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Linear decision rules for seasonal hydropower planning: modelling considerations

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Abstract

The seasonal hydropower planning problem is presented as a robust optimization model using linear decision rules (LDR). Uncertainty is present in multiple inflow series and the price of electricity. The objective is to generate weekly policies that maximize profit. The LDR approximation is effective at reducing computational complexity, and is well-suited to multistage problems. By restricting the decision variables to be affine reactions of the realizations of the uncertain parameters, the original intractable problem is transformed into a tractable one with shorter computational time. This article is intended as an introduction to LDR for practitioners within the hydropower industry. The basic results will be presented, along with explanations of the ideas behind the method. An example case is given to illustrate the solution method.

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Keywords: OR in energy; Robust optimization; Seasonal planning problem; Hydropower optimization; Linear decision rules

1. Introduction

This paper is intended as an introductory guide to understanding, modelling and implementing linear decision rules (LDR) in practice, specifically in the context of hydropower scheduling. It presents a new approach to LDR in reservoir management, with zero-mean forecast errors as uncertainty

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parameters, and slopes on multiple uncertainty types, aspects not found in previous publications[1]. Additionally, it discusses issues related to limited dependencies on uncertainty parameters more thouroughly than earlier literature. These contributions to the field will be shown to give compact models that return sensible schedules, and intuitively understandable policies.

2. Problem formulation

The problem at hand involves optimally deciding a seasonal production plan. Given a forecast for weekly prices and inflow, the aim is to maximize profit. Specifically, discharge for generation, x^q_{rt}, pumping, x^p_{rt}, and spill, x^s_{rt} should be decided for all reservoirs r at all times t. For simplicity of notation, the three types of decisions constitute the *decision set* $\mathbb{D} = \{q, p, s\}$. From these decisions, the reservoir levels m_{rt} are deduced. All the variables need to be within certain bounds, that may be time dependent. i.e. water for generation together with water that bypass the connected power station.

$$\max \quad T^H \sum_{t\in\mathcal{T}} \beta_t \pi_t \sum_{r\in\mathcal{R}} \sum_{d\in\mathbb{D}} E^d_{rt} x^d_{rt} \tag{1}$$

$$\text{s.t.} \quad m_{r0} = M^0_r \qquad r \in \mathcal{R} \tag{2}$$

$$m_{rt} = m_{r(t\text{-}1)} + F_{rt} + T^S \sum_{d\in\mathbb{D}} \left(\sum_{\rho\in\mathbb{C}^d_r} x^d_{\rho t} - x^d_{rt} \right) \qquad r \in \mathcal{R}, t \in \mathcal{T} \tag{3}$$

$$\underline{M}_{rt} \leq m_{rt} \leq \overline{M}_{rt} \qquad r \in \mathcal{R}, t \in \mathcal{T} \tag{4}$$

$$L^q_{rt} \leq E^q_{rt} x^q_{rt} \leq U^q_{rt} \qquad r \in \mathcal{R}, t \in \mathcal{T} \tag{5}$$

$$L^p_{rt} \leq E^p_{rt} x^p_{rt} \leq U^p_{rt} \qquad r \in \mathcal{R}, t \in \mathcal{T} \tag{6}$$

$$\underline{D}_{rt} \leq x^q_{rt} + C^B_{rt} \leq \overline{D}_{rt} \qquad r \in \mathcal{R}, t \in \mathcal{T} \tag{7}$$

