

# Spin pumping, dissipation, and direct and alternating inverse spin Hall effects in magnetic-insulator/normal-metal bilayers

André Kapelrud and Arne Brataas

*Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway*

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We theoretically consider the spin-wave mode- and wavelength-dependent enhancement of the Gilbert damping in magnetic insulator–normal metal bilayers due to spin pumping as well as the enhancement’s relation to direct and alternating inverse spin Hall voltages in the normal metal. In the long-wavelength limit, including long-range dipole interactions, the ratio of the enhancement for transverse volume modes to that of the macrospin mode is equal to two. With an out-of-plane magnetization, this ratio decreases with both an increasing surface anisotropic energy and mode number. If the surface anisotropy induces a surface state, the enhancement can be an order of magnitude larger than for the macrospin. With an in-plane magnetization, the induced dissipation enhancement can be understood by mapping the anisotropy parameter to the out-of-plane case with anisotropy. For shorter wavelengths, we compute the enhancement numerically and find good agreement with the analytical results in the applicable limits. We also compute the induced direct- and alternating-current inverse spin Hall voltages and relate these to the magnetic energy stored in the ferromagnet. Because the magnitude of the direct spin Hall voltage is a measure of spin dissipation, it is directly proportional to the enhancement of Gilbert damping. The alternating spin Hall voltage exhibits a similar in-plane wave-number dependence, and we demonstrate that it is greatest for surface-localized modes.

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## I. INTRODUCTION

In magnonics, one goal is to utilize spin-based systems for interconnects and logic circuits [1]. In previous decades, the focus was to gain control over these systems by exploiting long-range dipole interactions in combination with geometrical shaping. However, the complex nature of the nonlinear magnetization dynamics persistently represents a challenge in using geometrical shaping alone to realize a variety of desired properties [1].

In magnonic systems, a unique class of materials consists of magnetic insulators. Magnetic insulators are electrically insulating, but localized magnetic moments couple to form a long-range order. The prime example is *yttrium iron garnet* (YIG). YIG is a complex crystal [2] in the Garnet family, where  $\text{Fe}^{3+}$  ions at different sites in the unit cell contribute to an overall ferrimagnetic ordering. What differentiates YIG from other ferromagnetic (ferrimagnetic) systems is its extremely low intrinsic damping. The Gilbert damping parameter measured in YIG crystals is typically two orders of magnitude smaller than that measured in conventional metallic ferromagnets (Fe, Co, Ni, and alloys thereof).

The recent discovery that the spin waves in magnetic insulators strongly couple to spin currents in adjacent normal metals has reinvigorated the field of magnonics [3–12]. Although there are no mobile charge carriers in magnetic insulators, spin currents flow via spin waves and can be transferred to itinerant spin currents in normal metals via spin transfer and spin pumping [13,14]. These interfacial effects open new doors with respect to local excitation and detection of spin waves in magnonic structures. Another key element is that we can transfer knowledge from conventional spintronics to magnonics, opening possibilities for novel physics and technologies. Traditionally, spin-wave excitation schemes have focused on the phenomenon of resonance or the use of Ørsted fields from microstrip antennas.

A cornerstone for utilizing these systems is to establish a good understanding of how the itinerant electrons in normal metals couple across interfaces with spin-wave dynamics in magnetic insulators. Good models for addressing uniform (macrospin) magnetization that agrees well with experiments have been previously developed [13–15]. We recently demonstrated that for long-wavelength magnons the enhanced Gilbert damping for the transverse volume modes is twice that of the uniform mode, and for surface modes, the enhancement can be more than ten times stronger. These results are consistent with the theory of current-induced excitations of the magnetization dynamics [16] because spin pumping and spin transfer are related by Onsager reciprocity relations [17]. Moreover, mode- and wave-vector-dependent spin pumping and spin Hall voltages have been clearly observed experimentally [4].

In this paper, we extend our previous findings [18] in the following four aspects. (i) We compute the influence of the spin backflow on the enhanced spin dissipation. (ii) We also compute the induced direct and alternating inverse spin Hall voltages. We then relate these voltages to the enhanced Gilbert damping and the relevant energies for the magnetization dynamics. The induced voltages give additional information about the spin-pumping process, which can also be directly measured. (iii) We also provide additional information on the effects of interfacial pinning of different types in various field geometries. (iv) Finally, we explain in more detail how the numerical analysis is conducted for a greater number of in-plane wave numbers.

It was discovered [19–23] and later quantitatively explained [13,15,24,25] that if a dynamic ferromagnetic material is put in contact with a normal metal, the magnetization dynamics will exert a torque on the spins of electrons in the immediate vicinity of the magnet. This effect is known as spin pumping (SP) [13,15,25]. As the electrons are carried away from the ferromagnet-normal metal interface, the electrons spin with respect to each other, causing an overall loss of angular

momentum. The scattering formalism and a dynamic magnetic susceptibility equivalently describe the resulting pumped spin current [26,27]. The inverse effect, in which a spin-polarized current can affect the magnetization of a ferromagnet, is called spin-transfer torque (STT) [14,28,29].

The discovery that a precessing magnetization in magnetic insulators [3], such as YIG, also pumps spins into an adjacent metal layer was made possible by the fact that the mixing conductance in YIG-normal metal systems is of such a size that the extra dissipation of the magnetization due to the spin pumping is of the same order of magnitude as the intrinsic Gilbert damping. A consequence of this effect is that the dissipation of the magnetization dynamics is enhanced relative to that of a system in which the normal metal contact is removed. The detection of the antiferromagnet-paramagnet phase transition in CoO is a recent development of spin pumping from YIG [30]. The CoO spacer layer is sandwiched between YIG and a normal metal. Distinguishing the dynamics in the antiferromagnet from the dynamics from the YIG-normal metal system also requires a robust understanding of the spin pumping properties of the YIG-normal metal bilayer system.

This paper is organized in the following manner. Section II presents the equation of motion for the magnetization dynamics and the currents in the normal metal and the appropriate boundary conditions, both for general nonlinear excitations and in the fully linear response regime. In Sec. III, we derive approximate solutions to the linearized problem, demonstrating how the magnetization dissipation is enhanced by the presence of an adjacent metal layer. Section IV presents our numerical method and results. Finally, we summarize our findings in Sec. V.

## II. EQUATIONS OF MOTION

The equation of motion for the magnetization is given by the Landau-Lifshitz-Gilbert equation [31] (presented here in CGS units)

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}, \quad (1)$$

where  $\gamma = |g\mu_B/\hbar|$  is the magnitude of the gyromagnetic ratio;  $g \approx 2$  is the Landé  $g$ -factor for the localized electrons in the ferromagnetic insulator (FI), and  $\alpha$  is the dimensionless Gilbert damping parameter. In equilibrium, the magnitude of the magnetization is assumed to be close to the saturation magnetization  $M_s$ . The magnetization is directed along the  $z$  axis in equilibrium. Out of equilibrium, we assume that we have a small transverse dynamic magnetization component, such that

$$\mathbf{M} = \mathbf{M}(\mathbf{r}, t) = M_s \hat{\mathbf{z}} + \mathbf{m}(\mathbf{r}, t) = M_s \hat{\mathbf{z}} + \mathbf{m}(\mathbf{r}, t), \quad (2)$$

where  $|\mathbf{m}| \ll M_s$  and  $\mathbf{m} \cdot \hat{\mathbf{z}} = 0$ . Furthermore, we assume that the dynamic magnetization can be described by a plane wave traveling along the in-plane  $\zeta$  axis. In the  $(\xi, \eta, \zeta)$  coordinate system (see Fig. 1), we have

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\xi, \zeta, t) = \mathbf{m}_Q(\xi) e^{i(\omega t - Q\zeta)}, \quad (3)$$

where  $\omega$  is the harmonic angular frequency,  $Q$  is the in-plane wave number, and  $\mathbf{m}_Q(\xi) = X_Q(\xi)\hat{\mathbf{x}} + Y_Q(\xi)\hat{\mathbf{y}}$ , where  $X_Q$  and

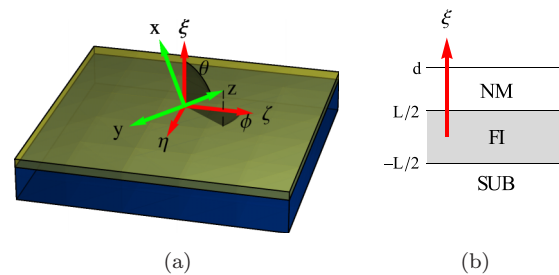


FIG. 1. (a) The coordinate system.  $\hat{\xi}$  is the film normal and  $\hat{\zeta}$  is the spin-wave propagation direction.  $\xi\eta\zeta$  form a right-handed coordinate system. The  $\hat{\mathbf{z}}$  axis is the direction of the magnetization in equilibrium, such that  $xy$  is the magnetization-precession plane. (b) The film stack is in the normal direction.

$Y_Q$  are complex functions. Note that  $\mathbf{m}$  is independent of the  $\eta$  coordinate due to translational invariance. The in-plane wave number,  $Q$ , can be engineered by lateral dimensioning of the sample in such a way as to induce standing waves in the film plane [4,32].

$\mathbf{H}_{\text{eff}}$  is the effective field, given as the functional derivative of the free energy [31,33]

$$\mathbf{H}_{\text{eff}}(\mathbf{r}, t) = -\frac{\delta U[\mathbf{M}(\mathbf{r}, t)]}{\delta \mathbf{M}(\mathbf{r}, t)} = \mathbf{H}_i + \frac{2A}{M_s^2} \nabla^2 \mathbf{M}(\mathbf{r}, t) + 4\pi \int_{-\frac{L}{2}}^{\frac{L}{2}} d\xi' \hat{\mathcal{G}}_{xy}(\xi - \xi') \mathbf{m}(\xi', \zeta, t), \quad (4)$$

where  $\mathbf{H}_i$  is the internal field, which is composed of the applied external field and the static demagnetization field. The direction of  $\mathbf{H}_i$  defines the  $z$  axis (see Fig. 1). The second term of Eq. (4) is the field,  $\mathbf{H}_{\text{ex}}$ , induced by the exchange interaction (assuming cubic symmetry), where  $A$  is the exchange stiffness parameter. The last term is the dynamic field,  $\mathbf{h}_d(\mathbf{r}, t)$ , induced by dipole-dipole interactions, where  $\hat{\mathcal{G}}_{xy}$  is the upper  $2 \times 2$  part of the dipole-dipole tensorial Green's function  $\hat{\mathcal{G}}_{\xi\eta\zeta}$  in the magnetostatic approximation (see Ref. [34]) rotated to the  $xyz$  coordinate system (see Appendix for coordinate-transformation matrices) [35].

