A Delay Vector Variance based Approach for Detecting and Isolating the Non-linearity Induced Oscillations in Control Loops ⋆

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Abstract: Non-linear time series analysis based methods are a popular choice for industrial control loop data analysis. In this paper a delay vector variance (DVV) based approach is presented to analyze the source of oscillations in an industrial control loop. The method is capable of differentiating between the linear and non-linear causes of oscillations and can also help in isolating the source of non-linearity. The automatic determination of embedding dimensions is augmented with the DVV analysis to make it more robust and reliable. The efficacy of the proposed method is established using simulation as well as industrial case studies.

Keywords: Delay vectors, embedding dimensions, rank statistics, target variance, scatter plot

1. INTRODUCTION

Data driven approaches for control loop performance monitoring (CPM) offer considerable advantages over conventional model based diagnosis. These data driven methods, dealing exclusively with recorded data, are more flexible, robust and are easy to automate. There are numerous factors affecting the performance of industrial control systems, such as poor controller tuning, external disturbances, equipment non-linearities etc. Therefore differentiating among these factors leads to enhanced profitability and reduced shut down time. The fact that only a third of industrial control loops are reported to be functioning correctly, while rest are suffering from one problem or the other [Choudhury et al. (2008)], makes performance monitoring an important part of any industrial control systems.

Non-linearities, whether stemming from a sticking valve or a faulty sensor, are reported to be one of the major sources of performance issues [Srinivasan et al. (2005)]. Therefore much focus has been on the detection and isolation of non-linearities in the control loops. Different methods are being explored and reported in literature [Thornhill and Horch (2007) and di Capaci and Scali (2015)], a brief overview of some procedures are discussed here.

Some methods, like the one proposed by Srinivasan et al. (2005), Hägglund (2011) and Yamashita (2005), try to find similarity of the waveform with predetermined sets.

A popular method to distinguish between non-linear and linear sources of oscillations in control loops based on higher order spectra is proposed by Choudhury et al. (2008). Inability to detect non-linearities in symmetric waveforms(exhibiting odd harmonics) such as triangular or square waves are major limitations of this method [Thornhill (2005) and Zang and Howell (2003)].

Quite recently non-linearity detection methods based on the Hilbert Huang Transform (HHT) have been presented [Babji et al. (2009) and Aftab et al. (2016)]. These method use the intra-wave frequency modulation to detect the presence of non-linearity induced oscillations. Though these methods are applicable to non-stationary time series and are fully data driven, the mode mixing in the empirical mode decomposition (the precursor of HHT) in the presence of noise, make these methods prone to falsely report non-linearities.

Non-linear time series analysis methods make use of the fact that time series from nonlinear systems exhibit phase coupling and hence are more predictable. Analysis based on surrogate time series is therefore used to differentiate between linear and nonlinear nature of time series. Surrogate time series are time series that share the same power spectrum with the original time series, but contain random phase. One such method, to detect the presence of non-linearity in control loops, is proposed by Thornhill (2005). The algorithm compares the predictability of the time series from control loops with its surrogate counterparts. The accuracy of this method depends on the tuning of different parameters like embedding dimensions, prediction horizon, number of nearest neighbors, etc. Moreover the method relies on a test statistic that assumes the prediction error to follow the Gaussian distribution, that may not be the case in practice [1].

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[1] Details can be seen in Thornhill (2005)

This paper presents the delay vector variance method (DVV) for detecting the presence of non-linearity in the control loops. This method is also based on surrogate data, but is more general and doesn't require the tuning of different parameters. Surrogate data is generated using the iterative amplitude adjusted fourier transform method (IAFFT) proposed by Schreiber and Schmitz (1996). Moreover a rank based test statistic is adopted to test the Null hypothesis, i.e. whether the data is from a linear source, as recommended by Kantz and Schreiber (2004). Another important feature is the automatic determination of embedding dimensions that is augmented with this analysis to make it more robust and reliable.