$$x^s_{rt} \geq 0 \qquad r \in \mathcal{R}, t \in \mathcal{T} \tag{8}$$

All reservoirs are considered to have a power station immediately downstream, either real, or a dummy station without generation capabilities if the reservoir discharges directly to another reservoir. Power stations can therefore be omitted from the model formulation, as seen in the simplified topology in Fig. 1. The objective function (1) maximizes the present value of the difference between revenues and costs for all decision types. Here, prices, π, are discounted with a discount rate, β, and are given per MW h, while decisions are in m^3/s. Thus, energy coefficients E^d are needed to convert volumetric sizes to power levels, and the number of hours in a time period, T^H, is used to get the total energy level. The energy coefficients are greater than zero for generation, less than zero for pumping, and zero for spillage. Equalities (2) and (3) ensure that the reservoir balance starts at its initial level and is updated between time periods. The reservoir level in one period depends on the previous level and the natural inflow, as well as the decisions made both in connected upstream reservoirs, $\mathbb{C}^d_r$, and in the current reservoir. Constraints (4)-(8) impose upper and lower bounds on reservoir level, generation, pumping, discharge, and spill, respectively. Pumping and generation limits are given in MW, so energy coefficients are also needed in the generation constraints (5) and pumping constraints (6). Notably, pumping is considered to be decided at the upstream reservoir, and can simply be seen

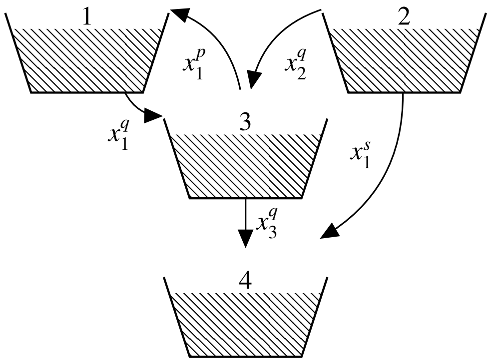

Fig. 1: An illustration of some of the variables used in the formulation.

as negative generation at a different energy coefficient. Thus, inequalities (6) typically have an upper bound of zero and always a non-positive lower bound. Since reservoir levels and natural inflows are given in Mm^3, the decisions that are given in m^3/s need to be multiplied by the length of a time period. Constraints (7) are the total discharge limits, so here bypass (C^B_{rt}) is considered in addition to the water for generation.

3. Linear decision rules and reformulation

Understanding the uncertain parameter δ, decisions of how much to discharge, pump and spill at reservoir r at time t can be defined as affine functions. As an example, the amount of discharge for generation becomes

$$x^q_{rt} = \hat{x}^q_{rt} + \sum_{u \in \mathbb{U}} \sum_{\tau \in \mathcal{M}^u_t} K^q_{rtu\tau} \delta_{u\tau}. \tag{9}$$

Here, the intercept $\hat{x}^q_{rt}$ and slopes $K^q_{rt\tau\upsilon}$ uniquely define the function, as long as the uncertainty parameters are linearly independent [2]. The intercepts represent scheduled values, while the slopes represent the real-time adjustments. Note that these adjustments depend on the realizations of the uncertain parameter $\delta_{u\tau}$ for all time periods τ in the memory set $\mathcal{M}^u_t$.

The memory set contains the periods a decision is dependent on, and can include all or some of the previous time periods, but never future ones. Allowing the LDR to depend on only a subset of the uncertain parameters will speed up computation, but reduce the flexibility of decisions. Some ways to reduce the size of the memory set include limiting the number of periods the decision can look backwards, and neglecting the uncertainty in time periods where the reservoir level typically is well within its upper and lower limits. Similarly, one could restrict what uncertainty series a variable is allowed to depend on, but in the following, it is assumed that every decision depends on every uncertainty series.

Now, a reformulation of the deterministic model to a robust optimization problem with LDR is described, following the setup in [3]. The aim is to show how a purely linear model can be used to find the intercepts and slopes as decision variables, and thus identify the optimal affine reactions.

First, the inequalities for discharge, pumping and spill are converted, before the reservoir volume inequalities and the objective are transformed.

