Impact of Time Varying Angular Frequency on the Separation of Instantaneous Power Components in Stand-alone Power Systems

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***Abstract*—The paper addresses the impact that time varying angular frequencies observed in electrical signals can have on the calculation and separation of components from the instantaneous electric power signal. Instantaneous power theories provide various methods for calculating the instantaneous power components in an electrical network. These methods are based on the basic assumption of constant fundamental frequency and harmonics that are multiple of the fundamental frequency. Recent field measurements in isolated electrical systems have however reported the existence of time varying angular frequencies or instantaneous frequencies. This new observation will affect the very foundation of the established methods for instantaneous power calculation and components separation. This paper analyses the separation of instantaneous average and oscillatory components of powers by using linear and non-linear filtering approaches in systems that exhibit time varying angular frequencies. The results of this comparison reveals the limitations of the assumption of fundamental and harmonic frequency when using linear filtering techniques in the presence of time varying angular frequencies. Non-linear filtering may offer a more robust and accurate estimation of the instantaneous values of powers and a power quality assessment that better reflects the actual system conditions.**

***Index Terms*—Instantaneous power theories, stand-alone systems, time varying angular frequency, linear filters, non-linear filters, Empirical Mode Decomposition**

I. INTRODUCTION

In classical power systems, frequency seldom exhibit large excursions from the fundamental frequency (50 Hz or 60 Hz) under steady-state operating conditions. In principle, deviations are caused by imbalances between generation and load, since the behaviour of these systems are defined by the properties of the large synchronous machines that typically dominate such systems [1]. It is the droop characteristic of synchronous machines that determines the load-frequency response which translates in reduced frequency when the load increases and in increased frequency when the load is reduced. The slope of the droop characteristics determines the rate at which the generator contributes to the total generated power in the network.

An example of the principle of the droop characteristics is shown in fig. 1 for active power.

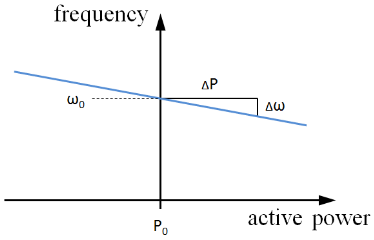

Fig. 1: Droop characteristic for active power generation and frequency

Time varying frequencies have been observed in data collected from stand-alone power systems (e.g. isolated microgrids, marine vessel power systems) and they originate from sources that are fundamentally different from the sources of frequency variations in classical power systems [2], [3]. In stand-alone power systems like microgrids, the generation source is usually interfaced with the load by “static” power electronic equipment (e.g. inverters). This means that no “rotating” masses are present, but e.g. solar panels, batteries or fuel cells supply DC power that is then inverted to AC power. The inverter output (voltage- and current waveform’s frequency) depends on the modulation and control algorithms applied to the unit. This control unit determines the gating signals to the power electronic switches. The dynamics of these controllers are defined by their inherent time-delay and will have different responses under dynamic loading conditions or changes in the set-point of controllers. Then, the frequency variations observed in these networks are caused by the response of controllers during dynamic operating conditions either on the generation side or the load side. To better illustrate the changing scenario for microgrids, an example of constantly changing generation is shown in fig. 2. The sun irradiation data shows the high intermittency of the input to a PV panel. This intermittency will affect the instantaneous delivery of power that will be processed by the power electronic controllers. The dynamics of the controllers in conjunction with the continuous change of input power from the source, will be reflected on the output voltage waveform of the inverter.

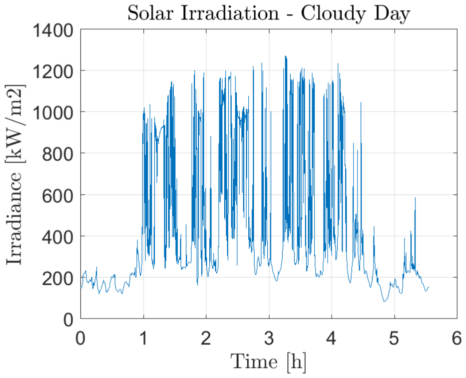

Fig. 2: Solar irradiance measured in Switzerland at a cloudy day. X-axis does not match actual day time, but serves as a time scale. Data acquired by EPFL, Lausanne, Switzerland.

