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RANDOM WAVE-INDUCED BURIAL AND SCOUR OF SHORT CYLINDERS AND TRUNCATED CONES ON MILD SLOPES

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ABSTRACT

A stochastic approach calculating the random wave-induced burial and scour depth of short cylinders and truncated cones on mild slopes is provided. It assumes the waves to be a stationary narrow-band random process and a wave height distribution for mild slopes is adopted, also using formulae for the burial and scour depths for regular waves on horizontal beds for short cylinders and for truncated cones. Examples of results are also provided.

Keywords: Burial; Scour; Short cylinders; Truncated cones; Sloping beds; Random waves; Stochastic approach.

INTRODUCTION

Random wave-induced burial and scour of short cylinders and truncated cones on mild slopes are addressed. Examples of short cylinders and truncated cones are sea mines on sloping seabeds. Such bodies, e.g. originally installed on a plane bed, may experience different seabed conditions; flat or rippled, being surrounded by scour holes, and be self-buried. This is due to complicated three-dimensional flow caused by the interaction between the flow velocity, i.e. the relative magnitude between current and waves, the bed geometry and material, the ratio between the near-bed oscillatory fluid particle excursion amplitude and the body dimensions. Furthermore, waves are stochastic, making the problem more challenging.

Details on the background and complexity of scour in the marine environment including reviews of the subject are given in, e.g., Whitehouse [1] and Sumer and Fredsøe [2]. To the present authors knowledge, no studies, except for Myrhaug and Ong [3] and Myrhaug et al. [4], exist in the open literature dealing with random wave-induced burial and scour of short cylinders. Details related to scour around and self-burial of short cylinders exposed to currents and regular waves are addressed in Catano-Lopera and Garcia [5, 6]. Voropayev et al. [7] addressed burial of short cylinders exposed to shoaling waves over a sloped sandy bottom. Myrhaug and Ong [3] presented results for burial and scour of short cylinders exposed to combined long-crested random waves plus currents, also including second-order wave asymmetry effects. Myrhaug et al. [4] presented a method giving the burial and scour depths of short cylinders exposed to long-crested (2D) and short-crested (3D) nonlinear random waves.

Furthermore, to the present authors' knowledge, no works, except for Myrhaug and Ong [8], are published in the open literature related to random wave-induced burial and scour of truncated cones. Details of scour around truncated cones and self-burial of such bodies exposed to combined currents and regular waves are dealt with in Catano-Lopera et al. [9]. They carried out laboratory tests and studied the burial and scour of truncated cones systematically. Related studies are numerical modelling (Jenkins et al. [10]) and field investigations (Guyonic et al. [11], Mayer et al. [12]). Catano-Lopera et al. [9] also reviewed other works related to truncated cones. Myrhaug and Ong [8] presented a stochastic method calculating the burial and scour depths of truncated cones exposed to 2D and 3D nonlinear random waves.

The aim of this study is to provide an analytical method where the random wave-induced burial and scour of short cylinders and truncated cones on mild slopes is derived. The empirical formulae for regular waves on horizontal beds given by Catano-Lopera and Garcia [5, 6] for short cylinders and Catano-Lopera et al. [9] for truncated cones are used; the waves are assumed to be a stationary narrow-band process, and the Battjes and Groenendijk [13] wave height distribution for mild slopes is adopted. The application of the method is demonstrated by providing examples of results.

The present stochastic method is reviewed in Myrhaug and Ong [14], and has recently been extended to the scour caused by 2D and 3D nonlinear random waves; below pipelines (Myrhaug and Ong [15]); below spherical bodies (Myrhaug and Ong [16]); around vertical piles (Myrhaug and Ong [17, 18], Ong et al. [19]); burial and scour of short cylinders (Myrhaug et al. [4]); burial and scour of truncated cones (Myrhaug and Ong [8]); scour around vertical piles due to random waves alone and random waves plus currents on mild slopes (Ong et al. [20]).

