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Econometric analysis of the determinants of electricity wholesale prices in Switzerland and Germany

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Zusammenfassung

Die wesentlichen Ergebnisse dieses Projekts sind einerseits eine methodische Innovation spezifischer Schätzverfahren und andererseits eine empirische Analyse der Schweizer und deutschen Strompreise.

Aus methodischer Sicht wird hier ein neuartiges Schätzverfahren zur Lösung von Quantils-Regressionsmodellen mit zeitvariablen Koeffizienten vorgestellt, basierend auf einem parametrischen Ansatz in einer multifaktoriellen Modellspezifikation. Mit Hilfe dieses Ansatzes lassen sich die zeitvariablen Koeffizienten rekursiv mit einem Kalman-Filter durch Maximum-Likelihood schätzen. Wegen der Nicht-Differenzierbarkeit der Likelihood-Funktion wird das Schätzproblem zunächst umformuliert zu einem nicht-linearen restringierten Optimierungsproblem und dann nach Relaxation der Nebenbedingungen mit der Augmented Langrangian Method gelöst. Dieser allgemeine Ansatz ist nützlich für viele Anwendungen im Risikomanagement und der Schätzung von Quantilen, bei denen komplexe dynamische Beziehungen bei der Preisbildung und plausible exogene Einflussfaktoren vorliegen.

Mit dem so hergeleiteten dynamischen Modell werden die Quantile der Strompreise als nicht-lineare Funktion von Fundamentalvariablen beschrieben: Brennstoffpreise, erneuerbare Energien (Wind und Photovoltaik) und Verhalten der Marktteilnehmer. Durch genaue Berücksichtigung der Form der Angebotsfunktion mit ihren konkaven, flachen und konvexen Abschnitten in Verbindung mit jenen Informationen über Brennstoffpreise und Einspeisung erneuerbarer Energien, die den Marktteilnehmern jeweils am Vortag zur Verfügung stehen, ergeben sich plausible Vorhersagen von Strompreis-Quantilen. Insbesondere lässt sich belegen, dass jene Fundamentalvariablen, welche üblicherweise deutsche Strompreise beeinflussen, grenzüberschreitend externe Effekte auf Schweizer Preise haben und bis zu 80% ihrer Variabilität erklären. Die Ergebnisse zeigen eine Preisanpassung an Markt-Fundamentaldaten im Zeitverlauf, je nach Lage des Schnittpunkts von Angebots- und Nachfragekurve zu einer bestimmten Stunde des Tages. Diese Preisanpassung resultiert aus einem Substitutionseffekt zwischen einzelnen Brennstoffen (Kohle/Gas) oder zwischen traditionellen Brennstoffen und unbeständiger Einspeisung aus erneuerbaren Energien.

Insgesamt importiert die Schweiz im Zeitverlauf günstigere Strompreise, zu einem grossen Teil wegen des Ausbaus erneuerbarer Energien im Nachbarland Deutschland. So fanden sich zeitvariable negative marginale Effekte von Wind und Photovoltaik auf Schweizer Strompreise für jede Tageszeit und jeden Wochentag. Dieses Ergebnis ist von grosser Bedeutung für die Schweizer Energiepolitik hinsichtlich der Gesetzgebung zu erneuerbaren Energien: Die Schweiz importiert tiefere Strompreise aufgrund der Energiewende in Deutschland. Insbesondere verengten sich im Zeitverlauf die Swissix-Preisdifferenzen zwischen Grund- und Spitzenlaststunden aufgrund der Einspeisung von Photovoltaik zur Spitzenlastzeit signifikant aufgrund der Vernetzung mit Deutschland. Weitere Anreize für Investitionen in erneuerbare Energien in der Schweiz sollten unter diesen Gesichtspunkten betrachtet werden.

Die Analyse wurde auf die Preise der Viertelstundenprodukte am Intraday-Markt für Deutschland erweitert. Dazu stand ein spezifischer Datensatz aus Intraday-Preisen kombiniert mit im Tagesverlauf aktualisierten Prognosen für Wind- und Photovoltaik-Einspeisung zur Verfügung. Die Preisgebote wurden modelliert durch die aktuell verfügbaren Informationen über die Fundamentalvariablen. Das Modell unterscheidet den Einfluss von Markt-Fundamentalfaktoren auf Preise in Abhängigkeit des vorherrschenden Regimes der Nachfragequote sowie abhängig von der Tageszeit. Die für den Markt relevanten Fundamentalvariablen beeinflussen stärker

die Gebote zur Tagesmitte als morgens oder abends. Es zeigt sich eine asymmetrische Anpassung der Strompreise sowohl bezüglich Handelsvolumen als auch Prognosefehler der Einspeisung aus erneuerbaren Energien. Im Regime hoher Nachfragequoten, wenn im Day-Ahead-Markt eine zu geringe Menge konventioneller Kapazität vermarktet wurde, reagieren Händler schneller auf neue Prognosen zur Einspeisung aus erneuerbaren Energien. Somit ist der historisch geschätzte Schwellenwert für die Nachfragequote einer spezifischen Lieferperiode eine wesentliche Information für das strategische Bieterverhalten am Intraday-Markt. Durch Vergleich der beobachteten Nachfragequote mit dem historischen Schwellenwert und in Abhängigkeit des am Markt vorherrschenden Regimes (hohe/niedrige Quote) können Marktteilnehmer das Modell für Preisvorhersagen nutzen.

Zusammenfassung der wesentlichen Ergebnisse:

- Markt-Fundamentalfaktoren, welche traditionell Strompreise in Deutschland beeinflussen, zeigen grenzüberschreitende Wirkung auf den benachbarten Schweizer Markt.
- Swissix-Preise passen sich kontinuierlich an Brennstoffpreise, Einspeisung erneuerbarer Energien, Verhalten der Marktteilnehmer sowie Angebots- und Nachfragekurven, welche spezifisch für den deutschen Markt sind, an.
- Der Swissix gleicht sich an das aufgrund kohlebasierter Produktion und Einspeisung aus erneuerbaren Energien im Nachbarland tiefere deutsche Preisniveau an.
- Verschiedene Preisquantile reagieren unterschiedlich auf Fundamentaldaten.
- Die übliche Methodik zur Lösung von Quantils-Regressionsmodellen mit einem nicht-parametrischen Ansatz wird zu einem parametrischen Verfahren erweitert.

Résumée

La version française sera présentée plus tard.

Abstract

The main results of this project consist of methodological innovation and empirical analysis of the Swiss and German electricity prices.

We innovated methodologically by introducing a novel estimation procedure for solving quantile regressions with time-varying coefficients based on a fully parametric approach in a multi-factor specification. A novel general methodology has therefore been developed in which time-varying multifactor coefficients are recursively estimated with a Kalman filter using maximum likelihood. Since the likelihood function is non-differentiable, the problem is reformulated as a non-linear optimization problem with constraints, and furthermore reformulated again by moving the constraints into the objective function and solved with the augmented Lagrangian method. As a general approach, we would expect this to be useful in many applications of risk management and quantile estimation where there is dynamic complexity in price formation and plausible exogenous price drivers.

The derived dynamic model has been used to describe electricity price quantiles as a nonlinear function of market fundamentals: fuel prices, renewable energies (wind and photovoltaic) and participant conduct. A careful consideration of the shape of the supply function with its concave, flat and convex regions, together with the information on fuel prices or expected infeed from renewable energy available to market participants on the previous day, allow plausible forecasts of electricity price quantiles. In particular, we found evidence that fundamental factors which traditionally impact German electricity prices have spillover effects cross-border on the Swiss prices and explain up to 80% of their total variation. Results show that there is price adaption to market fundamentals over time, depending on the intersection point of demand and supply curves at a certain hour during the day. The price adaption occurs due to the substitution effect between fuels (coal/gas) or between traditional fuels and the volatile renewable energies.

Overall Switzerland imports cheaper electricity prices over time, to a large extent due to the increase in renewable energies in the neighboring country Germany. Indeed, we found time-varying negative marginal effects of wind and PV on Swiss electricity prices for any time of the day and any day of the week. This result is of great relevance for the Swiss energy policy concerning the local law for renewable production: Switzerland imports lower electricity prices due to the energy transition in Germany. In particular, due to the high infeed of photovoltaic during peak hours the spread between peak/base Swissix prices narrowed significantly over time, as a consequence of market interconnectedness with Germany. Additional incentives to invest in renewable energies in Switzerland should therefore be considered in the light of our results.

The analysis has been extended to the prices of quarter-hourly products at the intraday market for Germany. A unique data set of 15-minute intraday prices and intraday-updated forecasts of wind and photovoltaic has been employed and price bids are modelled by prior information on fundamentals. Our model disentangles the effect of market fundamentals dependent on the regime of the demand quote and further dependent on the time of the day. Market fundamentals influence more the bidding behavior in the middle of the day than during mornings and evenings. There is an asymmetric adjustment of electricity prices with respect to both volume of trades and forecasting errors in renewables. Namely, in the high regime of the demand quote, where there is too little planned traditional capacity in the day-ahead market, traders react faster to the latest available forecasts of renewable infeed. Thus, the historically derived threshold in the demand quote for a specific delivery period is a highly relevant information for strategically bidding in the intraday market. The observed demand quote can be compared to the historical threshold value and, dependent whether the market is in the low or high demand quote regime, market participants can use the model for price forecasts accordingly.

Summary of main findings:

- Market fundamentals that traditionally impacted electricity prices in Germany show cross-border effects on the neighboring Swiss market.
- Swissix prices adapt continuously to fuel prices, renewable energies, participant conduct and demand/supply curves that are specific for the German market.
- Swissix adapts to the lower German price level due to the coal-based production and infeed from renewable energies in the neighbor country.

- Different price quantiles are impacted differently by market fundamentals.
- We extended the common methodology for solving quantile regressions by a non-parametric approach to a fully parametric approach.

Overview

Paper 1: “Cross-border effects of the German Electricity Market Fundamentals on the Swiss Electricity Prices”

Paper 2: “Estimation and Application of Fully-Parametric Multifactor Quantile Regression with Dynamic Coefficients”

Paper 3: “Econometric Analysis of 15-minute Intraday Electricity Prices”

Cross-border effects of the German electricity market fundamentals on the Swiss electricity prices

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October 24, 2016

Abstract

Given the perspective of the Swiss energy policy to invest in renewable energy sources in the future, it becomes highly relevant at this point to understand the traditional fundamental factors that impact cross-border Swiss electricity prices. Empirical evidence shows that German (Phelix) and Swiss (Swissix) electricity prices share a common long-term trend, given that the two markets are interconnected. Furthermore, it has been shown that shocks in Phelix prices are transmitted to Swissix. In this study, we examine the cross-border effects of the German market fundamentals that influence electricity prices in Switzerland, taking into account seasonal effects. In the context of a dynamic fundamental model, we investigate the continuous price adaption effect of electricity prices to market fundamentals and how this effect depends on the season of the year and the time of the day. The understanding of the risk drivers of electricity prices is of great importance for risk management, production planning, as well as for policy makers for the derivation of long-term energy scenarios.

Keywords: Swiss electricity prices, renewable energy, fundamental model, cross-border effects

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1 Motivation and project overview

In this research plan, we identify the market fundamental factors that impact Swiss electricity wholesale prices (Swissix). Given the perspective of the Swiss energy policy to invest in new installations of wind and PV as well as flexible storage devices, an understanding of the fundamental factors that impact Swiss prices becomes highly relevant at this point in time.

Liberalised markets are typically characterized by more market competition, higher economic efficiencies and lower prices. Altogether, its result is a higher total welfare, which is also the aim of the European Electricity Market. The first steps of the liberalisation process were implemented in December 1996 by means of the EU Directive 96/92/EC (European-Parliament, 1997). The main objective of this directive was to increase competition and to regulate existing monopolies. Switzerland also reacted in the mid-1990s and started to draft a law called "Electricity Market Law" (German: *Elektrizitätsmarktgesetz, EMG*), whose purpose was the market liberalisation within six years and the development of a national private-law grid company (Schweizerische Eidgenossenschaft, 2015).

The EMG was rejected in 2002, whereupon the Swiss Federal Council embedded the stepwise liberalisation in the "Power Supply Law" *(Stromversorgungsgesetz, StromVG)*. The latter was passed in 2007, and in preparation of its implementation the Transmission System Operator (TSO) Swissgrid was already established one year earlier. The law became effective in 2008 and allowed large clients to choose their power supplier freely from January 2009 on. Five years later, the market should have been opened for all clients, but because of the Fukushima nuclear disaster, the Energy Strategy 2050 had to be revised and, therefore, the full liberalisation in Switzerland was postponed.

It is important to distinguish between the regulatory opening of the market and the physical interconnections within the countries. The electricity grids of 34 countries in Europe are already physically interconnected, as shown in Figure 1. These countries are members of the European Network of Transmission System Operators for Electricity

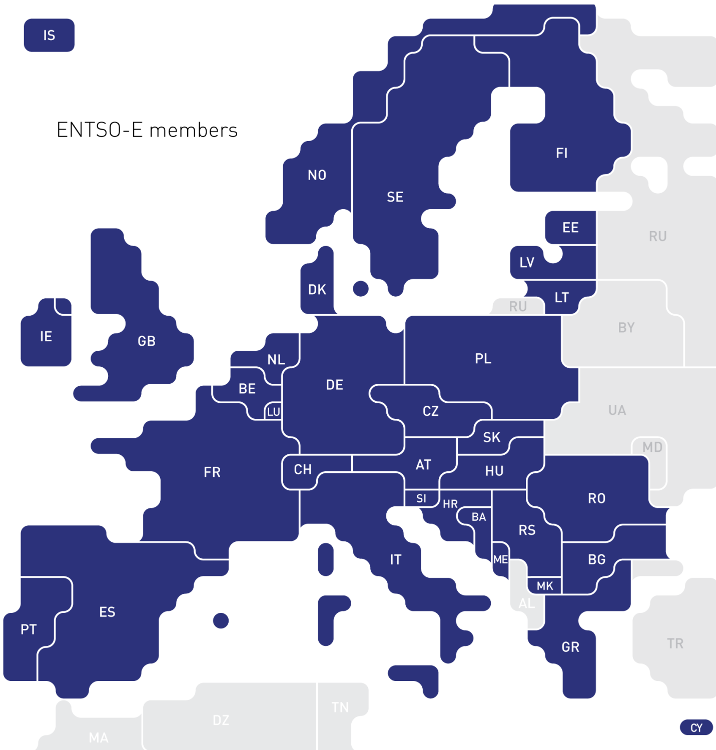


Figure 1: 34 member countries of the ENTSO-E are already physically interconnected. Source: ENTSO-E ENTSO-E (2015)

(ENTSO-E), an organisation of different TSOs. Switzerland participates as well, as it also exchanges electricity with other countries (ENTSO-E, 2015). Depending on the season, significant volumes are imported according to the statistics of the Bundesamt für Energie (2016), particularly from Germany.

The liberalisation of power markets enables in addition energy trading across international borders, but due to limited physical capacities of the transmission lines at the borders, price differences across countries still remain. These prices are strongly influenced by the capacity auctioning mechanism between the different countries involved. Parallel to this contractual layer, the increasing importance of sustainable environment management led to an expansion of fluctuant renewable power infeed, especially in Germany.

Renewable energy infeed in Germany is proven to have a high impact on the day-ahead electricity price in Austria and Germany (Paraschiv, Erni, and Pietsch, 2014). Furthermore, the intraday trading activities to adjust energy production- and consumption forecasts have increased significantly over the last years, partially because of the high share of fluctuating renewable power which needs to be balanced out (Kiesel and Paraschiv, 2015). As these technologies have extremely low variable costs and their production is fed into the grid with priority, they cause a shift in the merit order curve, which results in lower electricity prices (Erni, 2009).

This development influences the traditional relation between fossil fuel prices and electricity prices since coal, gas or oil are partially substituted by renewable energies in the production mix (Paraschiv, Erni, and Pietsch, 2014). The introduction of market coupling in many European countries initiated also a price shift, as a striking convergence of electricity prices took place among the countries participating in this process (EPEX-SPOT, 2015). Switzerland is not yet included in the market coupling mechanism of Germany and Austria, but the German and the Swiss markets are interconnected and cross-border effects are expected to occur.

It is known that Swiss electricity prices are mainly determined by German Phelix prices. This can be seen from the reaction of the Swissix on 15th January 2015, when the Swiss National Bank unpegged the Franc, and Swissix prices did not react to this policy change. In fact, the German, Swiss and Austrian electricity markets are interconnected, and it has been shown empirically that prices are cointegrated (Erni, 2009). In addition, it has also been shown that Swissix prices are Granger-caused by Phelix. It is therefore important to analyze the effect of the German market fundamentals on Swiss wholesale prices.

A recent study by Paraschiv, Erni, and Pietsch (2014) provided evidence that there is a continuous price adaption effect of Phelix prices to traditional market fundamentals: coal, gas, oil, CO2 prices, demand and power plant availability. The price adaption comes from two sources: adaption of electricity prices to prices of the input fuels and replacement in production of traditional fuels (in particular gas and oil) by renewable energies, i.e., wind and photovoltaic (PV). Renewables substituted the more expensive technologies in production and, thus, decreased electricity prices due to the merit order effect. In this study, we investigate the impact of the German market fundamentals on Swiss electricity prices. We focus particulary on the influence of the increasing infeed from renewable energies in Germany. Furthermore, we examine whether the electricity price adaption process to fundamentals differs among different hours within one day as well as between working versus weekend days.

2 Basic concept: Market coupling

The power transmission capacity of electric grids is limited. This requires the existence of a market where, as in case of power trading, the capacity to transmit this power is traded as well. In this context, there are two different capacity auction mechanisms: *explicit* and *implicit* capacity auctions. The concept of market coupling is related to the *day-ahead* market. Referring to the *intraday* market, there is no market coupling mechanism (operating with implicit *auctions*), but implicit cross-boarder capacity *allocations* are used instead[1]. In explicit auctions, transmission capacity is traded separately from the energy, which means that capacity needs to be bought before knowing the price for the electricity. This lack of information can lead to an economic disadvantage and, consequently, a decrease of market efficiency and total social welfare.

Market coupling uses the implicit auction mechanism, which allows players to buy capacity and power at the same marketplace: the capacity auction is *implicitly* embedded in the electricity auction. The price that is paid therefore reflects both, the price for the electricity and for the congestion on high voltage lines. This mechanism ensures an efficient energy flow from the low price area with a surplus of energy to the high price area. The consequence hereof is a convergence of prices of the different coupled markets. This mechanism is illustrated in Figure 2.

[1]At the border between Switzerland and Germany, implicit capacity allocations have been used since June 2013.

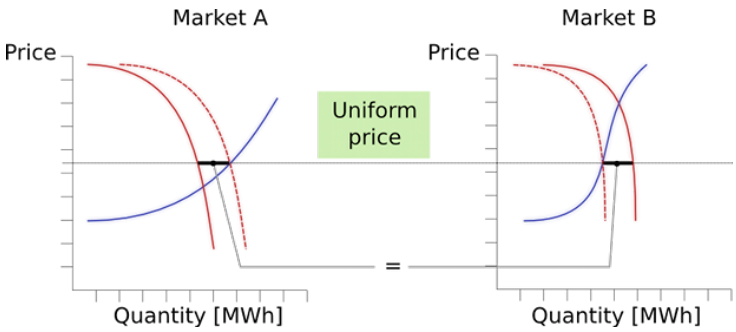


Figure 2: Market coupling leads to a convergence of the prices of different areas. The continuous lines show the demand curves before and the dotted lines after market coupling. Source: N-SIDE (2015).

The precondition for Switzerland for a participation in the market coupling mechanism is the termination of the bilateral agreement on electricity with the European Union, which has been suspended. In the light of the current status of Switzerland and with the perspective of a future introduction of market coupling, an investigation of the impact of fundamental factors for electricity prices in the neighbouring countries on Swiss power prices is highly relevant. Our analysis will refer to cross-border effects of German fundamental factors on Swissix prices.

3 Data

In this section we introduce a dynamic fundamental model for Swiss electricity prices and quantify their time-varying sensitivities with respect to the market fundamentals that traditionally impact the neighbouring power market of Germany. We investigate to which extent the use of wind and PV energy contributed to a decrease in electricity prices over time. Furthermore, we assess how the sensitivity of electricity prices to traditional input fuels like gas, oil or coal changed after 2011, given the increasing role of renewable energies.

In addition, we include the lagged electricity market clearing price for the same hour of the previous relevant delivery day and the price of the same hour with the lag of one week. This helps to reduce autocorrelation in our data and, furthermore, incorporates

Variable units	**Description**	**Data Source**
Lag Spot Price (1d) Swissix in EUR/MWh	Market clearing price for the same hour of the previous relevant delivery day	European Energy Exchange: http://www.eex.com
Lag Spot Price (7d) Swissix in EUR/MWh	Market clearing price for the same hour of the same weekday in the previous week	European Energy Exchange: http://www.eex.com
Coal Price EUR/t	Latest available price (daily auctioned) of the front-month Amsterdam-Rotterdam-Antwerp (ARA) futures contract before the electricity price auction takes place	European Energy Exchange: http://www.eex.com
Gas Price EUR/MWh	Last price of the NCG Day Ahead Natural Gas Spot Price on the day before the electricity price auction takes place	Bloomberg, Ticker: EGTHDAHD Index
Oil Price EUR/bbl	Last price of the active ICE Brent Crude futures contract on the day before the electricity price auction takes place	Bloomberg, Ticker: CO1 Comdty
Price for EUA EUR/EUA	Latest available price of the daily auctions at the EEX Emission Market (EUA)	European Energy Exchange: http://www.eex.com
Expected Wind and PV Infeed MW	Sum of expected infeed of wind electricity into the grid, published by German transmission system operators in the late afternoon following the electricity price auction	Transmission system operators: http://www.50Hertz.com, http://www.amprion.de, http://www.transnetbw.de, http://www.tennettso.de
Expected Power Plant Availability MW	Ex ante expected power plant availability for electricity production (voluntary publication) on the delivery day (daily granularity), daily published at 10:00 am	European Energy Exchange & transmission system operators: ftp://infoproducts.eex.com
Expected Demand MW	Sum of total vertical system load and actual wind infeed for the same hour on the last relevant delivery day	Transmission system operators: http://www.50Hertz.com, http://www.amprion.de, http://www.transnetbw.de, http://www.tennettso.de

Table 1: Overview of fundamental variables

Variable	**Daily**	**Hourly**
Lag Spot Price (1d)		×
Lag Spot Price (7d)		×
Coal Price	×	
Gas Price	×	
Oil Price	×	
Price for EU Emission Allowances	×	
Expected Wind		×
Expected PV Infeed		×
Expected Power Plant Availability	×	
Expected Demand		×

Table 2: Data granularity of fundamental variables

historic price and risk signals, which usually influence agents' price expectations and risk aversion. The rational behind choosing these fundamental variables is given in detail in section 3.1 ("Price formation fundamentals") of the study by Paraschiv, Bunn, and Westgaard (2016) for the market area Germany/Austria. We give a detailed overview of the sources and granularity of the relevant data in Tables 1 and 2.

4 Model formulation

We assess the price adaption process of Swissix hourly day-ahead electricity prices to market fundamentals which traditionally influence German power prices. To this end, we formulate a regression model with time-varying coefficients estimated by a Kalman Filter. In this section, we follow the discussion and apply the same methodology as in Paraschiv, Erni, and Pietsch (2014). Preliminary stability tests (following Karakatsani and Bunn (2010)) show strong evidence for time-varying parameters.

The daily seasonality pattern of prices is taken into account by deriving one individual model for each hour of the day. The weekly and yearly seasonality is incorporated in the demand variable. In this way, we distinguish between different load levels, where power plants with different marginal costs of production operate. Typically, in hours with a low level of demand (night hours) the power production in Germany is mainly coal-based, while during peak hours the *excess demand*[2] is covered by more expensive plants like gas and oil.

We formulate a state space model that allows for changing regression coefficients over time and estimate it with a Kalman Filter approach and maximum likelihood. The model formulation reads:

$$y_{i,t} = z'_{i,t}\gamma_{it} + v_{i,t} \tag{1}$$

$$\gamma_{i,t} = \gamma_{i,t-1} + w_{i,t} \tag{2}$$

[2]The demand which is not yet covered by the infeed from renewable energies (wind and PV).

where for $i \in \{1, ..., 24\}$

$$
\begin{aligned}
&v_{i,t} \sim \mathcal{N}(0, R_i) \\
&\gamma_{i,t} = (\gamma_{i,1,t}, \gamma_{i,2,t}, ..., \gamma_{i,k,t})' \\
&w_{i,t} = (w_{i,1,t}, w_{i,2,t}, ..., w_{i,k,t})' \\
&w_{i,t} \sim \mathcal{N}(0, Q_i) \\
&E(v_{i,t} w_{i,t}) = 0 \\
&Q_i = \text{diag}\{\sigma^2_{w_{i,1}}, \ldots, \sigma^2_{w_{i,k}}\}
\end{aligned}
$$

The dimension of the vector of exogenous variables is given by k. The measurement noise variance R_i and transition noise covariance matrix Q_i are assumed to be constant over time.

Equation (1) represents the measurement equation of the state space model. It relates the observed quantity $z_{i,t}$ (vector of exogenous, fundamental variables) to the variable $y_{i,t}$, which represents the day-ahead electricity price for hour i. Equation (2) is known as the transition equation and describes the dynamics of the time-dependent regression coefficients. In the above state-space formulation, the regression coefficients are not unknown constants, but latent, stochastic variables that follow random walks, estimated by a Kalman Filter algorithm (Kalman (1960)).

The intuition behind the random walk assumption is that the coefficients react to new information and are not predictable (see for example Karakatsani and Bunn (2010), Kim (2007)). Such an evolving price structure is likely to emerge in general due to agents' learning, regulators' announcements, mergers and acquisitions in the electricity industry, or stress events in electricity markets. The choice of a random walk is justified by the uncertainty related to future regulations and institutional policies related to renewable energies which impacted the electricity market over the investigated sample period.

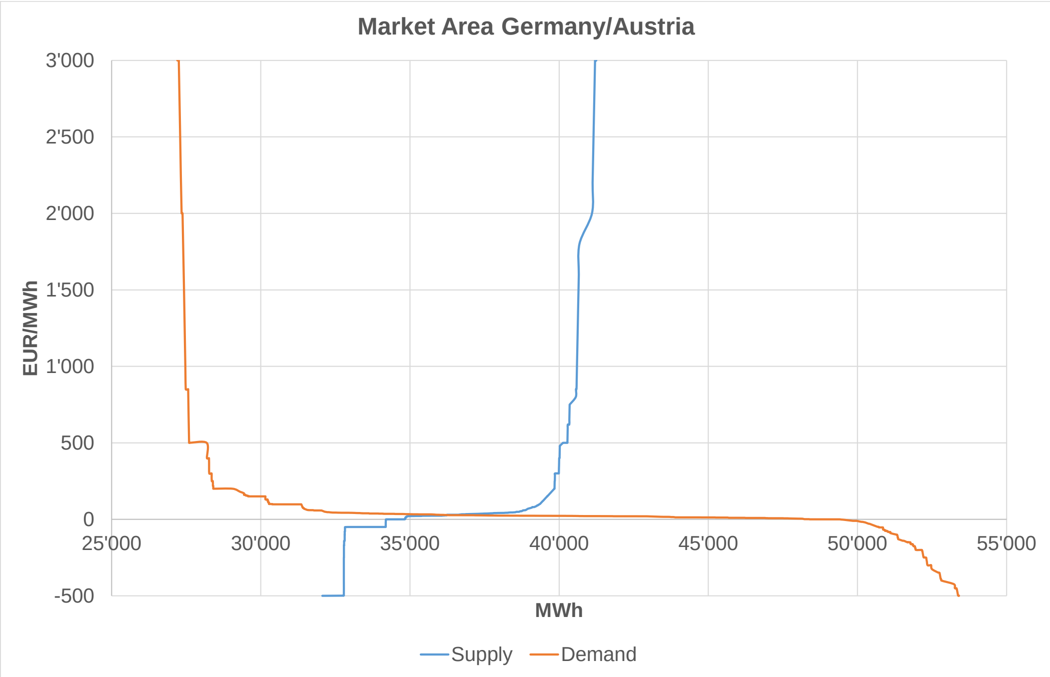
Throughout the estimation algorithm, as we run from $t = 1$ to $t = T$, we distinguish between two possible states of knowledge, namely the *a priory state*, when the electricity price is known up to $t-1$: $\hat{\gamma}_t^- = E(\gamma_t \mid y_{t-1})$, and the *posterior state*, when observations up to t are available: $\hat{\gamma}_t = E(\gamma_t \mid y_t)$. The predicted day-ahead electricity spot price $y_{i,t}$ is projected applying the a priori estimated regression coefficient of this stage to the observed exogenous variables. For a detailed derivation of the Kalman Filter and for the derivation of the likelihood function, see Karakatsani and Bunn (2010).

5 Results

The above introduced regression model has been estimated individually for each hour of the day. We comment in detail on the salient features of three hourly products[3], namely hour 4 (03:00–04:00), hour 13 (12:00–13:00), and hour 18 (17:00–18:00) in order to illustrate the main idea of our modeling approach: market fundamentals impact the Swiss electricity prices differently, depending on the steepness of the supply curve and on the demand profile at different trading periods within one day. We estimated the model separately for working versus weekend days, given that the demand slope is steeper during the week (Monday to Thursday) than on the weekend.

The inclusion of demand in our formulation encompasses weather and seasonal effects (as in Karakatsani and Bunn (2010)). Results will be interpreted in the context of the particularities of demand and supply curves for electricity in Germany. In Figure 3 we display for exemplification a typical price clearing result of the auctions for the German/Austrian and the Swiss market for a specific hour in 2016. The supply function, which represents the stack of offers for production quantities in ascending order of prices, distinctively increases through concave, flat and convex regions (see Paraschiv, Bunn, and Westgaard (2016)).

[3]Results for other hours of the day are available on request.



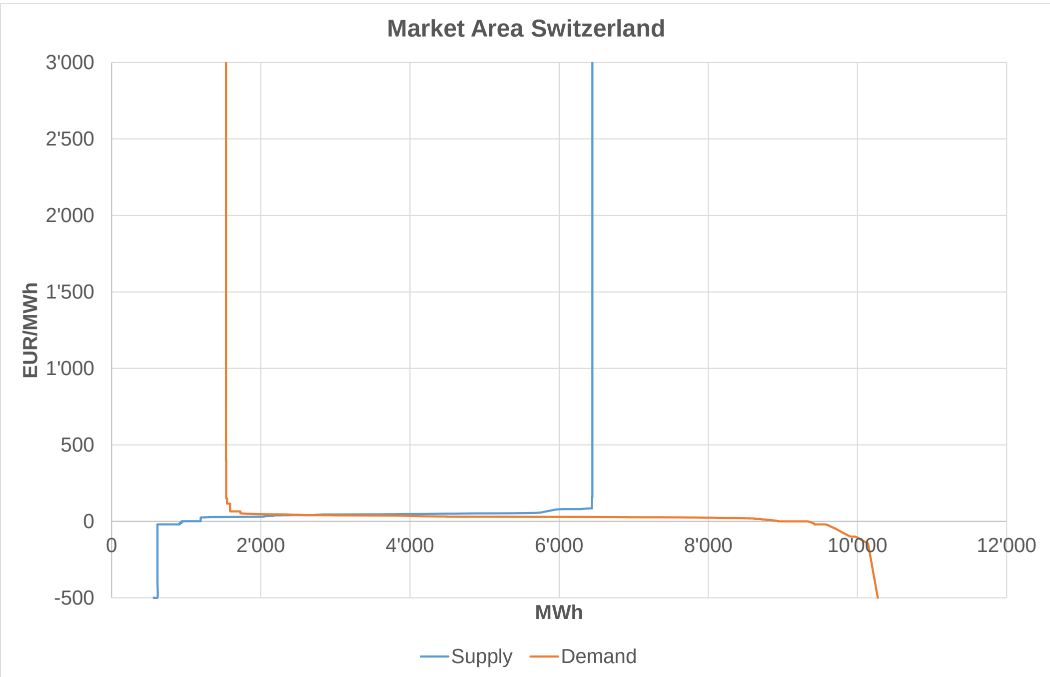


Figure 3: Actual supply and demand functions from the German/Austrian (top) and the Swiss (bottom) market for hour 11:00–12:00 on 31/08/2016. Source: EPEX.

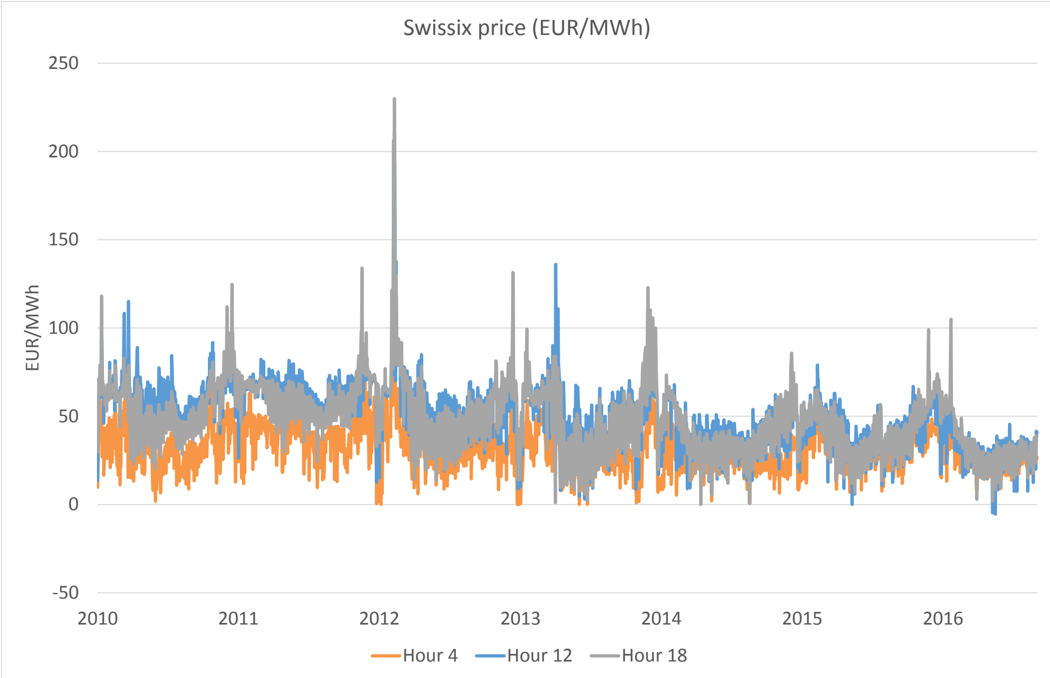


Figure 4: Swissix electricity price evolution 01/01/2010 till 31/08/2016.

Often particularly large or small values of the estimated coefficients correspond to extreme electricity price levels. Therefore, we included also the evolution of Swissix prices from January 2010 to August 2016 in Figure 4 for a comparison with the estimation results.

5.1 Learning effect

In Figure 5 (upper graph) we observe that the sign of the coefficient of the 1d-lagged spot price for hours 12 and 18 is negative most of the time while the coefficient for hour 4 is nearly zero over the entire sample period. The negative sign is intuitive since it reverts the level of electricity prices for a specific hour in the next day, which reflects the typical mean reverting behavior of electricity prices (see Paraschiv, Erni, and Pietsch (2014)). The coefficients of lagged spot prices reflect the so-called "participant conduct" (see Karakatsani and Bunn (2010) for the UK market and Paraschiv, Bunn, and Westgaard (2016) for the market area German/Austrian).

