Complexity reduction in motion cueing algorithm for the ULTIMATE driving simulator

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Abstract: The performance of the driving simulator depends on the efficiency of the motion cueing algorithm. An explicit model predictive control was established recently for the motion cueing algorithm. The complexity of the explicit solution increases manifold when the human vestibular model is considered. This paper focuses on the complexity reduction of explicit solution using low complexity contractive sets for the motion cueing algorithm. The low-complexity explicit controller is formulated for the efficient control of the motion cueing algorithm for the ULTIMATE driving simulator at Renault.

Keywords: Motion cueing algorithm, Explicit model predictive control, Complexity reduction, Driving simulator, Contractive set

1. INTRODUCTION

The automobile manufacturers are focusing on the development of high performance driving simulators in order to optimize the vehicle design (Fang,2016). The performance of a driving simulator can be improved by increasing the efficiency of the motion cueing algorithm (MCA). The motion cueing algorithm is used to reproduce the motion of a simulated vehicle by computing input commands to the simulator, subject to workspace limits and performance constraints. The efficiency of the MCA can be given in terms of performance to track the vehicle's accelerations and rotational speeds without perceived undesirable motion, safe and stable control of the driving simulator (Fang,2016).

Different kinds of motion cueing algorithms have been used in flight and driving simulators over the years. Many modern MCAs use some variant of MPC, while in the past various filter-based MCA strategies were common (Maran,2015). The classical filter based motion cueing algorithm is a combination of frequency filters. A high pass filter is used to remove the low frequency components of the acceleration signal in order to keep the linear motion system working within its bounds, whereas a low frequency filter is used to extract the low frequency signal of the acceleration which is then converted to the tilt angle. However, the classical filter must be designed by considering the worst case, i.e. the step signal. In consequence, the classical filter usually does not use all the available workspace. Another defect is its backlash effect at the end of the step signal which can result in simulator sickness (Reymond,2000). To overcome the shortcomings of the classical filter, adaptive filter (a classical filter with variable gain) and optimal filters(based on optimal control theory) are used to design the motion cueing algorithm. In these approaches, a cost function is defined and optimized without considering the system constraints, which may give the sub-optimal performance. Model predictive control overcomes these limitations as it performs optimization accounting for the system constraints. However, MPC is generally limited to the systems with slow dynamics and non-safety critical nature due to its computational complexity. Explicit MPC overcomes these limitations of standard MPC by formulating the optimization problem

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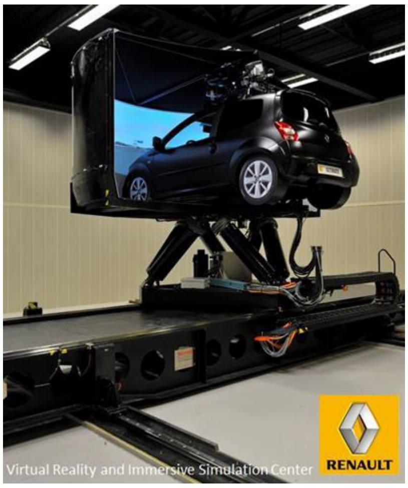

Fig. 1. ULTMIATE driving simulator

as a multi-parametric problem. The optimization problem can be solved offline and the control law can be given as a piecewise affine (PWA) function of the current state. This transforms the online MPC computations into simple evaluation of a PWA function. Explicit MPC has been used for MCA in (Fang,2012). A braking condition has been introduced in (Fang,2016) which helps in reducing the complexity of the explicit solution. The explicit MPC is feasible for systems with a modest number of states, as its complexity increases very fast with an increase in the number of system states. The explicit solution for the motion cueing algorithm is very complex when the effect of the human vestibular system is considered. In this paper, contractive sets are used to compute an explicit solution with reduced complexity. This makes it feasible to use a more detailed model with an increased number of states in the motion cueing algorithm.

In section 2, a model for the ULTIMATE driving simulator of Renault is established. Two models are presented for the perceived acceleration in the driving simulator; one accounting for the specific force only (gravito-linear acceleration inertial force) and the other one taking into account the human vestibular model. A contractive set is formulated which is then used in the designed controller for motion cueing algorithm in section 3. Section 4 presents the results obtained by applying the controller designed in section 3 to the models given in section 2 and the conclusion is drawn in section 5.

2. SYSTEM MODEL

The VR and Immersive Simulation Center of Renault has developed a 8 degree of freedom (DOF) driving simulator named ULTIMATE. The simulator comprises of a hexapod (Bosch-Rexroth) and an X-Y rail actuator. The software SCANeR uses a motion cueing algorithm to control the motion system in ULTIMATE.