BURIAL AND SCOUR IN REGULAR WAVES

Burial and scour of short cylinders

Burial and scour of short (finite length) cylinders exposed to regular waves plus currents at a horizontal bed were investigated in laboratory tests by Catano-Lopera and Garcia [5]. They provided empirical formulae for the equilibrium burial depth B and the equilibrium length L of the scour hole of the short circular cylinder with diameter D and length L_c with cylinder aspect ratios $a_r = L_c / D$ in the range 2 to 4. Figure 1 depicts a definition sketch of the burial depth, the scour lengths and the scour width of the scour hole around a short cylinder on a mild slope. Let Y represent the burial depth B and the length L of the scour hole. For waves alone the formulae can be represented as

$$\frac{Y}{D} = pKC^r\theta^s \tag{1}$$

Here KC is the Keulegan-Carpenter number defined by

$$KC = \frac{UT}{D} \tag{2}$$

where U is the undisturbed linear near-bed orbital velocity amplitude, T is the wave period, and p, r, s are given in Table 1 for the scour characteristics B and L; L represents the downstream length of the scour hole, L_{sd}, and the total length of the scour hole, L_{st}; the upstream length of the scour hole is then $L_{su} = L_{st} - L_{sd}$. Equation (1) is valid for live-bed scour, for which $\theta > \theta_{cr}$ where θ is the undisturbed Shields parameter defined by

$$\theta = \frac{\tau_w}{\rho g(\gamma - 1)d_{50}} \tag{3}$$

where τ_w is the maximum bed shear stress under waves, ρ is the fluid density, g is the acceleration due to gravity, $\gamma = \rho_s / \rho$ is the sediment grain density to fluid density ratio, ρ_s is sediment grain density, d_{50} is the median grain size diameter, and θ_{cr} is the critical value of motion at the bed, i.e. $\theta_{cr} \approx 0.05$. The scour process attains its equilibrium stage through a transition period, and therefore the approach is valid when it is assumed that the storm has lasted longer than the time-scale of the scour.

The maximum bed shear stress in a wave cycle is

$$\frac{\tau_w}{\rho} = \frac{1}{2} f_w U^2 \tag{4}$$

where f_w is the friction coefficient taken as (valid for waves plus current for wave-dominant situations; see Myrhaug et al. [21], Table 3)

$$f_w = c\left(\frac{A}{z_0}\right)^{-d} ; \begin{cases} (c,d) = (18,1) \text{ for } 20 \lesssim A/z_0 \lesssim 200 \\ (c,d) = (1.39, 0.52) \text{ for } 200 \lesssim A/z_0 \lesssim 11000 \\ (c,d) = (0.112, 0.25) \text{ for } 11000 \lesssim A/z_0 \end{cases} \tag{5}$$

Here $z_0 = d_{50}/12$ is the bed roughness.

The KC number can alternatively be expressed by

$$KC = \frac{2\pi A}{D} \tag{6}$$

Here A is related to the linear wave amplitude a by

$$A = \frac{a}{\sinh kh} \tag{7}$$

where h is the water depth, and k is the wave number determined from the dispersion relationship $\omega^2 = gk \tanh kh$.

Furthermore, the width W of the scour hole (see Fig. 1) is in the range 1.3 to 2.5 times the cylinder length L_c with a mean value of $1.8L_c$, regardless of KC.

The main mechanisms causing the burial and scour of a short cylinder were observed and described by Catano-Lopera and Garcia [5, 6] and Demir and Garcia [22]; see these references for more details, together with a summary given by Myrhaug and Ong [3].

For the burial depth Eq. (1) is based on data where $2 \le KC \le 48$; for the lengths of the scour hole Eq. (1) is valid for $2 \le KC \le 71$. Since Eq. (1) appears to be physically sound for $KC \ge 0$, i.e., B and L equal zero for $KC = 0$, the formulae are taken to be valid from $KC = 0$. However, this extension of Eq. (1) relies on that the threshold of sediment motion is exceeded, which for small values of KC may not be the case.