Marine Vessel's power system is another example of a stand-alone microgrid that has experienced time varying frequency as reported by Tarasiuk et al. in [4], [5]. In this case, the dynamic load demand during navigation through rough sea is arguably one of the sources of the observed time varying angular frequency. In his paper, Tarasiuk highlights the inability of the majority of commercial power quality analysers to truly reflect the condition of the electrical system when substantial frequency variations are present. These insights hint that the major impact of time-varying frequency will be seen in the extraction of accurate measurements of electrical parameters in these systems.

In this paper, we analyse the impact that this time varying frequency will have on the methods typically used to separate the oscillating power components from the instantaneous power measurements. The research is then extended on calculations based on current and voltage measurements.

The p-q Instantaneous Power Theory, widely used in industry, is one of the core algorithms used to separate and control (or filter) the oscillatory component of the total instantaneous power. Linear filters have been the standard technique for performing this separation and they have shown to perform well for constant frequency components of the electrical signals. In the following sections, the paper discusses a non-linear filter, the Empirical Mode Decomposition-based (EMD [6]) Filter, as alternative separation technique and compares its performance with the classical linear filter approach. Three different types of signals are analysed by using Low-pass, High-pass and a non-linear filter based on the EMD. Two synthetic signals are designed to recreate time-varying angular frequencies in the instantaneous power in order to show the separation properties of the different filter techniques. A signal from the real world, obtained from measurements of a marine vessel during dynamic sea operation, is analysed to test the performance of the linear filter and the EMD-based Filter. Results of these comparisons indicate that, in the presence of time varying angular frequency in power signals, EMD-based non-linear filtering can be a promising technique for power component's separation and minimum distortion of information. The authors foresee an impact of the technique presented in this paper in providing a better estimation of power quality in isolated power systems in remote areas, where microgrids are emerging as a way to mitigate energy poverty. If developed under the open source principle, this solution can empower the users from simple households to a village microgrid, by giving them access to more accurate measurement of power quality and power consumption. These will ultimately show its benefits in increased efficiency and therefore cost reduction by minimizing losses and increasing reliability and life-time of connected appliances.

II. EXTRACTION OF OSCILLATING POWER COMPONENTS BASED ON P-Q THEORY

Fig. 3 shows a flow chart indicating the necessary steps for the separation of average and oscillating power components from the calculated instantaneous power in a three phase power system. The separation of these components is usually done by using linear filters, as indicated by the green box in the figure. This technique has proven to be effective when the signal to which the filter is applied does not contain time varying frequencies, as also reported by Tarasiuk in [4]. In the presence of time varying frequency as the ones reported in [7], [4], the classical determination of oscillating and average power components needs to be revised. The instantaneous power theory (p-q Theory, [2]) provides a foundation that is widely implemented in industry for various applications, ranging from active filters, grid tied inverter control, and any other application that requires a separation of oscillating and average power components from the total instantaneous power calculated from voltage and current measurements. For doing this, the voltage- and current measurements are transformed into a stationary reference frame (Clarke Transformation shown in euqations 1 and 2) and the instantaneous real- and imaginary powers are calculated according to equations 3 and 4. The stationary reference frame (with α- and β-axis), in contrast to the rotating reference frames of the Park Transformation (with d- and q-axis) has the advantage that a synchronisation with the system frequency is not necessary.

