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Joint maintenance and controller reconfiguration for a gradually deteriorating control system

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Abstract

This paper tackles with the maintenance decision-making of gradually deteriorating control systems. The main feature of these systems is their ability within a closed-loop to drive actuators in order to set the controlled process in a given state. Usually, the literature deals with the diagnosis of faults of a control system and the means for recovering system performances after their appearance. The controller reconfiguration is one of these means. The root cause of a fault is rarely argued nor its occurrence time. Before designing maintenance policies, this paper proposes a stochastic modeling framework of a degrading control system focusing on the actuator deterioration. It is assumed a close relationship between the controller setting, the actuator deterioration and finally its faulty situations. These latter are related to given degradation thresholds. Due to the stochastic nature of the deterioration, the corresponding hitting times are thus random. The obtained models allow to assess the conditional reliability of the actuator and then the prognosis of its residual useful life namely the RUL. This RUL is finally used to achieve two maintenance policies based upon the reconfiguration of the controller considered as a new maintenance action.

Keywords

Actuator, Condition-based maintenance, Control system, Degradation, Loss of effectiveness, Prognosis, Stochastic process

Introduction

This paper is connected to the field of control systems. In few words, the structure of such a system is a feedback closed loop within which the desired process output is compared with the real one. A difference between both may exist due for instance to disturbances in the system, measurement errors or faulty components. Whatever the reason, this difference supplies a controller whose aim is to adjust an input signal controlling a set of actuators. By their action, the objective is to reduce this difference and to achieve the desired process output in a relatively short time. Here, this work focuses on faulty components and especially actuators. The main reason is that an actuator is a costly component in the interface between the controller and the system. Generally speaking, the research effort is devoted to the design of powerful algorithms coupled with automatic fault detection methods to overcome a faulty situation Zhang (2008). When an actuator is in such a case, it experiences difficulties in implementing the control input. The aim is then to smartly balance the control effort with the reconfiguration of the controller in order to preserve the desired dynamic and static performances of the system as well as its stability. The control is then fault tolerant and the literature deals with fault-tolerant control systems or FTCS Zhang (2002).

The control reconfigurability allows performance restoration in the presence of faults whereas the reliability of a system is its ability to perform a required function over a stated period of time under given operational conditions. In Zhang (2008), Zhang is wondering if such FTCS techniques have an influence on the system reliability and if there exists a mean to measure it. As a matter of fact, the reliability of such a system is never seen as an objective criterion when designing FTCS Wu (2004). In order to give some details in response to these questions, the first goal of this paper is to apply reliability theory to control design engineering with a consistent modeling framework

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for deteriorating control systems. The actuator is assumed to be the only degrading component. In FTCS literature, an actuator fault is modeled with a loss of effectiveness to fully implement the control input. This loss is usually (i) independent on the controller setting, (ii) independent on any kind of physical degradation process and above all (iii) governed by deterministic fault occurrence dates Casavola (2006). Here, it is assumed a close relationship between the control input, the actuator degradation and its loss of effectiveness. The control input is seen as a source of stress deteriorating the actuator. First, a stochastic modeling framework is considered to take into account the usage of a control system. Each variation of the usage implies a variation of the control input. The actuator needs to provide an additional effort. For instance in Khelassi et al. (2010), the actuator lifetime is modeled with a failure rate according to its variable usage during the mission. Second, stochastic degradation models are integrated to describe the physical deterioration of an actuator due to the control input and then its loss of effectiveness. In Grosso et al. (2012); Pereira et al. (2010), the authors assume a deterministic relationship between the degradation and the control input. Unfortunately, their modeling does not account for any uncertainty nor actuator loss of effectiveness.

The second objective of this paper is to study the maintenance of such a system. As stated above, in works related to FTCS, the control reconfigurability enables to recover performances after the detection of a fault. Here, this possibility is used as a third maintenance action completing the classic preventive and corrective ones. The modelling framework allows to assess the conditional reliability of the actuator used to trigger either the preventive action or the update of the controller setting.

This paper is organized as follows. In Section , control systems are described in general terms. The criteria for assessing the performances of such a system are given as well as a brief description of a LQR controller commonly used in industrial automation. A model for the actuator loss of effectiveness is also proposed. In Section , the way an actuator accumulates degradation is depicted as the combination of two independent wear and shock processes. In Section , the conditional reliability of an actuator as well as its remaining useful life are derived and used in Section for decision making purposes in two condition-based maintenance policies. These policies include an additional action with the possibility to update the controller setting. The case study of a linear motion system is depicted in Section . The impact of this new action on the maintenance cost and on the control performances is assessed.

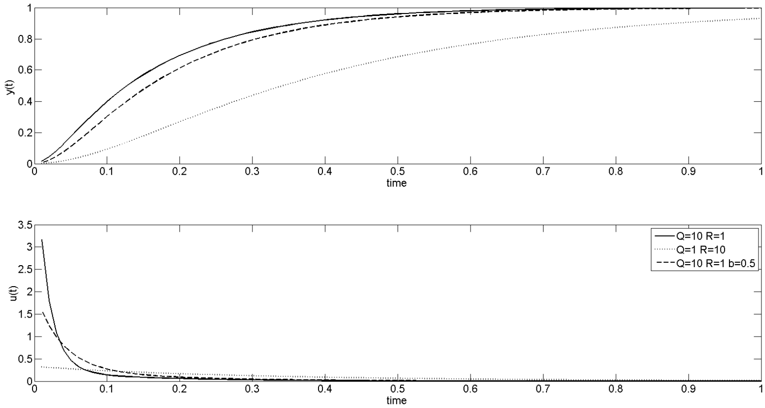

Figure 1. System output $y(t)$ (up) and corresponding control input $u(t)$ (down) for two LQR controller settings. The setpoint $s(t)$ (not shown here) is a Heaviside step $s(t) = 1 \forall t \geq 0$.