The effect of the dipolar interaction on the spin-wave spectrum depends on the orientation of the internal field with respect to both the interface normal of the thin film,  $\hat{\xi}$ , and the in-plane spin-wave propagation direction,  $\hat{\zeta}$ . Traditionally, the three main configurations are the out-of-plane configuration ( $\theta = 0$ ), in the *forward volume magnetostatic wave* (FVMSW) geometry [see Fig. 2(a)]; the in-plane and parallel-to- $\hat{\zeta}$  configuration, in the *backward volume magnetostatic wave* (BVMSW) geometry [see Fig. 2(b)]; and the in-plane and perpendicular-to- $\hat{\zeta}$  configuration, in the *magnetostatic surface wave* (MSSW) geometry [see Fig. 2(c)] [1,35–39]. Here, the term “forward volume modes” denotes modes that have positive group velocities for all values of  $QL$ , whereas backward volume modes can have negative group velocities in the range of  $QL$ , where both exchange and dipolar interactions are significant. Volume modes are modes in which  $\mathbf{m}_Q(\xi)$  is distributed across the thickness of the entire film, whereas the surface modes are localized more closely near an interface.

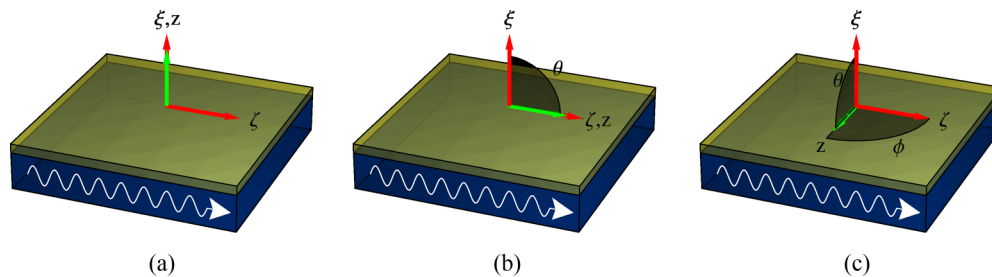


FIG. 2. Laboratory field configurations, i.e., directions of  $\hat{\mathbf{z}}$  (green arrow) in relation to film normal  $\hat{\xi}$  and the spin-wave propagation direction  $\hat{\zeta}$ , resulting in the different geometries: (a) FVMSW geometry, (b) BVMSW geometry, and (c) MSSW geometry.

### A. Spin-pumping torque

We consider a *ferromagnetic insulator* (FI) in contact with a *normal metal* (NM) (see Fig. 1). If the magnetization in the FI close to the interface is precessing around the effective field, electron spins in the NM reflected at the interface will start to precess due to the local exchange coupling to the magnetization in the FI. The reflected electrons carry the angular momentum away from the interface, where the spin information can get lost through dephasing of the spins within a typical spin diffusion length  $l_{sf}$ . This loss of angular momentum manifests itself as an increased local damping of the magnetization dynamics in the FI. The magnetization dissipation due to the spin-pumping effect can be taken into account by adding the local dissipation torque [15],

$$\boldsymbol{\tau}_{sp} = \frac{\gamma \hbar^2 g_{\perp}}{2e^2 M_s^2} \delta\left(\xi - \frac{L}{2}\right) \mathbf{M}(\mathbf{r}, t) \times \frac{\partial \mathbf{M}(\mathbf{r}, t)}{\partial t}, \quad (5)$$

to the right-hand side (rhs) of Eq. (1). Here,  $g_{\perp}$  is the real part of the spin-mixing conductance per area, and  $e$  is the electron charge. We neglect the contribution from the imaginary part of the mixing conductance, because this has been shown to be significantly smaller than that of the real part, in addition to affecting only the gyromagnetic ratio [15]. The spin-current density pumped from the magnetization layer is thus given by

$$\mathbf{j}_{sp}^{(s)} = -\frac{\hbar^2 g_{\perp}}{2e^2 M_s^2} \left[ \mathbf{M}(\mathbf{r}, t) \times \frac{\partial \mathbf{M}(\mathbf{r}, t)}{\partial t} \right]_{\xi=L/2}, \quad (6)$$

in units of erg. Next, we will see how the spin pumping affects the boundary conditions.

### B. Spin-pumping boundary conditions

Following the procedure of Rado and Weertman [40], we integrate Eq. (1) with the linear expansion of Eq. (2) over a small pill-box volume straddling one of the interfaces of the FI. Upon letting the pill box thickness tend to zero, only the surface torques of the equation survive. Accounting for the direction of the outward normal of the lid on the different top and bottom interfaces, we arrive at the exchange-pumping boundary condition

$$\left( \frac{2A}{M_s^2} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial \xi} + \frac{\hbar^2}{2e^2 M_s^2} g_{\perp} \mathbf{M} \times \frac{\partial \mathbf{m}}{\partial t} \right)_{\xi=\pm L/2} = 0. \quad (7)$$

There is no spin current pumped at the interface to the insulating substrate; thus a similar derivation results in a boundary condition that gives an unpinned magnetization,

$$\left. \frac{\partial \mathbf{M}(\mathbf{r}, t)}{\partial \xi} \right|_{\xi=-L/2} = 0. \quad (8)$$

In the next section, we will generalize the boundary conditions of Eq. (7) by also considering possible surface-anisotropy energies.

#### 1. Including surface anisotropy:

In the presence of surface anisotropy at an interface with an *easy-axis* (EA) pointing along the direction  $\hat{\mathbf{n}}$ , the surface free energy is

$$U_s[\mathbf{M}(\mathbf{r}, t)] = \int dV K_s \left[ 1 - \left( \frac{\mathbf{M}(\mathbf{r}, t) \cdot \hat{\mathbf{n}}}{M_s} \right)^2 \right] \delta(\xi - \xi_i), \quad (9)$$

where  $K_s$  is the surface-anisotropy energy density at the interface, which is assumed to be constant;  $\hat{\mathbf{n}}$  is the direction of the anisotropy *easy* axis; and  $\xi_i$  is the transverse coordinate of the interface. The contribution from the EA surface-anisotropy energy to the effective field is determined by

$$\mathbf{H}_s = -\frac{\delta U_s[\mathbf{M}(\mathbf{r}, t)]}{\delta \mathbf{M}(\mathbf{r}, t)} = \frac{2K_s}{M_s^2} (\mathbf{M} \cdot \hat{\mathbf{n}}) \delta(\xi - \xi_i) \hat{\mathbf{n}}.$$

However, if we have an *easy-plane* (EP) surface anisotropy with  $\hat{\mathbf{n}}$  being the direction of the *hard* axis, the effective field is the same as that for the EA case, except for a change of sign of  $K_s$ . We unify both cases by defining  $K_s > 0$  to imply that we have an EA surface anisotropy with its easy axis along  $\hat{\mathbf{n}}$ , whereas  $K_s < 0$  implies that we have an EP surface anisotropy with its hard axis along  $\hat{\mathbf{n}}$ .

Following the approach from Sec. II B, the total boundary condition, including exchange, pumping, and surface anisotropy, becomes

$$\left[ \pm \frac{2A}{M_s^2} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial \xi} - \frac{2K_s}{M_s^2} (\mathbf{M} \cdot \hat{\mathbf{n}}) (\mathbf{M} \times \hat{\mathbf{n}}) + \frac{\hbar^2}{2e^2 M_s^2} g_{\perp} \mathbf{M} \times \frac{\partial \mathbf{m}}{\partial t} \right]_{\xi=\pm L/2} = 0, \quad (10)$$

where the positive (negative) sign in front of the exchange term indicates that the bulk FI is located below (above) the interface coordinate.

### C. Linearization

We linearize the equation of motion using Eq. (2) with respect to the dynamic magnetization  $\mathbf{m}$ . The linearized equation of motion for the bulk magnetization, Eq. (1), becomes [35]

$$\left\{ i \frac{\omega}{\omega_M} \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix} + \mathbb{1} \left[ \frac{\omega_H}{\omega_M} + 8\pi \frac{\gamma^2 A}{\omega_M^2} \left( Q^2 - \frac{d^2}{d\xi^2} \right) \right] \right\} \mathbf{m}_Q(\xi) = \int_{-\frac{L}{2}}^{\frac{L}{2}} d\xi' \widehat{\mathcal{G}}_{xy}(\xi - \xi') \mathbf{m}_Q(\xi'), \quad (11)$$

where  $\omega_H \equiv \gamma H_i$ ,  $\omega_M \equiv 4\pi\gamma M_s$ , and  $\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ .

Next, we linearize the boundary conditions of Eq. (10). We choose the anisotropy axis to be perpendicular to the film plane,  $\hat{\mathbf{n}} = \hat{\xi}$ , which in the  $xyz$  coordinate system is given by  $\hat{\xi}_{xyz} = (\sin\theta, 0, \cos\theta)$ , where  $\theta$  is the angle between the  $z$  axis and the film normal (see Fig. 1). The finite surface anisotropy forces the magnetization to be either perpendicular or coplanar with the film surface so that  $\theta = 0, \pi/2, \pi$ . Linearizing to first order in the dynamic magnetization, we arrive at the linearized boundary conditions for the top interface:

$$\left( L \frac{\partial}{\partial \xi} + i \frac{\omega}{\omega_M} \rho + d \cos(2\theta) \right) m_{Q,x}(\xi) \Big|_{\xi=\frac{L}{2}} = 0, \quad (12a)$$

$$\left( L \frac{\partial}{\partial \xi} + i \frac{\omega}{\omega_M} \rho + d \cos^2(\theta) \right) m_{Q,y}(\xi) \Big|_{\xi=\frac{L}{2}} = 0, \quad (12b)$$

where  $d \equiv LK_s/A$  is the dimensionless surface-pinning parameter that relates the exchange to the surface anisotropy and the film thickness and  $\rho \equiv \omega_M L \hbar^2 g_{\perp} / 4Ae^2$  is a dimensionless constant relating the exchange stiffness and the spin-mixing conductance.

### D. Spin accumulation in NM and spin backflow

The pumped spin current induces a spin accumulation,  $\mu^{(s)} = \mu^{(s)} \hat{\mathbf{s}}$ , in the normal metal. Here,  $\hat{\mathbf{s}}$  is the spin-polarization axis, and  $\mu^{(s)} = (\mu_{\uparrow} - \mu_{\downarrow})/2$  is half of the difference between chemical potentials for spin-up and spin-down electrons in the NM.

