# Risk Aversion in Imperfect Natural Gas Markets

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**Abstract**

This paper presents a natural gas market equilibrium model that considers uncertainty in shale gas reserve exploration. Risk aversion is modeled using a risk measure known as the Average Value-at-Risk (also referred to as the Conditional Value-at-Risk). In the context of the European natural gas market, we show how risk aversion affects investment behavior of a Polish and a Ukrainian natural gas supplier. As expected, increased risk aversion leads generally to lower investment, and a larger share of investments in the form of lower risk alternatives, i.e., conventional resources. However, in our market setting where multiple risk-averse agents each maximize their own profits we do observe some counter-intuitive, non-monotonic results. It is noteworthy that in a competitive market, risk aversion leads to significantly lower reserve exploration, which may be interpreted as a *credible threat* by a large dominating supplier (such as Russia). A threat to *flood* natural gas markets could deter importing countries from extending their own reserve bases.

**Keywords:** Stochastic Equilibrium, Nonlinear Programming, Risk Measures, Natural Gas Markets

**Classification:** 90C15, 60B05, 62P05

## 1 Introduction

The European gas market faces declining production resulting in the expectation of increased imports. In the USA, natural gas production has increased by one third since 2005 due to extraction of natural gas from shale formations. This is projected to increase further by about 50 % until 2035 (United States Department of Energy (2015)). Some European countries have potentially

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large shale resources (cf. Holz et al. (2015)) and European countries eager to reduce their import dependency are considering shale gas development. Shale gas production is characterized by low up-front exploration and investment costs, but high operational costs, compared to conventional gas production. Shale gas is also fraught with environmental concerns, uncertainties in resource estimates, economic viability and political climate issues, as well as "not-in-my-backyard" issues that surround hydraulic fracturing's potential to cause environmental damage and undesired seismic activity. Some of these issues may reduce even the long-term potential for U.S. domestic shale gas production (cf. Richter (2015)) and are major concerns in European countries. In Europe, some countries have allowed fracking, while others have forbidden it, and some are still contemplating (Boersma and Johnson (2012)). The United Kingdom has allowed shale gas exploration (cf. Stevens (2013)).[1] France[2] and Bulgaria[3] have banned shale gas production altogether, whereas Germany does not allow it, at least until 2021.[4]

In the Netherlands, the only EU member which is a net gas exporter, the national government is strongly in favor of shale gas production and is likely to allow test drilling in the next few years.[5] In contrast, some local governments are seeking statuses as *shale-gas-free* provinces or counties and have announced court cases against the national government if they attempt to overrule local choices. Meanwhile, several companies have halted shale gas exploration in Poland due to disappointing finds.[6] These points illustrate how regulatory and resource risk are main characteristics of the European shale gas market, and thereby, the natural gas market in general. Risk management and aversion are likely to affect investment and trade in ways deterministic or expected value maximization models cannot predict. Unfortunately, very few models consider risk aversion by multiple agents in a single market framework.

Stochastic equilibrium models for (imperfect) energy markets typically represent uncertainty by limiting the number of possible future outcomes to a finite set of outcomes represented in a scenario tree. A stochastic problem contains at least two stages, where some decisions must be made in an earlier stage, rather than the stage at which uncertain events become known. Typical first or earlier-stage decisions concern concepts such as long-term contracts and investments. In later stages, uncertain demand or price levels, or other aspects become known and so-called recourse decisions can be made, for instance production and sales decisions. Two-stage stochastic complementarity problems for natural gas markets are considered in a number of publications. Haurie et al. (1990) studied long-term contracting decisions in the European natural gas market under oil price uncertainty. Haurie and Zaccour (2005) reconsidered these results. Zhuang (2005) and Zhuang and Gabriel (2008) investigated long-term contracting under gas spot price uncertainty. Abada (2012) presented a two-stage stochastic model for the European gas market (S-GAMMES)

[1] http://www.eia.gov/countries/cab.cfm?fips=UK

[2] http://www.theguardian.com/environment/2013/oct/11/france-fracking-ban-shale-gas

[3] http://www.theguardian.com/world/2012/feb/14/bulgaria-bans-shale-gas-exploration

[4] http://www.wsj.com/articles/germany-shelves-shale-gas-drilling-for-next-seven-years-1404481174

[5] http://www.volkskrant.nl/binnenland/kabinet-houdt-optie-proefboren-schaliegas-open~a3804551/ (in Dutch)

[6] http://www.nytimes.com/2015/01/31/business/international/chevron-to-abandon-shale-venture-in-poland-a-setback-to-fracking-europe.html

for infrastructure investments under gas demand uncertainty.

In contrast to the two-stage problems above, in multi-stage problems there are several moments in time where actors make decisions facing uncertainty. Egging (2010, 2013) develops a stochastic mixed complementarity problem and applies it to an aggregate regional representation of the global natural gas market. Egging and Holz (2015, 2016) use the model to analyze eight scenarios up to 2040 with a data set containing 79 individual countries. A similar stochastic model for the European market (S-GASTALE) was developed by Bornaee (2012) and was based on the model developed by Boots et al. (2004); Egging and Gabriel (2006); Lise and Hobbs (2008).[7]

Suppliers in natural gas markets are greatly affected by uncertainty associated with, for instance, market size, prices, and the competitive environment. Specifically when considering shale gas exploration and production, the playing field is very unclear. A total ban on production after a major investment has been made will lead to huge unrecoverable costs and in turn jeopardize company profit. Avoiding such drastic losses, or at least balancing them rationally versus the upward potential of risky decisions, gives rise to the use of risk averse decision making. The literature concerning market equilibrium models with risk-averse agents is very limited. Ehrenmann and Smeers (2008) present a stylized example in a broader discussion of the regulatory regime in the liberalized European electric power market. Cabero et al. (2010) look at a stylized example containing three electricity producers inspired by the Spanish wholesale electricity market. García-González et al. (2007) consider the unit commitment of hydropower plants for a single price-taking company considering detailed operational characteristics. They implement risk aversion through a minimum profit constraint. Chin and Siddiqui (2014) develop a two-stage electricity market model with generation capacity expansions, and forward and spot markets; they analyze a monopoly, duopoly, and perfectly competitive setting with risk-averse suppliers. To the best of our knowledge, Luna et al. (2016) is the only reference considering risk aversion in a natural gas market in a two-stage setting, however for one type of natural gas resources only. They develop an approximation scheme to solve two model variants. We implement the same risk measure as Luna et al. (2016), but show that an approximation-decomposition approach is not needed to solve the resulting model.

In this paper, we develop a multistage model for a natural gas market that accounts for market power and risk averse behavior among suppliers using the Average Value-at-Risk ($\mathsf{AV@R}$, also referred to as Conditional Value-at-Risk). This approach incorporates risk in an appropriate way and is not as conservative as the robust game theory approach addressed in Aghassi and Bertsimas (2006).

Our model considers different types of natural gas resources, each with specific investment and production cost characteristics, and it accounts for endogenous expansions of production capacity, pipelines, and natural gas reserves. The model can be viewed as an extension of the stochastic Global Gas Model in Egging (2010, 2013). The functional contributions are (i) reserve limitations,

[7] In the above we have limited ourselves to discussing extensive-stochastic mixed complementarity models presented in the literature. Readers interested in solution methods to address scalability challenges of such models can refer to, e.g., Fuller and Chung (2005, 2008), decomposition; Gabriel and Fuller (2010) decomposition; Gabriel et al. (2009) scenario reduction; Egging (2010, 2013) decomposition; Lise and Hobbs (2008), rolling horizon; Devine et al. (2014, 2016) rolling horizon.

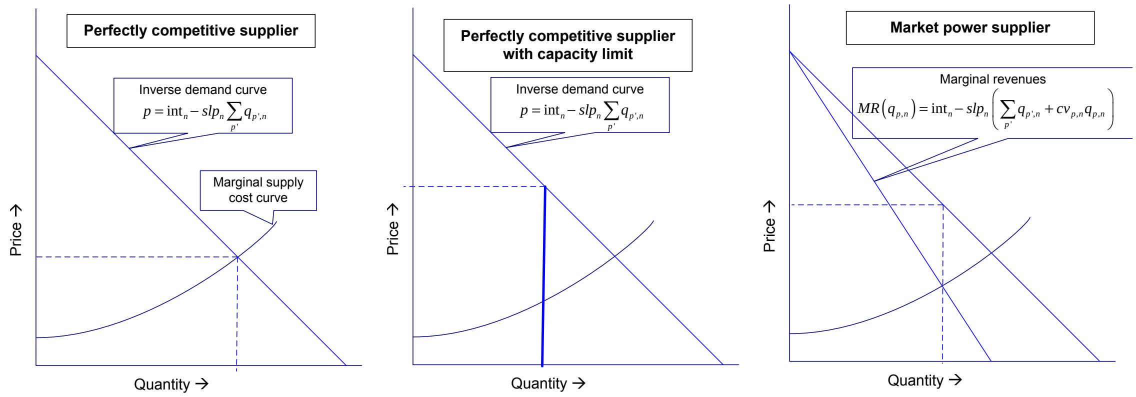


Figure 1: Illustration how capacity limits and market power exertion may drive up market prices

(ii) multiple resource types, (iii) endogenous reserve exploration and production capacity expansions, and (iv) allowing for risk averse AV@R objectives.

The model is solved as a multistage stochastic mixed complementarity problem (MCP). Furthermore, we compute the derivatives of the risk measures explicitly. This allows us to solve the problem directly, thus avoiding smoothed auxiliary variants of the genuine problem (cf. Luna et al. (2016)). Focusing on shale gas, and the uncertainties and risks connected to it in a realistic representation of the north and central European gas market, we show how risk aversion affects reserve exploration and production capacity expansion in conventional and shale gas by the suppliers. Generally, we find that larger risk aversion decreases investment in the more risky alternative and lowers expected and risk-adjusted profits. However, sometimes increased risk-aversion leads to higher objective values. In the gaming setting, lower capacity investment effectively reduces market supply, driving up prices beyond what risk-neutral suppliers would have obtained. To provide an intuition, Figure 1 illustrates in three graphs how capacity limits and market power exertion result in a higher market price than a perfectly competitive equilibrium.

**The remainder of this paper is organized as follows.** In Section 2, we present the model formulation. Section 3 introduces and discusses several case studies that analyze the impact of uncertainty and risk aversion on investment behavior of natural gas suppliers. Section 4 provides conclusions on the model. The appendix contains details concerning the risk measure, a brief discussion of computational approach and times, an extensive overview of the input data, and selected case study results.

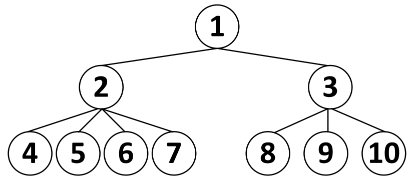


Figure 2: Example of scenario tree with three stages and seven scenarios

## 2 The model

This paper models the market for natural gas as a game theoretic, multistage stochastic mixed complementarity problem.[8] To this end, we consider different individual agents in the natural gas market: suppliers, consumers, and a transmission system operator (TSO) who manages the gas transportation network of pipelines. All risk-neutral suppliers and the TSO maximize their individual expected discounted profit and decide on investments within their area of influence.[9] Decisions for the suppliers concern investment in exploration activities (to find new gas fields), investment in production capacity (to allow production from gas fields), and production and sales of gas. Investments induce costs, and resulting capacities can be brought online after a specified delay corresponding to the leadtime typical for capacity expansion projects. The TSO operates the network to transport gas. The TSO is considered a price-taking agent maximizing its expected profit, which is achieved by renting out transportation capacity to suppliers and collecting fees when doing so. Whenever deemed profitable, the TSO will invest in enlarging pipeline network capacities. Consumers are modeled through linear inverse demand functions by regional node. Volumes and capacities are denoted in billion cubic feet (bcf) per day, cost and prices in $ per thousand cubic feet (mcf).

The market equilibrium depends on the (optimal) decisions of all individual agents (cf. Bjørndal and Jörnsten (2008)). Market equilibrium is characterized by the fact that a different decision by a single agent cannot increase the profit of that agent (Nash equilibrium, cf. Aubin (1998) and Luna et al. (2013)).

In this paper, we consider an investment (planning) horizon which spans consecutive periods with a multistage structure (cf. Pflug and Pichler (2014, 2015b)). The agents play a noncooperative game wherein they simultaneously consider their decisions over the whole time horizon at the beginning (for a discussion on simultaneous versus sequential decisions refer to Wogrin et al. (2013)). The evolution of the market is not deterministic. Instead, different (stochastic) scenarios model the information structure in a multistage tree (See Figure 2 for an example), and at each node of the tree new decisions are possible.

The stochastic tree regarding the evolution of the market is the same for each agent, but their decisions are individual. We model the evolution of the market by economic scenarios and

[8]The model was developed in the master thesis Kalvø and Walle-Hansen (2014), but presented here more succinctly.
[9]Risk-averse suppliers maximize a risk measure as explained in the following sections.

uncertainty concerning shale gas finds. We analyze yearly average volumes, and hence, ignore seasonality and storage aspects.