3.1. General inequalities

Consider inequality (5) rewritten with the function valued variable for generation,

$$\hat{x}^q_{rt} + \sum_{u\in\mathbb{U}} \sum_{\tau\in\mathcal{M}^u_t} K^q_{rt\tau u}\delta_{u\tau} \leq \frac{U^q_{rt}}{E^q_{rt}}, \quad r\in\mathcal{R}, t\in\mathcal{T}, \delta\in\mathcal{U}. \tag{10}$$

Rewriting (1)-(8) this way would yield an LP where the intercepts and slopes are the decision variables, so the function valued search is reduced to an LP. However, since there should be one such constraint for each point in the set of values the uncertainty can take $\mathcal{U}$, and there are uncountably many points in a polytope, the problem will still be impossible to solve. As noted in [3], this problem can be circumvented, because only constraints describing the worst case is really needed. It is not trivial, though, to tell which constraint will be the strictest. To this end, the shape of the uncertainty set can be utilized. As a polytope defined by $H\delta \leq h$, it can be used as the feasible region for a linear program determining the parameter values maximizing the LHS.

$$\max_{\delta}\left\{\begin{array}{l} \displaystyle\sum_{u\in\mathbb{U}} \sum_{\tau\in\mathcal{M}^u_t} K^q_{rt\tau\upsilon}\delta_{u\tau} \\ \displaystyle s.t. \sum_{u\in\mathbb{U}} \sum_{\tau\in\mathcal{T}} H_{ju\tau}\delta_{u\tau} \leq h_j, \quad \forall j \quad : \mu^{Uq}_{rtj} \end{array}\right\} \leq \frac{U^q_{rt}}{E^q_{rt}} - \hat{x}^q_{rt} \qquad r\in\mathcal{R}, t\in\mathcal{T} \tag{11}$$

This formulation of the constraints is equivalent with (10), but has finite cardinality. However, it is not linear as both the slopes and the uncertainties are decision variables in the objective. To regain a completely linear structure, LP duality theory [4] can be exploited to unlink the different variables K and δ (Note the dual variables in gray with their corresponding constraint). An equivalent formulation to (11) is

$$\min_{\mu^{Uq}}\left\{\begin{array}{lll} \displaystyle\sum_j h_j\mu^{Uq}_{rtj} & & \\ \displaystyle s.t. \sum_j H_{ju\tau}\mu^{Uq}_{rtj} = K^q_{rt\tau\upsilon}, & u\in\mathbb{U}, \tau\in\mathcal{M}^u_t & : \delta_{u\tau} \\ \displaystyle\sum_j H_{ju\tau}\mu^{Uq}_{rtj} = 0, & u\in\mathbb{U}, \tau\notin\mathcal{M}^u_t & : \delta_{u\tau} \\ \mu^{Uq}_{rtj} \geq 0, \quad \forall j & & \end{array}\right\} \leq \frac{U^q_{rt}}{E^q_{rt}} - \hat{x}^q_{rt} \qquad r\in\mathcal{R}, t\in\mathcal{T}. \tag{12}$$

Here, the equalities come from the fact that the δ-variables are free. Now, this formulation only requires that $\sum_j h_j\mu^{Uq}_{rtj} \leq \frac{U^q_{rt}}{E^q_{rt}} - \hat{x}^q_{rt}$ for *some* set of μ^{Uq}. That is, the set of constraints that needed to hold true *for all* realizations is replaced with a set of constraints that only need to be satisfied *once*. Thus, the original (10) holds whenever the following set of constraints have a solution:

$$\sum_j h_j\mu^{Uq}_{rtj} \leq \frac{U^q_{rt}}{E^q_{rt}} - \hat{x}^q_{rt}, \quad r\in\mathcal{R}, t\in\mathcal{T} \tag{13}$$

$$\sum_j H_{ju\tau}\mu^{Uq}_{rtj} = K^q_{rt\tau\upsilon}, \qquad r\in\mathcal{R}, t\in\mathcal{T}, u\in\mathbb{U}, \tau\in\mathcal{M}^u_t \tag{14a}$$

$$\sum_j H_{ju\tau}\mu^{Uq}_{rtj} = 0, \qquad r\in\mathcal{R}, t\in\mathcal{T}, u\in\mathbb{U}, \tau\notin\mathcal{M}^u_t \tag{14b}$$

$$\mu^{Uq}_{rtj} \geq 0, \qquad r\in\mathcal{R}, t\in\mathcal{T}, j. \tag{15}$$

Similar transformations can be carried out for all the original constraints, to formulate a robust linear program in which (4)-(8) holds for any realization of the uncertainty. Further examples are omitted here for brevity, as the transformations are very similar to the one shown above. However, there are some subtleties to be noted.