As the spin accumulation is a direct consequence of the spin dynamics in the FI [see Eq. (6)], the spin accumulation cannot change faster than the magnetization dynamics at the interface. Thus, assuming that spin-flip processes in the NM are much faster than the typical precession frequency of the magnetization in the FI [25], we can neglect the precession of the spin accumulation around the applied field and any decay in the NM. With this assumption, the spin-diffusion equation  $\frac{\partial \mu^{(s)}}{\partial t} = D \nabla^2 \mu^{(s)} - \frac{\mu^{(s)}}{\tau_{\text{sf}}}$ , where  $D$  is the spin-diffusion constant, and  $\tau_{\text{sf}}$  is the material-specific average spin-flip relaxation time, becomes

$$\mu^{(s)} \approx l_{\text{sf}}^2 \nabla^2 \mu^{(s)}, \quad (13)$$

where  $l_{\text{sf}} \equiv \sqrt{\tau_{\text{sf}} D}$  is the average spin-flip relaxation length.

The spin accumulation results in a backflowing spin-current density, given by

$$\mathbf{j}_{\text{bf}}^{(s)}(L/2) = \frac{\hbar g_{\perp}}{e^2 M_s^2} [\mathbf{M}(\mathbf{r}, t) \times (\mathbf{M}(\mathbf{r}, t) \times \boldsymbol{\mu}^{(s)}(\mathbf{r}, t))] \Big|_{\xi=L/2}, \quad (14)$$

where the positive sign indicates flow from the NM into the FI. This spin current creates an additional spin-transfer torque on the magnetization at the interface:

$$\boldsymbol{\tau}_{\text{bf}} = -\frac{\gamma \hbar g_{\perp}}{e^2 M_s^2} \delta \left( \xi - \frac{L}{2} \right) \mathbf{M}(\mathbf{r}, t) \times (\mathbf{M}(\mathbf{r}, t) \times \boldsymbol{\mu}^{(s)}). \quad (15)$$

Because the spin accumulation is a direct result of the pumped spin current, it must have the same orientation as the  $\mathbf{M}(\mathbf{r}, t) \times \partial_t \mathbf{M}(\mathbf{r}, t)$  term in Eq. (5). That term is comprised of two orthogonal components: the first-order term  $M_s \hat{\mathbf{z}} \times \dot{\mathbf{m}}$ , in the  $xy$  plane, and the second-order term  $\mathbf{m} \times \dot{\mathbf{m}}$ , oriented along  $\hat{\mathbf{z}}$ . Because the magnetization is a real quantity, care must be taken when evaluating the second-order term. Using Eq. (3), the second-order pumped spin current is proportional to

$$\begin{aligned} \text{Re}\{\mathbf{m}\} \times \partial_t \text{Re}\{\mathbf{m}\} \Big|_{\xi=L/2} \\ = e^{-2\text{Im}\{\omega\}t} \text{Re}\{\omega\} \hat{\mathbf{z}} [\text{Im}X_Q \text{Re}Y_Q - \text{Re}X_Q \text{Im}Y_Q], \end{aligned} \quad (16)$$

which is a decaying *direct-current* (DC) term. This is in contrast to the first-order term, which is an *alternating-current* (AC) term. Thus we write the spin accumulation as

$$\boldsymbol{\mu}^{(s)} = \mu_{\text{AC}}^{(s)} (\hat{\mathbf{z}} \times \hat{\mathbf{m}}_t) + \mu_{\text{DC}}^{(s)} \hat{\mathbf{z}}, \quad (17)$$

where we have used the shorthand notation  $\mathbf{m}_t = \dot{\mathbf{m}}(\xi = L/2)$ , such that  $\hat{\mathbf{m}}_t = \mathbf{m}_t / |\mathbf{m}_t|$ , which in general is not parallel to  $\mathbf{m}$  but guaranteed to lie in the  $xy$  plane. Inserting Eq. (17) into Eq. (13) gives one equation each for the AC and DC components of the spin accumulation,

$$\frac{\partial^2 \mu_j^{(s)}}{\partial \xi^2} = l_{\text{sf},j}^{-2} \mu_j^{(s)}, \quad (18)$$

where  $j$  denotes either the AC or DC case and  $l_{\text{sf},\text{DC}} = l_{\text{sf}}$  while  $l_{\text{sf},\text{AC}} = l_{\text{sf}}(1 + l_{\text{sf}}^2 Q^2)^{-1/2}$  because  $\mathbf{m}_t \propto \exp(i(\omega t - Q\xi))$ . Equation (18) can be solved by demanding spin-current conservation at the NM boundaries: at the free surface of the NM, there can be no crossing spin current; thus the  $\xi$  component of the spin-current density must vanish there,  $\partial_{\xi} \mu_j^{(s)} \Big|_{\xi=L/2+d} = 0$ . Similarly, by applying conservation of angular momentum at the FI-NM interface, the net spin-current density crossing the interface, due to spin pumping and backflow, must equal the spin current in the NM layer, giving

$$\begin{aligned} \left[ -\frac{\hbar^2 g_{\perp}}{2e^2 M_s^2} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} + \frac{\hbar g_{\perp}}{e^2 M_s^2} \mathbf{M} \times (\mathbf{M} \times \boldsymbol{\mu}^{(s)}) \right] \Big|_{\xi=L/2} \\ = -\frac{\hbar \sigma}{2e^2} \partial_{\xi} \boldsymbol{\mu}^{(s)} \Big|_{\xi=L/2}, \end{aligned} \quad (19)$$

TABLE I. Typical values for the parameters used in the calculations [6,7,11,42,43].

Parameter	Value	Unit
$A$	$3.66 \times 10^{-7}$	erg cm <sup>-1</sup>
$\alpha$	$3 \times 10^{-4}$	–
$K_s$	0.05	erg cm <sup>-2</sup>
$g_{\perp}$	$8.18 \times 10^{22}$	cm <sup>-1</sup> s <sup>-1</sup>
$\gamma$	$1.76 \times 10^7$	G <sup>-1</sup> s <sup>-1</sup>
$4\pi M_s$	1750	G
$\sigma$	$8.45 \times 10^{16}$	s <sup>-1</sup>
$d$	50	nm
$l_{sf}$	7.7	nm
$\Theta$	0.1	–

where  $\sigma$  is the conductivity of the NM. Using these boundary conditions, we recover the solutions (see, e.g., Refs. [25,41])

$$\mu_j^{(s)} = \mu_{j,0}^{(s)} \frac{\sinh(l_{sf,j}^{-1}[\xi - (L/2 + d)])}{\sinh(-\frac{d}{l_{sf,j}})}, \quad (20)$$

where  $\mu_{j,0}^{(s)}$  is time dependent and depends on the  $\zeta$  coordinate only in the AC case. We find that the AC and DC spin accumulations  $\mu_{j,0}^{(s)}$  are given by

$$\mu_{AC,0}^{(s)} = -\frac{\hbar m_t}{2 M_s} \left[ 1 + \frac{\sigma}{2g_{\perp} l_{sf,AC}} \coth\left(\frac{d}{l_{sf,AC}}\right) \right]^{-1}, \quad (21)$$

$$\mu_{DC,0}^{(s)} = -\frac{l_{sf}\hbar}{\sigma M_s^2} \tilde{g}_{\perp} \tanh\left(\frac{d}{l_{sf}}\right) \hat{\mathbf{z}} \cdot [\mathbf{m} \times \dot{\mathbf{m}}]_{\xi=L/2}, \quad (22)$$

where  $\tilde{g}_{\perp}$  is a renormalized mixing conductance, which is given by

$$\tilde{g}_{\perp} = g_{\perp} \left\{ 1 - \left[ 1 + \frac{\sigma}{2g_{\perp} l_{sf,AC}} \coth\left(\frac{d}{l_{sf,AC}}\right) \right]^{-1} \right\}. \quad (23)$$

This scaling of  $g_{\perp}$  occurring in the DC spin accumulation originates from the second-order spin backflow due to the AC spin accumulation that is generated in the normal metal.

Adding both the spin-pumping and the backflow torques to Eq. (1) and repeating the linearization procedure from Sec. II C, we find that the AC spin accumulation renormalizes the pure spin-mixing conductance. Thus the addition of the backflow torque can be accounted for by replacing  $g_{\perp}$  with  $\tilde{g}_{\perp}$  in the boundary conditions of Eq. (12), making the boundary conditions  $Q$ -dependent in the process.

Using the values from Table I, which are based on typical values for a YIG-Pt bilayer system, we obtain  $\tilde{g}_{\perp}/g_{\perp} \sim 0.4$  for  $QL \ll 1$ , whereas  $\tilde{g}_{\perp}/g_{\perp} \rightarrow 1$  for large values of  $QL$ . Thus AC backflow is significant for long-wavelength modes and should be considered when estimating  $g_{\perp}$  from the linewidth broadening in *ferromagnetic resonance* (FMR) experiments [11].

### E. Inverse spin Hall effect

The *inverse spin Hall effect* (ISHE) converts a spin current in the NM to an electric potential through the spin-orbit coupling in the NM. For a spin current in the  $\hat{\xi}$

direction, the ISHE electric field in the NM layer is  $\mathbf{E}_{ISHE} = -e^{-1} \Theta \langle (\partial_{\xi} \boldsymbol{\mu}^{(s)} \times \hat{\xi})_{\xi} \rangle$ , where  $\Theta$  is the dimensionless spin-Hall angle, and  $\langle \cdot \rangle_{\xi}$  is a spatial average across the NM layer, i.e., for  $\xi \in (L/2, L/2 + d)$ . Using the previously calculated spin accumulation, we find that the AC electric field is

$$\mathbf{E}_{ISHE}^{AC} = -\Theta \frac{\hbar}{2deM_s} \left[ 1 + \frac{\sigma}{2g_{\perp} l_{sf,AC}} \coth\left(\frac{d}{l_{sf,AC}}\right) \right]^{-1} \times [-\hat{\eta}(-m_{t,y} \cos \theta \cos \phi + m_{t,x} \sin \phi) + \hat{\xi}(-m_{t,x} \cos \phi - m_{t,y} \cos \theta \sin \phi)], \quad (24)$$

where

$$m_{t,i} = -[\text{Im}\omega \text{Re}m_i + \text{Re}\omega \text{Im}m_i]_{\xi=L/2}, \quad (25)$$

and  $i = x, y$ . For BVMSW ( $\theta = \pi/2, \phi = 0$ ) modes, the AC field points along  $\hat{\xi}$ , whereas for MSSW ( $\theta = \phi = \pi/2$ ) modes, it points along  $\hat{\eta}$  (i.e., in plane, but transverse to  $\zeta$ ; see Fig. 1). Notice that for both BVMSW and MSSW mode geometries, only the  $x$  component of  $\mathbf{m}_t$  contributes to the field. In contrast, for FVMSW ( $\theta = 0$ ) modes, the field points somewhere in the  $\eta\zeta$  plane, depending on the ratio of  $m_{t,x}$  to  $m_{t,y}$ .

Similarly to the AC field, the DC ISHE electric field is given by

$$\mathbf{E}_{ISHE}^{DC} = \Theta \frac{\mu_{DC,0}^{(s)}}{de} \sin \theta (\hat{\eta} \cos \phi - \hat{\xi} \sin \phi), \quad (26)$$

which is perpendicular to the AC electric field and zero for the FVMSW mode geometry.

