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Influence of computer-aided assessment on ways of working with mathematics

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This paper is based on an on-going project for modernizing the basic education in mathematics for engineers at the Norwegian University of Science and Technology. One of the components in the project is using a computer-aided assessment system (Maple T.A.) for handling students' weekly hand-ins. Successful completion of a certain number of problem sets in Maple T.A. is required for being admitted to the final examination. This also gives partial credit towards the final grade. In this paper, I will look at possible influence Maple T.A. may have on the ways students engage with the mathematical problems.

1. Introduction

The computer-aided assessment system Maple T.A.[1] is used as a platform for weekly problem sets in calculus courses for students in the Master of Technology programmes at the Norwegian University of Science and Technology, NTNU. Maple T.A. is based on the kernel of the computer algebra system (CAS) Maple and can evaluate answers given in numerical or algebraic form, and it can also be used for multiple-choice questions. The use of Maple T.A. is part of a larger project for innovative education in mathematics (see Rønning, 2014, 2015). Maple T.A. has been used in education for many years and there are several reports on experiences from using it (e.g. Brito *et al.*, 2009; Winkler *et al.*, 2012; Blackman, 2014; Barana *et al.*, 2015).

Students at NTNU are given one problem set, or test, in Maple T.A. each week, 12 tests altogether per course. To be accepted for the final examination at least 6 out of 12 tests have to be passed (which means that four out of six problems must be correctly solved). Each test is open for two weeks and within this time limit students have an unlimited number of attempts to pass the test. Passing 10 or more tests gives 10% credit towards the final grade. It is therefore to be expected that the Maple T.A. tests are highly prioritized in the students' work and that they will spend much time and effort in passing the tests. In addition to the Maple T.A. tests the students get four problem sets in each course as classical written hand-ins, graded and commented on by teaching assistants. Two out of four hand-ins must be passed to be admitted to the examination, and passing all four will give another 10% towards the final grade. In addition to the compulsory problem sets the students get a number of "recommended exercises" each week. These are to a large extent more routine type exercises, meant as an introduction to the topic of the week. The courses I am addressing are the two first calculus

1 https://www.maplesoft.com/products/mapleta/.

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courses, Calculus 1 (one variable calculus) in the first semester and Calculus 2 (multi-variable calculus) in the second semester, for engineering (Master of Technology) students.

In this paper, I will in particular address the following question: *In what ways can this use of Maple T.A. be seen to influence the students' ways of working with mathematics?* Other aspects of using Maple T.A. will also be discussed. The data I will draw on are collected from surveys and interviews of students.

2. A developmental project for mathematics in engineering education

In 2013 a project to modernize the basic education in mathematics for engineering students at NTNU was initiated. This project is referred to as KTDiM, which is an acronym (in Norwegian) for *Quality, Accessibility and Differentiation in the Basic Education in Mathematics.* The project is focusing on reforming the structure and framework of the teaching and learning resources of the first two calculus courses and also the introductory course in statistics. The content of the courses has been kept basically the same as before. At NTNU around 1600 students are admitted each year to a wide range of 5-year integrated master programmes in technology. All these students take Calculus 1 in their first semester, and the majority of them, around 1200, take Calculus 2 in their second semester. The large number of students and the wide variety of study programmes that they are attending naturally lead to a wide variety in background, interest, preferences and motivation for learning mathematics. This variety has motivated the production of a wide range of learning resources, for example, different type of videos, available at all times and to be used instead of or in addition to more traditional learning resources, such as lectures and textbook. The project also includes a change in the lecture structure. The main change has been to introduce so-called *interactive lectures*. Here, the work is based on problems published a couple of days before the lecture. The students are given time to work on these problems during class, with the possibility to interact directly with fellow students and with the teacher. The teacher will use part of the time on the board, discussing issues that have come up during the interaction with the students.