Burial and scour of truncated cones

Burial and scour of conical frustums, or truncated cones (hereafter referred to as cones) at a horizontal bed exposed to regular waves plus currents were investigated in laboratory tests by Catano-Lopera et al. [9]. Figure 2 depicts a definition sketch of the burial depth, the scour width and the scour lengths of the scour hole around a truncated cone on a mild slope. For waves alone Catano-Lopera et al. [9] obtained empirical formulae represented by Eq. (1). For the equilibrium relative burial depth B_d of the cone with height h_c, base diameter D_b and top diameter D_t, with $D_t / D_b = 0.5$ (see Fig. 2), the results are given by Eq. (1) for $Y = B_d$ and $D = h_c$; KC is defined as in Eq. (2) with D as a typical diameter, taken as the average, i.e., $D = (D_b + D_t)/2$; θ is as defined in Eq. (3); p, r, s are given in Table 1 for B_d and for the geometric dimensions of the equilibrium scour hole; the scour hole width W_s, the relative downstream length L_{sd}, and the total scour hole length L_{st} (see Fig. 2); the relative upstream length of the scour hole is then $L_{su} = L_{st} - L_{sd}$.

The main mechanisms of the burial and scour process for a truncated cone were observed and described by Catano-Lopera et al. [9]; see this reference for more details, together with a summary given by Myrhaug and Ong [8].

The results for the burial and scour of cones are based on data where $1.5 \le KC \le 16$ and $0.04 \le \theta \le 0.34$. Since Eq. (1) appears to be physically sound for $KC > 0$, i.e., the scour characteristics equal zero for $KC = 0$, the formulae can be taken to be valid from $KC = 0$. However, this extension of Eq. (1) relies on that the threshold of sediment motion is exceeded, which for small values of KC may not be the case. Also in this case the scour process attains its equilibrium stage through a transition period, and therefore the approach is valid when it is assumed that the storm has lasted longer than the time scale of the scour.

Table 1. Summary of the scour responses *Y* and the object dimensions *D* for waves alone and the coefficients in Eq. (27) for *d* = 0.25.

Y	D	p	r	s	t
Short cylinder					
B	D	0.24	0.4	0.4	1.1
L_{sd}	D	0.75	0.56	0	0.56
L_{st}	D	$0.75a_r^{0.3}$	0.6	0	0.6
Truncated cone					
B_d	h_c	0.0125	0.8	0.1	0.975
W_c	$(D_b+D_t)/2$	1.03	0.1	0.05	0.1875
L_{sd}	$(D_b+D_t)/2$	0.43	0.46	0	0.46
L_{st}	$(D_b+D_t)/2$	0.80	0.34	0	0.34