The paper is organized as follows. Section 2 gives and overview of the delay vector variance (DVV) method. The steps involved in testing the Null hypothesis using rank based statistics are outlined in section 3. Automatic determination of embedding dimensions is explained in section 4. Section 5 gives the detailed algorithm. Simulation and industrial case studies are presented in sections 6 and 7 followed by conclusions.

2. DELAY VECTOR VARIANCE (DVV) METHOD

The delay vector variance method, developed recently [Gautama et al. (2004a)], tries to ascertain the nature of the time series via estimation of predictability and determinism. The method, as the name suggests, is based on the time delay embedding representation of time series $\mathbf{X(k)} = \{\mathbf{x(k)} \mid k = 1 \ldots N\}$. The time series is represented as a set of delay vectors (DVs), the method of so called *phase space reconstruction* [Kantz and Schreiber (2004)], with embedding dimension m and time delay τ, given by $\mathbf{y(k)} = [x_{k-m\tau}, \ldots, x_{k-\tau}]$. The each DV has a corresponding target, i.e. the next sample $x(k)$ [Gautama et al. (2004b)].

The delay vectors so obtained are the state space representation of the actual time series. The target variance σ^* is then calculated using the following steps [Gautama et al. (2004a),Gautama et al. (2004b)].

(1) The distance between DVs is calculated using the Euclidean norm, denoted by $||\mathbf{y(i)} - \mathbf{y(j)}||$ for $i \neq j$.
(2) The mean μ_d and standard deviation σ_d are computed for the calculated DV distances.
(3) All the DVs that lie within a certain distance τ_d are collected in sets Ω_k , written mathematically as
$$\Omega_k = \{\mathbf{y(i)} \mid \|\mathbf{y(i)} - \mathbf{y(j)}\| \leq \tau_d\} \quad (1)$$
(4) The threshold τ_d is taken from the uniformly sampled interval $[\mu_d - n_d\sigma_d; \mu_d - n_d\sigma_d]$; with n_d specifying the span over which the analysis is performed.
(5) The target variance σ_k^{*2}, is calculated for each set Ω_k that contains at least N_0 DVs.
(6) The average of these target variances σ_k over all sets divided by the variance of time series σ_x^2 gives the overall target variance σ^{2*}, given by Equation 2.
$$\sigma^{*2} = \frac{(1/K)\sum_{k=1}^{K} \sigma_k^{*2}}{\sigma_x^2} \quad (2)$$

The target variance σ^* is the inverse measure of the predictability of the time series. The target variance is plotted against the standardized distance measure r_d given by

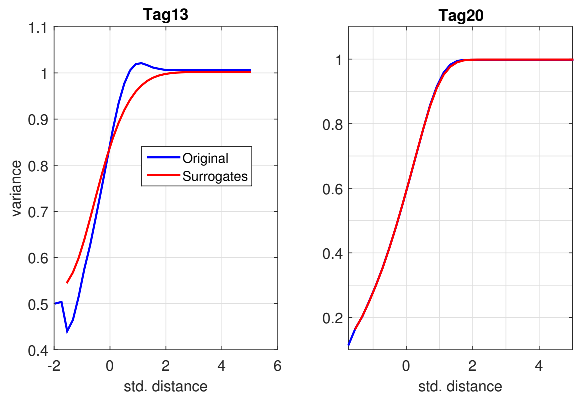

Fig. 1. A snapshot of target variance

$$r_d = \frac{\tau_d - \mu_d}{\sigma_d} \quad (3)$$

The greater the determinism in the time series the lower would be the value of target variance at smaller spans and vice versa. The target variance in (2) converges to unity for larger spans as for the maximum span all DVs belong to same set Ω_k and variance of Ω_k matches the variance of the data itself. The span parameter n_d must be increased if the target variance does not converge to unity. Figure 1 illustrates this effect graphically.

This target variance parameter not only gives the estimate of the determinism in the time series but also helps in classifying the time series as an output of linear or non-linear systems. This fact can be used to detect presence of non-linearity induced oscillations in industrial control loops as explained in subsequent sections.