Traditionally, the students have submitted compulsory written hand-ins on a set of problems every week, to be marked by teaching assistants. This system requires a large number of assistants and is therefore rather expensive. It has also been noted that the feedback given by the teaching assistants often is not very helpful. Also it comes several days after the problem set was handed in, so the students may have lost interest in it. This could be observed from the fact that a large portion of the returned hand-ins were not picked up by the students. It has been well known for many years that there is a widespread practice that students copy solutions from each other in order to be able to hand in the problem sets by the deadline that is set. One motivation for the change was a feeling that time and money spent on teaching assistants marking hand-ins could be put to better use in other ways, ways that would have better effect on students' learning. Therefore, more resources have been directed towards direct contact with the students, mainly by introducing a student support centre, and by outsourcing much of the weekly marking to a computer-aided assessment (CAA) system. For this, the system Maple T.A. was chosen. The requirement that a certain number of problem sets (6 out of 12) had to be submitted and approved of was kept also in Maple T.A. Written hand-ins were still maintained but on a smaller scale, monthly instead of weekly hand-ins. As of spring 2015 successful completion of at least 10 Maple T.A. sets also counts 10% towards the final grade. This feature is introduced to make it attractive to work on, and submit, more problem sets than strictly necessary to be admitted to the final examination. Although the 10% value of the Maple T.A. sets may not mean a great deal for the final grade it will still give the signal that the Maple T.A. tests are rather important.

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Let the function f be given by $f(x,y,z)=\dfrac{2xz}{5y+2}$.

Compute the third order partial derivative $\dfrac{\partial^3 f}{\partial y \partial z \partial x}(x,y,z)$.

The answer should be an expression in x, y and z.

FIG. 1. Example of Maple T.A. problem from Calculus 2.

3. The Maple T.A. system

Maple T.A. is a product from the Maplesoft company and it is based on the kernel from the CAS Maple. Maple T.A. is described as an Online Automated Assessment Tool and as advantages of such tools are mentioned "that students get instant feedback on their assignments" and "the ability to generate many similar questions from a single question template" (Maplesoft, 2016, p. 2). In the same source it is also stated that "these systems limit cheating while encouraging productive collaboration. Instead of sharing answers for their assignments, students discuss the process of finding the solution,. . ." (p. 2). The ability of variation described above consists mostly of varying parameters in the problems. An example of a Maple T.A. problem from Calculus 2 is shown in Fig. 1. A variation on this problem could, for example, be that the numbers 2 and 5 are substituted for other numbers, or also to introduce powers of the variables involved.

In some cases, variations of this type can create significantly different problems, as shown by the example to calculate $\int \frac{x^n}{a+x^m}\,dx$, where the values of a, n and m may vary (Brito *et al.*, 2009). In other cases, variations may be rather trivial, as in the example in Fig. 1.

In Maple T.A. it is possible to give problems where the answer is multiple choice or "fill in the blank". For "fill in the blank" the answer can be a numerical value or an algebraic expression. In the earlier versions of Maple T.A. algebraic expressions had to be entered using the particular syntax of Maple but in the present version an equation editor has been included, which makes entering algebraic expressions easier. Maple T.A. can also recognize mathematical equivalence, for example, if the intended answer is $\tan(x)$, it will be accepted if the student enters $\frac{\sin(x)}{\cos(x)}$. A correct answer is indicated by displaying a green disc with the mark ✓ inside and an incorrect answer with a red disc with the mark $\times$. There is no indication of the reason for an error.

4. Theoretical framework

An underlying assumption for the analysis of the findings in this article is that knowledge is not passively transferred to the learner but rather actively constructed by the learner in interaction with other learners, the teacher and various learning resources provided. In this context, a learning resource will be seen as a mediating artefact, mediating between the learner and the material that is assumed to be learnt. This leads to an assumption that the mediating artefact influences what is being learnt and how the learner works with the given material. This way of thinking can be traced back to the work of Vygotsky, who writes about a *mediated activity* (Vygotsky, 1978, p. 54). Leon'tev developed the work of Vygotsky further, into what has become known as *Activity Theory.* Leon'tev (1979) has described an activity system in three levels; an *activity*, which is driven by a *motive* (level 1), the activity is seen as consisting of *actions* that are *goal*-related (level 2). Actions are mediated by *operations*, which are subject to certain *conditions* (level 3). In an activity there are different actors, for example, teachers and students, and typically these actors may have different motives and goals for the activities and actions.

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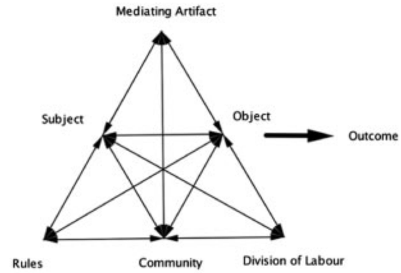

FIG. 2. Engeström's model of an activity system.