### 2.1 The supplier's problem

Every supplier can invest in reserve exploration and production infrastructure, i.e., production capacity. It takes one stage (a period of five years) from an exploration or investment decision to availability of reserve additions or production capacity expansions to allow actual production of gas. Decisions regarding production, transportation, and sales rates are made at every node in the scenario tree. The supplier sells gas to consuming countries and pays the TSO for transportation services to transport gas to export markets. Market prices depend on sales by all suppliers and scenario and stage-dependent inverse demand curves. Actual reserve additions due to exploration materialize as a random process over time. The supplier takes risk into account by picking a risk-adjusted objective to weigh the potential returns (the Average Value-at-Risk, cf. Appendix B and Ahmed et al. (2007)).

Total earnings are composed of the quantity $q^S$ sold daily (the sales rate) at the price $\pi$, the total revenues thus are $\pi \cdot q^S$.

Our model further involves a parameter $cv$ modeling market power (implementing so-called *conjectured variation*, with values $0 \leq cv_{p,n} \leq 1$; see Egging and Gabriel (2006) for further explanation) and the corresponding market price is $\pi = \mathrm{int} - \mathrm{slp} \cdot \sum q^S$ (cf. Eq. (1) below and Figure 1), where *int* is an intercept and *slp* the slope, both of which are exogenously given parameters.

Investment costs reduce today's total earnings, but investment may allow higher earnings in later periods. The two types of investment costs we consider are production capacity expansion costs and cost to expand reserves.

To this end, we model the future profit $Y$ as a variable with random outcomes on some probability space with samples $\omega \in \Omega$. The profit materializes for every sample $\omega \in \Omega$ and consists of sales revenues minus cost for production (and processing), transportation, exploration, and capacity expansion.

$$Y_P(\omega) := dr_\omega \left\{ \begin{array}{l} D \left( \begin{array}{l} \sum_{n \in N} \left( \begin{array}{l} \left( cv_{p,n} \left( \mathrm{int}_{n\omega} - \mathrm{slp}_{n\omega} \cdot \sum_{p' \in P} q^S_{p'n\omega} \right) \right. \\ \left. +(1 - cv_{p,n}) \pi^M_{n\omega} \right) q^S_{pn\omega} \\ - \sum_{r \in R} \left( C^L_{prn} q^P_{prn\omega} + C^Q_{prn} (q^P_{prn\omega})^2 \right) \end{array} \right) \\ - \sum_{a \in A} \tau_{a\omega} f_{pa\omega} \end{array} \right) \\ - \sum_{r \in R} \left( C^{\Delta P}_{prn} e^P_{prn\omega} + C^{\Delta R}_{prn} e^R_{prn\omega} \right) \end{array} \right\}, \tag{1}$$

The symbol $dr_\omega$ stands for the discount rate, $D$ is the number of days in a stage, gas is sold at rate $q^S_{pn\omega}$ and produced at rates $q^P_{prn\omega}$, $\pi^M_{n\omega}$ is the market price for demand node $n$, and $cv_{p,n}$ is the market power parameter for the supplier $p$ in country node (market) $n$. Parameters *int* and *slp* are the slope and the intercept of the inverse demand curve (cf. Table 7). Production costs are assumed to be quadratic, $C^L_{prn} q^P_{prn\omega} + C^Q_{prn} (q^P_{prn\omega})^2$. Transportation costs are determined by the market clearing price for transmission $\tau_{a\omega}$ and the flow rate $f_{pa\omega}$ on the arc $a$ for supplier $p$ in scenario $\omega \in \Omega$. Next, $e^P_{prn}$ and $e^R_{prn}$ are investments in production capacity expansion and reserve exploration in resource $r$ by supplier $p$ in the country node $n$, respectively, and $C^{\Delta P}_{prn}$ and $C^{\Delta R}_{prn}$ are the corresponding unit costs.

The objective of the supplier is the expected discounted profit, which is the probability-weighted sum $\mathbb{E}\, Y_P = \sum_\omega p_\omega Y_P(\omega)$. He maximizes the profit with respect to the investments $e^R$ and $e^P$, and with respect to the rates $q^P$, $q^S$ and $f$. $Y_P$ is a quadratic function in the quantities produced and quantities sold and is linear in transportation and both investment types. The random outcome $Y_P(\cdot)$ is *concave* in its decision variables, and so is the objective function of the risk-neutral supplier.

In the subsequent discussion we extend the objective and incorporate risk in the analysis. To this end we replace the expected value of the future profit and consider the Average Value-at-Risk, i.e., the supplier maximizes $\mathsf{AV@R}(Y_P)$ instead of $\mathbb{E}\, Y_P$ (cf. Appendix B for the implementation we employ Eq. (12). This particular reformulation of the Average Value-at-Risk is a maximization itself, and thus in line with profit maximization of a concave function for the supplier).

### Constraints

The supplier faces economic and physical constraints. Table 1 collects the dual variables corresponding to the subsequent constraints employed in the formulation as an optimization problem.

**Production capacity constraints.** The (daily) production rate of each resource type $r$ in country node $n$, and every supplier $p$ and scenario tree node $\omega$ cannot exceed the capacity to extract gas from the resource, including expansions in previous time periods. Hence the constraints

$$q^P_{prn\omega} \leq CAP^{P0}_{prn} + \sum_{\omega' \in pred(\omega)} e^P_{prn\omega'}; \qquad \text{dual: } \lambda^P_{prn\omega} \tag{2}$$

have to hold for every $r$ and $p$, on every node $n$ in each scenario $\omega$.

**Mass balance constraints.** In every node in the network, the amount a supplier sends into a country plus the amount it produces there must equal the amount it sends out of the country plus the amount it sells there.

$$\sum_{r \in R} q^P_{prn\omega} + \sum_{a \in a^+(n)} f_{pa\omega} - q^S_{pn\omega} - \sum_{a \in a^-(n)} f_{pa\omega} = 0; \qquad \text{dual: } \phi^P_{pr\omega}. \tag{3}$$

After production, the different resource types $r$ are considered indistinguishable, hence in Eq. (3) we provide the summation over resources $r$ to determine the total production. The constraints thus hold for every $p$ on every node $n$ and in each scenario $\omega$.

**Reserve constraints.** The cumulative production in a field cannot exceed the initial reserves in that field plus additional reserves due to exploration.[10] Here, $\Delta R$ represents the different random

[10]Because in the model production capacity is not connected to specific reserves, existing or added production capacity can be used to produce reserve additions in later stages.

Table 1: Dual variables

| dual variable | unit | description |
|---|---|---|
| $\pi^M_{n\omega}$ | bUS\$/ bcf | duals to the sales market clearing constraint (9) |
| $\tau_{a\omega}$ | bUS\$/ bcf | dual to the arc flow market clearing constraint (10) |
| $\lambda^P_{prn\omega}$ | $\frac{\text{bUS\\$}}{\text{bcf/ d}}$ | dual to the production capacity constraints (2) |
| $\phi^P_{pn\omega}$ | $\frac{\text{bUS\\$}}{\text{bcf/ d}}$ | dual to the supplier's mass balance restriction (3) |
| $\mu^P_{p\omega}$ | 1 | duals to the supplier's budget restriction (5) |
| $\mu^T_{\omega}$ | 1 | duals to the TSO's budget restriction |
| $\rho^P_{prn\omega}$ | $\frac{\text{bUS\\$}}{\text{bcf}}$ | dual to the supplier's reserve constraint (4) |

outcomes as a consequence of reserve exploration.

$$\sum_{\omega' \in pred(\omega)} Dq^P_{prn\omega'} + Dq^P_{prn\omega} \leq R^{P0}_{prn} + \sum_{\omega' \in pred(\omega)} \Delta R_{prn\omega'} e^R_{prn\omega'}; \qquad \text{dual: } \rho^P_{prn\omega}. \tag{4}$$

As for (2), these constraints hold for $r$ and $p$ on every node $n$ and in each scenario $\omega$.

**Budget constraints.** Suppliers may have limited access to funding to finance investment. The costs of exploration and construction of production facilities have to be within the limits of the capital budget available to the supplier, so that

$$\sum_{r \in R, n(p) \in N} C^{\Delta P}_{prn} e^P_{prn\omega} + C^{\Delta R}_{prn} e^R_{prn\omega} \leq B^P_{\omega p}; \qquad \text{dual: } \mu^P_{p\omega} \tag{5}$$

has to hold for each individual supplier $p$ irrespective of the scenario $\omega$.

## 2.2 The transmission system operator

The transmission system operator (TSO) is responsible for managing and expanding the gas transportation network. The European regulatory framework concerning third-party access of infrastructure aims at a network access at competitive prices (EC (2009)). To reflect this, the TSO is modeled as a price-taking agent. This means that transport capacity will be made available to the suppliers at marginal cost as long as capacity is not restrictive. When capacity is restrictive, a market-determined congestion fee is charged on top of the marginal cost, reflecting the marginal value of transportation services to the suppliers.[11] We assume that both operational and expansion costs are linear. The congestion fee $\tau_{a\omega}$ is the dual of the market clearing condition, Eq. (10) below. The congestion fee can only be positive when capacity is binding; the higher its value for an arc, the more the specific arc is a bottleneck. The TSO maximizes its expected, probability weighted ($p_\omega$), discounted ($dr_\omega$) profit by employing the objective

$$\sum_{a,\omega} p_\omega dr_\omega \left( D \left( \tau_{a\omega} - C_a^F \right) s_{a\omega} - C_a^{\Delta A} e_{a\omega}^A \right), \tag{6}$$

where $s_{a\omega}$ is the capacity offered to the suppliers. The TSO decides on the capacities offered ($s_{a,\omega}$) and the network expansions ($e_{a\omega}$).

**Flow capacity constraint.** Pipeline capacities depend on flows and pressure differences in neighboring pipes, which can be modeled by the Weymouth equation for high pressure gas flow. In this paper, we replace the nonlinear physical restrictions with capacity limits on flow rates (cf. Figure 3). Our model allows for expansions, so these capacities may increase over time. The constraints

$$s_{a\omega} \leq CAP_a^{A0} + \sum_{\omega' \in pred(\omega)} e_{a\omega'}^A. \tag{7}$$

have to hold on every arc $a$ and in each scenario $\omega$.

**Budget constraint.** The TSO's budget limits network expansions in each stage:

$$\sum_a C_a^{\Delta A} e_{a\omega}^A \leq B_\omega^{TSO}. \tag{8}$$

### 2.3 Market clearing constraints

Final consumers are represented as inverse demand curves by node, so that the market clearing constraints

$$\pi_{n\omega}^M = \mathrm{int}_{n\omega} - \mathrm{slp}_{n\omega} \cdot \sum_{p \in P} q_{pn\omega}^S; \qquad \text{dual: } \pi_{n\omega}^M \tag{9}$$

have to hold in each node $n$ in every scenario $\omega$.

Further, the transport capacity offered by the TSO must be equal to the capacity demanded by the suppliers

$$s_{a\omega} = \sum_{p \in P} f_{pa\omega}; \qquad \text{dual: } \tau_{a\omega} \tag{10}$$

on every arc $a$ and in each scenario $\omega$.

[11] This is a common approach, see e.g., Zhuang (2005), Zhuang and Gabriel (2008), Holz et al. (2008), or Lise and Hobbs (2008).

In sum, every supplier $p$ produces natural gas in one or more countries (nodes $n$) and sells the gas to various countries, including domestic sales. A supplier can export gas by transporting gas through a pipeline network at a unit cost $\tau_{au}$, consisting of a base cost plus a market-determined congestion fee. The supplier maximizes his expected discounted profit given a preference towards risk. This risk preference is expressed through the $\mathsf{AV@R}_{p,\alpha_p}$, meaning that supplier $p$ maximizes the profits by considering all $\alpha_p \cdot 100\,\%$ worst scenarios.

Uncertainty is present in many forms in the natural gas market, for example in demand, expansion costs, and political climate regarding bans or licenses for developing shale gas. In this paper, we concentrate on the risk associated with shale gas development. Stage-wise uncertainty is modeled by a multistage scenario tree. Of course, every agent could base its future planning on an individual tree, but here the evolution of time (the information structure) is the same for all agents. Every agent bases its decisions on a single multistage scenario tree, which is valid for all.

# 3 Case study

In the case studies presented, the input data used in Kalvø and Walle-Hansen (2014) have been thoroughly revised to more carefully reflect actual supply and demand in the 2014 European market. Additionally, in the cases presented in this paper, we vary the risk aversion levels of different agents independently. For the risk measure we allow small values for $\alpha$ while setting $\kappa = 0$ in Eq. (11) (cf. Appendix B) to better display the influences of (extreme) risk aversion.

Here, we analyze the impact of risk aversion on reserve exploration, capacity investment, and trade in the Northern European natural gas market, specifically considering uncertainty in the amount of shale gas available after exploration investment by Poland and Ukraine. We also solve and analyze the deterministic problem[12] and the stochastic problem wherein all agents are risk neutral. We do not discuss these cases extensively due to space limitations, but mention some results for contrast.

## 3.1 Input data

In this section, we present the natural gas market network and some other input data (see Figure 3 and Tables 6, 7, and 8 in Appendix C for further details).

Past production and consumption data are based on BP (2014). We consider shale gas in Poland and Ukraine only. Russia, Norway, and the Netherlands are exporting countries over the whole time horizon. Poland is an importer in the first year, but may become a net exporter in later years. All other countries are, and continue to be, importers. Two characteristics affect the analysis: (i) compared to Poland, in the first year, Ukraine covers a much smaller part of demand by imports, (ii) for Poland, shale gas makes up a much larger part of the total production potential.