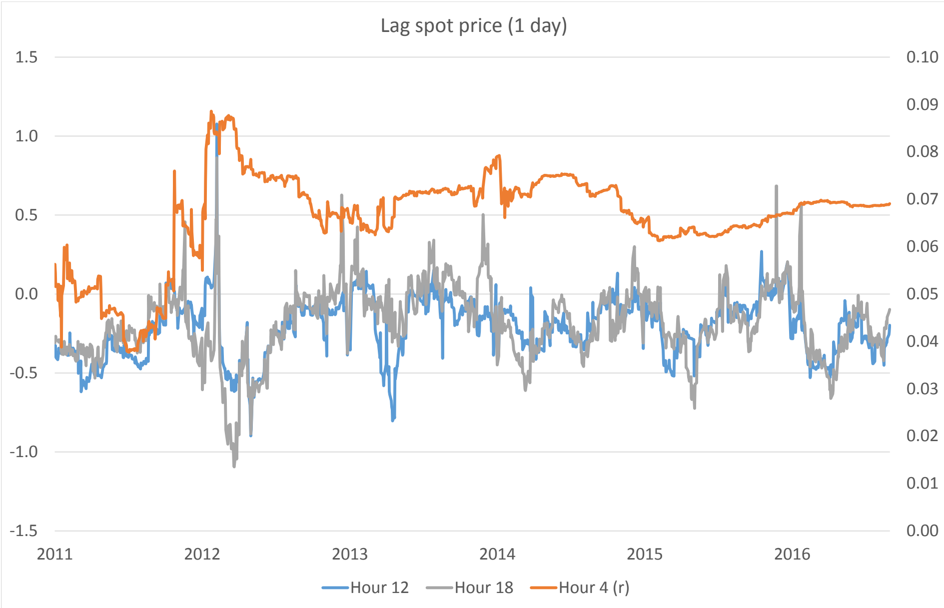
During the peak hours 12 and 18, there is a high demand for electricity which is more difficult to balance out and, thus, prices are more volatile. This explains the more pronounced price adaption to 1d-lagged prices for these two hours than for hour 4. However, typically electricity prices revert towards their production costs, which explains the negative sign of the coefficients most of the time. Additionally, for hour 18 we observe that coefficients cross the zero axis and become positive, especially in winter months of each year, thus, during evening peaks, when prices are in the upper region of the supply curve and Switzerland imports electricity from Germany. The positive sign of the coefficients in these winter days reflect the clustering effect of extremely large electricity prices (positive price spikes).

In Figure 5 (lower graph) we observe that there is price adaption of Swissix to its 7d-lagged values as observed in the same hour of the same weekday in the previous week. The spot price lagged by one week corrects the weekly seasonality pattern not fully reflected by the demand curve. In particular, the positive sign of the coefficients indicates that market participants tend to reinforce successful bids previously placed in the market, which is consistent with evidence for the use of market power in electricity price bids found in other studies (e.g., see Karakatsani and Bunn (2010)).

Summing up, the upper graph of Figure 5 shows the typical mean reversion pattern in electricity prices for the short-term (1-day lag), but in addition there is evidence for the use of market power on a medium-term (1-week lag) as illustrated in the lower graph.

5.2 Influence of fuel prices on Swiss electricity prices

As discussed in Paraschiv, Bunn, and Westgaard (2016), the mid-region of the supply function in the German electricity market, which is characterized by a mix of coal- and gas-based production, is flat and price volatility relatively low. For both generation



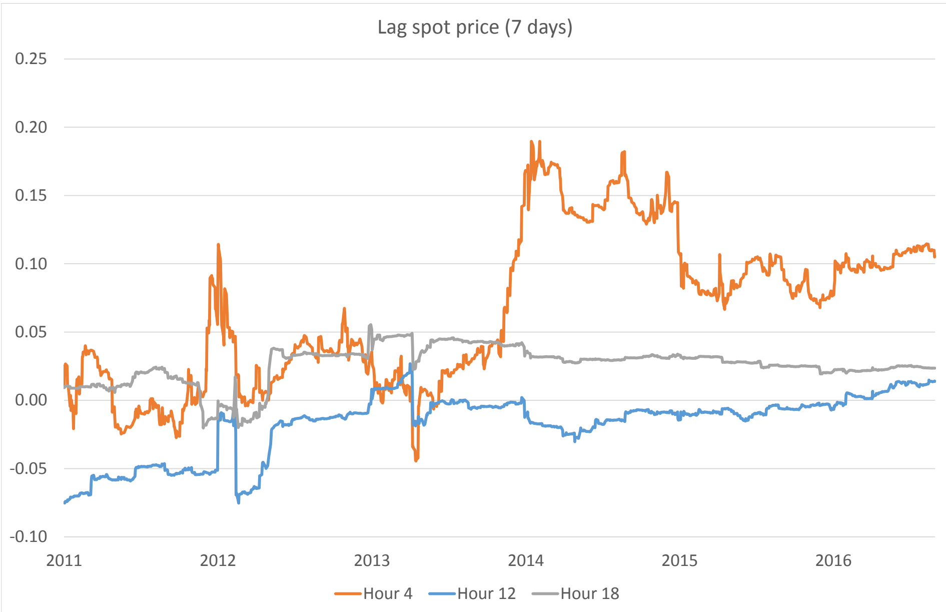


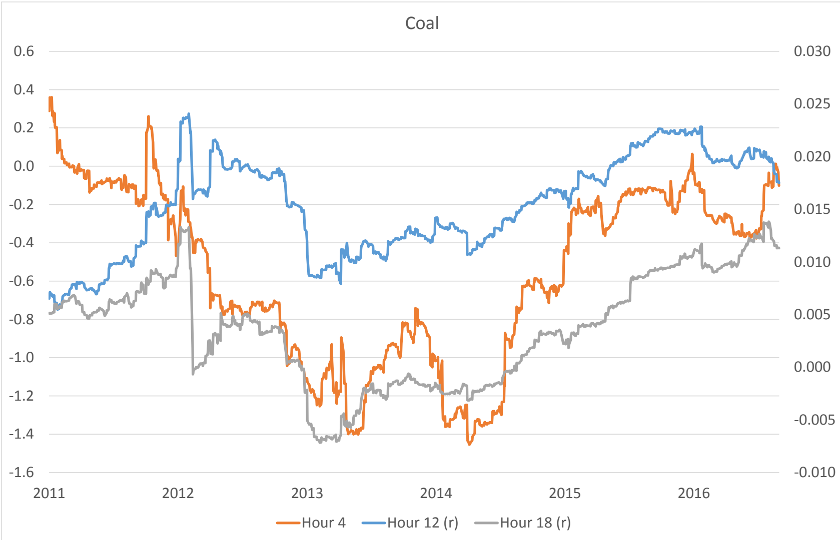
Figure 5: Time-varying coefficients of the 1d-lagged price (same hour, previous working day) and the 7d-lagged price (same hour, one week ago). The coefficients have been estimated for **working days** during the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18. Observations of the year 2010 have been used for an initial estimation of the unknown covariances R and Q. Note that in the upper graph the values of H4 are scaled on the right axis.

technologies, coal and gas, producers must hand in emission certificates for every emitted tonne of CO2. Per unit of generated power, coal requires about twice the number of emission certificates compared to gas. Also the operational efficiencies of the various coal and gas plants vary, so that the order of marginal costs tends to be a mix of the technologies: Not all coal plants are located below all gas plants in the supply function.

Thus, as commodity prices for coal and gas fluctuate, the sequence of the various gas and coal facilities in this section of the supply function may also interchange. This leads to a competitive and intricate relationship of power prices to gas and coal commodity prices. As a consequence, we expect an adaption process of Swiss electricity prices to prices of fuels used as input in Germany, given that the two markets are interconnected and, furthermore, prices are cointegrated.

In Figures 6 and 7 we observe that indeed Swiss electricity prices adapt continuously to prices for coal, gas and oil over time. Price adaption to coal and gas is, as explained above, due to the interchange between fuels in the mid-region of the supply curve and, furthermore, due to their interaction with renewable energies. An interchange in production between gas and oil, and from here implied price adaption to these fuels prices, occurs for example in the upper convex region of the supply function, which is characteristic for power markets at times of high demand and increasingly scarce supply. The technologies in this supply region, like gas and oil (diesel), tend to have low capital but high marginal cost (see Paraschiv, Bunn, and Westgaard (2016)).

The signs of the coefficients for coal are near zero for the peak hours 12 and 18 but they remain negative over longer time in case of hour 4. As discussed above, coal is a cheap production technology, which is situated at the left end of the merit order curve, and electricity production in Germany during off-peak hours (night) is mainly coal based. As Switzerland imports cheap electricity from Germany, there is an adaption to the lower German price level. In consequence, coal prices have therefore a negative marginal effect on Swissix electricity prices.



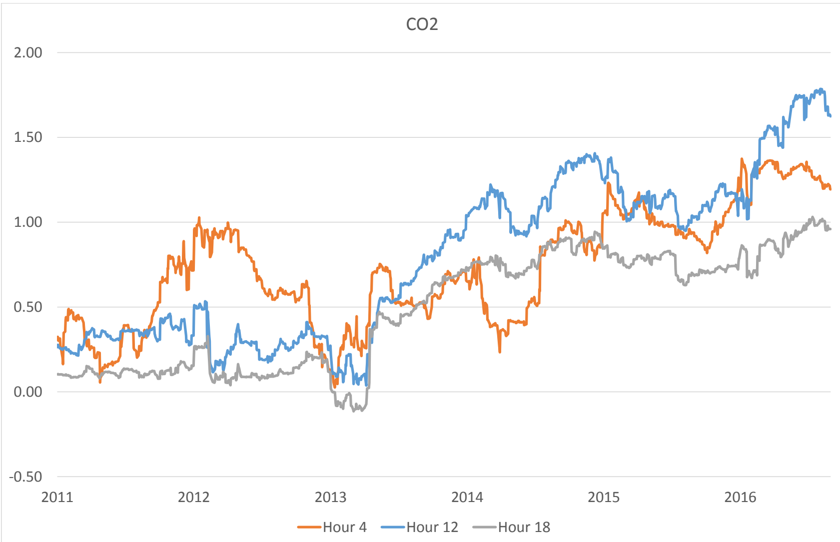
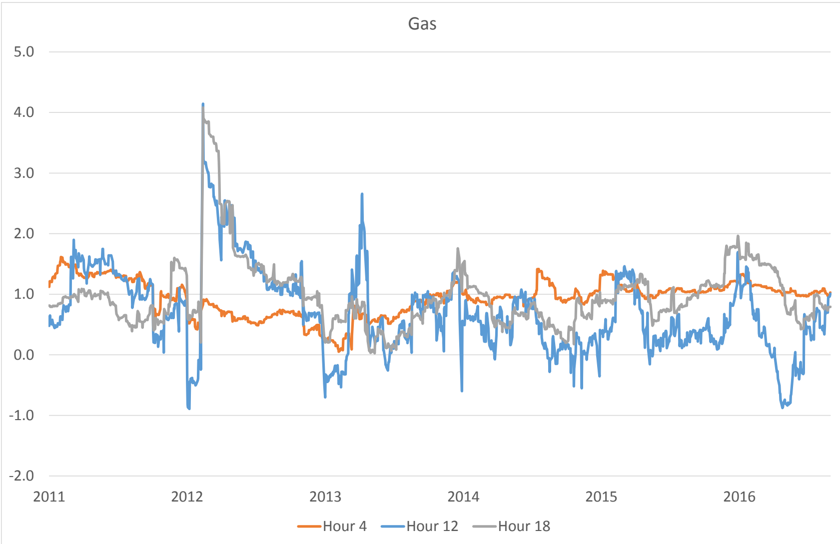


Figure 6: Time-varying coefficients of coal and CO2 prices. The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **working days**. Note that in the upper graph values for H12 and H18 are scaled on the right axis.



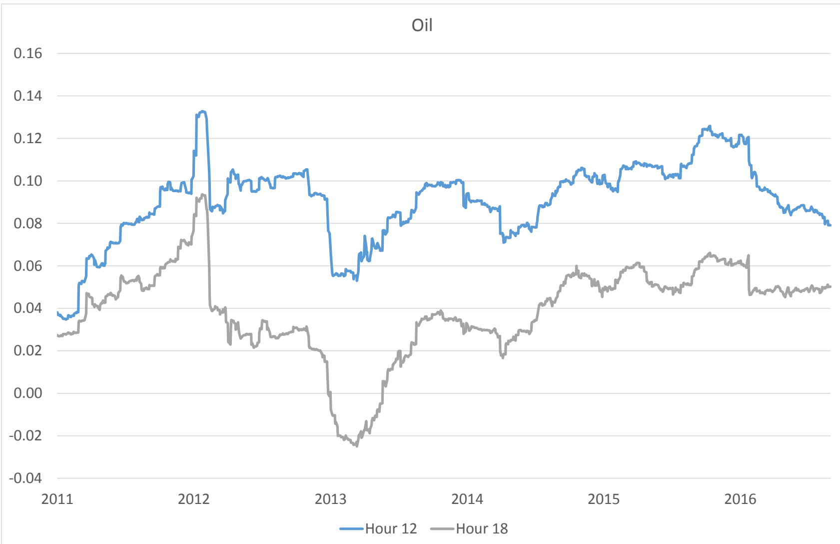


Figure 7: Time-varying coefficients of gas and oil prices. The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **working days**.

	2009	2010	2011	2012	2013	2014
Coal	42.6	41.5	42.8	44	45.2	43.2
Nuclear	22.6	22.2	17.6	15.8	15.4	15.8
Natural Gas	13.6	14.1	14	12.1	10.5	9.5
Oil	1.7	1.4	1.2	1.2	1	1
Renewable energies from which	**15.9**	**16.6**	**20.2**	**22.8**	**23.9**	**25.9**
Wind	6.5	6	8	8.1	8.4	8.9
Hydro power	3.2	3.3	2.9	3.5	3.2	3.3
Biomass	4.4	4.7	5.3	6.3	6.7	7.0
Photovoltaic	1.1	1.8	3.2	4.2	4.7	5.7
Waste-to-energy	0.7	0.7	0.8	0.8	0.8	1
Other	3.6	4.2	4.2	4.1	4	4.3

Table 3: Electricity production in Germany by source (%), as shown in *Paraschiv, Bunn, and Westgaard (2016).*

Switzerland and the EU operate separate emissions trading schemes. In Switzerland, the so-called CO2 steering taxes (German: *Lenkungssteuer*) must be paid, which is intended to change the behaviour of consumers and the industry towards a more economic energy usage. However, with respect to the electricity supply side, steering taxes are not a very important price determinant, since Switzerland produces most of its electricity by means of CO2 neutral technologies like nuclear and hydropower. In Figure 6 (lower graph) we observe that the Swissix shows price adaption to the German CO2 prices with an increasing trend after 2013. This can be explained by an increasing share of fossil-based fuels among the electricity imports from Germany after 2011 (see Table 3), which leads to a pass-through effect of CO2 prices.

In Figure 7 (upper graph) we observe cross-border positive marginal effects of gas prices on Swissix. The price adaption comes from the interchange between gas/coal in the mid-region or gas/oil in the convex regions of the supply curves. Further adaption comes from the substitution between gas (flexible technologies) and the volatile photovoltaic and wind infeed. There are no increasing marginal effects from gas over time since gas power plants have been shut down in Germany due to the competition from renewable energies.

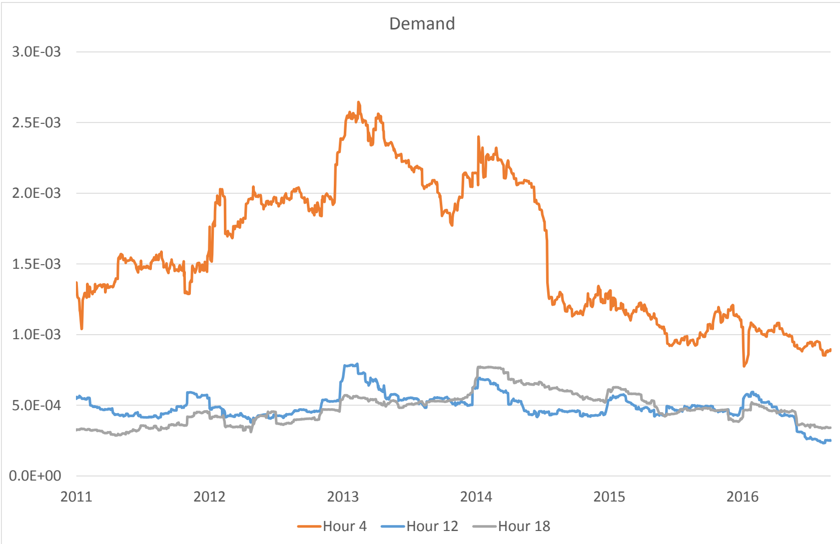
We also observe spikes in the evolution of gas coefficients, especially during the winter days, which correspond to electricity price spikes in Figure 4. This shows that Swissix price spikes were due to the more expensive gas-based production imported from the German market. Often in winter days the coefficients for hour 18 are higher, due to the so-called “evening peak”, than for hour 12. For hour 4, when prices are in the concave region of the supply curve and gas is typically not used, coefficients for gas are less volatile and induce no price adaption.

For the interpretation of coefficients for oil we should keep in mind that the percentage of oil in the German electricity production is negligible (see Table 3). This effect is due to the increasingly competitive conditions in the German market, where traditional gas and oil plants have been replaced gradually in production by photovoltaic and wind. Because oil is not burned in the night, we display the coefficients only for the peak hours. We observe little price adaption to oil and furthermore marginal effects close to zero.

5.3 Influence of demand and supply

Since electricity is produced to meet demand instantaneously, with yet very little storage options by end-users, hourly variations in price are due to fluctuations in demand that are mapped through the nonlinear supply function to prices, and also through changes in the shape of the supply function itself due to availabilities of wind, solar and other sources of power, as well as the pricing strategies of generators (see the discussion in Paraschiv, Bunn, and Westgaard (2016)). In Figure 8 we observe that Swissix prices adapt to shocks in demand and power plant availability (as a measure for electricity supply in Germany). The marginal effects of demand are positive for all hours. A high expected demand in the German market increases power prices there, and this increase will be passed to the prices of neighboring importing countries.

In the night demand and also the planned capacity are generally low. As a consequence, the system is more sensitive to the fluctuant infeed from wind. Since the inflexible coal facilities have high shut-down and start-up costs, producers are willing to



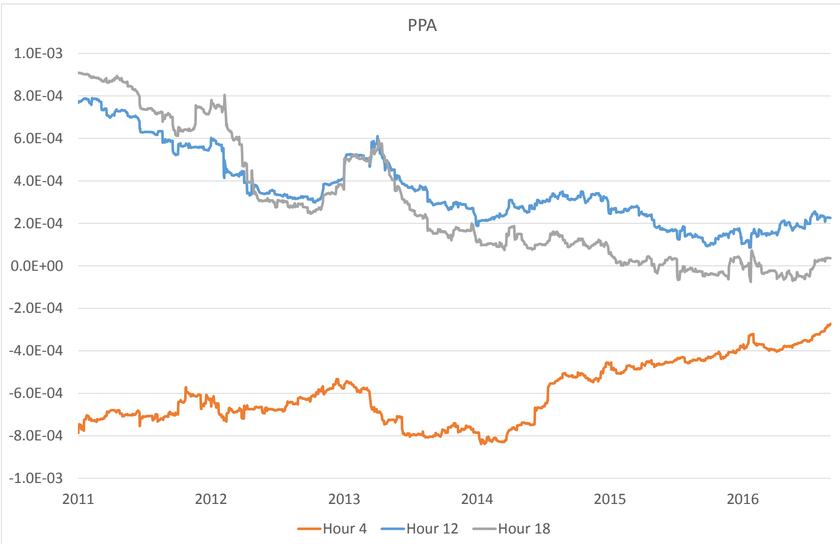


Figure 8: Time-varying coefficients of German demand and supply. The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **working days**.

accept prices below their marginal costs in order to generate continuously. Hence, the large (negative) marginal effects of power plant availability (PPA) for hour 4 reflects the deeply discounted and even negative offer prices at the low end of the supply function.

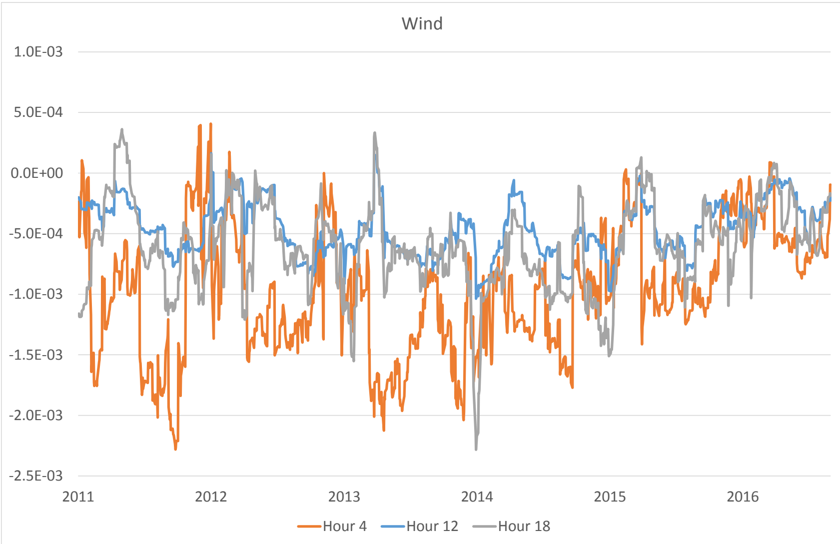
5.4 Influence of renewable energies

The infeed from renewable energies has been increasing continuously over the investigated period, as shown in Table 3. At the time of this analysis, in Germany renewable technologies receive fixed feed-in tariffs and priority dispatch, which effectively means that their total production is sold at a fixed price[4]. Renewables are situated in the lower region of the merit order curve and shift it to the right. Hours with high renewables supply cause difficulties for other generating facilities that might be inflexible and should run continuously (nuclear, district heating and industrial co-generation facilities, as well a some large coal power stations). As outlined above, these inflexible facilities accept negative marginal returns in order to generate continuously which leads to lower electricity prices.

In Figure 9 we show that wind and PV in Germany have a decreasing effect on Swiss prices as well. The continuous price adaption to renewables reflects the substitution in production with traditional fuels as gas or oil in the flat mid-region and upper regions of the supply curve (similar results have been found in the analysis of Paraschiv, Bunn, and Westgaard (2016) for the German price index): When wind and PV production is high (low), the supply function is moved to the right (left), and gas facilities with higher marginal cost are turned off (on). As expected, there are larger marginal effects of the wind during night hours, due to the inflexibility of coal plants to adapt to unexpected extra supply from wind.

This result is of significant relevance for the Swiss policy concerning the local law for renewable production: Switzerland imports lower electricity prices due to the renewables policy in Germany. In particular, due to the high infeed of photovoltaic during peak hours,

[4]Meanwhile Germany has adopted feed-in premiums.



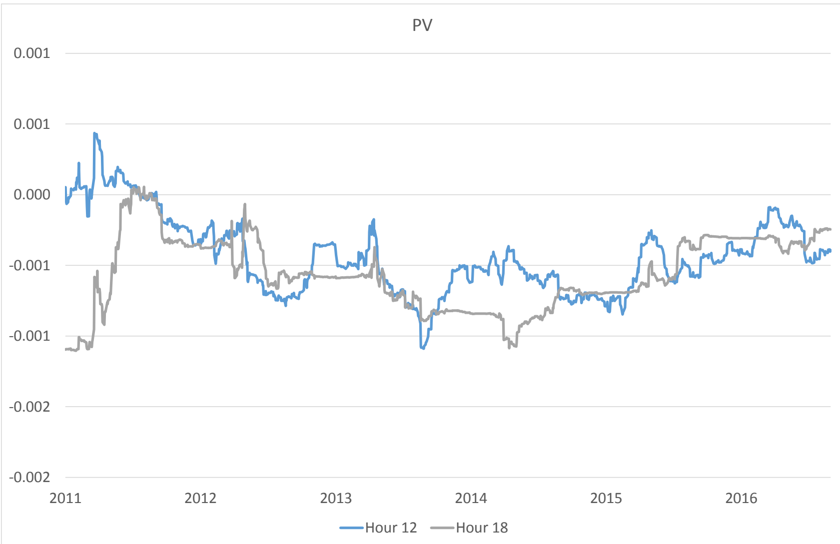


Figure 9: Time-varying coefficients of German wind and PV infeed. The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **working days**.

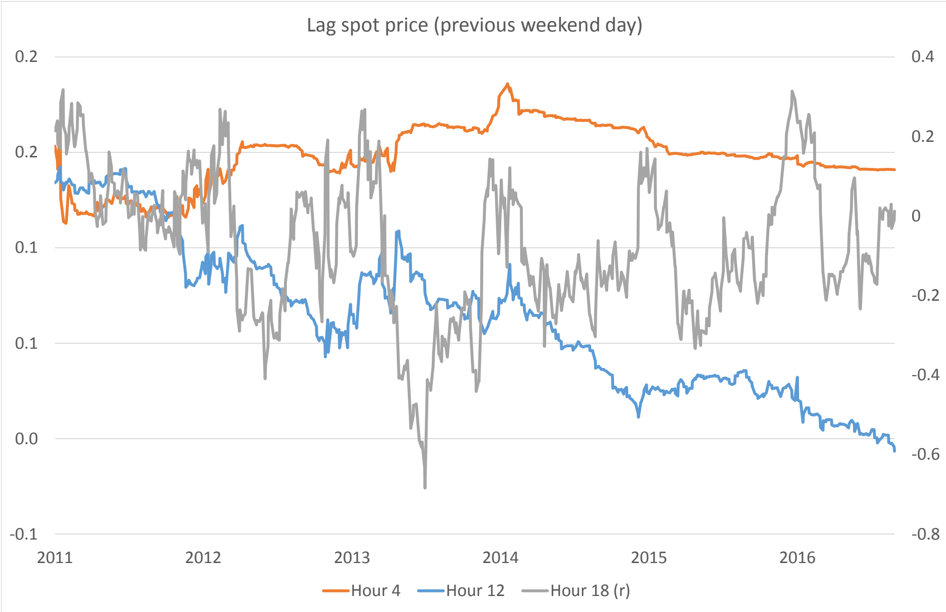
the spread between Swissix peak and off-peak prices narrowed significantly over time, which results from the market interconnectedness with Germany. The reduced spreads impacted the profitability of pumped-storage hydropower plants severely. Additional incentives for investments in renewable energies or subsidies for hydropower in Switzerland should therefore be considered in the light of this insight.

5.5 Weekend effect

Figures 10 to 14 replicate the results for weekend days. Overall we observe that Swiss electricity prices react differently to German market fundamentals for weekend than for working days. In Figure 10 (upper graph) we observe that the coefficients of 1d-lagged spot prices (here: previous weekend day in the sample) are positive for hours 4 and 12 but oscillate around zero in case of hour 18.

On the other hand, there is more price adaption of Swissix to 1d-lagged spot prices for hours 12 and 18, when the electricity production is situated in the convex region of the supply function (plants with higher marginal costs run to supplement the residual demand), and prices are more sensitive to changes in demand and, thus, more volatile. This is consistent with our interpretation of the similar graph for working days (Figure 5): At the convex region of the supply function, conduct (1d-lagged spot prices) tends to be a more important feature of price formation than demand, supply or fuel price fundamentals.

There is a descending trend in the marginal effects of 1d-lagged spot prices for hour 12. This reflects that prices tend to loose the autoregressive nature since more photovoltaic has been fed into the electricity grid over time and, therefore, has higher impact on the price formation process. The positive coefficients for hours 4 and 12 suggest the exercise of market power. However, overall the absolute values of the coefficients are lower for weekend than for working days. Still, the exercise of market power becomes more obvious during weekend than working days since for the latter coefficients for hour 12 stay positive



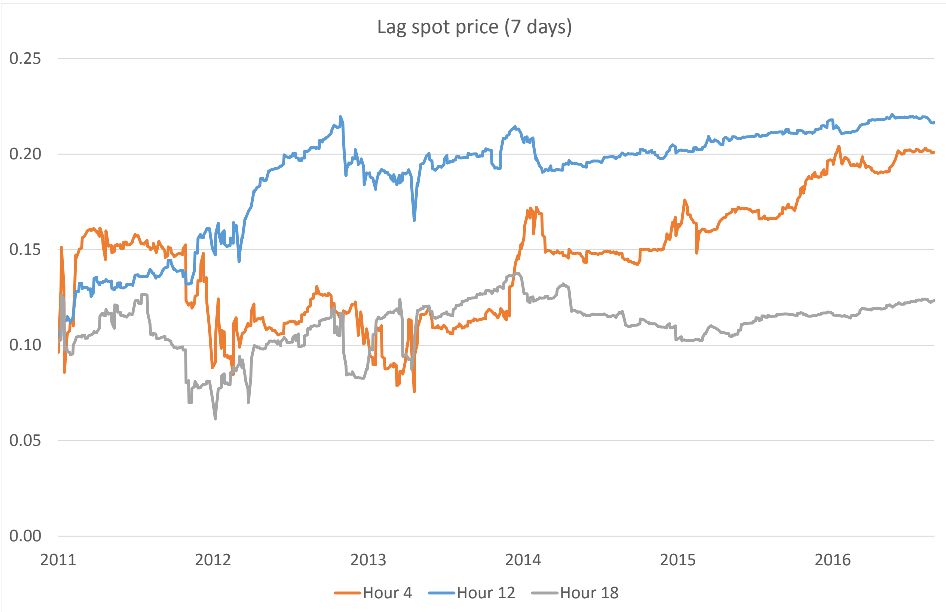


Figure 10: Time-varying coefficients of the 1d-lagged price (same hour, previous weekend day) and the 7d-lagged price (same hour, one week ago). The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **weekend days**.

over time.

Similarly to the working days (Figure 5), the coefficients of 7-day lagged spot prices remain positive, which confirms our previous results that market participants tend to reinforce previously successful bids (use of market power as shown in the study of Karakatsani and Bunn (2010) for the UK market).

The marginal effects of coal on the Swiss electricity prices in the weekend are negative over almost the entire sample period 2011–2016 for all hours (see Figure 11, upper graph). This shows that, independent where the intersection point of the demand and supply curves is located in various hours of the day, Swiss electricity prices are marginally reduced by the low price level of coal-based electricity in Germany. The suppressing effect on prices is more obvious for the night hour 4, when the electricity in Germany is mainly produced by coal. This is similar with our insights from the analysis of working days (see Figure 6). In Figure 11 (lower graph) coefficients of CO2 show no clear trend and have overall a very small magnitude, so the effect of CO2 on Swiss electricity prices is negligible for the weekend.

In Figure 12 (upper graph) we observe that there are positive coefficients of gas prices and the price adaption is more pronounced for hour 12. A similar picture has been obtained in the case of working days: At noon gas is the marginal unit and, thus, more adaption of electricity to gas prices is observed. As mentioned already before, in the convex region of the merit order gas and oil are typically interchanged in production, and this explains partially the price adaption pattern of coefficients. In addition, adaption occurs when there is photovoltaic infeed which is fed with priority in the electricity grid and replaces temporarily the flexible gas power plants in production. Still, gas power plants are kept on running to balance out demand in days when PV is unavailable.

In Figure 12 (lower graph) we observe that an increase in oil prices causes a decrease in Swissix prices. The intuition is that, as gas and oil are complementarily used in production in the convex region of the merit order and gas plants are flexible, increasing

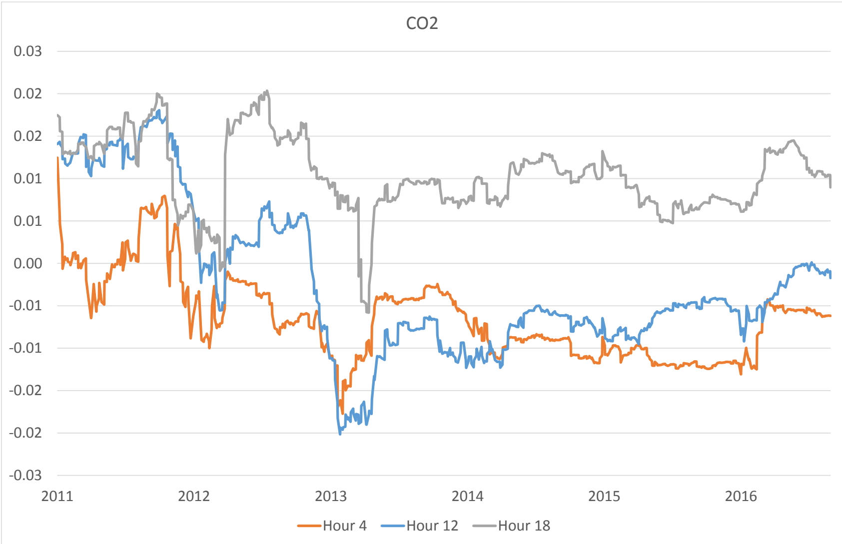


Figure 11: Time-varying coefficients of coal and CO2 prices. The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **weekend days**.

oil prices create incentives to switch to gas as the cheaper technology. This explains the indirect decreasing effect of oil prices on Swissix.

The coefficients of demand for the weekend in Figure 13 resemble the pattern observed during working days (Figure 8). As expected, positive shocks in demand increase prices. The coefficients for power plant availability are also negative for hours 12 and 18. If an excess of capacity meets a low and inelastic demand for electricity, weekend power prices decrease significantly. Due to the low demand on Saturdays and Sundays, this effect is even more pronounced than for working days where negative coefficients of power plant availability occurred only for hour 4.

In Figure 14 we observe that wind and PV have dampening marginal effects on the Swiss electricity prices also during weekend days. The pattern of wind and PV coefficients resemble our results in Figure 9 for working days. Again, the decreasing marginal effect of wind on Swissix is most pronounced during the night, which is consistent with previous results which link high frequency of wind infeed in the night to very low levels of electricity prices (see Paraschiv, Erni, and Pietsch (2014) for the German/Austrian market).

With respect to PV, we observe slightly larger marginal effects (in absolute values) of PV on Swissix in the weekend than during working days. This is intuitive since during working days the intersection point of the demand and supply curves is located in the convex region of the merit order and, thus, gas and oil are turned on to supplement the higher demand. When there are higher levels of PV infeed, generation with the latter is reduced accordingly to avoid oversupply, which is reflected in price adaption and substitution effects between market fundamentals. However, in the weekend the supply is mainly coal based, so any excess of PV infeed will decrease prices more since coal plants are costly to shut down. In other words, high levels of PV infeed reduce prices faster during weekend than during working days, which is reflected in the absolute values of the coefficients.