Engeström (1999) has developed the *expanded mediational triangle* (Fig. 2), where the classical mediational triangle is expanded downwards by introducing additional elements in the activity system that can be seen to have influence on the activity. In this system the activity is seen as the *subject's* work on the *object* under the influence of the *mediating artefact*. The motive of the activity is represented by the *outcome*. However, the activity is influenced not just by the mediating artefact, but also by certain *rules* that apply and by other actors in the context, *community*, and by how various actors contribute to the outcome, *division of labour*. Engeström describes this figure as a way to "graphically expand Vygotsky's original model into a collective activity system" (2001, p. 134) and he recognizes that this expansion is in fact due to Leon'tev, although he did not provide the graphical representation. Engeström denotes the uppermost triangle in Fig. 2 as "the 'tip of the iceberg' representing individual and group actions embedded in a collective activity system" (2001, p. 134).

In my analysis I will consider Maple T.A. as a *mediating artefact*, mediating between the students (subject) and the mathematical problems that they are working on (object). The intended outcome of the activity is a satisfactory solution to the problems. In this process, there are a number of other factors that influence the students' work and these factors constitute the lower part of the mediational triangle. I will describe these in more detail in the analysis.

5. Use of CAA systems and Maple T.A. in particular

Maple T.A. is a widespread system that has been available for several years and there are reports from experiences of its use from a number of countries, here demonstrated by examples from Austria (Winkler *et al.*, 2012), Italy (Barana *et al.*, 2015), Portugal (Brito *et al.*, 2009) and USA (Blackman, 2014). Among the advantages of using Maple T.A., these authors report that it saves resources used for correction, that it enables rapid feedback, and that randomization of examples will reduce copying of solutions. These features match well with the motivation to introduce Maple T.A. at NTNU. Also reports on better results attributed to the use of Maple T.A. are given. Brito *et al.* reports that " [t]he homeworks implemented using Maple T.A. helped students to achieve a better result" (p. 168) and Winkler *et al.* report that "[i]n the last four years the examples and their grading were improved very well" and that "the use of Maple T.A. is now also appreciated by the students" (2012, p. 911).

Greenhow (2015) discusses some principles that should be met in order that computer-aided assessment in mathematics can be said to be effective. One of these principles is that "[s]uch assessment should not only simply grade students, but should also promote learning" and to obtain this "it is clearly essential to write clear and full feedback" (p. 117). This is, however, very challenging to achieve as it requires that the system is able to handle specific types of incorrect answers, again requiring that the one who is creating the problems is able to predict likely incorrect answers to a

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TABLE 1. *Number of respondents to surveys*

Semester	*n*	Semester	*n*
Autumn 2013	662	Spring 2015	594
Spring 2014	554	Autumn 2015	799
Autumn 2014	739	Spring 2016	545

given problem. Clearly, CAA systems have limitations compared with a human being when it comes to what kind of problems it can handle. Problems handled with CAA must have clear, objective answers, whereas human markers can handle both objective and subjective problems, and also human markers can act flexibly when faced with ill-posed or unanticipated student responses, as Greenhow points out (p. 118).

Much work has been done on more comprehensive systems to support students' learning processes, for example, with so-called Intelligent Tutoring Systems. These are computer programmes that model learners' psychological states to provide individualized instruction (Ma *et al.*, 2014; see also Graesser *et al.*, 2012). Such systems are constructed for self-regulated learning and the meta-study by Ma *et al.* (2014) reports high achievement outcomes when using such systems.

6. Empirical data and how they are analysed

The empirical data that this paper is built on come from surveys administered to the whole cohort of students at the end of each semester as well as from focus group interviews with a small number of students. The two sources of data will be discussed below.

6.1 Information from surveys

Since starting to use Maple T.A. in 2013 as part of a pilot to KTDiM (KTDiM runs from 2014 to 2016), surveys have been conducted to monitor the students' experiences with the various parts of the reformed structure both in Calculus 1 (autumn) and Calculus 2 (spring). In addition students have been interviewed in focus groups. One interview, in the spring of 2015, was in particular addressing the Maple T.A. problems. Every year approximately 1600 students take Calculus 1 and around 1200 students take Calculus 2. The number of respondents to the surveys is shown in Table 1. The surveys were published online when approximately 3–4 weeks of the semester remained, and they were closed the day before the final examination. Some time has been set aside during one lecture to answer the survey, otherwise the students have submitted answers on their own time, using any device, from computer to smartphone.