2.1 Detection of Non-linearity using DVV Method

The surrogates of the input time series are used within the DVV framework to assess the linear or non-linear nature of the data from the oscillating control loops. The difference in the predictability of the original time series and its surrogate counterpart is taken as an indication of the non-linearity in the time series.

Figure 1 shows the target variance of two control loops from an industrial plant, along with the corresponding target variance of surrogate time series, to illustrate the DVV analysis results graphically. It can be seen that the target variance of data from Tag 13 is significantly different from its surrogate and is also more predictable; whereas for Tag 20 the loop data and its surrogates exhibit almost similar target variance. Moreover, another way of visualizing this relationship is using the so called DVV scatter plot; wherein the target variance of the time series is plotted versus that of its surrogates. For the linear case the target variance of the original time series and its surrogates would be similar and the scatter plot will be in close proximity of the bisector line whereas in the case of the non-linear signal this plot will diverge from the bisector line. This fact is shown in Figure 2 where Tag 13 scatter plot deviates clearly from the bisector line whereas for Tag 20 it is in close match with the bisector line.

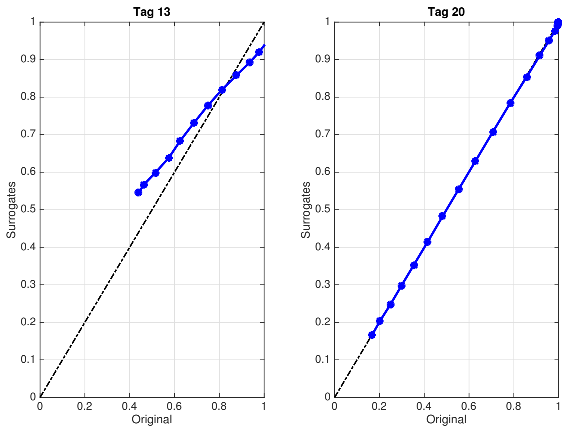

Fig. 2. DVV scatter plot ::A glance

2.2 *Measuring the Extent of Non-linearity*

The extent of non-linearity can be adjudged using the deviation of the target variance from the bisector line in the scatter plot. For this purpose the root mean square error (RMSE) from the bisector line seems to be an automatic choice to compare the extent of non-linearity in different time series. The rank based statistical test (described next) is used to differentiate between the linear and non-linear nature of time series and then the extent of non-linearity is calculated using the RMSE value.

2.3 *Isolating the Source of Non-Linearity*

Once the control loops suffering from the non-linearity induced oscillations are identified, the next step is to isolate the source for targeted maintenance and remedial actions. In a multi-loop environment, non-linearity induced at one point may propagate to other variables so the correct identification of the origin of the oscillations can significantly reduced the effort required in maintaining the control system. It is a well known fact that the different parts of plant act as low pass mechanical filter and tend to filter out the higher harmonics as we move away from the source of non-linearity [Thornhill et al. (2001)].

The RMSE measure discussed in section 2.2 can be used to compare the extent of non-linearity in different variables. The loop with maximum RMSE value is taken to be the source of non-linearity. The results, using RMSE, for the industrial case study are discussed in section 7.

3. TESTING OF THE NULL HYPOTHESIS: THE TEST STATISTICS

The rank based statistical test, as recommended by Kantz and Schreiber (2004), is adopted here to test the Null hypothesis that the time series is an output of linear Gaussian process. The rank based test is chosen as the probability distribution of the non-linearity measure is not known in advance; so a non-parametric rank based statistical test from Theiler and Prichard (1996) is adopted here to accept or reject the Null hypothesis.

The discriminating statistics chosen here is the RMSE value of the target variance. The RMSE η_0 is computed for the original time series. Then B surrogates are generated and corresponding statistics η_k are calculated for each $k = 1, \ldots, B$. Then it is checked whether the η_0 is on the tail of distribution. For one sided test the null hypothesis rejected at level α, if η_0 is among largest $(B+1)\alpha$ in the sorted list containing η_0 and all $\eta_k s$.