In the *market power* (*Cournot-oligopoly*) cases, suppliers act as Nash-Cournot agents in all export markets, but do not exert market power in their respective domestic markets. Suppliers and

[12] In deterministic cases, we solved the so-called expected value problems wherein the values of parameters which are uncertain in the stochastic analyses are taken as the probability-weighted average values.

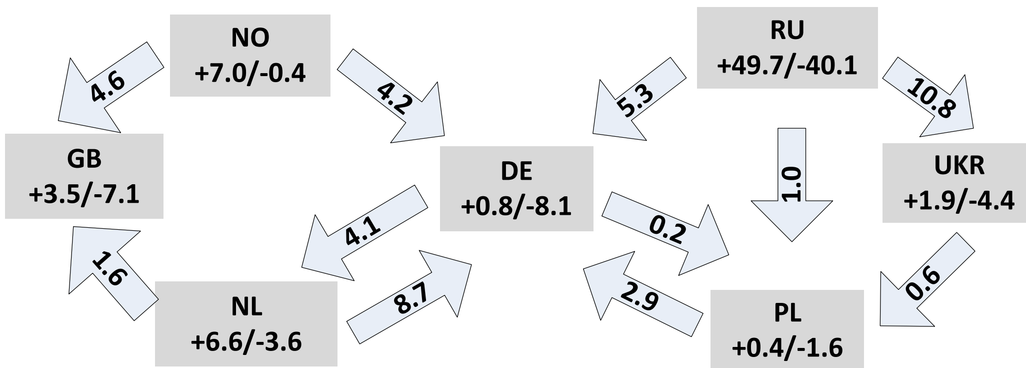


Figure 3: Natural gas network with base year data (bcf/ d) for production (positive values in rectangles), consumption (negative values), and pipeline capacity.

the TSO discount future profits at 5 % per year. Demand growth for Russia, Poland, and Ukraine relative to 2014 is +5 % in 2019 and +10 % in 2024. Reference prices in all countries are inflated relative to 2014 by +5 % in 2019 and +10 % in 2024. Price-demand elasticity is assumed to be 0.5 in all countries. Intercepts and slopes of the linear inverse demand curves are calculated based on reference prices, demand, and elasticities (see Table 8).

In stochastic problems, shale gas exploration activities can result in low (−80%) or high (+80 %) additions relative to expectation, or a complete ban (−100%) on shale gas production in future periods. The latter renders the shale gas found and shale production capacity invested useless. The ban probability for shale gas production is an input parameter $Prob_{ban}$. The probabilities for the events low (subscript L) and high (H, resp.) shale gas finds are equal and calculated as $Prob_L = Prob_H = \frac{1}{2}(1 - Prob_{ban})$. As such, if a supplier invests in shale gas exploration expecting to find an amount allowing 1.0 bcf/ day of production, he will find 0.2 or 1.8. However, if a shale gas ban is put into place by the regulator, none of the shale reserves can be produced.[13]

For succinctness we often write Poland, Ukraine, etc. when referring to the Polish and the Ukrainian suppliers.

### 3.2 Scenario Tree

In Poland and Ukraine, there can be a ban on shale gas, or high or low finds are possible. Therefore, the scenario tree splits into nine branches after the first stage (see Table 2 below). If a ban is imposed in a country, it will carry on through the third stage; if not, there will not be a ban imposed in the third stage. Using B, L, and H (ban, low, and high, resp.) to denote the event states for both the

[13]Capacity investment and exploration cost values have been tuned relative to actual values to allow *enough* investment. Still, due to the short time horizon there is some under investment, especially by the TSO in the second stage due to lack of pay-back period.

second and third stages, BHBL indicates a scenario with a ban in Poland (in *both* stages) and high finds in Ukraine in the second stage and low finds in the third stage. To reduce the number of rows in the last column in Table 2, an X is used as shorthand to indicate that both L and H can happen in the third stage. Figure 4 (cf. Pflug and Pichler (2015a)) illustrates the scenario tree.

Table 2: Main scenario characteristics

| **stage 2** | | **scenario** | |
|---|---|---|---|
| **POL** | **UKR** | | **stage 3** |
| H | H | HH | XX |
| L | H | LH | XX |
| B | H | BH | BX |
| H | L | HL | XX |
| L | L | LL | XX |
| B | L | BL | BX |
| High | B | HB | XB |
| Low | B | LB | XB |
| Ban | B | BB | BB |

In the stochastic cases, we assume a 10 % probability in either country that a ban on shale gas production will be announced, independent from one another. This means that the BBBB scenario has a $Prob_{ban} \cdot Prob_{ban} = 10\% \cdot 10\% = 1\%$ chance to happen. If no ban is imposed, the probabilities for low and high finds are symmetrical, i.e., 50 %–50 %. Hence the probability of the LBHB scenario, e.g., to occur is $(45\% \cdot 10\%) \cdot (50\%) = 2.25\%$ (with $Prob_L = \frac{1}{2}(1 - Prob_{ban}) = 45\%$) and the HHHH (and any other XXXX) scenario, $(45\% \cdot 45\%) \cdot (50\% \cdot 50\%) = 5.0625\%$.

## Computational approach

We combine Karush-Kuhn-Tucker (KKT) conditions from the model formulation presented in Section 2 with the market clearing conditions (collected in Appendix F). After taking the KKT, we then substitute the inverse demand curve for the market prices in the KKT of the suppliers. The KKT are implemented in GAMS (Brook et al. (1988)) and we use the solver PATH to solve the data instances (version 24.2.1, Dirkse and Ferris (1995); Ferris and Munson (2000)).

During our experiments, we found that for problems that deviate from the expected value problems, the default PATH solver settings are not always well-suited to find equilibrium solutions. For moderate risk aversion level ($\alpha$-values close to 100 %), solutions were still found quickly. By stepwise gradually changing the level parameter $\alpha$ of the Average Value-at-Risk for one supplier at a time, allowing several solution attempts with different solver settings, and allowing slightly larger-than-default solution tolerances up to $10^{-5}$, we managed to solve all attempted problem instances. (See Appendix D for some details.) We discuss selected results in the following sections.

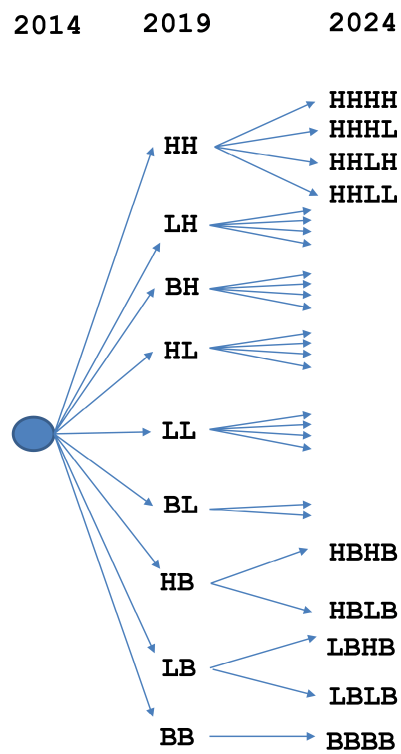

Figure 4: Scenario Tree

### 3.3 AV@R Results

We have implemented risk aversion by letting the suppliers from Poland and Ukraine consider their profits in some part of the scenarios with the worst outcomes only. The share is indicated by a percentage (the level $\alpha$). A value of 100 % means that all scenarios are considered and a value of 80 % indicates that the worst-outcome scenarios with a total probability of 80 % are considered in the objective function. A lower percentage indicates a higher level of risk aversion, or being more risk averse. Note: worst-outcome scenarios are not known a-priori and may be different for different agents. In fact, if $\alpha_{POL} + \alpha_{UKR} \leq 100\%$ the set of worst-case scenarios may not overlap at all.

#### 3.3.1 Modest risk aversion

Table 3 displays the values of the Average Value-at-Risk for all suppliers for combinations of risk aversion values for the Polish and Ukraine suppliers. Except for Poland and Ukraine, all suppliers are risk neutral. (For $\alpha = 100\%$ , the AV@R values equal the expected profit.)

**Varying own risk aversion.** The first column and first three rows in Table 3 show the AV@R values for Poland in the Cournot market case where Ukraine is risk-neutral ($\alpha_{UKR} = 100\%$). In line with our expectations, we see that increased risk aversion ($\alpha_{POL} = 100\%, 90\%, 80\%$) leads to a decrease in AV@R$_{POL}$: from 23.51 (\$bln) to 23 and 22.46, both decreases are slightly over 2 %. The outcomes for the competitive case are presented in the last three columns. In column $\alpha_{UKR} = 100\%$, we see that for gradually higher Polish risk aversion, AV@R$_{POL}$ decreases from 10.75 to 10.52 and 10.32 in stepwise decreases of approximately 2 %.

To do the same analysis for Ukraine, we look at the values for $\alpha_{UKR} = 100\%$ : 49.24, 49.05, 48.80 and 12.33, 12.32, 12.29,[14] and the AV@R of Ukraine for increasing risk aversion ($\alpha_{UKR} = 100\%, 90\%, 80\%$) in the Cournot market and competitive market respectively. As absolute and relative values, the decreases are much smaller than for Poland. This can be explained by the fact that for Poland, shale gas forms a much larger share of total production potential, whereas Ukraine's supply contains much more conventional reserves. As such, for increasing risk aversion, Ukraine has the option to shift part of its investment to conventional supplies, which in our case are not exposed to uncertainty. Additionally, Ukraine has a much larger resource base to start with, and the expansions are smaller relative to these existing resources.

When we look at a moderately risk averse Ukraine ($\alpha_{UKR} = 90\%$ and $\alpha_{UKR} = 80\%$), we see that increasing Polish risk aversion (going down one or two rows) leads to similar decreases in Polish AV@R for $\alpha_{UKR} = 100\%$: 23.50, 22.99, 22.46 and 23.52, 22.99, 22.44. Likewise, for $\alpha_{POL} = 90\%$ and $\alpha_{POL} = 80\%$, increasing Ukrainian risk aversion (going right one or two columns) leads to similar decreases in Ukrainian AV@R as for $\alpha_{POL} = 100\%$: 49.21, 49.01, 48.72, etc.

**Varying the other's risk aversion.** Considering the Polish expected profit when it is risk neutral (AV@R$_{POL}$, $\alpha_{POL} = 100\%$) in the Cournot case and decreasing $\alpha_{UKR}$ from 100% to 90% to 80% (going to the right), shows that Polish expected profit first slightly *decreases* and then *increases* (23.51, 23.50, 23.52). However, for $\alpha_{POL} = 90\%$, the same decrease in $\alpha_{UKR}$ leads to AV@R$_{POL}$ values of 22.997, 22.988, and 22.994: first a slight *decrease* and then a slight *increase*. For $\alpha_{POL} = 80\%$, we get 22.462, 22.458, and 22.439: two *decreases*.

In the competitive case, for the value $\alpha_{POL} = 100\%$, decreasing $\alpha_{UKR}$ leads to increasing AV@R$_{POL}$ values: 10.747, 10.818, and 10.842. For $\alpha_{POL} = 90\%$, there is first an *increase* from 10.524 to 10.573, and then a *decrease* to 10.563. For $\alpha_{POL} = 80\%$, we see two *increases*, from 10.317 to 10.427 and to 10.460. Clearly, given an own risk-aversion level and increasing the other's risk-aversion level leads to *non-monotonic* changes in the own AV@R.

