)$ (16)

e is equally likely to appear at any point in the ordered set of e_i's, i.e. each has a probability of $\frac{1}{\left(N_{resampling} + 1\right)}$.

Then, $u = \alpha \times \left(N_{resampling} + 1\right)$ and

$l = \left(1\text{-}\alpha\right) \times \left(N_{resampling} + 1\right)$ (17)

Where, α = ½ of the significance level.

The procedures for balanced bootstrap resampling based on regional *L*-moment parameter estimation algorithm to construct 100(1-2α) % confidence intervals of quantile estimates, following Faulkner and Jones (1999), Burn (2003) and Hailegeorgis and Burn (2009) is given as below:

i. Prepare original sample "pooled" from homogeneous region;

ii. By repeating each year of data $N_{resampling}$ times we would get a matrix of (N_{years}*$N_{resampling}$) rows by N_{sites} columns, where N_{years} is the number of years for which data is available at one or more data stations and N_{sites} is the number of homogeneous sites for regional analysis;

iii. Balanced bootstrap resamples are then obtained from random permutation of N_{years} rows of data from which *L*-moments, *L*-moment ratios, parameters and quantiles corresponding to a return period of interest can be estimated for the selected "best-fit" distributions given in Table 2. This process is then repeated $N_{resampling}$ times;

iv. Calculate bootstrapped residuals (e_i), which are the deviations of each quantile estimates from the quantile estimate of the original sample. $e_i = Q_i - Q_{sam}$, where Q_i is quantiles estimated from bootstrapped samples and Q_{sam} is quantile estimated from the original sample;

v. Rank these deviations in ascending order to find e_u and e_l for 95% confidence interval where u and l are defined as above and correspond to the upper and the lower confidence levels respectively. For $N_{resampling}$ = 999 used in this study, u corresponds to 25th and l corresponds to 975th bootstrap residuals; and

vi. Finally, the confidence intervals for the estimated quantiles are computed from (16).

4. Results

Since the annual maximum precipitation intensity data series from the other sites considered are short and/or don't include recent extremes (Table 1), only the data series for the target site of Risvollan has been tested for trend patterns and stationarity to check the validity of stationarity assumption and to demonstrate the dependency of trend patterns on the data series used. In this study, the method by Zhang et al. (2010) is adopted and a trend test based on varying starting period and fixed end period is used. Both the parametric Mann-Kendall and the non-parametric regression tests have produced similar results for trend patterns. For the target site, the data used for the analysis of extreme precipitation can be said to be stationary (Fig. 2 and 3) and hence stationary frequency analysis is valid.

For this study, no site has appeared to be discordant based on the discordancy measure explained earlier. Results of homomgenity tests based on H-statistics (Table 2) indicated that H-values range from -1.75 to 1.22.

Results for the selection of statistical distribution are given in Table 2 and Fig. 4. Four different types of three parameter distributions, the Generalised extreme value (GEV), Generalised logistic (GLO), Pearson Type III (PIII) and Generalized Pareto (GPAR) are tested. Different types of statistical distributions appeared to be the "best-fit" for extreme precipitation of different durations. The "best-fit" distribution for precipitation durations of 5 min., 45min. and 120 min. is the Pearson Type III; Generalised Pareto distribution is the "best-fit" and also the only fit for extreme precipitation of 10 min., 20 min. and 30 min. durations. Generalised logistic distribution is the "best-fit" distribution for extreme precipitations of 60 min., 90 min. and 180 min. durations. Identification of distribution based on a regional *L*-moment ratio diagram (Fig. 4) also resulted in similar "best-fit" distributions as that of the Z-statistics for all durations of extreme precipitation events. IDF curves with uncertainty bounds (95 % confidence intervals) for the target site are given in Fig. 10 and 11.

Percentage differences of the 95% lower and upper confidence levels of quantiles (which are estimated based on bootstrap resampling) and the existing IDF curve (i.e. estimated from at-site analysis for the target site of Risvollan by the Metrological Institute of Norway: www.eklima.no and labeled as Imetno in Fig. 10 and 11), from the quantiles estimated from regional analysis in this study are given in Table 3. The differences in quantile estimates from this study as revealed from the 95% confidence bounds range from -32.9 % to +25.1 % for a return period of 2 years and rises to -43 % to +31% for a return period of 100 years. The percentage differences in the existing IDF quantiles and the quantiles estimated from this regional analysis ranges from +25.8 % for a return period of 2 years to - 40 % for a return period of 100 years.

5. Discussion

Trend pattern and stationarity

The varying starting periods used for trend tests help to identify the start year of significant trend patterns. The fixed end period is used since the objective is to assess the patterns of the trend for recent extremes to detect the recent trends and to utilize the updated information for design and management. It can be indicated that different results for significant test for trends are obtained from data set from varying starting years until recent extremes. But the extreme precipitation data set used for the target site for regional frequency analysis covers from 1987 to 2009 and trend patterns vanish for the data series containing recent extremes (Fig. 2 and 3). Therefore, based on the analysis of data series from the target site, stationarity assumption is valid and *L*-moments based frequency analysis can reasonably be applied.

Discordancy test and homogeneity tests based on H-statistics

All the H-values are less than one for durations of 5 min. to 120 min. which shows that the region is “acceptably homogeneous” and the H-value is slightly greater than one and less than two (H_1 = 1.22) for duration of 180 min. which shows that the region is “ possibly heterogeneous”. Therefore, the data used from the study region can be ”pooled” based on the criterion presented by Hosking and Wallis (1997) for data augementation and hence reliable estimation of quantiles. This study subsatntiates that it is worth testing the homogenity of extreme precipitation from a further wide spatial extent for the climate regime in Norway for reliable estimation of quantiles of high return periods and also for estimation of regional IDF curves or regional quantile maps to be able to estimate the design values at ungaged locations in the region.

Selection of distributions based on Z-statistics and L-moment diagram

It can be indicated that two or more distributions ($Z \leq 1.64$) may fit the extreme precipitation data but the “best-fit” distribution for which the quantile is estimated is the one with Z-value closer to Zero. Therefore, it is indicated that it is very important to follow thorough statistical distributions selection procedures rather than fitting a single distribution for all extreme precipitations of different durations in order to reduce the uncertainty in quantile estimation pertinent to the selection of the “best-fit” statistical distribution for the extreme data considered.

Quantile estimations and uncertainty bounds

From the confidence bounds of estimated quantiles, it can be observed that there is large sampling uncertainty which increases with the return period. These uncertainty ranges have inevitable impact on the design magnitudes of urban drainage infrastructure. The existing IDF curves for the city of Trondheim is based on at-site fitting of the two parameter extreme value Type I (EV1) or Gumbel distribution for the whole durations of extreme precipitation events. The EV1 (Gumbel) distribution is the special case of the Generalised extreme value (GEV) distribution when the shape parameter is zero (‘k’ = 0). But the tail behavior of a distribution is largely influenced by its shape parameter(s). In the contrary reliable prediction of the rare extreme quantiles of higher recurrence intervals, which are located at the tails of a distribution are of main interest to minimize the risks pertinent to the occurrence of extreme events. Despite its drawbacks, the Gumbel distribution is usually appealing to hydrology practitioners and for teaching purposes due to its simplicity in parameter estimation by the method of moments, method of maximum likelihood, and *L*-moments.