The total time-averaged energy in the ferromagnet  $\mathcal{E}_{\text{total}}$  (see, Morgenthaler [44]) is given by

$$\langle \mathcal{E}_{\text{total}} \rangle_T = \int_{\text{ferrite}} \text{Re} \left[ -i\pi \frac{\omega^*}{\omega_M} (\mathbf{m} \times \mathbf{m}^*) \hat{\mathbf{z}} \right] dV, \quad (27)$$

where the integral is taken over the volume of the ferromagnet.

Because the DC ISHE field is in-plane, the voltage measured per unit distance along the field direction,  $\hat{\Lambda} = \hat{\eta} \cos \phi - \hat{\xi} \sin \phi$ , can be used to construct an estimate of the mode efficiency. Taking the one-period time average of Eq. (26) using Eq. (22) and normalizing it by Eq. (27) divided by the in-plane surface area,  $\mathcal{A}$ , we find an amplitude-independent measure of the DC ISHE:

$$\epsilon_{DC}^{DC} = \frac{\langle e \hat{\Lambda} \cdot \mathbf{E}_{ISHE}^{DC} \rangle_T}{\langle \mathcal{E}_{\text{total}} \rangle_T / \mathcal{A}} = -2\gamma \Theta \frac{l_{sf}\hbar}{d\sigma M_s} \tilde{g}_{\perp} \tanh\left(\frac{d}{l_{sf}}\right) \sin \theta \times \frac{\text{Re}[-i\frac{\omega^*}{\omega_M} (\mathbf{m} \times \mathbf{m}^*) \hat{\mathbf{z}}]_{\xi=L/2}}{\int_{-L/2}^{L/2} \text{Re}[-i\frac{\omega^*}{\omega_M} (\mathbf{m} \times \mathbf{m}^*) \hat{\mathbf{z}}] d\xi}, \quad (28)$$

given in units of cm, and where  $\{\cdot\}^*$  denotes complex conjugation.

Similarly, the AC ISHE electric field, being time-varying, will contribute a power density that, when normalized by the

power density in the ferromagnet, becomes

$$\begin{aligned} \epsilon^{\text{AC}} &= \frac{\langle \sigma (\mathbf{E}_{\text{ISHE}}^{\text{AC}})^2 \rangle_T}{\frac{\text{Re}\{\omega\}}{2\pi AL} \langle \mathcal{E}_{\text{total}} \rangle_T} \\ &= \frac{\pi \sigma}{\text{Re}\{\omega\}} \left( \frac{\Theta \hbar}{2deM_s} \right)^2 \left[ 1 + \frac{\sigma}{2g_{\perp} l_{\text{sf,AC}}} \coth \left( \frac{d}{l_{\text{sf,AC}}} \right) \right]^{-2} \\ &\quad \times \frac{|m_{t,x}|^2 + \cos^2 \theta |m_{t,y}|^2}{\frac{1}{L} \int_{-L/2}^{L/2} \text{Re} \left[ -i \frac{\omega^*}{\omega_M} (\mathbf{m} \times \mathbf{m}^*) \cdot \hat{\mathbf{z}} \right] d\xi}. \end{aligned} \quad (29)$$

To be able to calculate explicit realizations of the mode-dependent equations (28) and (29), one will need to first calculate the dispersion relation and mode profiles in the ferromagnet.

### III. SPIN-PUMPING THEORY FOR TRAVELLING SPIN WAVES

Because the linearized boundary conditions [see Eqs. (12)] explicitly depend on the eigenfrequency  $\omega$ , we cannot apply the method of expansion in the set of pure exchange spin waves, as was performed by Kalinikos and Slavin [35]. Instead, we analyze and solve the system directly for small values of  $QL$ , whereas the dipole-dipole regime of  $QL \sim 1$  is explored using numerical computations in Sec. IV.

#### A. Long-wavelength magnetostatic modes

When  $QL \ll 1$ , Eq. (11) is simplified to

$$\begin{aligned} &\left\{ \begin{pmatrix} \sin^2 \theta & 0 \\ 0 & 0 \end{pmatrix} + i \frac{\omega}{\omega_M} \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix} \right. \\ &\quad \left. + \mathbb{1} \left[ \frac{\omega_H}{\omega_M} - 8\pi \frac{\gamma^2 A}{\omega_M^2} \frac{d^2}{d\xi^2} \right] \right\} \cdot \mathbf{m}_Q(\xi) = 0, \end{aligned} \quad (30)$$

where the first-order matrix term describes the dipole-induced shape anisotropy and stems from  $\hat{\mathcal{G}}_{xy}$  (see Ref. [35]). We make the ansatz that the magnetization vector in Eq. (3) is composed of plane waves, e.g.,  $\mathbf{m}_Q(\xi) \propto e^{ik\xi}$ . Inserting this ansatz into Eq. (30) produces the dispersion relation

$$\begin{aligned} \left( \frac{\omega}{\omega_M} \right)^2 &= \left( \frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 + i\alpha \frac{\omega}{\omega_M} \right) \\ &\quad \times \left( \frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 + \sin^2 \theta + i\alpha \frac{\omega}{\omega_M} \right), \end{aligned} \quad (31)$$

where  $\lambda_{\text{ex}} \equiv \sqrt{8\pi\gamma^2 A/\omega_M^2}$  is the *exchange length*. Keeping only terms to first order in the small parameter  $\alpha$ , we arrive at

$$\begin{aligned} \frac{\omega(k)}{\omega_M} &= \pm \sqrt{\left( \frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 \right) \left( \frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 + \sin^2 \theta \right)} \\ &\quad + i\alpha \left( \frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 + \frac{\sin^2 \theta}{2} \right). \end{aligned} \quad (32)$$

The boundary conditions in Eq. (12) depend explicitly on  $\omega$  and  $k$  and give another equation  $k = k(\omega)$  to be solved simultaneously with Eq. (32). However, in the absence of spin

pumping, i.e., when the spin-mixing conductance vanishes,  $g_{\perp} \rightarrow 0$ , it is sufficient to insert the constant  $k$  solutions from the boundary conditions into Eq. (32) to find the eigenfrequencies.

Different wave vectors can give the same eigenfrequency. It turns out that this is possible when  $\omega(k) = \omega(i\kappa)$ , which has a nontrivial solution relating  $\kappa$  to  $k$ :

$$\lambda_{\text{ex}}^2 \kappa^2 = \sin^2 \theta + \lambda_{\text{ex}}^2 k^2 + 2 \frac{\omega_H}{\omega_M} \pm i2\alpha \omega(k)/\omega_M. \quad (33)$$

With these findings, a general form of the magnetization is

$$\begin{aligned} \mathbf{m}_Q(\xi) &= \begin{pmatrix} 1 \\ r(k) \end{pmatrix} \left\{ \left[ C_1 \cos \left[ k \left( \xi + \frac{L}{2} \right) \right] \right. \right. \\ &\quad \left. \left. + C_2 \sin \left[ k \left( \xi + \frac{L}{2} \right) \right] \right] \right\} + \begin{pmatrix} 1 \\ r(i\kappa) \end{pmatrix} \\ &\quad \times \left\{ C_3 \cosh \left[ \kappa \left( \xi + \frac{L}{2} \right) \right] \right. \\ &\quad \left. + C_4 \sinh \left[ \kappa \left( \xi + \frac{L}{2} \right) \right] \right\}, \end{aligned} \quad (34)$$

where  $\{C_i\}$  are complex coefficients to be determined from the boundary conditions, and where  $\kappa = \kappa(k)$  is given by Eq. (33). The ratio between the transverse components of the magnetization,  $r(k) = Y_Q/X_Q$ , is determined from the bulk equation of motion [see Eq. (30)] and is in linearized form,

$$r(k) = - \frac{\alpha \sin^2 \theta \pm 2i \sqrt{\left( \frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 \right) \left( \frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 + \sin^2 \theta \right)}}{2 \left( \frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 \right)}, \quad (35)$$

implying elliptical polarization of  $\mathbf{m}_Q$  when  $\theta \neq 0$ .

Inserting Eq. (34) into Eq. (8) only leads to a solution when  $k = 0$ , such that  $C_2 = C_4 = 0$  in the general case. By solving Eq. (12b) for  $C_3$ , we find

$$\begin{aligned} \frac{C_3}{C_1} &= - \frac{\frac{\omega_H}{\omega_M} + \lambda_{\text{ex}}^2 k^2 + \sin^2 \theta + i\alpha \frac{\omega}{\omega_M}}{\frac{\omega_H}{\omega_M} - \lambda_{\text{ex}}^2 \kappa^2 + \sin^2 \theta + i\alpha \frac{\omega}{\omega_M}} \\ &\quad \times \frac{\left( i \frac{\omega}{\omega_M} \tilde{\rho} + d \cos^2 \theta \right) \cos(kL) - kL \sin(kL)}{\left( i \frac{\omega}{\omega_M} \tilde{\rho} + d \cos^2 \theta \right) \cosh(\kappa L) + \kappa L \sinh(\kappa L)}, \end{aligned} \quad (36)$$

where  $\tilde{\rho} \equiv \rho|_{g_{\perp} \rightarrow \tilde{g}_{\perp}}$  is the pumping parameter altered by the AC spin backflow from the NM (see Sec. IID).  $C_1$  is chosen to be the free parameter that parameterizes the dynamic magnetization amplitude, which can be determined given a particular excitation scheme. The linearization of Eq. (36) with respect to  $\alpha$  is straightforward, but the expression is lengthy; we will therefore not show it here.

Inserting the ansatz with  $C_2 = C_4 = 0$  and  $C_3$  given by Eq. (36) into Eq. (12a) gives the second equation for  $k$  and  $\omega$  [the first is Eq. (32)]. In the general case, the number of terms in this equation is very large; thus we describe it as

$$f(k, \omega, \alpha, \tilde{\rho}) = 0, \quad (37)$$

i.e., an equation that depends on the wave vector  $k$ , frequency  $\omega$ , Gilbert damping constant  $\alpha$ , and the spin-pumping parameter  $\tilde{\rho}$ .