For Ukraine, we see similar mixed trends in how AV@R$_{UKR}$ depends on Poland's risk aversion. If we look at the expected profits of all other suppliers, we might expect that for increasing risk aversion of either agent (going to the right and/ or down), expected profits would increase. But looking in the table, we see mixed results in both the Cournot and the competitive market. For instance, in the competitive market, for $\alpha_{POL} = 90\%$ and an increasing $\alpha_{UKR}$, both Dutch and Norwegian expected profits decrease slightly. The interplay of agents in the gaming setting affects

[14]The values in the fourth row below the heading, with row indication UKR 100%

Table 3: AV@R ($bln) all suppliers for varying risk-aversion levels Poland and Ukraine

| | | Cournot | | | competitive | | |
|---|---|---|---|---|---|---|---|
| | | $\alpha_{UKR}$ | | | $\alpha_{UKR}$ | | |
| | $\alpha_{POL}$ | 100% | 90% | 80% | 100% | 90% | 80% |
| POL | 100% | 23.51 | 23.50 | 23.52 | 10.75 | 10.82 | 10.84 |
| | 90% | 23.00 | 22.99 | 22.99 | 10.52 | 10.57 | 10.56 |
| | 80% | 22.46 | 22.46 | 22.44 | 10.32 | 10.43 | 10.46 |
| UKR | 100% | 49.24 | 49.05 | 48.80 | 12.33 | 12.32 | 12.29 |
| | 90% | 49.21 | 49.01 | 48.72 | 12.34 | 12.33 | 12.30 |
| | 80% | 49.20 | 49.05 | 48.62 | 12.34 | 12.32 | 12.30 |
| GER | 100% | 10.75 | 10.75 | 10.75 | 6.23 | 6.23 | 6.23 |
| | 90% | 10.75 | 10.75 | 10.75 | 6.23 | 6.23 | 6.23 |
| | 80% | 10.75 | 10.75 | 10.75 | 6.23 | 6.23 | 6.23 |
| NED | 100% | 76.51 | 76.48 | 76.52 | 49.71 | 49.72 | 49.72 |
| | 90% | 76.47 | 76.43 | 76.36 | 49.70 | 49.66 | 49.56 |
| | 80% | 76.54 | 76.54 | 76.31 | 49.69 | 49.71 | 49.70 |
| NOR | 100% | 89.95 | 89.93 | 90.01 | 45.83 | 45.84 | 45.84 |
| | 90% | 89.93 | 89.92 | 89.89 | 45.82 | 45.78 | 45.67 |
| | 80% | 90.13 | 90.07 | 89.99 | 45.81 | 45.83 | 45.83 |
| RUS | 100% | 348.28 | 348.11 | 348.24 | 304.89 | 306.73 | 307.07 |
| | 90% | 348.12 | 347.90 | 347.46 | 304.98 | 306.81 | 307.22 |
| | 80% | 348.56 | 348.26 | 347.62 | 304.95 | 306.74 | 307.09 |
| UKD | 100% | 54.95 | 54.95 | 54.98 | 26.78 | 26.78 | 26.78 |
| | 90% | 54.93 | 54.94 | 54.96 | 26.78 | 26.78 | 26.78 |
| | 80% | 55.05 | 55.00 | 55.09 | 26.77 | 26.77 | 26.77 |

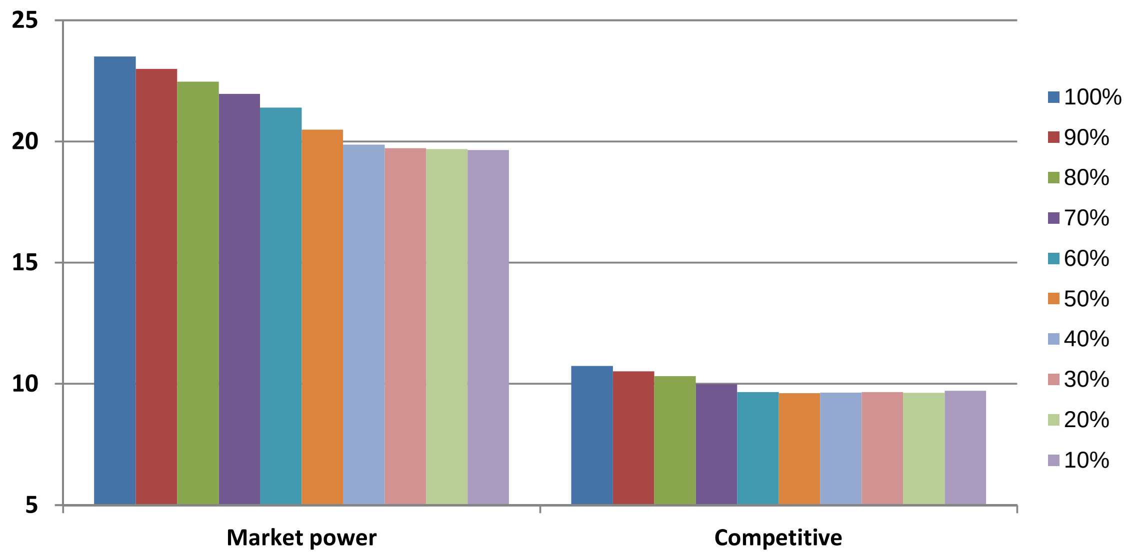


Figure 5: AV@R Poland (bln\$) - from left to right increasing risk aversion

capacity expansions, supplied volumes, and trade patterns in a non-transparant way, which results in non-monotonic results.

### 3.3.2 Extreme risk aversion

This subsection discusses AV@R values when only one supplier is risk averse: first Poland and then Ukraine. Figure 5 illustrates how Polish AV@R changes with increasing risk-aversion given that all other suppliers are risk neutral (including Ukraine). The left-hand side presents results for the Cournot case and the right-hand side presents results for the competitive case.

The general trend for both market structures is that starting from expected profit maximization and increasing risk aversion, the main decrease in AV@R occurs when reducing the $\alpha$-value from $100\%$ to $50\%$. When considering even lower values, the AV@R levels out. Surprisingly, for very low $\alpha$-values in the competitive case, the AV@R increases again, albeit very modestly. From $\alpha_{POL} = 50\%$ with AV@R 9.62, the values for further decreasing the values of $\alpha_{POL}$ are 9.65 (+0.03), 9.67 (+0.02) 9.63 (−0.04), and 9.72 (+0.09), respectively. The order of magnitude of these values is about $1\ \%$.

Figure 6 illustrates how Ukraine AV@R changes as risk-aversion increases, given that all other suppliers are risk neutral. The left-hand side shows results for the Cournot case, the right-hand side shows competitive case results.

The figures show that the AV@R values for Ukraine are much less sensitive to increasing risk aversion than the Polish values. As discussed above, Ukraine has the option to invest in conventional rather than shale resources, which turns out to be a great hedging opportunity that can be tuned

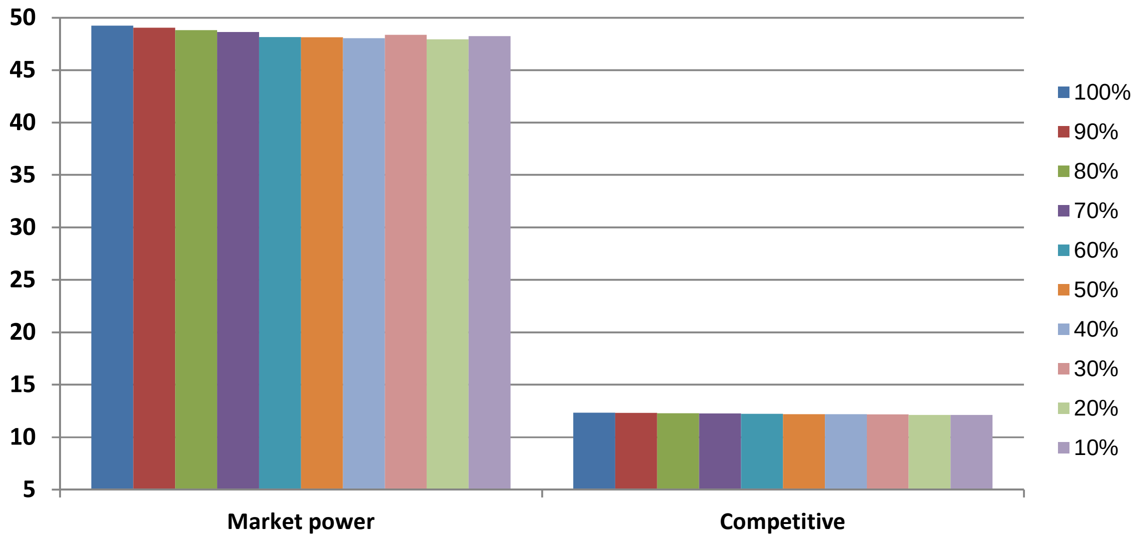


Figure 6: AV@R Ukraine (bln$) - from left to right increasing risk aversion

perfectly towards the risk attitude.

Interestingly, for Ukraine, the AV@R value in the competitive case monotonically decreases the $\alpha$-value, but the Cournot case AV@R values do not. For $\alpha_{UKR} = 40\%$, the AV@R is 48.0, and for increasing risk attitude, the AV@R becomes 48.4 (+0.4), 47.9 (−0.5), and 48.3 (+0.4). Similar to the Polish case, the order of magnitude shown for the increases is about 1 %.

Support for the possibility of this non-monotonic result is provided by Chin and Siddiqui (2014) who find that suppliers in a duopoly market may earn higher than expected profits when becoming more risk averse. In the perfectly competitive setting, the increasing AV@R can be explained by the price effect of lower capacity investment (cf. Figure 1), while the nonmonotonic result is likely due to the multi-agent setting, wherein agent's profits are affected by the risk-averse behavior of others.

### 3.3.3 Investment behavior by risk-averse suppliers

In this section, we investigate the part of the investment budget that is used under various levels of risk aversion. We also investigate how the budget is used for actual investment in reserve exploration and production capacity expansion in both conventional and shale gas. According to our assumptions, both countries have an investment budget of $4 bln in each stage. We consider first cases where all suppliers are risk neutral, except Poland.

Table 4 contains the budgets used by Poland under different market structures and risk aversion levels. Table 11 in the appendix contains the actual investments by Poland. Here, the second-stage scenario nodes are reordered so that the three Polish B, L, and H scenarios are grouped together.

In the market power case, Poland uses its complete budget in the first and second stages, except in some instances of extreme risk aversion—although even then 95 % is still used. In the first stage,

Table 4: Poland Budget use under varying risk aversion

| Case | Scenario Node | **100%** | **80%** | **60%** | **40%** | **20%** |
|---|---|---|---|---|---|---|
| Cournot | stage 1 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | BB | 4.0 | 4.0 | 4.0 | 4.0 | 3.9 |
| | BL | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | BH | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | LB | 4.0 | 4.0 | 4.0 | 4.0 | 3.8 |
| | LL | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | LH | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | HB | 4.0 | 4.0 | 4.0 | 3.9 | 4.0 |
| | HL | 4.0 | 4.0 | 4.0 | 3.9 | 4.0 |
| | HH | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | stage 2 average | 4.0 | 4.0 | 4.0 | 3.974 | 3.979 |
| competitive | stage 1 | 4.0 | 4.0 | 3.0 | 1.91 | 1.90 |
| | BB | | | | 0.13 | 0.10 |
| | BL | | | | 0.13 | 0.10 |
| | BH | | | | 0.13 | 0.10 |
| | LB | 4.0 | 4.0 | 4.0 | 0.7 | 0.3 |
| | LL | 4.0 | 4.0 | 4.0 | 0.6 | 0.5 |
| | HL | 4.0 | 4.0 | 4.0 | 1.1 | 1.0 |
| | HB | 4.0 | 3.1 | 3.0 | 1.2 | 0.4 |
| | HL | 4.0 | 4.0 | 3.2 | 1.1 | 0.8 |
| | HH | 4.0 | 4.0 | 3.5 | 1.0 | 0.9 |
| | stage 2 average | 3.60 | 3.56 | 3.29 | 0.866 | 0.692 |

for none or moderate risk aversion, the 0.36 investment in conventional is limited to a capacity expansion to bring production capacity in line with reserves (so that all can be produced in the model horizon; see Table 11, Appendix E[15]). Only cases of extreme risk aversion lead to exploration of conventional reserves (and additional production capacity expansion), at the expense of lower shale investment. In the second stage, if Poland is facing a ban (BX), all investment is allocated towards conventional. For extreme risk aversion in the first stage, the conventional production capacity has been increased, and additional conventional reserve exploration is performed in the second stage. (Due to how the model is set up, production capacity added in the first stage can be used for later added reserves.)

In the competitive case, with lower prevailing market prices, the investment budget is only fully used at low risk aversion levels, both in the first and second stage. In the second stage, if there is a ban, there will only be very modest reserve additions in conventional. In contrast, if there is no ban, investments are made in shale gas exploration and capacity expansions. However, at modest risk aversion levels, the budget is not fully used, and at higher risk aversion levels less than 10 % of the budget is used.

Next, we discuss cases where all suppliers are risk neutral, except Ukraine. Table 5 shows budget use and Table 12 (Appendix E) shows detailed investment decisions for Ukraine for varying risk aversion levels and market structures.

In none of the scenarios, under any of the risk aversion levels, will Ukraine invest significantly in *conventional reserves exploration* (expansions less than 0.01, less than 2 % of the budget); however, it will invest in *conventional production capacity*.

In the market power case, Ukraine uses the full investment budget in the first stage. However, the lion's share (84 %–95 %) of the budget is allocated towards conventional production capacity expansion, rather than shale investment. In the second stage, in all scenarios where shale is banned in Ukraine (XB), a large part of the investment budget (more than 50 % of the budget) is used for further conventional production capacity expansion, except for the extreme risk averse $\alpha = 20\%$ when expansions are very modest. In contrast, in scenarios where shale gas is not banned in Ukraine, a large share of the budget is used in the second stage. For risk-neutral cases, all of the budget goes to (further) shale gas exploration and production capacity additions; whereas, for higher risk aversion, more is invested in conventional production capacity expansions. Only for extreme risk aversion levels, a low share of the budget is used, and generally, higher investment in conventional occurs, but not monotonic. For instance, for $\alpha_{UKR} = 60\%$ in scenario nodes LB and HB, there is a "dip" in the conventional investments: values 2.8 (vs. 3.1) and 2.7 (vs. 2.9) in Table 5. In both scenario nodes, the investment in conventional production capacity is *higher* at the more risk averse $\alpha_{UKR} = 40\%$ (LB: 0.85 vs 0.78; HB 0.79 vs. 0.73 in Table 12).

In the competitive case, the investment budget is never fully used and that which is used gets geared towards shale reserves. In the first stage, only in the risk-neutral case, a very large part of the budget is used (3.3/4=82.4 %). In all other scenarios and/ or higher risk aversion, Ukrainian

[15]Values that illustrate non-monotonicity or that are discussed in the text have been highlighted in the tables for convenience.