Control system

LQR controller

In the context of industrial automation, systems are governed with dedicated controllers. The aim of a controller is to allow a system output $y(t)$ to track a given target value namely a setpoint $s(t)$. The difference between the measured variable $y(t)$ and the desired setpoint $s(t)$ allows the controller to generate a control input $u(t)$ that is applied to a set of actuators. The aim of actuators is then to put the system in a given state.
The performances of a control system are commonly deemed with three criteria that are the rise time t_r, the settling time t_{st} and the steady state error position ε_p Ogata (2001). They are defined as follows:

$$\begin{aligned} t_r &= \inf\{t : y(t) = 0.9\, y_\infty\} - \inf\{t : y(t) = 0.1\, y_\infty\} \\ t_{st} &= \inf\{t : \frac{y(s) - y_\infty}{y_\infty} <= 5\%\} \;\forall\, s \geq t \\ \varepsilon_p &= s_\infty - y_\infty \end{aligned} \tag{1}$$

where y_∞ and s_∞ are respectively the asymptotic values of $y(t)$ and $s(t)$, assuming they exist i.e. the system is stable.

LQR controller standing for Linear Quadratic Regulator is one means for controlling a system Lavretsky and Wise (2013). The control input $u(t)$ is obtained by minimizing a quadratic criterion J on a time horizon $[t_0, t_h]$ so that

$$J(u) = \int_{t_0}^{t_h} [y(t) - s(t)]^T \; Q \; [y(t) - s(t)] + u(t)^T \; R \; u(t) \, dt \tag{2}$$

The goal of matrices Q and R is to find a tradeoff between desired performances and actuator effort. Figure 1 depicts the effect of two settings of a controller for a given system (solid and dotted lines). It can be seen that a large magnitude of Q implies higher dynamic and static performances. In this case, the control input $u(t)$ is such that the actuator is strongly solicited in a very short time. The potential consequence may be a reduction of its lifetime. On the contrary, the actuator is smoothly solicited with a large magnitude of R. The actuator health is preserved but at the expense of reduced performances Gokdere et al. (2006).

Actuator loss of effectiveness model

A control system is commonly described with a linear time invariant state space representation given by

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) \tag{3}$$

$$y(t) = \mathbf{C}x(t) \tag{4}$$

with $x(t) \in \mathbb{R}^n$ standing for the state of the system at time t and $u(t) \in \mathbb{R}^p$, $y(t) \in \mathbb{R}^q$ the multi-dimensional input and output Kwakernaak (1972). $\mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times p}$, $\mathbf{C} \in \mathbb{R}^{q \times n}$ are assumed constant real matrices standing respectively for the state, the command and the output matrix. In such a representation, $x(t)$ is not related to a physical state as an intrinsic deterioration but rather to a set of features (e.g angular speed, engine torque for an electric motor) contributing to the system output $y(t)$.
The literature dealing with control systems focuses mainly on the loss of effectiveness of the actuator for implementing the control input $u(t)$. Once this faulty situation is diagnosed, the aim of the related works is to update the controller setting in order to counteract the fault and then to preserve the initial performances. These systems are then referred to Fault Tolerant Control Systems or FTCS Zhang (2008, 2002). It is worth noticing that the root cause of a fault and its occurrence time are never argued in the existing works. The model of loss of effectiveness proposed hereafter intends to give more realism to the faulty situation.

Once a fault appears, the state equation (3) is reformulated with

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}[\mathbf{I} - \text{diag}(l_1, \ldots, l_p)]\, u(t). \tag{5}$$

It means that each actuator i implements $(1 - l_i)u_i(t)$ instead of the full control input $u_i(t)$ Zhao (1998). $l_i \in [0, 1]$ is the loss of effectiveness.

The first assumption is about the ability for an actuator to implement $u(t)$ depending on its level of physical degradation $D(t)$. In this paper, a fault is the symptom of a deterioration accumulated with time and appears as soon as a given degradation level is hit. The occurrence time is then the hitting time of this level. Generally, only one fault is considered in the literature dealing with FTCS. Here for more realism, the model assumes a sequence of faulty situations from the least to the most severe with three degradation levels L_1, L_2 and L_f. Initially, $l_i = 0$ the actuator implements fully the control input $u(t)$ while it is being degraded. Once L_1 is reached, the actuator experiences its first fault with $l_i = 1 - b_i$. In such a case, the actuator i becomes sensitive to its own deterioration. Then, a second fault appears when $D(t) \geq L_2$. The actuator implements $u(t)$ with $l_i = 1 - a_i$ in a minimal way until the level of failure L_f is reached. In this latter case, the actuator is unable to carry out its role. An example of loss of effectiveness is illustrated in the figure 1 (dashed line) with a loss of 50%. The aim of the proposed model is to align the sequence of faults shifting toward the failure with the physical degradation process $D(t)$. It is obviously feasible to take into account more degradation levels and faults but this possibility could complicate some calculations such as the actuator reliability $R(t)$ which will be detailed further.

The second assumption claims that the degradation process $D(t)$ is of random nature Bérenguer and Grall (2008); Virkler et al. (1979). This assumption allows to state a generic model for the degradation phenomenon and thus to be independent of material and technological features as well as to take uncertainties into account Sobczyk (1992). Only few works tackle the degradation of an actuator as a random phenomenon. In Weber et al. (2012a), the failure of an actuator is modeled with a lifetime distribution whereas in Lefebvre (1996) a stochastic process is used. In any case, the successive hitting times of degradation thresholds L_1, L_2 and L_f are then random while in FTCS literature an arbitrary date is often chosen when a faulty situation occurs Casavola (2006).

Actuator degradation model

As it can be seen in figure 1, the actuator needs to provide a transient effort when it is solicited during a setpoint change. This effort depends obviously on the controller setting and also on the potential loss of effectiveness for implementing $u(t)$. Starting from this observation, the control input $u(t)$ is then assumed playing a key role in the degradation process $D(t)$. Some papers share the same view on that subject. In Vieira et al. (2015), the degradation increment is proportional to the control effort $u(t)$ with a constant proportionality coefficient. The authors use the degradation information for designing a life extending control of a linear motion system. In Pereira et al. (2010); Grosso et al. (2012), the actuator degradation has a positive deterministic progression. The degradation increment depends on $u(t)$ as well as on its variations. Again, this valuable information is used for designing a controller whose aim is to achieve a mission without failure or to reach the next date of a scheduled maintenance. In Brown et al. (2009), a brushless DC motor is selected as the component of interest of an electro-mechanical system. Its failure mechanism is the breakdown of the winding insulation due to temperature. The estimated level of degradation is proportional to the temperature according to the Arrhenius law. One means for controlling the temperature and then to slow down the winding deterioration is to smartly control the motor current $u(t)$. These few examples show that the control input $u(t)$ is a real source of stress for the actuator and has a substantial impact on its degradation $D(t)$ assumed in the following as a continuous monotone increasing degradation phenomenon with time.