Because both the bulk and interface-induced dissipation are weak,  $\alpha \ll 1$ ,  $\tilde{\rho} \ll 1$ , the wave vector is only slightly perturbed with respect to a system without dissipation, i.e.,  $k \rightarrow k + \delta k$  where  $\lambda_{\text{ex}} \delta k \ll 1$ . It is therefore sufficient to expand  $f$  up to first order in these small quantities:

$$f(k, \omega, 0, 0) + (\tilde{\rho}) \left. \frac{\partial f}{\partial \tilde{\rho}} \right|_0 + \alpha \left. \frac{\partial f}{\partial \alpha} \right|_0 + (\lambda_{\text{ex}} \delta k) \left. \frac{\partial f}{\partial (\lambda_{\text{ex}} \delta k)} \right|_0 \approx 0, \quad (38)$$

where the sub-index 0 means evaluation in a system without dissipation, i.e., when  $(\alpha, \tilde{\rho}, \delta k) = (0, 0, 0)$ . By solving the system of equations in the absence of dissipation,  $f(k, \omega, 0, 0) = 0$ , the dissipation-induced change in the wave vector  $\delta k$  is given by

$$\delta k \approx - \frac{\tilde{\rho} \left. \frac{\partial f}{\partial \tilde{\rho}} \right|_0 + \alpha \left. \frac{\partial f}{\partial \alpha} \right|_0}{\lambda_{\text{ex}} \left. \frac{\partial f}{\partial (\lambda_{\text{ex}} \delta k)} \right|_0}. \quad (39)$$

In turn, this change in the wave vector should be inserted into the dispersion relation of Eq. (31) to find the dissipation. Inspecting Eq. (31), we note that  $\delta k$ -induced additional terms proportional to  $\omega$  are of the form  $(k + \delta k)^2 - k^2 \approx 2k\delta k$ , which renormalize the Gilbert-damping term  $i\alpha \frac{\omega}{\omega_M}$ . Thus, in Eq. (39), there are terms proportional to the frequency in both terms in the numerator. We extract these terms  $\propto i \frac{\omega}{\omega_M}$  by differentiating with respect to  $\omega$  and define the renormalization of the Gilbert damping, i.e.,  $\alpha \rightarrow \alpha + \Delta\alpha$ , from spin pumping as

$$\Delta\alpha = \frac{i2\lambda_{\text{ex}}k\omega_M \partial_\omega (\lambda_{\text{ex}} \delta k|_{\alpha=0})}{i2\lambda_{\text{ex}}k\omega_M \partial_\omega (\lambda_{\text{ex}} \delta k|_{\tilde{\rho}=0}) - 1}, \quad (40)$$

where  $\partial_\omega$  represents the derivative with respect to  $\omega$  and  $k$  is the solution to the zeroth-order equation. Note that in performing a further local analysis around some point  $k_0$  in the  $k$  space of Eq. (37), a series expansion of  $f$  around  $k_0$  must be performed before evaluating Eqs. (39) and (40).

Equation (40) is generally valid, except when  $d = 0$  and  $kL \rightarrow 0$ , which we discuss below. In the following section, we will determine explicit solutions of the zeroth-order equation for some key cases, and map out the spin-wave dispersion relations and dissipation in the process.

### B. No surface anisotropy ( $d = 0$ )

Let us first investigate the case of a vanishing surface anisotropy. In this case, the zeroth-order expansion of Eq. (37) has a simple form and is independent of the magnetization angle  $\theta$ . The equation to determine  $k$  is given by

$$kL \tan(kL) = 0, \quad (41)$$

with solutions  $k = n\pi/L$ , where  $n \in \mathbb{Z}$ . Similarly, the expression for  $\delta k$  is greatly simplified,  $\delta k_n = i \frac{\omega}{\omega_M} \frac{\tilde{\rho}}{n\pi} \frac{\lambda_{\text{ex}}}{L}$ ,  $n \neq 0$ , such that the mode-dependent Gilbert damping is

$$\Delta\alpha_n = 2\tilde{\rho} \left( \frac{\lambda_{\text{ex}}}{L} \right)^2, \quad n \neq 0. \quad (42)$$

For the macrospin mode, when  $n = 0$ , the linear expansion in  $\delta k$  becomes insufficient. This is because  $kL \tan(kL) \sim (kL)^2$  for  $kL \rightarrow 0$ ; thus we must expand the function  $f$  to second order in the deviation  $\delta k$  around  $kL = 0$ . For  $d = 0$ , we find that the boundary condition becomes  $\delta k^2 L^2 = i \frac{\omega}{\omega_M} \tilde{\rho} \lambda_{\text{ex}}^2$ , and when inserted into Eq. (31), it immediately gives

$$\Delta\alpha_0 = \tilde{\rho} \left( \frac{\lambda_{\text{ex}}}{L} \right)^2 = \frac{1}{2} \Delta\alpha_n, \quad (43)$$

which is the macrospin renormalization factor found in Ref. [15]. Using a different approach, our results in this section reproduce our previous result that the renormalization of the Gilbert damping for standing waves is twice the renormalization of the Gilbert damping of the macrospin [18]. Next, we will obtain analytical results beyond the description in Ref. [18] for the enhancement of the Gilbert damping in the presence of surface anisotropy.

### C. Including surface anisotropy ( $d \neq 0$ )

In the presence of surface anisotropy, the out-of-plane and in-plane field configurations must be treated separately. This distinction is because the boundary condition (37) has different forms for the two configurations in this scenario.

#### 1. Out-of-plane magnetization

When the magnetization is out of plane, i.e.,  $\theta = 0$ , the spin-wave excitations are circular and have a high degree of symmetry. A simplification in this geometry is that the coefficient  $C_3 = 0$ . In the absence of dissipation, the boundary condition (37) determining the wave vectors becomes

$$kL \tan(kL) = d. \quad (44)$$

Let us consider the effects of the two different anisotropies in this geometry.

*a. Easy-axis surface anisotropy ( $d > 0$ ).* When  $d \sim 1$  or larger, the solutions of Eq. (44) are displaced from the zeros of  $\tan(kL)$ , i.e., the solutions we found in the case of no surface anisotropy, and towards the upper poles located at  $k_u L = (2n + 1)\pi/2$ , where  $n = 0, 1, 2, \dots$ . We therefore expand  $f$  in Eq. (37) [and thus also in Eq. (44)] into a Laurent series around the poles from the first negative order up to the first positive order in  $kL$  to solve the boundary condition for  $kL$ , giving

$$kL \approx \frac{\lambda_{\text{ex}}}{L} \frac{3(1+d) + 2(k_u L)^2 - \sqrt{12(k_u L)^2 + 9(1+d)^2}}{2k_u L}. \quad (45)$$

Using this result and the Laurent-series expansion for  $f$  in Eqs. (39) and (40), we find the Gilbert-damping renormalization term ( $\alpha \rightarrow \alpha + \Delta\alpha_{\text{EA},n}^{(\text{oop})}$ ) and the ratio between the modes

$$\frac{\Delta\alpha_{\text{EA},n}^{(\text{oop})}}{\Delta\alpha_0} \approx 3(3(1+d) + 2(k_u L)^2 - \sqrt{12(k_u L)^2 + 9(1+d)^2}) \times \frac{(\sqrt{4(k_u L)^2 + 3(1+d)^2} - \sqrt{3(1+d)})}{2(k_u L)^2 \sqrt{4(k_u L)^2 + 3(1+d)^2}}. \quad (46)$$

This ratio is plotted in Fig. 3 for  $n \leq 5$ . We see that the ratio vanishes for large values of  $d$ . For small values of the

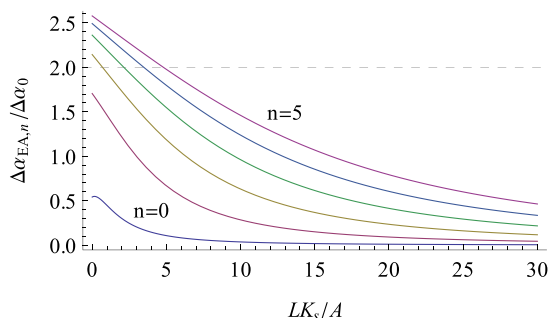


FIG. 3. The ratio of enhanced Gilbert damping  $\Delta\alpha_{EA,n}/\Delta\alpha_0$  in a system with easy-axis surface anisotropy vs the enhanced Gilbert damping of macrospin modes in systems with no surface anisotropy as a function of surface-anisotropy energy.  $n$  refers to the mode number, where  $n = 0$  is the uniformlike mode. The dashed line represents the ratio  $\Delta\alpha_n/\Delta\alpha_0$  in the case of no surface anisotropy [see Eq. (42)].

anisotropy energy  $d$ , the approximate ratio exceeds the exact result of the ratio we found in the limiting case of no surface anisotropy [see Eq. (42)]. For moderate values of  $d \sim 5$ , the expansion around the upper poles is sufficient, but only for the first few modes. This implies that moderate-strength easy-axis surface anisotropy quenches spin pumping for the lowest excited modes but does not affect modes with higher transverse exchange energy.

*b. Easy-plane surface anisotropy ( $d < 0$ ).* Easy-plane surface anisotropy is represented by a negative surface anisotropy  $d$  in Eq. (44). In this case, the boundary condition must be treated separately for the uniformlike ( $n = 0$ ) mode and the higher excitations. When  $|d| > 1$ , we can obtain a solution by expanding along the imaginary axis of  $kL$ . This corresponds to expressing the boundary condition in the form  $-ikL \tanh(ikL) = -|d|$ , with the asymptotic behavior  $kL \approx -i|d|$ . Using the asymptotic form of the boundary condition in Eq. (39) and calculating the renormalization of the Gilbert damping using Eq. (40), we find that the renormalization is  $\alpha \rightarrow \alpha + \Delta\alpha_{EP,0}^{(oop)}$ , where

$$\frac{\Delta\alpha_{EP,0}^{(oop)}}{\Delta\alpha_0} = 2|d|. \quad (47)$$

Thus the Gilbert damping of the lowest mode is much enhanced by increasing the surface anisotropy. The surface-anisotropy mode is localized at the surface because it decays from the spin-active interface and into the film. Because the effective volume of the mode is reduced, spin pumping more strongly causes dissipation out of the mode and into the normal metal.