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2}$$

$$p = v_\alpha i_\alpha + v_\beta i_\beta \tag{3}$$

$$q = v_\beta i_\alpha \quad v_\alpha i_\beta \tag{4}$$

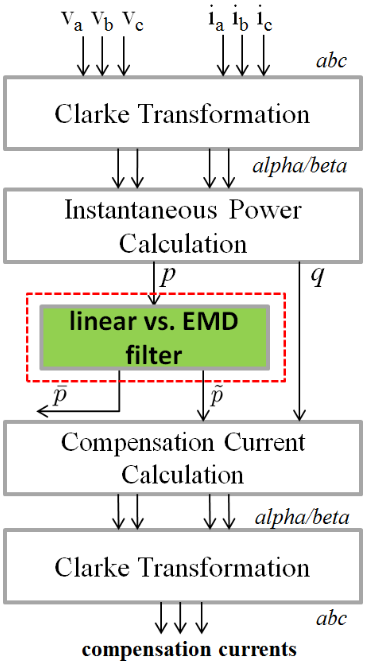

Fig. 3: Flow chart of the implementation steps of the p-q Theory where filters are used to separate instantaneous real power into oscillating ($\tilde{p}$) and average ($\bar{p}$) components. The imaginary power (q) is not separated if entirely compensated.

Distinction Between Harmonics and Time Varying Angular Frequency

Under balanced and steady-state conditions, the power flow in a three phase system is constant. This is also represented by the real- and imaginary instantaneous power contents. However, in the presence of harmonics (integer multiple of the fundamental frequency) in the voltages and currents, the instantaneous power exhibits oscillating power components depending on the harmonic content of the three phase voltages and currents and the phase shift of the harmonic content between the three phases. For the sake of simplicity, assuming sinusoidal voltage waveforms, these oscillating components of the instantaneous power are given by equation 5.

$$
\begin{aligned}
p_{oscillation} &= \frac{V_{peak} I_{pharm}}{2} f \\
cos[(\omega_{base} \quad \omega_{harm})t][1 + 2cos(\frac{2}{3}\pi \quad \alpha)] \\
cos[(\omega_{base} + \omega_{harm})t][1 + 2cos(\frac{2}{3}\pi + \alpha)] g
\end{aligned}
\tag{5}
$$

where I_{pharm} is the amplitude of the current harmonic and α the phase shift of the harmonic content between the balanced three phases. Fig. 4 shows the harmonic components in a frequency spectrum representation based on the Fourier series, assuming constant frequency components.

When time varying angular frequencies are present in the calculated instantaneous power, the formulation presented in equation 5 is no longer valid and a complete revision of the instantaneous power formulations will be necessary. This re-definition is beyond the scope of this paper. However, to illustrate and visualize the existence of this time varying frequency, instantaneous frequency obtained from the voltage measurements from a stand-alone PV microgrid in Bhutan is shown in a Hilbert spectrum in fig. 5. The instantaneous frequency results from applying the HHT approach to the voltage signal [8].

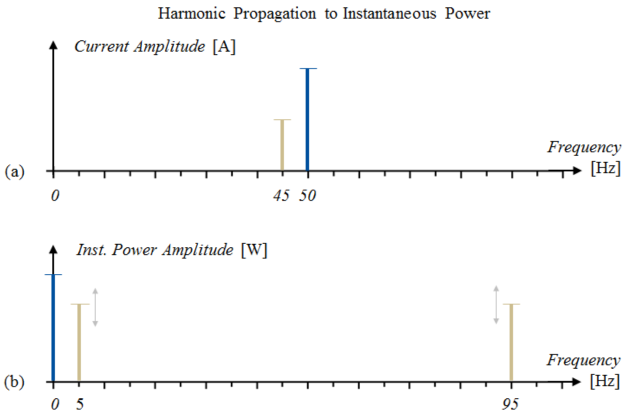

Fig. 4: Propagation of current harmonics on power oscillations. Ideal sinusoidal voltages are assumed. (a) Harmonic- and base current of the three phases in frequency domain. (b) Depending on the phase shift of the harmonics the power oscillation appear more or less strong on either or only one of the two frequencies.

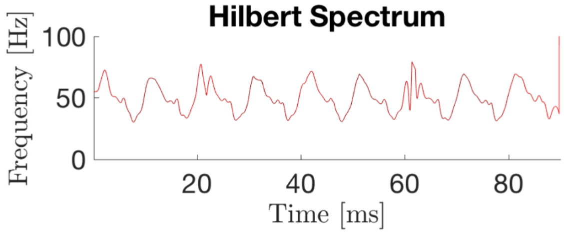

Fig. 5: Hilbert Spectrum of the base frequency IMF from field measurements of voltage in a stand-alone PV microgrid in Bhutan. The amplitude of the base frequency is constant within a 2.5% range. [8]

For a conceptual proof of the capability of the HHT to extract time varying frequency components from a signal, a synthetic signal containing both constant and time varying frequencies is generated (fig. 6a). The HHT is then implemented to extract its intrinsic modes (fig. 6b) and to visualize the frequency components (fig. 6c) of the originally generated signal [8].