In order to model the impact of $u(t)$ on $D(t)$, the actuator is subject to independent markovian wear and shock processes respectively denoted $Z(t)$ and $S(t)$. For that, the degradation model developed in Kharoufeh et al. (2006); Kharoufeh (2003) is considered. The wear phenomenon $Z(t)$ is an accumulation of positive, independent, homogeneous in time degradation increments. This is the intrinsic deterioration when the actuator evolves around its operating setpoint in steady state. It comes

$$
\begin{aligned}
&Z(0) = 0 \\
&Z(s) - Z(t) = Acc_{\Theta_1(s-t)} \quad \forall s > t \geq 0\cdot
\end{aligned}
\tag{6}
$$

The increment of deterioration between times t and s is a positive random variable $Acc_{\Theta_1(t-s)}$. It depends on the parameter $\Theta_1(t-s)$, which is a function of the time interval $t-s$. The corresponding probability law is denoted by $f_{Acc}(.;\Theta_1(t-s))$.

The second process $S(t)$ corresponds to a discret time shock phenomenon. Each time the system is solicited with a new operating setpoint, the actuator needs to provide an additional effort. This energy spent in a relatively short time yields a variation of degradation resumed as a shock with amplitude Y occurring at setpoint change. Another way for modeling the impact of this additional effort would be to consider degradation increments with a parameter $\Theta_1(t-s)$ modified during the effort Ponchet et al. (2010). For a given controller setting, the different Y_i are i.i.d random variables with distribution $f_Y(.;\Theta_u)$. Up to time t, the total degradation due to the second phenomenon is then the sum $S(t)$ such as

$$S(t) = \sum_{j=0}^{N_t} Y_j \quad \text{with } Y_0 = 0\cdot \tag{7}$$

Θ_u is the intensity of shocks experienced by the actuator during a setpoint change. This intensity depends on the controller setting. For example, in case of a LQR controller, a large magnitude of Q involves a high energy expense and then a shock with a large amplitude. Whatever is the controller setting, Θ_u is also impacted by the potential loss of effectiveness of the actuator. This loss of ability to fully implement the control input $u(t)$ is similar to a gain reduction (see figure 1). In such a case, the actuator experiences the shocks with a decreasing amplitude as it deteriorates so that

$$\Theta_u = \Theta_u^1 \mathbb{I}_{\{D \leq L_1\}} + \Theta_u^2 \mathbb{I}_{\{L_1 < D \leq L_2\}} + \Theta_u^3 \mathbb{I}_{\{D > L_2\}} \tag{8}$$

where the Θ_u^i, $i = 1, 2, 3$ are parameters related to the average amplitude of shocks in the three successive degradation phases described in the previous section.

The frequency of use of a control system such as an automated machine may be highly variable or completely deterministic. It depends on the type of industry (e.g. packaging, metallurgy, automotive). To keep within an overall framework, the different changes of setpoint corresponding to the usage profile of a control system are assumed distributed with a homogeneous Poisson process $\{N_t : t >= 0\}$ with intensity μ Babai et al. (2011). The probability that the actuator experiences n shocks (i.e n solicitations) in a time interval $\tau > 0$ is

$$\mathbb{P}(N_{t+\tau} - N_t = n) = \frac{(\mu\tau)^n}{n!} e^{-\mu\tau} \quad \forall t \geq 0\cdot \tag{9}$$

The damage jumps Y_i coupled with Poisson arrivals yield a well-known compound Poisson process $\{S(t) : t \geq 0\}$.

Up to now, the degradation model for $D(t) = Z(t) + S(t)$ is in a generic form. The next step is to choose some distributions for $f_{Acc}(.;\Theta_1(t-s))$ and $f_Y(.;\Theta_u)$. According to the features of $Z(t)$ detailed above, a Gamma process seems to be a good candidate for characterizing a deterioration increment $Acc_{\Theta_1(t-s)}$ van Noortwijk (2009). It allows tractable computations and also to be applied easily to real data Grall-Maës (2012). On a time interval $t-s$, an increment is a Gamma distributed r.v. with parameters $\Theta_1(t-s) = \big(\alpha.(t-s), \beta\big)$ such that

$$f_{Acc_{\Theta_1(t-s)}}(r) = \frac{(\frac{1}{\beta})^{\alpha(t-s)}}{\Gamma(\alpha(t-s))} r^{\alpha(t-s)-1} e^{\frac{-r}{\beta}} \quad r > 0 \tag{10}$$

Γ is the Gamma function defined with $\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt$. α stands for the shape parameter and β for the scale parameter.
The damage jump Y is in essence a positive real value. Hence, without loss of generality, Y is assumed exponentially distributed with intensity Θ_u and the distribution is

$$f_Y(r) = \Theta_u \, e^{-\Theta_u \, r} \quad r > 0 \tag{11}$$

Obviously, the availability of degradation data combined with expert judgement should refine our choice of distributions for Y and Z. A few authors made a different choice. For instance in Weber et al. (2012b), a proportional hazard model is considered for modelling the lifetime of an actuator. The failure rate is modified accordingly with the energy of the control input $u(t)$. In Lefebvre (1996); Rishel (1991), the wear $Z(t)$ is modeled with a stochastic process having continuous increasing paths coupled with a second random process of influencing environmental variables as $u(t)$. For now, the given distributions allow to state the probability that the total amount of damage D incurred at time t is strictly lower than a level L for a given controller setting Θ_u as follows

$$\mathcal{F}(L,\Theta_1(t),\Theta_u) = \sum_{n=0}^{\infty} \frac{(\mu t)^n}{n!} e^{-\mu t} \int_0^L \int_0^r \frac{(1/\beta)^{\alpha t}}{\Gamma(\alpha t)} (r-h)^{\alpha t - 1} e^{-(r-h)/\beta} \frac{\Theta_u^n}{\Gamma(n)} h^{n-1} e^{-h\Theta_u} \, dh \, dr. \tag{12}$$

Actuator reliability and its RUL

L_f is the degradation level for which the actuator is unable to carry out its mission. The corresponding failure time T_f is the first hitting time defined with

$$T_f = \inf\{t : D(t) \geq L_f\} \tag{13}$$

$D(t)$ is assumed observable and measurable with a dedicated monitoring equipment. Thus, if the actuator is still working at time t_i with $D(t_i) = d_i$ then the conditional reliability $R_i(t)$ is

$$R_i(t) = \mathbb{P}(T_f > t \,|D(t_i) = d_i) = \mathbb{P}(D(t) < L_f \,|D(t_i) = d_i) \tag{14}$$