For the higher modes ( $n > 0$ ), the negative term on the rhs of Eq. (44) forces the  $kL$  solutions closer to the negative, lower poles of  $\tan(kL)$ , located at  $k_n^{(l)}L = (2n - 1)\pi/2$ , where  $n = 1, 2, 3, \dots$ . We repeat the procedure used for the EA case by expanding  $f$  into a Laurent series around these lower poles,

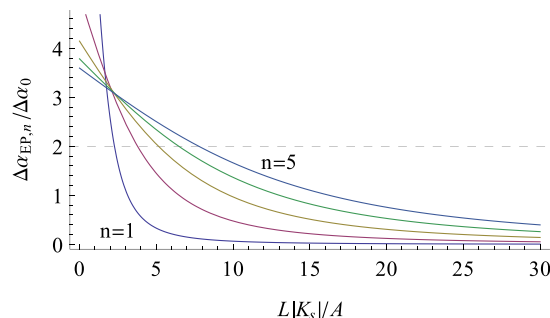


FIG. 4. Plot of  $\Delta\alpha_{EP,n}^{(oop)}/\Delta\alpha_0$ . The dashed line represents the ratio  $\Delta\alpha_n/\Delta\alpha_0$  in the case of no surface anisotropy [see Eq. (42)].

arriving at

$$kL \approx \frac{3(1 - |d|) + 2(k_n^{(l)}L)^2 + \sqrt{12(k_n^{(l)}L)^2 + 9(1 - |d|)^2}}{2k_n^{(l)}L}. \quad (48)$$

Using this relation and the new lower-pole Laurent expansion for  $f$ , Eqs. (39) and (40) give us the renormalization of the Gilbert damping ( $\alpha \rightarrow \alpha + \Delta\alpha_{EP,n}^{(oop)}$ ) and the ratio

$$\begin{aligned} \frac{\Delta\alpha_{EP,n}^{(oop)}}{\Delta\alpha_0} &\approx 3(3(1 - |d|) + 2(k_u L)^2) \\ &\quad + \sqrt{12(k_u L)^2 + 9(1 - |d|)^2} \\ &\quad \times \frac{(\sqrt{4(k_u L)^2 + 3(1 - |d|)^2} + \sqrt{3(1 - |d|)})}{2(k_u L)^2 \sqrt{4(k_u L)^2 + 3(1 - |d|)^2}}. \end{aligned} \quad (49)$$

This ratio is plotted in Fig. 4 from  $n = 1$  up to  $n = 5$ . We see that the ratio vanishes for large values of  $|d|$ . Similar to the case of EA surface anisotropy, the approximation breaks down for large  $n$  and/or small values of  $|d|$ .

Whereas the  $n = 0$  mode exhibits a strong spin-pumping enhanced dissipation in this field configuration, the DC ISHE field vanishes when  $\theta = 0$  [see Eq. (26)]. This is one of the reasons why this configuration is seldom used in experiments. However, this configuration can lead to a significant AC ISHE, and a similar AC signal was recently detected [12]. Because of the strong dissipation enhancement, the EP surface anisotropy induced localized mode in perpendicular magnetization geometry could be important in future experimental work.

## 2. In-plane magnetization

We will now complete the discussion of the spin-pumping enhanced Gilbert damping by treating the case in which the magnetization is in plane ( $\theta = \pi/2$ ). For such systems, the coefficient  $C_3 \neq 0$ , and the zeroth-order expansion of Eq. (37) becomes

$$kL \tan kL = -\frac{d((\lambda_{ex}k)^2 + \frac{\omega_H}{\omega_M})\sqrt{1 + (\lambda_{ex}k)^2 + 2\frac{\omega_H}{\omega_M}}}{\sqrt{1 + (\lambda_{ex}k)^2 + 2\frac{\omega_H}{\omega_M}}(1 + 2(\lambda_{ex}k)^2 + 2\frac{\omega_H}{\omega_M}) - d\frac{\lambda_{ex}}{L}(1 + (\lambda_{ex}k)^2 + \frac{\omega_H}{\omega_M}) \coth\left(\frac{L}{\lambda_{ex}}\sqrt{1 + (\lambda_{ex}k)^2 + 2\frac{\omega_H}{\omega_M}}\right)}. \quad (50)$$



For typical film thicknesses, of some hundred nanometers, we have  $L/\lambda_{\text{ex}} \gg 1$  and  $(\lambda_{\text{ex}}k)^2 \ll 1$  for the lowest eigenmodes. Thus we take the asymptotic  $\coth \sim 1$  and neglect the  $(\lambda_{\text{ex}}k)^2$  terms, ridding the rhs of Eq. (50) of any  $k$  dependence. Equation (50) now becomes similar to the out-of-plane case,

$$kL \tan(kL) = d_{\text{eff}}, \quad (51)$$

where

$$d_{\text{eff}} = -\frac{d \frac{\omega_H}{\omega_M} \sqrt{1 + 2 \frac{\omega_H}{\omega_M}}}{\left(1 + 2 \frac{\omega_H}{\omega_M}\right)^{3/2} - d \frac{\lambda_{\text{ex}}}{L} \left(1 + \frac{\omega_H}{\omega_M}\right)}. \quad (52)$$

$d_{\text{eff}}$  is positive if  $d < 0$  and negative for  $d > 0$  up to a critical value  $d\lambda_{\text{ex}}/L = \lambda_{\text{ex}}K_s/A = (1 + 2 \frac{\omega_H}{\omega_M})^{3/2} / (1 + \frac{\omega_H}{\omega_M})$ , where the denominator becomes zero. For negative  $d$ ,  $|d_{\text{eff}}| < |d|$ , whereas for positive  $d$ ,  $|d_{\text{eff}}|$  is initially smaller than that of  $|d|$  but quickly approaches the critical value. With the value  $K_s$  from Table I, we have  $|d_{\text{eff}}| < |d|$ , independent of the sign of  $d$ .

With this relation, we can calculate an approximate Gilbert damping renormalization in both the EA and EP cases using the EP and EA relations, respectively, obtained in the out-of-plane configuration. Thus

$$\Delta\alpha_{\text{EA},0}^{\text{ip}} \approx \Delta\alpha_{\text{EP},0}^{\text{oop}}|_{d \rightarrow d_{\text{eff}}} = 2|d_{\text{eff}}|, \quad (53)$$

$$\Delta\alpha_{\text{EA},n}^{\text{ip}} \approx \Delta\alpha_{\text{EP},n}^{\text{oop}}|_{d \rightarrow d_{\text{eff}}}, \quad (54)$$

$$\Delta\alpha_{\text{EP},n}^{\text{ip}} \approx \Delta\alpha_{\text{EA},n}^{\text{oop}}|_{d \rightarrow d_{\text{eff}}}. \quad (55)$$

To summarize this section regarding the enhancement of Gilbert damping, we see that the enhancement can be very strong for the surface modes because their effective sizes are smaller than the thickness of the film. For all other modes, the enhancement decreases with increasing magnitude of the surface-anisotropy energy.

#### IV. NUMERICAL CALCULATIONS

The first step in the numerical method is to approximate the equation of motion of Eq. (11) into by finite-size matrix eigenvalue problem. We discretize the transverse coordinate  $\xi$  on the interval  $[-L/2, L/2]$  into  $N$  points labeled by  $j = 1, 2, \dots, N$ , and characterize the transverse discrete solutions of the dynamic magnetization vectors  $\mathbf{m}_Q$  by  $(m_{x,j}, m_{y,j})$  of size  $2N$ .

We approximate the second-order derivative arising from the exchange interaction using a  $n$ th-order central difference method. For the  $n - 2$  discretized points next to the boundaries, we also use  $n$ th-order methods, using forward (backward) difference schemes for the lower (upper) film boundary. This strategy avoids the introduction of “ghost” points outside the interval  $[-L/2, L/2]$  to satisfy the boundary conditions.

Thus the total operator acting on the magnetization on the left-hand side of Eq. (11) becomes a sparse  $2N \times 2N$  matrix operator. On the right-hand side of Eq. (11), we also represent the convolution integral as a  $2N \times 2N$  dense matrix operator, where each row is weighted according to the extended integration formulas for closed integrals to  $n$ th order [45]. The four  $N \times N$  subblocks of this integration operator correspond

to the four tensor elements of  $\widehat{\mathcal{G}}_{xy}$ . In the final discrete form, we obtained a  $2N \times 2N$   $\omega$ -dependent matrix.

Next, the four boundary conditions (at the left and right boundaries for the two components,  $m_x$  and  $m_y$ ) are used to reduce the number of equations to  $2N - 4$ . This is performed by algebraically solving the discretized boundary conditions with respect to the boundary points, i.e., by determining  $m_i$  where  $i \in \{1, N, N + 1, 2N\}$  in terms of the magnetizations at the interior points.

Finally, each  $(2N - 4) \times (2N - 4)$  matrix is separated into two parts: a term independent of the frequency  $\omega$  and a term proportional to  $\omega$ . The dipole interaction causes the eigenvalue problem to be non-Hermitian and therefore computationally more demanding than a generalized eigenvalue problem. We find the dispersion relation and magnetization vectors by solving this eigenvalue problem. The resulting eigenvectors are used to find the magnetization at the boundary by back-substitution into the equations for the boundary conditions.

We are interested in finding the mode and wave-vector dependence of the spin-pumping enhanced Gilbert damping. To obtain this information numerically, we perform two independent calculations of the (complex) eigenvalues. First, we calculate the complex eigenvalues  $\omega_d$  when there is no spin pumping, but dissipation occurs via the conventional bulk Gilbert damping. Second, we calculate the complex eigenvalues  $\omega_{\text{sp}}$  when spin pumping is active at the FI-NM interface but there is no bulk Gilbert damping. A mode- and wave-vector-dependent measure of the effective enhanced Gilbert damping enhancement is then given by

$$\Delta\alpha = \alpha \frac{\text{Im}\omega_{\text{sp}}}{\text{Im}\omega_d}. \quad (56)$$

To ensure that we treat the same modes in the two independent calculations, we check the convergence of the relative difference in the real part of the eigenvalues. Table I lists the values for the different system parameters that are used throughout this section.

Let us first discuss the renormalization of the Gilbert damping when there is no surface anisotropy. We will present the numerical results for the three main geometries described in Sec. I and compare the results to the analytical results of Sec. III A.

##### A. FVMSW ( $\theta = 0$ )

Figure 5 shows the wave-vector dependent renormalization of the Gilbert damping  $\Delta\alpha$  due to spin pumping at the FI-NM interface in the FVMSW geometry. In this geometry, waves traveling along  $\pm\hat{\xi}$  have the same symmetry; thus each line is doubly degenerate and corresponds to two waves of  $\pm\omega$ . The “spikes” in the figure are due to degeneracies, i.e., mode crossings, and upon inspection, these spikes can be observed in the dispersion relation.

##### 1. Easy-axis surface anisotropy ( $\hat{\xi}$ easy axis)

Figure 6 shows  $\Delta\alpha_{\text{EA}}$  for the FVMSW geometry with an EA surface anisotropy at the spin-active interface. As predicted in Sec. III C 1a, all modes exhibit a decreased  $\Delta\alpha$  compared with those in Eqs. (43) and (42). For small  $QL$  and the chosen value of  $K_s$  (see Table I), the first four modes match the

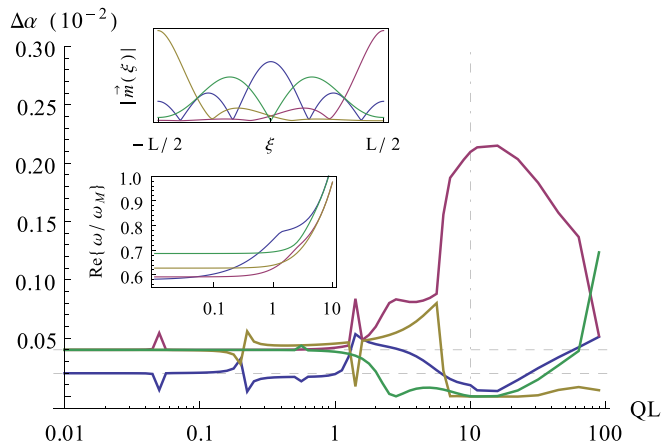


FIG. 5.  $\Delta\alpha$  vs wave vector for the FVMSW geometry of the four smallest eigenvalues. (Top inset) Magnitudes of eigenvectors (in arbitrary units) across the film at  $QL = 10$ . (Bottom inset) Dispersion relation in the dipole-dipole active regime.

analytical result of Eq. (46), which is consistent with the plot in Fig. 3. For even higher excited modes, the effect of the EA surface anisotropy becomes weaker due to the increase in transverse exchange energy. These modes (not shown in the figure) approach the value of  $\Delta\alpha_n$ .