Table 1. Rank statistics for rejecting NULL hypothesis

if rank $> (B+1)(1-\alpha)$
then reject Null hypothesis (non-linear source)
else
accept Null hypothesis (linear source)

In this work, to test the Null hypothesis, $B = 99$ surrogates and $\alpha = 0.1$ are chosen. Therefore the Null hypothesis is rejected if the RMSE of the original time series in among largest 10 RMSEs out of total $100(r > 90)$ and time series is classified as originating from a non-linear source (Table 1). The rank based test and the RMSE values for the two signals in Figure 2 are given in Table 2. This procedure will be applied to simulation and industrial examples later in the paper to show how effective this method is in detecting the non-linearities in the control loops and isolating the root source.

Table 2. Rank statistic and RMSE

Signal	rank	Type	RMSE
Tag 20	4	Linear	–
Tag 13	99	Non-Linear	0.041

4. DETERMINING THE EMBEDDING DIMENSIONS

The construction of delay vectors, for the phase space reconstruction, is carried out using delayed embedding. The embedding dimension m of the attractor manifold is the most important parameter in such a reconstruction. Too small m will mask some of the dynamics and will lead to erroneous results whereas too large m will result in unnecessary computation overload. An automatic method for determination of the embedding dimension, proposed by Cao (1997), is augmented with the DVV to make it more robust and reliable.

According to the embedding theorems [Sauer et al. (1991), Cao (1997)] m is the true embedding if any two points which are close in the m-dimensional reconstructed space stay close in the $(m+1)$-dimensional reconstructed space as well. Therefore some kind of a distance measurement between two points in m and $m+1$ dimensional space can be used to identify the required dimension m.

Here again we consider a time series $\mathbf{X}(\mathbf{k}) = \{\mathbf{x}(\mathbf{k}) \mid k = 1 \ldots N\}$ and i^{th} delay vector $y_i(m)$ with embedding dimension m is given by

$$y_i(m) = (x_i, x_{i+\tau}, \ldots, x_{i+(m-1)\tau}) \qquad i = 1, 2, \ldots, N - m\tau \tag{4}$$

Now a distance measure $a(i, m)$ is defined as

$$a(i,m) = \frac{\|y_i(m+1) - y_{n(i,m)}(m+1)\|}{\|y_i(m) - y_{n(i,m)}(m)\|} \tag{5}$$
$$i = 1, 2, \ldots, N - m\tau$$

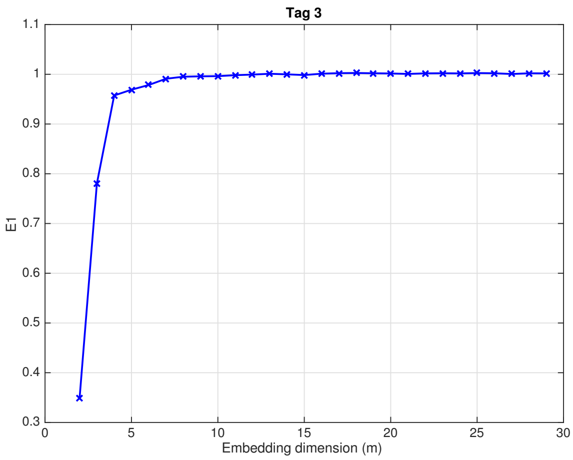

Fig. 3. Determining the embedding dimensions

where $\|.\|$ represents vector norm operation; $y_i(m+1)$ is the i^{th} delay vector with $m+1$ dimensions and $n(i,m)$ is integer in set $(1 \geq n \geq N - m\tau)$ such that $y_n(m)$ is the nearest neighbor of $y_i(m)$ in the m dimensional space.