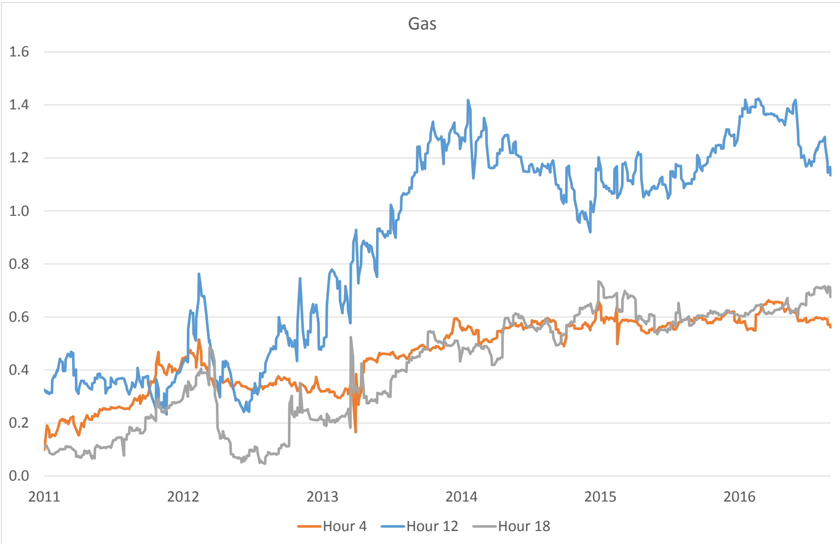
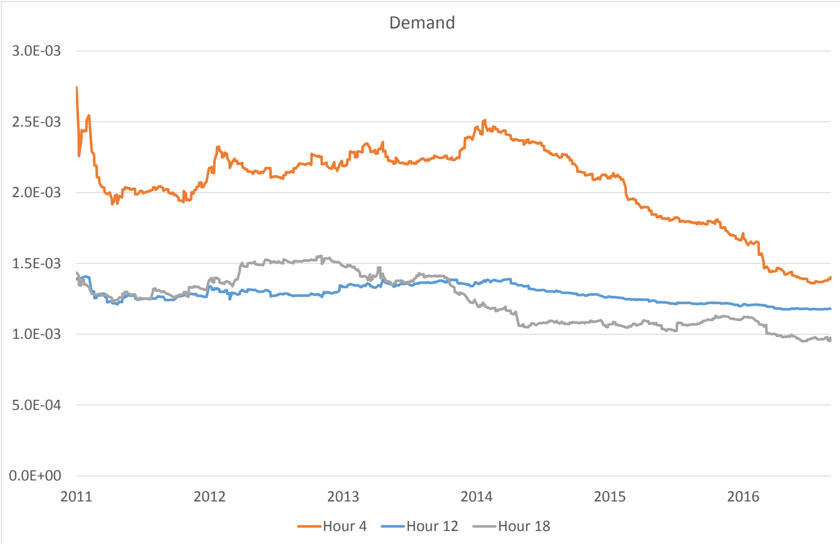


Figure 12: Time-varying coefficients of gas and oil prices. The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **weekend days**.



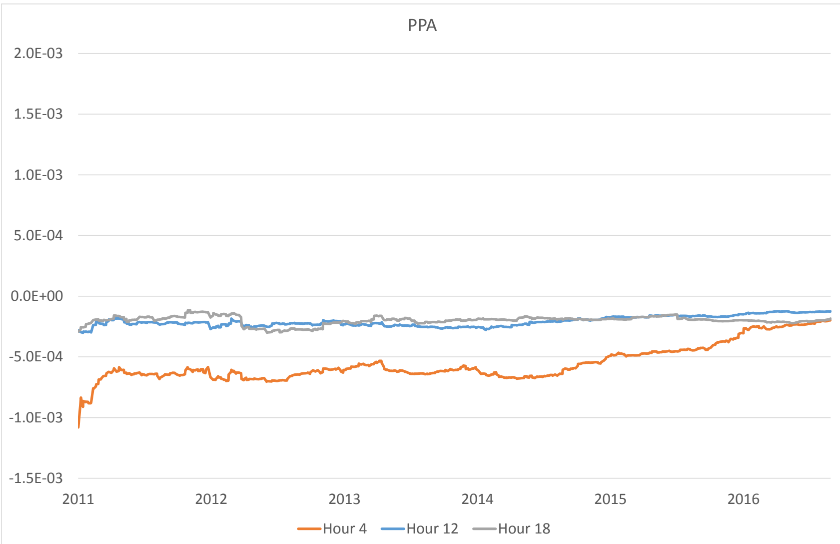
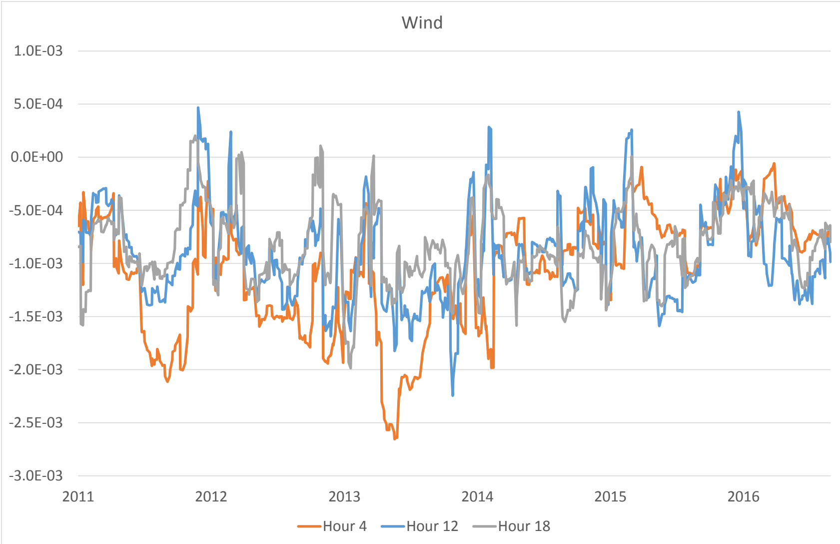


Figure 13: Time-varying coefficients of German demand and supply. The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **weekend days**.



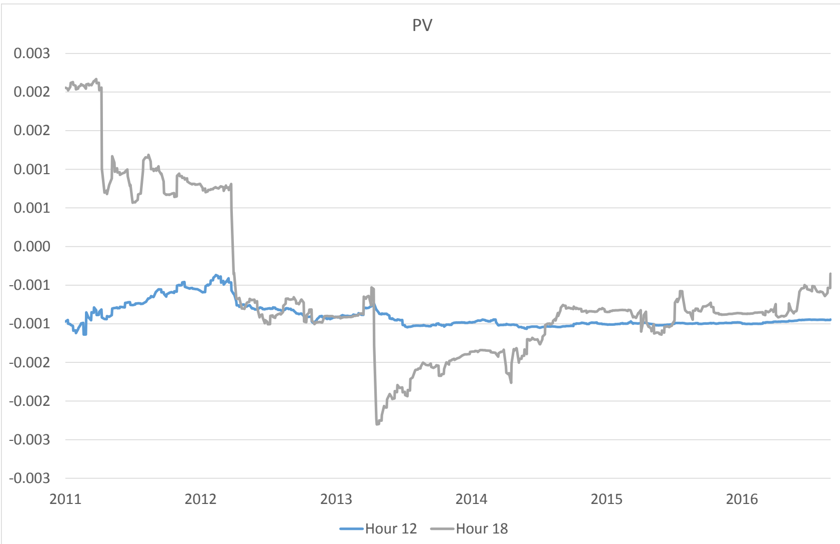


Figure 14: Time-varying coefficients of German wind and PV infeed. The coefficients have been estimated for the sample period from 1 January 2011 until 31 August 2016 with respect to the key hours H4, H12 and H18 on **weekend days**.

Hourly blocks	Morning 7-10	Noon 11-14	Afternoon 15-18	Evening 19-24	Night 1-6
R^2	0.491	0.644	0.766	0.785	0.741
adjusted R^2	0.467	0.627	0.754	0.774	0.728
MAE $\left(\frac{\text{EUR}}{\text{MWh}}\right)$	4.224	4.089	4.163	3.865	3.999
DW	2.354	2.349	2.330	2.238	1.946
LLF	-4086.534	-4050.761	-4051.969	-3931.799	-4060.185

Table 4: Goodness of fit, for morning, noon and afternoon hourly blocks for **working days**, *including* renewable energies wind and PV.

Hourly blocks	Morning 7-10	Noon 11-14	Afternoon 15-18	Evening 19-24	Night 1-6
R^2	0.736	0.806	0.813	0.822	0.714
adjusted R^2	0.710	0.787	0.794	0.804	0.685
MAE $\left(\frac{\text{EUR}}{\text{MWh}}\right)$	5.457	5.288	5.321	4.067	4.877
DW	2.178	1.948	1.932	2.100	2.119
LLF	-2124.511	-2112.721	-2110.272	-1949.686	-2062.411

Table 5: Goodness of fit, for morning, noon and afternoon hourly blocks for **weekend days**, *including* renewable energies wind and PV.

5.6 Model significance

In Tables 4 and 5 we assess the goodness of fit for our model by computing the R^2, mean average error (MAE) and Durbin Watson statistics. Log-likelihood function (LLF) values of the model estimates are shown in the last rows of the tables. During working days the variation of Swissix prices explained by the the time-varying fundamental model is 49% for morning hours and increases to almost 80% for the evening. Note that the R^2 values for working and weekend days cannot be compared directly since in the latter case there are significantly less observations available for the estimation. Indeed, the mean average error (MAE) expressed in EUR/MWh shows larger deviations for the weekend than for working days. Furthermore, we conclude that model residuals show no (or very little) serial correlation, as indicated by the Durbin-Watson (DW) test statistics.

Hourly blocks	Morning 7-10	Noon 11-14	Afternoon 15-18	Evening 19-24	Night 1-6
R^2	0.469	0.625	0.763	0.764	0.657
adjusted R^2	0.445	0.608	0.752	0.753	0.641
MAE $\left(\frac{\text{EUR}}{\text{MWh}}\right)$	4.646	4.346	4.392	4.260	4.676
DW	2.296	2.246	2.227	2.190	1.839
LLF	-4223.906	-4154.082	-4154.176	-4106.158	-4285.626

Table 6: Goodness of fit, for morning, noon and afternoon hourly blocks **working days**, *excluding* renewable energies wind and PV.

Hourly blocks	Morning 7-10	Noon 11-14	Afternoon 15-18	Evening 19-24	Night 1-6
R^2	0.677	0.747	0.756	0.785	0.627
adjusted R^2	0.645	0.722	0.733	0.764	0.590
MAE $\left(\frac{\text{EUR}}{\text{MWh}}\right)$	6.300	6.239	6.139	4.527	5.639
DW	1.957	1.912	1.851	2.048	2.110
LLF	-2231.725	-2220.441	-2227.014	-2027.281	-2152.368

Table 7: Goodness of fit, for morning, noon and afternoon hourly blocks **weekend**, *excluding* renewable energies wind and PV.

In addition to the results discussed above, we estimated also a version of the time-varying regression model where wind and PV were excluded. This should help assess to what extent infeed from renewable energies in Germany explains the variation of electricity prices in Switzerland and how relevant they are for the Swissix price formation process. The results in Tables 6 and 7 show that the explained variation drops particularly for the weekend when demand is low and the share of renewable energies in the overall production is relatively high compared to working days. In all cases, the MAE increases by roughly 10% when wind and PV are not taken into account. This illustrates again that the infeed of renewable energies in Germany clearly has an impact on electricity prices in Switzerland.

6 Conclusion and outlook

In this study, we extended the analytic framework proposed in Paraschiv, Erni, and Pietsch (2014) to assess the cross-border effects between German and Swiss electricity market prices. In the cited paper, the role of market fundamentals and their time-varying impact on Phelix electricity prices has been studied. Given the fact that markets are interconnected, we found that market fundamentals that traditionally impacted electricity prices in Germany show cross-border effects on the neighboring Swiss market. In addition to the original study, we disentangled the effect of fundamentals on prices not only between different hours of one day, but also between working versus weekend days.

It could be observed that Swissix prices adapt continuously to fuel prices, renewable energies (wind and photovoltaic), participant conduct and demand-supply curves that are specific for the German market. Results lead to an intuitive economic interpretation, where marginal effects of fundamentals depend on the different locations of the intersection point between demand and supply curves within a day. Hence, fundamentals impact Swissix prices differently, dependent on the time of the day, given that the input mix differs across peak and off-peak delivery periods.

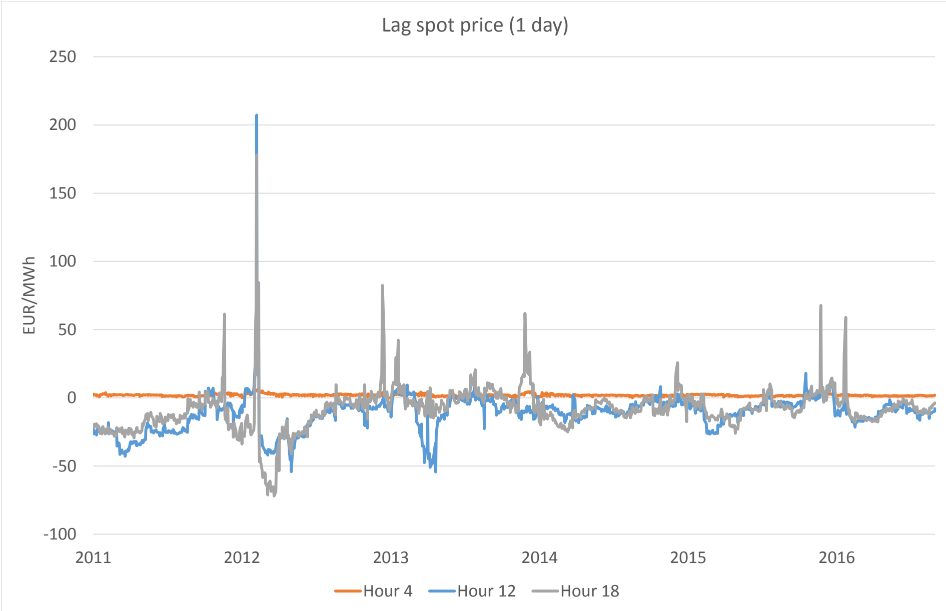
We found that during working days fuel prices of coal, gas and (to some extent) oil have time-varying marginal effects on Swiss electricity prices and their coefficients are comparatively large in absolute values. Depending on the time of the day where prices are observed, price adaption to fuels can be explained by the substitution among those, in particular coal and gas in the mid-region of the supply curve, or by their interaction with renewable energies such as gas/wind or gas/PV when prices are in the upper (steeper) segment of the merit order curve. However, the impact of fuel prices drops during weekends when demand is lower and, thus, the price formation arises mainly from the interaction between coal and renewable energies.

In summary, Switzerland imports cheaper electricity prices over time, marginally due to the increase in renewable energies in its neighbour country Germany. Indeed, we found time-varying negative marginal effects of wind and PV on Swiss electricity prices, independent on the time of the day and for all weekdays. This result is of great relevance for Swiss policy makers with respect to the local regulation of the promotion of renewable energies: Switzerland imports lower electricity prices due to energy transition in Germany. In particular, because of the high infeed of PV during peak hours the spread between Swissix peak and off-peak prices narrowed significantly over time, as a consequence of market interconnectedness with Germany. Incentives for investments in renewable energies in Switzerland as well as subsidies for hydropower should be considered in the light of these insights.

A Appendix: Marginal effects in EUR/MWh

On the following pages, we show the evolution of the marginal effects expressed in EUR/MWh. The corresponding numbers were derived by multiplication of the estimated time-varying coefficients with the corresponding input data. Fuel prices (coal, gas, oil) are given in EUR/MWh. Values for demand, power plant availability and expected infeed from renewable energies (wind, PV) are given in MW. Note that prices of CO2 emission allowances are quoted in EUR per metric ton. Therefore, the time-varying coefficients of CO2 shown in Figures 6 and 11 have the unit "tonne per MWh". This shows that the marginal effects reported here are also affected by the different efficiencies of the generation technologies that were in use at a particular time point.

We also report the mean values and standard deviations of the marginal effects, again measured in EUR/MWh, for each individual year as well as for the whole sample period in Table 8 for working days and Table 9 for weekends. This quantifies how much the various fundamentals contribute to the Swissix price in different periods. Finally, the standard deviations express the variability of the marginal effects within each year.



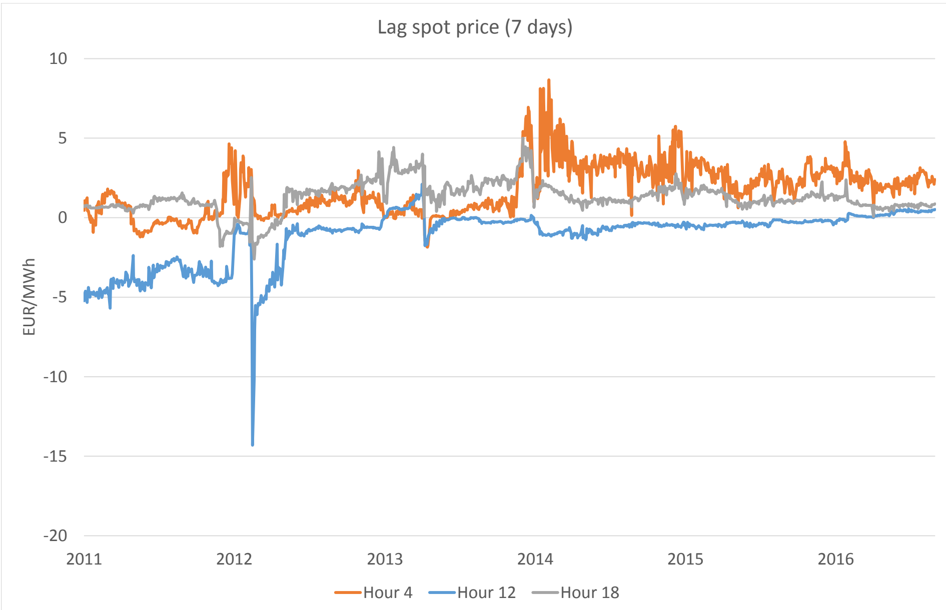


Figure 15: Time-varying marginal effects in EUR/MWh between the hourly day-ahead Swissix and the lagged price (same hour, previous day and same hour 7 days ago) for **working days**.

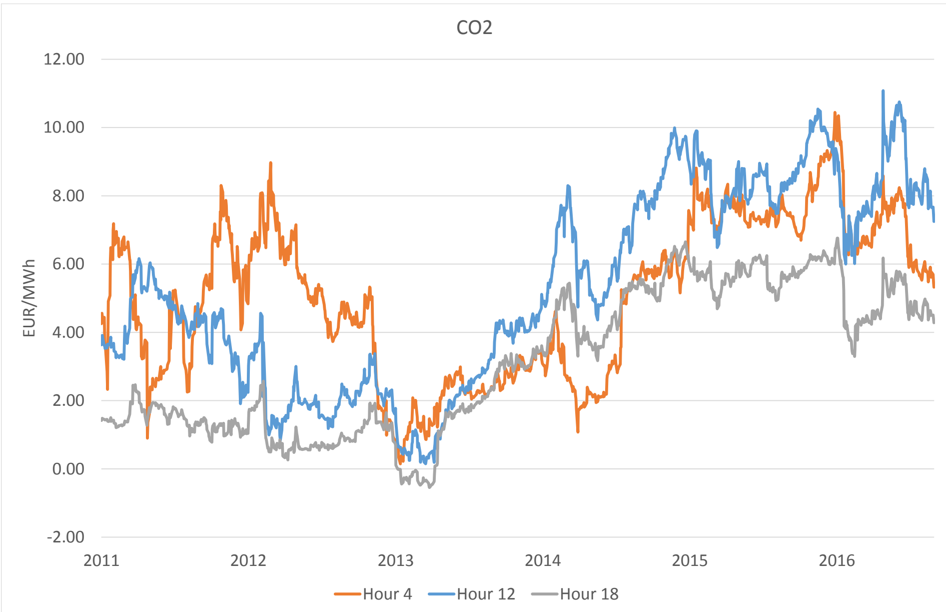
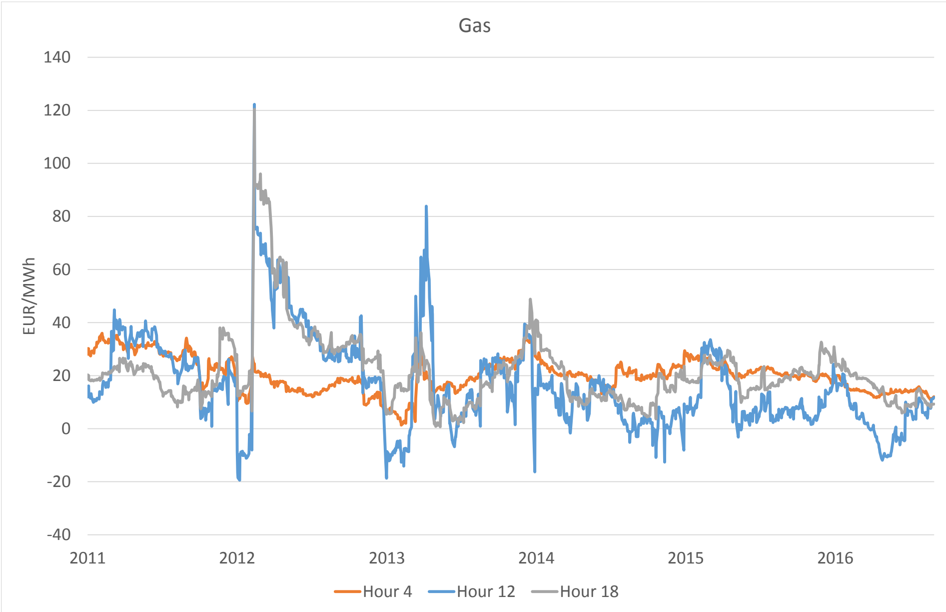


Figure 16: Time-varying marginal effects of coal and CO2 for **working days**.



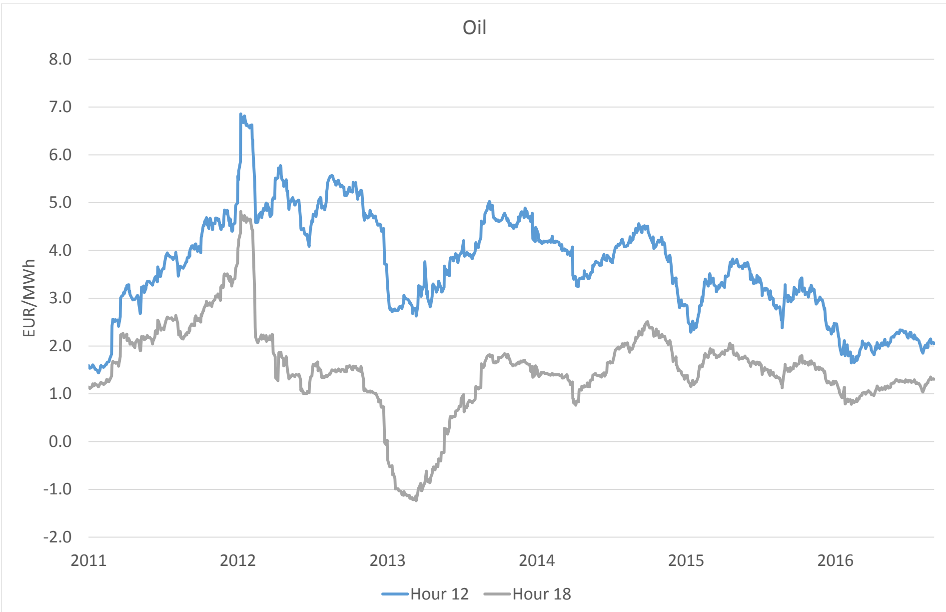
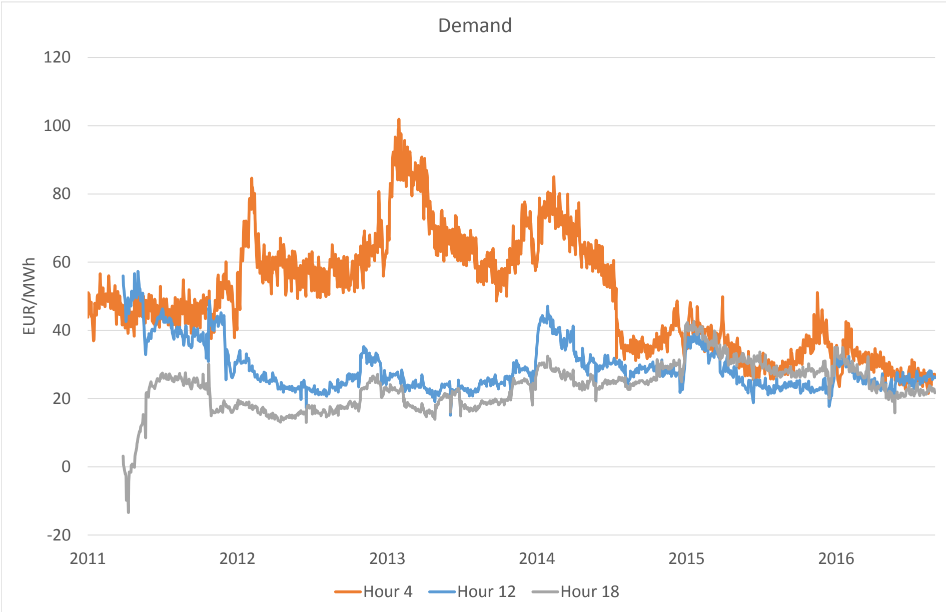


Figure 17: Time-varying marginal effects of gas and oil in EUR/MWh for **working days**.



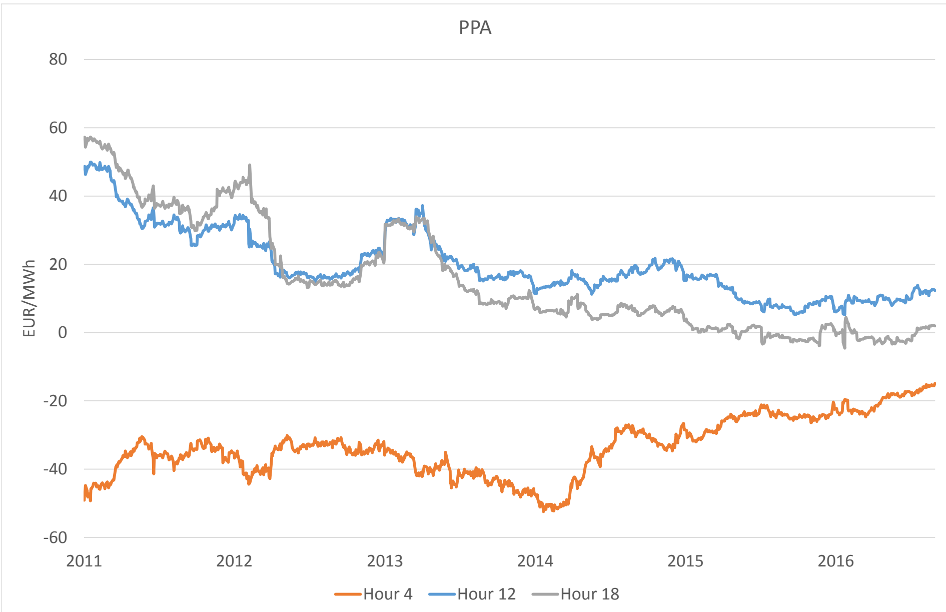
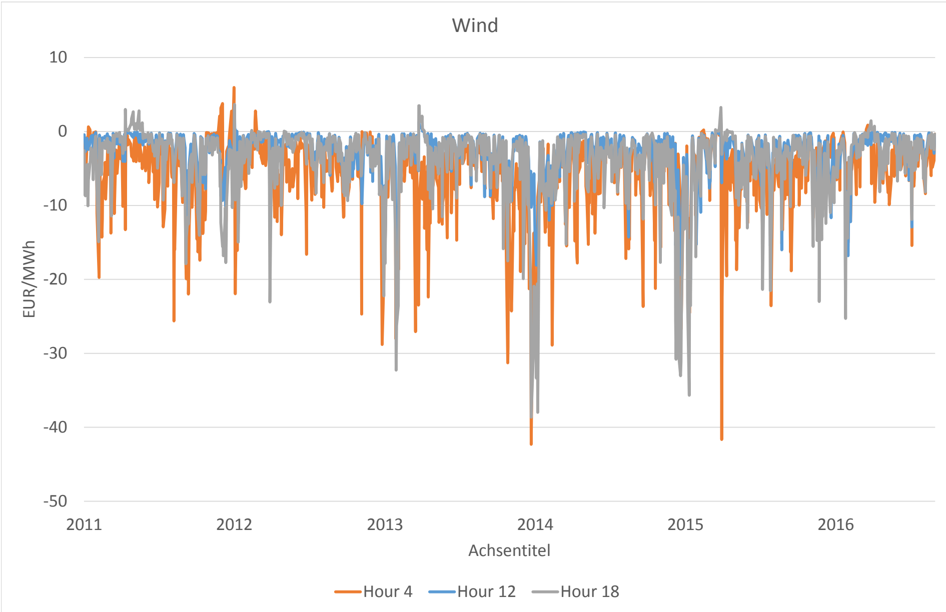


Figure 18: Time-varying marginal effects of demand and power plant availability (PPA) in EUR/MWh for **working days**.



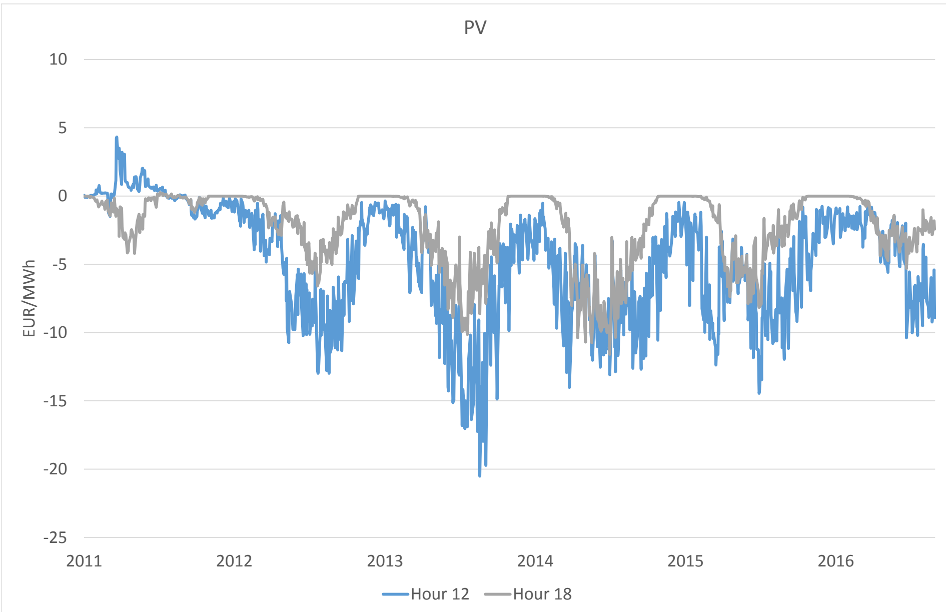
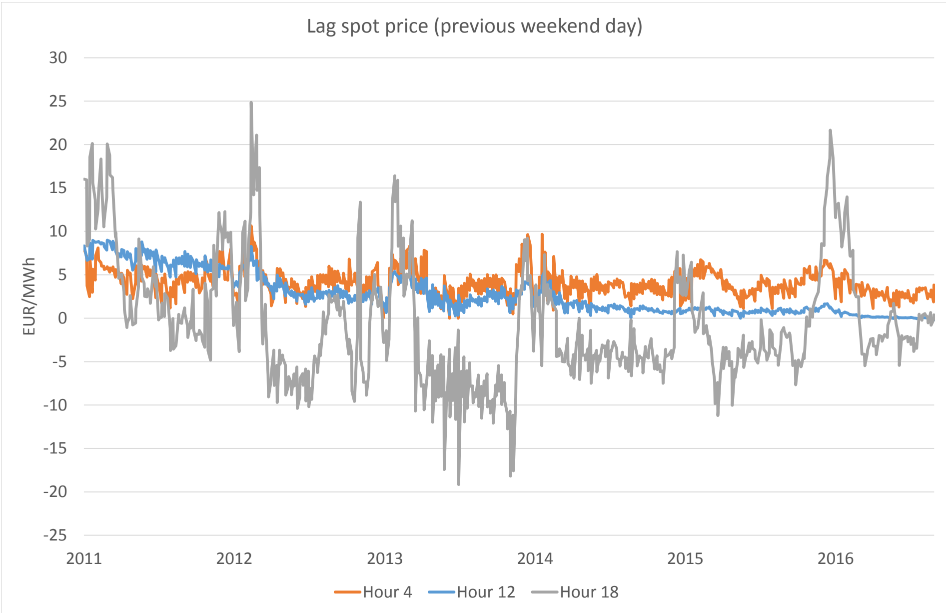


Figure 19: Time-varying marginal effects of wind and photovoltaic in EUR/MWh for **working days**.



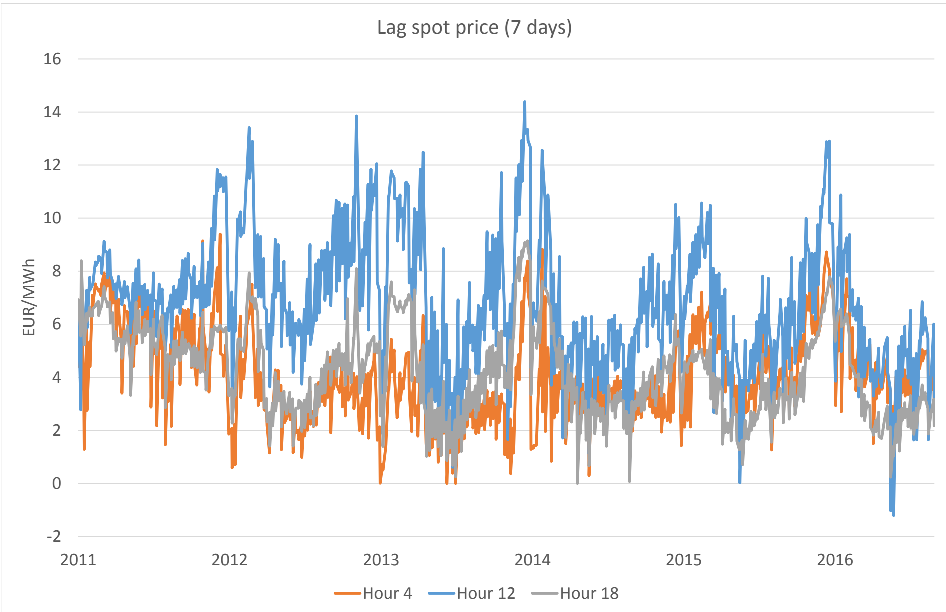
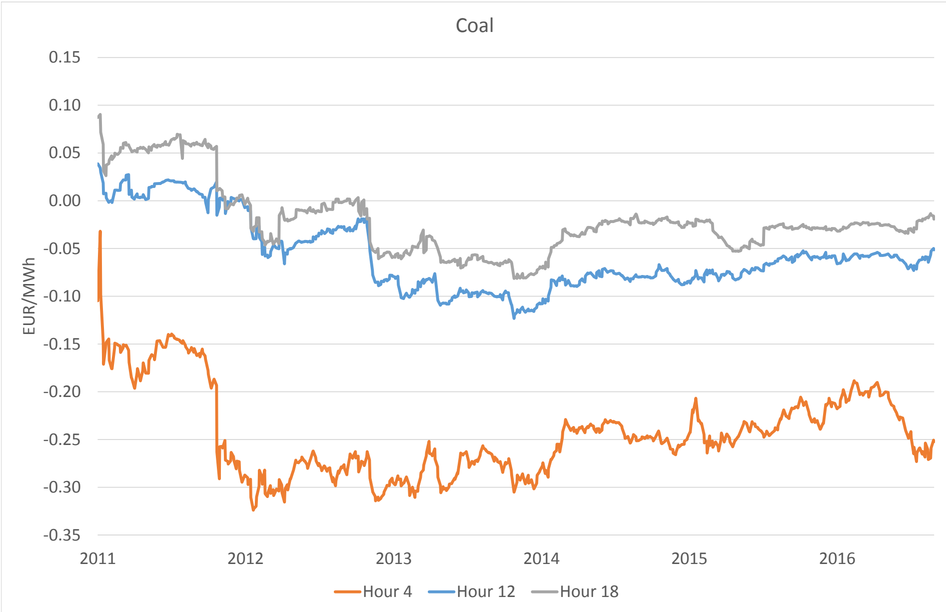


Figure 20: Time-varying marginal effects in EUR/MWh between the hourly day-ahead Swissix and the lagged price (same hour, previous day and same hour 7 days ago) for **weekend days**.