The same at-site data for Risvollan as the present study was used by the Norwegian Meteorological Institute to develop the existing IDF curves. The improvement obtained from the present work is due to the regional analysis based on the use of regional records rather than the at-site estimation from records of short length (i.e. at-site analysis). Plots of the existing IDF curves (Fig. 10 and 11) reveal that there is a sharp bend in the IDF curves above duration of 20 minutes which indicates that the statistical distribution fitted to the extreme precipitation of above 20 min. durations may not represent the parent distribution (i.e. there is “misspecification” of statistical distribution). In addition, the fitted two parameter distribution which has no shape parameter lacks robustness and hence “misspecification” of distribution affects the quantile estimation to a larger extent.

6. Conclusions

Regional frequency analysis of extreme precipitation events based on the method of *L*-moments has been reviewed and applied for the city of Trondheim for data augmentation and reliable estimation of quantiles. Extreme precipitation intensities of durations 5 min. to 180 min., which can be relevant for design and management of urban water infrastructure, are “pooled” from four gaging stations in the city of Trondheim for regional frequency analysis and estimation of quantiles corresponding to 2 to 100 years return period. *L*-moments are estimated directly from ranked and weighted ordered sample data series which is a contribution towards further understanding of the *L*-moment procedures of regional frequency analysis. The approach is not difficult and it helps for easy implementation of the *L*-moment procedures especially for extension with additional developments such as assessment of uncertainty as demonstrated in this study.

Check for stationarity of data and the dependency of the commonly used trend test procedures on the sample data used has been demonstrated and thorough trend pattern tests based on data from varying start years and also that include recent extremes should be followed and general conclusion on the stationarity of the data need to be drawn with caution.

It can be indicated that different statistical distributions fit to extreme precipitation events of different durations and hence careful choice of “best-fit” and robust statistical distributions for different durations is indispensable to reduce the uncertainty pertinent to selection of distributions.

The sampling uncertainty associated with the frequency analysis of extreme precipitation events is assessed and quantified in terms of interval estimate (i.e. 95% confidence bounds) based on non-parametric bootstrap resampling. The interval estimate showed that there is huge uncertainty in quantile estimation due to sampling of data which needs to be incorporated in any frequency analysis from historical data. The updated estimated quantiles and IDF curves with uncertainty bounds obtained from this study are found to be more reliable as compared to the existing IDF curves for Trondheim.

The methods and procedures followed in this study are expected to contribute to endeavors for estimating reliable quantiles and reducing the uncertainties associated with IDF curves. IDF curves with quantified uncertainty bounds would help the end users to be able to recognize the uncertainties behind the IDF curves and propagate the uncertainties pertinent to IDF curves for reliable derivation of IDF curves based design storm hyetographs and simulation of urban runoff in the design and management of urban drainage infrastructure or in any flood risk assessment tasks.

This study focuses on the assessment and quantification of sampling uncertainty pertinent to IDF curves and hence it can't be considered as a comprehensive uncertainty assessment. Also propagation of this uncertainty to simulation of urban runoff is not studied. This task is planned for future research.

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Appendices

Figure captions

Fig. 1. Locations of precipitation stations used for regional analysis

Fig. 2. Results of Mann-Kendal and regression methods for trend pattern at 95 % confidence intervals and check for stationarity for extreme precipitation of 5 min. to 30 min. durations at Risvollan site (Trondheim) for different data start years to data end year of 2009

Fig. 3. Results of Mann-Kendal and regression methods for trend pattern at 95 % confidence intervals and check for stationarity for extreme precipitation of 45 min. to 180 min. durations at Risvollan site (Trondheim) for different data start years to data end year of 2009

Fig. 4. Regional L-Moment ratio diagram for identification of "best-fit" regional distributions

Fig. 5. Mean of annual maximum precipitation intensity or "index storm" used (1 l/s.ha or 1 liter/second.hectar = 0.36 mm/hr or 0.36 millimeter/hour

Fig. 6. Annual maximum precipitation series for different durations at Risvollan site

Fig. 7. Annual maximum precipitation series for different durations at Moholt-Voll site (jumped years are missing data)

Fig. 8. Annual maximum precipitation series for different durations at Blakli site

Fig. 9. Annual maximum precipitation series for different durations at Tyholt site

Fig. 10. IDF curves and 95 % confidence intervals for Risvollan site (Trondheim) for quantile estimates of 2, 5 and 10 years return periods from regional frequency analysis of annual maximum extreme precipitation events of 5 min. to 180 min. durations

Fig. 11. IDF curves and 95 % confidence intervals for Risvollan site (Trondheim) for quantile estimates of 20, 50 and 100 years return periods from regional frequency analysis of annual maximum extreme precipitation events of 5 min. to 180 min. durations

Table 1: Climate stations (sites) and annual maximum extreme precipitation intensity used for regional analysis.

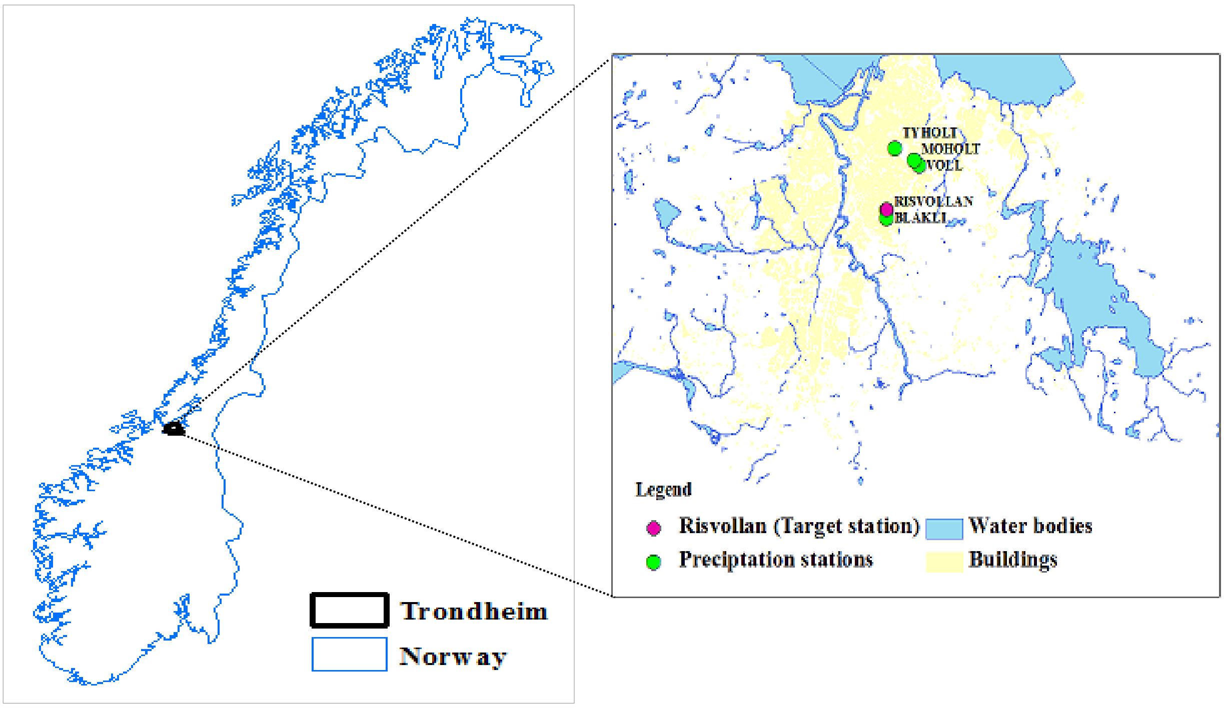
No.	Sites	Altitude, m amsl	Latitude (degree)	Longitude (degree)	Data range	No. of available data (years)	Mean annual total precipitation (mm)	Remarks
1	Risvollan	84	63.3987	10.4228	1987-2009	23	881	Target site (operational)
2	Moholt-Voll	127	63.4107	10.4535	1995-2009	13	855	Operational
3	Tyholt	113	63.4225	10.4303	1965-1993	25	740	Closed
4	Blakli	138	63.3960	10.4258	1974-1985	10	900	Closed
Total data used for regional analysis						71		