The longitudinal or lateral acceleration of the vehicle is reproduced using the motion of hexapod tilt or rail translation which is termed as tilt-coordination. It is based on the fact that the linear motion sensors, the otolith organs in the human vestibular system cannot differentiate between gravity and translational acceleration without other motion cues. The projected gravity component due to the hexapod tilt angle is considered as a linear acceleration by the otolith organs if the rotational motion is below the sense threshold. The motion cueing algorithm is designed to limit the rotational motion in the driving simulator below the sense threshold, so that the stimulated vehicle's motion feeling is perceived in the simulator as closely as possible to a real car by using the linear motion of the rail and tilt angle of the hexapod.

The continuous time simulator model is given as:

$$\dot{x}_{sim} = A_{sim}x_{sim} + B_{sim}u \tag{1}$$

where,

$$x_{sim} = \begin{bmatrix} p \\ v \\ \theta \\ \omega \end{bmatrix}, A_{sim} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{sim} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, u = \begin{bmatrix} u_{acc} \\ u_{tilt} \end{bmatrix}$$

Here p, v, θ and ω represent linear position, linear velocity, tilt angle and tilt velocity respectively, whereas u_{acc} and u_{tilt} represent the linear and tilt acceleration respectively of the driving simulator. The state and input constraints are given below:

$$-2.6 < p < 2.6 \quad -3 < v < 3 \quad -5 < u_{acc} < 5$$
$$-10 < \theta < 10 \quad -6 < \omega < 6 \quad -15 < u_{tilt} < 15$$

where p, v, u_{acc}, θ, ω and u_{tilt} are expressed in m, m/s, m/s^2, $degree$, $degree/s$ and $degree/s^2$ respectively.

The driving simulator model can be designed in two ways: using only the specific force to represent perceived acceleration or using the complete human vestibular model to formulate the perceived acceleration.

2.1 Driving simulator model with the effect of gravity

Generally the effect of gravity is taken into account while formulating the motion cueing algorithm. In this case, the perceived acceleration can be given as:

$$u_{per} = u_{acc} + \theta g \tag{2}$$

Here u_{acc} is the linear acceleration of the driving simulator while θ and g represent the tilt angle and the constant of gravity respectively. Note that in (2) the approximation $sin(\theta) \approx \theta$ is used, which is acceptable within the range of operation of the simulator.

If the perceived acceleration is considered as one of the states then the discrete time simulator model can be given as:

$$x_{sim,k} = A_{sim,k}x_{sim,k} + B_{sim,k}u_k \tag{3}$$

where,

$$x_{sim,k} = \begin{bmatrix} p_k \\ v_k \\ \theta_k \\ \omega_k \\ u_{per} \end{bmatrix}, A_{sim,k} = \begin{bmatrix} 1 & dt & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & dt & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & g & 0 & 0 \end{bmatrix}$$

$$B_{sim,k} = \begin{bmatrix} 0.5dt^2 & 0 \\ dt & 0 \\ 0 & 0.5dt^2 \\ 0 & dt \\ 1 & 0 \end{bmatrix}, u_k = \begin{bmatrix} u_{acc} \\ u_{tilt} \end{bmatrix}$$

Here u_per is the perceived acceleration (specific force) and dt is the sampling time.

A controller is formulated in section 3 and the results obtained with this model are given in section 4.1.

2.2 Driving simulator with human vestibular system model

The vestibular system situated in the inner ear of a human, consists of two parts, i.e. the semicircular canal and the otolith. The acceleration perceived is the combination of these two parts. The human sensation models were first used in the motion cueing algorithm in (Sivan,1982) which were useful for motion reproduction in driving simulators. The methods described in (Sivan,1982) were improved by (Telban,2000) and (Telban,2005) with the help of an in-line Riccarti equation solver in order to optimize the feedback control law for motion cueing algorithms. The otolith and semicircular canal models are given below:

Otolith Model The otolith model is responsible for sensing linear motion in human beings. The state space representation of the otolith model explained in (Fang,2016) is given below:

$$\dot{x}_{oto} = A_{oto}x_{oto} + B_{oto}u \tag{4}$$

where,

$$A_{oto} = \begin{bmatrix} -\frac{\tau_L + \tau_S}{\tau_L \tau_S} & 1 & 0 & 0 \\ -\frac{1}{\tau_L \tau_S} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_{oto} = \begin{bmatrix} \frac{k_{oto}\tau_a}{\tau_L \tau_S} & 0 \\ \frac{k_{oto}}{\tau_L \tau_S} & 0 \\ 0 & \frac{k_{oto}\tau_a \cdot g}{\tau_L \tau_S} \\ 0 & \frac{k_{oto} g}{\tau_L \tau_S} \end{bmatrix}$$

$$x_{oto} = [x_{oto1} \; x_{oto2} \; x_{oto3} \; x_{oto4}]^T, u = [u_{acc} \; u_{tilt}]^T$$

The first state of the otolith model x_{oto1} is the perceived acceleration in the driving simulator.