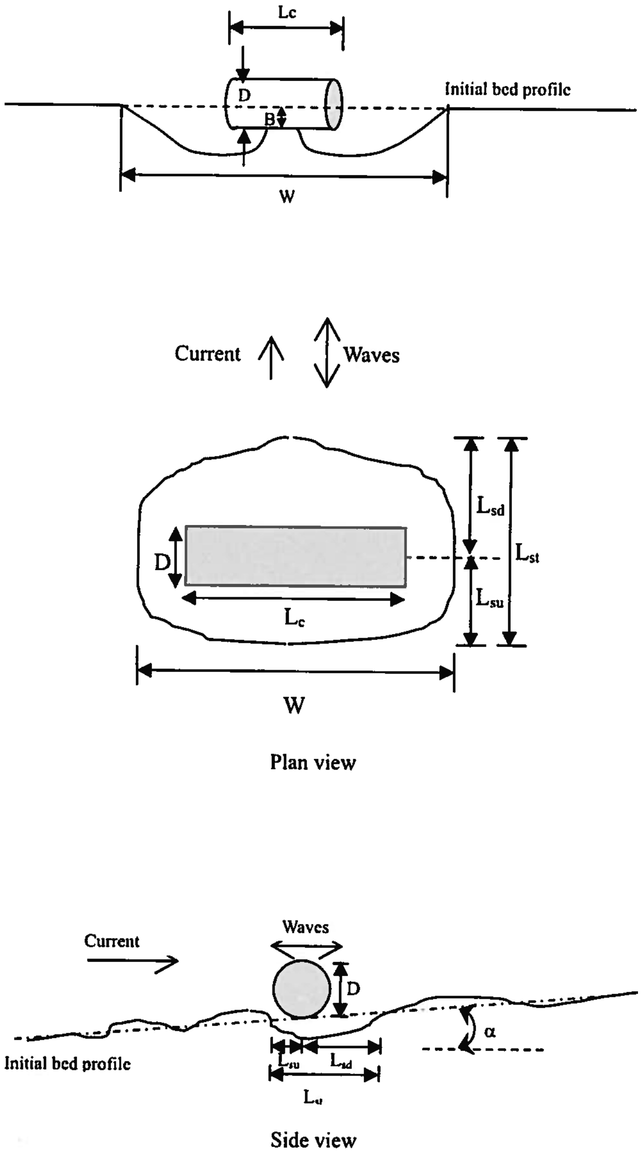

Figure 1. Definition sketch of the burial depth (*B*), the scour lengths (*Lsd*, *Lst*, *Lsu*) and the scour width (*W*) of the scour hole around a short cylinder on a mild slope (where *α* is the bed slope).

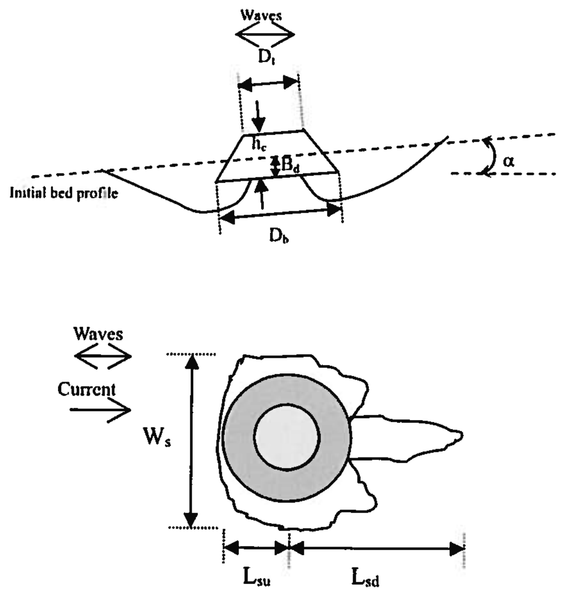

Figure 2. Definition sketch of the burial depth (B_d), the scour width (W_s) and the scour lengths (L_{sd}, L_{su}) of the scour hole around a truncated cone on a mild slope (where α is the bed slope).

BURIAL AND SCOUR IN RANDOM WAVES ON MILD SLOPES

A tentative stochastic approach will be outlined following the one given in Myrhaug et al. [4] and Myrhaug and Ong [8], except for the modification associated with that for mild slopes by adopting the Battjes and Groenendijk [13] wave height distribution. As a first approximation it is assumed that the scour characteristic formulae valid for a horizontal bed given in the previous section can be applied for mild slopes as well.

Theoretical background

At a given location in a sea state with stationary narrow-band random waves consistent with regular linear waves in finite water depth h, the near-bed orbital displacement amplitude, A, and the near-bed horizontal orbital velocity amplitude, U, can be taken as, respectively,

$$A = \frac{H}{2\sinh k_p h} \tag{8}$$

$$U = \omega_p A = \frac{\omega_p H}{2\sinh k_p h} \tag{9}$$

where $\omega_p = 2\pi / T_p$ is the spectral peak frequency, T_p is the spectral peak period, and k_p is the wave number corresponding to ω_p determined from the dispersion relationship

$$\omega_p^2 = g k_p \tanh k_p h \tag{10}$$

Furthermore, A and U are made dimensionless by defining $\hat{A} = A / A_{rms}$ and $\hat{U} = U / U_{rms}$, respectively, where

$$A_{rms} = \frac{H_{rms}}{2\sinh k_p h} \tag{11}$$

$$U_{rms} = \omega_p A_{rms} = \frac{\omega_p H_{rms}}{2\sinh k_p h} \tag{12}$$

Combination of Eqs. (8), (9), (11) and (12) gives

$$\omega_p = \frac{U}{A} = \frac{U_{rms}}{A_{rms}} \tag{13}$$

and consequently

$$\frac{U}{U_{rms}} = \frac{A}{A_{rms}} = \frac{H}{H_{rms}} \tag{14}$$

The Battjes and Groenendijk [13] parametric wave height distribution based on laboratory experiments on shallow foreshores is adopted; the cumulative distribution function (*cdf*) is composed of two two-parameter Weibull distributions of the non-dimensional wave height $\hat{H} = H / H_{rms}$:

$$P(\hat{H}) = \begin{cases} P_1(\hat{H}) = 1 - \exp[-(\frac{\hat{H}}{\hat{H}_1})^{k_1}] \; ; \; \hat{H} < \hat{H}_{tr} \\ P_2(\hat{H}) = 1 - \exp[-(\frac{\hat{H}}{\hat{H}_2})^{k_2}] \; ; \; \hat{H} \geq \hat{H}_{tr} \end{cases} \tag{15}$$

where $k_1 = 2$, $k_2 = 3.6$, $\hat{H}_1 = H_1 / H_{rms}$, $\hat{H}_2 = H_2 / H_{rms}$, $\hat{H}_{tr} = H_{tr} / H_{rms}$. Here H_{tr} is the transitional wave height corresponding to the change of wave height where there is a change of the distribution associated with depth-induced wave breaking, given by

$$H_{tr} = (0.35 + 5.8\tan\alpha)h \tag{16}$$

where α is the slope angle, and H_{rms} is related to the zeroth spectral moment m_0 by

$$H_{rms} = (2.69 + 3.24\sqrt{m_0} / h)\sqrt{m_0} \tag{17}$$

The values of H_1 and H_2 can either be read from Table 2 in Battjes and Groenendijk [13], or found by an iteration procedure solving two equations (see their paper for more details). The model is a so-called point model, i.e. depending on local parameters regardless of the history of the waves in deeper water. It should be noted that the effect of the bottom slope is of a secondary nature compared to the effect of water depth (more details are given in Battjes and Groenendijk [13]).

The zeroth spectral moment is

$$m_0 = \int_0^{\infty} S(\omega,h)d\omega \quad (18)$$

where $S(\omega,h)$ is the wave spectrum in finite water depth, obtained by multiplying the deep water wave spectrum $S(\omega)$ with a depth-correction factor $\psi(\omega,h)$ as

$$S(\omega,h) = \psi(\omega,h)S(\omega) \quad (19)$$

which, according to Young [23]

$$\psi(\omega,h) = \frac{[k(\omega,h)]^{-3}\partial k(\omega,h)/\partial\omega}{\{[k(\omega,h)]^{-3}\partial k(\omega,h)/\partial\omega\}_{kh\to\infty}} \quad (20)$$

ensuring that the frequency part of the wave spectrum becomes proportional to k^{-3} irrespectively of the water depth (see Young [23] for more details). From Eq. (20) it follows that (see Jensen [24])

$$\psi(\omega,h) = \frac{\omega^6}{(gk)^3[\tanh kh + kh(1-\tanh^2 kh)]} \quad (21)$$

In shallow water $(kh \ll 1)$, Eq. (21) becomes

$$\psi(\omega,h) = \frac{\omega^2 h}{2g} \quad (22)$$

Thus, by combining Eqs. (19) and (22), the shallow water wave spectrum is

$$S(\omega,h) = \frac{\omega^2 h}{2g}S(\omega) \quad (23)$$

Combining Eqs. (18) and (23) gives

$$m_0 = \frac{h}{2g}\int_0^{\infty}\omega^2 S(\omega)d\omega \quad (24)$$

Thus, it follows that m_{0h1} and m_{0h2} at two different water depths h_1 and h_2 in shallow water are related by

$$m_{0h2} = \frac{h_2}{h_1}m_{0h1} \quad (25)$$

Outline of stochastic method

Now, consider the highest among random waves in a stationary narrow-band sea-state, since it is reasonable to assume that it is mainly the highest waves which cause the scour response. The sea-state is also assumed to last long enough to develop the equilibrium scour depth. The highest waves considered here are those exceeding the probability $1/n$, $\hat{H}_{1/n}$ (i.e., $1-P(\hat{H}_{1/n})=1/n$). The parameter of interest is the expected (mean) value of the maximum equilibrium scour characteristics caused by the $(1/n)$ th highest waves given as

$$E[Y(\hat{H})\,|\,\hat{H} > \hat{H}_{1/n}] = n\int_{\hat{H}_{1/n}}^{\infty} Y(\hat{H})p(\hat{H})d\hat{H} \quad (26)$$

Here Y represents the scour characteristics, and $p(\hat{H})$ is the probability density function (*pdf*) of $\hat{H}$. It should be noted that the present approach assumes that: (1) the free surface elevation is a stationary narrow-band process with zero expectation, and (2) the scour response formulae for regular waves given in the previous section (Eq. (1)), are valid for irregular waves as well. Overall, these assumptions are the same as in e.g. Myrhaug et al. [25], where more details are given.