The assessment of the actuator reliability at time t depends on the effectiveness phase in which the degradation level d_i is measured. Therefore, the conditional reliability is finally

$$R_i(t) = R_i^{\#1}(t)\mathbb{I}_{\{d_i \leq L_1\}} + R_i^{\#2}(t)\mathbb{I}_{\{L_1 < d_i \leq L_2\}} + R_i^{\#3}(t)\mathbb{I}_{\{d_i > L_2\}} \tag{15}$$

whose expansion yields

$$\begin{aligned} R_i(t) \simeq \Big\{ & \mathcal{F}(L_1 - d_i, \Theta_1\,(t - t_i), \Theta_u^1) \\ & + \int_{t_i}^{t} \mathcal{F}(L_2 - L_1, \Theta_1\,(t - r), \Theta_u^2)\, f_{T_1}^{\#1}(r)dr \\ & + \int_{t_i}^{t}\int_{t_i}^{v} \mathcal{F}(L_f - L_2, \Theta_1\,(t - v), \Theta_u^3) f_{T_1,T_2}^{\#1}(r,v)\,dr\,dv \Big\} \\ & + \Big\{ \mathcal{F}(L_2 - d_i, \Theta_1\,(t - t_i), \Theta_u^2) \\ & + \int_{t_i}^{t} \mathcal{F}(L_f - L_2, \Theta_1\,(t - v), \Theta_u^3)\, f_{T_2}^{\#2}(v)dv \Big\} \\ & + \Big\{ \mathcal{F}(L_f - d_i, \Theta_1\,(t - t_i), \Theta_u^3) \Big\} \end{aligned} \tag{16}$$

The degradation process $D(t)$ is the combination of two stochastic jump processes $Y(t)$ and $Z(t)$. The equation (16) is an approximation of the conditional reliability $R_i(t)$. It can be derive assuming that the difference $T_2 - T_1$ between the hitting times of levels L_2 and L_1 is approximated by the hitting time of the level $L_2 - L_1$ as explained in Bérenguer et al. (2003).

Due to the random nature of $D(t)$, the residual useful lifetime of an actuator at time r noted RUL_r is then a stochastic variable Lorton et al. (2013); Liao and Tian (2013) so that

$$RUL_r = \inf\{t \geq r, D(t) \geq L_f\} - r \tag{17}$$

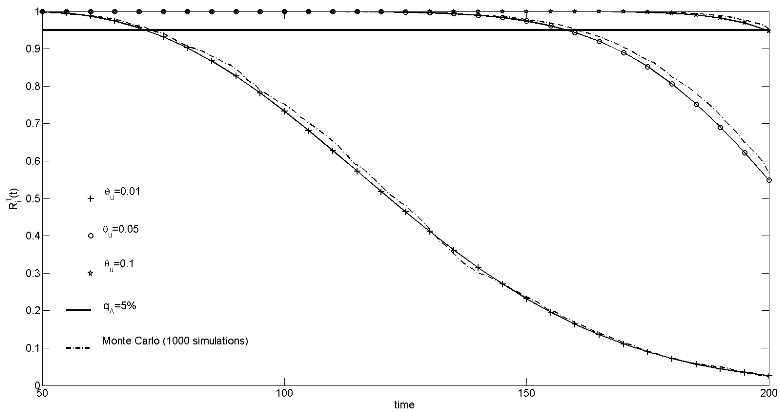

Figure 2. Reliability of the actuator when its degradation level $d_i = 50$ is measured in phase 1 at $t_i = 50$. $\alpha = 2, \beta = 1, a = 0.5, b = 0.75, L_1 = 100, L_2 = 300, L_f = 500$.

Given the knowledge of the degradation level d_i measured at time t_i, the probability law of RUL_i is the conditional probability such that

$$Pr(RUL_i > u \,|\, D(t_i) = d_i) = Pr(T_f - t_i > u \,|\, D(t_i) = d_i). \tag{18}$$

Its cdf is $1 - R_i(t)$ with $R_i(t)$ given by (16). Finally, for decision making purposes, a quantile q_A of RUL_i is considered such as

$$\text{QRUL}_i(q_A) = \inf\{t : R_i(t) \leq (1 - q_A)\} - t_i \tag{19}$$

Figure 2 depicts the reliability of an actuator $R_i^{\#1}(t)$ obtained from equation (16) with a degradation level $d_i = 50$ measured at $t_i = 50$ in phase 1. The densities f_{T1},f_{T2} and $f_{T1,T2}$ are numerically computed with partial derivatives of the function $\mathcal{F}$ (12). For example, the joint density $f_{T1,T2}$ is given as follows:

$$f^{\#1}_{T_1,T_2}(r,v) \simeq \frac{\mathcal{F}(L_1 - di, \Theta_1\,(r + \Delta t - t_i), \Theta_u^1) - \mathcal{F}(L_1 - di, \Theta_1\,(r - t_i), \Theta_u^1)}{\Delta t} \times \frac{\mathcal{F}(L_2 - L_1, \Theta_1\,(v + \Delta t - r), \Theta_u^2) - \mathcal{F}(L_2 - L_1, \Theta_1\,(v - r), \Theta_u^2)}{\Delta t} \tag{20}$$

The numerical results compared with the Monte Carlo simulations state (i) a good adequation between both for different values of intensity of shocks θ_u despite the approximation used for assessing $R_i(t)$ and (ii) that the remaining useful life given a quantile q_A is easily obtained with equation (19).