Table 5: Ukraine budget use under varying risk aversion

| Case | Scenario Node | **100%** | **80%** | **60%** | **40%** | **20%** |
|---|---|---|---|---|---|---|
| Cournot | stage 1 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | BB | 3.4 | 3.4 | 4.0 | 2.0 | 0.4 |
| | LB | 3.3 | 3.2 | 2.8 | 3.1 | 0.1 |
| | HB | 3.1 | 3.1 | 2.7 | 2.9 | 0.4 |
| | BL | 4.0 | 4.0 | 4.0 | 3.8 | 1.2 |
| | LL | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | HL | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| | BH | 4.0 | 3.8 | 4.0 | 3.8 | 0.9 |
| | LH | 4.0 | 4.0 | 4.0 | 3.5 | 2.9 |
| | HH | 4.0 | 4.0 | 4.0 | 4.0 | 2.8 |
| | stage 2 average | 3.92 | 3.91 | 3.89 | 3.76 | 2.89 |
| competitive | stage 1 | 3.3 | 0.37 | 0.34 | 0.20 | 0.24 |
| | BB | | | | 0.7 | |
| | BL | 0.8 | 0.1 | | 0.6 | 0.29 |
| | LL | 0.8 | 0.1 | 0.1 | | 0.03 |
| | HL | 0.8 | 0.1 | 0.1 | 0.0 | 0.07 |
| | BH | 0.9 | 0.2 | 0.8 | 0.8 | 0.13 |
| | LH | 0.9 | 0.0 | 0.0 | 0.0 | |
| | HH | 0.9 | 0.0 | 0.0 | 0.0 | |
| | stage 2 average | 0.76 | 0.07 | 0.09 | 0.09 | 0.04 |

investments are very low. Under risk neutrality or modest risk aversion in the XL scenarios, when shale finds have turned out to be low, there is significant additional shale gas exploration. In contrast, in XH scenarios when reserves have played out high, additional investment in production capacity occurs.

For both Poland and Ukraine, total expected investment over two stages decreases when risk aversion increases (with one exception: for Poland in the Cournot case, from $\alpha_{POL}$ = 40% to $\alpha_{POL}$ = 20% there is a minor increase of 0.005, from 3.974 to 3.979).

## 4 Conclusions and future work

In this paper, we present and implement a natural gas market equilibrium model considering risk aversion related to uncertainty in shale gas reserve exploration. Risk aversion is modeled using the Average Value-at-Risk of discounted profit. In the implementation, we analyze how risk aversion may affect investment behavior of a Polish and Ukrainian natural gas supplier in the context of a competitive or an oligopolistic European natural gas market. The risk concerning shale gas is that a shale gas ban may be imposed in either or both countries, and the added reserves can amount to lower or higher volumes than expected.

In general, our results confirm what intuition suggests, that increasing a supplier's risk aversion leads to lower investment by that supplier and a larger share of the investment goes towards the lower risk alternative investment (which in the model are conventional resources) and lower AV@R values. However, for extreme risk aversion when increasing the aversion so much that less than half of the (worst-case) scenarios are considered, sometimes the AV@R slightly *increases* again, compared to more modest levels of risk aversion. Next, given a supplier with a specific risk-aversion level (including risk neutrality), increasing the risk aversion of another supplier has unpredictable effects on the own AV@R (or expected profit if the considered agent is risk neutral).

To some extent, lower supply volumes due to risk averse behavior can have the same effect as market power exertion, causing market prices to go up. In our game setting with multiple agents where some are risk-averse, due to asymmetric cost characteristics and network bottlenecks, non-monotonic relations and effects make it is impossible to predict to what extent lower supply volumes by one supplier will be offset by others, and how trade patterns and expected profits will be affected.

In a competitive market, risk aversion leads to significantly lower reserve exploration in general, because of ample low-cost supply by major exporters. An alternative interpretation for this is the existence of a *credible threat* by a large dominating supplier (i.e., Russia) to flood natural gas markets if importing countries significantly extend their own reserve base. Such behavior by Saudi Arabia relative to US shale oil producers has been mentioned as one possible explanation for the extreme drop in global crude oil prices in the second half of 2014.[16]

[16] http://www.economist.com/news/finance-and-economics/21635505-will-falling-oil-prices-curb-americas-shale-boom-bind

Due to the yes/ no nature of shale gas bans, regulatory uncertainty has a large impact on risk perception of market participants. If European governments consider shale gas an appropriate means to increase self-sufficiency and reduce natural gas import dependency from Russia and others, consistent long-term policies and incentives or guarantees may be needed to provide a level playing field to ensure shale gas exploration is a potentially interesting endeavor for the industry. Naturally, environmental, water quality, and other concerns must similarly be addressed by policy to protect health and quality of life of inhabitants of production areas. Also considering earthquake damages due to the Groningen field production in the Netherlands in recent years, it is advisable to allocate a part of the revenues into a support fund managed by an independent authority where citizens can file claims in response to damage likely due to the production activities.

In future work, we want to consider model extensions and case studies involving the following aspects. In natural gas markets, risk can be allocated to different agents in the value chain, rather than completely carried by the suppliers. This mitigates some of the price risk due to market power exertion or other factors. The shale gas uncertainty represented in our cases is very extreme, with possible outcomes being nothing, a very low amount, or a very high amount. More balanced outcomes might reduce the perceived risk and cause less impact on investment for moderate risk aversion levels. A current model feature concerns independence of production capacity and reserves in each country. Production capacity is not depreciated, and when earlier reserves are depleted and new reserves are found, the existing production capacity can still be used for production of these newly found reserves. As long as producing equipment is relocatable, this can be a good representation of the real world; however, relocation cost and depreciation should be accounted for. More parameters can be considered to be uncertain in the model and case studies. Lastly, it would be interesting to consider alternate risk averse objectives, such as a power law utility approach.

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## A Nomenclature

| Name | Symbol | Unit of Measurement or Comment |
|---|---|---|
| **agents, superscripts** | | |
| suppliers | P | |
| transmission system operator | TSO | |
| **sets** | | |
| arcs in the pipeline network | $a \in A$ | |
| suppliers | $p \in P$ | |
| scenario tree node | $n \in N$ | |
| scenario | $\omega \in \Omega$ | |
| **parameters** | | |
| conjectured variation | $cv$ | |
| demand curve intercept | int | bcf/day |
| demand curve slope | slp | bcf/day/bUS$ |
| discount rate | $dr$ | |
| arc expansion cost | $C^{\Delta A}$ | bUS$/bcf/day |
| production capacity expansion cost | $C^{\Delta P}$ | bUS$/bcf/day |
| reserves exploration cost | $C^{\Delta R}$ | bUS$/bcf/day |
| number of days in a stage | $D$ | |
| supplier investment budget | $B^P$ | bUS$/stage |
| TSO investment budget | $B^{TSO}$ | bUS$/stage |
| production costs (linear, quadratic) | $C^L$, $C^Q$ | bUS$/bcf/day |
| risk measure | $\kappa$ | |
| **primal variables** | | |
| production quantity (supplier) | $q^P$ | bcf /day |
| sales quantity (supplier) | $q^S$ | bcf /day |
| investments in production capacity expansion | $e^P$ | bcf/day |
| investments in reserves exploration | $e^R$ | bcf/day |
| investments in arc capacity expansion | $e^A$ | bcf/day |
| supplier's arc flow rate | $f^P$ | bcf/day |
| aggregate arc flow rate | $s$ | bcf/day |
| AV@R-auxiliary variable | $q$ | |
| **dual variables** | | |
| risk measure | $\kappa$ | |
| AV@R-auxiliary variable | $y$ | |
| market price | $\pi$ | bUS$/ bcf |
| various | $\lambda$, $\tau$, $\varphi$, $\mu$, $\rho$,... | cf. Table 1 |

## B Average Value-at-Risk

Each agent maximizes its individual expected discounted profit. In order to account for risk in this setup, we do not consider a simple expected profit maximization of the variable $Y$ in Eq. (1). Instead, we consider the Average Value-at-Risk, $\mathsf{AV@R}_\alpha(Y)$. More generally, we have an acceptability functional

$$\mathcal{R}(Y) := \kappa \cdot \mathbb{E}Y + (1-\kappa)\cdot \mathsf{AV@R}_\alpha(Y), \tag{11}$$

where $\kappa \in [0,1]$, $\mathbb{E}Y$ is the expected profit, and $\mathsf{AV@R}_\alpha(Y)$ is the Average Value-at-Risk at level $0 < \alpha \leq 1$ defined by

$$\begin{aligned}\mathsf{AV@R}_\alpha(Y) &= \frac{1}{\alpha}\int_0^\alpha F_Y^{-1}(u)\mathrm{d}u \\ &= \max_{x\in\mathbb{R}}\, x - \frac{1}{\alpha}\mathbb{E}(x-Y)_+ \qquad (12)\\ &= \min\left\{\mathbb{E}\,YZ:\ 0\leq Z\leq \frac{1}{\alpha},\ \mathbb{E}Z=1\right\}, \qquad (13)\end{aligned}$$

where we have listed several equivalent reformulations (cf. Pflug and Römisch (2007); Pichler and Shapiro (2015)).

Note, that $\mathsf{AV@R}_\alpha(\cdot) \leq \mathsf{AV@R}_1(\cdot) = \mathbb{E}(\cdot)$. In this way, the worst outcomes of the random variable $Y$ matter more for the risk functional $\mathcal{R}(\cdot)$ and the expectation is an upper bound.

For the concrete implementation of the Average Value-at-Risk we apply Eq. (12) to the random variable in Eq. (1) representing the random profit. Note the particular advantage of formula (12): it is a maximum, so that the newly introduced variable $x$ in Eq. (12) can be interchanged with all other variables in profit maximization. In this way, just one variable is added and the problem structure stays the same. Employing the Average Value-at-Risk using this formula allows for risk to be incorporated into the profit maximization objective.

## B.1 The derivative of the acceptability functional $\mathcal{R}$

To compute the equilibrium in a mixed complementarity problem (MCP) formulation it is necessary to have the derivative of the objective accessible. The Average Value-at-Risk and the risk functional $\mathcal{R}$ are convex, and thus, sub-differentiable, although not differentiable in the strict sense. Luna et al. (2016) employ the identity Eq. (12) and replace the kink-function

$$x \mapsto x_+ = \max\{0,\ x\}$$

there with the smoothed approximation

$$x \mapsto \frac{1}{2}\left(\sqrt{x^2+4\tau^2}+x\right) \geq x_+,$$

and they let finally $\tau \to 0$ in a sequence of iterative computations.

Our approach avoids an additional parameter (as $\tau$) and iterative computations by addressing the subdifferential directly. The sub-derivative is given by the random variables $Z$ minimizing the dual representation Eq. (13),

$$Z \in \partial\,\mathsf{AV@R}(Y) = \arg\min\left\{\mathbb{E}\,YZ:\ 0\leq Z\leq\frac{1}{\alpha},\ \mathbb{E}Z=1\right\}.$$

The subgradient of the acceptability functional $\mathcal{R}$ consequently is

$$Z \in (1-\kappa) \cdot \partial \,\mathsf{AV@R}(Y) + \kappa \cdot \mathbb{1},$$

where $\mathbb{1}(\cdot) = 1$ and $Z$ is coupled in an anti-monotone way with $Y$ (cf. Nelsen (1998)).

For a discrete probability measure, the subderivative can be given explicitly. For this, let the outcomes $Y_1, Y_2, \ldots Y_n$ of $Y$ be ordered such that $Y_{(1)} \leq Y_{(2)} \leq \ldots Y_{(n)}$. With $i^*$ satisfying

$$p_{(1)} + p_{(2)} + \cdots + p_{(i^*-1)} < \alpha \leq p_{(1)} + \cdots + p_{(i^*)}$$

one may define

$$Z_{(i)} := \begin{cases} \frac{1}{\alpha} & (i) < (i^*) \\ Z^* & \text{else} \\ 0 & (i) > (i^*), \end{cases}$$

where $Z_{(i^*)} = Z^* \geq 0$ is adjusted such that $\sum_{i=1}^{n} p_{(i)} Z_{(i)} = 1$. Then it holds (cf. Pichler (2014); Pichler and Shapiro (2015)) that

$$(1-\kappa) \cdot Z + \kappa \cdot \mathbb{1} \in \partial \mathcal{R}(Y).$$

# C Input Data

The reference production and consumption data are based on BP (2014) and some additional information from Kalvø and Walle-Hansen (2014) concerning shale gas resources in Poland and Ukraine. Production data for the net exporting countries have been adjusted to reflect export supplies to countries not included in the data set. Data for initial reserves are transformed to reflect the production rate that would deplete the reserves in a single day. For instance, if the Netherlands produce at full capacity 6.6 bcf/ d in the first and second period and do not add any reserves, the maximum production rate in the last period can be 16.44 − 6.60 − 6.60 = 3.24 only. In the *market power* cases, suppliers act as Nash–Cournot agents in all export markets but do not exert market power in their respective domestic markets.

Demand growth for Russia, Poland, and Ukraine relative to 2014 is +5 % in 2019 and +10 % in 2024. Reference prices in 2014 are adjusted to let model outcomes for a deterministic model run reflect production, consumption, and net trade in 2013.