III. LINEAR AND NON-LINEAR FILTERS FOR EXTRACTION OF OSCILLATING POWER COMPONENT

As mentioned in an earlier section, the classical approach for separating the oscillating power component is to use linear filters [2]. Constant frequency and multiple of the fundamental frequency do not pose major challenges to the use of linear filters and the approach has proven

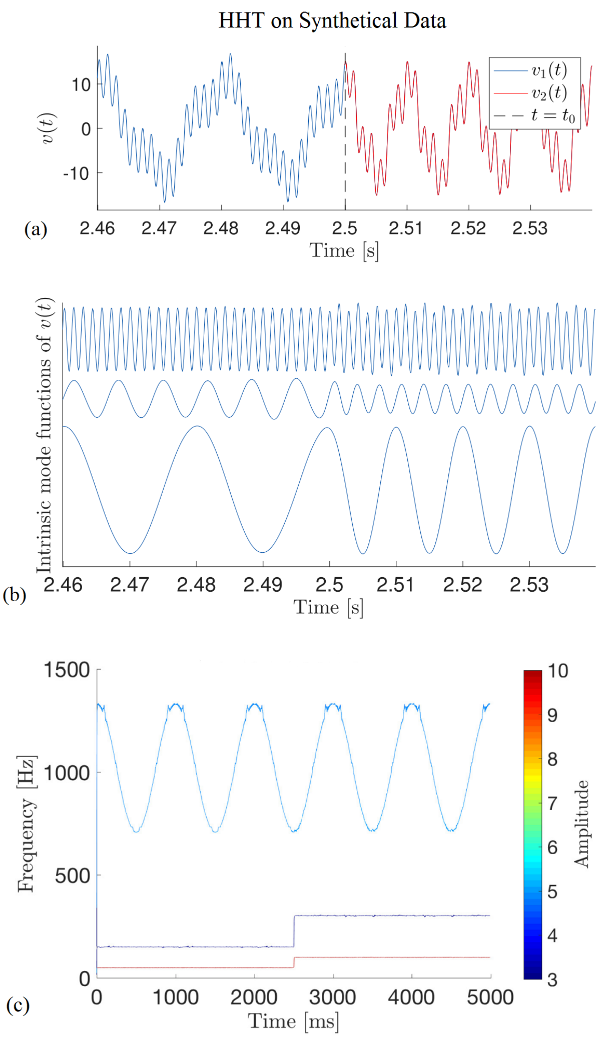

Fig. 6: Conceptual proof of the HHT capability [8]. (a) Synthetic voltage signal (v_t) containing both constant and time varying frequencies. (b) Intrinsic Mode Functions of the synthetic signal. (c) Hilbert spectrum of v_t using a median filter with a length of 2% of the sampling frequency to remove artifacts.

to work effectively in most real-life implementations. In the case of signals containing time varying frequencies, linear filters introduces undesirable features such as delay or phase shifts that are evidenced by the tests done in this paper. To cope with these undesirable features, the authors introduce a non-linear filter approach based on the EMD, developed by Norden E. Huang as part of the Hilbert-Huang Transform (HHT) [6]. The EMD is used instead of the linear filter block to extract the low oscillating component of the calculated instantaneous power. It decomposes the instantaneous power signal in its intrinsic modes called Intrinsic Mode Functions (IMFs) with the algorithm that is shown in fig. 7. Once the IMFs are obtained, the Hilbert spectrum visualizes the time varying angular frequency of each IMF.

The use of the EMD does not come however without disadvantages. To correctly separate the IMFs, each component needs to create their own extrema in the total signal composition. Their magnitude and frequencies must therefore be sufficiently different as discussed in [9]. Another problem appears when high frequency components are present only in certain time intervals and is known as “mode mixing”. A solutions to this problem is available in the literature (ensemble EMD [10]) and can be implemented case by case, depending on the type of signal being analyzed.