Maintenance policies

Two maintenance policies are proposed in this paper. They are based on the reconfiguration of the controller in order to be less stressful for the actuator and then to extend its lifetime. Starting from the initial controller setting, we assume two other feasible reconfigurations. Each time a reconfiguration is performed, the amplitude Y of future shocks is expected to decrease. According to the actuator degradation model, this amplitude is exponentially distributed with intensity Θ_u. Thus, the value of Θ_u after the reconfiguration step is greater than Θ_u^- the value just before in such a way that

$$\Theta_u = \varphi.\Theta_u^- \quad \text{with } \varphi > 1 \tag{21}$$

In Khelassi et al. (2010), the energy of the signal $u(t)$ is seen as the source of stress deteriorating the actuator. For instance, this energy cost may increase the temperature of the winding insulation of an electric motor or the temperature of its ball bearing. Here, the reconfiguration procedure is such that this energy follows the same linear rule (21).
The controller reconfiguration is seen as a third action completing the commonly used preventive and corrective actions. After replacements the actuator is assumed as good as new. The inspection cost is much lower than the replacement cost and the corrective replacement is carried out as soon as a failure is detected.
Different indicators are proposed in order to rate each maintenance policy and in particular the impact of the controller setting. The maintenance policy is assessed with the average maintenance cost per time unit

$$C_{av} = \left(C_c\, N_c + C_p\, N_p + C_i\, N_i + C_d\, d_d\right) / \, T_{av} \tag{22}$$

with $N_c = \mathbb{E}\left[N_c(t)\right]$ the average number of corrective replacements, $N_p = \mathbb{E}\left[N_p(t)\right]$ the average number of preventive replacements, $N_i = \mathbb{E}\left[N_i(t)\right]$ the average number of inspections, d_d the average unavailability duration of the actuator and $T_{av} = \mathbb{E}\left[T\right]$ the average duration of a renewal cycle. C_c and C_p stand for the unit corrective and preventive replacement costs, C_i for the unit inspection cost and C_d for the unavailability

cost per time unit. The controller reconfiguration is assumed to be cheap and its cost will be neglected.
Lastly, the static and dynamic performances of the control system are deemed with a dedicated indicator $\text{Perf}_{\text{av}} = \mathbb{E}\left[\text{Perf}(t_{sc})\right]$. $\text{Perf}(t_{sc})$ is assessed at each setpoint change t_{sc} and is a combination of the three criteria t_r, t_{st} and ε_p. It remains equal to 1 as long as there is no loss of actuator effectiveness nor controller reconfiguration.

The two proposed maintenance policies are described in the next two sections.

RUL-based maintenance policy

In the framework of the first maintenance policy, the actuator is inspected only when it is in faulty situations. Since the loss of effectiveness model assumes two faults, the controller may be updated at the most twice. This scenario of inspection matches the state diagnosis of a control system which is triggered following the appearance of a fault. Two decision thresholds are given. The first one $\text{RUL}_{\text{rconf}}$ is related to the controller reconfiguration and the second one RUL_{prev} to the preventive replacement. For a given quantile q_A, the following cases are considered

- if $d_i < L_f$ and $\text{QRUL}_i(q_A) \leq \text{RUL}_{\text{prev}}$ then a preventive maintenance is triggered with a cost C_p. $\text{QRUL}_i(q_A)$ is calculated with equation (19).
- if $d_i < L_f$ and $\text{RUL}_{\text{rconf}} \geq \text{QRUL}_i(q_A) > \text{RUL}_{\text{prev}}$ then the controller is updated.
- if $d_i < L_f$ and $\text{QRUL}_i(q_A) > \text{RUL}_{\text{rconf}}$ then nothing is done. The system is governed with the same controller until the next inspection t_{i+1}.
- if $d_i \geq L_f$ then the actuator has already failed and a corrective replacement is carried out with a cost C_c

(ρ, Δ) maintenance policy

In the framework of the second maintenance policy, the actuator is periodically inspected every Δ time units. At $t_i = i.\Delta$, a measure $D(t_i) = d_i$ is available. For a fixed threshold ρ, the decision rules are as follows

- if $d_i < L_f$ and $R_i(t_i + \Delta) > 1 - \rho$ then the decision is postponed until the next inspection t_{i+1}. $R_i(t)$ is assessed with equation (16).
- if $d_i < L_f$ and $R_i(t_i + \Delta) \leq 1 - \rho$ then the actuator is preventively replaced with a cost C_p unless a controller reconfiguration is feasible with a setting satisfying the previous rule.

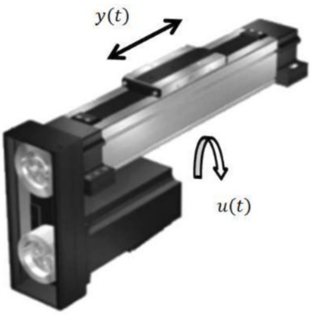

Figure 3. Drilling unit

- if $d_i \geq L_f$ then the actuator has already failed and a corrective replacement is carried out with a cost C_c

Numerical studies

Case study

A classic problem of position control is here considered. The studied system is a drilling unit whose role is to move forward in order to drill a hole in a part or component and then to move backward to release the drilled part. The incoming flow of parts is assumed random as is generally the case in the manufacturing industry. As it can be seen in figure 3, the drilling unit is moved with a brushless DC motor through a belt transmission. The process output $y(t)$ is the feed speed of the drilling unit. The control input $u(t)$ is the angular speed of the electric motor providing the necessary kinetic energy. $s(t)$ is the desired feed speed. The feed speed of such a machine is the key point of a satisfactory drilling quality Conrad and McClamroch (1986). In nominal mode, the kinetic energy is fully transferred. The motor and its ball bearing ensuring the link with the belt is the actuator. We assume that the ball bearing is the only degrading component whose deterioration implies a decreasing torque and then a loss of kinetic energy Nandi et al. (2005). In a real situation, vibration data would be gathered from the motor shaft and processed with the aim to detect typical vibration signatures characterizing a level of degradation Mohanty et al. (2014). Finally, a set of electrical and mechanical differential equations (not provided here) allows to model the drilling machine after a linearization procedure. It follows the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ for the corresponding state representation.

Degradation thresholds			Actuator effectiveness	
L_1	L_2	L_f	a	b
100	300	500	0.5	0.75

Gamma process		Maintenance policy			
α	β	C_c	C_p	C_i	φ
2	1	100	80	30	2

Poisson process intensity μ	T_{span}	q_A	(Q, R)
0.05	200	5%	$(1, 1)$

Table 1. Simulation data

This latter enables to simulate the behavior of the machine so as to assess its static and dynamic performances ε_p,t_r, t_{st}.