## 2. Easy-plane surface anisotropy ( $\hat{\xi}$ hard axis)

Figure 7 shows  $\Delta\alpha_{EP}$  for the FVMSW geometry with an EP surface anisotropy. We see that the mode corresponding to  $n = 0$  has been promoted to a surface mode with a large  $\Delta\alpha$ , which for small values of  $QL$  matches Eq. (47). For the higher excited modes, we observe a decrease in  $\Delta\alpha$  compared to the case with no surface anisotropy.

### B. BVMSW ( $\theta = \pi/2$ and $\phi = 0$ )

Figure 8 shows the  $QL$ -dependent renormalization of the Gilbert damping due to spin pumping at the FI-NM interface in the BVMSW geometry. We see that the enhancement  $\Delta\alpha$

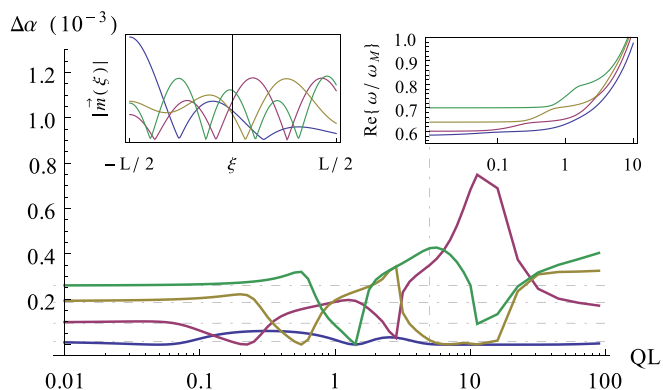


FIG. 6.  $\Delta\alpha_{EA}$  vs wave vector for the FVMSW geometry showing the four smallest eigenvalues. The horizontal dashed lines indicate solutions of Eq. (46). (Left inset) Magnitudes of eigenvectors (in arbitrary units) across the film at  $QL = 5$ . (Right inset) Dispersion relation in the dipole-dipole active regime.

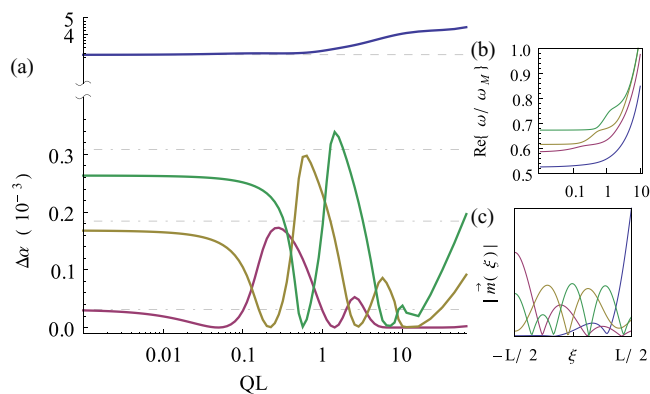


FIG. 7. (a)  $\Delta\alpha_{EP}$  vs wave vector for the FVMSW geometry, showing the four smallest eigenvalues. The dashed lines represent the analytic solutions from Sec. III C 1b. (b) Dispersion relation in the dipole-dipole active regime. (c) Magnitude of eigenvectors (in arbitrary units) across the film at  $QL = 5$ .

agrees with the analytic limits in Eqs. (43) and (42) for small values of  $QL$ . For large values of  $QL$ , we are in the strong exchange regime, in which the in-plane exchange energy becomes large compared to all other energy contributions. This in-plane exchange stiffness effectively quenches the coupling to the normal metal layer, causing  $\Delta\alpha \rightarrow 0$  for large values of  $QL$ .

Although Fig. 8 only appears to show the three first eigenvalues and eigenvectors, it actually contains double this amount. Because  $\hat{\mathbf{z}}$  is parallel to the wave-propagation direction  $\hat{\xi}$  in this geometry, there is no change in dipolar energies, regardless of whether the wave travels in the  $+\hat{\xi}$  direction or in the  $-\hat{\xi}$  direction; thus, the Gilbert damping is enhanced equally in both wave directions. A slight offset from this configuration, taking either  $\theta < \pi/2$  or  $\phi \neq 0$ , would result in a splitting of each line in Fig. 8 into two distinct lines.

### Including surface anisotropy

Figure 9 shows both the EA and the EP surface-anisotropy calculations in the BVMSW geometry. In the case of an EA

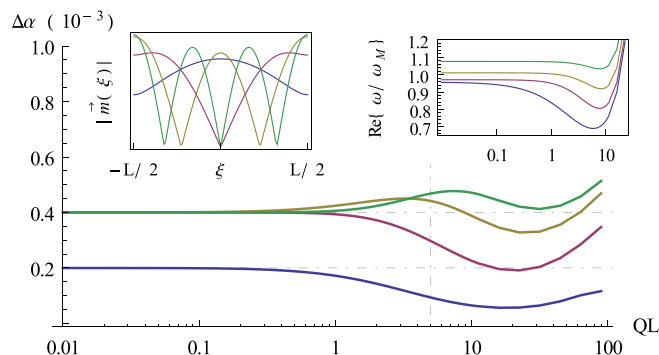


FIG. 8.  $\Delta\alpha$  vs wave vector for the BVMSW geometry ( $\theta = \pi/2$  and  $\phi = 0$ ) with  $K_s = 0$ , plotted for the four smallest eigenvalues. (Left inset) Magnitudes of normalized eigenvectors across the film at  $QL = 5$ . (Right inset) Dispersion relation in the dipole-dipole active regime.

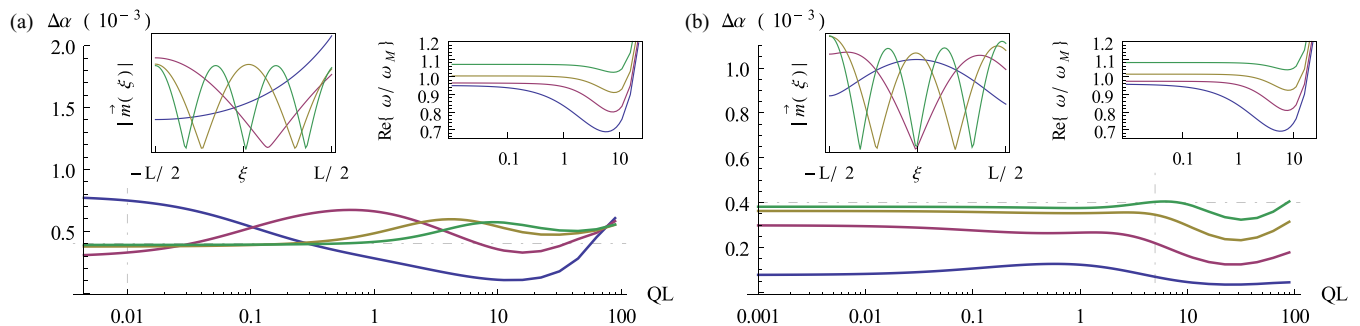


FIG. 9. (a) Dispersion relation vs wave vector for the BVMSW geometry ( $\theta = \pi/2$ ,  $\phi = 0$ ) for the four lowest eigenvalues in the case of EA surface anisotropy. (b) Dispersion relation in the case of EP surface anisotropy. In both figures, the horizontal dashed lines mark the value of  $\Delta\alpha_n$  in the case of no surface anisotropy.

surface anisotropy, the mode corresponding to  $n = 0$  gets promoted to a surface mode, similarly to the case in which there is EP surface anisotropy in the FVMSW geometry. The increase in  $\Delta\alpha$  is much smaller for the same magnitude of  $K_s$ , as explained in detail in Sec. III C. The higher modes, corresponding to  $n > 0$ , exhibit increased quenching of the Gilbert damping enhancement. In the case of EP surface anisotropy, all modes exhibit quenched Gilbert damping enhancement.

### C. MSSW ( $\theta = \phi = \pi/2$ )

Figure 10 shows the  $QL$ -dependent renormalization of the Gilbert damping due to spin pumping at the FI-NM interface in the MSSW geometry. The computed eigenvalues agree with Eqs. (43) and (42) for small values of  $QL$ . We see in the inset of Fig. 10 that in this geometry, the macrospinlike mode behaves as predicted by Damon and Eshbach [36], Eshbach and Damon [37], cutting through the dispersion relations of the higher excited modes for increasing values of  $QL$  in the dipole-dipole regime. A prominent feature of this geometry is the manner in which the modes with different signs of  $\text{Re}\{\omega\}$  behave

differently due to the dipole-dipole interaction. This is because the internal field direction ( $\hat{\mathbf{z}}$ ) is not parallel to the direction of travel ( $\hat{\boldsymbol{\xi}}$ ) of the spin wave. Hence, changing the sign of  $\omega$  is equivalent to inverting the externally applied field, changing the  $xyz$  coordinate system in Fig. 1 from a right-handed coordinate system to a left-handed system. In the middle of the dipole regime, the lack of symmetry with respect to propagation direction has different effects on the eigenvectors; e.g., in the dipole-dipole active region the modes with positive or negative  $\text{Re}\{\omega\}$  experience an increased or decreased magnitude of the dynamic magnetization, depending on the value of  $QL$ , as shown in Figs. 10(e) and 10(f). This magnitude difference creates different renormalizations of the Gilbert damping, as the plot of  $\Delta\alpha^{(\pm)}$  in Figs. 10(b) and 10(c) shows.