Table 2: Summary results for heterogeneity measures and goodness-of-fit measures (Z-statistics)

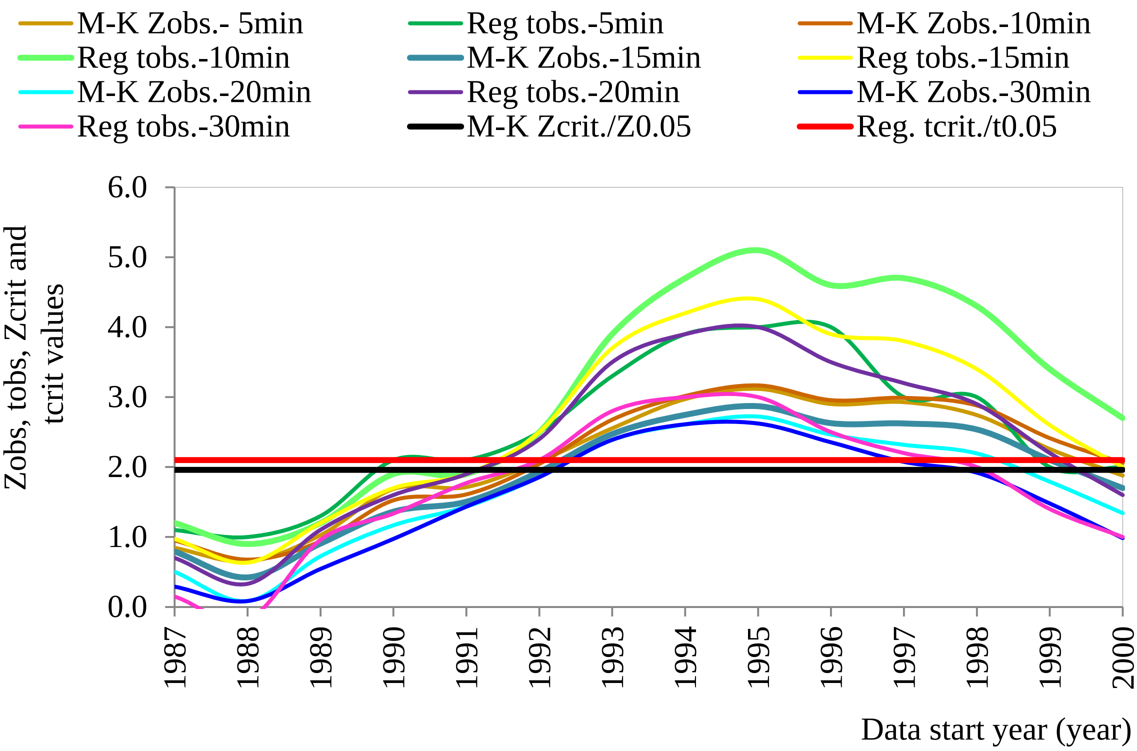
	Heterogeneity measures			Z-statistics				"Best-fit" distribution
Durations (min.)	H_1	H_2	H_3	GLO	GEV	PIII	GPAR	
5	-1	-0.45	0.15	1.33	0.69	**-0.04**	-0.87	PIII
10	0.21	0.17	-0.58	3.26	2.33	1.88	**0.28**	GPAR
15	-0.17	-0.07	-0.39	4.31	3.31	2.96	**1.17**	GPAR
20	0.11	-0.04	0.23	4.2	3.26	2.83	**1.2**	GPAR
30	0.37	-1.18	-0.74	3.37	2.48	1.95	**0.5**	GPAR
45	-0.83	-1.3	-0.39	1.6	0.82	**0.28**	-0.95	PIII
60	-1.75	-1.44	-0.75	**0.24**	-0.48	-0.9	-2.09	GLO
90	-0.61	-1.81	-1.57	**0.26**	-0.47	-0.89	-2.1	GLO
120	0.31	-1.51	-0.29	2.01	1.08	**0.73**	-0.93	PIII
180	1.22	-1.25	-1.29	**0.06**	-0.73	-1	-2.44	GLO

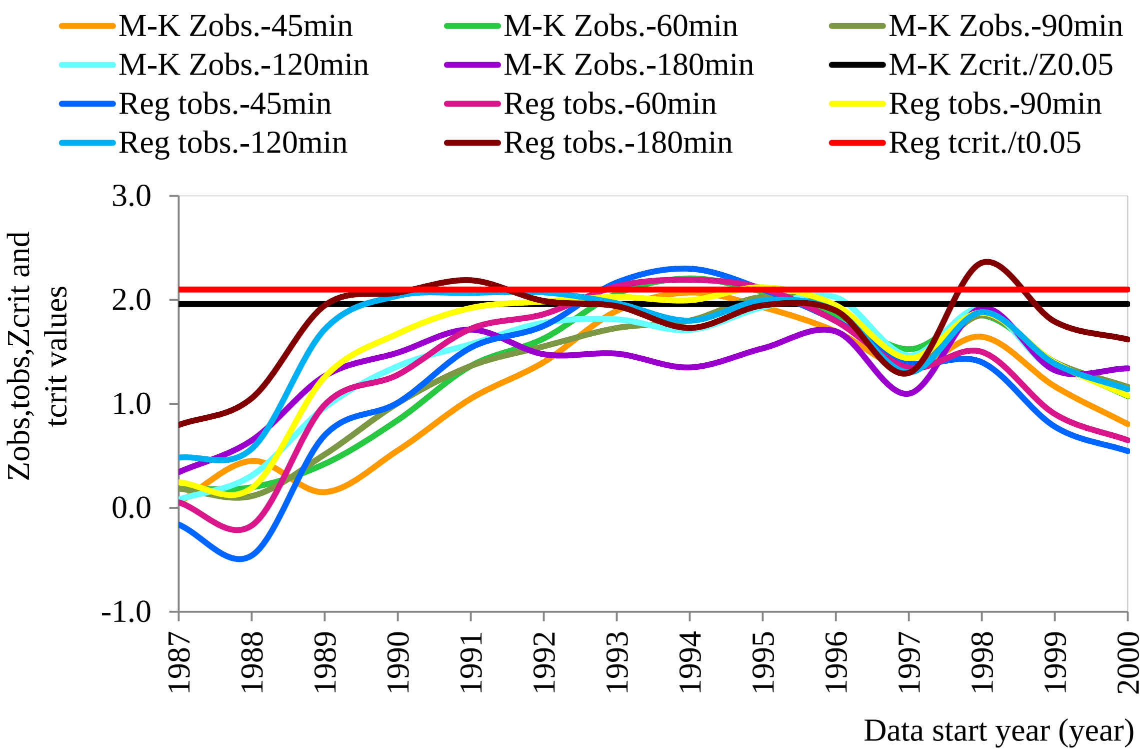
Table 3: Differences in percentages (%) for the lower and upper confidence levels estimated quantiles and the existing IDF curves from the estimated precipitation intensity quantiles from regional frequency analysis for a target site (Risvollan).