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -10 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \mathbf{C}^T = \begin{pmatrix} 1 \\ -10/9 \\ 1/9 \end{pmatrix}$$

Three LQR controller settings are available. The mission starts with an initial setting $(Q = 1, R = 1)$ corresponding to the most stressful control for the actuator. The intensity of shocks is assumed to be $\Theta_u = 0.05$. According to the proposed maintenance policies, the setting may be shifted toward a smooth control with $(Q = 1, R = 3) \Leftrightarrow \Theta_u = 0.1$ then to a non stressful one with $(Q = 1, R = 7) \Leftrightarrow \Theta_u = 0.2$. These settings are chosen so that the energy of the control $u(t)$ due to a Heaviside solicitation agrees with the same ratio $\varphi = 2$, (21). Lastly, the initial setting offers the best static and dynamic performances.

The simulation data are summarized in table 1.

RUL-based policy results

From figures 4, 5 and 6, the significant effect of the threshold RUL_{prev} respectively on N_c, N_p and T_{av} can be observed for $\text{RUL}_{\text{rconf}} = 0$. Hence when this threshold is low (e.g $\text{RUL}_{\text{prev}} = 20$), the preventive action is rarely triggered allowing a longer mission but with a higher risk for the actuator to fail and to be correctively replaced. In the case of a higher threshold (e.g $\text{RUL}_{\text{prev}} = 100$), it can be observed the opposite phenomenon. The average performances are also impacted by the choice of the RUL_{prev} value. Indeed, the more the mission lasts the more the actuator has the possibility to experience the

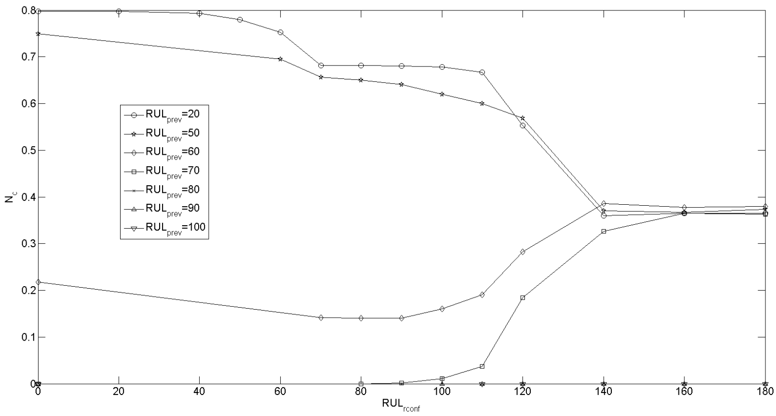

Figure 4. Average number of corrective replacements N_c vs threshold $\text{RUL}_{\text{rconf}}$

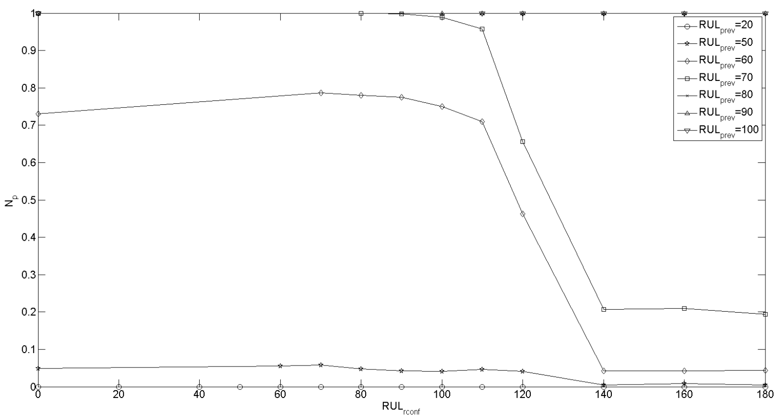

Figure 5. Average number of preventive replacements N_p vs threshold $\text{RUL}_{\text{rconf}}$

different faulty situations reducing the static and dynamic performances of the control system (dotted circle in figure 7).
The feature of the third maintenance action (i.e the controller reconfiguration) is to preserve the health of the actuator by decreasing the stress involved by the control input $u(t)$. Its degradation process is then slowed down. It can be seen that the higher the difference between the two thresholds the more the controller reconfiguration has

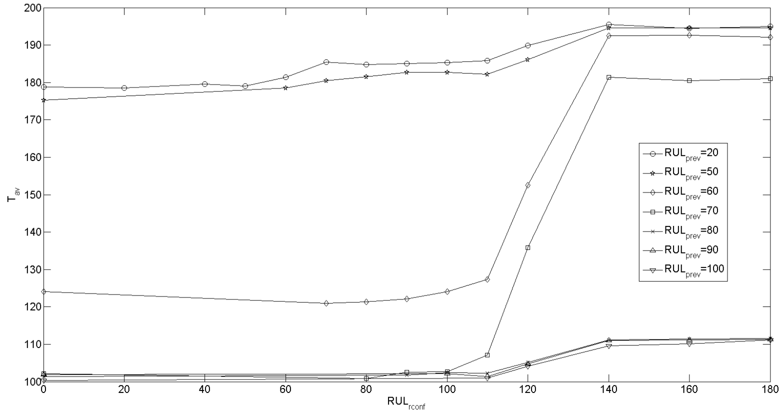

Figure 6. Average duration of a renewal cycle T_{av} vs threshold $\text{RUL}_{\text{rconf}}$

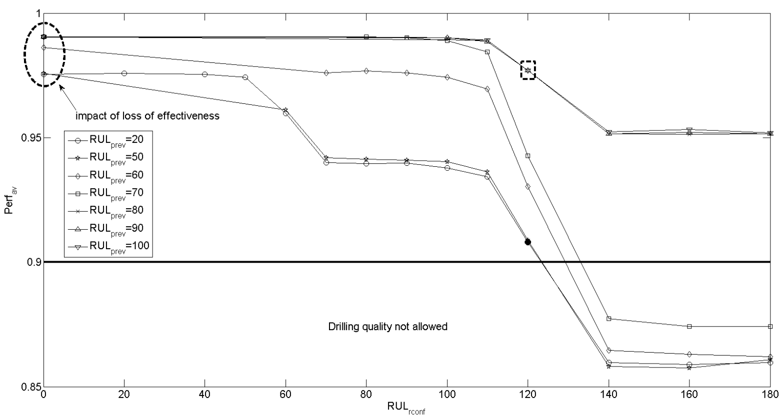

Figure 7. Average performances Perf_{av} vs threshold $\text{RUL}_{\text{rconf}}$

an influence on N_c. For instance with a threshold value $\text{RUL}_{\text{prev}} = 20$ or 50, the controller updates extend the lifetime of the actuator and the number of corrective replacements decrease. The threshold values $\text{RUL}_{\text{prev}} = 60$ and 70 are cases for which the reconfiguration delays the preventive replacement (figure 5) but a failure can not be avoided (figure 4). The last values $\text{RUL}_{\text{prev}} = 80$, 90 and 100 are cases for which the reconfiguration has not an impact on the average number of corrective and preventive