Prices in all countries are inflated relative to 2014 by +5 % in 2019, and +10 % in 2024. Price-demand elasticity is assumed to be 0.5 in all countries. Intercepts and slopes are calculated based on reference prices, demand, and elasticities.

The system wide TSO has a budget of 2 \$bln/ year for network expansions.

Supplier and TSO discount future profits at 5 % per year.

Table 6: Supplier data

| | | Constant per unit production cost ($ /bcf/d) | Linear increase in production cost ($ /bcf/d) | Initial production capacity (bcf/d) | Production capacity expansion cost ($ /bcf/d) | Initial reserves (scaled to bcf/d) | Reserves expansion cost ($ /bcf) | Yearly budget (bln$ ) |
|---|---|---|---|---|---|---|---|---|
| **GER** | **conv** | 3.0 | 1.00 | 0.80 | 2.0 | 0.93 | 4.0 | |
| **NED** | **conv** | 3.0 | 0.61 | 6.60 | 2.0 | 16.44 | 3.0 | $ 10.0 |
| **NOR** | **conv** | 3.0 | 0.57 | 7.00 | 2.0 | 39.45 | 3.0 | $ 20.0 |
| **POL** | **conv** | 3.0 | 1.00 | 0.40 | 2.0 | 1.92 | 3.5 | $ 4.0 |
| | **shale** | 4.0 | 0.10 | | 1.0 | 0.27 | 1.0 | |
| **RUS** | **conv** | 1.5 | 0.04 | 49.73 | 1.0 | 602.74 | 2.0 | $ 20.0 |
| **UKD** | **conv** | 3.0 | 1.00 | 3.50 | 2.0 | 4.71 | 3.0 | $ 8.0 |
| **UKR** | **conv** | 2.0 | 1.00 | 1.90 | 2.0 | 12.33 | 3.0 | $ 4.0 |
| | **shale** | 4.0 | 0.10 | | 1.0 | 0.11 | 1.0 | |

Table 7: Demand data

| | **Price** | | | **Demand** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Reference ($ /mcf)** | | | **Reference (bcf/d)** | | | **elast** | **intercept** | | | **Slope** | | |
| | **2014** | **2019** | **2024** | **2014** | **2019** | **2024** | | **2014** | **2019** | **2024** | **2014** | **2019** | **2024** |
| **GER** | 13.00 | 13.65 | 14.30 | 8.10 | 8.10 | 8.10 | 0.5 | 39.0 | 40.95 | 42.90 | 3.210 | 3.370 | 3.531 |
| **NED** | 10.00 | 10.50 | 11.00 | 3.60 | 3.60 | 3.60 | 0.5 | 30.0 | 31.50 | 33.00 | 5.556 | 5.833 | 6.111 |
| **NOR** | 9.00 | 9.45 | 9.90 | 0.40 | 0.40 | 0.40 | 0.5 | 27.0 | 28.35 | 29.70 | 45.000 | 47.250 | 49.500 |
| **POL** | 11.00 | 11.55 | 12.10 | 1.60 | 1.68 | 1.76 | 0.5 | 33.0 | 34.65 | 36.30 | 13.750 | 13.750 | 13.750 |
| **RUS** | 5.50 | 5.78 | 6.05 | 40.10 | 42.11 | 44.11 | 0.5 | 16.5 | 17.33 | 18.15 | 0.274 | 0.274 | 0.274 |
| **UKD** | 13.00 | 13.65 | 14.30 | 7.10 | 7.10 | 7.10 | 0.5 | 39.0 | 40.95 | 42.90 | 3.662 | 3.845 | 4.028 |
| **UKR** | 9.00 | 9.45 | 9.90 | 4.40 | 4.62 | 4.84 | 0.5 | 27.0 | 28.35 | 29.70 | 4.091 | 4.091 | 4.091 |

Table 8: Transmission System Operator data

| Pipeline | Flow cost ($ /bcf/d) | Initial cap (bcf/d) | Expansion cost ($ /bcf/d) |
|---|---|---|---|
| **GER_POL** | 0.5 | 0.2 | 1.0 |
| **NED_GER** | 0.5 | 8.7 | 1.0 |
| **NED_UKD** | 1.0 | 1.6 | 1.2 |
| **NOR_GER** | 1.0 | 4.2 | 2.0 |
| **NOR_UKD** | 1.0 | 4.6 | 1.5 |
| **POL_GER** | 0.5 | 2.9 | 1.0 |
| **RUS_GER** | 2.0 | 5.3 | 2.0 |
| **RUS_POL** | 1.5 | 1.0 | 1.5 |
| **RUS_UKR** | 0.5 | 10.8 | 1.0 |
| **UKR_POL** | 0.5 | 0.6 | 1.0 |

# D Calculation times for cold and hot starts

Here we provide selected calculation times and some details for the computational approach to solve data instances that did not solve using default PATH solver settings. Computer specifications: Intel i7-3770 CPU 3.4 GHz. Four cores with 2GB each; 64-bit operating system. PATH version 4.7.02.

We use the GAMS data exchange ('gdx') utility to store solution values in files that can be loaded to initialize variables before a solve attempt at a later moment. Using the solution values of a previous solve to initialize variables is called hot start. Solving the model starting with all variables with value zero is called a cold start. Here we consider hot starts using two types of previously obtained solutions for the same data instance with either 1. slightly different values for risk aversion or 2. assuming perfectly competitive behavior. The original model size is: 9527 variables/ equations (it is a square system). The preprocessed size is 6099. Times reported here are in seconds reported by GAMS in the listing file (resource usage). Table 9 and Figure 7 present calculation times. Of the instances presented, the instance with $\alpha_{UKR}$ = 100% and $\alpha_{POL}$ = 30% did not solve with default solver settings (marked **). It also did not solve when increasing various iteration limits in the solver settings (crash iteration limit, gradient step limit, major iteration limit, minor iteration limit, and cumulative iteration limit). However, as illustrated in the table, hot starts allowed the model to be solved. When using the solution for $\alpha_{UKR}$ = 100% and $\alpha_{POL}$ = 40% and stepwise decreasing $\alpha_{POL}$ values, we obtained a solution for the initially unsolvable data instance and obtained solutions much quicker for the other data instances (see the left column with headings 'cold' and 'hot'). The last column shows solution times for a hot start when initializing the variables using perfectly competitive data instances with the same risk aversion assumptions. Considering the previously unsolvable data instance (**), this proved to be a very efficient way to find a solution, with low solution times for both the perfectly competitive data instance and the hot-started market power case. Although for the other three hot starts solving the perfectly competitive data instances took longer than the market power variants, the created starting points did allow for solving the market power cases in just a few seconds.

We observed a general tendency for solution times to be larger for increasing risk aversion levels and when both Poland and Ukraine are risk averse, but this was not always the case. By using increasingly smaller steps for $\alpha$-values and automatically resolving data instances with other PATH solver settings, eventually all data instances attempted were solved. Solver settings that were modified (other than the iteration limits indicated above) include: crash method, crash perturb, lemke start, nms, nms initial reference factor, nms memory size; nms mstep_frequency, nms searchtype and merit function (http://pages.cs.wisc.edu/~ferris/path.html "Some Options for Path"). In some cases, using adjusted settings would not allow for a data instance to be solved, but these settings would provide an alternate starting point for a new try with the default solver settings (but with higher iteration limits); this would often allow the problem to be solved. For some difficult instances, what combination of solver settings allowed for solving the problem was very dependent on the problem parameterization, and we did not attempt to analyze this further.

Table 9: Calculation times for selected data instances with market power exertion, $\alpha_{UKR}$ = 100% and varying values for $\alpha_{POL}$

| $\alpha_{POL}$ | cold | hot | cold - perfectly competitive variant | hot |
|---|---|---|---|---|
| 100 | 1.2 | | 1.8 | 0.8 |
| 70 | 17 | | 121 | 4.8 |
| 50 | 17 | | 40 | 2.1 |
| 40 | 1.6 | start | | |
| 37.5 | 3.9 | 0.3 | | |
| 35 | 1.6 | 0.4 | | |
| 32.5 | 10 | 0.3 | | |
| 30 | ** | 0.4 | 2.5 | 4.5 |

** terminates after almost three minutes with ‘Other error’-‘Locally Infeasible’ and three (solver-determined) restarts.

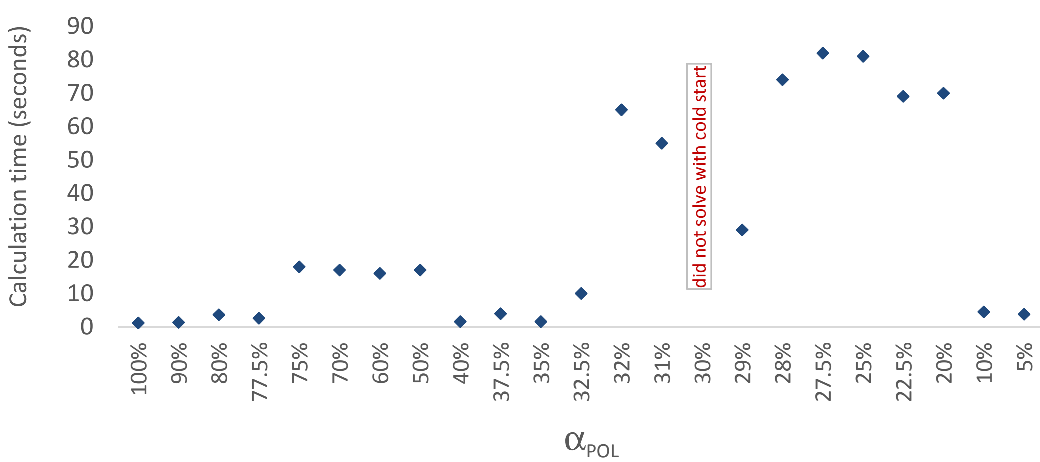


Figure 7: Cold-start solution times for Cournot cases. $\alpha_{UKR}$ = 100% and varying values for $\alpha_{POL}$

# E Results

## E.1 Selected deterministic results

Here we present and discuss the deterministic results as a backdrop for the discussion of the stochastic results.

### E.1.1 Exploration behavior

The deterministic results show that regardless of the competitive environment, Norway, the Netherlands, and Germany never invest in reserve exploration or production capacity additions. This reflects both the existing reserve situation (Norway has abundant reserves) and the high cost of reserve exploration (in the Netherlands and Germany). In contrast, Poland and the Ukraine use their respective budgets entirely in either market structure. Investments by Poland do not depend on the market structure, however, in the perfectly competitive market, Ukraine directs a larger share towards exploration compared to Cournot market situations. In a Cournot market, facing high import prices, the United Kingdom uses its full expansion budget toward reserve exploration in both periods. In a perfectly competitive market with lower import prices, the United Kingdom uses only a minor part of its expansion budget in the first period, but it uses all of its budget in the second period. As a perfectly competitive *at-marginal-cost* supplier, Russia would expand its production capacity in the first stage. In contrast, as a large market power-exerting supplier, it does not expand production capacity, thereby withholding supplies to the market in later periods compared to the competitive situation.

In sum, in a competitive market, there is plenty of import supply in later stages, reducing the need for import-dependent countries to invest in future domestic supply. However, when anticipating higher prices and lower supply by market power exerting agents, import dependent countries invest much more and earlier on in reserve exploration.

### E.1.2 Lower shale gas finds

Risk aversion cannot be modeled in a deterministic setting. However, some insight can be gained by reducing the expected finds from shale gas exploration. Lower finds can alternatively be viewed as higher per unit investment cost: finding an $x\%$ decreased amount is equivalent to a $\frac{1}{1-x\%}$ per unit investment cost increase.

Regardless of the market power assumptions, Germany produces at maximum capacity in the first stage and depletes reserves in the second stage. This is optimal behavior when price increases are lower than the discount rate (cf. Hotelling (1931)). In either market power case, Poland produces at full capacity in all stages. The Polish production levels are entirely determined by the limited supplier expansion investment budget (as mentioned, it uses its full expansion budget regardless of market power). Lower finds imply lower production.

Ukraine shifts some investment to conventional when shale gas returns are less promising. This effectively results in a more resilient domestic supply situation compared to Poland. However, in the perfectly competitive case, the decline in Ukrainian domestic production is much larger compared to the market power case, as import prices are much more competitive and drive out domestic supply.

Other countries have no shale gas, but are affected indirectly because total available gas and market prices depend on shale production.

In the competitive case, domestically produced volumes by the United Kingdom are significantly lower than in the high market power case, as cheap foreign supplies drive down market prices. In the market power case, the United Kingdom always entirely depletes its available and added conventional reserves. Lower shale gas availability in Ukraine and Poland results in modest shifts in UK production to later stages.

In the market power case, lower shale gas production causes other (exporting) countries to shift supplies to later stages. This drives up market prices in the second stage, causing Russian supplies to increase somewhat. Market prices in the third stage will have risen so much that Russia supplies at full capacity. In contrast, in the perfectly competitive case, Russia expands its production capacity in the first stage (as discussed above) and produces at full capacity in all stages.

The Netherlands deplete their reserves completely in all cases, with shifts in output to later periods for lower shale gas availability. Norway does not deplete reserves over the time horizon. In both cases, lower shale gas availability in continental Europe triggers higher Norwegian output.

