

**STOKES DRIFT ESTIMATION FOR SEA STATES BASED ON LONG-TERM  
VARIATION OF WIND STATISTICS**

**By**

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## **Abstract**

This paper provides a simple analytical method which can be used to give estimates of the Stokes drift within sea states based on long-term variation of wind conditions. This is exemplified by using long-term wind statistics from the Northern North Sea and the Northwest Shelf of Australia. The paper presents the mean values and the standard deviations of the surface Stokes drift and the Stokes transport. Based on, for example, global wind statistics, the present analytical results can be used to make estimates of the Stokes drift.

Keywords: Surface gravity waves; surface Stokes drift; Stokes transport; mean wind speed; long-term wind statistics.

## 1. Introduction

Simple and effective descriptions of transport mechanisms in operational ocean circulation models are often required, in which the Stokes drift represents an important transport component [McWilliams and Restrepo, 1999]. Locally the Stokes drift is responsible for transport of e.g. pollutants and biological material. It also contributes in mixing processes across the interface between the atmosphere and the ocean. The Stokes drift is the wave-averaged Lagrangian velocity derived from the water particle motion in the wave propagation direction. It has its maximum at the surface and decreases rapidly with the depth below the surface. By integrating the Stokes drift over the water depth the total mean mass transport is obtained, which is also referred to as the volume Stokes transport by Rascle et al. [2008] (see e.g. Dean and Dalrymple [1984] for more details of the Stokes drift).

The Stokes drift and the volume Stokes transport were originally defined for regular waves. However, Kenyon [1969] formulated the Stokes drift for short-crested random waves for an arbitrary constant water depth, and presented results for deep water waves. Furthermore, the characteristic quantities for Stokes drift and Stokes transport for random waves in terms of the sea state parameters significant wave height and characteristic wave periods are also defined and discussed (see e.g. Rascle et al. [2008]; Webb and Fox-Kemper [2011]; Rascle and Ardhuin [2013]; Myrhaug [2013, 2015]). Results for individual random waves have also been presented and discussed (see e.g. Myrhaug et al. [2014, 2016]; Myrhaug and Holmedal [2014]; Myrhaug and Ong [2015]). Recently, Webb and Fox-Kemper [2015] have explored the effects of directional spreading and multidirectional waves, including effects that occur when swell and wind waves coexist with different directional properties. Similar issues were addressed in Liu et al. [2014], Carrasco et al. [2014] and Breivik et al. [2014].

The purpose of this study is to demonstrate how long-term wind statistics can be used to provide the statistical properties of the surface Stokes drift and the Stokes transport within sea states. Examples of application are given based on long-term distributions of the mean wind speed at the 10 m elevation above the sea surface from the Northern North Sea and the Northwest Shelf of Australia. The main interest of the paper lies in the procedure demonstrating how wind statistics can be used to provide statistical properties of the wave-induced drift in sea states, and how this can be used to make assessment of the surface Stokes drift and the Stokes transport based on e.g. global wind statistics.

## 2. Background

The **unidirectional** surface Stokes drift velocity within a sea state of random waves in deep water is given by [Webb and Fox-Kemper, 2011]

$$U_s = \frac{\pi^3 H_s^2}{g T_3^3} \quad (1)$$

where  $g$  is the acceleration due to gravity,  $H_s = 4\sqrt{m_0}$  is the significant wave height, and  $T_3 = 2\pi(m_0 / m_3)^{1/3}$  is the wave period associated with the surface Stokes drift velocity. The spectral moment  $m_n$  is defined as

$$m_n = \int_0^{2\pi} \int_0^\infty \omega^n S(\omega, \theta) d\omega d\theta ; n = 0, 1, 2, 3, \dots \quad (2)$$

where  $S(\omega, \theta)$  is the directional wave spectrum which is used to describe short-crested (three-dimensional (3D)) waves,  $\omega$  is the cyclic wave frequency, and  $\theta$  is the angle of wave propagation direction. **Many formulations of the directional wave spectrum exist in the literature.** By following Rasche and Arduin [2013],  $S(\omega, \theta) = S(\omega)(1 - \sigma_\theta^2(\omega))$  where  $S(\omega)$  is the single-sided wave energy

spectrum, and  $\sigma_\theta(\omega)$  is the directional spreading in radians. The spectral moments for long-crested (two-dimensional (2D)) waves are obtained from Eq. (2) by neglecting the directional spreading,

that is, given by  $m_n = \int_0^\infty \omega^n S(\omega) d\omega, n = 0, 1, 2, 3, \dots$