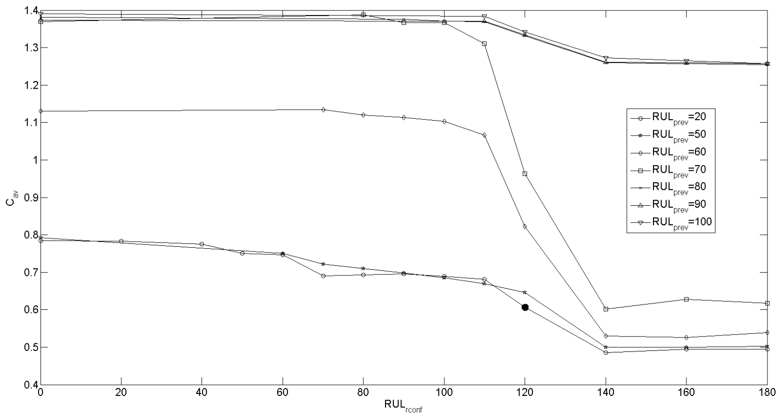

Figure 8. Average maintenance cost C_{av} vs threshold $\text{RUL}_{\text{rconf}}$ with $C_d = 0, C_i = 30, C_p = 80, C_c = 100.$

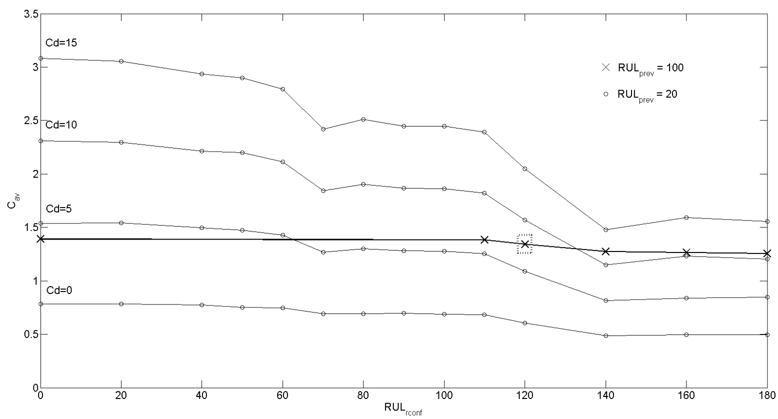

Figure 9. Impact of unavailability cost C_d on average maintenance cost vs threshold $\text{RUL}_{\text{rconf}}$ for $\text{RUL}_{\text{prev}} = 20$ and 100. $C_i = 30, C_p = 80, C_c = 100.$

replacements. The actuator is always preventively replaced.

In general, the main consequence of the third maintenance action is to delay the faulty situations and also the possibility of a failure. The maintenance cost tends to decrease (figures 8, 9) and the time cycle to increase (figure 6) but at the expense of decreasing

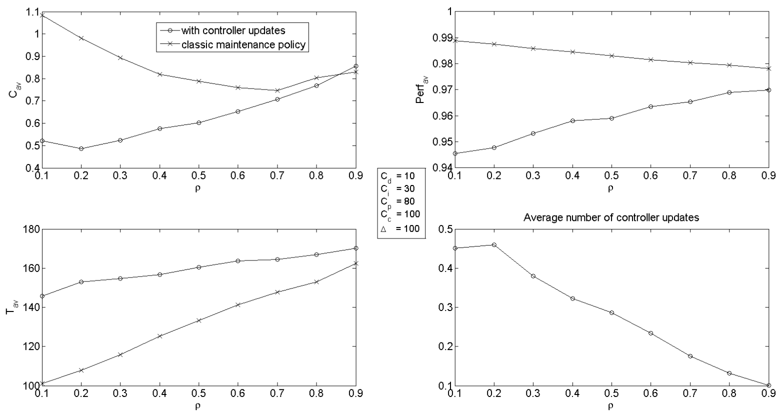

Figure 10. C_{av}, Perf$_{\text{av}}$, T_{av} and average number of updates vs threshold ρ for an inspection step $\Delta = 100$

performances (figure 7).
Since the actuator is inspected no more than twice, the maintenance cost of such a policy is highly sensitive to unavailability. In case of $C_d = 0$, a solution with $\text{RUL}_{\text{prev}} = 20$ and $\text{RUL}_{\text{rconf}} = 120$ may be a good trade off. Whatever is the cost value C_p, the preventive replacement is avoided with a total average maintenance cost $C_{av} = 0.6071$ representing a saving of 22.6% and with average performances $\text{Perf}_{\text{av}} = 0.91$ representing a loss of 6.8% (black filled circle in figures 8 and 7). This solution is obviously acceptable if and only if it does not affect the drilling quality. For instance, performances lower than a given threshold of 0.9 could be unauthorized because they induce a poor drilling quality of produced parts. Finally, for $C_d \neq 0$ figure 9 shows the limits of the successive reconfigurations of the controller when a low threshold value $\text{RUL}_{\text{prev}} = 20$ is chosen. Indeed, above a specific unavailability cost $C_d > 5$, a classic preventive maintenance is better. It offers an average maintenance cost $C_{av} = 1.341$ and $\text{Perf}_{\text{av}} = 0.977$ with $\text{RUL}_{\text{rconf}} = 120$ (dotted square in figures 9,7).