Across the board, we see that although potential shale gas supply is modest as a share in total market supply and only present in the two countries Poland and Ukraine, there is a clear impact of its availability on supply, consumption, and trade throughout the entire market in the various stages.

#### E.1.3 Pipeline Network Investment

In all cases, the TSO budget of $ 2 bln is binding in the first stage, however, not in the second period. Supposedly, in the second period, the remaining time horizon (the third stage only) does not allow for a long enough period to make a return on investment.

In the competitive cases, the complete budget is used to expand the connection Ukraine-Poland to provide more Russian supplies to Poland via Ukraine in later stages. In the second stage, anticipating the depletion of German reserves and when shale gas additions to the reserve base are significant, the connection from Poland to Germany is expanded (albeit modestly). However, if shale gas finds are expensive and low, the Ukraine to Poland pipeline is expanded with no expansion in the Poland to Germany pipeline.

This illustrates how in a competitive market the optimal network investment changes in response to modest changes in available shale gas amounts, and that the TSO in the model behaves as one would expect.

To explain the network expansions in the Cournot case, we remind the reader of a result in Egging and Gabriel (2006). The authors discuss that market power exerting suppliers have an incentive to supply markets *further away*. Compared to perfectly-competitive suppliers who minimize cost and help to achieve the maximum social welfare, market-power exerting suppliers will reduce domestic and near-by supplies to drive up market prices and supply more to other, *further away* markets instead. This can lead to higher per unit supply cost for lower total supplied quantities. Since the

TSO is a price-taker in all cases, its investment addresses the future transportation needs of the suppliers. As discussed in the previous section, in the Cournot case, Russia does not expand its production capacity and exports significantly lower amounts compared to when it is a price taker. When exerting market power, Norway exports smaller amounts to the United Kingdom, whereas Russia and possibly the Netherlands supply more. Also, when Russia supplies less to Poland, other countries will supply larger amounts to Poland. The network expansions to accommodate these trade patterns are, in the first stage, mostly dedicated to that which go from the Netherlands to the United Kingdom, with minor expansions in the capacities from Ukraine to Poland and Germany to Poland. The second stage network expansions are small.

### E.2 Value of Stochastic Solution for Expected Value Problems

The Value of Stochastic Solution (*VSS*) is a commonly used indicator for the added value of explicitly considering uncertainty in stochastic optimization problems, rather than using expected values of uncertain parameters in a deterministic optimization problem. The *VSS* for a single profit maximizing agent is

$$VSS \quad := \quad (\mathbb{E}_\omega \max_x Y(x,\omega)) - \max_x \mathbb{E}_\omega Y(x,\omega), \tag{14}$$

where the minuend in Eq. (14) is often referred to as the wait-and-see problem. Birge and Louveaux (1997) prove that the *VSS* is nonnegative. However, in contrast to the single agent perspective in Eq. (14), the market problems that we analyze in this paper contain multiple agents. From the microeconomic literature, we know that equilibria in perfectly competitive markets can be found by optimizing a single expected social welfare objective representing all agents in the markets. Additionally, equilibria in a monopolistic market can be found by maximizing the profit function of the monopoly agent. As such, for both of these market types, the standard *VSS* concept can be applied to the aggregate of all market agents. For perfectly competitive markets, the aggregate *VSS* will be (must be) nonnegative. However, we have an imperfect market with multiple agents that cannot be solved using a single-objective optimization problem.

The fourth row in column ESS Table 10 shows that the expected social welfare (in the stochastic case with a 10 % ban probability) is 3.2 % higher in a perfectly competitive market than in the market power case. Expected aggregate consumer surplus is almost 18 % higher in a perfectly competitive market than in the market power case. Almost 3/4 (73.5 %) of aggregate expected social welfare is consumer surplus in a competitive market (1408.8/ 1917.0), this drops to below 2/ 3 (64.4%) in the Cournot market (1195.8/ 1857.8). Expected aggregate supplier surplus is 43 % higher in the market power case than in the perfect competitive case. Supplier share is almost 1/4 (23.8%) in the competitive market and over 1/3 (35.2%) when they exert market power. TSO profit is much lower in the market power market situation. Its expected profits are almost 500 % higher in the competitive case versus the market power case. However, by itself the TSO profit is a modest share (0.5 % in Cournot vs. 2.7 % in competitive).

We calculate the *VSS* of the aggregate social welfare values by subtracting the values in column EVS from the value in column ESS. In both cases, the aggregate *VSS* is less than 0.1 %. (+0.09 % in perfect competition, +0.07 % with market power). These are small but positive values. When we consider the *VSS* by separate agent, we see variations. The *VSS* for the aggregate consumer surplus (over all countries) is +0.9 % in the market power case and +0.6 % in the competitive case. The *VSS* for aggregate supplier profit is -1.5 % (negative) in the competitive case and $-1.7\%$ (negative) under market power. The *VSS* for the TSO is quite large in the Cournot case (+9.1 %) and slightly negative in the competitive case ($-0.2\%$). Hence, on aggregate agent groups, the impact of solving the stochastic model is mixed. This mixed picture becomes more explicit when considering individual agents.

Table 10: VSS by agent group - 10% ban probability

| | | Deterministic | ESS | EVS | VSS | VSS-% |
|---|---|---|---|---|---|---|
| Aggregrate | | Deterministic | ESS | EVS | VSS | VSS-% |
| Social | Cournot | 1863.0 | 1857.8 | 1856.5 | 1.3 | 0.07% |
| Welfare | competitive | 1920.2 | 1917.0 | 1915.3 | 1.7 | 0.09% |
| Difference competitive vs. Cournot | | 3.1% | 3.2% | 3.2% | | |
| Consumers | | Deterministic | ESS | EVS | VSS | VSS-% |
| | Cournot | 1199.1 | 1195.8 | 1184.5 | 11.3 | 0.9% |
| | competitive | 1415.4 | 1408.8 | 1400.5 | 8.3 | 0.6% |
| Difference competitive vs. Cournot | | 17.8% | | | | |
| Suppliers | | Deterministic | ESS | EVS | VSS | VSS-% |
| | Cournot | 655.6 | 653.2 | 664 | -10.8 | -1.7% |
| | competitive | 453.9 | 456.5 | 463.2 | -6.7 | -1.5% |
| Difference Cournot vs. competitive | | 44% | 43.1% | | | |
| Transmission | | Deterministic | ESS | EVS | VSS | VSS-% |
| System | Cournot | 8.3 | 8.8 | 8.0 | 0.8 | 9.1% |
| Operator | competitive | 50.9 | 51.6 | 51.7 | -0.1 | -0.2% |
| Difference competitive vs. Cournot | | | 486% | | | |

The expected discounted profits and surpluses over the time horizon (in $-mln, scaled down by the number of days in each stage (=365*5=1625)

We see variations when considering the *VSS* by separate agent group. Generally, when solving the stochastic model, considering agent groups: the consumers benefit, suppliers loose (facing a *negative* (!) *VSS*) and the picture for the TSO is mixed. The *VSS* for the aggregate consumer surplus (over all countries) is +0.9 % in the market power case and +0.6 % in the competitive case. The *VSS* for aggregate supplier profit is $-1.5\%$ (negative) in the competitive case and $-1.7\%$ under market power. The *VSS* for the TSO is quite large in the Cournot case (+9.1 %) and slightly negative in the competitive case ($-0.2\%$). Hence, on agent groups, the impact of solving the stochastic model is mixed. This mixed picture becomes more explicit when considering individual agents. Figure 8 in the appendix presents the *VSS* of individual suppliers. In fact, Poland and Ukraine, the countries where a ban may be imposed, have a positive *VSS* in both market structures, whereas the supplier in the United Kingdom has a very modest positive *VSS* in the Cournot market case, but a negative *VSS* in the competitive case. All other suppliers face negative *VSS* values in both market structures.

Figure 8 shows the *VSS* for individual suppliers as a relative value ($\frac{ESS}{EVS} - 1$) for both market

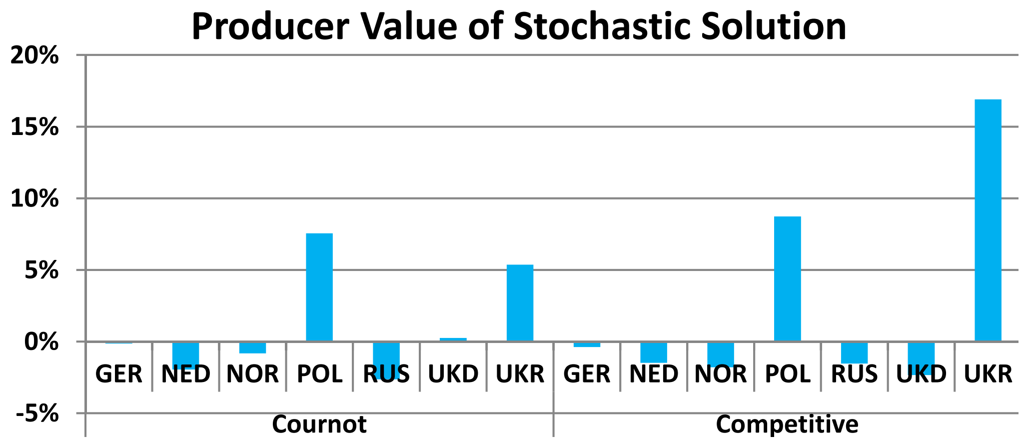


Figure 8: Value of stochastic solution by supplier

structures (still with a ban probability of 10 % in Poland and Ukraine). In both market structures for most suppliers, the *VSS* is negative (as was the aggregate). Only Poland and Ukraine, the countries where a ban may be imposed, have a positive *VSS* in both market structures, whereas the supplier in the United Kingdom has a very modest positive *VSS* in the Cournot market case, but a negative $VSS$ in the competitive case. Polish *VSS* is about 8 % in either market structure. The Ukraine *VSS* is more than 5 % in the Cournot market case and almost 17 % in a perfectly competitive market.

As noted, the *VSS* for the aggregate social welfare is positive. As discussed above, our model cannot be solved as an optimization problem, and the non-negativity result for *VSS* does not apply for each individual agent in this setting. Negative *VSS* values for individual agents in market models with multiple agents have been found previously, cf. Zhuang (2005); Genc et al. (2007); Zhuang and Gabriel (2008); Egging (2010, 2013). The game settings have led some researchers to speculate that some kind of prisoner dilemma may cause the result obtained by Genc et al. (2007). It may also be a distributional effect between the agents due to problem characteristics, as illustrated above.

### E.3 Polish Investment for varying AV@R

Cf. Table 11.

### E.4 Ukrainian Investment for varying AV@R

Cf. Table 12.

Table 11: Polish Investments for varying AV@R

| Case | 100% Reserves | | Prod cap | | 80% Reserves | | Prod cap | | 60% Reserves | | Prod cap | | 40% Reserves | | Prod cap | | 20% Reserves | | Prod cap | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Scen | conv | shal | conv | shal | conv | shal | conv | shal | conv | shal | conv | shal | conv | shal | conv | shal | conv | shal | conv | shal |
| Cour stage 1 | | 0.84 | 0.36 | 0.63 | | 1.00 | 0.36 | 0.47 | | 1.00 | 0.36 | 0.47 | 0.09 | 0.19 | 0.69 | 0.31 | 0.15 | 0.02 | 0.75 | 0.15 |
| BB | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.61 | | 0.03 | | 0.61 | | | |
| BL | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.61 | | 0.03 | | 0.63 | | | |
| BH | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.40 | | 0.61 | | 0.03 | | 0.63 | | | |
| LB | | 1.01 | | 1.18 | | 0.95 | | 1.24 | | 0.95 | | 1.24 | 0.57 | 0.18 | | 0.01 | 0.50 | 0.17 | | 0.15 |
| LL | | 1.01 | | 1.18 | | 0.95 | | 1.24 | | 0.95 | | 1.24 | 0.58 | 0.17 | | | 0.62 | 0.01 | | |
| LH | | 1.01 | | 1.18 | | 0.95 | | 1.24 | | 0.95 | | 1.24 | 0.58 | 0.17 | | | 0.63 | | | |
| HB | | 0.60 | | 1.60 | 0.19 | | 0.19 | 1.13 | 0.19 | 0.00 | 0.19 | 1.13 | 0.57 | 0.10 | | 0.06 | 0.61 | 0.02 | | 0.03 |
| HL | | 0.60 | | 1.60 | 0.19 | | 0.19 | 1.13 | 0.18 | 0.03 | 0.18 | 1.18 | 0.57 | 0.10 | | 0.02 | 0.62 | | | 0.00 |
| HH | | 0.60 | | 1.60 | 0.16 | 0.07 | 0.16 | 1.26 | 0.17 | 0.04 | 0.17 | 1.20 | 0.58 | 0.15 | | 0.03 | 0.63 | | | |
| st. 2 avg | 0.04 | 0.72 | 0.04 | 1.25 | 0.12 | 0.44 | 0.12 | 1.09 | 0.12 | 0.44 | 0.12 | 1.09 | 0.58 | 0.13 | 0.00 | 0.01 | 0.62 | 0.01 | | 0.01 |
| AV@R | 23.511 | | | | 22.462 | | | | 21.401 | | | | 19.876 | | | | 19.688 | | | |
| Comp stage 1 | | 0.89 | 0.36 | 0.59 | | 0.84 | 0.36 | 0.63 | | 0.55 | 0.36 | 0.38 | | 0.03 | 0.37 | 0.28 | | 0.03 | 0.37 | 0.28 |
| BB | | | | | | | | | | | | | 0.02 | | | | 0.02 | | | |
| BL | | | | | | | | | | | | | 0.02 | | | | 0.02 | | | |
| BH | | | | | | | | | | | | | 0.02 | | | | 0.02 | | | |
| LB | | 0.99 | | 1.20 | | 1.01 | | 1.18 | | 0.92 | | 1.27 | 0.02 | 0.22 | | 0.11 | 0.03 | 0.02 | 0.01 | |
| LL | | 0.99 | | 1.20 | | 1.01 | | 1.18 | | 0.92 | | 1.27 | 0.02 | 0.20 | | 0.08 | 0.02 | 0.16 | 0.01 | |
| LH | | 0.99 | | 1.20 | | 1.01 | | 1.18 | | 0.92 | | 1.27 | 0.02 | 0.28 | | 0.23 | 0.02 | 0.28 | | 0.23 |
| HB | | 0.54 | | 1.66 | 0.00 | 0.41 | 0.00 | 1.28 | 0.00 | 0.41 | 0.00 | 1.25 | 0.02 | 0.40 | | 0.19 | | 0.15 | | 0.08 |
| HL | | 0.54 | | 1.66 | | 0.59 | | 1.60 | 0.00 | 0.45 | 0.00 | 1.32 | 0.04 | 0.23 | 0.02 | 0.19 | | 0.23 | | 0.18 |
| HH | | 0.54 | | 1.66 | | 0.59 | | 1.60 | | 0.50 | | 1.40 | 0.02 | 0.25 | | 0.22 | 0.02 | 0.25 | | 0.21 |
| st. 2 avg | | 0.69 | | 1.28 | 0.00 | 0.71 | 0.00 | 1.24 | 0.00 | 0.62 | 0.00 | 1.18 | 0.02 | 0.22 | 0.00 | 0.16 | 0.01 | 0.19 | 0.00 | 0.13 |
| AV@R | 10.747 | | | | 10.317 | | | | 9.664 | | | | 9.646 | | | | 9.631 | | | |