The Stokes transport (i.e. the mass flux across the depth per unit width) within a sea state of random waves in deep water is given by [Myrhaug, 2013]

$$M = \frac{\rho\pi H_s^2}{8T_1} \quad (3)$$

where  $\rho$  is the fluid density, and  $T_1 = 2\pi m_0 / m_1$  is the spectral mean wave period.

### 3. Some statistical properties of surface Stokes drift and Stokes transport with examples

Many standard spectral formulations contain the mean wind speed at a given elevation above the sea surface as a parameter, e.g. the Pierson-Moskowitz and the Phillips spectra. Here **examples of results are given by choosing the Phillips deep water wave spectrum** for long-crested waves (see Tucker and Pitt [2001]; **Holthuijsen [2007, Section 8.3.2] for more details**)

$$S(\omega) = \alpha \frac{g^2}{\omega^5}, \omega \geq \omega_p = \frac{g}{U_{10}} \quad (4)$$

where  $\alpha = 0.0081$  is the Phillips constant,  $\omega_p = 2\pi / T_p$  is the spectral peak frequency, and  $U_{10}$  is the mean wind speed at the 10 m elevation above the sea surface. **For long-crested deep water waves it follows from Eq. (2) that**

$$H_s = 4\sqrt{m_0} = \frac{2\sqrt{\alpha}}{g} U_{10}^2 \quad (5)$$

$$T_1 = 2\pi \frac{m_0}{m_1} = \frac{3\pi}{2g} U_{10} \quad (6)$$

$$T_3 = 2\pi \left( \frac{m_0}{m_3} \right)^{1/3} = \frac{2^{1/3} \pi}{g} U_{10} \quad (7)$$

$$T_p = \frac{2\pi}{\omega_p} = \frac{2\pi}{g} U_{10} \quad (8)$$

By using Eqs. (5) to (7), Eqs. (1) and (3) are rearranged, respectively, to

$$U_s = 2\alpha U_{10} \quad (9)$$

$$\frac{M}{\rho} = \frac{\alpha}{3g} U_{10}^3 \quad (10)$$

Different parametric models for the cumulative distribution function (*cdf*) or the probability density function (*pdf*) of  $V = U_{10}$  are given in the literature. A recent review is given in Bitner-Gregersen [2015], where the joint statistics of  $V$  with significant wave height  $H_s$  and spectral peak period  $T_p$  are presented. In the present paper the long-term statistics of the wave-induced drift are exemplified by using two *cdfs* of  $V$ .

First, the *cdf* of  $V$  given by Johannessen et al. [2001] is used. This *cdf* is based on wind measurements covering the years 1973 – 1999 from the Northern North Sea (*NNS*). The database consists of composite measurements from the Brent, Troll, Statfjord and Gullfaks fields as well as the weather ship Stevenson representing a wide range of wind and wave conditions, i.e. ranging up to about  $H_s = 12$  m and  $T_p = 18$  s (see Johannessen et al. [2001] for more details), . Model data from the Norwegian hindcast archive (WINCH, gridpoint 1415) have been filled in for periods where measured data were missing. Thus a 25 year long continuous time series has been used upon which the *cdf* of the 1-hour values of  $V$  is described by the two-parameter Weibull model

$$P(V) = 1 - \exp \left[ - \left( \frac{V}{r} \right)^r \right]; V \geq 0 \quad (11)$$

with the Weibull parameters

$$r = 8.426 \quad , \quad t = 1.708 \quad (12)$$

Second, the conditional *cdf* of  $V$  given  $H_s$  given by Bitner-Gregersen [2015] is used. This conditional *cdf*,  $P(V | H_s)$ , is based on wind data from hindcast analysis from the Northwest Shelf of Australia (*NWSA*) covering the years 1994 – 2005 representing a wide range of wind and wave conditions, i.e. ranging up to about  $H_s = 8$  m and  $T_p = 20$  s (see Bitner-Gregersen [2015] for more details). The conditional *cdf* is described by the two-parameter Weibull model