First, constraints (18) are defined for *every* combination (u, τ), even if the decision in question does not depend on $\delta_{u\tau}$. This simply means that $\delta_{u\tau}$ has objective coefficient 0 in the primal in (11), but the variable is still present in the constraints below. Thus, the corresponding dual constraint (14b) should have a right hand side of 0. Second, completely analogous transformations could be done to $\geq$-restrictions. But here, the left hand side becomes a minimization problem in the primal space, so the restrictions $H\delta \leq h$ are non-standard and give rise to non-positive dual variables [4]. Third, equality constraints can be handled in a simpler way, see [3].

3.2. Reservoir volumes

The reservoir volumes m_{rt} are not decisions themselves, but merely consequenses of all previous discharge, pumping and spill decisions, which again depend on all previous realizations of uncertainty. This makes limited memory ($|\mathcal{M}_t^u| < t$) impossible[1]. The idea is that the reservoir balance is simply the initial level plus the sum of all natural inflows, adjusted for all decisions in the current and connected reservoirs. Note that the purpose of stating Eqs.(2)-(3) in the deterministic model is for readability, and they can be substituted into (4).

If the expressions of the linear decision rules for discharge, pumping and spill are inserted, then (2)-(4) can be desribed by the following inequalities[2],

$$\underline{M}_{rt} \leq \sum_{\tau=1}^{t} \left(\hat{F}_{r\tau} + \sum_{d\in\mathbb{D}} \Big[\sum_{\rho\in\mathbb{C}_r^d} \hat{x}_{\rho\tau}^d - \hat{x}_{r\tau}^d \Big] + \sum_{u\in\mathbb{U}} \left(\sum_{\substack{t^*\leq t \\ \tau\in\mathcal{M}_{t^*}^u}} \sum_{d\in\mathbb{D}} \Big[\sum_{\rho\in\mathbb{C}_r^d} K_{\rho t^* u\tau}^d - K_{rt^* u\tau}^d \Big] + I_{ur}^S F_{r\tau}^V \right) \delta_{u\tau} \right) + M_r^0 \leq \overline{M}_{rt}, \tag{16}$$

and similar transformations as described in Section 3.1 can be made.

3.3. The objective function

At last, the definitions in (9) are employed on the objective function (1), and the price variables are substituted with affine functions as well.

$$\mathbb{E}_\delta \left[T^H \sum_{t\in\mathcal{T}} \beta_t \Big(\hat{\pi}_t + \sum_{\tau\in\mathcal{T}_t} \Pi_\tau \delta_{P\tau} \Big) \sum_{r\in\mathcal{R}} \sum_{d\in\mathbb{D}} E^d \Big(\hat{x}_{rt}^d + \sum_{u\in\mathbb{U}} \sum_{\tau\in\mathcal{M}_t^u} K_{rtu\tau}^d \delta_{u\tau} \Big) \right]. \tag{17}$$

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[1] Similarly, if restrictions are imposed on which uncertainty series decisions are allowed to depend on, these restrictions cannot apply as strictly to the reservoir level. This is because the reservoir level is a consequence of decisions at upstream reservoirs as well, and these can depend on other series.

[2] Here, binary parameters I_{ur}^S are 1 if u is the inflow series reservoir r belongs to, i.e. $u = V(r)$.

