# Discussion/comments regarding "Sea spray icing phenomena on marine vessels and offshore structures: Review and formulation" by A.R. Dehghani-Sanij, S.R. Dehghani, G.F. Naterer, Y.S. Muzychka

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## Abstract

The purpose of this discussion is to demonstrate how the formula for the liquid water content in wind-generated seawater spray presented by Dehghani-Sanij et al. (2017) in conjunction with available wind speed statistics can be used to estimate the wind-generated seawater spray. Results are exemplified based on mean wind speed statistics for the Northern North Sea and the North Atlantic.

**Keywords:** Icing phenomena; Wind-generated seawater spray; Mean wind speed; Wind statistics

# 1. Discussion

The paper by Dehghani-Sanij et al. (2017) (hereafter referred to as D-S17) made a thorough review of sea spray icing phenomena on marine vessels and offshore structures, including formulations of marine icing models, where the liquid water content (LWC) in wind-generated seawater spray represents an important part of the modelling.

One of the formulae presented by D-S17 for estimating LWC in wind-generated seawater spray was that proposed by Horjen (1983) (i.e. given by Eqs. (16), (17) and (18) in D-S17) (with dimension kg/m$^3$)

$$w(z)=\frac{C}{z^2}A(V)H_s(V) \tag{1}$$

$$A(V)=A_0+A_1V+A_2V^2+A_3V^3 \tag{2}$$

$$H_s(V)=B_0+B_1V+B_2V^2+B_3V^3 \tag{3}$$

where $C=6.3185\times10^{-5}$, $V\equiv U_{10}$ is the mean wind speed 10 m above the sea surface, $z$ is the elevation above the sea surface, $H_s$ is the significant wave height, $A_n$ and $B_n$ are coefficients given by

$$(A_0,A_1,A_2,A_3)=(-53.5173,11.3119,-0.7934,0.01864) \tag{4}$$

$$(B_0,B_1,B_2,B_3)=(1.28311,-2.26480\times10^{-2},4.19756\times10^{-2},-6.05377\times10^{-4}) \tag{5}$$

It should be noted that the coefficients in Eq. (4) are valid for $V$ larger than about 12 m/s (see Fig. 3 in Itagaki (1984)), and that the coefficients in Eq. (5) correspond to the fetch 300 n.m. (see Table 2 in D-S17). Furthermore, the coefficients in Eq. (1) are valid for $V$ up to 32.4 m/s; see D-S17 for more details. Thus, according to Eqs. (1) to (5), *w(z)* will depend only on *V*. Therefore *w(z)* can be obtained from known wind statistics for an ocean area of concern.

Parametric models for the cumulative distribution function *(cdf)* (or the probability density function *(pdf)*) of $V$ are available in the literature; see e.g. Bitner-Gregersen (2015) for a recent review. In the present discussion results are exemplified by using two *cdfs* of $V$; from Johannessen et al. (2001) and Mao and Rychlik (2016), respectively. The Johannessen et al. (2001) *cdf* is based on 1 hourly values of $V$ from wind measurements covering the years 1973 – 1999 from the Northern North Sea (NNS); see the reference for more details. The Mao and Rychlik (2016) *cdfs* represent wind speed along different ship routes in the North Atlantic (NA) fitted to 10 years of wind speed data; the *cdf* used here are from the location 20°W 60°N (South of Iceland) (see the reference for more details). These *cdfs* of $V$ are given by the two-parameter Weibull model

$$P(V) = 1 - \exp\left[-\left(\frac{V}{\theta}\right)^{\beta}\right] \quad ; V \geq 0 \tag{6}$$

with the Weibull parameters $\theta$ and $\beta$ as

$$\text{NNS: } \theta = 8.426\,\text{m/s},\ \beta = 1.708 \tag{7}$$

$$\text{NA}(20^{\circ}\text{W}\,60^{\circ}\text{N})\text{: } \theta = 10.99\ \text{m/s},\ \beta = 2.46 \tag{8}$$

According to Eqs. (4) and (5), $V$ is defined for $V_1 = 12\,\text{m/s} \leq V \leq V_2 = 34.2\ \text{m/s}$, and thus $V$ follows the truncated Weibull *cdf* :

$$P_t(V) = \frac{1}{N}\left\{\exp[-(\frac{V_1}{\theta})^{\beta}] - \exp[-(\frac{V}{\theta})^{\beta}]\right\};\ V_1 \leq V \leq V_2 \tag{9}$$

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$$N = \exp\left[-(\frac{V_1}{\theta})^{\beta}\right] - \exp\left[-(\frac{V_2}{\theta})^{\beta}\right] \tag{10}$$

Now statistical quantities of *w(z)* can be calculated based on the *cdfs* of $V$; here results are exemplified by the expected (mean) value of *w(z)* :

$$E\left[w(z)\right]=\int_{0}^{\infty} w(z)\, p_t(V)dV \tag{11}$$

where $p_t(V)$ is the truncated *pdf* of $V$ given by $p_t(V)=dP_t(V)/dV$, $P_t(V)$ is given in Eqs. (9) and (10), and *w(z)* as given in Eqs. (1) to (3). Then the integral in Eq. (11) can be evaluated analytically as outlined in the following.