Parameters	Value
τ_L	6.1
τ_s	0.1
τ_a	30
k_{oto}	0.4

Table 1. Otolith parameter values

Semi-Circular Canal Model The semi-circular canal model is related to sensing rotational motion in human beings, and can be represented in state space form as described in (Fang,2016). The model given below is the state space representation of the semi-circular canal model.

$$\dot{x}_{scc} = A_{scc}x_{scc} + B_{scc}u \tag{5}$$

where,

$$A_{scc} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -T_2 & 1 & 0 \\ 0 & -T_1 & 0 & 1 \\ 0 & -T_0 & 0 & 0 \end{bmatrix}, B_{scc} = \begin{bmatrix} 0 & 0 \\ 0 & T_3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$x_{scc} = [x_{scc1} \; x_{scc2} \; x_{scc3} \; x_{scc4}]^T, u = [u_{acc} \; u_{tilt}]^T$$

$$T_0 = \frac{1}{T_L T_S T_a}, T_1 = \frac{T_L + T_s + T_a}{T_L T_S T_a}$$

$$T_2 = \frac{T_L T_S + T_L T_a + T_s T_a}{T_L T_S T_a}, T_3 = \frac{k_{scc}}{T_S}$$

The values of the semicircular canal model parameters mentioned in (Telban,2000) and (Reymond,2000) are used here which are given in table 2.

Parameters	Value
T_L	6.1
T_S	0.1
T_a	30
k_{scc}	1.0

Table 2. Semicircular canal parameters values

Combined model taking into account the vestibular system By considering the impact of human vestibular system and combining the models in systems (1), (4) and (5), the combined model can be given as:

$$\dot{x} = \begin{bmatrix} A_{scc} & 0 & 0 \\ 0 & A_{oto} & 0 \\ 0 & 0 & A_{sim} \end{bmatrix} x + \begin{bmatrix} B_{scc} \\ B_{oto} \\ B_{sim} \end{bmatrix} u \tag{6}$$

It is a continuous time model with redundant states. A 9-state model can be obtained by model reduction.

The braking conditions described in (Fang,2016) makes sure that the linear velocity v and the tilt velocity ω are reduced when linear position p and angular position θ approach their limits so that it always remain inside the constraints. The braking conditions are given as follows:

$$p_{min} < p + c_v v T_{brk,p} + 0.5c_u u_{acc} T_{brk,p}^2 \leq p_{max} \tag{7}$$

$$\theta_{min} < \theta + c_\omega \omega T_{brk,\theta} + 0.5c_u u_{tilt} T_{brk,\theta}^2 \leq \theta_{max} \tag{8}$$

where, p_{min}, θ_{min}, p_{max} and θ_{max} represent minimum and maximum values of p and θ.

3. CONTROLLER DESIGN

Explicit model predictive control has been used in (Fang,2016) for MCA. The problem occurs when human vestibular model is considered, as the complexity of explicit model predictive control (MPC) increases rapidly with an increase in the number of states. A simpler approach is to formulate explicit MPC using controlled contractive set as explained in (Munir,2016). In that case, the complexity of explicit solution depends on the complexity of the contractive set.

Let a discrete time system with state and input constraints given by $\mathcal{X} = \{x|Hx \leq h\}$ and $\mathcal{U} = \{u|H_u u \leq h_u\}$ respectively, then the contractive set can be defined as follows:

Definition 1: A compact polytopic set $\mathcal{P} \in \mathcal{X}$ having the origin in the interior is called controlled γ-contractive, with contraction factor $\gamma \in [0, 1)$ if for all $x \in \mathcal{P}$ there exists an $u \in \mathcal{U}$ such that $Ax_k + Bu_k \in \gamma\mathcal{P}$.

3.1 Contractive set formulation

A contractive set of desired complexity can be obtained by the procedure described in (Munir,2016).