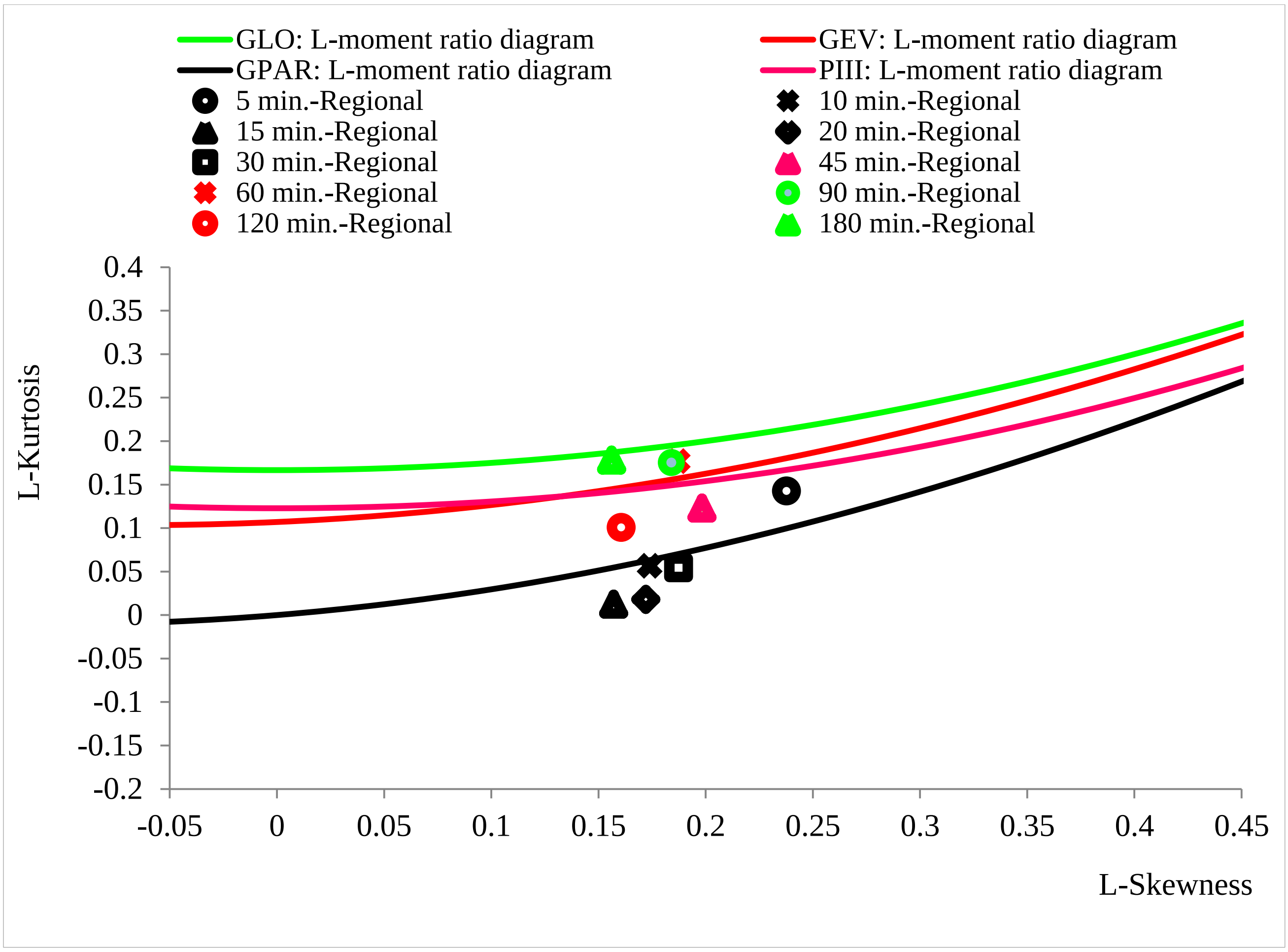
Return period (years)	Quantiles	Durations (min.)									
		5	10	15	20	30	45	60	90	120	180
2	95% LCL	-29.1	-28.1	-32.9	-31.4	-27.8	-30.7	-27.8	-26.3	-24.2	-25.1
	95% UCL	+25.1	+23.2	+23.0	+21.7	+19.7	+14.7	+9.6	+6.3	+6.9	0.0
	Existing IDF	+25.8	+23.4	+12.8	+16.4	+16.6	+9.1	+1.9	+8.7	+8.2	+12.0
5	95% LCL	-23.1	-21.0	-20.8	-22.1	-20.3	-19.4	-20.7	-17.1	-16.5	-17.8
	95% UCL	+24.7	+23.6	+23.3	+21.8	+21.7	+19.1	+15.3	+14.9	+12.2	+6.3
	Existing IDF	-1.1	-3.6	-4.9	+0.2	+7.0	-13.3	-16.7	-11.9	-11.0	+0.4
10	95% LCL	-28.7	-19.6	-17.1	-18.5	-18.0	-18.1	-18.1	-12.5	-17.5	-13.7
	95% UCL	+25.0	+22.2	+21.3	+19.9	+21.2	+20.2	+18.3	+19.0	+12.8	+10.6
	Existing IDF	-6.4	-8.5	-6.5	-1.6	+6.9	-20.4	-25.5	-20.8	-15.2	-6.0
20	95% LCL	-35.8	-21.0	-16.7	-16.9	-19.9	-19.0	-17.1	-11.3	-19.4	-11.6
	95% UCL	+24.5	+24.4	+20.1	+19.8	+23.4	+21.1	+20.5	+21.3	+13.3	+14.8
	Existing IDF	-8.9	-9.0	-4.4	+0.2	+9.7	-24.1	-31.4	-26.9	-17.9	-10.4
50	95% LCL	-42.4	-27.5	-21.4	-21.5	-26.7	-23.9	-23.0	-16.6	-21.9	-14.7
	95% UCL	+27.5	+27.2	+22.7	+23.6	+26.6	+20.4	+21.9	+22.8	+15.9	+19.7
	Existing IDF	-12.9	-6.1	+1.3	+5.6	+16.2	-25.9	-37.0	-32.0	-22.0	-15.0
100	95% LCL	-43.0	-36.5	-27.7	-26.7	-34.8	-30.3	-31.8	-24.3	-21.1	-19.3
	95% UCL	+31.1	+29.3	+24.0	+25.7	+29.3	+19.6	+23.4	+22.6	+19.6	+22.2
	Existing IDF	-18.3	-2.4	+7.1	+11.0	+22.4	-26.0	-40.0	-34.6	-26.0	-17.4