First, combination of Eqs. (1) to (3) gives

$$w(z)=\frac{C}{z^2}(c_0+c_1V+c_2V^2+c_3V^3+c_4V^4+c_5V^5+c_6V^6) \tag{12}$$

$$c_0=A_0B_0 \tag{13}$$

$$c_1=A_0B_1+A_1B_0 \tag{14}$$

$$c_2=A_0B_2+A_1B_1+A_2B_0 \tag{15}$$

$$c_3=A_0B_3+A_1B_2+A_2B_1+A_3B_0 \tag{16}$$

$$c_4=A_1B_3+A_2B_2+A_3B_1 \tag{17}$$

$$c_5=A_2B_3+A_3B_2 \tag{18}$$

$$c_6=A_3B_3 \tag{19}$$

Second, by combining Eqs. (11) and (12) the analytical results are obtained, requiring the calculation of $E[V^n]$, by using the formulae in Abramowitz and Stegun (1972, Chs. 6.5 and 26.4):

$$E[V^n] = \int_{V_1}^{V_2} V^n p_t(V)\, dV = \frac{\theta^n}{N}\left\{\Gamma[1+\frac{n}{\beta}, (\frac{V_1}{\theta})^\beta] - \Gamma[1+\frac{n}{\beta}, (\frac{V_2}{\theta})^\beta]\right\} \tag{20}$$

Here $\Gamma(r,s)$ is the incomplete gamma function and $n$ is a real number (not necessarily an integer); $\Gamma(r,0)=\Gamma(r)$ where $\Gamma$ is the gamma function, and $\Gamma(r,\infty)=0$.

Then, by using Eqs. (11) to (20) the results are

$$\text{NNS: } E[w(z)] = 1.59\cdot 10^{-3}\cdot z^{-2} \tag{21}$$

$$\text{NA: } E[w(z)] = 8.37\cdot 10^{-4}\cdot z^{-2} \tag{22}$$

Thus, in this example the LWC in the wind-generated seawater spray is a factor 1.9 larger in the Northern North Sea than at the North Atlantic location.

An alternative to the presented stochastic method is to estimate the LWC in wind-generated spray by using a deterministic method, i.e. to replace $V^n$ with $(E[V])^n$ in Eq. (12). For the NNS and NA data the values of $E[V]$ are 15.29 m/s and 14.97 m/s, respectively, giving the deterministic method values

$$\text{NNS: } w(z) = 3.19\cdot 10^{-4}\cdot z^{-2} \tag{23}$$

$$\text{NA: } w(z) = 2.91\cdot 10^{-4}\cdot z^{-2} \tag{24}$$

Thus, the deterministic to stochastic method ratios of *w(z)* are obtained as 0.20 for NNS and 0.35 for NA. This suggests that a stochastic method should be used.

The present method can also be applied using the fifth-degree polynomial of $H_s(V)$ in Eq. (19) in D-S17, as well as using the other formulae given by the authors. However, in those cases the integral in Eq. (11) might need to be evaluated by numerical integration.

Overall, the present results demonstrate how the LWC in wind-generated sea spray can be estimated using the formulae presented by the authors together with available mean wind speed statistics.

## References

Abramowitz, M., Stegun, I.A, 1972. *Handbook of Mathematical Functions.* Dover, New York.

Bitner-Gregersen, E.M., 2015. Joint met-ocean description for design and operations of marine structures. *Appl. Ocean Res.* 51, 279-292.

Dehghani-Sanij, A.R., Dehghani, S.R., Natarer, G.F., Muzychka, Y.S., 2017. Sea spray icing phenomena on marine vessels and offshore structures: Review and formulation. *Ocean Eng.* 132, 25-39.

Horjen, I., 1983. *Mobile Platform Stability. MOPS Subproject 02-Icing, MOPS Report No. 7,* Norwegian Hydrodynamic Laboratories, NHL 283021, Trondheim, Norway.

Itagaki, K., 1984. Icing rate on stationary structures under marine conditions. *US Army Corps of Engineers, Cold Regions Research and Engineering Laboratory, CRREL Report 84-12,* Hanover, New Hampshire.

Johannessen, K., Meling, T.S., Haver, S., 2001. Joint distribution for wind and waves in the Northern North Sea. In: *Proc. 11th International Offshore and Polar Engineering Conf.*, Vol. III, Stavanger, Norway, 19-28.

Mao, W., Rychlik, I., 2016. Estimation of Weibull distribution for wind speeds along ship routes. *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment,* doi: 10.1177/1475090216653495.