Table 1 shows that the response rate is around 45–50% in both courses. In Calculus 2, the students are asked about their grade in Calculus 1 and the answers here show a distribution in the sample that is skewed towards the good grades compared with the actual population. Also the failure rate in the sample is considerably lower than in the actual population. This indicates that the sample has a bias towards stronger students. The self-reporting of grades from upper secondary school gives the same indication. Apart from this we have no information about characteristic features of the sample compared with the whole population. The survey has been promoted in the lectures so there is reason to suspect that those who are regularly attending lectures may be overrepresented.

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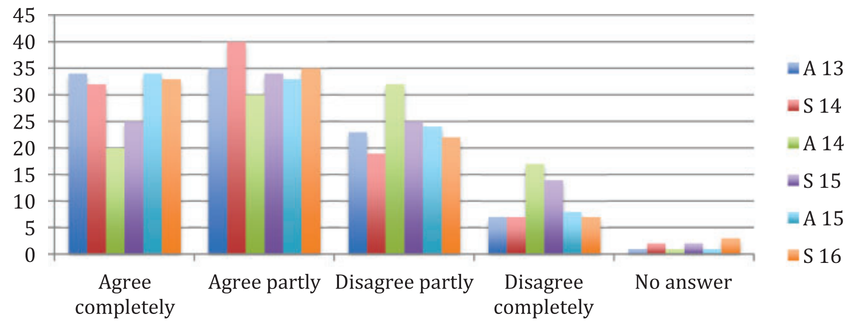

FIG. 3. The system is easy to use.

Below I will present the distribution of the answers to the following items about Maple T.A. on the six surveys completed from autumn (A) 2013 to spring (S) 2016:

Take a stance on the following statements about the Maple T.A. problem sets

(1) The system is easy to use.
(2) It is good to get a quick response if the answer is correct or not.
(3) I learn a lot from doing these problems.
(4) I miss getting feedback on what I have done wrong.
(5) I copy some problems from others each time.

Answers were given in a four-point Likert scale, ranging from "agree completely" to "disagree completely". The absence of a value in the middle forces the students to take a stance, either agree or disagree with the given statement, but there is a choice of two grades for the level of agreement or disagreement. The distribution of responses to these statements is shown in Figs. 3–7. In the diagrams "A13" means "Autumn 2013", "S14" means "Spring 2014" and so on.

Based on these data one can conclude that the students find the system relatively easy to use and that the most appreciated feature, not unexpectedly, is that the system provides quick response. On the other hand, one can see that the students are not so satisfied with the quality of the response. In all surveys 80–90% of the respondents agree to the statement "I miss getting feedback on what I have done wrong". Still the majority of students is of the opinion that they learn from the Maple T.A. problems, although there is a considerable group, 30–50%, that disagree (partly or completely) with the statement "I learn a lot from doing these problems".

One of the motivations for introducing Maple T.A. was to eliminate the phenomenon that students copy solutions from each other. Since Maple T.A. randomly generates problems it was believed that copying would not be an issue. However, after some time it became clear that the randomness of the problems was rather limited. It consisted mainly of changing values of parameters. Therefore, for most problems, it would be possible to create a general solution to the problem, and then in each case just substitute the particular values of the parameters into the answer. It also became known that students created Facebook groups where the general solutions to the weekly Maple T.A. problems were posted. This gave the incentive to include the statement "I copy some problems from others each time" in the survey as of autumn 2014. As can be seen from Fig. 7, more than 50% of the students agree (completely or partly) to this statement. There is no statement saying "I copy some problems some times", so it may be that "Agree partly" to the given statement is a way to say this. There is also information in the data suggesting that the fact that there is a deadline for each problem set and the requirement that a certain number of problem sets must be submitted strengthen the desire or need for copying.

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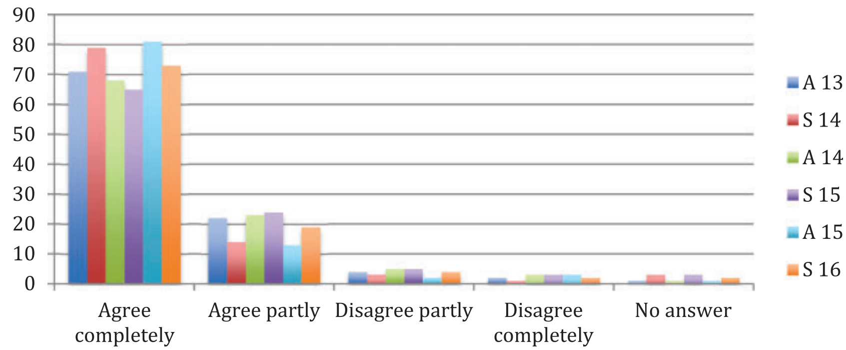

FIG. 4. It is good to get a quick response if the answer is correct or not.