It can be noticed from the models in (3) and (6) that the system is asymptotically stable except for the simulator states $[p\ v\ \theta\ \omega]$. Therefore, a contractive set is needed for simulator states only, as the overall system will be stable if the simulator states are forced to converge to the origin.

Contractive Constraints The braking constraints from eq (7) eq (8) are added to the contractive constraints described in (Munir,2016). The maximum contraction factor for the contractive set $\mathcal{P} = \{x \in \mathcal{R}^n | Fx \leq f\}$ is given as:

$$\max_{x} \gamma^* \tag{9a}$$

subject to

$$Hx_k \leq h \tag{9b}$$

$$Fx_k \leq f \tag{9c}$$

$$\gamma^* = \min_{u,\gamma} \gamma \tag{9d}$$

subject to

$$A_{brake}x_k + B_{brake}u_k \leq f_{brake} \tag{9e}$$

$$F(Ax_k + Bu_k) \leq \gamma f \tag{9f}$$

$$H_u u_k \leq h_u \tag{9g}$$

Here the pairs $[H, h]$ and $[Hu, hu]$ represent the state and the input constraints respectively. The bilevel optimization problem mentioned here can be transformed into single level optimization problem as described in (Hovd,2014).

Therefore, the contractive set given by pair of $[F, f]$ can be obtained by modifying the problem mentioned in (Munir,2016).

$$\max_{C,d,F,f,M} log(det(C)) \tag{10a}$$

subject to

$$C = C^T > 0 \tag{10b}$$

$$\begin{bmatrix} (f_i - F_i d)I & CF_i^T \\ F_i C & f_i - F_i d \end{bmatrix} \succeq 0 \forall i = 1, ...m \tag{10c}$$

$$MF = H \tag{10d}$$

$$Mf \leq h \tag{10e}$$

$$\max_{x,u,\gamma,\lambda,s} \gamma^* \tag{10f}$$

subject to

$$Hx_k \leq h \tag{10g}$$

$$Fx_k \leq f \tag{10h}$$

$$\lambda_a \geq 0 \tag{10i}$$

$$\lambda_b \geq 0 \tag{10j}$$

$$\lambda_c \geq 0 \tag{10k}$$

$$\lambda_a \leq M_a^{\lambda} s \tag{10ℓ}$$

$$\lambda_b \leq M_b^{\lambda} s \tag{10m}$$

$$\lambda_c \leq M_c^{\lambda} s \tag{10n}$$

$$F(Ax_k + Bu_k) - \gamma f \leq 0 \tag{10o}$$

$$\lambda(F(Ax_k + Bu_k) - \gamma f) \geq -M_a^u(1 - s) \tag{10p}$$

$$H_u u_k \leq h_u \tag{10q}$$

$$H_u u_k - h_u \geq -M_b^u(1 - s) \tag{10r}$$

$$A_{brake}x_k + B_{brake}u_k - f_{brake} \leq 0 \tag{10s}$$

$$\lambda(A_{brake}x_k + B_{brake}u_k - f_{brake}) \geq -M_c^u(1 - s) \tag{10t}$$

$$\nabla_{u,\gamma}\mathcal{L}(u, \gamma) = 0 \tag{10u}$$

where,

$$\mathcal{L} = \gamma^* + \lambda_a^T(FAx_k + FBu_k - \gamma f) + \lambda_b^T(H_u u_k - h_u) + \lambda_c^T(A_{brake}x_k + B_{brake}u_k - f_{brake})$$

$$A = \begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ dt & 0 \\ 0 & 0 \\ 0 & dt \end{bmatrix}, x = \begin{bmatrix} p \\ v \\ \theta \\ \omega \end{bmatrix}$$

$$A_{brake} = \begin{bmatrix} 1 & T_{brk,p} & 0 & 0 \\ -1 & -T_{brk,p} & 0 & 0 \\ 0 & 0 & 1 & T_{brk,\theta} \\ 0 & 0 & -1 & -T_{brk,\theta} \end{bmatrix}$$

$$B_{brake} = \begin{bmatrix} TG_p & 0 \\ -TG_p & 0 \\ 0 & TG_\theta \\ 0 & -TG_\theta \end{bmatrix}, f_{brake} = \begin{bmatrix} p_{max} \\ -p_{min} \\ \theta_{max} \\ -\theta_{min} \end{bmatrix}$$

$$TG_p = \frac{1}{2} c_u T_{brk,p}^2, TG_\theta = \frac{1}{2} c_u T_{brk,\theta}^2$$

where c_u =0.45, $T_{brk,p}$=2.5 and $T_{brk,\theta}$ =0.5

This is a highly non-convex problem. Particle swarm optimization (PSO) can be used to solve this problem as explained in (Munir,2016). It can be noted here that the simulator model consists of two independent systems, $[x, p]$ and $[\theta, \omega]$. Therefore, the contractive sets for these two systems can be formulated separately and merged together later. This can be done to achieve better results (larger contractive set) with the PSO.