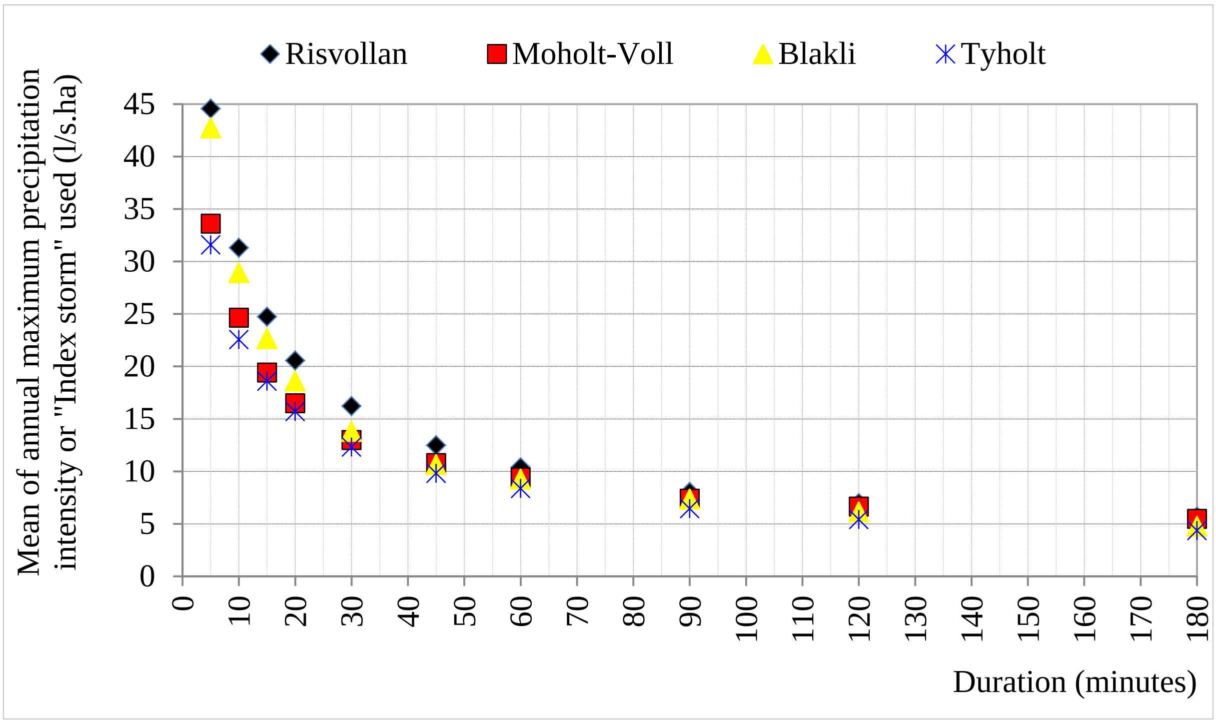


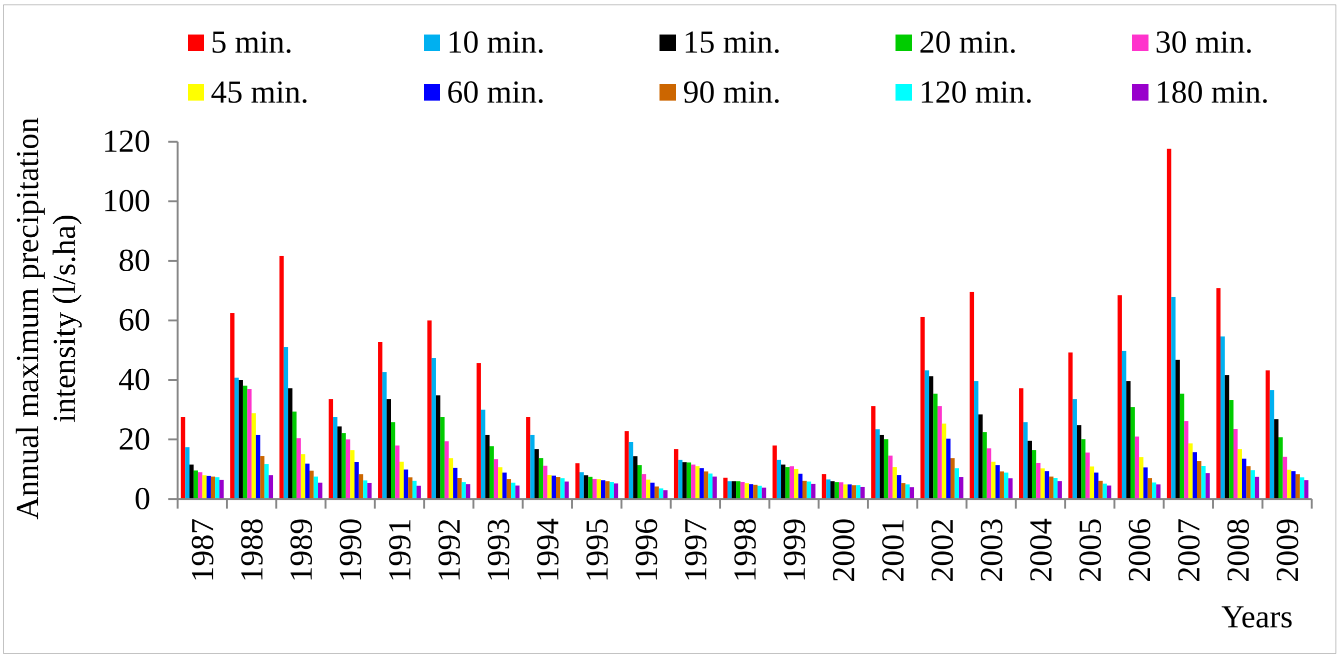
Trondheim
Norway
TYHOLT
MOHOLT
VOLL
RISVOLLAN
BLAKLI
Legend
Risvollan (Target station)
Water bodies
Preciptation stations
Buildings


M-K Zobs.- 5min
Reg tobs.-5min
M-K Zobs.-10min
Reg tobs.-10min
M-K Zobs.-15min
Reg tobs.-15min
M-K Zobs.-20min
Reg tobs.-20min
M-K Zobs.-30min
Reg tobs.-30min
M-K Zcrit./Z0.05
Reg. tcrit./t0.05
Zobs, tobs, Zcrit and tcrit values
6.0
5.0
4.0
3.0
2.0
1.0
0.0
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Data start year (year)