For a narrow-band process $T = T_p$ where $T_p = 2\pi/\omega_p = 2\pi A_{rms}/U_{rms}$ and where Eq. (13) has been used. By substituting this in Eq. (1) using Eqs. (2) to (5) and using Eq. (14), Eq. (1) can be re-arranged to give the scour characteristics for individual narrow-band random waves as

$$y \equiv \frac{Y/D}{pKC_{rms}^{r}\theta_{rms}^{s}} = \hat{H}^t; t = r + s(2-d) \quad (27)$$

where

$$KC_{rms} = \frac{U_{rms}T_p}{D} = \frac{2\pi A_{rms}}{D} \quad (28)$$

$$\theta_{rms} = \frac{\frac{1}{2}c(\frac{A_{rms}}{z_0})^{-d}U_{rms}^2}{g(\gamma-1)d_{50}} \quad (29)$$

Here KC_{rms} and θ_{rms} are uniquely defined for given values of U_{rms}, A_{rms} and T_p.

The mean of the maximum scour characteristics caused by the $(1/n)$ th highest waves is obtained from Eqs. (26) and (27) as

$$E[y(\hat{H})\,|\,\hat{H} > \hat{H}_{1/n}] = n\int_{\hat{H}_{1/n}}^{\infty} \hat{H}^t p(\hat{H})d\hat{H} \quad (30)$$

where $p(\hat{H}) = dP(\hat{H})/d\hat{H}$ with $P(\hat{H})$ as given in Eq. (15). The integral in Eq. (30) can be evaluated by using the results given in Abramowitz and Stegun ([26], Chs. 6.5 and 26.4) as

1. for $\hat{H}_{1/n} < \hat{H}_{tr}$

$$E[y(\hat{H})\,|\,\hat{H} > \hat{H}_{1/n}] = n\int_{\hat{H}_{1/n}}^{\hat{H}_{tr}} \hat{H}^t p_1(\hat{H})d\hat{H} + n\int_{\hat{H}_{tr}}^{\infty} \hat{H}^t p_2(\hat{H})d\hat{H}$$
$$= n\hat{H}_1^t\left\{\Gamma[1+\frac{t}{k_1},(\frac{\hat{H}_{1/n}}{\hat{H}_1})^{k_1}] - \Gamma[1+\frac{t}{k_1},(\frac{\hat{H}_{tr}}{\hat{H}_1})^{k_1}]\right\} \quad (31)$$
$$+ n\hat{H}_2^t\Gamma[1+\frac{t}{k_2},(\frac{\hat{H}_{tr}}{\hat{H}_2})^{k_2}]$$

2. for $\hat{H}_{1/n} > \hat{H}_{tr}$

$$E[y(\hat{H}) \mid \hat{H} > \hat{H}_{1/n}] = n\int_{\hat{H}_{1/n}}^{\infty} \hat{H}p_2(\hat{H})d\hat{H}$$
$$= n\hat{H}_2^t\Gamma\,[1+\frac{t}{k_2},(\frac{\hat{H}_{1/n}}{\hat{H}_2})^{k_2}] \qquad (32)$$

Here $\Gamma(\bullet,\bullet)$ is the incomplete gamma function. Note that $\hat{H}_{1/n}$ for $n = 3,10,50,100,1000$, are given in Battjes and Groenendijk ([13], Table 2).

A commonly used procedure for predicting random wave-induced burial and scour characteristics, is to use characteristic statistical values of the wave-related quantities in an otherwise deterministic approach, e.g. to substitute $\hat{H} = E[\hat{H}_{1/n}]$ in Eq. (27). This will be exemplified in the next section.

EXAMPLES OF RESULTS

To the present authors' knowledge no data exist in the open literature for random wave-induced burial and scour of short cylinders and truncated cones on mild slopes. Therefore examples of calculation for these conditions based on the results in the previous section are given. One should note that the formulation in the previous section is general, i.e. valid for a finite water depth. However, here the shallow water approximation is used to serve the purpose of illustrating the method for practical purposes using data typical for field conditions.