3.2 Controller design for reference tracking using contractive sets

Once the contractive set is found, it can be used in the controller formulation as described in (Hovd,2014) and (Munir,2016). By adding the reference tracking and braking condition, the controller is designed as follows.

$$\min_{u_k} (x_{k+1} - x_{ref})^T Q(x_{k+1} - x_{ref}) + u_k^T R u_k \tag{11a}$$

subject to

$$x_{k+1} = Ax_k + Bu_k \tag{11b}$$

$$H_u u \leq h_u \tag{11c}$$

$$A_{brake}x_k + B_{brake}u_k \leq f_{brake} \tag{11d}$$

$$F(x_{k+1} - x_{ref}) \leq \gamma\alpha f \tag{11e}$$

where

$$\alpha = max\{F_i(x_{k,i} - x_{ref})/f_i\}, \forall i = 1, \cdots, m \tag{11f}$$

This formulation ensures that the α will be reduced by factor γ at each time step, which will force the state trajectories to converge to the origin, consequently ensuring the stability of the system. The explicit solution to the formulation mentioned above can be obtained by solving it parametrically, with x_k and α as the parameters. If a parametric solution with only x_k as a parameter is desired then the contractive set can be subdivided. Each subdivision is defined by the origin and one of the facets of the contractive set. Inside each subdivision, α then depends linearly on x, and the parametric solution can be obtained with only x as a parameter (Koduri,2016).

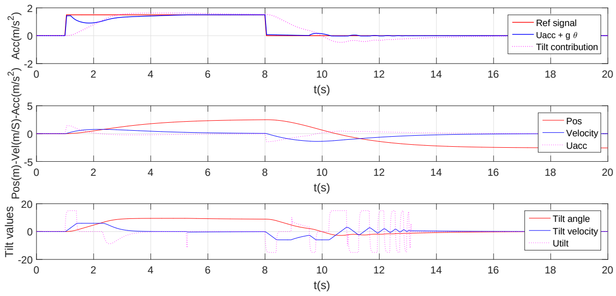

Fig. 2. 2-DOF motion rendering results using model (3) for step reference

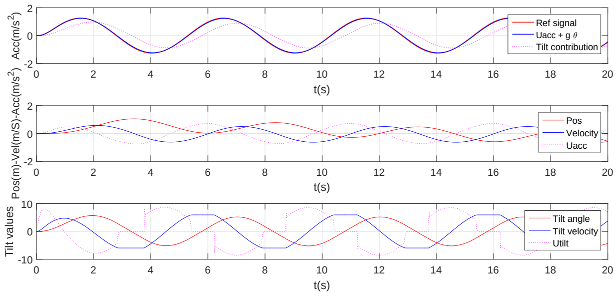

Fig. 3. 2-DOF motion rendering results using model (3) for sine reference

The number of different combinations of the constraints which may be active at the optimum determines the complexity of the explicit solution. The formulations with longer prediction horizons are expected to have a high number of constraints and typically more combinations of the constraints. The prediction horizon for the formulation mentioned above is 1, therefore the complexity of the explicit solution is expected to be low.

The contractive set considers the reference as origin and tries to converge the trajectories towards its origin which implies that the contractive is moved to a new origin whenever reference changes. There may arise a case that the current state is outside the contractive set when it is moved to the new origin in order to follow a certain reference. In that case, a sub-optimal reference must be selected such that the current state remains within the moved contractive set for some $\alpha \in [0, 1]$.

4. RESULTS

The explicit controller is designed for the models (3) and (6).