The mean $E(m)$ of all $a(i,m)$s is defined as

$$E(m) = \frac{1}{N - m\tau} \sum_{i=1}^{N-m\tau} a(i,m) \tag{6}$$

The variation in E for the increase in embedding dimension to $m+1$ is given by the quantity E_1 defined as

$$E_1(m) = \frac{E(m+1)}{E(m)} \tag{7}$$

Therefore when E_1 stops changing for some $m \geq m_0$; $m_0 + 1$ is taken as the minimum embedding dimension for the reliable phase space reconstruction [Cao (1997)]. The E_1 measure for one of the data (Tag 3) is plotted in Figure 3. It can be seen that the E_1 approaches the unity value after $m = 8$ and stays there for all subsequent values of m. Therefore embedding dimension $m > 8$ can be used for the correct analysis.

5. PROPOSED METHOD

The proposed method for the identification of the non-linearity induced oscillations and isolation of the source using DVV is listed below.

(1) Mean center and normalize the time series from the control loop to unit standard deviation.
(2) End match the data to avoid spurious modes in the surrogate. End matching requires that the difference between both initial and final values d_0 and difference between initial and final gradient d_1 is minimized. d_0 and d_1 are calculated using (8) [Thornhill (2005)].

$$d_0 = \frac{(x_i - x_{i+n-1})^2}{\sum_{j=i}^{i+n-1}(x_j - \bar{x})^2}$$
$$d_1 = \frac{[(x_{i+1} - x_i) - (x_{i+n-1} - x_{i+n-2})]^2}{\sum_{j=i}^{i+n-1}(x_j - \bar{x})^2} \tag{8}$$

where x_i is the i^{th} element and $\bar{x}$ is the mean of time series $x_i \ldots x_{i+n-1}$.

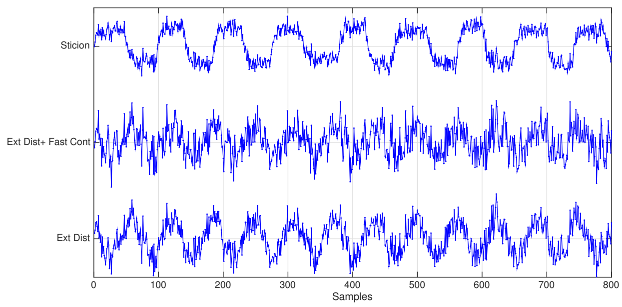

Fig. 4. Time trends from simulation example

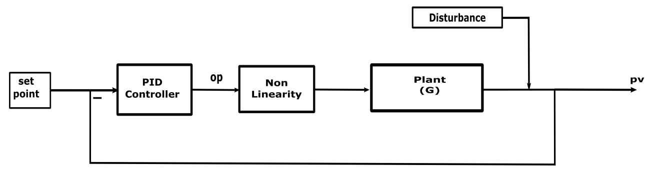

Fig. 5. Closed loop model

(3) Determine the minimum embedding dimension for the phase space reconstruction.
(4) Perform the DVV analysis on normalized, end matched data.
(5) Generate B surrogates of the loop data.
(6) Perform DVV analysis on all the surrogates.
(7) Compute the average target variance of all the surrogates.
(8) Compute the RMSE for input data and surrogates.
(9) Perform the rank based statistical test.
(10) Loops with RMSE rank $r > (B+1)(1-\alpha)$ are classified as non-linear at confidence level α.
(11) Perform steps 1-10 for all the loops under analysis.
(12) Once all the non-linear loops are identified look for the maximum RMSE value.
(13) The loop with maximum RMSE value is designated as the source of the non-linearity.

6. SIMULATION EXAMPLE

The simulation example used to distinguish non-linearity induced oscillations from the linear one is taken from our recent work [Aftab et al. (2016)]. The block diagram of the closed loop system is shown in Figure 6. Three scenarios, oscillations due to valve stiction(LuGre friction model), external sinusoidal disturbance and external dist added to a loop with a poorly tuned controller, are tested using the proposed method. Noise of variance 0.1 is also added to test the robustness of the proposed method. The time trends for these three test cases are shown in Figure 4.