The given flow conditions are:

- The water depths h = 5 m, 3.5 m, 2 m on the slope α = 1/100, and the water depths h = 5 m, 2 m on the slope α = 1/50 (see Fig. 3)
- Zeroth spectral moment at h = 5 m, $m_0 = 0.0625$ m^2
- Spectral peak period $T_p = 8.2$ s, corresponding to $\omega_p = 2\pi / T_p = 0.766$ rad/s
- k_p at $h = 5$ m from the shallow water dispersion relationship corresponding to ω_p, $k_p = \omega_p / \sqrt{gh}$ = 0.109 rad/m (which is 4.4 percent smaller than the finite water depth value; justifying the shallow water approximation used in this example).
- Median grain diameter (median sand/coarse sand according to Soulsby ([26], Fig. 4) $d_{50} = 0.5$ mm
- $\gamma = 2.65$ (as for quartz sand)
- Short cylinder diameter, $D = 0.5$ m
- Cylinder aspect ratio, $a_r = L_c / D = 4$
- Cone height, $h_c = 0.5$ m
- Cone base diameter, $D_b = 0.67$ m
- Cone top diameter, $D_t = 0.33$ m

The calculated quantities are given in Table 2. The results are exemplified for $n = 10$. Now m_0 at $h = 2$ m is obtained from Eq. (25); A_{rms} / z_0 (i.e., $z_0 = d_{50} / 12$) exceeds 11 000 and thus $(c,d) = (0.112, 0.25)$. Moreover, θ_{rms} exceeds the critical Shields parameter $\theta_{cr} \approx 0.05$, i.e., live-bed conditions. Note that the critical Shields parameter for a sloping bed is different from the horizontal bed value used here. However, for the mild slopes of 1/100 and 1/50 the present results are still valid since the critical Shields parameter is obtained as 0.051 and 0.052, respectively, by using the results in Soulsby ([27], Ch. 6.4). Now H_{tr} is obtained from Eq. (16); $\hat{H}_1, \hat{H}_2$ and $\hat{H}_{1/10}$ are read from Battjes and Groenendijk ([13], Table 2). For the slope 1/100 it appears that $\hat{H}_{1/10} < \hat{H}_{tr}$ for both h = 5m and h = 3.5 m, and thus the scour characteristics for the short cylinder and the cone are calculated from Eqs. (27), (31), Table 1. At $h = 2$ m for the slope 1/100 it appears that $\hat{H}_{1/10} > \hat{H}_{tr}$, and thus the burial and scour characteristics for the short cylinder and the cone are calculated from Eqs. (27), (32), Table 1. For the slope $\alpha = 1/50$ it appears that $\hat{H}_{1/10} < \hat{H}_{tr}$ for both $h = 5$ m and $h = 2$ m, and consequently the burial and scour characteristics are calculated from Eqs. (27), (31), Table 1. The deterministic values of the burial and scour characteristics are calculated by substituting $\hat{H} = E[\hat{H}_{1/10}]$ in Eq. (27).

Overall, it appears that the effects of the water depth and the slope are small, but as demonstrated by the examples the features of the maximum equilibrium burial and scour characteristics caused by the $(1/10)$th highest waves are: First, for both slopes the values are larger at $h = 2$ m than at $h = 5$ m due to the shoaling of waves, and dominated by the increase of KC_{rms} and θ_{rms}. Second, the effect of increasing the slope α from 1/100 to 1/50 is insignificant at $h = 5$m, but the values are slightly increased at h = 2 m. Third, the effect of changing the slope at a given horizontal location, e.g. by changing the water depth from h = 3.5 m on the slope $\alpha = 1/100$ to $h = 2$ m on the slope $\alpha = 1/50$, is to increase the values, which is dominated by the increase of KC_{rms} and θ_{rms} (see Fig. 3 and Table 2). Fourth, in most of these examples the stochastic method gives values which are slightly smaller than those obtained by the deterministic method. However, overall a stochastic method should be used since it takes the stochastic features into account in a consistent manner compared with what the deterministic method does.