The EMD software implemented to analyze the signals in this paper is supported by *Signal Analysis Lab*[1] and does not show features to compensate for mode mixing.

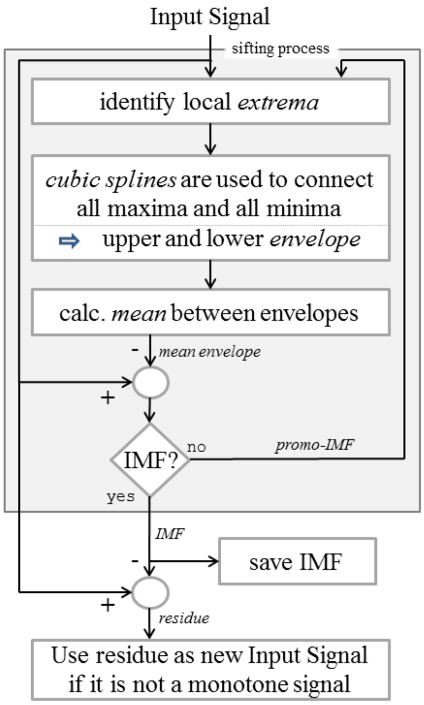

Fig. 7: The algorithms of the Empirical Mode Decomposition.

A. Performance Analysis of Linear Filters vs. EMD Filter

This section presents a qualitative comparison of the capabilities of linear and EMD filters for the separation of the oscillating power components from calculated or measured instantaneous power. To this aim, the well established low- and high-pass Butterworth filters are used to separate the oscillating components of the instantaneous power signal. This approach is the most common and accepted practice used in industry for the control of most shunt active filters [2], [11]. Subsequently, a non-linear EMD filter is implemented to the same power signal to compare with the results obtained with the linear filters. In contrast to the linear filter block, the EMD is not applied to a measurement stream in real-time, but to the data-set offline as seen later in fig. 9. A time-delay, depending

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[1] Organization number: 817 564 232

on the computational power of the processor used will be introduced with an EMD used on real-time data streams.

These three different linear and non-linear filters are implemented to three different types of signals. The first one is a synthetic signal, representing the superposition of two sine waves on the instantaneous power signal. The second one, also a synthetic signal, shows a step function with a superimposed sine wave on the instantaneous power and the last one is a field measurement of voltages and currents, of a marine vessel microgrid during sea voyage dynamic operation. The instantaneous power for this case was calculated according to the p-q Theory sketched in fig. 3.

1) Synthetic power composed by superposition of two sine waves: This signal is chosen to illustrate the steady-state performance of the linear filters and the EMD Filter. Three phase currents with two harmonic frequencies and the three phase voltages are processed according to the p-q Theory and result in the instantaneous power signal as in equation 6.

$$\begin{aligned} p_{inst(t)} = 34.5kW + cos(5Hz\;\;2\pi\;\;t)\;\;6.9kW \\ +cos(200Hz\;\;2\pi\;\;t)\;\;6.9kW \end{aligned} \tag{6}$$

The aim of the filter in the p-q theory sketched in fig. 3 is to extract or separate the higher frequency oscillation from the instantaneous power signal, so that the mean power value $p_{\mathrm{mean(t)}}$ equals the low frequency oscillation component or effective power. As the EMD Filter separates the two harmonics into two separate IMFs, the mathematical mean value is approached with computational errors and coincides with the EMD filter result. The linear filters, tuned according to table I show either time-delay or remains of high frequency oscillation on the obtained average power.

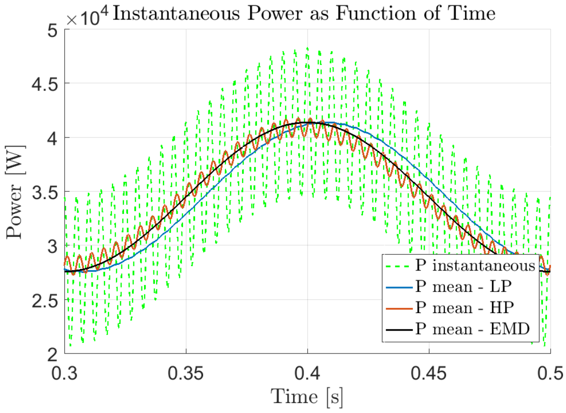

Fig. 8: Synthetic signal analysis: Steady State results of the linear low-, and high pass filter compared to the EMD Filter. Ripples on the high-pass filtered and phase shift of the low-pass filtered are clearly visible disadvantages of the linear filters.