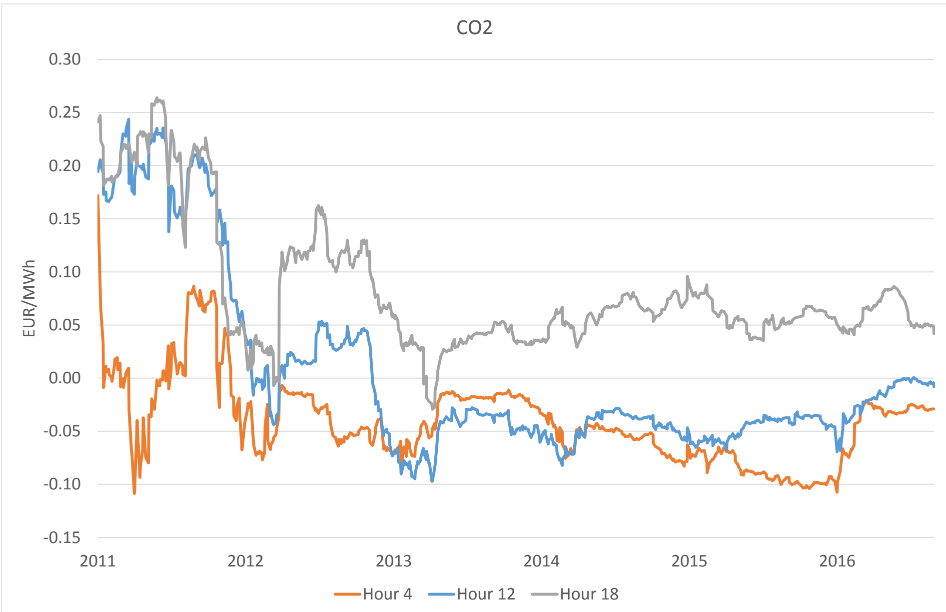


Figure 21: Time-varying marginal effects of coal and CO2 in EUR/MWh for **weekend days**.

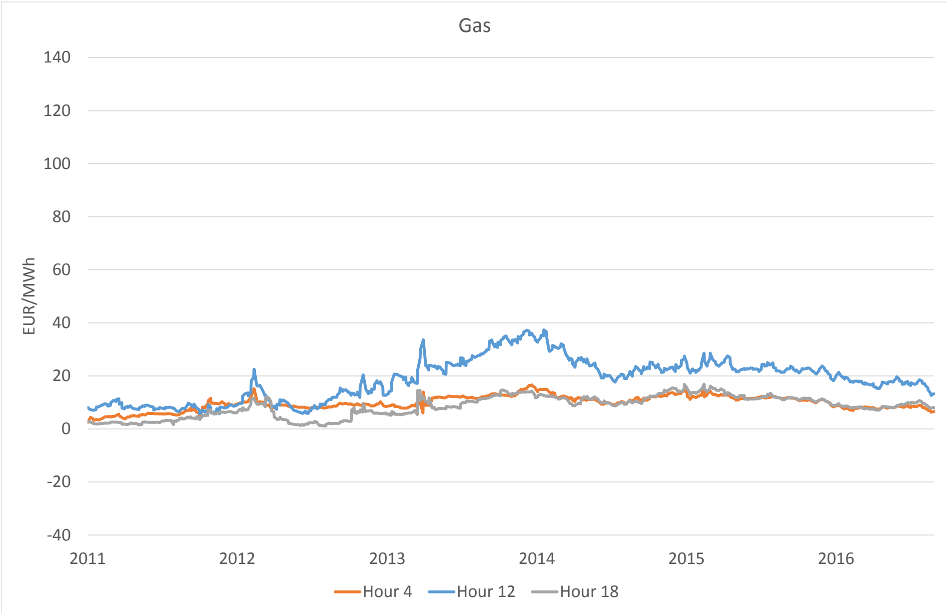
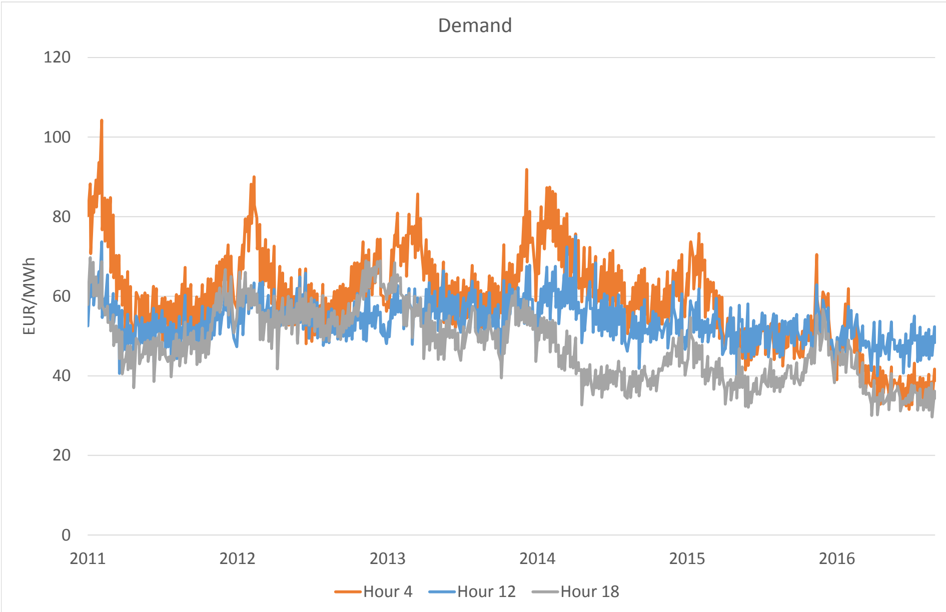


Figure 22: Time-varying marginal effects of gas and oil prices in EUR/MWh for **weekend days**.



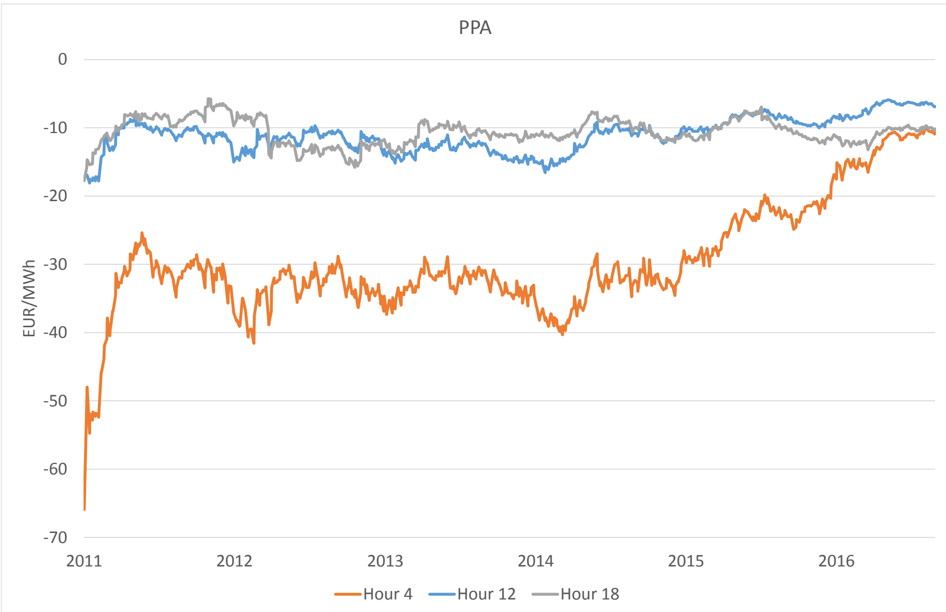
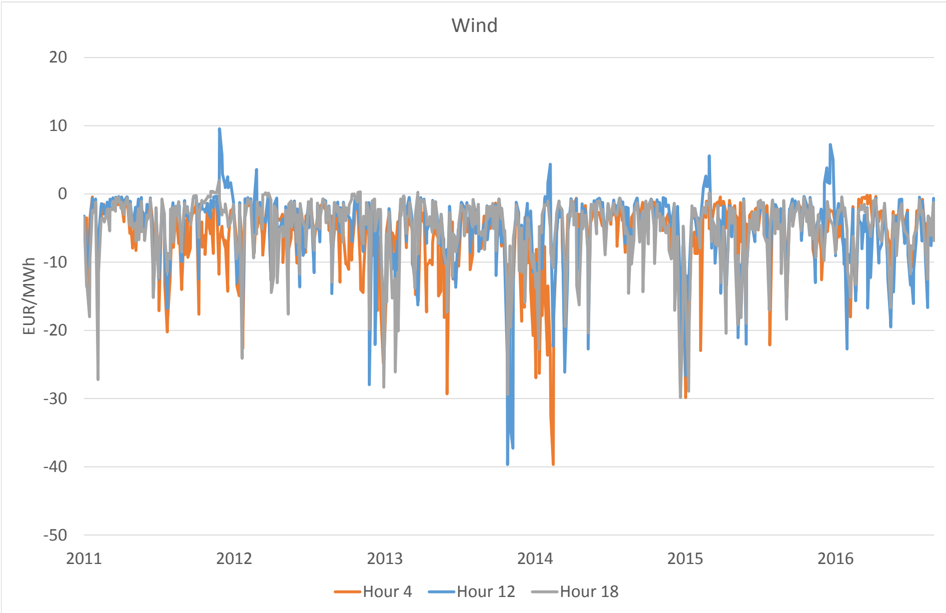


Figure 23: Time-varying marginal effects of demand and power plant availability (PPA) in EUR/MWh for **weekend days**.



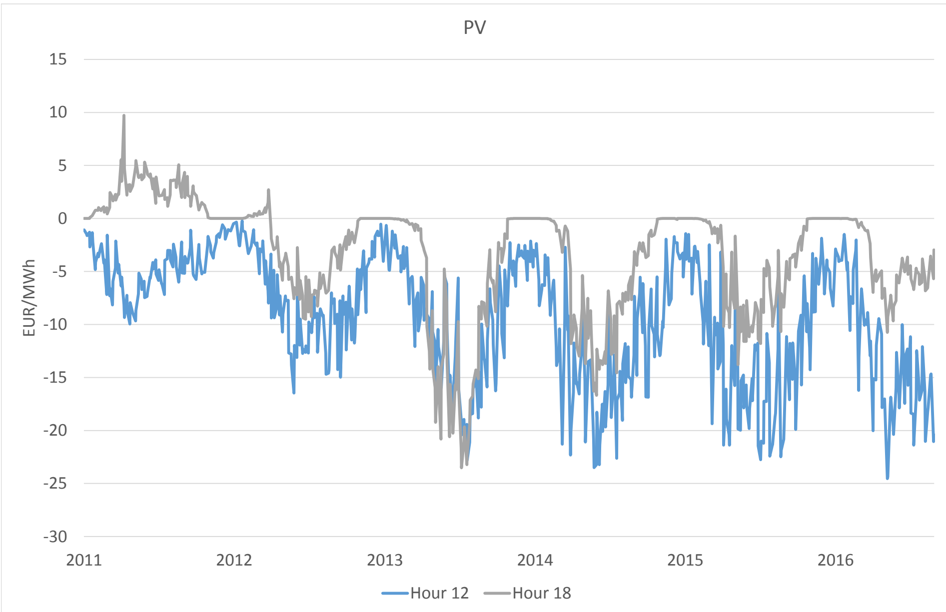


Figure 24: Time-varying marginal effects of wind and photovoltaic in EUR/MWh for **weekend days**.

Hour 4		Lag Spot (1d)	Lag Spot (7d)	Coal	Gas	Oil	CO2	Wind	PPA	Demand	PV
2011	mean	1.99	0.25	-0.52	28.21	n.a	4.88	-4.95	-37.27	47.46	n.a.
	std.	0.67	1.01	1.59	4.69		1.61	5.16	4.69	4.23	
2012	mean	2.44	1.02	-5.99	15.46	n.a	5.06	-4.42	-35.14	60.85	n.a.
	std.	1.02	0.81	1.83	3.62		1.76	4.27	3.38	6.77	
2013	mean	2.02	0.85	-8.07	18.52	n.a	2.18	-7.35	-41.26	70.58	n.a.
	std.	0.93	1.47	1.55	8.25		0.79	6.51	3.74	12.06	
2014	mean	1.78	3.70	-6.98	20.71	n.a	4.15	-6.41	-37.41	53.19	n.a
	std.	0.54	1.37	2.11	2.81		1.63	5.56	8.49	16.40	
2015	mean	1.92	2.57	-1.18	21.93	n.a	7.78	-5.73	-25.57	34.75	n.a
	std.	0.61	0.71	0.58	2.87		0.72	5.54	2.65	5.63	
2016	mean	1.62	2.41	-1.26	14.27	n.a	6.99	-4.00	-19.79	29.59	n.a
	std.	0.39	0.59	0.62	1.85		1.09	2.96	2.85	4.06	
2011	mean	1.98	1.77	-4.15	20.19	n.a	5.08	-5.56	-33.46	50.49	n.a
–2016	std.	0.78	1.62	3.46	6.46		2.27	5.33	8.54	16.72	

Hour 12		Lag Spot (1d)	Lag Spot (7d)	Coal	Gas	Oil	CO2	Wind	PPA	Demand	PV
2011	mean	20.62	-3.82	0.11	23.82	3.40	4.26	-2.24	35.67	26.01	0.11
	std.	11.27	0.67	0.03	10.10	1.01	0.93	2.49	7.09	3.78	1.09
2012	mean	13.27	-1.70	0.17	33.34	5.14	2.13	-2.02	21.08	24.52	-5.06
	std.	22.23	1.98	0.03	23.24	0.61	0.70	2.11	5.56	2.44	3.62
2013	mean	-8.10	0.01	0.08	15.47	3.84	2.42	-2.87	22.85	31.89	-6.98
	std.	13.12	0.57	0.01	18.68	0.75	1.48	3.11	6.94	5.45	4.99
2014	mean	-7.80	-0.69	0.10	7.89	3.93	7.06	-3.58	16.53	26.71	-6.57
	std.	4.60	0.29	0.01	6.99	0.40	1.53	3.94	2.68	5.21	3.35
2015	mean	-6.76	-0.36	0.12	11.07	3.11	8.64	-3.98	10.50	25.71	-5.97
	std.	8.78	0.15	0.01	8.81	0.39	0.87	4.17	3.65	2.74	3.28
2016	mean	-11.01	0.23	0.11	3.97	2.05	8.27	-2.28	9.87	21.24	-3.95
	std.	4.32	0.21	0.01	8.23	0.18	1.17	2.96	1.72	6.73	2.76
2011	mean	-11.27	-1.13	0.12	16.60	3.66	5.31	-2.86	19.95	26.28	-4.78
–2016	std.	13.49	1.66	0.03	17.32	1.09	2.88	3.31	10.10	5.41	4.20

Hour 18		Lag Spot (1d)	Lag Spot (7d)	Coal	Gas	Oil	CO2	Wind	PPA	Demand	PV
2011	mean	-15.43	0.65	0.07	19.78	2.28	1.46	-3.43	42.32	17.65	-0.65
	std.	10.74	0.65	0.01	6.84	0.65	0.33	4.19	7.92	3.21	0.99
2012	mean	-14.99	1.03	0.03	39.98	1.83	1.01	-2.77	22.62	19.87	-1.70
	std.	27.73	1.33	0.04	22.24	1.06	0.49	3.17	10.68	2.92	1.77
2013	mean	1.85	2.39	-0.03	16.90	0.48	1.61	-4.22	18.74	26.41	-2.94
	std.	10.37	0.87	0.02	9.98	1.12	1.35	5.85	9.76	2.50	3.08
2014	mean	-8.52	1.39	0.00	15.24	1.68	4.85	-5.65	6.45	31.36	-3.59
	std.	7.77	0.41	0.02	7.68	0.44	0.91	6.58	1.43	4.58	3.59
2015	mean	-5.26	1.27	0.04	20.89	1.58	5.75	-4.76	-0.02	24.06	-2.26
	std.	10.59	0.34	0.01	4.96	0.22	0.37	5.39	1.78	3.99	2.44
2016	mean	-8.41	0.84	0.06	14.51	1.11	4.79	-2.83	-0.82	19.88	-1.88
	std.	8.43	0.27	0.02	5.65	0.15	0.71	3.51	1.89	3.55	1.46
2011	mean	-8.47	1.29	0.03	21.60	1.52	3.17	-4.01	15.76	23.40	-2.19
–2016	std.	15.78	0.95	0.04	14.41	0.93	2.09	5.11	16.60	5.90	2.62

Table 8: Means and standard deviations of the marginal effects in EUR/MWh per year and for the whole sample period for **working days**.

Hour 4		Lag Spot (1d)	Lag Spot (7d)	Coal	Gas	Oil	CO2	Wind	PPA	Demand	PV
2011	mean	4.87	5.66	-0.18	6.17	n.a.	0.01	-5.28	-34.19	62.58	n.a.
	std.	1.45	1.65	0.05	2.03		0.05	4.49	7.71	11.49	
2012	mean	4.27	3.28	-0.29	9.08	n.a.	-0.04	-6.51	-33.78	62.67	n.a.
	std.	1.79	1.37	0.02	1.20		0.02	5.08	2.62	8.98	
2013	mean	4.15	2.89	-0.28	11.64	n.a.	-0.03	-7.63	-32.90	65.00	n.a.
	std.	2.20	1.71	0.01	2.24		0.02	5.18	1.78	8.42	
2014	mean	3.63	3.24	-0.25	11.76	n.a.	-0.06	-6.57	-33.88	64.86	n.a.
	std.	1.34	1.26	0.01	1.59		0.01	7.41	3.07	8.90	
2015	mean	4.17	4.79	-0.23	11.69	n.a.	-0.09	-5.01	-23.99	53.44	n.a.
	std.	1.35	1.64	0.01	1.05		0.01	5.04	3.12	8.07	
2016	mean	3.04	4.22	-0.22	8.00	n.a.	-0.04	-5.28	-12.91	40.65	n.a.
	std.	0.97	1.33	0.03	0.62		0.02	4.36	2.28	6.52	
2011 –2016	mean	4.08	3.99	-0.24	9.83	n.a.	-0.04	-6.10	-29.54	59.25	n.a.
	std.	1.68	1.81	0.05	2.69		0.04	5.47	8.15	11.92	

Hour 12		Lag Spot (1d)	Lag Spot (7d)	Coal	Gas	Oil	CO2	Wind	PPA	Demand	PV
2011	mean	6.90	7.53	0.01	8.11	0.88	0.18	-3.08	-11.54	53.35	-4.41
	std.	1.19	1.52	0.01	1.12	0.20	0.05	4.28	2.38	5.70	2.31
2012	mean	3.36	8.04	-0.04	11.88	-1.20	0.01	-5.11	-11.84	54.88	-7.21
	std.	1.36	2.32	0.02	3.70	0.63	0.03	5.32	1.16	4.33	4.20
2013	mean	2.75	7.24	-0.10	26.71	-2.38	-0.05	-6.69	-13.44	56.71	-9.29
	std.	1.29	3.42	0.01	6.19	0.28	0.02	6.95	1.04	4.69	5.73
2014	mean	1.44	5.90	-0.08	24.63	-1.82	-0.05	-4.83	-12.00	55.43	-11.73
	std.	0.97	2.28	0.01	4.51	0.52	0.01	5.63	2.00	5.99	5.83
2015	mean	0.88	6.68	-0.07	23.05	-0.99	-0.05	-4.85	-9.06	51.43	-12.09
	std.	0.30	2.54	0.01	1.78	0.23	0.01	5.76	0.95	4.41	6.26
2016	mean	0.19	4.97	-0.06	17.38	-1.00	-0.02	-7.06	-7.09	48.60	-12.69
	std.	0.24	2.20	0.00	1.69	0.16	0.02	5.13	0.93	3.72	6.00
2011 –2016	mean	2.72	6.83	-0.06	18.72	-1.10	0.00	-5.17	-11.06	53.69	-9.39
	std.	2.41	2.64	0.04	7.93	1.11	0.09	5.72	2.50	5.52	5.96

Hour 18		Lag Spot (1d)	Lag Spot (7d)	Coal	Gas	Oil	CO2	Wind	PPA	Demand	PV
2011	mean	4.94	5.54	0.05	3.48	0.65	0.19	-3.90	-9.23	50.82	2.04
	std.	6.46	0.93	0.02	1.61	0.29	0.06	4.38	2.36	7.17	1.77
2012	mean	-1.10	4.19	-0.02	4.95	-0.46	0.09	-4.56	-12.21	56.98	-2.69
	std.	7.28	1.45	0.02	3.07	0.44	0.04	5.18	2.19	5.49	3.06
2013	mean	-4.13	4.45	-0.06	10.11	-1.52	0.03	-6.15	-11.01	53.18	-6.82
	std.	7.75	2.26	0.01	3.01	0.23	0.02	5.43	1.18	5.62	7.57
2014	mean	-2.86	3.76	-0.03	11.55	-0.81	0.06	-5.92	-10.28	42.47	-4.96
	std.	3.46	1.55	0.01	1.85	0.39	0.01	5.76	1.23	5.22	5.17
2015	mean	-1.29	3.77	-0.03	12.38	-0.62	0.06	-6.03	-10.10	41.49	-4.07
	std.	5.90	1.55	0.01	1.62	0.24	0.01	5.51	1.44	5.15	4.16
2016	mean	0.83	3.19	-0.03	8.56	-0.37	0.06	-5.46	-10.97	37.13	-3.87
	std.	5.22	1.50	0.00	0.89	0.07	0.01	3.86	1.03	4.97	2.98
2011 –2016	mean	-0.70	4.20	-0.02	8.50	-0.53	0.08	-5.33	-10.62	47.61	-3.38
	std.	6.88	1.74	0.04	4.03	0.73	0.06	5.18	1.92	8.90	5.38

Table 9: Means and standard deviations of the marginal effects in EUR/MWh per year and for the whole sample period for **weekend days**.

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Estimation and application of fully parametric multifactor quantile regression with dynamic coefficients

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October 10, 2016

Abstract

This paper develops and applies a novel estimation procedure for quantile regressions with time-varying coefficients based on a fully parametric, multifactor specification. The algorithm recursively filters the multifactor dynamic coefficients with a Kalman filter and parameters are estimated by maximum likelihood. The likelihood function is built on the Skewed-Laplace assumption. In order to eliminate the non-differentiability of the likelihood function, it is reformulated into a non-linear optimisation problem with constraints. A relaxed problem is obtained by moving the constraints into the objective, which is then solved numerically with the Augmented Lagrangian Method. In the context of an application to electricity prices, the results show the importance of modelling the time-varying features and the explicit multi-factor representation of the latent coefficients is consistent with an intuitive understanding of the complex price formation processes involving fundamentals, policy instruments and participant conduct.

JEL Classification: *C01*; *C13*; *C22*
Keywords: *Quantile Regression*; *Dynamic Coefficients*; *Parametric Estimation*; *Electricity Prices*

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1 Introduction

The estimation of price quantiles in financial, commodity and product markets has attracted substantial and increasing methodological research not only because of its technical challenges but also as a practical consequence of risk management and regulatory compliance procedures that require explicit controls at specific probability levels. Whilst tail probabilities in time series densities typically pose problems of robust estimation and therefore require substantial empirical histories, the underlying price formations in many markets often have evolutionary properties as technology, market structure and participant conduct change over time, and so a crucial technical task is the estimation of extreme quantiles that have dynamic dependences upon fundamental factors. In its fully specified form, this dynamic representation is an under-researched topic.

Various researchers have developed quantile models with, to some extent, dynamic characteristics. These include the use of a lagged dependent variable (ie quantile) to model dynamic adaptation, or a dependence upon exogenous factors which induce dynamic quantiles through the evolution of the factors, or via a stochastic representation of the time series heteroscedasticity, eg by means of a GARCH feature. None of these approaches estimate time varying quantile coefficients upon fundamental factors, and the contribution of this paper is therefore motivated by these requirements. We suggest that this context represents an important class of problems in practice and provide a detailed application to electricity prices in which the quantile coefficients change dynamically within the day as demand, wind and solar variables fluctuate, as well as monthly as gas and coal prices change, and annually as underlying demand and supply characteristics evolve. The Maximum Likelihood estimation of these multi-factor quantile coefficients is awkward, being nonlinear and nondifferentiable, and we develop a new ML estimation process for this purpose based upon nonlinear optimisation solved by the augmented Lagrangian method.

The conventional quantile regression formulation usually proceeds as follows. Given a set of T observations, $y_t, t = 1, ..., T$, define $q(\alpha)$ as the α-th quantile. The probability that an observation is less than $q(\alpha)$ is α, where $0 < \alpha < 1$. In quantile regression, the quantile $q_t(\alpha)$ corresponding to the t-th observation is a linear function of explanatory variables, x_t, that is $q_t = x_t'\beta$. The quantile regression estimates are obtained by minimising $\sum_t \rho_\alpha(y_t - x_t'\beta)$ with respect to the parameter vector β (see Rossi & Harvey (2006)). Different weights ρ_α are applied, depending on whether y_t is to the left or to the right of $q_t(\alpha)$. Estimates computed by linear programming, following Koenker & Bassett (1978), provide a semi-parametric approach as no distributional assumptions are required. However parametric methods that make distributional assumptions at each quantile level can provide alternative estimators, are analytically attractive and facilitate extended specifications to include, for example, dynamic characteristics.

Thus, Rossi & Harvey (2006) formulate a dynamic model with autoregressive quantiles Q_t where the likelihood function at each quantile is built on the assumption of an asymmetric double exponential distribution for the observations. The authors show that maximising the log-likelihood function is equivalent to minimising the criterion function in the linear programming approach. Other parametric formulations have used a Skewed-Laplace (SL) distribution for the observations to achieve a similar equivalence, eg Yu & Moyeed (2001), Tsionas (2003) and Gerlach et al. (2011). In the latter case the dynamic quantiles result from the inclusion of time-dependent volatility. Where exogenous fundamental factors are known to influence price formation, it is often the case that the autocorrelation apparently present in the time series is better reflected in the price adaption process to market fundamentals (electricity prices are a good example, as in Karakatsani & Bunn (2010) and Paraschiv et al. (2014)). Hence there are benefits in formulating dynamic quantiles with time-varying coefficients on multiple factors, and an intuitive attraction to modelling explicitly the factors that might influence market participant behaviour prior to market clearing.

Variable coefficient models for conditional quantiles applied to independent data were developed in non time series contexts by Honda (2004) and Kim (2007), using local polynomials and polynomial-splines, respectively. In our time series context, instead of predefining polynomials to describe the variation in coefficients, we model them as unobserved latent variables in a state-space formulation. Furthermore, in a multi-factor Kalman filter (Kalman (1960)) representation both the observations and latent states can be expressed as functions of exogenous fundamental variables. Quantiles can be formulated using the parametric Skewed-Laplace assumption, but maximum likelihood estimation is awkward. Facing a similar problem in estimating a T-CAViaR model, Gerlach et al. (2011) emphasize the difficulty of solving the non-linear, stepwise likelihood function based on the SL-density of observations and they used a pragmatic Bayesian specification. In order to eliminate the non-differentiability of the likelihood function caused by the indicator function, we reformulated the problem as a non-linear optimisation with constraints. We obtained a relaxed problem by moving the constraints into the objective, which is then numerically solved with the Augmented Lagrangian Method. The application to electricity prices demonstrates the value of this approach in terms of the detailed specification and the explicit, plausible transparency of the time-varying quantile drivers.

In the next section we describe the model specification and estimation process in general. Then we present a fundamental description of price formation in the German wholesale electricity market, which is the main reference for electricity trading in Europe. The results follow, with some new interpretations that reflect the complexity of fundamental interactions and advance our understanding of electricity price risk drivers, as revealed by this methodology. Section four concludes the paper.

2 Model specification

2.1 Preliminaries

The general specification of a dynamic time series quantile model is:

$$y_t = f_t(\beta, x_t) + u_t \tag{1}$$

with $t = 1, ..., n$ the time dimension; $y_t \in \mathbb{R}$ is the dependent variable; $x_t = (x_{1t}, ..., x_{pt})' \in \mathbb{R}^p$ are the explanatory variables; β is a p-vector of unknown parameters and u_t is an error term.

The conditional $\alpha \in (0, 1)$ level quantile is then

$$q_\alpha(y_t|\beta^\alpha, x_t) = f_t(\beta^\alpha, x_t) \tag{2}$$

For any $\alpha \in (0, 1)$ the distance from y_t to a given quantile level q_α is measured by the absolute distance, but different weights are applied depending on whether y_t is to the left or to the right of q_α (see Hao & Naiman (2007)). Thus, the distance from y_t to a given q_α is defined:

$$d_\alpha(y_t, q_\alpha) = \begin{cases} (\alpha - 1)|y_t - q_\alpha|, & y_t < q_\alpha \\ \alpha|y_t - q_\alpha|, & y_t \geq q_\alpha \end{cases} \tag{3}$$

We look for the value q_α that minimises the mean distance from y_t: $\mathbb{E}[d_\alpha(y_t, q_\alpha)]$. As shown in Hao & Naiman (2007) and Gerlach et al. (2011) the minimum occurs when q_α is the α quantile of y_t. q_α depends on β^α which is the solution to:

$$\min_\beta \sum_t \rho_\alpha(y_t - f_t(\beta, x_t)), \tag{4}$$

where $\rho(\cdot)$ is a loss function specified as:

$$\rho_\alpha(u_t) = u_t \cdot (\alpha - \mathbf{1}(u_t < 0)) \tag{5}$$

with $u_t = y_t - f_t(\beta, x_t)$.

Below we show the link between the minimisation problem given in Equation (4) and the likelihood function derived based on the assumption of Skewed-Laplace (SL) distributed residuals u_t. Based on the density of the Skewed Laplace distribution, the likelihood function is derived as:

$$\text{Ł}_\alpha(\beta, \tau; y, x) \propto \tau^{-n} \exp\left\{-\tau^{-1}\sum_{t=1}^{n}(y_t - f_t(\beta, x_t)) \times [\alpha - \mathbf{1}_{(-\infty,0)}(y_t - f_t(\beta, x_t))]\right\}. \quad (6)$$

We observe that the summation to be minimised in Equation (4) is contained in the exponent of the likelihood. Thus, the maximum likelihood estimation for β is equivalent to the quantile estimator in (4) (see Gerlach et al. (2011)).

2.2 Extension to quantile regression with time-varying coefficients

We extend the model version in Equation (1) by introducing time-varying coefficients. We therefore formulate a state space model with time-dependent coefficients recursively filtered with a Kalman filter (Kalman (1960)) and parameters estimated by maximum likelihood. The likelihood function is similar to the specification in (6) but we allow for time-varying β_t. The assumption that u_t follow a $SL(\mu, \tau, \alpha)$ distribution is used here to estimate parametrically the quantiles of the dependent variables y_t. The SL assumption is a realistic choice for distributions with fat tails.

The state space formulation reads:

$$y_t^\alpha = (\beta_t^\alpha)' x_t + u_t \quad (7)$$

$$\beta_t^\alpha = c + D\beta_{t-1}^\alpha + w_t \quad (8)$$

where $u_t \sim \mathrm{SL}(0, \tau, \alpha)$, Ξ is the variance of residuals u_t and $w_t \sim \mathrm{N}(0, \Omega)$. Equation (7) is the measurement equation, which relates a known quantity (vector of exogenous variables) x_t to an observed variable y_t, while Equation (8) describes the process governing the unobserved state β_t^α and is a latent transition equation. In our model specification, we do not predefine the process driving the coefficients β_t^α as random walk, but we formulate a more flexible model to allow for mean reversion and subsequently exogenous influences on the latent states as well.

In Figure 1 an overview of the recursive algorithm for filtering the coefficients is provided, as pre-requisite for the maximisation of the likelihood function. Essentially it consists of a sequence of equations that implement a predictor-corrector type estimator for the unobserved state variables with two alternating steps: In the first *prediction step*, the Kalman filter forms an optimal predictor of the unobserved state β_t given all the information available up to time $t-1$. To this end, the state vector is extrapolated and prior estimates for time t are obtained. In the subsequent *updating step*, new information that becomes available at time t is used to update the prior estimates for β_t to obtain the posterior estimates. Below, we denote by $\beta_{t|t-1}$ the prior estimate and by $\beta_{t|t}$ the posterior estimate for the state β_t (here the specific quantile level α is skipped in the notation for simplicity).

The likelihood function is built on the prediction errors $u_t := y_t - f_t(\beta_t, x_t)$, where $f_t(\beta_t, x_t) := (\beta_{t|t-1})' x_t$ due to the linear model structure and $\beta_{t|t-1}$ are the prior estimates of the time-varying coefficients. There are several challenges for the maximisation procedure to obtain the optimal likelihood value and parameter estimates. Firstly, in each iteration of the optimisation procedure the likelihood function is updated with estimates of the time dependent state β_t which requires a new run through the Kalman filter and enhances the complexity of the whole procedure. Another difficulty comes from the shape of the likelihood function which is *piecewise* (V-shape) and *nonlinear* due to the indicator function.

The advantage of solving quantile regressions by a fully parametric approach is that by filtering the coefficients with the Kalman Filter, this is generally considered to be robust with respect to the variance and distribution of residuals. However, one important consideration is that in this approach, each quantile is independently estimated. Whilst it might be thought there should be a monotonic ordering constraint across quantiles, it is not necessarily the case that there should be a monotonic relationship across the quantile coefficients for each factor. We did not impose any ordering constraints and return to this point when discussing the results of the application to the electricity prices.