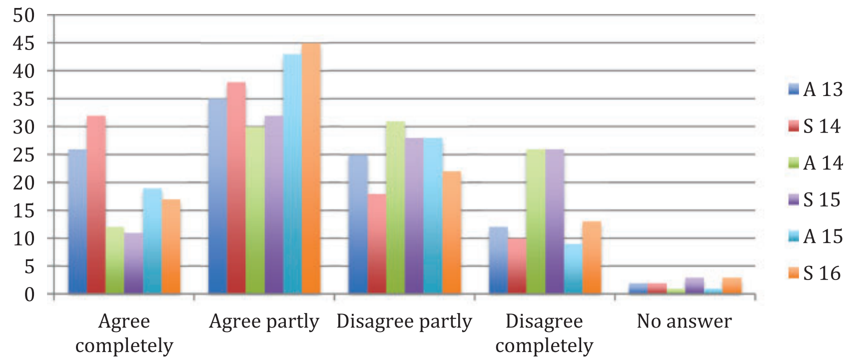

FIG. 5. I learn a lot from doing these problems.

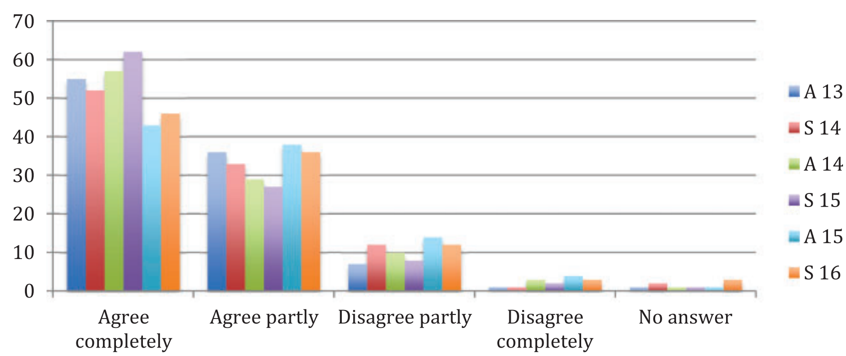

FIG. 6. I miss getting feedback on what I have done wrong.

6.2 Information from interviews

The interviews took place in April 2015 and seven students participated. They were interviewed in groups of 2 + 2 + 3. Each interview lasted around one and a half hours and was audio recorded. Before coming to the interview the students were informed that I wanted to hear about their experiences with

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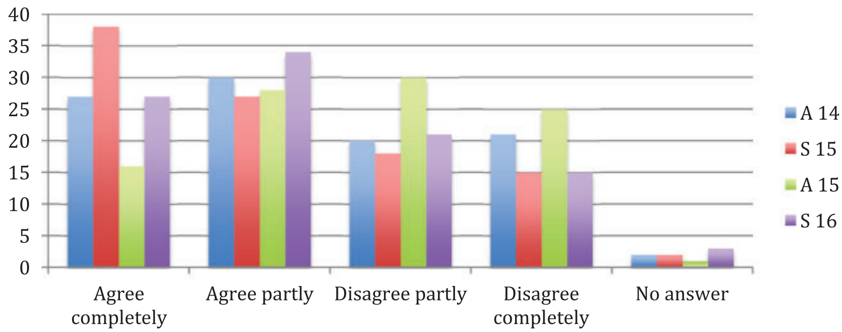

FIG. 7. I copy some problems from others each time.

the Maple T.A. system. At the interview I encouraged them to talk freely about their experiences and come up with suggestions for changes. Apart from this I had not prepared questions in advance. The recordings have been played and partly transcribed, focusing on statements that are relevant for answering the research question. The findings from this process have been coded and grouped in categories using a constant comparison method (Cohen *et al.*, 2011).

After the questions shown in Figs. 3–7 the survey presents this open-ended question: “If you have other comments to the Maple T.A. problem sets, you may write them here”. This may in a given survey result in more than 30 pages of text and together with the focus group interviews this has given me more detailed insight into certain aspects of how Maple T.A. is used and perceived by the students. Students report that they work with problems in a slightly different way in the Maple T.A. environment compared with the written hand-ins. Some of these differences are described below, documented by student utterances, which are also used as headings. From the interviews and open-ended answers, I have been searching for answers to how the use of Maple T.A. influences the students’ ways of working with mathematics. Using a coding and categorization procedure I have ended up with four categories that will shed light on this question. These categories are presented below.