#### Including surface anisotropy

Figure 11 shows  $\Delta\alpha$  computed for modes in the MSSW geometry with EA and EP surface anisotropies. We can clearly see that for small  $QL$  an exponentially localized mode exists in the EA case, and as predicted in Sec. III C, all the lowest-energy modes have spin pumping quenched by EP surface

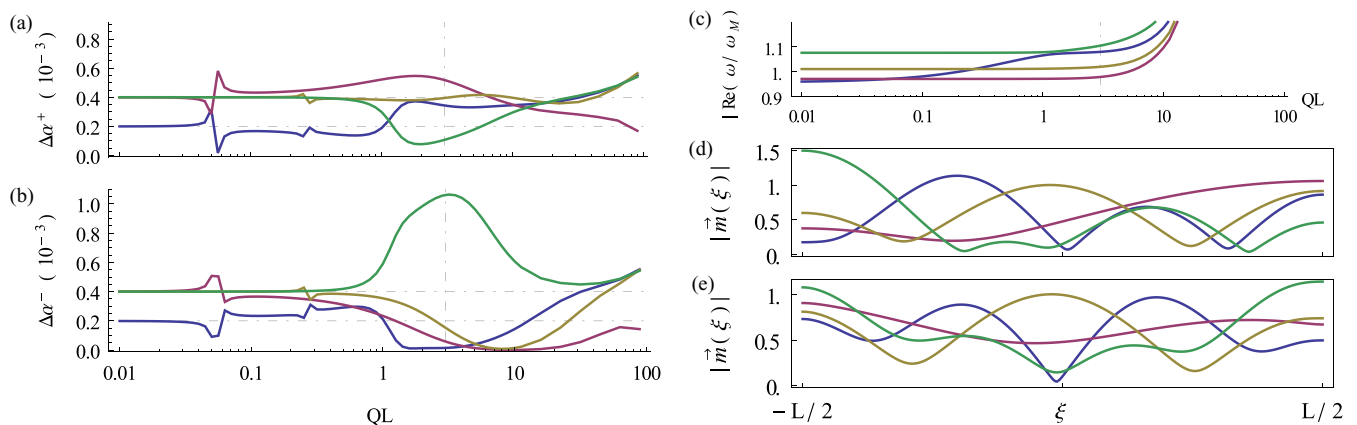


FIG. 10. Gilbert damping renormalization in the MSSW geometry. Subplots (a) and (b) show Gilbert damping renormalization  $\Delta\alpha$  for modes with positive (negative)  $\text{Re}\{\omega\}$ . The horizontal dashed lines represent the analytical values  $\Delta\alpha_0$  and  $\Delta\alpha_n$  for small  $QL$ . (c) Dispersion relation vs wave vector for the MSSW geometry ( $\theta = \phi = \pi/2$ ) for the four smallest eigenvalues, colored pairwise in  $\pm\omega$ . Subplot (d) [(e)] shows the magnitude of normalized eigenvectors (in arbitrary units) at  $QL = 3$  across the film modes with positive [(negative)]  $\text{Re}\{\omega\}$ .

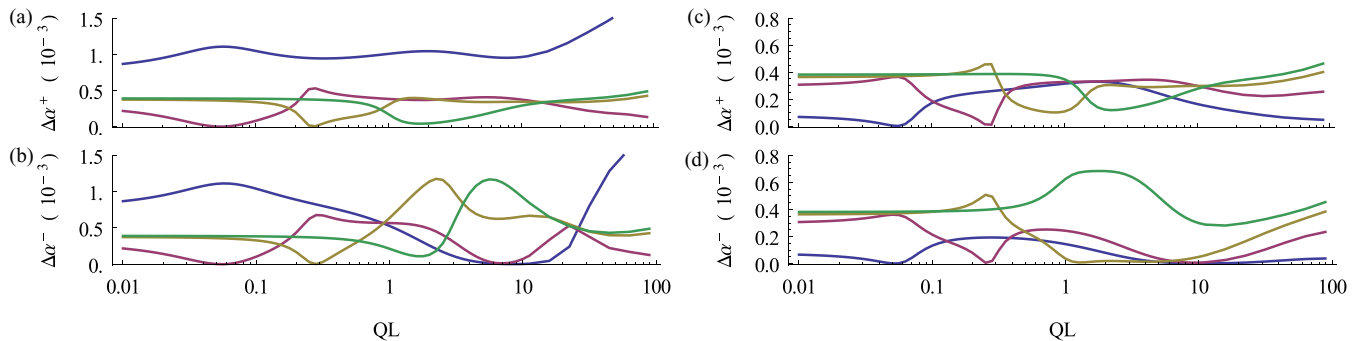


FIG. 11. (a) and (b) Gilbert damping renormalization from spin pumping in the MSSW geometry ( $\theta = \phi = \pi/2$ ) for modes with positive (negative)  $\text{Re}\{\omega\}$  in the case of EA surface anisotropy. The four smallest eigenvalues are colored pairwise in  $\pm\omega$  across the plots. (c) and (d) show the Gilbert damping renormalization in the case of EP surface anisotropy.

anisotropy. This is similar to the corresponding case in the BVMSW geometry.

#### D. AC and DC ISHE

Figure 12 shows the DC and AC ISHE measures for the BVMSW geometry corresponding to the data represented in Fig. 8. In this geometry, the angular term,  $\sin\theta$ , in Eq. (28) is to equal one, ensuring that the DC measure is nonzero. This is not the case for all geometries because the DC electric field vanishes in the FVMSW geometry. The mode-dependent DC ISHE measure exhibits the same  $QL$ -dependence as the spectrum of the Gilbert damping enhancement in all geometries where  $\sin\theta \neq 0$ . We have already presented the renormalization of the Gilbert damping in the most general cases above. Therefore we restrict ourselves to presenting the simple case of the BVMSW geometry with no surface anisotropy here.

The AC ISHE measure plotted in Fig. 12 exhibits a similar  $QL$  dependence to the Gilbert damping renormalization (and hence the DC ISHE measure), but with a slight variation in the spectrum towards higher values of  $QL$ . Note that

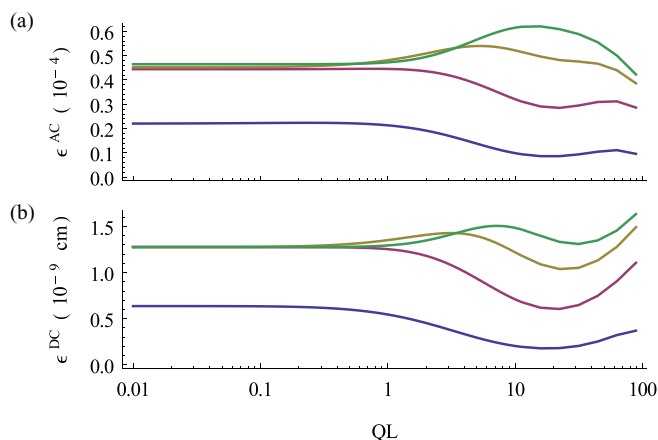


FIG. 12. ISHE as a function of in-plane wave vector in the BVMSW geometry with  $K_s = 0$ . (a) AC ISHE measure of Eq. (28) and (b) DC ISHE measure of Eq. (28).

because Eq. (24) is nonzero for all values of  $\theta$ , the AC effect should be detectable in the FVMSW geometry. By comparing the computed renormalization of the Gilbert damping for the different geometries in the previous subsections, we see that the strong renormalization of the  $n = 0$  induced surface mode that occurs in the FVMSW geometry with easy-plane surface anisotropy (see Sec. IV A 2 and Fig. 7) can have a proportionally strong AC ISHE signal in the normal metal.

#### V. CONCLUSION

In conclusion, we have presented analytical and numerical results for the spin-pumping-induced Gilbert damping and direct- and alternating terms of the inverse spin-Hall effect. In addition to the measures of the magnitudes of the DC and AC ISHE, the effective Gilbert damping constants strongly depend on the modes through the wave numbers of the excited eigenvectors.

In the long-wavelength limit with no substantial surface anisotropy, the spectrum is comprised of standing-wave volume modes and a uniformlike (macrospin) mode. These results are consistent with our previous findings [18]: in the long-wavelength limit, the ratio between the enhanced Gilbert damping for the higher volume modes and that of the macrospin mode is equal to two. When there is significant surface anisotropy, the uniform mode can be altered to become a pure localized surface mode (in the out-of-plane geometry and with EP surface anisotropy), a blend between a uniform mode and a localized mode (in-plane geometries and EA surface anisotropy), or quenched uniform modes (out-of-plane field configuration and EA surface anisotropy, or in-plane field configuration and EP surface anisotropy). The effective Gilbert damping is strongly enhanced for the surface modes but decreases with increasing surface-anisotropy energies for all the other modes.

The presented measures for both the AC and DC inverse spin-Hall effects are strongly correlated with the spin-pumping renormalization of the Gilbert damping, with the DC effect exhibiting the same  $QL$  dependency, whereas the AC effect exhibits a slightly different variation for higher values of  $QL$ . Because the AC effect is nonzero in both in-plane and out-of-plane geometries and because both EP and EA surface

anisotropies induce surface-localized waves at the spin-active interface, the AC ISHE can be potentially large for these modes.

#### ACKNOWLEDGMENTS

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#### APPENDIX: COORDINATE TRANSFORMS

The transformation for vectors from  $\xi\eta\zeta$  to  $xyz$  coordinates (see Fig. 1) is given by an affine transformation matrix  $T$ , so that

$$\mathbf{f}_{(xyz)} = \mathbf{T} \cdot \mathbf{f}_{(\xi\eta\zeta)},$$

for some arbitrary vector  $\mathbf{f}$ . Tensor-vector products are transformed by inserting a unity tensor  $\mathbf{I} = \mathbf{T}^{-1}\mathbf{T}$  between the tensor and vector and by left multiplication by the tensor  $\mathbf{T}$ , such that the tensor transforms as  $\mathbf{T}\hat{\mathcal{G}}\mathbf{T}^{-1}$  for some tensor  $\hat{\mathcal{G}}$  written in the  $\xi\eta\zeta$  basis.

$\mathbf{T}$  is given by the concatenated rotation matrices  $\mathbf{T} = \mathbf{R}_2 \cdot \mathbf{R}_1$ , where  $R_1$  is a rotation  $\phi$  around the  $\xi$  axis, and  $R_2$  is a rotation  $\theta - \frac{\pi}{2}$  around the new  $\eta$  axis/ $y$  axis. Hence

$$\mathbf{R}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{pmatrix}, \quad (\text{A1})$$

$$\mathbf{R}_2 = \begin{pmatrix} \sin \theta & 0 & -\cos \theta \\ 0 & 1 & 0 \\ \cos \theta & 0 & \sin \theta \end{pmatrix}, \quad (\text{A2})$$

such that

$$\mathbf{T} = \begin{pmatrix} \sin \theta & -\cos \theta \sin \phi & -\cos \theta \cos \phi \\ 0 & \cos \phi & -\sin \phi \\ \cos \theta & \sin \theta \sin \phi & \sin \theta \cos \phi \end{pmatrix}. \quad (\text{A3})$$

This transformation matrix consists of orthogonal transformations; thus the inverse transformation, which transforms  $xyz \rightarrow \xi\eta\zeta$ , is just the transpose,  $\mathbf{T}^{-1} = \mathbf{T}^T$ .

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