(ρ, Δ) *policy results*

Figure 10 depicts the evolution of C_{av}, Perf_{av} and T_{av} for different values of $\rho \in [0.1, 0.9]$ and a fixed inspection step $\Delta = 100$. As in the previous study, the benefits brought by the updating of the controller setting can be emphasized. The reconfiguration

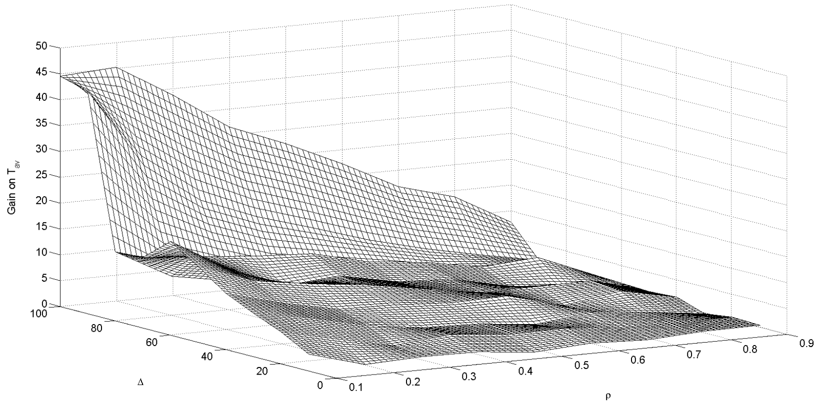

Figure 11. Gain on average renewal cycle T_{av} provided by controller updates

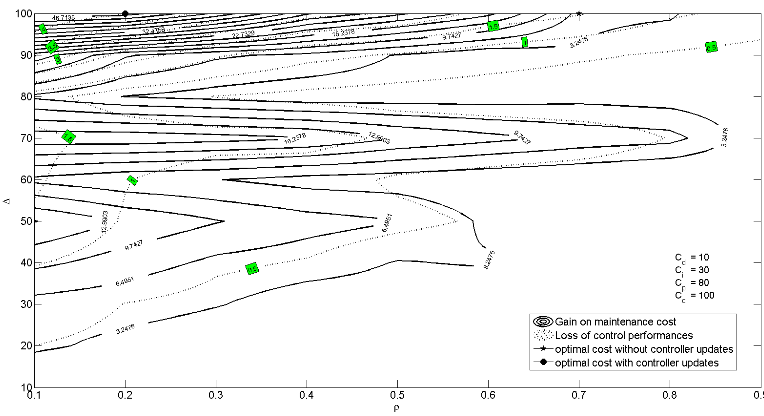

Figure 12. Bold (dotted) isolines delimit gain on maintenance cost (loss of control performances) areas provided by controller updates

extends the lifetime of the actuator by slowing down its physical deterioration. The duration of the mission is increased (see also figure 11) and the optimal cost tends to be lower than in a classic maintenance policy without reconfiguration. In the same figure (below right), it can also be noticed that as the risk of @failure raises the possibility to update the controller decreases. It leads to corrective or preventive replacements. In

	$\Theta_u = 0.05$	$\Theta_u = 0.1$	$\Theta_u = 0.2$	Combining updates
C_{av}	0.7472	0.2753	0.1530	0.4873
$Perf_{av}$	0.9804	0.8980	0.7867	0.9477

Table 2. Optimal maintenance cost and Control performances obtained with different controller settings

other words, for the given set of controllers, the opportunity to find one setting satisfying $R_i(t_i + \Delta) > 1 - \rho$ decreases with higher values of ρ. The same reason explains the increasing tendency of Perf_{av} that converges toward the level of performances without controller reconfiguration. For this latter case, the loss of effectiveness is the only cause of decreasing performances.

Lastly, figure 12 illustrates the gain on the maintenance cost as well as the loss of control performances provided by the successive controller reconfigurations. The optimal cost when no updates are considered is obtained with $(\rho, \Delta) = (0.7, 100)$ (black star). In such a case, $C_{av} = 0.7472$ and $\text{Perf}_{\text{av}} = 0.9804$. With the same inspection step Δ and risk level ρ, the reconfigurations of the controller allow to obtain $C_{av} = 0.7067$ representing a saving of 5.42% and $\text{Perf}_{\text{av}} = 0.9653$ representing a loss of 1.54%. The optimal cost with updates is assessed with $(\rho, \Delta) = (0.2, 100)$ (black disc) that yields $C_{av} = 0.4873$ and $\text{Perf}_{\text{av}} = 0.9477$ representing respectively a saving of 34.78% and a loss of 3.33% against the classic maintenance policy. These results are summarized in table 2. It can be seen that the sole use of the first ($\Theta_u = 0.1$) or the second ($\Theta_u = 0.2$) setting instead of the initial one ($\Theta_u = 0.05$) yields a substantial saving on the maintenance cost but with a significant loss of control performances. As previously if performances below 0.9 are forbidden because they introduce a poor drilling quality then these settings used exclusively are not acceptable. Starting with the initial setting and then shifting towards less stressful updates for the actuator seems to be a satisfactory compromise between extending actuator lifetime and preserving adequate control performances.

Conclusion

This paper proposes a framework to model the deterioration of control systems focusing on the actuator. The main idea is that the way an actuator is controlled is the root cause of its degradation. The control input $u(t)$ related to the controller setting plays a central role in the actuator degradation process. Two stochastic deterioration phenomena are considered. The first one is a natural wear when the system is in a steady state evolving around its operating setpoint. The second one is a shock process when the system is solicited during a setpoint change. An actuator fault is a symptom of a given level of accumulated deterioration. Two faults are considered from the less severe to the most severe one before the actuator fails whereas the literature dealing with control systems focuses on the performances restoration after the appearance of only one fault.

The degradation model allows to assess the reliability of the actuator conditionally to its level of deterioration which is assumed to be measurable. The residual useful life is derived and used for decision making in the framework of two condition-based maintenance policies where the reconfiguration of the controller is suggested as a third action. The aim of this new maintenance action is to update the controller setting in order to be less stressful for the actuator and then to extend its lifetime. The actuator degradation is slowed down, the maintenance cost is reduced and the availability of the system is increased. However these benefits are at the expense of control performances.

Given the importance placed on the control input $u(t)$, the achieved models allow to explain the origin of an actuator fault and its failure as well as their random occurrence dates. The stochastic point of view gives more realism to the actuator behavior within the closed-loop structure of the system. These models also propose an answer to the questions given in the introduction by Zhang (2008). In FTCS literature, the aim is to restore the control performances when an actuator fault occurs. This recovering is achieved with the reconfiguration of the controller setting whose aim is to compensate the loss of actuator effectiveness. Consequently, the reconfiguration can be more stressful for the actuator than before the fault. In the framework of this paper, the control reconfigurability as seen in FTCS literature doesn't improve the actuator reliability. A fault suggests that the system is being degraded and that it must be less solicited in the remaining of its mission. The reliability of a control system must be a central criterion at design stage. The paper tackles this point and shows that a trade-off is necessary between the reliability of a control system and its performances.

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