Table 3. DVV analysis (Simulation example)

Case	rank	type	RMSE
Stiction	100	Non-Linear	0.042
External Dist+Fast Cont	49	Linear	–
Ext Dist	62	Linear	–

The results of these three scenarios are summarized in Table 3. The method is able to classify the test scenarios correctly. The stiction case is correctly identified as non-linear; the fact endorsed both by the rank statistics and

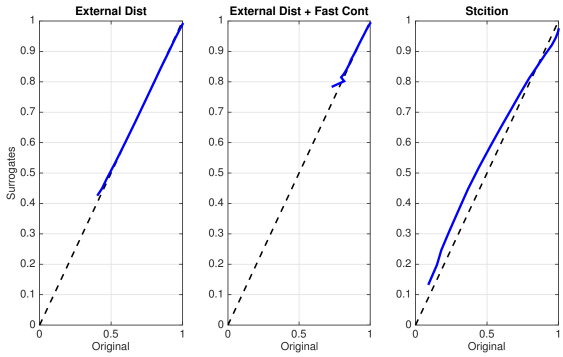

Fig. 6. DVV scatter plots for simulation example

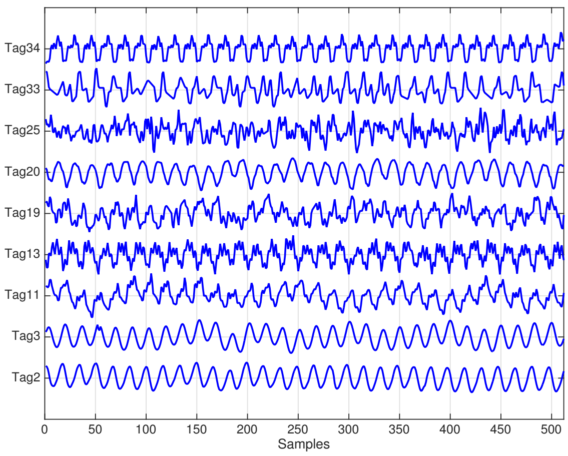

Fig. 7. Time trends of industrial data

the DVV scatter plot. The application of the proposed method to the industrial case study is presented next.

7. INDUSTRIAL CASE STUDY

The case study is taken from Thornhill (2005) and Zang and Howell (2005). The analysis of 37 different tags reveal that a plant wide oscillation with fundamental frequency of $0.06min^{-1}$ is found to be present in 9 Tags. The objective is to analyze these tags and to find the root cause of oscillation. The time trends of these 9 Tags are shown in Figure 7.

All these tags are analyzed using the DVV method proposed in this paper and results are summarized in Table 4. The analysis consists of two parts.

Table 4. DVV analysis (industrial case study)

Tag	rank	type	RMSE
Tag 2	79	Linear	–
Tag 3	38	Linear	–
Tag 11	95	Non-Linear	0.0378
Tag 13	99	Non-Linear	0.0415
Tag 19	94	Linear	0.0318
Tag 20	4	Linear	–
Tag 25	86	Linear	–
Tag 33	92	Non-Linear	0.0299
Tag 34	100	Non-Linear	0.0743

7.1 Detection of Non-Linearity

First of all the loops are analyzed for the presence of non-linearity. The DVV analysis points out that the Tags 11, 13, 19, 33, and 34 exhibit non-linearity induced oscillations; whereas no non-linearity is detected in other tags using the rank based statistics. The findings confirm the already reported findings that the oscillations are due to non-linearity in the control loops. The results are similar to the ones reported by Thornhill (2005) and Zang and Howell (2005) except that Tag19 which is classified here as non-linear.

7.2 Isolating the Source of Non-Linearity

Once it is concluded that the plant wide oscillation with frequency $0.06min^{-1}$ is due to non-linearity, the next step is to find the source of this non-linearity. As discussed in section 2.3 that different components of plant act as mechanical filter and tend to filter the higher order harmonics and the signature of non-linearity loses strength as we move away from the source. This fact dictates that the most non-linear element will be the source of the oscillation.

The RMSE values from the DVV analysis (Table 4) reveal that Tag34 has the largest non-linearity and is thus the root cause of the oscillation. The same finding is reported in other studies conducted by Thornhill (2005) for the same data set.