$$P(V | H_s) = 1 - \exp[-(\frac{V}{r})^t] ; V \geq 0 \quad (13)$$

with the Weibull parameters

$$r = 0.050 + 5.514 H_s^{0.280} \quad (14)$$

$$t = 1.250 + 5.600 H_s^{0.660} \quad (15)$$

Now the long-term statistics of the wave-induced drift can be derived by using these *cdfs* of  $V$ . Thus, it follows that  $U_s / 2\alpha$  is distributed as  $V$  (see Eq. (9)), i.e. given by the *cdfs* in Eqs. (11) and (13). Furthermore, it follows that  $m = (M / \rho) / (\alpha / 3g)$  is distributed as  $V^3$  (see Eq. (10)), i.e. given by the *cdfs*

$$P(m) = 1 - \exp[-(\frac{m}{r^3})^{t/3}] ; m \geq 0 \quad (16)$$

with the Weibull parameters in Eq. (12), and

$$P(m | H_s) = 1 - \exp[-(\frac{m}{r^3})^{t/3}] ; m \geq 0 \quad (17)$$

with the Weibull parameters in Eqs. (14) and (15).

Statistical quantities of interest are the expected value and the variance of the wave-induced drift. This requires calculation of  $E[V]$  and  $Var[V^n]$  for  $n=1$  and  $n=3$  according to Eqs. (11), (12) and (13) to (15). For a Weibull distributed quantity this is given by [Bury, 1975]

$$E[V^n] = r^n \Gamma\left(1 + \frac{n}{t}\right) \quad (18)$$

where  $\Gamma$  is the gamma function. Furthermore, [Bury, 1975]

$$\sigma^2[V^n] \equiv Var[V^n] = E[V^{2n}] - (E[V^n])^2 \quad (19)$$

Here  $H_s = 3$  m has been chosen as an example using the *NWSA cdf* giving  $r = 7.55$  and  $t = 12.81$  according to Eqs. (14) and (15), respectively. Table 1 gives a summary of the results using the wind statistics from *NNS* and *NWSA*. The consequences of the results in Table 1 combined with Eqs. (9) and (10) are; (i) for *NNS*: the standard deviation is 60% of the mean value of  $U_s$ ; corresponding value is 54% for  $M$ ; (ii) for *NWSA* the conditional standard deviation is 88% of the conditional mean value of  $U_s$ ; the corresponding value for  $M$  is 26%.

An alternative approximate method (i.e. a deterministic method) of estimating the Stokes drift is to substitute  $E[V]$  in Eqs. (1) and (3) (i.e. replace  $V$  with  $E[V]$  in Eq. (1) and replace  $V^3$  with  $(E[V])^3$  in Eq. (3)). For  $U_s$  the deterministic method and the stochastic method are identical. For *NNS* the deterministic to stochastic method ratio for  $M$  is obtained as (see Table 1)  $(E[V])^3 / E[V^3] = 0.44$ . For *NWSA* the corresponding ratio is 0.97 (see Table 1). Overall, a stochastic method should be used since it takes into account the stochastic features in a consistent manner compared with what the deterministic method does.

A quantity which often is of interest is the ratio between the surface Stokes drift and  $V$ . The statistical quantities associated with this ratio based on Eqs. (9), (10) and the results in Table 1 are



given in Table 2; i.e. the mean value, the standard deviation and the mean value  $\pm$  one standard deviation.

To the present author's knowledge there are limited results from observations and models to compare with. However, for 2D waves these examples give results which are consistent with those obtained by Rascle et al. [2008, Fig. 8] for both *NNS* and *NWSA*; they found that the surface Stokes drift is in the range 0.7% - 1.6% for  $V$  in the range 7 m/s – 8 m/s in the open ocean.