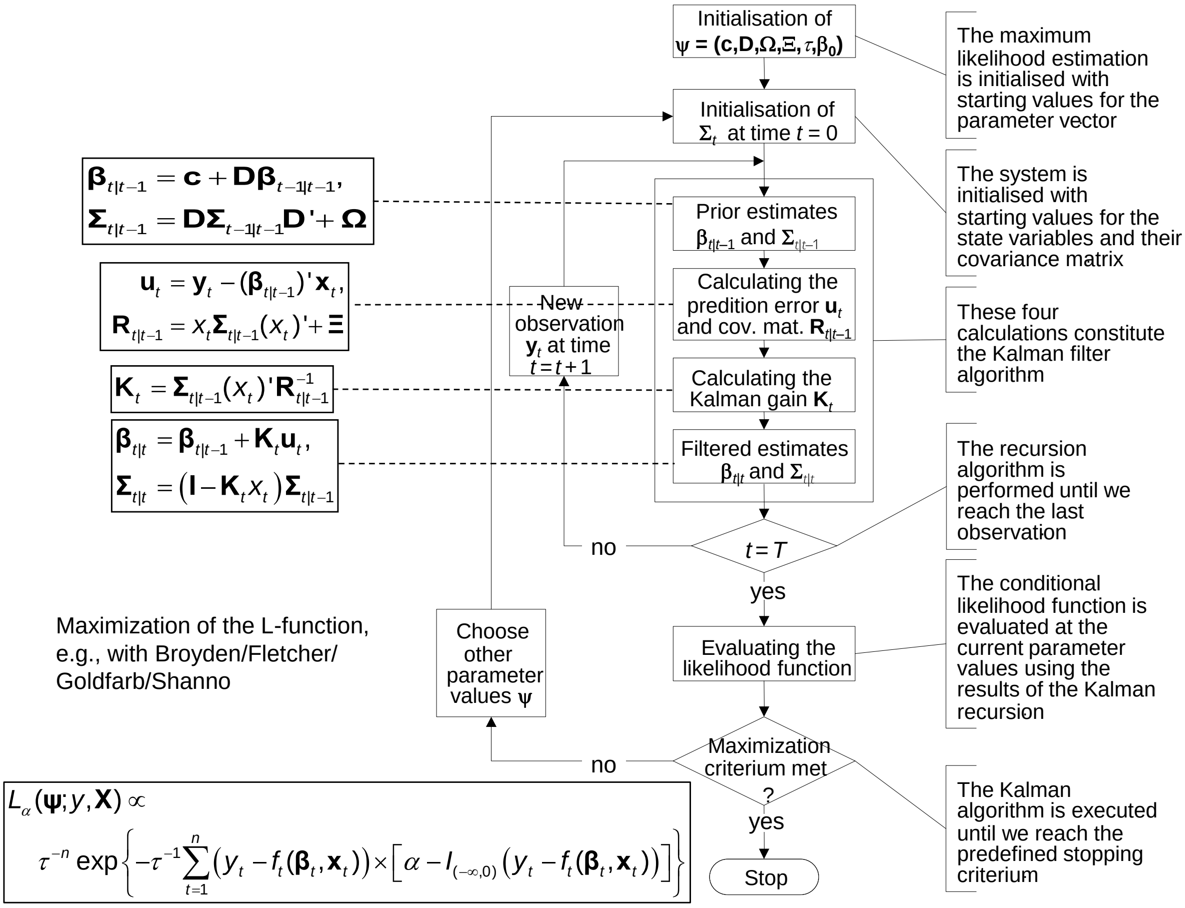
2.3 Estimation procedure

As outlined in the above subsection, solving the non-linear, stepwise likelihood function based on the SL-density of observations poses several challenges for the optimisation. Since the function is piecewise, we proceed with the elimination of the non-differentiability of the likelihood function caused by the indicator function. We further reformulate the problem as a non-linear optimisation with constraints and obtain a relaxed problem by moving the constraints into the objective, which is then solved numerically with the Augmented Lagrangian Method.

Recall from the Kalman filter that the definition of the prediction error based on the prior estimates is:

$$u_t := y_t - (\beta_{t|t-1})' x_t \tag{9}$$

Given the vector of unknown parameters $\Psi = (c, D, \Omega, \Xi, \tau, \beta_0)$, the logarithm of the likelihood function can be written as:



$$\boldsymbol{\beta}_{t|t-1} = \mathbf{c} + \mathbf{D}\boldsymbol{\beta}_{t-1|t-1}, \quad \boldsymbol{\Sigma}_{t|t-1} = \mathbf{D}\boldsymbol{\Sigma}_{t-1|t-1}\mathbf{D}' + \boldsymbol{\Omega}$$

$$\mathbf{u}_t = \mathbf{y}_t - (\boldsymbol{\beta}_{t|t-1})'\mathbf{x}_t, \quad \mathbf{R}_{t|t-1} = x_t\boldsymbol{\Sigma}_{t|t-1}(x_t)' + \boldsymbol{\Xi}$$

$$\mathbf{K}_t = \boldsymbol{\Sigma}_{t|t-1}(x_t)'\mathbf{R}_{t|t-1}^{-1}$$

$$\boldsymbol{\beta}_{t|t} = \boldsymbol{\beta}_{t|t-1} + \mathbf{K}_t\mathbf{u}_t, \quad \boldsymbol{\Sigma}_{t|t} = \left(\mathbf{I} - \mathbf{K}_t x_t\right)\boldsymbol{\Sigma}_{t|t-1}$$

$$L_\alpha(\boldsymbol{\psi}; y, \mathbf{X}) \propto \tau^{-n} \exp\left\{-\tau^{-1}\sum_{t=1}^{n}\left(y_t - f_t(\boldsymbol{\beta}_t, \mathbf{x}_t)\right)\times\left[\alpha - I_{(-\infty,0)}\left(y_t - f_t(\boldsymbol{\beta}_t, \mathbf{x}_t)\right)\right]\right\}$$

Figure 1: Algorithm estimation procedure

$$
\begin{aligned}
\ln \mathrm{L}_\alpha(\Psi; y, x) \propto & - n \ln \tau - \frac{1}{\tau} \sum_{t=1}^{n} u_t \times \left[\alpha - \mathbf{1}_{(-\infty,0)}(u_t)\right] \\
= & - n \ln \tau - \frac{1}{\tau} \left[\alpha \sum_{t=1}^{n} \{u_t | u_t \geq 0\} + (\alpha - 1) \sum_{t=1}^{n} \{u_t | u_t < 0\} \right] \\
= & - n \ln \tau - \frac{1}{\tau} \left[\alpha \sum_{t=1}^{n} \{u_t | u_t \geq 0\} + (1 - \alpha) \sum_{t=1}^{n} \{-u_t | u_t < 0\} \right] \\
= & - n \ln \tau - \frac{\alpha}{\tau} \sum_{t=1}^{n} v_t^1 - \frac{(1-\alpha)}{\tau} \sum_{t=1}^{n} v_t^2
\end{aligned} \tag{10}
$$

where $v_t^1 := \max\{u_t, 0\}$, $v_t^2 := \max\{-u_t, 0\}$ are introduced as linear constraints in the optimisation problem. Thus, the maximisation of the log-likelihood function (LF) is equivalent to a non-linear constrained problem (due to the indicator function in the original formulation of LF). Writing this optimisation problem in the standard formulation as non-linear minimisation problem with linear constraints leads to:

$$
\min n \ln \tau + \frac{\alpha}{\tau} \sum_{t=1}^{n} v_t^1 + \frac{(1-\alpha)}{\tau} \sum_{t=1}^{n} v_t^2 \tag{11}
$$

$$
\begin{aligned}
\text{s.t.} \quad & v_t^1 \geq y_t - (\beta_{t|t-1})' x_t, && t = 1, ..., n \\
& v_t^2 \geq (\beta_{t|t-1})' x_t - y_t, && t = 1, ..., n \\
& v_t^1, v_t^2 \geq 0, && t = 1, ..., n
\end{aligned}
$$

However, the formulation as defined in (11) is not ready to be passed to a solver, since the coefficients $\beta_{t|t-1}$ are not constant. Instead, the time-varying coefficients are functions of the parameter vector Ψ that should be estimated, i.e., $\beta_{t|t-1} := \beta_{t|t-1}(\Psi)$. As a consequence, the estimates $\beta_{t|t-1}$ for $t = 1, ..., n$, which are now part of the constraints, must always be updated with the Kalman filter when the parameter vector changes in one iteration of the optimisation algorithm before the log-likelihood function (i.e., the objective) is evaluated. For that reason, we integrate the constraints into the objective function. In a first step, we rewrite the optimisation problem in 11 as one with equality constraints:

$$\min n \ln \tau + \frac{\alpha}{\tau} \sum_{t=1}^{n} v_t^1 + \frac{(1-\alpha)}{\tau} \sum_{t=1}^{n} v_t^2 \tag{12}$$

$$\begin{aligned} \text{s.t.} \quad & -v_t^1 + v_t^2 + y_t - (\beta_{t|t-1})' x_t = 0, && t = 1, ..., n \\ & v_t^1, v_t^2 \geq 0, && t = 1, ..., n \end{aligned}$$

Now we define $\widetilde{\Psi}$ as a new parameter vector into which also the auxiliary variables $v_1^i, ..., v_n^i$, $i \in \{1, 2\}$, are integrated and

$$\begin{aligned} f(\widetilde{\Psi}) &:= n \ln \tau + \frac{\alpha}{\tau} \sum_{t=1}^{n} v_t^1 + \frac{(1-\alpha)}{\tau} \sum_{t=1}^{n} v_t^2 \\ c_t(\widetilde{\Psi}) &:= -v_t^1 + v_2^1 + y_t - \left(\beta_{t|t}(\widetilde{\Psi})\right)' x_t \end{aligned}$$

so that the problem can be rewritten shortly as

$$\min_{\widetilde{\Psi}} f(\widetilde{\Psi}) \quad \text{s.t.} \quad c_t(\widetilde{\Psi}) = 0; \quad v_t^1, v_t^2 \geq 0, \quad t = 1, ..., n. \tag{13}$$

By moving the equality constraints into the objective as a penalty term, we can solve the problem with the Augmented Lagrangian Method (e.g., see Noecdal/Wright, 2006, ch 17). The objective of the relaxed problem reads as:

$$\Phi(\widetilde{\Psi}, \Lambda; \mu) := f(\widetilde{\Psi}) + \frac{\mu_k}{2} \sum_{t=1}^{n} \left(c_t(\widetilde{\Psi})\right)^2 - \sum_{t=1}^{n} \lambda_t c_t(\widetilde{\Psi}) \tag{14}$$

where μ_k is a penalty coefficient and $\lambda_1, \ldots, \lambda_n$ are Lagrange multipliers. The relaxed problem (14), together with the bounds $v_t^1, v_t^2 \geq 0$, $t = 1, \ldots, n$, is solved first with a small penalty μ_0 (we used $\mu_0 = 1$) and initial values for the model parameters Ψ_0. The latter imply a sequence of prior estimates for the state variables $\beta_{t|t-1}$ from a cycle of the Kalman filter, then initial values for the auxiliary variables v_t^1, v_t^2 in the extended parameter vector $\widetilde{\Psi}_0$ can be set to $v_t^1 := \max\{u_t, 0\}$, $v_t^2 := \max\{-u_t, 0\}$, where u_t is defined in (9).

Efficient algorithms exist for the numerical optimisation of bound-constrained non-linear problems. Thus, we used the L-BFGS-B algorithm (see Zhu et al. (1997)). After a solution $\widetilde{\Psi}_1$ of the problem was found, the penalty parameter μ_k is increased and the corresponding problem is solved in subsequent iterations $k = 1, 2, \ldots$. In each iteration, the resulting solution $\widetilde{\Psi}_{k+1}$ is used as initial solution for the next iteration $k+1$ and the variables λ_t are updated according to

$$\lambda_t = \lambda_t - \mu_{k+1} c_t(\widetilde{\Psi}_{k+1}).$$

This is repeated until in some iteration k' the absolute value of the constraint functions $c_t(\widetilde{\Psi}_{k'+1})$ than some tolerance level for all $t = 1, \ldots, n$.

We start the estimation for the 50%-quantile ($\alpha = 0.5$) on the basis that this is the most robust. The initial values for the parameters stacked in the vector Ψ are obtained from the estimation of a conventional mean regression model with time-varying coefficients as used in Paraschiv et al. (2014). The resulting parameters are then used to construct the starting values for the solution of the quantile regression model for 40% and 60%, the solution of the 40% (60%) quantile regression provides the input for the 30% (70%) model, and the algorithm proceeds iteratively in this manner.

3 Dynamic MultiFactor Electricity Price Quantiles

3.1 Price Formation Fundamentals

The European Power Exchange provides the main trading platform for electricity prices in Europe and, amongst a wide range of products, the day ahead, hourly prices for

Germany ("Phelix") are the major reference prices and the most actively researched (https://www.epexspot.com/en/market-data/dayaheadauction). Each day, an auction clears offers from generators and bids from retailers (and large users) at midday and sets separate prices for delivery in each hour of the subsequent day. Figure 2 displays a typical price clearing result of the auction for a specific hour in 2014. The supply function, representing the stack of offers for production quantities in ascending prices, distinctively increases through concave, flat and convex regions. Since electricity is produced to meet demand instantaneously, with very little storage by end-users, hourly variations in price are due to fluctuations in demand being mapped through the nonlinear supply function into prices and also through changes in the shape of the supply function itself due to availabilities of wind solar and other sources of power, as well as the pricing strategies of generators. Weather and patterns of consumer behaviour are the main drivers of demand and their periodic nature thereby maps into prices, whilst transient episodes at the more steeply convex or concave regions of the supply function induce price volatility.

Thus, both periodic and heteroscedastic components feature in statistical specifications, (eg Koopman & Cornero (2007)), as well as non Gaussian price densities and hourly co-movements (eg Panagiotelis & Smith (2008)). The distinctive bi-inflexioned shape of the supply function is a result of various generating technologies, policy support and participant conduct. The lower concave region, which often extends to negative prices, is becoming common in power markets where there is a substantial amount of renewable technologies (wind, solar, biomass) incentivised by policy support. In Germany at the time of this analysis, renewable technologies were given fixed feed-in tariffs and priority dispatch which effectively meant that whatever could be produced could be sold at a fixed price. At times of high supply has the effect of shifting the supply function to the right, and if demand is low as well, this presents difficulties to other generating facilities that might be inflexible and expect to run continuously (nuclear, district heating and industrial co-generation facilities, as well a some large coal power stations). The inflexible facilities have high shut-down and start-up costs and will therefore pay in order to generate continuously. Hence the deeply discounted and even negative offer prices at the low end of the supply function.

In the mid-region of the supply function, where demand tends to be most of the time, the supply function is flat and price volatility relatively low. The generating technologies here are a mixture of coal and gas. Both are subject to carbon emission supplements, with coal using roughly twice the number of emission certificates compared to gas, per unit of power generated. Furthermore, the operational efficiencies of the various coal and gas plants vary, so that the marginal cost ordering tends to be a mixture of the technologies. It is not the case that all the coal plants are located below all the gas plant in the supply function. Thus, as commodity prices for coal and gas fluctuate, the ordering of the various gas and coal facilities in this section of the supply function also interchange. This leads to a competitive and intricate relationship of power prices to gas and coal commodity prices. The upper convex region of the supply function is characteristic of power markets at times of high demand and increasingly scarce supply. The technologies in this section tend to be low capital cost, high marginal cost, gas and oil (diesel). Not only do these facilities have higher running costs, they need to recover fixed costs over fewer running hours in the year. Furthermore, they tend to be owned by the larger generators and with imperfect competition at times of relative scarcity, offers substantially above marginal costs frequently emerge ("price spikes").

3.2 Model Specification

With the above considerations in mind, we specify a regression model for price quantiles which recognises that the above factors are likely to have varying effects over time as market structure, technologies, polices and commodity prices emerge. The key exogenous variables are: demand, reserve margin (being defined as the difference between the demand for electricity in a particular hour and the power plant availability, "PPA", for that day), coal price, gas price, oil price, carbon allowance prices (CO2), wind production and solar production (PV). Since the market prices are set from a single day ahead auction, and all exogenous variables are taken either as forecasts or day-ahead prices known ahead of the auction to the market participants, it is sufficient to model the price quantiles in reduced form as

$$\begin{aligned} P_t^{\alpha} = \beta_{0t}^{\alpha} + \beta_{1t}^{\alpha} P_{t-1} + \beta_{2t}^{\alpha} Wind_t + \beta_{3t}^{\alpha} PV_t + \beta_{4t}^{\alpha} Demand_t + \\ + \beta_{5t}^{\alpha} Coal_t + \beta_{6t}^{\alpha} Gas_t + \beta_{7t}^{\alpha} Oil_t + \beta_{8t}^{\alpha} CO2_t + \beta_{9t}^{\alpha} Reserve_t + u_t \qquad (15) \end{aligned}$$

$$\beta_{jt}^{\alpha} = c + D\beta_{j(t-1)}^{\alpha} + w_t \quad \text{for} \quad j = 1, ..., 9. \qquad (16)$$

We first estimated a simpler version of the transition equation by modelling the time-varying coefficients as a random walk and this allows a stochastic representation of the coefficients to emerge. A similar time-varying approach applied to expected electricity prices was used in Karakatsani & Bunn (2010) and Paraschiv et al. (2014). Subsequently, we considered mean-reversion and exogenous variables in the transition equation.

For analysis, 24 time series data sets have been constructed for each hour of a day between 01/01/2010–31/05/2014 to apply this model to the German electricity prices. Tables 2 and 1 show the sources of data employed and descriptive statistics of price indexes. In Table 3 we show the data granularity.

In the above formulation, the renewable energies are assumed to have a direct impact on electricity prices; more (less) renewable output will shift the supply curve to the right (left) and thereby lower (increase) prices. Thus we included wind and PV in the observation equation. However, we also envisage that renewable output will have an indirect effect on prices at various quantiles through their influence on the coefficients of gas and coal. The intuition is that as the supply function shifts, the relative influences of the gas and coal technologies will change.

	Phelix spot	Demand forecast	Expected PPA	Coal	Gas	Oil	CO2	PV forecast	Wind forecast
Mean	46	42448	54764	76	22	76	11	3674	5043
Median	46	42801	55249	74	23	76	12	1985	3691
Maximum	210	57625	63981	99	38	91	17	21862	24690
Minimum	-222	24818	40016	52	11	59	3	0	229
Std. Dev.	16	7633	5008	10	4	8	4	4227	4228
Skewness	-2	0	0	0	-1	0	0	1	2
Kurtosis	30	2	2	2	3	2	2	4	5
Jarque-Bera	837967	1406	42	22	120	59	112	4699	18027
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Observations	27720	27720	1155	1155	1155	1155	1155	13860	27720

Table 1: Descriptive statistics for the level of the input variables, between 1 January 2010 and 28 February 2013. Data granularity is consistent with Table 3.

To test for both the direct and indirect effects, we estimated a version in which the transition equations for the coefficients of coal and gas contain, as exogenous variables, wind and PV[1].

$$P_t^\alpha = \beta_{0t}^\alpha + \beta_{1t}^\alpha P_{t-1} + \beta_{2t}^\alpha Wind_t + \beta_{3t}^\alpha PV_t + \beta_{4t}^\alpha Demand_t + \\ + \beta_{5t}^\alpha Coal_t + \beta_{6t}^\alpha Gas_t + \beta_{7t}^\alpha Oil_t + \beta_{8t}^\alpha CO2_t + \beta_{9t}^\alpha Reserve_t + u_t \tag{17}$$

$$\beta_{jt}^\alpha = c + D\beta_{j(t-1)}^\alpha + \varepsilon_j^\alpha Wind_t + \gamma_j^\alpha PV_t + \phi_j^\alpha Reserve_t + w_t \quad \text{for} \quad j = \{1, ..., 9\}; \tag{18}$$

where for $j = \{1, ..., 5; 8, 9\}$ ε_j and γ_j take the value 0.

[1]We tested also an additional model in which wind and PV were used only as exogenous variables in the transition equation for the coefficients of coal and gas and excluding them from the observation equation. However, the estimation procedure did not yield significant coefficients. This revealed that it is important to account for a direct effect of renewables on the electricity prices.

Variable units	**Description**	**Data Source**
Spot Price EUR/MWh	Market clearing price for the same hour of the last relevant delivery day	European Energy Exchange: `http://www.eex.com`
Coal Price EUR/12'000 t	Latest available price (daily auctioned) of the front-month Amsterdam-Rotterdam-Antwerp (ARA) futures contract before the electricity price auction takes place	European Energy Exchange: `http://www.eex.com`
Gas Price EUR/MWh	Last price of the NCG Day Ahead Natural Gas Spot Price on the day before the electricity price auction takes place	Bloomberg, Ticker: GTHDAHD Index
Oil Price EUR/bbl	Last price of the active ICE Brent Crude futures contract on the day before the electricity price auction takes place	Bloomberg, Ticker: COA Comdty
Price for EUA EUR 0.01/EUA 1'000 t CO2	Latest available price of the EEX Carbon Index (Carbix), daily auctioned at 10:30 am	European Energy Exchange: `http://www.eex.com`
Expected Wind and PV Infeed MWh	Sum of expected infeed of wind- and PV electricity into the grid, published by German transmission system operators in the late afternoon following the electricity price auction	Transmission system operators: `http://www.50Hertz.com`, `http://www.amprion.de`, `http://www.transnetbw.de`, `http://www.tennettso.de`
Expected Power Plant Availability MWh	Ex ante expected power plant availability for electricity production (voluntary publication) on the delivery day (daily granularity), daily published at 10:00 am	European Energy Exchange & transmission system operators: `ftp://infoproducts.eex.com`
Expected Demand MWh	Demand forecast for the relevant hour on the delivery day	European Network of Transmission System Operators (ENTSOE): `https://transparency.entsoe.eu/`

Table 2: Overview of fundamental variables used in the analysis. Note: For oil, the price of this contract is typically used as a reference for derivatives contracts and thus most relevant for trading. For coal, gas, and carbon, the prices result from a daily auction which does not exist for oil.

Variable	**Daily**	**Hourly**
Spot Price		×
Coal Price	×	
Gas Price	×	
Oil Price	×	
Price for EU Emission Allowances	×	
Expected Wind and PV Infeed		×
Expected Power Plant Availability	×	
Expected Demand		×

Table 3: Data granularity of fundamental variables

3.3 Results

From the 24 separate hourly models estimated, we comment in detail on the salient features one hour[2], hour 13 (12:00–13:00), in order to justify the value of the modelling framework presented, the estimation process and the underlying concept of factor influences from consideration of the bi-inflexioned shape of the supply function. The inclusion of demand in our formulation encompasses weather and seasonal effects (as in Karakatsani & Bunn (2010)). Estimation results and optimised initial values for the random walk transition equations for all coefficients are shown in Table 4.

In Figure 3 we show the time-varying coefficients of gas at a range of quantiles. We observe that gas prices have a positive effect on electricity prices at all quantiles and more so in consideration of the risks of high (P90, P80, P70) prices. These events occur in the upper region of the supply function where gas is the marginal technology. These three quantile coefficients have shown a smooth decline over time reflecting the increasingly competitive conditions in the German market. In contrast, the middle quantiles (P40,P50, P60) show considerable volatility and this reflects the middle part of the supply function where gas and coal facilities intermingle and the marginal technologies will switch according to gas and coal price spreads as well as the overall supply function shifts induced by wind and solar generation.

For comparison, Figure 4 shows the quantile coefficients for wind, all of which are negative as expected. The smoothest effect is at P10 and reveals the influence of wind in driving lower electricity prices when high wind and low demand induce price formation in the lower concave region of the supply function. For the mid quantiles we again see a more volatile sequence of coefficient estimates as price formation emerges from the flat mid region of the supply function, and in Figure 5 the highly sensitive interaction of the estimates for wind and gas at the P50 quantile is clearly illustrated. When the wind production is high (low), it moves the supply function to the right (left) and the higher

[2] Results for other hours of the day are available upon request.

cost gas facilities are pushed out of (into) action. In this Figure, the correlation coefficient between gas and wind coefficients at the P50 quantile is accordingly -0.256. We found a similar, though weaker negative correlation for PV and gas coefficients.

Table 5 provides further support for this intricate explanation of the quantile coefficient drivers as we introduce exogenous variables into explaining the transition equation for the gas coefficients. For all electricity price quantiles except the lowest, we see that increased wind and solar production lowers the effect of gas prices. We also see that for price spikes (P90) in the upper convex region of the supply function, the high offers from gas generators are moderated by a higher reserve margin; conduct we would expect from reduced scarcity.

With regard to conduct at the extremes, in Figure 6 we observe that for hour 13 the coefficients of the lagged spot prices have a positive sign at all quantiles, but only at P10 and P90 do they appear to be substantial. This is an intuitive corroboration of the observations advanced earlier that at the lower and upper regions of the supply function, conduct tends to be a more important feature of price formation than demand, supply or fuel price fundamentals. Very low prices tend to reflect the discounting of inflexible facilities and high price spikes reflect the exercise of market power. As a behavioural feature we would expect to see some adaptive behaviour and the results that the coefficients for lagged spot only stand out at P10 and P90 are a reinforcement of this concept. Furthermore, the P10 coefficient is larger and increasing, illustrative of the increasing difficulty, and as a consequence conduct co-ordination, in dealing with low extreme prices as more renewable capacity has been introduced into the power system.

The results from the other hours revealed similar intricate dynamics, the main differences being that in the night hours of low demand, coal featured more strongly than gas and of course solar PV was absent as a driving factor. One of the modelling areas of concern was the separate estimation of the quantiles and whether this might lead to incoherent estimation of the distribution as a whole. Whilst it is clear from the supply

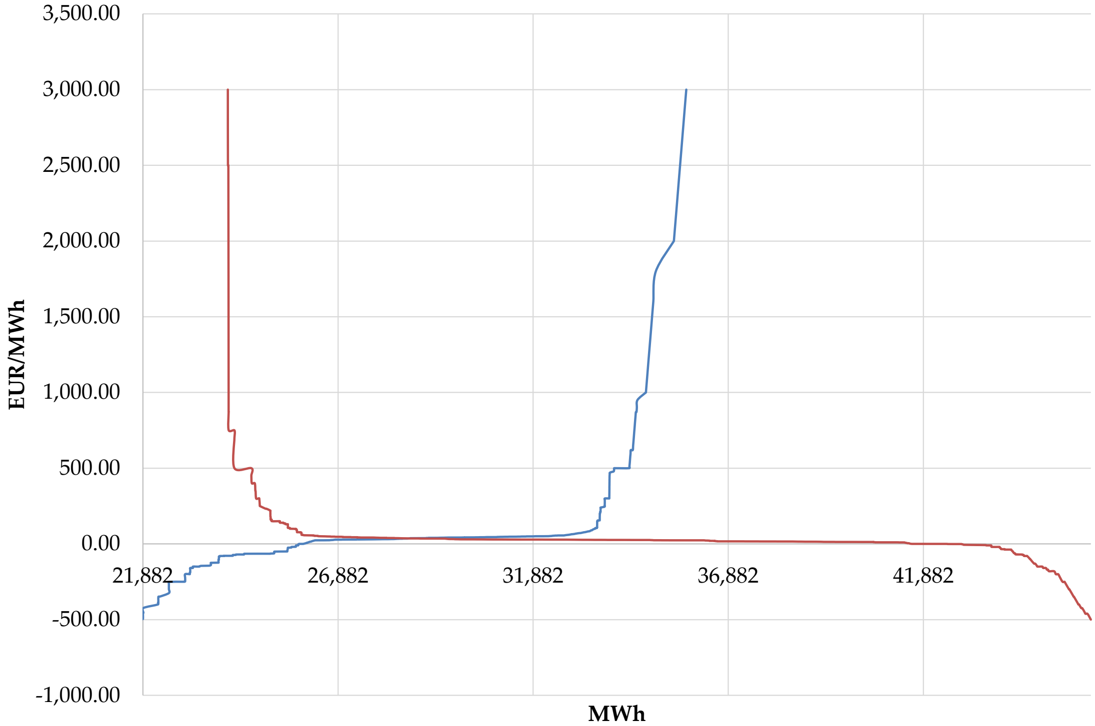


Figure 2: Actual supply and demand functions from the German market. Hour 11-12 on 22 Sept 2015. Source: EPEX Day ahead.

function concept and the discussion of the above results that we would not expect the factor coefficients to necessarily show a monotonic relationship with the levels of the quantiles, we would expect the model as a whole to produce a coherent predictive distribution for prices with a monotonic ordering of price quantile levels. Figure 7 displays the hourly quantile bands for a typical pair of days in the sample. Overall, in the data as a whole, less than 3% of the quantiles for the hourly price distributions were incoherent in the sense of violating a monotonic ordering. We take that to be reassuring for the model specification and a lack of overfitting.

4 Conclusion

We demonstrated the value of a well specified dynamic model for quantile estimation by means of an application to electricity price risk. Electricity prices are a commodity in which price formation is nonlinear in its relationship to fundamentals, dynamic in the relative influences of drivers, with further complications introduced by policy interventions for supporting specific technologies and opportunities for participant conduct to be influential at high and low prices. Despite these complications careful consideration of the shape of the supply function with its concave, flat and convex regions, together with the information that is available to market participants day ahead allows plausible expectations for the price dynamics to be considered, and these explain very well the signs and significance of the parameters in the estimated models. Nevertheless, the models need to have a detailed specification with the various quantiles being related to multiple factors through coefficients which have dynamic properties themselves related to some of the exogenous factors. This modelling requirement motivates the development of quantile models that need fully parametric specifications to capture dynamics through exogenous factors and time-varying coefficients.

A novel general methodology has therefore been developed in which time-varying multi factor coefficients are recursively estimated with a Kalman filter using maximum likelihood. Since the likelihood function is non-differentiable, the problem is re-formulated as a non-linear optimsation with constraints, and furthermore re-formulated again by moving the constraints into the objective function to solve an augmented Lagrangian method. With careful selection of starting values, maximum likelihood estimates were thereby acquired. As a general approach, we would expect this to be useful in many applications of risk management and quantile estimation where there is dynamic complexity in price formation and plausible exogenous price drivers.

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	Q 10%	Q 20%	Q 30%	Q 40%	Q 50%	Q 60%	Q 70%	Q 80%	Q 90%
$\Omega * 10^{-10}$	0.0004	0.0006	0.0011	0.0049	0.0005	0.0013	0.0001	0.0003	0.0053
	0.0000	2.9611	4.3414	3.8028	10.5005	0.0063	0.0001	0.0001	0.0001
	0.0000	2.5867	1.6459	1.4661	3.1369	0.0028	0.0006	0.0006	0.0001
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0178	0.0156	0.0308	0.0054	0.0151	0.0901	0.0003	0.0009	0.0072
	1.0510	29683.9052	2510432.0731	2953519.3948	8093839.7540	6061.2393	10.6293	31.1348	3.5574
	4.5723	1.8411	0.2306	0.0493	0.0062	0.0363	0.3076	0.0001	0.0006
	0.0001	0.0003	0.0005	0.0120	0.0301	0.1902	0.6769	0.6900	0.1968
	0.0000	3.9643	2.1434	2.7245	17.8127	0.0172	0.0459	0.0928	0.0000
β_0	-0.00404	-0.00508	-0.0067	-0.0048	0.0004	-0.0053	-0.0011	-0.0043	-0.0249
	0.00003	0.00014	0.0002	0.0004	0.0004	0.0002	0.0001	0.0001	0.0001
	0.00003	0.00028	0.0003	0.0002	0.0002	0.0001	0.0000	0.0000	0.0000
	-0.00001	-0.00005	-0.0001	-0.0002	-0.0001	-0.0001	0.0000	0.0000	-0.0001
	-0.00449	-0.01603	-0.0510	-0.0742	-0.0429	-0.0202	-0.0074	-0.0060	-0.0349
	-0.00008	-0.01774	-0.0596	-0.1820	-0.2000	-0.1573	-0.2064	-0.2025	-0.2828
	0.00034	0.01504	0.0505	0.0484	-0.0410	-0.0201	-0.0450	-0.0431	-0.0218
	-0.02572	-0.13382	-0.2397	-0.1619	-0.0449	0.0121	0.0263	0.0782	0.2058
	0.00001	0.00006	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001
Ξ	19.5589	16.9401	3.0117	7.2805	16.1200	0.0178	0.0263	0.0540	0.2401
τ	1.1148	2.0776	2.1053	2.1354	2.1344	2.1563	2.1621	1.9651	1.4764
Obj. Values	1819.9297	2777.0406	2816.3499	2827.7250	2835.4923	2834.2591	2788.7955	2692.8638	2130.9757

Table 4: Parameter estimates for hour 13, all quantiles. The coefficients shown in vectors Ω and β_0 follow the order indexed in Equation (15)