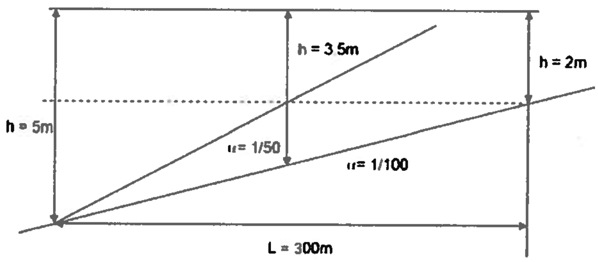

Figure 3. Definition sketch of mild slope conditions with bed slopes α = 1/100 and 1/50.

SUMMARY

This paper provides an analytical stochastic method calculating the random wave-induced maximum equilibrium burial and scour characteristics around short cylinders and truncated cones on mild slopes. The formulation is valid for finite water depths, but examples of application are based on the shallow water approximation representing realistic field conditions serving the purpose of illustrating the method for practical purposes.

Although simple, this is a practical tool which should be useful as a first approximation to represent the stochastic features of the random wave-induced maximum equilibrium burial and scour characteristics around short cylinders and truncated cones on mild slopes. However, comparisons with data are required before a conclusion regarding the validity of this method can be given. In the meantime it should be useful as an engineering tool for the assessment of random wave-induced burial and scour around short cylinders and truncated cones on mild slopes based on available wave statistics.

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Table 2. Example of calculations with h = 2 m, 5 m, d_{50} = 0.5 mm, D = 0.5 m, h_c = 0.5 m, slopes 1/100 and 1/50, $(D_b + D_t)/2$ = 0.5 m, n = 10.

	h = 5 m		h = 2 m		h = 3.5 m
	1/100	1/50	1/100	1/50	1/100
$m_0(m^2)$	0.0625		0.025		0.0438
$H_{rms}(m)$, Eq. (17)	0.71		0.47		0.61
$k_p h$	0.547		0.346		0.458
$A_{rms}(m)$, Eq. (11)	0.65		0.68		0.67
$U_{rms}(m/s)$, Eq. (12)	0.50		0.52		0.51
A_{rms}/z_0	15600		16320		16080
(c,d), Eq. (5)	(0.112, 0.25)		(0.112,0.25)		(0.112,0.25)
KC_{rms}, Eq. (28)	8.2		8.5		8.4
θ_{rms}, Eq. (29)	0.155		0.166		0.160
$\hat{H}_{tr} = H_{tr}/H_{rms}$, Eq. (16)	2.87	3.28	1.74	1.98	2.34
$\hat{H}_1 = H_1/H_{rms}$	1.0	1.0	1.011	1.004	1.001
$\hat{H}_2 = H_2/H_{rms}$	1.60	1.63	1.287	1.357	1.459
$E[\hat{H}_{1/10}]$	1.800	1.800	1.757	1.785	1.797
$\hat{H}_{1/10}$	1.517	1.517	1.534	1.523	1.519
STOCHASTIC					
Short cylinder					
$B(m)$	0.252	0.252	0.256	0.260	0.257
$L_{sd}(m)$	1.689	1.688	1.703	1.714	1.711
$L_{st}(m)$	2.851	2.850	2.876	2.895	2.890
Truncated cone					
$B_d(m)$	0.0496	0.0495	0.501	0.0508	0.0505
$W_s(m)$	0.646	0.646	0.648	0.648	0.648
$L_{sd}(m)$	0.740	0.740	0.745	0.748	0.747
$L_{st}(m)$	0.996	0.996	1.002	1.004	1.004
DETERMINISTIC					
Short cylinder					
$B(m)$	0.252	0.252	0.256	0.260	0.257
$L_{sd}(m)$	1.693	1.693	1.705	1.720	1.715
$L_{st}(m)$	2.858	2.858	2.878	2.906	2.897
Truncated cone					
$B_d(m)$	0.0495	0.0495	0.0501	0.0509	0.0506
$W_s(m)$	0.647	0.647	0.648	0.650	0.649
$L_{sd}(m)$	0.742	0.742	0.746	0.751	0.749
$L_{st}(m)$	1.016	0.999	1.003	1.008	1.007

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