Furthermore, the results for 3D waves given in Table 2 are obtained by reducing the values for 2D waves by 20%, which is a typical reduction of the surface Stokes drift when the directional spreading is taken into account (Rascle and Ardhuin [2013]; Webb and Fox-Kemper [2015]). Thus, by reducing the values for 2D waves by 20%, these examples give results which are consistent with those obtained by Rascle and Ardhuin [2013, Fig. 6(a)]; they found that the surface Stokes drift is approximately in the range 0.9% - 1.2% of  $V$  in the range 7 m/s – 8 m/s for  $H_s$  in the range 1 m to 3 m (i.e.  $H_s$  is in the range 1 m – 1.4 m according to Eq. (5) depending on calculating  $E[H_s]$  or on calculating  $H_s$  by substituting  $E[V]$ ).

It should be noted that the deterministic method results for  $U_s$  are the same as the stochastic method results. For  $M$  the results are given in Table 3, showing that for *NNS* the deterministic method value is outside the range of the mean value  $\pm$  one standard deviation, while for *NWSA* it is within this range.

#### 4. Summary

A simple analytical method is provided which can be used to give estimates of the Stokes drift within sea states based on statistics of long-term observations of wind conditions is. Results are exemplified by using long-term wind statistics from the Northern North Sea and the Northwest

Shelf of Australia by calculating the mean values and the standard deviations of the surface Stokes drift and the Stokes transport. The present analytical method can be used to give estimates of the wave-induced drift based on, for example, global wind statistics.

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**Table 1. Example of results using wind statistics from the Northern North Sea (NNS) and the Northwest Shelf of Australia (NWSA).**

NNS ; Eqs. (11), (12), (18), (19)		NWSA ; Eqs. (13), (14), (15), (18), (19)	
$E[U_{10}](\text{m/s})$	7.52	$E[U_{10}   H_s = 3 \text{ m}](\text{m/s})$	7.25
$\sigma[U_{10}](\text{m/s})$	4.53	$\sigma[U_{10}   H_s = 3 \text{ m}](\text{m/s})$	6.35
st. dev./m.v.	0.60	st. dev./m.v.	0.88
$E[U_{10}^3](\text{m}^3/\text{s}^3)$	967.19	$E[U_{10}^3   H_s = 3 \text{ m}](\text{m}^3/\text{s}^3)$	391.57
$\sigma[U_{10}^3](\text{m}^3/\text{s}^3)$	520.52	$\sigma[U_{10}^3   H_s = 3 \text{ m}](\text{m}^3/\text{s}^3)$	103.45
st. dev./m.v.	0.54	st. dev./m.v.	0.26

**Table 2. Ratios of surface Stokes drift parameters and  $U_{10}$  for 2D (long-crested) and 3D (short-crested) waves; all values are in %.**

	NNS		NSWA	
	2D waves	3D waves	2D waves	3D waves
$\frac{E[U_s]}{E[U_{10}]}$	1.62	1.30	—	—
$\frac{\sigma[U_s]}{E[U_{10}]}$	0.98	0.78	—	—
$\frac{\text{m.v.} \pm \text{std.dev.}}{E[U_{10}]}$	0.64, 2.60	0.52, 2.10	—	—
$\frac{E[U_s   H_s = 3 \text{ m}]}{E[U_{10}   H_s = 3 \text{ m}]}$	—	—	1.62	1.30
$\frac{\sigma[U_s   H_s = 3 \text{ m}]}{E[U_{10}   H_s = 3 \text{ m}]}$	—	—	1.42	1.14
$\frac{\text{m.v.} \pm \text{std. dev.}}{E[U_{10}   H_s = 3 \text{ m}]}$	—	—	0.200, 3.04	0.160, 0.244

**Table 3. Volume Stokes transport parameters for 2D waves; Eq. (10) and Table 1;**

**all dimensions are in m<sup>2</sup>/s.**

	<i>NNS</i>	<i>NSWA</i>
$E[\frac{M}{\rho}]$	0.266	—
$\sigma[\frac{M}{\rho}]$	0.143	---
m.v. $\pm$ std.dev.	0.123, 0.409	—
$E[\frac{M}{\rho}   H_s = 3\text{m}]$	--	0.108
$\sigma[\frac{M}{\rho}   H_s = 3\text{m}]$	--	0.028
m.v. $\pm$ std. dev.	--	0.080, 0.136
$(\frac{M}{\rho})$ deterministic	0.117	0.105