6.2.1 Hunting for the answer In Maple T.A. the only thing that counts is the correct answer. The students know that only the answer is evaluated and that nobody will look at how they arrived at the answer. In addition the students get immediate feedback on whether the answer is correct or not. This makes them pay less attention to writing out the solution process carefully. “When you do it on paper you do it more properly” as one student said. When you solve Maple problems “it is like scribbling because you can just sit there and test out solutions”. On the written hand-ins, however, “you have to trust what you have done”. Students report that in particular when they don’t get the correct answer at the first attempt they start trying out different ways of solving it, gradually more at random, because they don’t know what causes the error. The error could have to do with the mathematics but it could also just be a syntax error. (This was before the equation editor was introduced in Maple T.A.) Then “you start to hunt for the correct answer.”

6.2.2 It is more important when a person looks at the solution Students report that they learn much from the written hand-ins. This includes both learning from doing the problems (almost 90% agree) and learning from the feedback (close to 60% agree). Important reasons for this can be that the

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students want to have feedback not only on the answer but also, and perhaps mainly, on the process: “I want feedback on my thinking”. “When I write the solution for a written hand-in I include more explanations and that is also helpful for myself. It becomes clearer then.” Because they know that a person will look at and evaluate what they have done, they take greater care in formulating the solution, which in itself is seen as helpful for their own learning process.

6.2.3 I lose confidence in myself if I get it wrong The students explain that the main thing is to pass the examination and this can be taken as an important explanation for the desire for “feedback on the thinking”. They know that at the final examination they get credit both for the correct answer and for “the thinking”. So when Maple T.A. responds with “Incorrect solution” this is frustrating because the student does not know *why* the solution is wrong. “I lose confidence in myself if I get it wrong. Then I start asking questions about whether everything I have done is wrong and that makes me uncertain”, as one student said. The fact that the feedback from Maple T.A. provides no details, it shows only correct or incorrect, may make the Maple T.A. problems very sensitive to level of difficulty. In the beginning there were many complaints against the level of difficulty and although a lot of effort has been done to make the problems easier, still about 65% of the students agree to the statement “The problems are too difficult”. Statements from interviews indicate that this may be connected to the feedback from the system being just correct/incorrect.” The written hand-ins may be more difficult because then you get feedback from the teacher. If the Maple problems are too difficult you don’t know what you have done wrong”. This is experienced as frustrating and leads to loss of confidence. “It is like a roller-coaster. In the end you just sit and guess”. Getting “sharp comments” on a written hand-in is not perceived as bad for the self-efficacy because then the comment is addressed to a particular part of the solution and being corrected is then seen as helpful because it is valued as helpful for being able to pass the examination.

6.2.4 We do maple first because we have to The recommended exercises given every week are meant to be rather easy, intended to become familiar with the material. For these exercises worked solutions are provided, written or presented in the lecture room, but there are no hand-ins and also no requirements for the students to do them. The Maple T.A. test, however, has to be completed before a certain deadline and therefore the work with this is prioritized. This has as a consequence that less attention is paid to the recommended exercises. Even if the students know that it would be a good idea to do these first, the fact that the Maple T.A. test has to be completed makes them start with this. This in turn has the effect that students start to copy. “You copy solutions to the Maple T.A. test to get it done away with and then you do the recommended exercises”. From the surveys one can see that the copy rate is lower for the written hand-ins than for Maple T.A. Typically less than 10% “agree completely” with the statement “I copy some problems from others each time” for the written hand-ins. For Maple T.A. the corresponding numbers can be seen in Fig. 7. This can be explained with the fact that it is much easier to copy Maple T.A. since only the answer is needed. The written hand-ins are considered more important for the learning process and therefore copying does not help, and besides it involves more work. Comparing solutions on written hand-ins, however, is more widespread, and accepted among the students.