8. CONCLUSIONS

A novel approach based on delay vector variance (DVV) is presented to detect and isolate non-linearities in control loops. The proposed scheme is simple and effective; and doesn't require tuning of the parameters like prediction horizon, number of nearest neighbors etc as the case with existing methods like the one presented by Thornhill (2005). An automatic method to find the suitable embedding dimensions is also augmented with the DVV analysis to make it more robust than existing tools. Moreover, instead of assuming the Gaussian distribution of the test statistics, a non-parametric rank based statistic is used to test the Null hypothesis. The efficacy of the proposed scheme is established using both simulation and industrial case studies.

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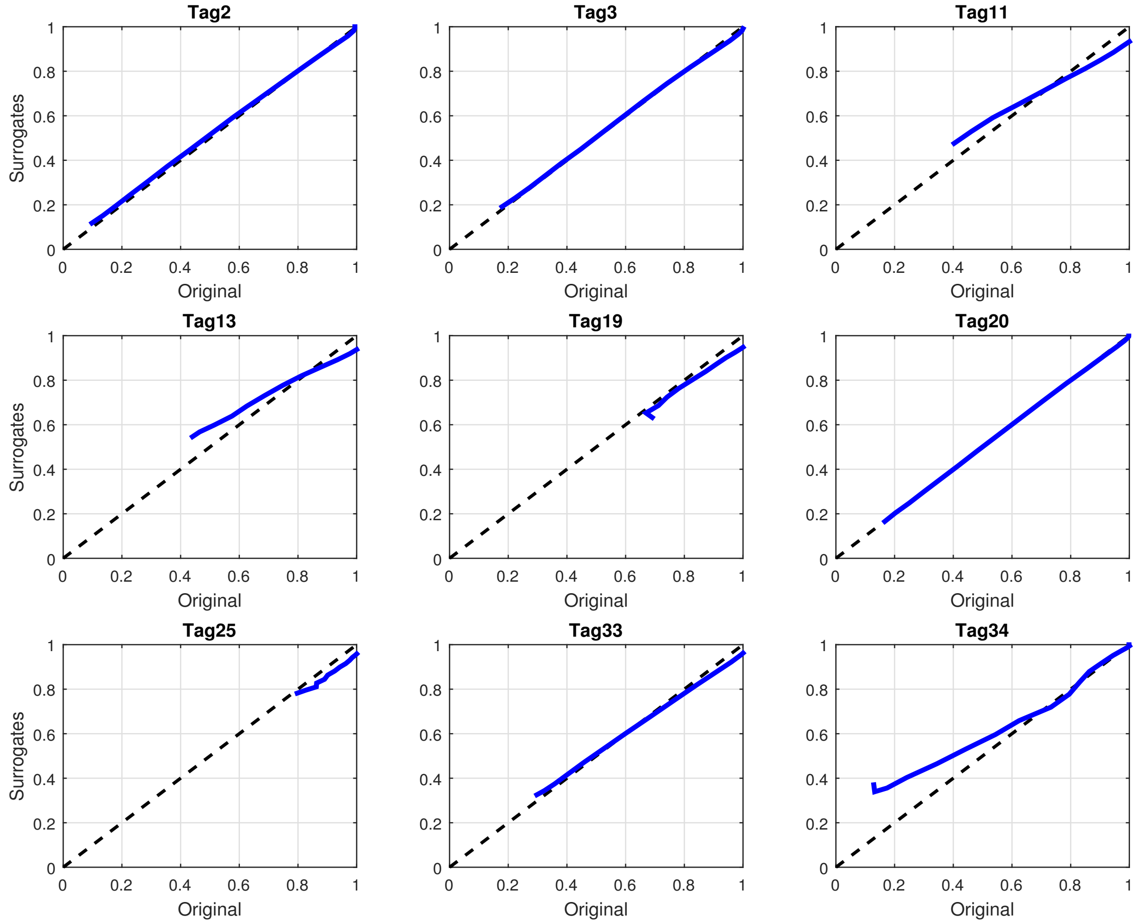

Fig. 8. DVV scatter plot (industrial case study)

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