2) Synthetic power composed by Step Function with harmonic component: Although an ideal step function is not likely to be recorded in real power system, this synthetic signal is used to compare the linear filter and the EMD Filter performances. This comparison will reveal the time responses of these filters during transient changes in power. For this analysis low- and high-pass Butterworth filter of third order are used to competitively run against the EMD Filter (filters tuning information is listed in table I). The superposition of a harmonic signal to the unit-step function is done with the purpose of making the EMD-Filter effective. With a purely step function, it is not possible to obtain meaningful results from the EMD Filter implementation as only one minimum (at t=0) and one maximum (at the step time) is identified and the cubic spline of the EMD method (refer to fig. 7) can not be set up properly. To overcome this limitation, a 70 Hz sine wave with 5% amplitude is added to the unit step function. The instantaneous power signal and the filter responses are depicted in fig. 9. The low- and high pass filters show long time-constants or ripples in the steady state output respectively. The EMD Filter shows a fast rise and zero deviation in steady-state. The transients before the step will only be observed when the EMD is applied to recorded data. In a real time application prediction is obviously not possible.

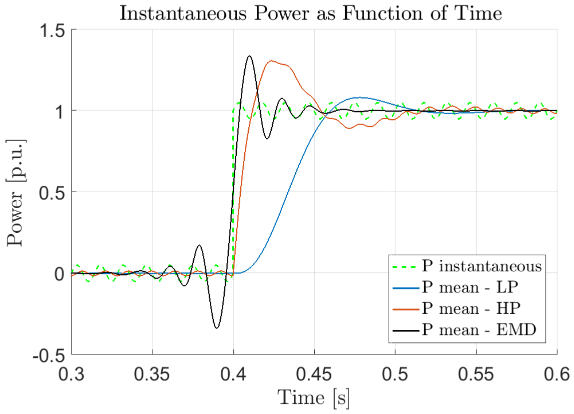

Fig. 9: Synthetic signal analysis: Mean instant. power calculation using linear and EMD filter on a unit step and a superimposed 70 Hz, 5 % amplitude oscillation.

3) Measured signals in a marine vessel during sea voyage: Measured voltage and current signals in a marine vessel during voyage in rough sea are analysed in two steps with the intention to compensate only for high frequency oscillations of 100 Hz in the first step and to perform a more coarse compensation of also 12 Hz oscillation in the second step. This is done to illustrate the flexibility of the filter methods by adjusting the cut-off frequency and order in the linear filters and on the other hand by adjusting the number of IMFs that are summed to form the average signal in the EMD Filter.

For the first step, fig. 10a shows the instantaneous power where oscillations at approximately 12 Hz and 100 Hz can be revealed and the results of the filter methods. The aim of the filter is to compensate for the 100 Hz oscillations only and the filters are tuned according to table I. The table shows that the EMD Filter splits the instantaneous power signal in 13 IMFs and a residue, from which the sum of the residue and the 6 last (low frequency) IMFs constitutes the mean value. Fig. 10b shows that the EMD filter in contrary to the Butterworth filter does not lag the actual mathematical average of the instantaneous power. For better understanding, the compensated IMFs and the residue are shown in fig. 12a and 12b.