7. An activity theory approach

In an educational context, it is certainly possible to see activity on many levels. Here I will see the *activity* as participating in a mathematics course (Calculus 1 or 2) and for the students the main *motive*

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TABLE 2. *The elements of the mediational triangle for the students*

Nodes in the mediational triangle	
Subject	Students
Object	The Maple T.A. problems
Mediating artefact	The Maple T.A. system
Rules	Requirements for passing the test
	Min. number of tests required
	Tests count towards the final grade
	Deadline
	The Maple T.A. syntax
	Feedback only on the answer (right/wrong)
Community	Other students
	Teachers
Division of labour	Students and teachers have different goals and motives
	Compulsory work is done first
	Facebook groups for worked solutions

is passing the examination. This activity can be seen as consisting of several *actions*, one of them being the Maple T.A. tests. The students' work with these tests is directed by the *goal* of passing the test. To pass the test they have to complete the problems. These are the *operations*, which are constrained by *conditions*, such as giving the answer in a form that Maple T.A. can recognize.

In Table 2, I have identified important elements in the activity system and mapped them to the various nodes in the mediational triangle. This is inspired by Jaworski *et al.* (2012) who have used Activity Theory in a university setting to analyse a situation seen both from the students' and the teachers' point of view.

In my analysis I see everything from the point of view of the students, based on what they report in surveys and interviews. From the point of view of the teachers the main motive of the activity can be viewed somewhat differently, namely that the students should learn mathematics. More precisely, the teachers want the students to develop good conceptual and procedural knowledge in mathematics (Hiebert & Lefevre, 1986) and engage in deep learning (Marton & Säljö, 1976). The teachers also want to maintain high requirements to pass the examination, or at least to get a good grade. Therefore, since the Maple T.A. tests count towards the grade the problems cannot be *too* easy. The typical training exercises are placed among the recommended exercises. However, the students do the Maple T.A. problems first (division of labour) because they are compulsory (rules). When the problems are seen as too difficult, copying from worked solutions comes in as another element in the division of labour, necessary to comply with the rules of the system.

The mediating artefact is a computer system, not a person. This affects the relation between the subject and the object, as expressed by the statements "it is more important when a person looks at the solution" and "I lose confidence in myself if I get it wrong". It is also a feature of the mediating artefact that it only evaluates the answer. This feature can also be seen as being part of the rules. That the students do not get feedback on the process or on their thinking leads to the situation "hunting for the answer". This can be seen as encouraging guessing or trying out different solutions until you get it right. Whereas with written hand-ins "you have to trust what you have done", and therefore the Maple T.A. system may encourage a way of working with mathematics that is not in line with the teachers' motive for the activity (deep learning).

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8. Discussion

The main purpose of this research has been to identify and characterize the impact that using a system like Maple T.A. under certain conditions may have on students' ways of working with mathematics. The conditions under which the system is used is that the Maple T.A. problems are compulsory in the sense that a certain number of them must be passed to be allowed to sit the examination. In addition they count towards the final grade, albeit not much. From the survey studies, it can be seen that feedback from the system, apart from right or wrong, is something that is missed by the students. This can be linked to the findings from the qualitative analysis represented by the categories *Hunting for the answer* and *It is more important when a person looks at the solution.* The fact that only the final answer is evaluated, and only with right or wrong, seems to encourage a way of working that, at least after some failures, pays little attention to how the answer is obtained and turns into a "random hunt for the correct answer". Note that the students have an unlimited number of attempts for each problem. I will suggest that this way of working is not beneficial for obtaining deep learning (Marton & Säljö, 1976). It may be argued that getting the correct answer in computations is indeed important, so the problem with the CAA is not that the answer has to be correct but that the system does not care about how one has arrived at the answer. Then, the computer turns into an external authority (Povey & Burton, 1999, p. 233) and the need for the student to trust his/her own answer is reduced. In real life applications of mathematics, that engineers are expected to encounter, it can safely be said that an important competency is to be able to trust that one's own answer is correct. When comparing Maple T.A. problems to written hand-ins, the students are focusing on the difference between presenting the answer to the computer and presenting the solution to a person. They expect the person to look at the whole argument in the problem and not just the final answer and therefore they take greater care with the presentation. An important effect of this is that they realize that they learn more from the actual procedure of presenting the argument to an assumed reader. This is consistent with a view on learning as a cycle consisting of appropriation, transformation, publication and conventionalization. In the publication phase the person makes "a personal public statement" (Ernest, 1994, p. 209), which is seen as important in the learning process. From the surveys it can be observed that around 60% agree, partly or completely, that they learn a lot from doing the Maple T.A. problems. That can be seen as a strong result but compared with other learning resources, it is low. In the two last surveys, less than 20% "completely agree" that they learn a lot from doing the Maple T.A. problems. In comparison, around 50% "completely agree" that they learn a lot from the so-called interactive lectures.