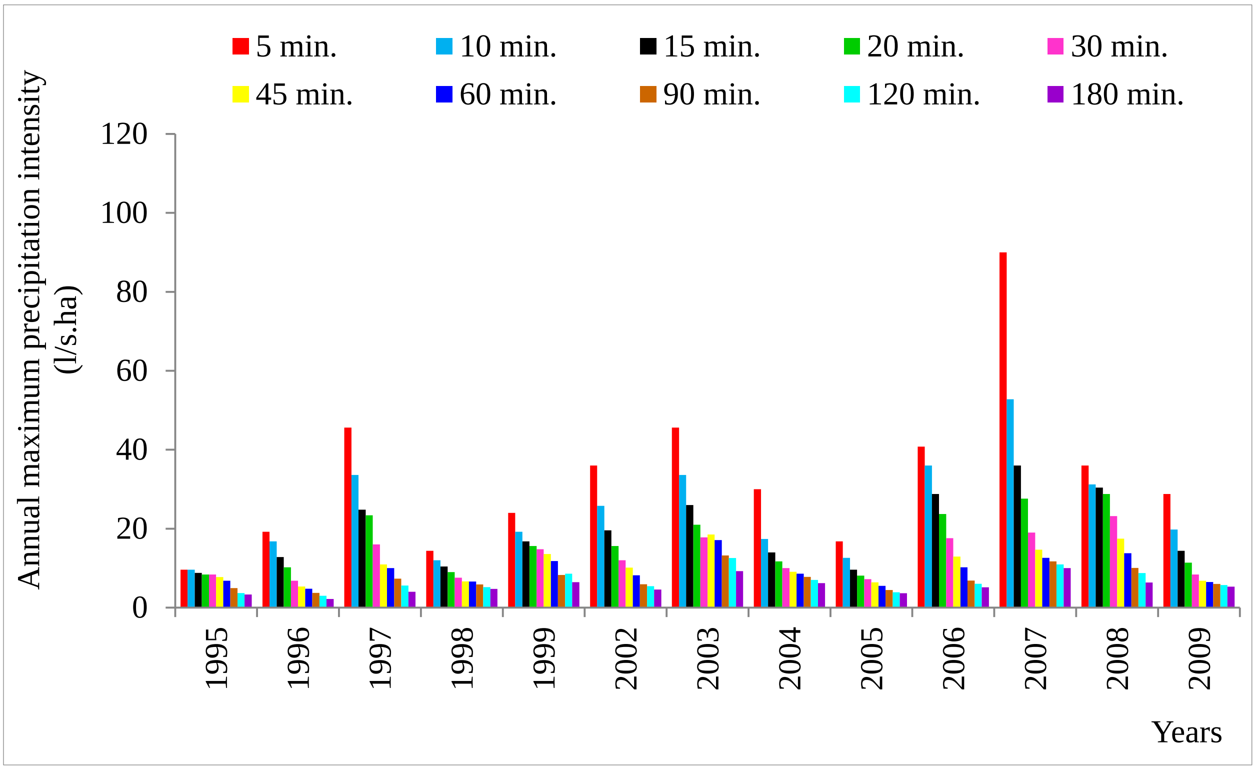

M-K Zobs.-45min
M-K Zobs.-60min
M-K Zobs.-90min
M-K Zobs.-120min
M-K Zobs.-180min
M-K Zcrit./Z0.05
Reg tobs.-45min
Reg tobs.-60min
Reg tobs.-90min
Reg tobs.-120min
Reg tobs.-180min
Reg tcrit./t0.05
Zobs,tobs,Zcrit and tcrit values
3.0
2.0
1.0
0.0
-1.0
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
Data start year (year)

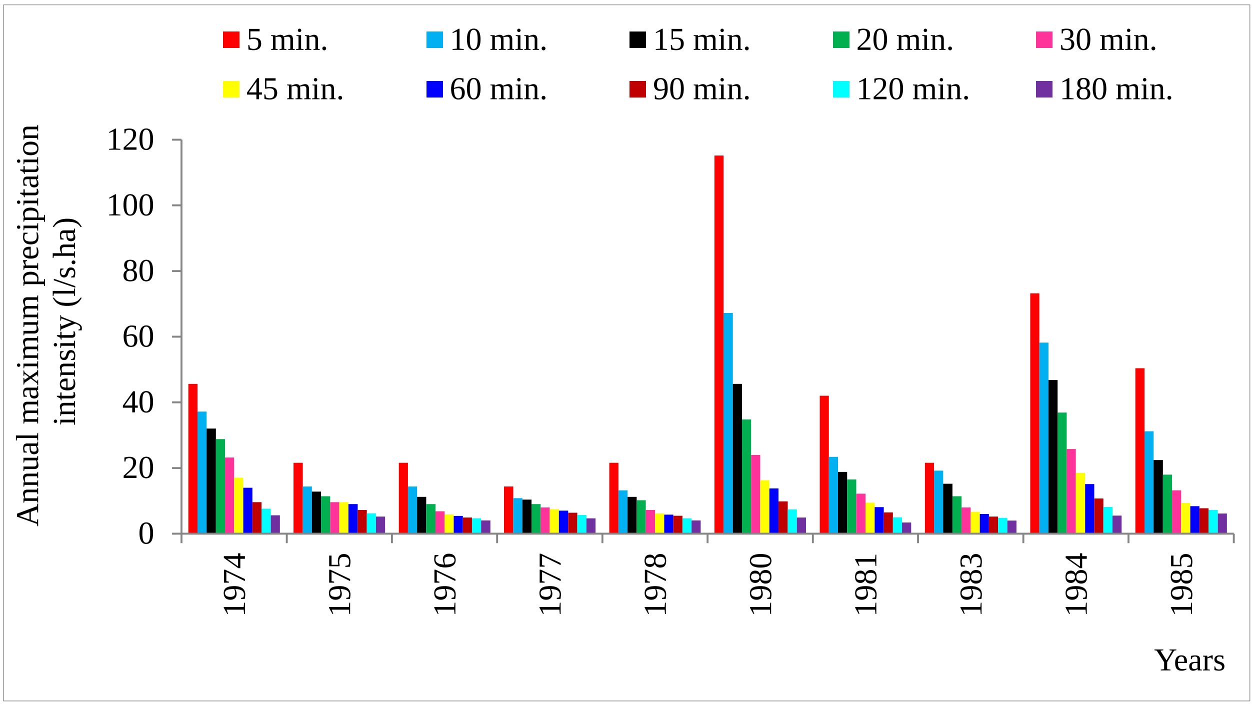

GLO: L-moment ratio diagram
GEV: L-moment ratio diagram
GPAR: L-moment ratio diagram
PIII: L-moment ratio diagram
5 min.-Regional
10 min.-Regional
15 min.-Regional
20 min.-Regional
30 min.-Regional
45 min.-Regional
60 min.-Regional
90 min.-Regional
120 min.-Regional
180 min.-Regional
L-Kurtosis
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
L-Skewness


Risvollan
Moholt-Voll
Blakli
Tyholt
Mean of annual maximum precipitation intensity or "Index storm" used (l/s.ha)
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
Duration (minutes)