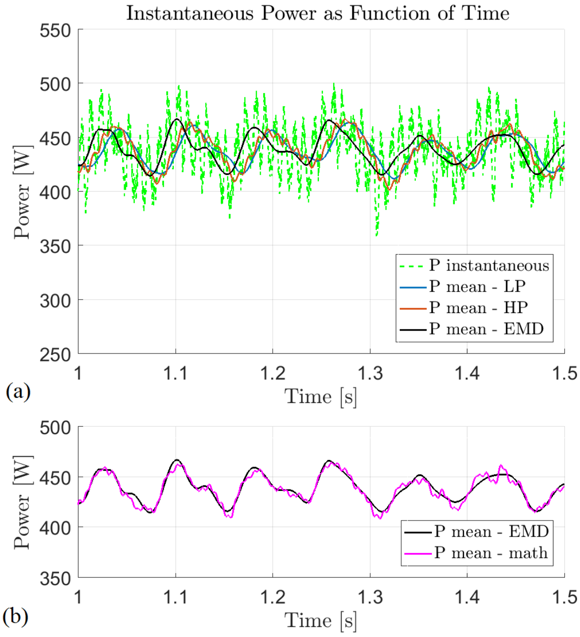

Fig. 10: Sea voyage marine vessel measurements analysis. (a) The EMD Filtered signal shows no phase-lag or ripples in comparison to linear filters. (b) For validation purpose of the EMD-average the mathematical average (calculated for fix window sizes of 0.01s) is shown

In fig. 11, the second step trial shows the analysis of the same sea voyage instantaneous power as above, but treated with filters of different settings. In this case, also the (approximately) 12 Hz oscillations are intended to be compensated. Only IMFs 12, 13 and the residue (shown in fig. 12b) are summed to form the mean value of the EMD filter. The results of the analysis show that the linear filter introduces a remarkable phase shift compared to the EMD filter solution and the mathematical average.

TABLE I: Butterworth Filter and EMD settings for the printed plots

	LP (f_c;order)	HP (f_c;order)	EMD (residue+IMF)
Fig. 8	70 Hz; 5^{th}	8 Hz; 5^{th}	res.+3 last (of4) IMF
Fig. 9	10 Hz; 3^{rd}	10 Hz; 3^{rd}	res.+4 last (of5) IMF
Fig. 10	23 Hz; 3^{rd}	7 Hz; 3^{rd}	res.+6 last (of13) IMF
Fig. 11	3,8 Hz; 3^{rd}	not applied	res.+2 last (of13) IMF

B. Non-linear EMD Filter Applied to Current and Voltage Measurements

The major motivation behind the analysis of voltage and current waveforms instead of calculated power signal, is that noise pollution will be amplified in the power calculation process and mixed to the relevant information of the measurement. The transformation and multiplication of the raw measurements might mask, bury and transform the relevant information in the data. Oscillation on the instantaneous power can be originated by different sets of harmonic components on the current signal where the phase of the harmonic plays a key role. From the power

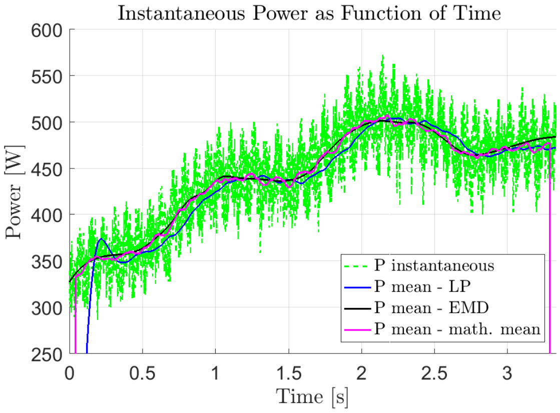

Fig. 11: Sea voyage marine vessel measurement analysis: For the LP filter again, the phase lag is clearly visible. The mathematical average (calculated for a fix window sizes of 0.083s) is closer to the EMD-mean. The mean value from HP filter is not shown because the transients endure the whole 3.33 s.