The Maple T.A. system is rather unforgiving in the way it evaluates an answer, it presents a green disk with a mark when the answer is correct and a red disc with a cross when the answer is wrong. In addition to just missing information about why the answer is wrong, the rather abrupt feedback from the system also has an effect on the students' self-confidence and self-efficacy. This can be assumed to have negative effect on the quality of the students' learning process (Bandura, 1997). Greenhow states that "a CAA system should be able to encode algorithms, not only just for the correct answer (the key) but also for incorrect answers (distracters) based on specific mistakes that are likely" (2015, p. 121). For rather simple problems this may be feasible but for more complicated problems it would be very challenging to provide an exhaustive list of anticipated wrong solutions. Another possibility is to provide hints to get further with the problem if the answer is wrong. This is mentioned as an important feature by Barana *et al.* (2015). They write:

> If they [the students] fail, they are guided into a step-by-step path to the solution; then a similar problem, with different numbers and data, is presented, so that they can repeat and learn the procedure to face and solve the problem. (Barana *et al.*, 2015, p. 670)

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Connected to this it will be easy to reduce the score on a problem if the student has made use of one or more hints for a given problem. This may sound like a tempting solution but it is important to be aware of what such a way of engaging with a problem entails. When providing hints, the original problem is changed to a different, and easier, problem. So even if the student arrives at the correct solution with hints, the problem that is actually solved is not the intended problem. Seeing the communication between the student and the computer as an interaction pattern this can be seen as akin to what Bauersfeld (1988) describes as a *funnelling pattern of interaction*. The computer leads the student towards the answer by providing new questions that break the original question down in bits and pieces, thereby losing the original challenge given by the problem.

One of the original motivations for introducing Maple T.A. was to eliminate the issue of students copying solutions from each other. It is clear that this did not happen. The variations in the problems from one student to another are such that it is possible to produce a general solution that can be copied. It can be seen that this happens to a rather large extent. One of the categories coming out of the qualitative analysis was *We do Maple first because we have to*. The fact that the Maple T.A. problem sets are compulsory will give them a high priority. In the very first survey (autumn 2013), the students were presented with nine typical activities for a Calculus 1 student and they were asked to rank them from 1 to 9 in order of priority. Here, 46% ranked "going to lectures" as 1 but "doing the Maple T.A. problems" was ranked 1 by 18% and 2 by 22%. It is reasonable to believe that the high importance of the Maple T.A. problems, combined with time pressure, may stimulate the desire to copy and therefore "create a market" for the development and distribution of general solutions. It may be that using Maple T.A. in less high-stake situations can turn out to be a better use of the system.

A final comment is concerned with the view on learning that I will claim can be seen in the CAA systems and more widely also in the Intelligent Tutoring Systems (Graesser *et al.*, 2012). The idea lying behind such systems is not a new one. In the 1960s the ideas of *programmed learning* were strong. These ideas were based on a behaviouristic view on learning, where learning was assumed to take place by stimulus-response processes (Bower & Hilgard, 1981, p.15). Introducing hints, combined with reducing the score, is also well in line with this view on learning. Gagné (1962) discusses self-instructional programmes and programmed learning as "one particular form of ordering the stimulus and response events designed to bring about productive learning" (p. 355). In another paper (Gagné, 1963) he talks about programmed learning material in a paper and pencil environment. He writes:

> [S]uch materials are designed to present information to the learner, and he is required to make a response to it by filling in a blank or answering a question. Once he has done this, an answer frame is exposed which informs him of the correct response; he then proceeds to the next frame, and so on throughout the program. (Gagné, 1963, p. 620)

These ideas were soon transferred to a computer environment and led to the development of several systems for so-called computer-aided instruction in the 1960s and 1970s. The same ideas are still alive as can be seen in the 2016 presentation of Maple T.A. below:

> One of the most obvious advantages is that students get instant feedback on their assignments. While students are still actively engaged in learning the material, they can find out if they understand it as well as they think they do, and take corrective action immediately if they don't. (Maplesoft, 2016, p. 2)

The feature of instant feedback is certainly much appreciated by the students in my study (see Fig. 4) but at the same time they are not satisfied with the *quality* of the feedback (see Fig. 6). I think there is still a lot of work to do to get really helpful feedback from a CAA system and it is important to be

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aware of its limitations and the ways it may have an impact on the students' learning processes when introducing such a system into a learning environment.

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