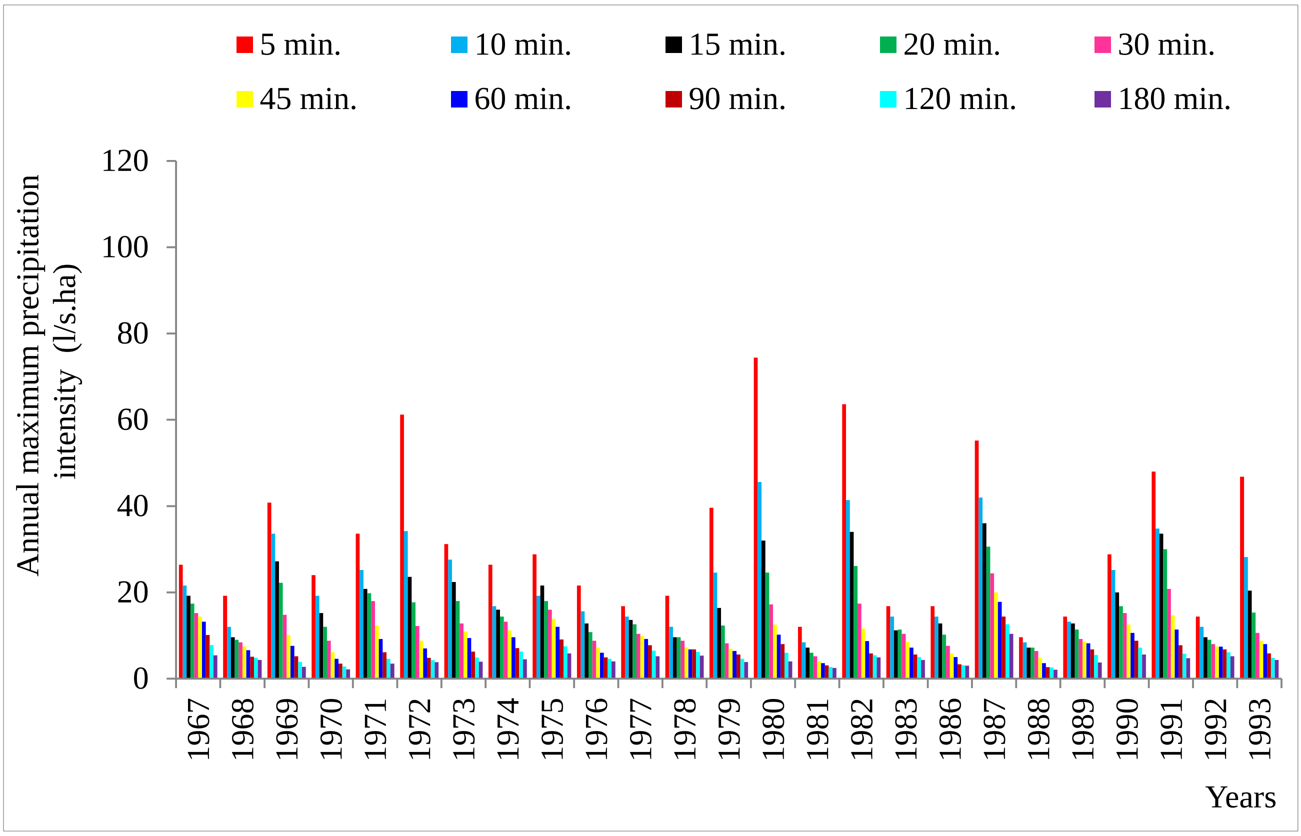


5 min.
10 min.
15 min.
20 min.
30 min.
45 min.
60 min.
90 min.
120 min.
180 min.
Annual maximum precipitation intensity (l/s.ha)
0
20
40
60
80
100
120
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
Years

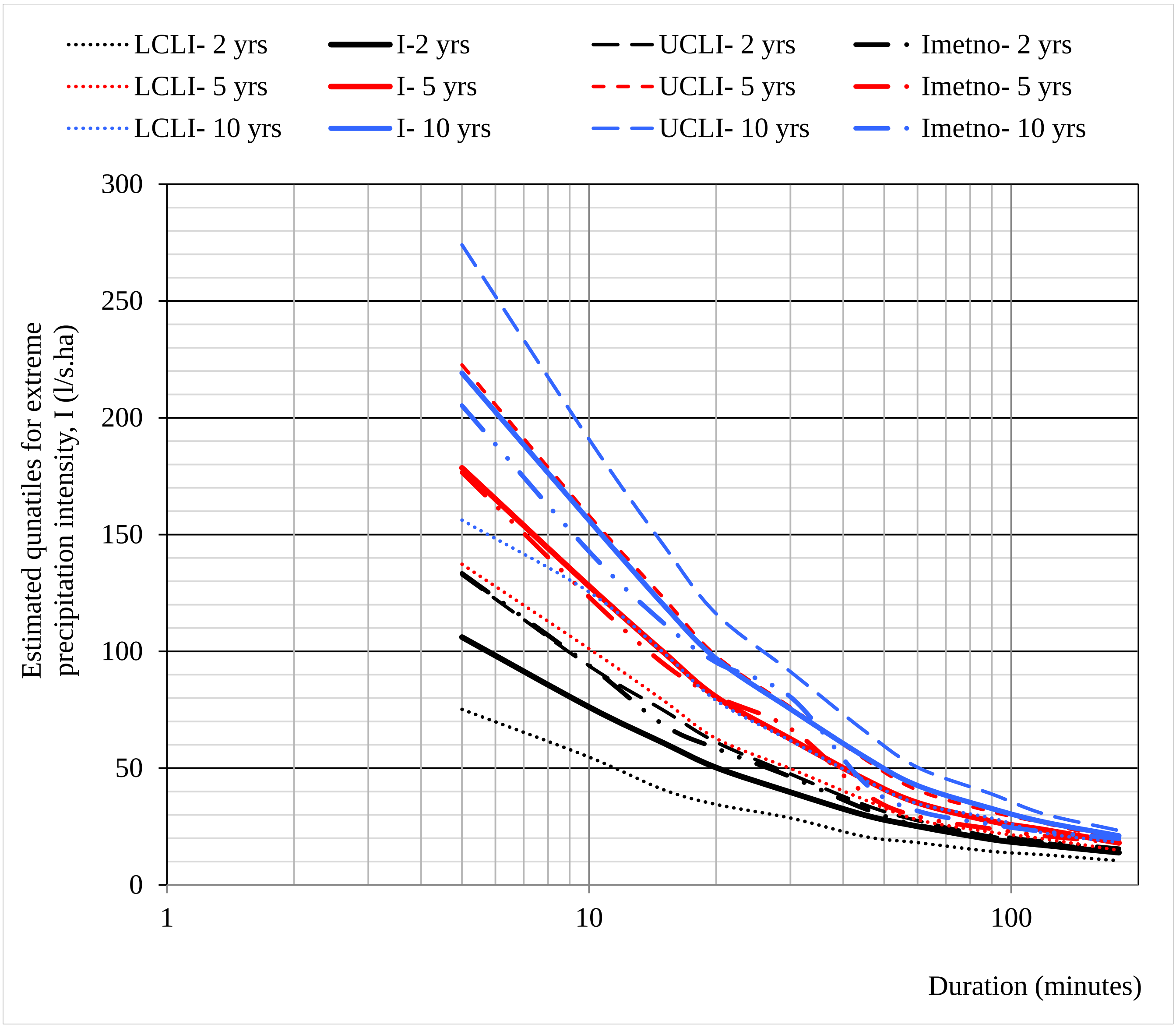

5 min.
10 min.
15 min.
20 min.
30 min.
45 min.
60 min.
90 min.
120 min.
180 min.
Annual maximum precipitation intensity (l/s.ha)
120
100
80
60
40
20
0
1995
1996
1997
1998
1999
2002
2003
2004
2005
2006
2007
2008
2009
Years