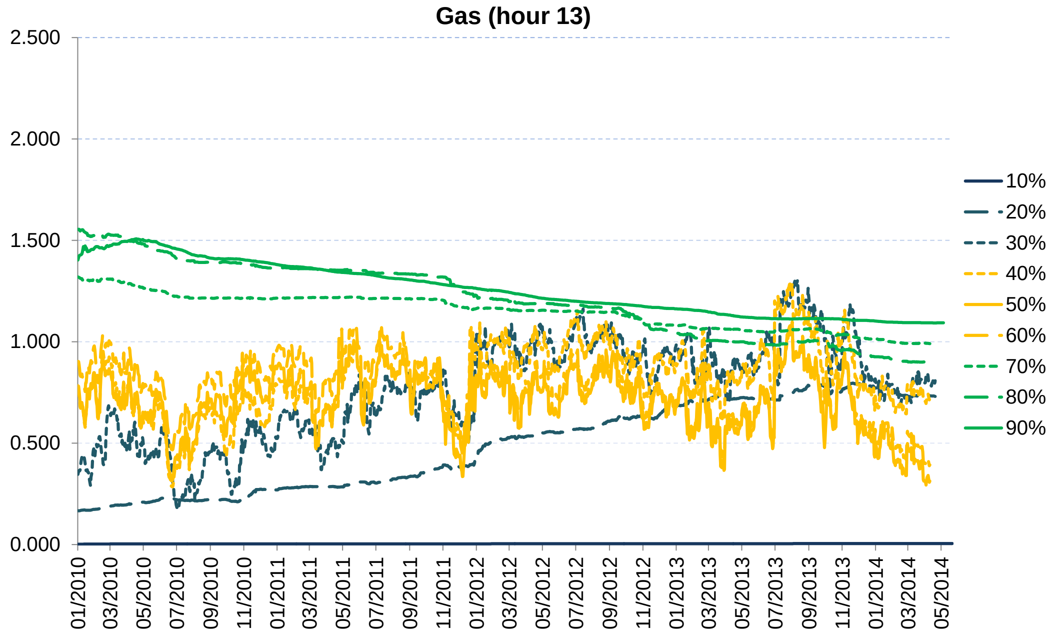

Figure 3: Gas Quantile Coefficients for Hour 13

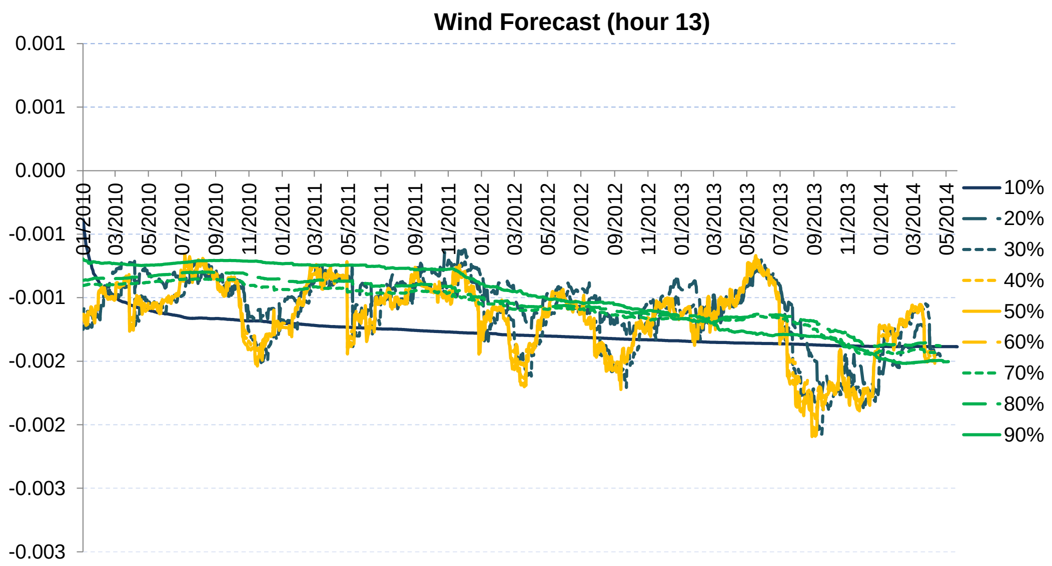

Figure 4: Wind Quantile Coefficients for Hour 13

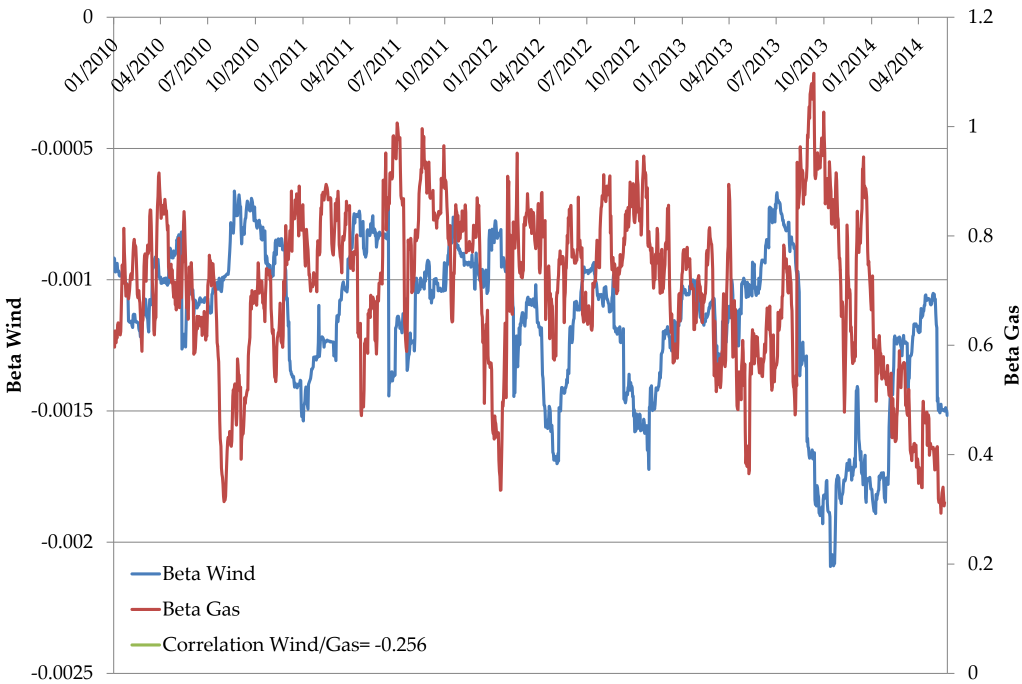

Figure 5: Correlation of Gas and Wind P50 Quantile Coefficients

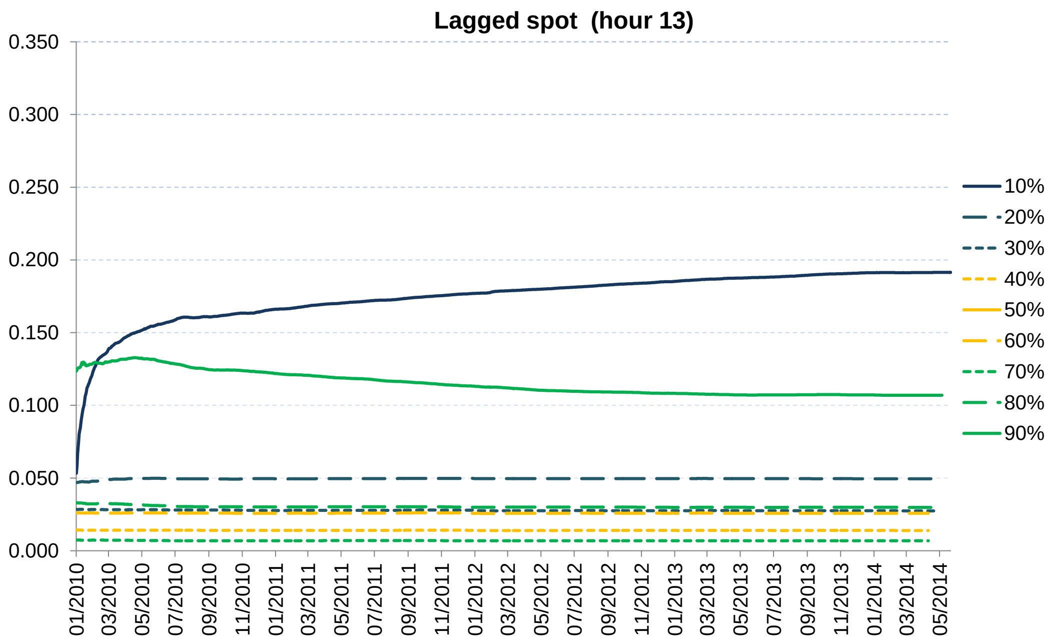

Figure 6: Lagged Price Quantile Coefficients for Hour 13

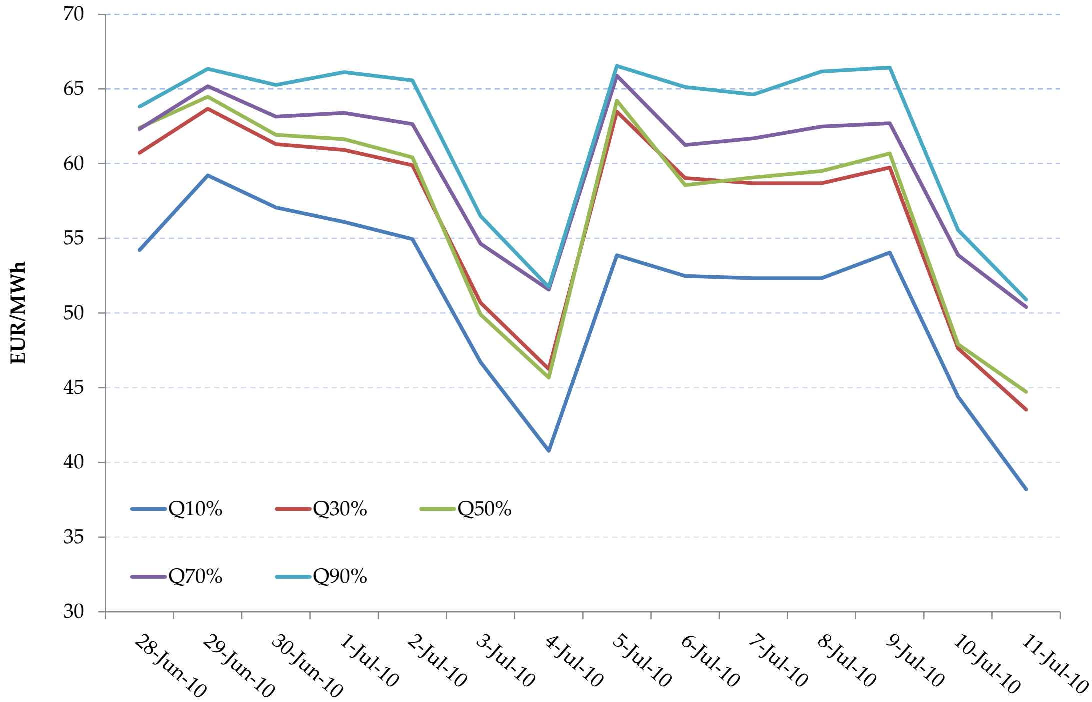

Figure 7: Predictive Hourly Price Quantiles for two days

Dependent variable: β_{gas}					
Method: Least Squares					
	Q 10%	**Q** 30%	**Q** 50%	**Q** 70%	**Q** 90%
C	0.0000	0.0103	0.0370	0.2507	0.2131
	(23.0451)	(4.1064)	(7.1436)	(21.0069)	(17.5405)
D_{gas}	0.9888	0.9872	0.9561	0.8012	0.8510
	(1795.4470)	(277.3021)	(149.5016)	(84.3599)	(99.6584)
$\varepsilon^{\alpha}_{Wind} * 10^8$	0.0018	-1.4500	-3.8600	-10.0000	-11.8000
	(2.8295)	(-0.9128)	(-1.8863)	(-6.1723)	(-5.9069)
$\gamma^{\alpha}_{PV} * 10^8$	0.0037	2.3300	-2.3200	-21.7000	-25.4000
	(5.6391)	(1.4540)	(-1.3332)	(-13.3793)	(-12.2649)
$\phi^{\alpha}_{Reserve} * 10^8$	0.0011	1.7500	2.4600	-3.6200	-3.1600
	(2.8067)	(1.7686)	(1.9042)	(-3.5875)	(-2.5796)
R-squared	0.9823	0.9863	0.9338	0.8889	0.9229
Test for omitted variables (wind, pv, reserves), p-values					
F-statistic	0.0000	0.1225	0.0279	0.0000	0.0000
Likelihood ratio	0.0000	0.1216	0.0276	0.0000	0.0000

Table 5: Parameter estimates for Equation (18), Hour 13, having as dependent variable the coefficient β for gas. To overcome autocorrelation, correlation, and heteroskedasticity in the error terms we employed the Newey-West estimator. t-statistics of coefficients are shown in paranthesis. We further performed a test for omitted variables and added the set of variables Wind, PV and Reserve Margin to Equation (16) to ask whether the set makes a significant contribution to explaining the variation in the dependent variable. In the lower panel of the table we show the p-values of the related F-statistic and Likelihood ratio.

Econometric analysis of 15-minute intraday electricity prices

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Abstract

The trading activity in the German intraday electricity market has increased significantly over the last years. This is partially due to an increasing share of renewable energy, wind and photovoltaic, which requires power generators to balance out the forecasting errors in their production. We investigate the bidding behaviour in the intraday market by looking at both last prices and continuous bidding, in the context of a fundamental model. A unique data set of 15-minute intraday prices and intraday-updated forecasts of wind and photovoltaic has been employed and price bids are modelled by prior information on fundamentals. We show that intraday prices adjust asymmetrically to both forecasting errors in renewables and to the volume of trades dependent on the threshold variable demand quote, which reflects the expected demand covered by the planned traditional capacity in the day-ahead market. The location of the threshold can be used by market participants to adjust their bids accordingly, given the latest updates in the wind and photovoltaic forecasting errors and the forecasts of the control area balances.

Keywords: intraday electricity prices, bidding behavior, renewable energy, forecasting model

*Part of the work was done while the author was visiting Center of Advanced Study, Norwegian Academy of Sciences and Letters, Oslo, as a member of the group: Stochastics for Environmental and Financial Economics

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1. Introduction

Trading in the intraday electricity markets increased rapidly since the opening of the market. This may be driven by the need of photovoltaic and wind power operators to balance their production forecast errors, i.e. deviations between forecasted and actual production. Evidence for this is a jump in the volume of intraday trading as the direct marketing of renewable energy was introduced. Furthermore, there may be a generally increased interest in intraday trading activities due to proprietary trading. We study the structure of intraday trading of electricity and identify the price-driving factors. Our main goal is to identify market fundamental factors that influence the bidding behavior in the 15-minute intraday market at the European Power Exchange (EPEX).

Along the basic timeline of electricity trading activities, see Figure 1, the intraday activities relate mostly to further adjustments of positions after the closure of the day-ahead market.

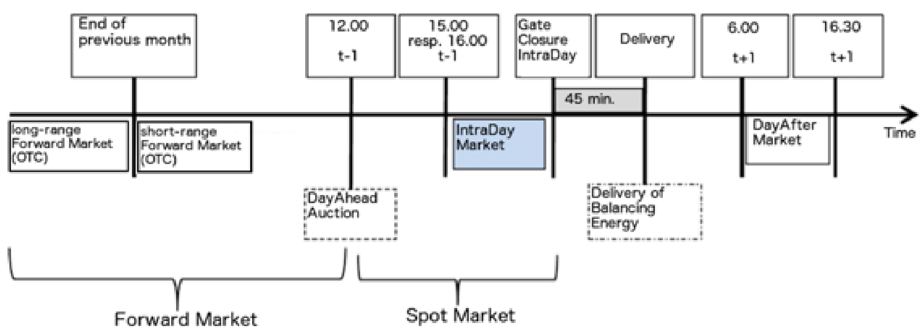


Figure 1: Timing Electricity Trading

While day-ahead trading offers the possibility to correct the long-term production schedule (build on the forward markets) in terms of hourly production schedule of power plants (Delta Hedging) and to adjust for the residual load profiles on an hourly basis, the increasing share of renewable energy sources (wind, solar) in electricity markets requires a finer adjustment.

According to the Equalization Mechanism Ordinance (ger.: Verordnung zur Weiterentwicklung des bundesweiten Ausgleichsmechanismus, abbr.:

AuglMechV) all electricity generated by renewable sources has to be traded day-ahead. This is usually done by the transmission system operator (TSO) with the plant operator receiving a legally guaranteed feed-in-tariff. From 2012 on the inclusion of a market premium led direct marketers within the feed-in premium support scheme to enter the market as well. Trading of electricity from a renewable energy source is based on forecasts which may have a horizon of up to 36 h (taking some data-handling into account). To correct errors in forecasts the AusglMechV requires the marketers of renewable energy to use the intraday market to balance differences in actual and updated forecasts. Intraday trading starts at 3 pm and takes place continuously until up to 30 min before the start of the traded quarter-hour. As forecasts change regularly, marketers may sell and buy the same contract at different times during the trading period.

After the closure of the intraday market balancing energy has to be used to close differences between available and forecasted electricity. As a smaller number of power plants are used for balancing energy the merit-order curve is steeper than that in the intraday market. Thus on average larger prices are paid and marketers aim at minimising this difference, see [5]. In addition, TSOs may impose sanctions on marketers who frequently require balancing energy.

Balancing energy is supplied by generators with the necessary flexibility to balance the market. In case generation is below demand positive balancing energy is used, otherwise negative balancing energy. [6] and [13] contain a detailed description of the integration of renewable energy in electricity markets and the regulatory requirements and we refer the reader to these sources for further information.

The day-ahead market (spot market) and the balancing markets have been investigated extensively. For example, [22] show that the day-ahead price formation process at EPEX depends on the interaction/substitution effect between the traditional production capacity (coal, gas, oil) with the fluctuant renewable energies (wind and photovoltaic (PV)). Further empirical studies on intraday/balancing markets include [1], [16]. Also, [18] studies strategic behaviour linking day-ahead and balancing markets.

An investigation in the merit-order effect is given by [2], who find that electricity generation by wind and PV has reduced spot market prices considerably by 6 €/MWh in 2010 rising to 10 €/MWh in 2012. They also show that merit order effects are projected to reach 14-16 €/MWh in 2016.

Recent studies of the intraday high-frequency electricity prices at EPEX are [8] and [9] who look at liquidity effects and forecast determinants on a hourly basis. Also, [3] considers trading strategies to minimise costs from imbalances for both PV and wind, but generates price changes in terms of a reduced-form model (using a stochastic process). The focus lies in developing a trading strategy for a given setting, and not on explaining the relevant price process. Several studies have discussed the effects of prognosis errors for wind generation (see [15] and [20]). As Figure 2 suggests, a PV production introduces quarter-hour ramps quite naturally. In addition, changes in forecasts of renewable energy production require a timely correction of day-ahead positions. However, photovoltaic has not been investigated so far.

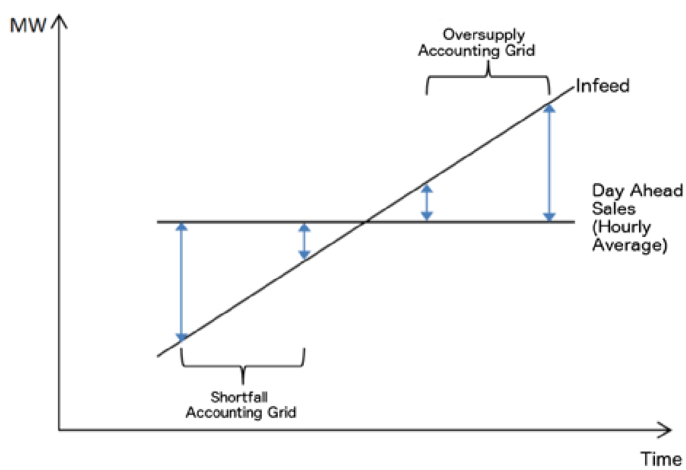


Figure 2: Quarter Hour Ramps

[8] and [9] used the ex-post published wind infeed data to explain ex-ante their impact on the day-ahead market. These are publicly available data from the Transparency Platform EPEX. However, the actual infeed is only known ex-post and therefore it cannot be used directly to explain the price formation on the intraday market. In fact, the intraday market participants have access to updated forecasts of wind. In our study, we will extend the existing literature by taking into account the intraday updated forecasts for wind and PV, which have been supplied by EWE Trading GmbH.

Each day, hourly day-ahead electricity prices are revealed around 2 pm at EPEX (see [23]). At the same time, market participants have access to forecasts for wind and PV published by each Transmission System Operator (TSO) in 15-minute intervals for the next day. However, wind and PV forecasts are updated frequently during the trading period. Thus, at the time when market participants place their bids for a particular intraday delivery period (hour, quarter of hour), updated information about the forecasting errors of renewables becomes available. In consequence, also deviations between the intraday prices and the day-ahead price for a specific hour are expected to occur. Our main research question is, thus, to which extent do market participants change their bidding behavior when new information on wind and PV forecasts becomes available. We will employ a unique data set of the latest forecasts of wind and PV available at the time of the bid.

Our analysis is twofold: Firstly, we derive an asymmetric fundamental model for the difference between the last price bid for a certain quarter of hour and the day-ahead price for that hour. We distinguish between summer/winter, peak/off-peak hours. We test for asymmetric behavior of prices to forecasting errors of renewable energy dependent on the demand quote regime and further investigate the typical jigsaw pattern of intraday prices. Thus, we identify a seasonality shape that provides traders important information about the time of the day when they can bid, dependent on their demand/supply profiles. Furthermore, the effect of volume of trades/market liquidity are investigated. Secondly, we are interested in the bidding behavior of market participants in the continuous intraday electricity market. We thus analyse the continuous trades and disentangle the effect of market fundamentals dependent on the time of the day. The econometric model is replicated for several traded hourly quarters, in different time of the day. In particular, we are interested to see how delta bid prices change when new information becomes available in the intraday renewable forecasts for wind and PV. We look at the trade-off between autoregressive terms and fundamental factors impacting the intraday price formation process.

Our contribution to the existing literature is twofold: we use ex-ante forecasts of fundamental variables and employ high-frequency intraday prices for specific quarter hours. Overall, our paper aims at understanding historically the continuous bidding in the intraday market, and proposes a one-period forecasting model based on fundamental variables which are observed by market participants at the time of the bid. We show that estimation results are stable over time, but it is highly relevant to reestimate the model separately for summer/winter, peak/off-peak periods. We benchmark our model by an autoregressive model and show that the price formation process is rather fundamentals driven, especially for mid-day delivery periods. Market participants access updated forecasts in renewables to have more private information and thus to bid more accurately.

The rest of the paper is organized as follows: In Section 2 we explain the modeling assumptions. Sections 3 and 4 show the data used as input and the theoretical model. Section 5 proceeds with the formulation of our specific model. Results and their interpretation are given in Section 6 and Section 7 concludes.

2. Model architecture

Our main assumption is that the electricity intraday price formation process depends on how much traditional capacity has been allocated in the day-ahead market and in which proportion it covers the forecasted demand. Let us consider two possible market regimes:

1. The traditional capacity planned for the day-ahead satisfies the expected demand for a certain hour;

2. There is a certain demand quote uncovered by the planned capacity.

Thus, in scenario 2, negative forecasting errors of wind and PV will increase faster the intraday prices than in scenario 1, due to the excess demand pressure. Viceversa, in scenario 1, positive forecasting errors in renewables will put pressure on traditional suppliers to reduce the production, since renewables are fed into the grid with priority (on average, 20% of electricity production in Germany is wind and PV based). Thus, prices will decrease faster than in scenario 2, where the excess of renewables (positive updated forecasts) will balance out the excess demand. Therefore, in the context of a threshold model, we investigate whether there is an asymmetric adjustment of the intraday prices to forecasting errors in renewables, dependent on the demand quote regime (proportion of the forecasted demand for electricity in the planned traditional capacity for the day-ahead). The location of the threshold in the demand quote is estimated and this gives an indication of the bidding behavior in the intraday market. Market participants can compare the historically derived threshold value to the currently computed forecasted demand quote for a certain hour to identify the market regime and to further define a bidding strategy.

Employing the demand quote as threshold variable is supported by the literature as several papers have found that total electricity demand influences price behaviour strongly. In [14] it is shown that the ratio between wind and conventional power production affects the electricity price most (the so-called wind penetration). [19] identify the residual load, the electricity demand that needs to be met by conventional power, as an important variable.

To include the trading volume as fundamental variable is also supported by the literature, as e.g. [6] find that the forecast balancing costs in intraday trading are linked to the trading volume. This is in line with earlier papers, such as [17] and [4], who estimate asymmetric GARCH models and include traded electricity volume in the variance equation to study its impact on price volatility.

In a first part of our analysis, we aim at a model for the difference between the last intraday bid price for a certain quarter of an hour and the day-ahead price for that specific hour. As a prerequisite for our modeling approach, we investigate the typical jigsaw pattern of the 15-minute intraday prices and control for seasonality. Figures 3 and 4 show the long-term mean of last prices and average prices bid for a certain quarter of an hour between 01/01/2014–01/07/2014 for peak and off peak hours, respectively. During the day, the jigsaw pattern is mainly explained by the following situation: Renewable energy providers sell day-ahead the full hour (average of all quarters). In the first part of the day, up to 1400, as the sun goes up, there is a buy-pressure on them in the first quarter as they are not able to produce the hourly average (see Figure 3, upper graph). On the other hand, in the fourth quarter they produce too much and have to sell. By contrary, in the second part of the day (between 1400–2000) the ramping down effect of the sun determines a sell-pressure in the first quarter, which turns into buy-pressure in the last quarter. The buy/sell pressure becomes obvious in the evolution of volume of trades (see Figure 3, lower graph): we observe that the volume of trades is highest during the first and last quarters of each peak hour, reflecting demand/supply side pressures.

We also found a persistent jigsaw pattern of prices during off-peak hours (between 2000–0800), as shown in Figure 4. This is driven by the production design of fossil power plants (supply side: when it starts low and ends high) or power-intensive industry (demand side: when it starts high and ends low). A reason for that may be inter-temporal restrictions in using fossil plants. In addition to fuel costs, these plants have ramp-up and ramp-down costs, which prevent plant operators from shutting down plants in case of drops in demand or starting up plants in case of spikes in demand. The short-term marginal costs from this may dominate fuel costs.

The typical jigsaw seasonality pattern of intraday quarter-hourly prices will be corrected by dummy variables in our model specification.

3. Input variables: definition and data sources

As motivated in section 2, for the analysis we employed historical day-ahead and intraday electricity prices for 15-minute products in the continuous trading system between 01/01/2014–30/06/2014. As fundamental variables selected in this study we refer to demand forecast, power plant availability, intraday updated forecasts for wind and photovoltaic, volume of trades in the continuous trading, and the control area balance. The latter represents the corresponding use of balancing power in the balancing market[1]. In particular, the control area balance corresponds to the sum of all balance group deviations of balance groups registered at the Transmission System Operator and of the relevant balance groups owned by the transmission system operator (e.g. EEG, grid losses, unintentional deviation)[2]. In Tables 1 and 2 we give an overview of the data sources and their frequency, respectively.

4. Asymmetric model for intraday prices

4.1. Threshold model specification

The technical specification of our model follows [21] and reads:

$$y_i = \theta_1^{'} x_i + \varepsilon_i, \quad \omega_i \le \tau, \tag{1}$$

$$y_i = \theta_2^{'} x_i + \varepsilon_i, \quad \omega_i > \tau, \tag{2}$$

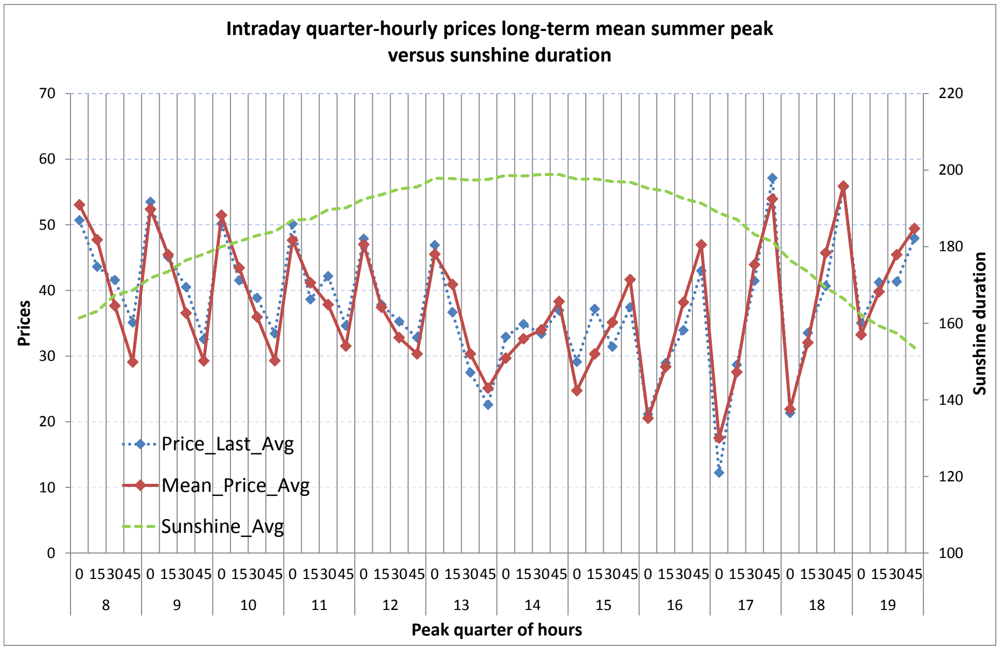
where ω_i is the threshold variable used to split the sample into two regimes. The random variable ε_i is a regression error.

Our observed sample is $\{y_i, x_i, \omega_i\}_{i=1}^{n}$, where y_i represent the dependent variable and x_i is an m-vector of independent variables. The *threshold variable* ω_i may be an element of x_i and is assumed to have a continuous distribution. To write the model in a single equation[3], we define the dummy

[1]As balance group deviations are not immediately available online the control area balance is calculated on the basis of the corresponding use of balancing power. The published data are values from operating measurements that are adjusted by measurement corrections if necessary. The actual settlement-relevant data can be retrieved under the prices for grid balancing.

[2]see `http://www.tennettso.de`

[3]see Hansen (2000)



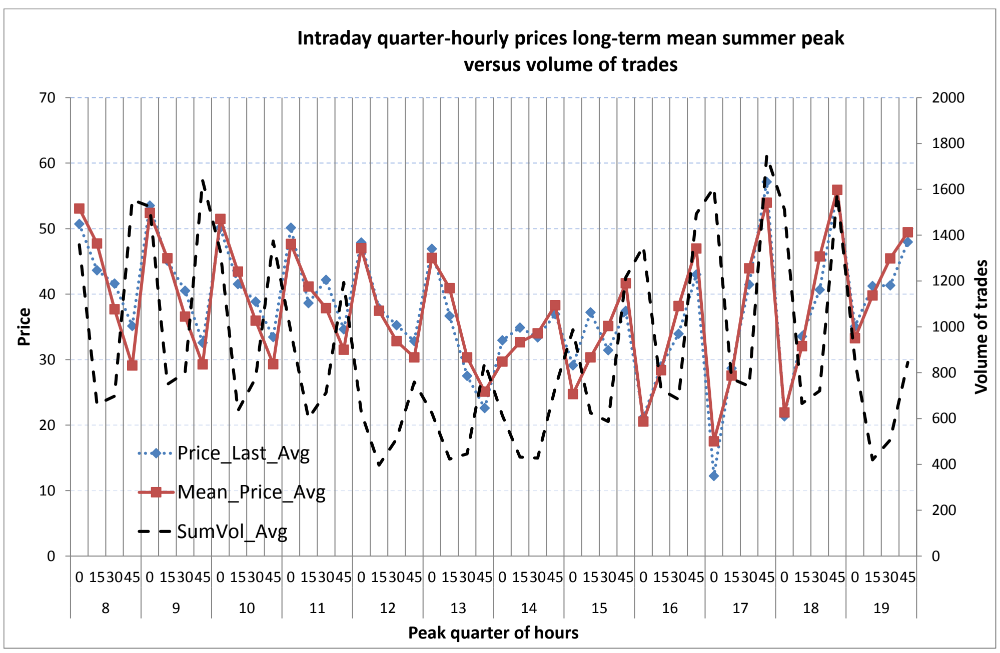


Figure 3: Seasonality pattern of the last prices and average prices bid for a certain quarter of an hour during the peak hours in summer. The right axes show the sunshine duration (upper graph) and the sum of volumes traded (lower graph).

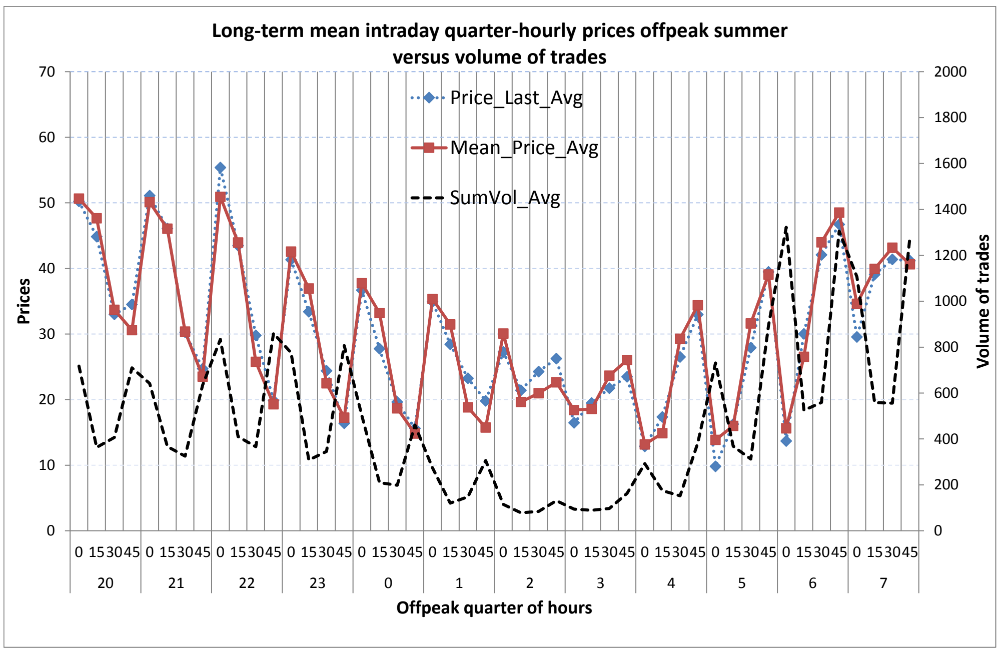


Figure 4: Seasonality pattern of the last prices and average prices bid for a certain quarter of an hour during the off-peak hours in summer. The right axis shows the sum of volumes traded.

Variable units	**Description**	**Data Source**
Day-ahead Price EUR/MWh	Market clearing price for a certain hour in the day-ahead auctions (Phelix)	European Power Exchange (EPEX) `https://www.epexspot.com/en/`
Intraday Price EUR/MWh	Intraday electricity prices for 15-minute products in the continuous trading	European Energy Exchange Transparency Platform: `http://www.eex-transparency.com/de`
Intraday Volume Trades MWh	Intraday volume trades for 15-minute products in the continuous trading	European Energy Exchange Transparency Platform: `http://www.eex-transparency.com/de`
Wind Forecast MW	Sum of intraday forecasted infeed of wind electricity into the grid	EWE TRADING GmbH `http://www.ewe.com/en/`
PV Forecast MW	Sum of intraday forecasted infeed of PV electricity into the grid	EWE TRADING GmbH `http://www.ewe.com/en/`
Expected Power Plant Availability MW	Ex-ante expected power plant availability for electricity production on the delivery day (daily granularity), daily published at 10:00 am	European Energy Exchange & transmission system operators: `ftp://infoproducts.eex.com`
Expected Demand MW	Demand forecast for the relevant hour on the delivery day	European Network of Transmission System Operators (ENTSOE): `https://transparency.entsoe.eu/`
Control area balance MW	Balancing market margins, available ex-post for a certain delivery period	Transmission system operators: `http://www.50Hertz.com`, `http://www.amprion.de`, `http://www.transnetbw.de`, `http://www.tennettso.de`

Table 1: Overview of fundamental variables used in the analysis

Variable	**Daily**	**Hourly**	**quarter-hourly**
Day-ahead Price		×	
Intraday Price			×
Intraday Volume Trades			×
Wind Forecast			×
PV Forecast			×
Expected Power Plant Availability	×		
Expected Demand		×	
Control area balance			×

Table 2: Data granularity of fundamental variables

variable $d_i(\tau) = \mathbf{1}[\omega_i \leq \tau]$, where $\mathbf{1}[\cdot]$ is the indicator function and we set $x_i(\tau) := x_i d_i(\tau)$. Furthermore, let $\lambda_n^{'} = \theta_2^{'} - \theta_1^{'}$ denote the threshold effect. Thus, equations (1) and (2) become:

$$y_i = \theta' x_i + \lambda_n' x_i(\tau) + \varepsilon_i \tag{3}$$

In order to simplify the threshold estimation procedure, we rewrite equation (3) in matrix notation. We define the vectors $Y \in \mathbb{R}^n$ and $\varepsilon \in \mathbb{R}^n$ by stacking the variables y_i and ε_i, and the $n \times m$ matrixes $X \in \mathbb{R}^{n\times m}$ and $X(\tau) \in \mathbb{R}^{n\times m}$ by stacking the vectors x_i' and $x_i(\tau)'$. Then (3) can be written as:

$$Y = X\theta + X(\tau)\lambda_n + \varepsilon \tag{4}$$

The regression parameters are $(\theta, \lambda_n, \tau)$ and the natural estimator is least squares (LS).

4.2. Hansen's grid search to locate the most likely threshold

To determine the location of the most likely threshold, we will apply Hansen's grid search. In the implementation of this threshold estimation procedure, we follow [11] and [12]. This paper develops a statistical theory for threshold estimation in the regression context. As mentioned in the previous section, the regression parameters are $(\theta, \lambda_n, \tau)$. Let

$$S_n(\theta, \lambda, \tau) = (Y - X\theta - X(\tau)\lambda)'(Y - X\theta - X(\tau)\lambda) \tag{5}$$

be the sum of squared errors function. Then, by definition, the LS estimators $\hat{\theta}, \hat{\lambda}, \hat{\tau}$ jointly minimize (5). For this minimization, τ is assumed to be restricted to a bounded set $[\underline{\tau}, \bar{\tau}] = \Omega$. The LS estimator is also the MLE when ε_i is i.i.d. $N(0, \sigma^2)$. Following [11], the computationally easiest method to obtain the LS estimates is through concentration. Conditional on τ, equation (4) is linear in θ and in λ_n, yielding the conditional OLS estimators $\hat{\theta}(\tau)$ and $\hat{\lambda}(\tau)$ by regression of Y on $X(\tau)^* = [X X(\tau)]$. The concentrated sum of squared errors function is

$$S_n(\tau) = S_n(\hat{\theta}(\tau), \hat{\lambda}(\tau), \tau) = Y'Y - Y'X(\tau)^*(X(\tau)^{*'}X(\tau)^*)^{-1}X(\tau)^{*'}Y,$$

and $\hat{\tau}$ is the value that minimizes $S_n(\tau)$, i.e.,

$$\hat{\tau} = \operatorname{argmin} S_n(\tau)$$

To test the hypothesis $H_0 : \tau = \tau_0$, a standard approach is to use the likelihood ratio statistic under the auxiliary assumption that ε_i is i.i.d. $N(0, \sigma^2)$.