4.1 Simulator model with effect of the gravity

Consider the system described in (3). The contractive set is obtained for simulator states using the method described in section 3.1. The contractive set obtained is used in controller design in order to obtain the explicit solutions as discussed in section 3.2. Figure 2) shows that the braking/acceleration (step) reference signal is reproduced by driving simulator and Figure 3 shows the tracking of a sinusoidal reference signal. In these figures,

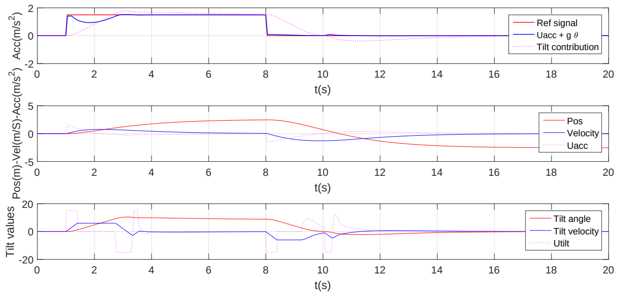

Fig. 4. 2-DOF motion rendering results using the method described in (Fang,2016) for step reference

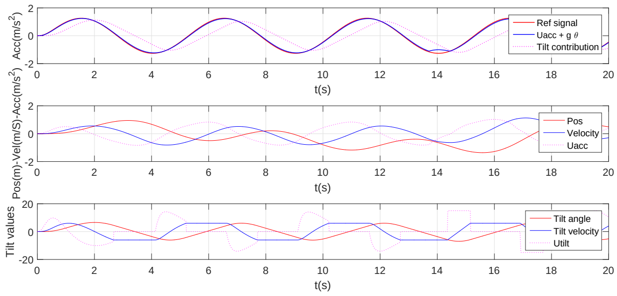

Fig. 5. 2-DOF motion rendering results using the method described in (Fang,2016) for sine reference

the top sub-figure shows the vehicle acceleration(red) and the perceived acceleration (blue) in the driving simulator. The remaining two sub-figures shows the linear and tilt motion of the simulator. The total number of regions of the explicit solution comes out to be 137 by employing the method described in section 3.

By using the technique explained in (Fang,2016), the number of regions of the explicit solution comes out to be 796. Figures 4 and 5 show the perceived acceleration and the reference signal. It can be noted that the reference tracking is similar in both techniques but the complexity of the motion cueing algorithm in terms of the number of regions of the explicit solution has decreased significantly by using the controller described in section 3.

4.2 Simulator model with human vestibuar model

The contractive set is obtained for simulator states using the method described in section 3.1 for the model (6). The contractive set obtained is used in controller design in order to obtain the explicit solutions as discussed in 3.2. The acceleration perceived by the driving simulator is shown in the Figure 6 for step reference while Figure 7 shows the tracking of a sinusoidal reference signal. The reference signal (magenta) is the vehicle's reference acceleration. The human vestibular model perceives (perceived reference) it as the red signal shown in the top subfigures, while blue signal shows the tracking of perceived reference.

The total number of regions of the explicit solution comes out to be 130 by the method described in section 3.

The total number of regions of the explicit solution using the method described in (Fang,2016) comes out to be

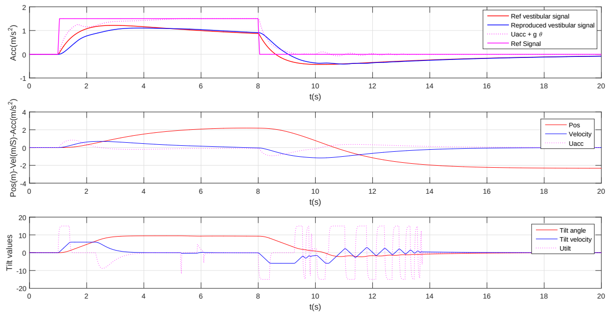

Fig. 6. 2-DOF motion rendering results using model (6) for step reference

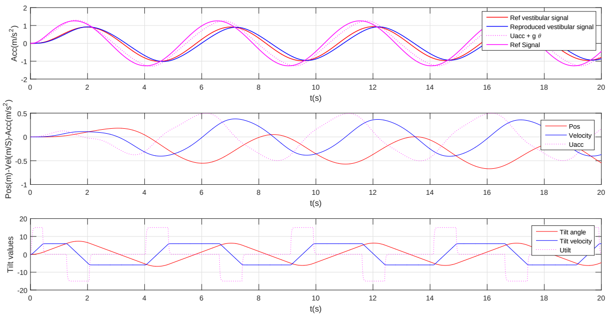

Fig. 7. 2-DOF motion rendering results using model (6) for sine reference

29843 for a control horizon of 1 which shows there is a significant reduction in complexity of the solution of explicit MPC in terms of the number of regions.

5. CONCLUSION

The contractive sets are used to formulate the explicit controller for motion cueing algorithm in this paper. Two models are given which describe the dynamics of perceived acceleration in Renault's driving simulator ULTIMATE. The proposed technique significantly reduces the complexity of the explicit solution without sacrificing performance.

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