Table 12: Ukrainian investment varying AV@R

| | 100% | | | 80% | | | | 60% | | | | 40% | | | 20% | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Reserves | Prod cap | | Reserves | | Prod cap | | Reserves | | Prod cap | | Reserves | Prod cap | | Reserves | | Prod cap | |
| Cour | | | | | | | | | | | | | | | | | | |
| | shal | conv | shal | conv | shal | conv | shal | conv | shal | conv | shal | shal | conv | shal | conv | shal | conv | shal |
| stage 1 | 0.20 | 0.92 | 0.15 | | 0.18 | 0.93 | 0.15 | | | 1.04 | 0.11 | 0.11 | 0.98 | 0.13 | | 0.20 | 0.92 | 0.15 |
| BB | | 0.94 | | | | 0.93 | | | | 1.09 | | | 0.56 | | 0.00 | | 0.10 | |
| LB | | 0.90 | | | | 0.89 | | | | 0.78 | | | 0.85 | | | | 0.04 | |
| HB | | 0.84 | | | | 0.84 | | | | 0.73 | | | 0.79 | | | | 0.11 | |
| BL | 0.84 | | 1.36 | | 0.56 | 0.39 | 0.86 | 0.01 | 0.68 | 0.17 | 1.14 | 0.24 | 0.89 | 0.05 | | 0.27 | 0.09 | 0.19 |
| LL | 0.84 | | 1.36 | | 0.65 | 0.25 | 1.03 | | 0.63 | 0.27 | 1.02 | 0.66 | 0.77 | | | 0.40 | 0.84 | 0.12 |
| HL | 0.84 | | 1.36 | | 0.82 | 0.03 | 1.32 | | 0.66 | 0.23 | 1.07 | 0.78 | 0.06 | 1.28 | | 0.65 | 0.27 | 1.01 |
| BH | 0.72 | | 1.47 | 0.00 | 0.50 | 0.26 | 1.04 | | 0.69 | 0.22 | 1.06 | 0.35 | 0.84 | 0.05 | | 0.11 | | 0.36 |
| LH | 0.72 | | 1.47 | | 0.52 | 0.30 | 1.08 | | 0.82 | | 1.37 | 0.33 | 0.73 | 0.11 | | | 0.71 | 0.16 |
| HH | 0.72 | | 1.47 | | 0.68 | 0.07 | 1.37 | | 0.66 | 0.22 | 1.08 | 0.77 | | 1.42 | | | 0.68 | 0.17 |
| st. 2 avg | 0.70 | 0.09 | 1.27 | 0.00 | 0.59 | 0.25 | 1.06 | 0.00 | 0.62 | 0.24 | 1.02 | 0.54 | 0.47 | 0.57 | 0.00 | 0.23 | 0.52 | 0.32 |
| AV@R | 49.236 | | | 48.804 | | | | 48.156 | | | | 48.048 | | | 47.936 | | | |
| Comp | | | | | | | | | | | | | | | | | | |
| stage 1 | 1.06 | | 0.75 | | 0.08 | | 0.13 | | 0.06 | | 0.12 | 0.03 | | 0.08 | | 0.04 | | 0.09 |
| BB | | | | | | | | | | | | | 0.19 | | | | | |
| BL | 0.42 | | | | 0.07 | | | | | | | 0.09 | 0.06 | 0.12 | | 0.08 | | 0.08 |
| LL | 0.42 | | | | 0.07 | | | | 0.07 | | | 0.03 | | | 0.00 | 0.00 | 0.00 | |
| HL | 0.42 | | | | 0.07 | | | | 0.07 | | | 0.03 | | | | 0.04 | | |
| BH | | | 0.51 | 0.01 | 0.03 | 0.01 | 0.01 | 0.00 | 0.04 | 0.12 | 0.13 | 0.14 | 0.02 | 0.26 | 0.00 | 0.01 | 0.00 | 0.04 |
| LH | | | 0.51 | | | | | | 0.01 | | | | | | 0.00 | | | |
| HH | | | 0.51 | | | | | | 0.01 | | | | | | | | | |
| st. 2 avg | 0.19 | | 0.23 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 | 0.03 | 0.01 | 0.01 | 0.02 | 0.01 | 0.02 | 0.00 | 0.01 | 0.00 | 0.01 |
| AV@R | 12.334 | | | 12.293 | | | | 12.237 | | | | 12.188 | | | 12.124 | | | |

# F KKT–Conditions

Table 13: Additional symbols

| Symbol | description |
|---|---|
| $\alpha_p$ | risk level of the AV@R |
| $k_p$ | dual variable for $(Y - x)_+$, cf. (13) |
| $\sigma_{p\omega}$ | $(Y - x)_+$, cf. (13) |
| $\text{term}(\omega)$ | terminal nodes: end nodes succeeding from $\omega$. In the last stage $\text{term}(\omega) = \omega$ |

## F.1 KKT conditions for the producers

Refer to Section 3.6.1.2 of Kalvø and Walle-Hansen (2014) for an extensive discussion on the derivation of the KKT conditions of the AV@R objective.

$$0 \leq -1 + \sum_{\omega \in \Omega} \sigma_{p\omega} \perp k_p \geq 0 \quad \forall \quad p \tag{15}$$

$$0 \leq \frac{1}{\alpha_p} p_\omega - \sigma_{p\omega} \perp y_{p\omega} \geq 0 \quad \forall \quad p, \omega \tag{16}$$

$$\begin{aligned} 0 \leq dr_\omega D \left( C^{PL}_{prn} + 2C^{PQ}_{prn} q_{prn\omega} \right) & \sum_{\omega' \in \mathrm{term}(\omega)} \sigma_{p\omega'} + \lambda^P_{prn\omega} - \phi^P_{n\omega} \\ + \rho^P_{prn\omega} D + & \sum_{\omega' \in succ(\omega)} D \rho^P_{prn\omega'} \perp q_{prn\omega} \geq 0 \quad \forall \quad p, r, n, \omega \end{aligned} \tag{17}$$

$$\begin{aligned} 0 \leq -dr_\omega D_\omega \Bigg[ \delta_{pn} \Bigg( \mathrm{int}_{n\omega} - \mathrm{slp}_{n\omega} ( \sum_{p' \in P} q^S_{p'n\omega} + q^S_{pn\omega} ) \Bigg) \\ + (1 - \delta_{pn}) \pi^M_{pn\omega} \Bigg] \sum_{\omega' \in \mathrm{term}(\omega)} \sigma_{p\omega'} + \phi^P_{pn\omega} \perp q^S_{pn\omega} \geq 0 \forall \quad p, n, \omega \end{aligned} \tag{18}$$

$$\begin{aligned} 0 \leq dr_\omega C^{\Delta P}_{prn} \sum_{\omega' \in \mathrm{term}(\omega)} \sigma_{p\omega'} - \sum_{\omega' \in succ(\omega)} \lambda^P_{prn\omega'} \\ + \mu^P_{p\omega} C^{\Delta P}_{prn} \perp e^P_{prn\omega} \geq 0 \quad \forall \quad p, r, n, \omega \end{aligned} \tag{19}$$

$$\begin{aligned} 0 \leq dr_\omega C^{\Delta R}_{prn} \sum_{\omega' \in \mathrm{term}(\omega)} \sigma_{p\omega'} - \sum_{\omega' \in succ(\omega)} \Delta R_{prn, b(\omega, \omega')} \rho^P_{prn\omega'} \\ + \mu^P_{p\omega} C^{\Delta R}_{prn} \perp e^R_{prn\omega} \geq 0 \quad \forall \quad p, r, n, \omega \end{aligned} \tag{20}$$

$$0 \leq dr_\omega D_\omega \tau_{a\omega} \sum_{\omega' \in \mathrm{term}(\omega)} \sigma_{p\omega'} - \phi^P_{pn^+(a)\omega} + \phi^P_{pn^-(a)\omega} \perp f_{pa\omega} \geq 0 \quad \forall \quad p, a, \omega \tag{21}$$

$$0 \leq y_{p\omega} - k_p + X_{p\omega} \perp \sigma_{p\omega} \geq 0 \quad \forall \quad p, \omega \in E \tag{22}$$

$$0 \leq CAP^{P0}_{prn} + \sum_{\omega' \in pred(\omega)} e^P_{prn\omega'} - q_{prn\omega} \perp \lambda^P_{prn\omega} \geq 0 \quad \forall \quad p, r, n, \omega \tag{23}$$

$$\sum_{r \in R} q_{prn\omega} + \sum_{a \in a^+(n)} f_{pa\omega} - q^S_{pn\omega} - \sum_{a \in a^-(n)} f_{pa\omega} = 0 \quad \phi^P_{pn\omega} \quad free \quad \forall \quad p, n, \omega \tag{24}$$

$$0 \leq B^P_{p\omega} - \sum_{r \in R, n(p) \in N} \left( C^{\Delta P}_{prn} e^P_{prn\omega} + C^{\Delta R}_{prn} e^R_{prn\omega} \right) \perp \mu^P_{p\omega} \geq 0 \quad \forall \quad p, \omega \tag{25}$$

$$\begin{aligned} 0 \leq R^{P0}_{prn} - \sum_{\omega' \in pred(\omega)} D_{\omega'} q_{prn\omega'} - D_\omega q_{prn\omega} \\ + \sum_{\omega' \in pred(\omega)} \Delta R_{prn\omega'}(\xi) e^R_{prn\omega'^-} + \Delta R_{prn\omega}(\xi) e^R_{prn\omega^-} \perp \rho^P_{prn\omega} \geq 0 \quad \forall \quad p, r, n, \omega \end{aligned} \tag{26}$$

## F.2 KKT conditions for the TSO

$$0 \leq p_\omega dr_\omega C_a^{\Delta A} - \sum_{\omega' \in succ(\omega)} \lambda_{a\omega'}^T + C_a^{\Delta A} \mu_\omega^T \perp e_{a\omega}^A \geq 0 \quad \forall \quad a, \omega \tag{27}$$

$$0 \leq -p_\omega dr_\omega D_\omega (\tau_{a\omega} - C_a^F) + \lambda_{a\omega}^T \perp s_{a\omega} \geq 0 \quad \forall \quad a, \omega \tag{28}$$

$$0 \leq CAP_a^{A0} - s_{a\omega} + \sum_{\omega' \in pred(\omega)} e_{a\omega'}^A \perp \lambda_{a\omega}^T \geq 0 \quad \forall \quad a, \omega \tag{29}$$

$$0 \leq B_\omega^{TSO} - \sum_{a \in A} C_a^{\Delta A} e_{a\omega}^A \perp \mu_\omega^T \geq 0 \quad \forall \quad \omega \tag{30}$$

## F.3 KKT conditions for market clearing

$$\text{int}_{n\omega} - \text{slp}_{n\omega} \sum_{p \in P} q_{pn\omega}^S - \pi_{n\omega} = 0 \quad \pi_{n\omega}^M \text{ free} \quad \forall \quad n, \omega \tag{31}$$

$$s_{a\omega} - \sum_{p \in P} f_{pa\omega} = 0 \quad \tau_{a\omega} \text{ free} \quad \forall \quad a, \omega \tag{32}$$