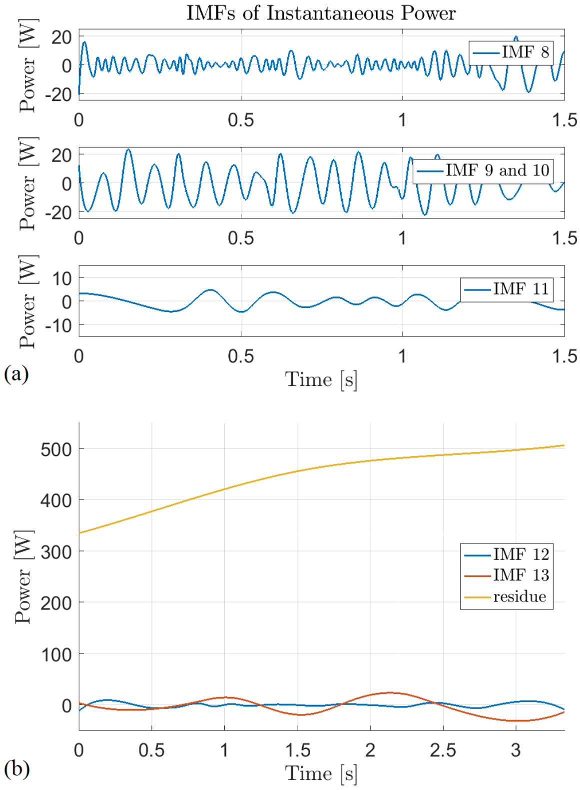

Fig. 12: IMFs from instantaneous power from the ship measurements. (a) IMFs 8 to 11. IMFs 9 and 10 cover the same frequency of approximately 12 Hz and are grouped. (b) IMFs 12, 13 and the residue are shown.

analysis, it can not be stated which harmonic is present on the current measurements.

However, when the EMD filter is applied to voltage and current measurement, frequencies with low amplitude and close to the system fundamental frequency are unlikely to be separated in an IMF. They are likely to appear mixed with the IMF that includes the fundamental frequency component [9].

When instantaneous power signal is analysed instead of voltage and current signals, this feature is not present because under normal operation conditions the power flow in a 3-phase system is constant and the major task is to separate harmonic contents from the total calculated power. A possible solution to this problem is the application of the Park transformation to current and voltage signals [12]. The reference frame of the d- and q axis in the Park transformation rotates at the instantaneous fundamental frequency of the system, given by a Phase Locked Loop (PLL). A correctly tuned PLL will return the instantaneous frequency of the fundamental positive sequence component of the measured voltage signal. Once this is done, the signal on d- and q- axis will not contain the fundamental frequency component and any additional modes present in the voltage and current can be clearly identified and separated by implementing an EMD Filtering.

As described in section II, the Park Transformation can replace the Clarke Transformation for the use in the p-q Theory without restrictions. A suggested approach will be to apply the Park transformation in the p-q Theory instead of the Clarke transformation originally introduced by Akagi and evaluate the measurement additionally and simultaneous to the instantaneous power signal.

IV. CONCLUSION

With the results obtained in section III, a discussion of the application of the EMD Filter in the practical implementation of the instantaneous p-q Theory is presented. Analysis of two representative synthetic data and measurements from a marine vessel during sea voyage show that in stand-alone electrical systems exposed to dynamic operation (variation in generation or loading), the control of the static power converters can give rise to time varying frequencies in the system voltages. Results from a comparison between linear and EMD-based Filters show that the EMD-based Filter brings advantages in comparison to classical low and high pass filter implementations. Further investigation is necessary to determine the feasibility of the EMD Filter in real-time and to ensure consistent robust performance for various types of network distortions. Such implementation will require high computational power to manage the signal with appropriate sampling rates and to act in a timely manner so that an actual benefit in comparison to the phase-lag, introduced by linear filters, can be achieved. As Hardware costs are decreasing, an affordable implementation for projects with limited financial resources can be hoped for.

The implications of the findings of this investigation can be critical for the quality of power delivered by these networks. On one hand, the measurement algorithms will need a thorough revision when the system voltage exhibits time varying angular frequencies. On the other hand, the implementation of control algorithms are designed on the assumption of constant fundamental frequency and harmonics with linear filters that are incapable of extracting time varying frequency components and introduce new distortions into the control signal, affecting the overall performance of corrective measures. Measurement equipment and power quality analyzers will need to address the problem with new algorithms that can take into account time varying frequencies in the definition and determination of power quality indexes such as voltage and current THDs and distortion power factor among others. These aspects of power quality are currently under investigation by the authors and will be reported in future publications.

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