Let

$$LR_n(\tau) := n\frac{S_n(\tau) - S_n(\hat{\tau})}{S_n(\hat{\tau})}.$$

The likelihood ratio test of H_0 is to reject for large values of $LR_n(\tau_0)$. Using the $LR_n(\tau)$ function, asymptotic p-values for the likelihood ratio test are derived:

$$p_n = 1 - \left(1 - \exp(-1/2 \cdot LR_n(\tau_0)^2)\right)^2.$$

5. Fundamental modeling of intraday prices

In our model, we examine whether intraday prices in the continuous bidding system are caused by market fundamentals. As already mentioned earlier in this study, marketers of renewable energy use the intraday market to balance out differences between actual/updated forecasts of wind and PV. Indeed, discussions with energy traders revealed that at the time of the bid market participants have private access to the freshest weather forecasts for a certain quarter of an hour (delivery period) and use this information for adjusting their bids accordingly. Intuitively, this adjustment causes deviations between the intraday and day-ahead prices for a certain delivery period. An understanding of these deviations is furthermore important for strategic bidding.

The impact of forecasting errors in renewables on intraday prices should however not be judged in isolation, but dependent on the demand quote, meaning the extent at which forecasted demand for a certain hour is covered by the traditional capacity planned in the day-ahead market. Keeping in mind that renewables are fed with priority into the electricity grid, accordingly, more or less traditional capacity is planned (and more or less demand gap or demand quote is realized). Thus, intuitively, the higher the expectation from the renewables in the market day-ahead, the higher the demand quote: power producers plan overall less traditional capacity, since the residual demand is expected to be covered by wind/PV infeed.

As discussed in section 2, dependent on the demand quote regime, thus, if there is excess demand or not in the market, positive and negative forecasting errors in wind and PV are expected to have different impact on price deviations. In the context of a threshold model specification, where the threshold variable is the demand quote, we will examine these dynamics.

5.1. Modeling deviations of last prices from the day-ahead price

In the first part of our analysis, we analyze the differences between the historical last prices bid for a certain 15-minute delivery period in the intraday market and the day-ahead price for the corresponding hour. We used historical last prices sorted for quarter-hourly products between 01/01/2014–30/06/2014. As market fundamentals we include positive/negative forecasting errors in wind and PV, defined as deviations between the latest forecast available at the time when the last prices are observed and the day-ahead available forecasts. The last prices for a certain delivery period are placed in the market not later than 30 minutes before the delivery period starts[4]. At this time, market participants also forecast the volume in the balancing market, namely positions that could not be filled in the intra-day market. These positions are defined by the Transmission System Operators as "control area balances"[5].

We derive the forecasts of control area balances based on an autoregressive model.[6] Results are shown in Table 3. The order of lags has been identified by examining the autocorrelation function and we further performed Akaike (AIC) and Bayesian (BIC) information criteria to select the best model[7]. We found that the control area balances for a certain 15-minute delivery period can be forecasted based on the last 8 observations (up to 2 hours ago). Forecasts based on this model are further included in our model estimation.

The demand quote is defined as:

$$DemandQuote_t = DemandForecast_t / PPA_{dt} \tag{6}$$

where d is the day-ahead and t one hour in day d. $DemandForecast_t$ is the demand forecast for the relevant hour t on the delivery day d overall Transmission System Operators (source ENTSOE[8]). Based on the expected demand, power producers plan traditional capacity day-ahead. The PPA is the ex-ante expected power plant availability for electricity production on the delivery day (daily granularity), daily published at 10:00 am (see Table 1 for the exact data sources). These data exclude the renewable capacity and include only the traditional plants[9]. EPEX publishes data on installed and

[4]Since 16th July, 2015, EPEX Spot will shorten the lead time from 45- to 30 minute before delivery (see European Power Exchange (EPEX) `https://www.epexspot.com/en/`).

[5]see `http://www.tennettso.de`

[6]Discussions with traders revealed that this is a common praxis in the industry.

[7]Results are available upon request

[8]European Energy Exchange & Transmission System Operators

[9]The PPA includes: coal, gas, lignite, oil, pumped-storage, run-of-the-river, seasonal-store and uranium planned capacity day-ahead.

Table 3: Autoregressive model for control area balances

Dependent Variable: Balances				
Method: Least Squares				
Included observations: 2535 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	18.551*	6.228	2.978	0.002
Balances(-1)	0.818	0.019	41.195	0
Balances(-2)	0.055	0.025	2.160	0.031
Balances(-3)	-0.072	0.025	-2.809	0.005
Balances(-4)	0.162	0.025	6.359	0
Balances(-5)	-0.132	0.025	-5.166	0
Balances(-6)	-0.013	0.025	-0.543	0.586
Balances(-7)	-0.004	0.025	-0.185	0.852
Balances(-8)	0.047	0.019	2.369	0.017
R-squared	0.727	Mean dependent var		131.686
Adjusted R-squared	0.726	S.D. dependent var		577.588
S.E. of regression	301.8479	Akaike info criterion		14.261
Sum squared resid	2.30E+08	Schwarz criterion		14.281
Log likelihood	-18067.2	Hannan-Quinn criter.		14.268
F-statistic	844.035	Durbin-Watson stat		1.998
Prob(F-statistic)	0			

The order of lags has been identified by examining the autocorrelation function and we further performed Akaike (AIC)and Bayesian (BIC) information criteria to select the best model.

available capacities. Although these publications are voluntary, participating companies have tripled in 2010 and by the end of the year represented 89% of all relevant companies (see [22]). Thus, the numbers provided can be considered a reasonable approximation for the entire market. We use ex-ante demand quote as explanatory variable to our model to take into account to which extent the expected demand for electricity for the day-ahead is covered by the planned traditional capacity.

In Tables A.10 and A.11 we show descriptive statistics of the selected input variables. We distinguish between summer/winter, peak/off peak hours (as shown in [23]). We observe that, independent on the season, on average the intraday last price for 15-minute delivery periods is below the day-ahead price for the corresponding hour. Furthermore, the difference becomes larger and more volatile for peak than for off-peak hours and in winter than in summer. The control area balances are, on average, negative in winter and turn into positive in summer. On average, the demand quote is higher and more volatile during peak than in off-peak hours.

To test for stationarity we perform an augmented Dickey-Fuller test (ADF test). For all variables we reject the null hypothesis of a unit root at a 95% significance level meaning that the data is stationary.

As shown in Figures 3 and 4, there is a clear jigsaw seasonality in the last prices, independent of the season. Based on the information of the long-term dynamics of historical last prices, we control for the seasonal pattern by introducing dummy variables as follows:

- **Summer peak**
  - We introduce one Dummy variable for each of the Q1–Q4 quarters for the interval 08:00–13:00 (*Morning pattern*)
  - We introduce one Dummy variable for each of the Q1–Q4 quarters for the interval 14:00–18:00 (*Afternoon pattern*)
- **Winter peak**
  - We introduce one Dummy variable for each of the Q1–Q4 quarters for the interval 08:00–12:00 (*Morning pattern*)
  - We introduce one Dummy variable for each of the Q1–Q4 quarters for the interval 13:00–17:00 (*Afternoon pattern*)
- **Summer off-peak**
  - We introduce one Dummy variable for each of the Q1–Q4 quarters for the interval 20:00–01:00 (*Evening descending pattern*)
  - We introduce one Dummy variable for each of the Q1–Q4 quarters for the interval 03:00–07:00 (*Early morning ascending pattern*)
- **Winter off-peak**
  - We introduce one Dummy variable for each of the Q1–Q4 quarters for the interval 20:00–21:00 and 04:00–07:00 (*Descending pattern*)
  - We introduce one Dummy variable for each of the Q1–Q4 quarters for the interval 23:00–03:00 (*Night, ascending pattern*)

The model specification reads:

$$
\begin{aligned}
(P_t^{ID} - P_t^{Dahd})^h = {} & c^h + \beta^h ControlAreaBalance_t \mathbf{1}_t^h + \theta^h DemandQuote_t \mathbf{1}_t^h \\
& + k^{hn}(Wind_t^{ID} - Wind_t^{Dahd})\mathbf{1}_t^h \mathbf{1}_t^n + k^{hp}(Wind_t^{ID} - \\
& - Wind_t^{Dahd})\mathbf{1}_t^h \mathbf{1}_t^p + k^{hn}(PV_t^{ID} - PV_t^{Dahd})\mathbf{1}_t^h \mathbf{1}_t^n \\
& + k^{hp}(PV_t^{ID} - PV_t^{Dahd})\mathbf{1}_t^h \mathbf{1}_t^p + \sum_{j=1}^{8} \delta_j^h DQ_j
\end{aligned}
$$

$$
\begin{aligned}
(P_t^{ID} - P_t^{Dahd})^l = {} & c^l + \beta^l ControlAreaBalance_t \mathbf{1}_t^l + \theta^l DemandQuote_t \mathbf{1}_t^l \\
& + k^{ln}(Wind_t^{ID} - Wind_t^{Dahd})\mathbf{1}_t^l \mathbf{1}_t^n + k^{lp}(Wind_t^{ID} - \\
& - Wind_t^{Dahd})\mathbf{1}_t^l \mathbf{1}_t^p + k^{ln}(PV_t^{ID} - PV_t^{Dahd})\mathbf{1}_t^l \mathbf{1}_t^n \\
& + k^{lp}(PV_t^{ID} - PV_t^{Dahd})\mathbf{1}_t^l \mathbf{1}_t^p + \sum_{j=1}^{8} \delta_j^l DQ_j
\end{aligned} \tag{7}
$$

As threshold variable, the demand quote splits the data in two regimes: high/sufficient demand quote ("h") or low ("l"). The indicator function $\mathbf{1}_t^{p/n}$ further distinguishes in each regime between positive/negative forecasting errors in renewables.

5.2. Model for the continuous trades for quarter-hourly products

In the second part, we examine the continuous trades for several quarter-hourly products. In particular, we are interested to see how delta bid prices for a certain quarter of an hour change when new information on the forecasts for wind and PV becomes available. We look at the trade-off between autoregressive terms and fundamental factors impacting the intraday price formation process.

The model specification reads:

$$
\begin{aligned}
(\Delta P_t^{ID})^h = {} & c^h + \alpha_1^h \Delta P_{t-1}^{ID} \mathbf{1}_t^h + \alpha_2^h \Delta P_{t-2}^{ID} \mathbf{1}_t^h + \alpha_3^h \Delta P_{t-3}^{ID} \mathbf{1}_t^h \\
& + k_w^{hn}(\Delta Wind_t^{ID})\mathbf{1}_t^h \mathbf{1}_t^n + k_w^{hp}(\Delta Wind_t^{ID})\mathbf{1}_t^h \mathbf{1}_t^p \\
& + k_{PV}^{hn}(\Delta PV_t^{ID})\mathbf{1}_t^h \mathbf{1}_t^n + k_{PV}^{hp}(\Delta PV_t^{ID})\mathbf{1}_t^h \mathbf{1}_t^p \\
& + \gamma^h DemandQuote_t^{Dahd} \mathbf{1}_t^h + \epsilon^h Volume_t^{ID} \mathbf{1}_t^h + \beta_h \sqrt{\Delta t}
\end{aligned}
$$

$$
\begin{aligned}
(\Delta P_t^{ID})^l = {} & c^l + \alpha_1^l \Delta P_{t-1}^{ID} \mathbf{1}_t^l + \alpha_2^l \Delta P_{t-2}^{ID} \mathbf{1}_t^l + \alpha_3^l \Delta P_{t-3}^{ID} \mathbf{1}_t^l \\
& + k_w^{ln} (\Delta Wind_t^{ID}) \mathbf{1}_t^l \mathbf{1}_t^n + k_w^{lp} (\Delta Wind_t^{ID}) \mathbf{1}_t^l \mathbf{1}_t^p \\
& + k_{PV}^{ln} (\Delta PV_t^{ID}) \mathbf{1}_t^l \mathbf{1}_t^n + k_{PV}^{lp} (\Delta PV_t^{ID}) \mathbf{1}_t^l \mathbf{1}_t^p \\
& + \gamma^l DemandQuote_t^{Dahd} \mathbf{1}_t^l + \epsilon^l Volume_t^{ID} \mathbf{1}_t^l + \beta_l \sqrt{\Delta t} \qquad (8)
\end{aligned}
$$

The examination of autocorrelation function of price changes for a certain quarter of an hour shows that the first 3 lags of price changes should be selected in the autoregressive part of the model. Changes in the wind, $\Delta Wind_t^{ID}$, and in the PV, ΔPV_t^{ID}, are real time updated forecasts, available at the time when bids are placed.[10] $Volume_t^{ID}$ is the volume trade at the time when the price change is observed. The bids for a certain quarter of an hour do not occur at equal time intervals in the continuous bidding. In fact, market participants start bidding around 4 pm, after the day-ahead prices are published at EPEX and continuous trades go up to 30 minutes before the beginning of the delivery period. Thus, the time steps between consecutively placed bids are not equal, but can vary from some seconds to several hours. We take into account this time discontinuity by including in our list of explanatory variables the control variable $\sqrt{\Delta t}$.

In Tables A.12 and A.13 we show descriptive statistics for the price changes and volume of trades for the 15-minute continuous trading for delivery periods at different times of the day. We observe that the volatility of intraday price changes increases continuously between the morning quarter of hours (H7Q1) up to noon (H12Q4) and decreases again towards the evening (quarters of hour 18). Thus, the higher the demand, the larger the average price changes in the continuous trading. The volume of trades is on average the highest and most volatile for the first and last quarters of each one of the investigated hours, independent on the time of the day. This explains the sell/buy pressure, as explained in Section 2.

[10]Results are available upon request

6. Estimation results and interpretation

6.1. Modeling deviations of last prices from the day-ahead price

The model shown in Equation (7) has been estimated for the historical differences between the last prices and the day-ahead prices separately for winter and summer and we further distinguished between peak (8 am and 8 pm) and off-peak hours. This approach is justified by the different price levels in summer compared to the winter time and by the different demand profiles during peak and off-peak hours (see [23] for an extensive discussion on the seasonality of electricity prices).

As a preliminary analysis, we estimated the model without allowing for a threshold effect, to assess whether there is a linear adjustment of intraday (last) prices to market fundamentals. The overall OLS estimation results for each case study are shown in Table 4.

Throughout all variables are significant and show the expected sign (see Table 4). Dummy variables which explain the jigsaw pattern are statistically significant and their inclusion still allows significant marginal effects of fundamental variables on delta prices. The coefficients of positive/negative forecasting errors in wind and PV are significant at 1% significance level. Positive forecasting errors of wind/PV signal market participants more capacity available in the market than planned. This will have a decreasing effect on the residual demand and will further decrease last price bids. Viceversa, when updated forecasts signal less infeed from renewables than planned in the day ahead (negative forecasting errors), market participants will increase their bid prices intraday accordingly.

At the time of the last price bids, market participants do not know yet the real control area balances, but forecasts of those are used in practice. This is reflected in the coefficients of balances forecasts which are statistically significant in all case studies and have a positive sign. Higher control area balances are a signal of excess demand which has not been yet balanced out in the intraday market, and this will be reflected in higher intraday last prices.

We observe that the coefficient of demand quote is negative during the off-peak regimes, but it turns into positive during peak hours. The mean value of demand quote in the *off-peak hours* is slightly below one, touching a maximum of 1.291 and 1.178, respectively (as shown in Tables A.10 and A.11). Thus, on average, the traditional capacity planned in the market covers the expected demand for the day-ahead. In Figure 5, the upper graph illustrates such a theoretical case, where the demand quote is 1. However, at higher levels of demand quote (up to a maximum observed in off-peak hours of about 1.2), power producers plan less capacity for the day ahead, due to a higher expectation of renewables infeed in the market (see Figure 5, lower graph).[11]. That means, less expensive capacity is planned, which situates the prices in the less convex area of the merit order. The input from renewable energies is expected to be, on average, 20% of the total input production mix in Germany (see [22]). Renewables will be fed with priority into the grid, decreasing the residual demand and thus market participants will bid lower prices intraday. This assumption is confirmed by the negative sign of the coefficients of demand quote in the off-peak hours winter/summer, as shown in Table 4.

For the *peak hours* descriptive statistics show that on average, the demand quote exceeds 1.2 (see Table A.11), which means that there is more than 20% of the expected demand uncovered by the planned traditional capacity. Thus, power producers plan less capacity in the market, given a high market expectation for renewables infeed in peak hours. We illustrate graphically this situation in Figure 6, lower graph. However, demand quotes above 1.2 reflect the situation where the 20% expected infeed from renewables will not suffice and there will be still high residual demand in the market. This will have an increasing effect on intraday prices in general and on the last prices in particular, which is confirmed by the positive sign of the coefficient of demand quote (see Table 4)[12].

We further tested for a threshold effect in the demand quote in each case. The threshold variable is the demand quote and the threshold location is estimated using the methodology described in section 4.2. All model parameters in Equations (7) are allowed to vary among regimes. We found evidence for significant threshold effect only in the case of winter peak case study. Results are available in Table 5.

We found no significant threshold effect in the demand quote in summer-related case studies and in winter off-peak. This shows that in those seasons, market participants adjust linearly last prices (and implicitly the spreads last prices-day-ahead prices) to market fundamentals. However, in winter peak time we found evidence for asymmetric behavior (see Table 5). Thus,

[11] It is known that in the night hours extreme wind infeed occur (see [23]).

[12] This is reflected in the high maximum spreads between the last prices and day-ahead prices observed in summer peak, as shown in Table A.11.

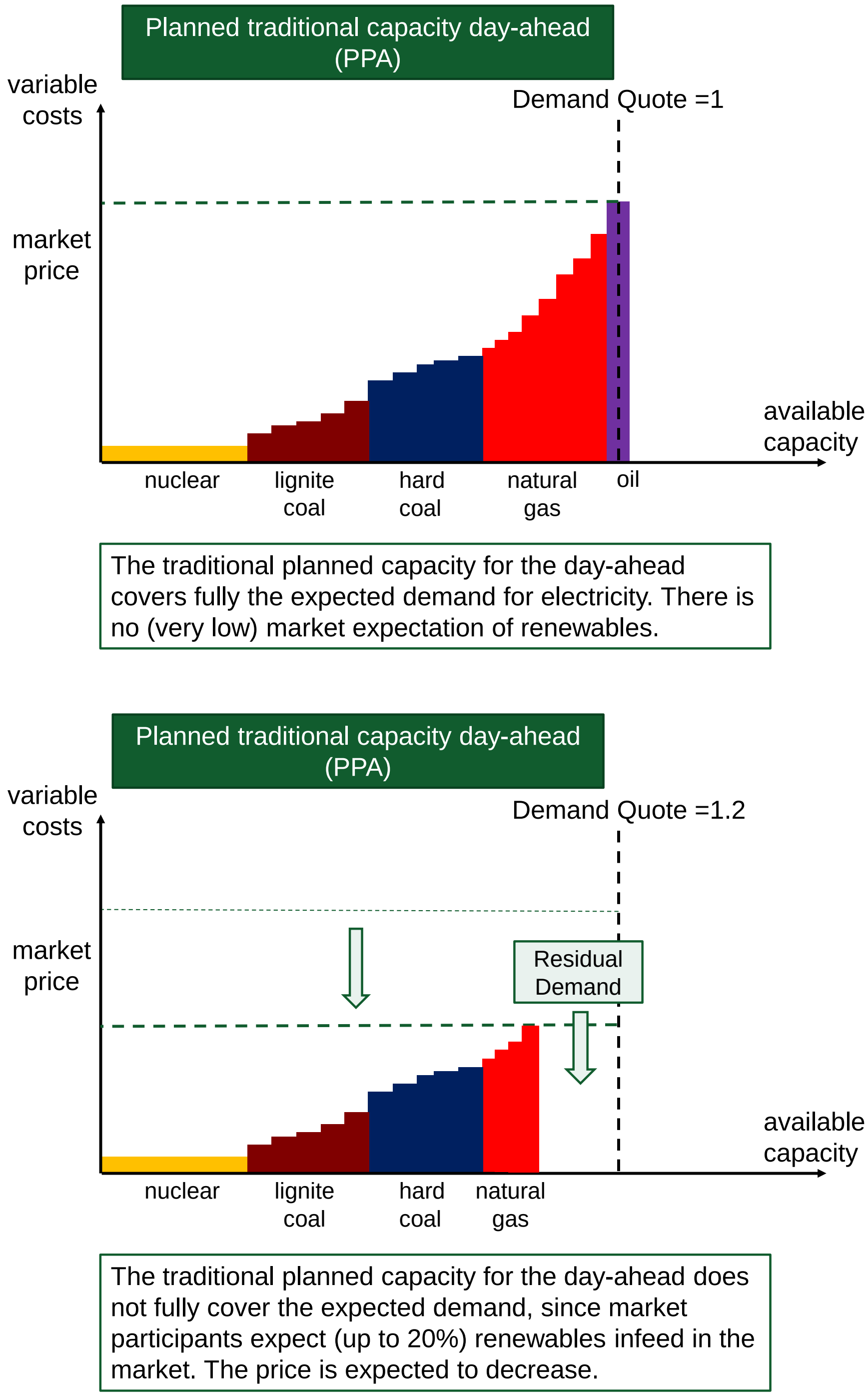


Figure 5: Theoretical model explaining the impact of ex-ante demand quote on intraday electricity prices.

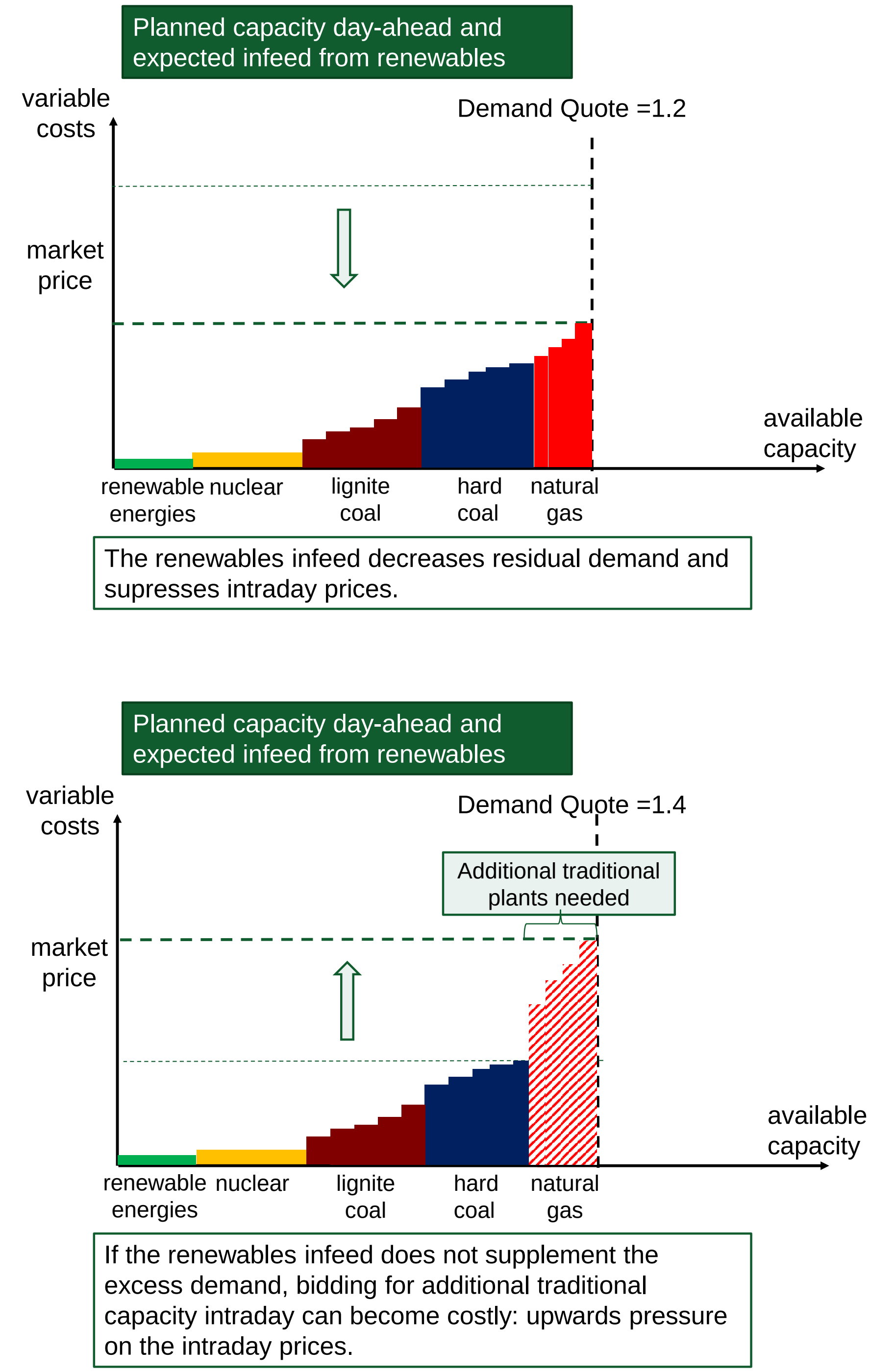


Figure 6: Theoretical model explaining the impact of ex-ante demand quote on intraday electricity prices (continuation).

a threshold in the demand quote was found significant at the level of 1.058. In the regime of low levels of demand quote (regime 1, < 1.058), we observe that coefficients are generally not statistically significant. That is, power producers have low expectation of renewable infeed in the day-ahead, and in consequence plan sufficient traditional capacity to satisfy expected demand. However, when demand levels are high, thus in regime 2, delta prices adjust linearly to forecasting errors in renewable energy, to control area balances and to demand quote. An increase in demand quote in this regime will furthermore suppress bid prices in the intraday market, since again higher demand quote levels reflect a high expectation of infeed from renewable energies, which will lower the price level. The coefficient of control area balances is positive and significant. This reflects two situations: if there is high infeed from renewables in the market, negative forecasts of control area balances will suppress the intraday last prices. By contrary, in the presence of high demand quote not fully covered by renewables infeed, positive forecasts in control area balances will increase intraday price bids.

The model can be used to forecast the last prices submitted for a certain quarter of one hour intraday. This is based on a rigourous forecasting model for the control area balances. This model is highly relevant for practitioners: the main goal of market participants is to clear their positions in the day-ahead and intraday markets and avoid participating in the more expensive balancing market.

6.2. Model for the continuous trades for quarter-hourly products

In this section, we show the impact of fundamental variables on the (continuous) bidding behavior. We checked for both linear and asymmetric adjustment of intraday price changes to explanatory variables, dependent on the time of the day. We therefore replicated the analysis to different delivery periods (peak/off-peak) corresponding to different demand profiles: quarters 1–4 of hours 7, 12 and 18 have been investigated. The estimation results of (OLS) linear estimation, without threshold, of Equation (8) are shown in Table 6, B.14 and B.15. The main threshold estimation results following the specification in Equation (8) are shown in Tables 7–9. In all cases the demand quote has been found to be the only significant threshold variable.[13]

[13]The threshold values are significant, accordingly to the likelihood ratio test, as discussed in section 4.1. The graphs and calculations corresponding to each threshold values

Table 4: Estimation results of the model shown in Equation 7. Global OLS without threshold

Dependent variable Delta Last Price- Price DayAhedd

	Summer off-peak		**Summer peak**		**Winter off-peak**		**Winter peak**	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	7.388*	(1.971)	-20.956*	(6.128)	14.469*	(4.762)	-9.015	(10.354)
DemandQ	-7.438*	(2.159)	10.929**	(4.852)	-12.715*	(4.605)	-0.354	(8.728)
Balancing	0.007*	(0.001)	0.008*	(0.001)	0.014*	(0.001)	0.009*	(0.001)
DeltaWindP	-0.005*	(0.001)	-0.002**	(0.001)	-0.003*	(0.001)	-0.003*	(0.001)
DeltaWindN	-0.007*	(0.001)	-0.012*	(0.001)	-0.004*	(0.001)	-0.004*	(0.001)
DeltaPVP	–	–	-0.003*	(0.001)	–	–	-0.003*	(0.001)
DeltaPVN	–	–	-0.004*	(0.001)	–	–	-0.005*	(0.001)
DQ1M	10.170*	(1.112)	10.022*	(1.462)	-4.561*	(1.729)	23.808*	(2.340)
DQ2M	3.515*	(1.144)	2.192	(1.507)	-5.094*	(1.717)	11.336*	(2.148)
DQ3M	-6.519*	(1.122)	-1.486	(1.463)	-3.148	(1.704)	2.740	(2.207)
DQ4M	-10.454*	(1.139)	-6.031*	(1.622)	-1.187	(1.719)	-0.548	(2.296)
DQ1A	-13.845*	(1.219)	-8.111*	(1.539)	3.114	(1.848)	-6.098*	(2.173)
DQ2A	-6.852*	(1.229)	0.268	(1.374)	-0.948	(1.802)	3.203	(2.016)
DQ3A	0.349	(1.161)	3.458**	(1.341)	-4.578**	(1.793)	16.773*	(2.118)
DQ4A	4.842*	(1.203)	13.132*	(1.451)	-4.568**	(1.825)	25.588*	(2.294)
Rsquared	35.43%		37.99%		28.76%		36.63%	
No. Obs.	2543		2483		2447		2363	

Standard errors are shown in parenthesis. * and **, denote a test statistic is statistically significant at the 1% and 5% level of significance, respectively. DemandQ=Demand Quote defined in Equation (6); Balancing=control area balances; DeltaWindIntrP/N=positive/negative forecasting errors in wind; DeltaPVIntraP/N=positive/negative forecasting errors in PV; DQ1M–DQ4M=Dummies for the four quarters of each morning hours (where morning defines the first part of the day: 0800–1400); DQ1A–DQ4A=Dummies for the four quarters of each afternoon hours (where afternoon defines the second part of the day: 1400–2000)

Table 5: Winter peak, threshold estimation results. Threshold variable: Demand Quote

Threshold estimation (threshold variable DemandQ)				
Dependent variable Delta Last Price- Price Dahd				
	Regime 1		**Regime 2**	
Threshold value	<= 1.058		> 1.058	
	Coeff	Std. Err.	Coeff	Std. Err.
Co	-48.973*	(15.527)	63.563*	(22.987)
DemandQ	26.810**	(12.806)	-61.545*	(19.412)
Balancing	0.003	(0.002)	0.010*	(0.001)
DeltaWindP	-0.004	(0.003)	-0.002**	(0.001)
DeltaWindN	-0.006**	(0.003)	-0.004*	(0.001)
DeltaPVP	-0.003	(0.002)	-0.004*	(0.001)
DeltaPVN	-0.006*	(0.001)	-0.006*	(0.001)
DQ1M	41.322*	(8.710)	21.500*	(2.324)
DQ2M	21.880*	(7.985)	10.443*	(2.129)
DQ3M	4.806	(7.948)	3.682	(2.205)
DQ4M	2.266	(8.284)	0.298	(2.329)
DQ1A	-8.175	(7.420)	-1.367	(2.340)
DQ2A	8.898	(7.325)	3.440	(2.207)
DQ3A	30.651*	(7.536)	12.192*	(2.235)
DQ4A	45.249*	(7.616)	17.453*	(2.369)
Rsquared	48.61%		35.93%	
No. Obs.	652		1711	

Standard errors are shown in parenthesis. * and **, denote a test statistic is statistically significant at the 1% and 5% level of significance, respectively. DemandQ=Demand Quote defined in Equation (6); Balancing=control area balances; DeltaWindIntrP/N=positive/negative forecasting errors in wind; DeltaPVIntraP/N=positive/negative forecasting errors in PV; DQ1M–DQ4M=Dummies for the four quarters of each morning hours (where morning defines the first part of the day: 0800–1400); DQ1A–DQ4A=Dummies for the four quarters of each afternoon hours (where afternoon defines the second part of the day: 1400–2000)

In Table 6 we benchmarked the performance of our model by a version excluding fundamentals (see lower panel). By comparing the values of the R^2 between the lower and upper panels we observe that at noon fundamental variables increase the explanatory power of the model by up to 4 times. This effect is however less obvious in the case of morning and evening peak quarter-hourly products (see Tables B.14 and B.15).

More specifically, results reveal that during morning and evening the information from lagged price changes become more relevant for the price formation process than fundamental variables. However, fundamentals become significant during noon (see Table 6). This can be due to the fact that over noon, given the high demand for electricity in the market, the merit order (MO) curve is usually very steep, since more expensive plants are on use. Thus, market participants become more sensitive to market fundamentals, in particular to forecasting errors of renewable energies, given their low marginal costs of production. Negative forecasting errors in wind and PV would further increase the steepness of the MO, which leads to increased intraday prices, while positive forecasting errors in renewables will have a suppressing effect on prices.

In Table 8 we allow for threshold effect in the demand quote for quarters 1–4 of hour 12. Similarly to the results in section 6.1, a threshold has been found significant when the demand quote is around 1.2, which allows a nice interpretation, given the 20% expected infeed from renewables in the German power market. Given this expectation, less traditional plants are planned day-ahead (see Figures 5 and 6). Also in this case, we conclude an asymmetric adjustment of intraday price changes to forecasting errors of wind and PV, dependent on the demand quote regime. In particular, results reveal that market participants adjust their intraday bids to updated forecasts moreover in the high demand quote regime. Thus, when there is a high expected infeed from renewables day-ahead, market participants follow updated forecasted errors in wind and PV and incorporate this information in adjusting their bids accordingly intraday. This effect becomes more obvious for noon hours, when the demand is high and the MO is usually steeper than during morning and evening hours. Thus, Tables 7 and 9 show that the role of forecasting errors of renewables for the morning and evening quarters drops, independent

are available upon request. We have tested for threshold significance also in the other fundamental variables, but no conclusive results were obtained.

of the demand quote regime.

Still, during morning and evening delivery periods (Tables 7 and 9) we observe that market fundamentals help explaining the jigsaw pattern of intraday prices: positive forecasting errors in PV decrease prices in quarter 4 of hour 7 in regime 2, which reflects the *ramping up effect of the sun*. By contrary, forecasting errors of wind and PV impact intraday prices in the first 3 quarters of hour 18. After this quarter, however, the role of forecasting errors of PV drops, showing the *ramping down effect of the sun*.

Results reveal further evidence for the ramping up/down effects of the sun, reflected in the sign of the volume of trades. We observe that the corresponding coefficient is significant only for quarter 4 of hour 7 (see Table B.14) and has a negative sign. This pattern is again observed in the threshold model for hour 7 (see Table 7) in regime 1, when the demand quote is below 1.415 (see Tables 7). For the last quarter of hour 7 the intraday price is below the average price bid for hour 7 in the day-ahead due to the sun ramping up effect, reflecting an oversupply of the accounting grid (see Figure 2). However, for hour 18 this effect is reverted. As shown in Tables B.15 and 9, the coefficient of volume of trades is significant and has a negative sign for the first quarter of hour 18 and turns into positive in the last quarter. This reflects the sun ramping down effect, which causes the jigsaw pattern for the evening hours: the intraday price for quarter 1 is below the average price bid in the day-ahead for the respective hour (oversupply of the accounting grid) and it ends above it for quarter 4, reflecting shortfalls in the accounting grid.