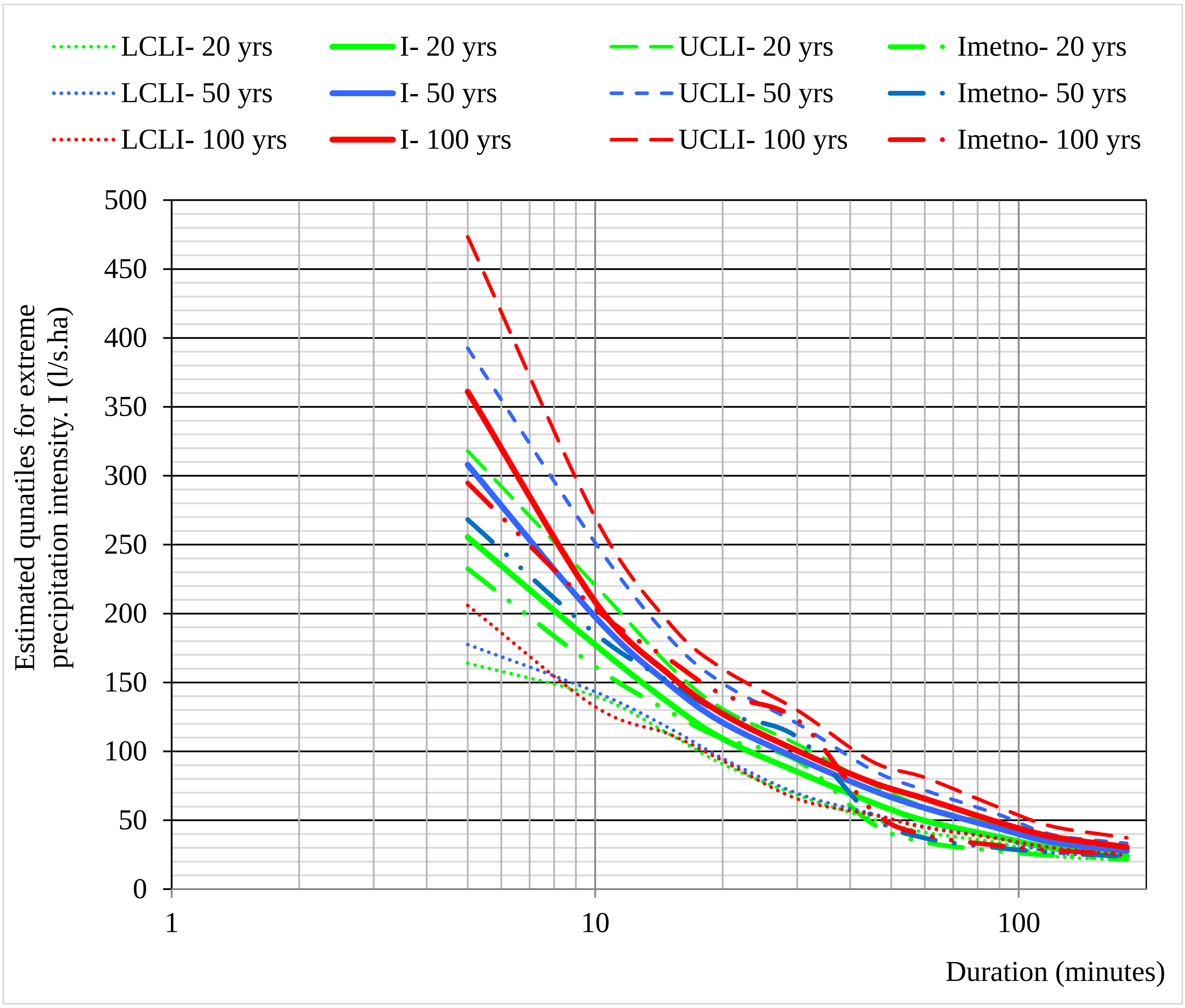


5 min.
10 min.
15 min.
20 min.
30 min.
45 min.
60 min.
90 min.
120 min.
180 min.
Annual maximum precipitation intensity (l/s.ha)
0
20
40
60
80
100
120
1974
1975
1976
1977
1978
1980
1981
1983
1984
1985
Years



5 min.
10 min.
15 min.
20 min.
30 min.
45 min.
60 min.
90 min.
120 min.
180 min.
Annual maximum precipitation intensity (l/s.ha)
120
100
80
60
40
20
0
1967
1968
1969
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1986
1987
1988
1989
1990
1991
1992
1993
Years


LCLI- 2 yrs
I-2 yrs
UCLI- 2 yrs
Imetno- 2 yrs
LCLI- 5 yrs
I- 5 yrs
UCLI- 5 yrs
Imetno- 5 yrs
LCLI- 10 yrs
I- 10 yrs
UCLI- 10 yrs
Imetno- 10 yrs
Estimated qunatiles for extreme precipitation intensity, I (l/s.ha)
300
250
200
150
100
50
0
1
10
100
Duration (minutes)



LCLI- 20 yrs
I- 20 yrs
UCLI- 20 yrs
Imetno- 20 yrs
LCLI- 50 yrs
I- 50 yrs
UCLI- 50 yrs
Imetno- 50 yrs
LCLI- 100 yrs
I- 100 yrs
UCLI- 100 yrs
Imetno- 100 yrs
Estimated qunatiles for extreme precipitation intensity. I (l/s.ha)
500
450
400
350
300
250
200
150
100
50
0
1
10
100
Duration (minutes)

Appendix A: Probability density functions (PDF), cumulative distribution functions (CDF) and quantile functions (QF) for some statistical distributions (Hosking & Wallis, 1997).

Distributions	$f_X(x)$ or PDF	$F_X(x)$ or CDF	$x(F)$ or QF
GEV	$\frac{1}{\alpha} e^{-(1-k)y - e^{-y}}$ $y = -k^{-1} \ln\left\{1-k\left(\frac{(x-\xi)}{\alpha}\right)\right\}$	$e^{-e^{-y}}$	$\xi + \frac{\alpha}{k}\left\{1 - (-\ln F)^k\right\}$
Pearson Type III	$\frac{1}{\beta\Gamma(\alpha)} \left(\frac{x-\xi}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x-\xi}{\beta}\right)}$	$\frac{1}{\Gamma(\alpha)} \int_0^{\left(\frac{x-\xi}{\beta}\right)} u^{\alpha-1} e^{-u} du$	No explicit analytical form: Approximation by Wilson-Hilferty Transformation
Kappa	$\frac{1}{\alpha}\left\{1-\frac{k}{\alpha}(x-\xi)\right\}^{\frac{1}{k}-1} \{F(x)\}^{1-h}$	$\left[1-h\left\{1-\frac{k}{\alpha}(x-\xi)\right\}^{\frac{1}{k}}\right]$	$\xi + \frac{\alpha}{k}\left\{1-\left(\frac{1-F^h}{h}\right)^k\right\}$
Wakeby	No explicit analytical form	No explicit analytical form	$\xi + \frac{\alpha}{\beta}\left\{1-(1-F)^{\beta}\right\} - \frac{\gamma}{\delta}\left\{1-(1-F)^{-\delta}\right\}$
GLOG	$\frac{\alpha^{-1} e^{-(1-k)y}}{\left(1+e^{-y}\right)^2}$ $y = -k^{-1} \log\left[1-k\frac{(x-\xi)}{\alpha}\right], \quad k \neq 0$ $y = \frac{(x-\xi)}{\alpha}, \quad k = 0$	$\frac{1}{1+e^{-y}}$	$\xi + \frac{\alpha}{k}\left\{1-\left(\frac{1-F}{F}\right)^k\right\}, \quad k \neq 0$ $\xi - \alpha \log\left(\frac{1-F}{F}\right), \quad k = 0$

GPAR	$\alpha^{-1}e^{-(1-k)y}$ $y=-k^{-1}\log\left[1-k\frac{(x-\xi)}{\alpha}\right],\ k\neq 0$ $y=\frac{(x-\xi)}{\alpha},\ k=0$	$1-e^{-y}$	$\xi+\frac{\alpha}{k}\{1-(1-F)^{k},\ k\neq 0$ $\xi-\alpha\log(1-F),\quad k=0$

Appendix B: Parameters for the statistical distributions in Appendix A (Hosking & Wallis, 1997).

Distributions	Parameters			
	Location	Scale	Shape	
GEV	ξ	α	k	
Pearson Type III	ξ	β	α	
Kappa	ξ	α	k & h	
Wakeby	ξ			α, β, γ & δ
GLOG	ξ	α	k	
GPAR	ξ	α	k	