Table 6: Estimation results hour 12, Quarters 1–4, global OLS without threshold

OLS estimation of the model including fundamental variables

Dependent variable Delta Price

	H12Q1		**H12Q2**		**H12Q3**		**H12Q4**	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	-0.558	(0.672)	-0.674	(0.977)	-0.111	(0.765)	-0.032	(0.799)
DeltaPrice1	-0.175**	(0.086)	-0.167*	(0.043)	-0.207*	(0.038)	-0.140*	(0.020)
DeltaPrice2	-0.071**	(0.032)	-0.040	(0.023)	-0.077**	(0.036)	-0.079*	(0.020)
DeltaPrice3	-0.102	(0.060)	-0.018	(0.017)	-0.039	(0.021)	-0.020	(0.013)
DemandQuote	0.109	(0.499)	0.408	(0.755)	0.156	(0.578)	0.088	(0.635)
Volume	0.053*	(0.019)	0.012	(0.009)	-0.012	(0.009)	-0.013**	(0.006)
SqrTimeStep	0.423	(1.570)	1.868	(1.365)	1.010	(1.348)	1.683	(1.853)
DeltaWindIntrP	-0.001*	(0.000)	-0.001	(0.001)	-0.001*	(0.000)	-0.001*	(0.000)
DeltaWindIntrN	-0.001*	(0.000)	-0.001	(0.001)	-0.001	(0.001)	-0.002**	(0.001)
DeltaPVIntraP	-0.002**	(0.001)	-0.002**	(0.001)	-0.002**	(0.001)	-0.004*	(0.001)
DeltaPVIntraN	0.000	(0.001)	-0.001	(0.001)	-0.002**	(0.001)	-0.002**	(0.001)
Rsquared	7.296%		4.705%		7.011%		8.411%	
No. Obs.	6859		5449		6558		7931	

OLS estimation of the autoregressive model excluding fundamental variables

Dependent variable Delta Price

	H12Q1		**H12Q2**		**H12Q3**		**H12Q4**	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.006	(0.077)	0.004	(0.099)	0.005	(0.092)	0.003	(0.066)
DeltaPrice1	-0.172*	(0.012)	-0.167*	(0.014)	-0.206*	(0.012)	-0.137*	(0.011)
DeltaPrice2	-0.065*	(0.012)	-0.041*	(0.014)	-0.077*	(0.013)	-0.078*	(0.011)
DeltaPrice3	-0.099*	(0.012)	-0.018	(0.014)	-0.041*	(0.012)	-0.019	(0.011)
Rsquared	3.715%		2.733%		4.219%		2.187%	
No. Obs.	6859		5449		6558		7931	

Standard errors are shown in parenthesis. *, and ** denote a test statistic is statistically significant at the 1% and 5% level of significance, respectively. The interpretation of variables is: DeltaPrice(x)=lagged price changes 1–3; DemandQuote=demand quote; Volume=volume of trades; SqrTimeStep=$\sqrt{\Delta_t}$; DeltaWindIntrP/N=positive/negative forecasting errors in wind; DeltaPVIntraP/N=positive/negative forecasting errors in PV.

Table 7: Estimation results hour 7, Quarters 1–4, First Sample Split

Dependent variable Delta Price								
	H7Q1		**H7Q2**		**H7Q3**		**H7Q4**	
Regime 1								
Threshold value	<= 1.161*		<= 0.757*		<= 0.828*		<= 1.415*	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.765	(1.365)	16.416*	(7.688)	-16.689	(13.279)	-1.561**	(0.822)
DeltaPrice1	-0.184*	(0.036)	-0.155**	(0.073)	-0.221*	(0.083)	-0.255*	(0.030)
DeltaPrice2	-0.193*	(0.038)	-0.187*	(0.044)	-0.087	(0.085)	-0.169*	(0.020)
DeltaPrice3	-0.098*	(0.022)	-0.005	(0.051)	-0.075	(0.057)	-0.086*	(0.017)
DemandQuote	-0.844	(1.253)	-21.980**	(10.706)	19.229	(17.252)	1.416**	(0.700)
Volume	0.010	(0.007)	0.044	(0.108)	-0.061	(0.053)	-0.018*	(0.006)
SqrTimeStep	0.054	(1.959)	1.370	(9.574)	44.873*	(12.333)	3.820**	(1.571)
DeltaWindIntrP	0.000	(0.000)	-0.056*	(0.018)	-0.134*	(0.025)	-0.001	(0.001)
DeltaWindIntrN	0.000	(0.001)	-0.013	(0.017)	0.014**	(0.007)	0.001	(0.001)
DeltaPVIntraP	0.001	(0.002)	0.001	(0.013)	0.007	(0.024)	0.003*	(0.001)
DeltaPVIntraN	0.000	(0.001)	0.012	(0.011)	0.011	(0.008)	0.000	(0.001)
Rsquared	6.081%		67.460%		63.497%		9.053%	
No. Obs.	4090		82		111		6984	
	H7Q1		**H7Q2**		**H7Q3**		**H7Q4**	
Regime 2								
Threshold value	> 1.161*		> 0.757*		> 0.828*		> 1.415*	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.388	(1.305)	-0.368	(1.062)	-0.172	(1.095)	-58.038	(120.183)
DeltaPrice1	-0.233*	(0.050)	-0.318*	(0.031)	-0.236*	(0.035)	-0.363*	(0.135)
DeltaPrice2	-0.081	(0.049)	-0.156*	(0.022)	-0.109*	(0.020)	-0.231*	(0.088)
DeltaPrice3	-0.047	(0.025)	-0.084*	(0.019)	-0.081*	(0.018)	-0.093**	(0.047)
DemandQuote	-0.210	(1.023)	0.302	(0.904)	-0.096	(0.914)	39.713	(83.769)
Volume	0.004	(0.006)	0.014	(0.009)	0.002	(0.009)	-0.035	(0.039)
SqrTimeStep	-3.034	(1.930)	-0.905	(1.372)	4.528*	(1.291)	43.401*	(17.220)
DeltaWindIntrP	-0.002**	(0.001)	0.000	(0.000)	-0.001	(0.001)	-0.052	(0.036)
DeltaWindIntrN	-0.001	(0.001)	0.000	(0.000)	0.000	(0.001)	-0.006	(0.036)
DeltaPVIntraP	0.001	(0.002)	0.001	(0.001)	0.000	(0.001)	-0.029*	(0.004)
DeltaPVIntraN	0.001	(0.001)	0.002**	(0.001)	-0.001	(0.001)	-0.027	(0.055)
Rsquared	10.094%		10.659%		7.349%		47.604%	
No. Obs.	2889		4791		4850		191	

Standard errors are shown in parenthesis. *, and ** denote a test statistic is statistically significant at the 1% and 5% level of significance, respectively. The interpretation of variables is: DeltaPrice(x)=lagged price changes 1–3; DemandQuote=demand quote; Volume=volume of trades; SqrTimeStep=$\sqrt{\Delta_t}$; DeltaWindIntrP/N=positive/negative forecasting errors in wind; DeltaPVIntraP/N=positive/negative forecasting errors in PV.

Table 8: Estimation results hour 12, Quarters 1–4, First Sample Split

Dependent variable Delta Price								
Regime 1								
	H12Q1		**H12Q2**		**H12Q3**		**H12Q4**	
Threshold value	<= 1.245*		> 0.757*		<= 1.146*		<= 1.197*	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	-0.669	(1.982)	-0.693	(3.302)	0.421	(2.418)	0.365	(3.418)
DeltaPrice1	-0.202	(0.118)	-0.126*	(0.043)	-0.191**	(0.075)	-0.108*	(0.031)
DeltaPrice2	-0.065	(0.043)	-0.042**	(0.021)	-0.142	(0.085)	-0.082**	(0.040)
DeltaPrice3	-0.099	(0.078)	-0.010	(0.018)	-0.023	(0.078)	-0.030	(0.017)
DemandQuote	0.163	(1.685)	0.518	(2.798)	0.036	(2.104)	-0.378	(3.069)
Volume	0.070**	(0.028)	0.022	(0.012)	-0.007	(0.029)	0.003	(0.016)
SqrTimeStep	-1.363	(2.119)	-0.205	(1.886)	-9.905	(5.560)	0.880	(2.436)
DeltaWindIntrP	0.000	(0.001)	0.000	(0.001)	0.005*	(0.002)	-0.001	(0.001)
DeltaWindIntrN	-0.001	(0.001)	-0.001	(0.001)	-0.006*	(0.001)	0.002	(0.002)
DeltaPVIntraP	-0.003*	(0.001)	-0.003*	(0.001)	-0.007**	(0.003)	-0.002	(0.002)
DeltaPVIntraN	0.001	(0.001)	-0.001	(0.001)	-0.002	(0.002)	-0.003*	(0.001)
Rsquared	9.155%		3.806%		27.371%		7.764%	
No. Obs.	3911		3052		487		2438	
Regime 2								
	H12Q1		**H12Q2**		**H12Q3**		**H12Q4**	
Threshold value	> 1.245*		> 0.757*		> 1.146*		> 1.197*	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.125	(1.349)	-1.036	(1.809)	-0.037	(0.928)	0.405	(0.944)
DeltaPrice1	-0.094**	(0.040)	-0.256*	(0.060)	-0.208*	(0.040)	-0.155*	(0.022)
DeltaPrice2	-0.108	(0.040)	-0.046	(0.053)	-0.072	(0.038)	-0.075	(0.020)
DeltaPrice3	-0.099**	(0.043)	-0.035	(0.035)	-0.039	(0.022)	-0.011	(0.018)
DemandQuote	-0.216	(0.965)	0.630	(1.304)	0.065	(0.693)	-0.163	(0.692)
Volume	0.018**	(0.008)	-0.006	(0.013)	-0.012	(0.010)	-0.021*	(0.006)
SqrTimeStep	1.140	(1.439)	3.942**	(1.758)	2.263	(1.191)	-0.097	(1.700)
DeltaWindIntrP	-0.002*	(0.000)	-0.002**	(0.001)	-0.001*	(0.000)	-0.001	(0.001)
DeltaWindIntrN	-0.001*	(0.000)	-0.002**	(0.001)	-0.001	(0.001)	-0.002**	(0.001)
DeltaPVIntraP	0.000	(0.001)	-0.001	(0.001)	-0.002**	(0.001)	-0.002**	(0.001)
DeltaPVIntraN	-0.001	(0.001)	-0.002**	(0.001)	-0.001	(0.001)	-0.004*	(0.001)
Rsquared	8.868%		10.760%		6.590%		11.624%	
No. Obs.	2948		2397		6071		5493	

Standard errors are shown in parenthesis. *, and ** denote a test statistic is statistically significant at the 1% and 5% level of significance, respectively. The interpretation of variables is: DeltaPrice(x)=lagged price changes 1–3; DemandQuote=demand quote; Volume=volume of trades; SqrTimeStep=$\sqrt{\Delta_t}$; DeltaWindIntrP/N=positive/negative forecasting errors in wind; DeltaPVIntraP/N=positive/negative forecasting errors in PV.

Table 9: Estimation results hour 18, Quarters 1–4, First Sample Split

Dependent variable Delta Price

Regime 1

	H18Q1		**H18Q2**		**H18Q3**		**H18Q4**	
Threshold value	<= 0.915*		<= 1.221*		<= 1.219*		<= 1.442*	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	46.694	(152.240)	0.020	(2.024)	-5.932*	(2.012)	-0.481	(1.031)
DeltaPrice1	-0.510*	(0.116)	-0.258*	(0.035)	-0.252*	(0.032)	-0.198*	(0.037)
DeltaPrice2	-0.284*	(0.105)	-0.197*	(0.030)	-0.154*	(0.028)	-0.088*	(0.022)
DeltaPrice3	-0.137	(0.086)	-0.079**	(0.031)	-0.111*	(0.029)	-0.148*	(0.049)
DemandQuote	-52.391	(170.802)	0.296	(1.758)	4.995*	(1.757)	0.142	(0.855)
Volume	-0.051	(0.085)	-0.038*	(0.008)	0.041*	(0.008)	0.035*	(0.005)
SqrTimeStep	6.124	(19.295)	-1.137	(1.179)	-0.772	(1.032)	-3.303*	(1.266)
DeltaWindIntrP	0.019	(0.026)	0.000	(0.000)	-0.001*	(0.000)	0.000	(0.000)
DeltaWindIntrN	-0.027	(0.020)	-0.001	(0.001)	0.000	(0.000)	-0.001	(0.001)
DeltaPVIntraP	-0.340	(0.224)	0.038	(0.052)	-0.006	(0.014)	-0.053	(0.032)
DeltaPVIntraN	0.159	(0.321)	0.024	(0.029)	-0.036	(0.045)	0.086	(0.106)
Rsquared	30.618%		8.668%		8.109%		6.356%	
No. Obs.	133		3571		3553		8776	

Regime 2

	H18Q1		**H18Q2**		**H18Q3**		**H18Q4**	
Threshold value	> 0.915*		> 1.221*		> 1.219*		> 1.442*	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.460	(0.670)	0.944	(2.590)	-1.882	(3.752)	-10.224	(43.509)
DeltaPrice1	-0.181*	(0.025)	-0.284*	(0.064)	-0.247*	(0.061)	0.008	(1.892)
DeltaPrice2	-0.161*	(0.035)	-0.095*	(0.039)	-0.171*	(0.055)	-0.090	(0.990)
DeltaPrice3	-0.119*	(0.023)	-0.098*	(0.035)	-0.106*	(0.029)	-0.011	(0.992)
DemandQuote	-0.165	(0.526)	-0.568	(1.970)	1.163	(2.876)	-39.818	(57.807)
Volume	-0.025*	(0.004)	-0.008	(0.012)	0.042*	(0.014)	0.156	(0.506)
SqrTimeStep	-0.212	(1.319)	-3.076	(1.815)	0.507	(1.533)	-48.774	(122.258)
DeltaWindIntrP	0.000	(0.000)	-0.001	(0.001)	0.000	(0.001)	0.000	(0.043)
DeltaWindIntrN	-0.003*	(0.001)	-0.002**	(0.001)	-0.002*	(0.000)	0.204	(0.301)
DeltaPVIntraP	0.012	(0.009)	-0.010	(0.015)	-0.019	(0.014)	0.332	(7.980)
DeltaPVIntraN	-0.014**	(0.007)	-0.008	(0.013)	0.005	(0.031)	-2.765	(8.155)
Rsquared	11.003%		11.252%		9.295%		25.624%	
No. Obs.	8299		2411		2397		160	

Standard errors are shown in parenthesis. *, and ** denote a test statistic is statistically significant at the 1% and 5% level of significance, respectively. The interpretation of variables is: DeltaPrice(x)=lagged price changes 1–3; DemandQuote=demand quote; Volume=volume of trades; SqrTimeStep=$\sqrt{\Delta_t}$; DeltaWindIntrP/N=positive/negative forecasting errors in wind; DeltaPVIntraP/N=positive/negative forecasting errors in PV.

7. Conclusion

In this study, we investigate the bidding behavior in the intraday electricity market, in the context of a fundamental model. In particular, we shed light on the impact of updated forecasting errors of wind and photovoltaic (PV) on the 15-minute electricity price changes in the continuous bidding. We employ a unique data set of the latest forecasts of wind and PV available to traders prior to the placements of their price bids intraday. To our knowledge, this is the first study in the literature which models intraday prices based on prior information on fundamentals. We further control for the demand/supply disequilibria, volume of trades, forecasts of control area balances and model the typical jigsaw seasonality pattern of 15-minute prices.

Our analysis is twofold. We firstly propose a forecasting model for the changes between last prices bid intraday for a certain quarter of an hour and the corresponding day-ahead price. This is highly relevant, since market participants are mainly interested in squeezing their positions in the day-ahead or intraday markets and avoid ending into the control area balancing market. Secondly, a fundamental model for the price changes in the continuous bidding is derived. We found clear evidence that the bidding behavior is influenced by forecasting errors in renewables, available at the time of the bid. Intuitively, intraday prices increase in negative forecasting errors, while positive forecasting errors have a suppressing effect on prices.

We account for both linear and asymmetric adjustments of price changes to market fundamentals. The asymmetries are driven by the threshold variable demand quote. This shows market participants the proportion in which the expected demand is covered by the planned traditional capacity in the day-ahead market. Our model disentangles the effect of market fundamentals dependent on the regime of the demand quote and further dependent on the time of the day. Tangentially, market fundamentals influence more the bidding behavior in the middle of the day than during mornings and evenings. There is an asymmetric adjustment of electricity prices with respect to both volume of trades and forecasting errors in renewables. Namely, in the high regime of the demand quote, where there is too little planned traditional capacity in the day-ahead market, traders incorporate the information of the latest available forecasting errors of renewables in their bids with a higher speed. This effect is more obvious for the mid-day quarters, but less obvious during morning and evening hours. Thus, the historically derived threshold in the demand quote for a specific delivery period is a highly relevant information for strategically bidding in the intraday market. The actual demand quote can be compared to the historical threshold value and, dependent whether the market is in the low/high demand quote regime, market participants can use the model for one-period forecasts accordingly.

The identification of regimes in the demand quote helps also to disentangle the demand/supply side volume of trades. In the regime of high demand quote, demand-side volume of trades have an increasing effect on prices. Vice versa, supply-side volumes have a suppressing effect on intraday prices, which becomes obvious in the low regime of the demand quote.

Outlook

Our model aims at understanding the bidding behavior historically speaking and offers a solid basis for one-period forecast of last intraday prices and continuous bids. Since all variables used as input can be computed based on the information available at the time of the bid (demand quote, updated forecasts in renewables), the model can be used for forecasting the (next) continuous bid. We prove the superiority of such a fundamental model over the classical AR model representation. Still, we do not offer evidentiary support that this is the best model for intraday prices, but given that it is the first study which employs intraday-updated renewables forecasts, it is certainly the most realistic representation existing in the literature up to present. Practitioners use in reality updated forecasted errors as private information to bid more accurately in the intraday electricity market. In this context, our one-period proposed forecasting model is highly relevant for both academics and practitioners.

Appendix A. Descriptive statistics

Table A.10: Descriptive statistics of the differences between the historical last prices for 15-minute delivery periods and the day-ahead price and of fundamental variables at the time of the last bid during the winter time (01/01/2014–01/04/2014), for working days Monday–Thursday.

Winter Monday to Thursday, peak hours							
	DeltaPriceLast	ControlAreaBalance	DemandQuote	DeltaWindN	DeltaWindP	DeltaPVN	DeltaPVP
Mean	-0.379	-158.279	1.155	-484.003	264.214	-301.559	373.034
Median	-0.640	-163.671	1.165	-125.000	0.000	0.000	0.000
Maximum	299.290	3697.952	1.266	0.000	5180.000	0.000	4188.000
Minimum	-101.970	-3012.049	0.649	-4165.000	0.000	-7557.000	0.000
Std. Dev.	26.738	713.387	0.069	781.715	626.864	849.927	710.205
Skewness	1.514	0.447	-3.316	-2.313	4.584	-4.660	2.380
Kurtosis	15.535	5.940	20.110	8.346	29.278	29.969	8.695
Jarque-Bera	16956.260	962.823	34334.330	5095.960	78971.970	83011.470	5615.863
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Observations	2447.000	2447.000	2447.000	2447.000	2447.000	2447.000	2447.000
ADF test t-Statistic	-7.653	-12.988	-7.208	-5.731	-6.318	-8.844	-11.928
CV 1% level	-3.433	-3.433	-3.433	-3.433	-3.433	-3.433	-3.433
CV 5% level	-2.863	-2.863	-2.863	-2.863	-2.863	-2.863	-2.863
CV 10% level	-2.567	-2.567	-2.567	-2.567	-2.567	-2.567	-2.567
Winter Monday to Thursday, off–peak hours							
	DeltaPriceLast	ControlAreaBalance	DemandQuote	DeltaWindN	DeltaWindP	DeltaPVN	DeltaPVP
Mean	-1.088	-150.579	0.934	-393.945	256.662	na	na
Median	-0.300	-136.937	0.908	-88.000	0.000	na	na
Maximum	152.810	2320.693	1.178	0.000	4670.000	na	na
Minimum	-110.350	-2139.298	0.634	-4012.000	0.000	na	na
Std. Dev.	20.224	456.092	0.122	632.799	488.188	na	na
Skewness	0.342	-0.017	0.178	-2.512	3.500	na	na
Kurtosis	5.129	4.620	1.981	10.353	21.523	na	na
Jarque-Bera	510.016	267.770	118.916	8087.061	39977.890	na	na
Probability	0.000	0.000	0.000	0.000	0.000	na	na
Observations	2447.000	2447.000	2447.000	2447.000	2447.000	na	na
ADF test t-Statistic	-7.812	-14.549	-8.909	-6.764	-9.406	na	na
CV 1% level	-3.433	-3.433	-3.433	-3.433	-3.433	na	na
CV 5% level	-2.863	-2.863	-2.863	-2.863	-2.863	na	na
CV 10% level	-2.567	-2.567	-2.567	-2.567	-2.567	na	na

We treat separately peak hours (from 08:00–20:00), as shown in panel 1 and off-peak hours (20:00–08:00), panel 2. The fundamental variables include: "DeltaPriceLast" = Difference between the historical last prices for 15-minute delivery periods and the day-ahead prices for the corresponding hour; "ControlAreaBalance"=Historical balancing market volumes for the corresponding hour; "DemandQuote"=The quote of demand in the power plant availability, as defined in Equation 6; "DeltaWindN/P" and "DeltaPVN/P" represent changes in the forecasts of renewables, wind and photovoltaic, between the time of the last price bid and the forecast available at 2 o'clock in the previous day

Table A.11: Descriptive statistics of the differences between the historical last prices for 15-minute delivery periods and the day-ahead price and of fundamental variables at the time of the last bid during summer time (01/04/2014–01/07/2014), for working days Monday–Thursday.

Summer Monday to Thursday, peak hours							
	DeltaPriceLast	ControlAreaBalance	DemandQuote	DeltaWindN	DeltaWindP	DeltaPVN	DeltaPVP
Mean	-0.060	130.313	1.259	-329.796	190.448	-357.785	314.296
Median	-1.730	99.908	1.249	-56.000	0.000	0.000	0.000
Maximum	255.710	3494.669	1.467	0.000	2473.000	0.000	2900.000
Minimum	-56.820	-1829.939	1.082	-3027.000	0.000	-4726.000	0.000
Std. Dev.	22.892	577.670	0.080	507.571	344.782	676.016	599.469
Skewness	3.888	0.855	0.373	-1.921	2.403	-2.896	2.186
Kurtosis	33.493	6.619	2.486	6.411	9.573	12.964	7.247
Jarque-Bera	104929.000	1697.612	86.909	2796.685	7026.268	14074.430	3935.850
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Observations	2543.000	2543.000	2543.000	2543.000	2543.000	2543.000	2543.000
ADF test t-Statistic	-6.875	-12.907	-3.433	-7.132	-9.796	-9.485	-10.162
Critical value: 1% level	-3.433	-3.433	-2.663	-3.433	-3.433	-3.433	-3.433
Critical value: 5% level	-2.862	-2.862	-2.862	-2.862	-2.862	-2.862	-2.862
Critical value: 10% level -2.567	-2.567	-2.567	-2.567	-2.567	-2.567	-2.567	
Summer Monday to Thursday, off–peak hours							
	DeltaPriceLast	ControlAreaBalance	DemandQuote	DeltaWindN	DeltaWindP	DeltaPVN	DeltaPVP
Mean	-0.619	72.547	0.979	-245.913	179.044	na	na
Median	0.020	82.760	0.955	0.000	9.000	na	na
Maximum	82.910	2286.065	1.291	0.000	2142.000	na	na
Minimum	-65.010	-1454.723	0.714	-2569.000	0.000	na	na
Std. Dev.	16.148	447.547	0.137	448.846	288.142	na	na
Skewness	0.087	0.182	0.210	-2.449	2.453	na	na
Kurtosis	4.130	3.800	1.941	8.926	10.916	na	na
Jarque-Bera	138.469	81.799	137.655	6262.849	9189.520	na	na
Probability	0.000	0.000	0.000	0.000	0.000	na	na
Observations	2543.000	2543.000	2543.000	2543.000	2543.000	na	na
ADF test t-Statistic	-7.402	-13.318	-8.048	-6.784	-9.466	na	na
Critical value: 1% level	-3.433	-3.433	-3.433	-3.433	-3.433	na	na
Critical value: 5% level	-2.862	-2.862	-2.862	-2.862	-2.862	na	na
Critical value: 10% level	-2.567	-2.567	-2.567	-2.567	-2.567	na	na

We treat separately peak hours (from 08:00–20:00), as shown in panel 1 and off-peak hours (20:00–08:00), panel 2. The fundamental variables include: "DeltaPriceLast" = Difference between the historical last prices for 15-minute delivery periods and the day-ahead prices for the corresponding hour; "ControlAreaBalance"= Historical balancing market volumes for the corresponding hour; "DemandQuote"= The quote of demand in the power plant availability, as defined in Equation 6; "DeltaWindN/P" and "DeltaPVN/P" represent changes in the forecasts of renewables, wind and photovoltaic, between the time of the last price bid and the forecast available at 2 o'clock in the previous day

Table A.12: Descriptive statistics of the intraday price changes between two consecutive bids for the 15-minute delivery periods in the continuous trading. We selected 4 delivery periods during morning (H7Q1–4), noon peak (H12Q1–4) and evening peak (H18Q1–4) quarter of hours.

	H7Q1	H7Q2	H7Q3	H7Q4	H12Q1	H12Q2
Mean	0.002	0.003	0.007	0.008	0.007	0.008
Median	0.000	0.000	0.000	0.000	0.000	0.000
Maximum	62.000	51.000	74.290	84.980	80.000	67.690
Minimum	-73.900	-71.700	-101.680	-73.790	-282.000	-247.340
Std. Dev.	5.306	6.335	6.284	6.404	6.906	7.249
Skewness	-0.288	-0.940	-0.507	0.732	-14.328	-8.138
Kurtosis	29.557	22.154	35.209	31.139	584.780	291.760
Jarque-Bera	143358.300	75254.870	210973.800	161306.400	68932280.000	16994366.000
Probability	0.000	0.000	0.000	0.000	0.000	0.000
Observations	4876.000	4876.000	4876.000	4876.000	4876.000	4876.000
ADF test t-Statistic	-38.895	-36.297	-27.598	-37.781	-39.001	-41.789
Critical value: 1% level	-3.431	-3.432	-3.431	-3.431	-3.431	-3.431
Critical value: 5% level	-2.862	-2.862	-2.862	-2.862	-2.862	-2.862
Critical value: 10% level	-2.567	-2.567	-2.567	-2.567	-2.567	-2.567

	H12Q3	H12Q4	H18Q1	H18Q2	H18Q3	H18Q4
Mean	0.006	0.002	-0.004	0.000	0.008	0.002
Median	0.000	0.000	0.000	0.000	0.000	0.000
Maximum	89.000	120.000	110.990	55.900	84.000	112.120
Minimum	-180.000	-92.000	-91.900	-68.000	-85.990	-112.120
Std. Dev.	8.011	6.576	6.167	5.988	6.350	6.939
Skewness	-3.725	0.754	2.275	-0.358	-0.087	-1.551
Kurtosis	121.892	55.360	68.092	24.433	28.764	58.012
Jarque-Bera	2883104.000	557458.100	865012.600	93434.750	134859.800	616793.700
Probability	0.000	0.000	0.000	0.000	0.000	0.000
Observations	4876.000	4876.000	4876.000	4876.000	4876.000	4876.000
ADF test t-Statistic	-53.756	-72.044	-46.798	-33.827	-49.234	-26.363
Critical value: 1% level	-3.431	-3.431	-3.431	-3.431	-3.431	-3.431
Critical value: 5% level	-2.862	-2.862	-2.862	-2.862	-2.862	-2.862
Critical value: 10% level	-2.567	-2.567	-2.567	-2.567	-2.567	-2.567

Table A.13: Descriptive statistics of the volume trades between two consecutive bids for the 15-minute delivery periods in the continuous trading. We selected 4 delivery periods during morning (H7Q1–4), noon peak (H12Q1–4) and evening peak (H18Q1–4) quarter of hours.

	H7Q1	H7Q2	H7Q3	H7Q4	H12Q1	H12Q2
Mean	15.048	8.213	8.394	14.029	10.004	6.976
Median	12.000	5.000	5.200	10.200	5.000	2.500
Maximum	150.000	60.600	70.000	100.000	234.900	75.000
Minimum	0.100	0.100	0.100	0.100	0.100	0.100
Std. Dev.	12.897	8.876	8.856	12.525	11.770	9.735
Skewness	1.455	1.823	1.820	1.414	2.979	2.177
Kurtosis	7.696	7.159	7.485	6.062	33.388	8.574
Jarque-Bera	6201.308	6215.672	6778.603	3528.371	194828.900	10163.740
Probability	0.000	0.000	0.000	0.000	0.000	0.000
Observations	4876.000	4876.000	4876.000	4876.000	4876.000	4876.000
ADF test t-Statistic	-33.183	-30.176	-24.859	-34.669	-37.050	-28.199
Critical value: 1% level	-3.431	-3.432	-3.431	-3.431	-3.431	-3.431
Critical value: 5% level	-2.862	-2.862	-2.862	-2.862	-2.862	-2.862
Critical value: 10% level	-2.567	-2.567	-2.567	-2.567	-2.567	-2.567

	H12Q3	H12Q4	H18Q1	H18Q2	H18Q3	H18Q4
Mean	8.975	11.606	13.690	8.480	8.136	12.688
Median	4.200	6.300	10.000	4.100	4.000	9.000
Maximum	100.000	100.000	179.000	95.500	195.600	200.000
Minimum	0.100	0.100	0.100	0.100	0.100	0.100
Std. Dev.	11.145	12.917	13.546	10.368	10.328	13.099
Skewness	1.845	1.661	1.788	1.960	3.200	2.842
Kurtosis	7.007	6.717	10.450	7.533	30.815	26.207
Jarque-Bera	6026.335	5050.212	13874.880	7295.622	165508.500	115984.000
Probability	0.000	0.000	0.000	0.000	0.000	0.000
Observations	4876.000	4876.000	4876.000	4876.000	4876.000	4876.000
ADF test t-Statistic	-26.156	-25.007	-34.258	-33.775	-31.587	-37.025
Critical value: 1% level	-3.431	-3.431	-3.431	-3.431	-3.431	-3.431
Critical value: 5% level	-2.862	-2.862	-2.862	-2.862	-2.862	-2.862
Critical value: 10% level	-2.567	-2.567	-2.567	-2.567	-2.567	-2.567

Appendix B. OLS estimation without threshold, morning and evening delivery periods

Table B.14: Estimation results hour 7, Quarters 1–4, global OLS without threshold, entire sample

OLS estimation of the model including fundamental variables
Dependent variable Delta Price

	H7Q1		**H7Q2**		**H7Q3**		**H7Q4**	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.288	(0.645)	-0.450	(0.965)	-1.392	(1.139)	-1.102	(0.858)
DeltaPrice1	-0.208*	(0.030)	-0.320*	(0.032)	-0.244*	(0.035)	-0.281*	(0.033)
DeltaPrice2	-0.157*	(0.032)	-0.159*	(0.021)	-0.121*	(0.027)	-0.175*	(0.020)
DeltaPrice3	-0.084*	(0.017)	-0.080*	(0.018)	-0.084*	(0.019)	-0.086*	(0.016)
DemandQuote	-0.300	(0.543)	0.381	(0.829)	0.966	(0.965)	1.011	(0.736)
Volume	0.008	(0.005)	0.015	(0.009)	0.001	(0.009)	-0.020*	(0.006)
SqrTimeStep	-0.833	(1.420)	-1.212	(1.359)	4.101*	(1.319)	4.127*	(1.547)
DeltaWindIntrP	0.0001	(0.0002)	0.0002	(0.0002)	-0.001	(0.001)	-0.001	(0.001)
DeltaWindIntrN	-0.001*	(0.0001)	0.0001	(0.0002)	0.0002	(0.001)	0.001	(0.001)
DeltaPVIntraP	0.0001	(0.001)	0.001	(0.001)	0.0002	(0.001)	0.002	(0.002)
DeltaPVIntraN	0.001	(0.001)	0.002**	(0.001)	-0.001	(0.001)	0.000	(0.001)
Rsquared	5.989%		10.930%		7.333%		9.481%	
No. Obs.	6979		4873		4977		7175	

OLS estimation of the autoregressive model, excluding fundamental variables
Dependent variable Delta Price

	H7Q1		**H7Q2**		**H7Q3**		**H7Q4**	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	0.004	(0.061)	0.005	(0.086)	0.010	(0.086)	0.007	(0.072)
DeltaPrice1	-0.207*	(0.012)	-0.321*	(0.014)	-0.243*	(0.014)	-0.276*	(0.012)
DeltaPrice2	-0.158*	(0.012)	-0.159*	(0.015)	-0.119*	(0.014)	-0.175*	(0.012)
DeltaPrice3	-0.083*	(0.012)	-0.080*	(0.014)	-0.085*	(0.014)	-0.082*	(0.012)
Rsquared	5.055%		9.718%		6.170%		8.085%	
No. Obs.	6979		4873		4977		7175	

Standard errors are shown in parenthesis. *, and ** denote a test statistic is statistically significant at the 1% and 5% level of significance, respectively. The interpretation of variables is: DeltaPrice(x)=lagged price changes 1–3; DemandQuote=demand quote; Volume=volume of trades; SqrTimeStep=$\sqrt{\Delta_t}$; DeltaWind-IntrP/N=positive/negative forecasting errors in wind; DeltaPVIntraP/N=positive/negative forecasting errors in PV.

Table B.15: Estimation results hour 18, Quarters 1–4, global OLS without threshold

OLS estimation of the model including fundamental variables

Dependent variable Delta Price

	H18Q1		H18Q2		H18Q3		H18Q4	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	-0.156	(0.809)	0.068	(0.941)	-1.861	(0.980)	-1.160	(1.087)
DeltaPrice1	-0.206*	(0.032)	-0.276*	(0.036)	-0.254*	(0.033)	-0.214*	(0.036)
DeltaPrice2	-0.163*	(0.033)	-0.149*	(0.025)	-0.173*	(0.030)	-0.105*	(0.023)
DeltaPrice3	-0.131*	(0.024)	-0.090*	(0.024)	-0.101*	(0.020)	-0.149*	(0.045)
DemandQuote	0.324	(0.642)	0.186	(0.772)	1.274	(0.806)	0.708	(0.908)
Volume	-0.025*	(0.004)	-0.028*	(0.006)	0.041*	(0.007)	0.037*	(0.005)
SqrTimeStep	0.143	(1.319)	-1.628	(1.062)	-0.233	(0.921)	-3.565*	(1.258)
DeltaWindIntrP	0.000	(0.000)	0.000	(0.000)	-0.001*	(0.000)	0.000	(0.000)
DeltaWindIntrN	-0.003*	(0.001)	-0.001	(0.001)	-0.001	(0.001)	-0.001	(0.001)
DeltaPVIntraP	0.011	(0.009)	-0.006	(0.013)	-0.004	(0.011)	-0.055	(0.033)
DeltaPVIntraN	-0.014**	(0.007)	0.004	(0.011)	-0.012	(0.027)	0.087	(0.105)
Rsquared	11.135%		8.929%		8.048%		7.037%	
No. Obs.	8507		5982		6162		8936	

OLS estimation of the autoregressive model excluding fundamental variables

Dependent variable Delta Price

	H18Q1		H18Q2		H18Q3		H18Q4	
	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.	Coeff	Std. err.
Co	-0.005	(0.058)	-0.001	(0.073)	0.005	(0.082)	0.005	(0.078)
DeltaPrice1	-0.201*	(0.011)	-0.276*	(0.013)	-0.252*	(0.013)	-0.207*	(0.010)
DeltaPrice2	-0.163*	(0.011)	-0.146*	(0.013)	-0.170*	(0.013)	-0.100*	(0.011)
DeltaPrice3	-0.131*	(0.011)	-0.088*	(0.013)	-0.098*	(0.013)	-0.144*	(0.010)
Rsquared	6.099%		7.715%		7.247%		5.859%	
No. Obs.	8507		5982		6162		8936	

Standard errors are shown in parenthesis. *, and ** denote a test statistic is statistically significant at the 1% and 5% level of significance, respectively. The interpretation of variables is: DeltaPrice(x)=lagged price changes 1–3; DemandQuote=demand quote; Volume=volume of trades; SqrTimeStep=$\sqrt{\Delta_t}$; DeltaWind-IntrP/N=positive/negative forecasting errors in wind; DeltaPVIntraP/N=positive/negative forecasting errors in PV.

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