Exploiting that (17) is a linear expression and expanding the multiplication, it is possible to take advantage of the facts that $\mathbb{E}[\delta_{ut}] = 0$ and thus $\mathbb{E}[\delta_{Pt}\delta_{u\tau}] = \mathrm{Cov}(\delta_{Pt}, \delta_{u\tau})$, to get

$$\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\sum_{d\in\mathbb{D}} T^H \beta_t E^d_{rt}\left(\hat{\pi}_t\hat{x}^d_{rt} + \sum_{u\in\mathbb{U}}\sum_{\tau\in\mathcal{M}^u_t} \Pi_t K^d_{rtu\tau}\mathrm{Cov}(\delta_{Pt}, \delta_{u\tau})\right). \tag{18}$$

This objective function contains two terms, the first one equivalent to the deterministic objective in Section 2, while the second one accounts for the uncertainty. The price depends on $\Pi_t\delta_{Pt}$, and the decisions depend on $K^d_{rtu\tau}\delta_{u\tau}$, so the expected value of their product is needed. This is exactly what the covariance provides for zero-mean variables, so this term allows the objective function to take into account both cross- and autocorrelations between the stochastic parameters.

4. Illustrative example

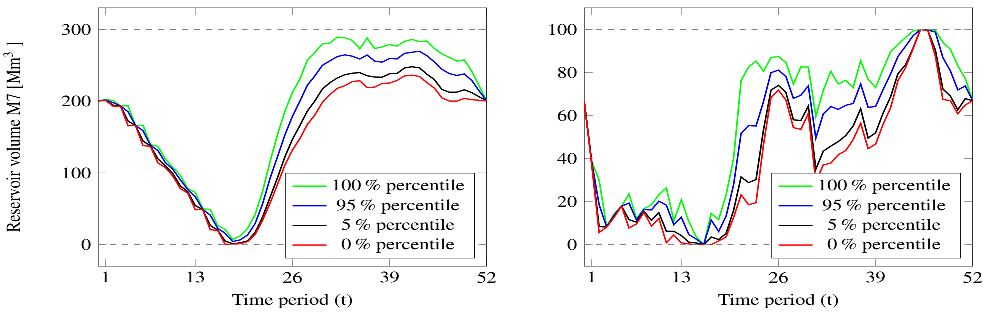
A two-reservoir system in cascade is considered with a 52-week planning horizon starting in January. The larger reservoir upstream (M7) is connected to a pumped-storage facility, and has the highest degree of regulation (DoR). The power station below reservoir (M6) can only generate power. M7 and M6 belong to different inflow series. As price and inflow forecasts, the mean of 3000 scenarios is used. They are generated by a time series model, tuned to match data from a sofisticated market model [5]. The time series formulation is similar to what is outlined in [6]. The uncertainty polytope is defined as the convex hull of these scenarios, so the problem is feasible for each of them. The LDR formulated in Section 2 have been implemented and solved in IBM ILOG CPLEX Optimization Studio 12.6.1.0 on a 64-bit Windows 7 Enterprise, Intel® Core™i7-3770 3.40 GHz, 16.0 GB RAM. The formulations yield an LP of 131 021 rows, 295 128 columns, and a run time of 249 seconds. The model is implemented with a full memory set, i.e. every previous time period is in the memory of every decision. Additionally, every reservoir has a target volume of 66.8 % in week 52, a normal reservoir level at this time of year.

The optimal policies are simulated on the 3000 scenarios, and percentiles of the resulting reservoir trajectories are plotted in Fig. 2a and Fig. 2b.

As can be seen, the LDR yield rather similar trajectories regardless of the inflow and price development. This is indicative of the model trying to counteract deviations. A natural conclusion is that it is easier to find robust continuations if the reservoir level is known with some degree of certainty. In M7 a gradual dispersal of scenarios can be seen, while in M6 the weeks 22-40 see the biggest spread of trajectories. In weeks of greater uncertainty, such as the snow melting season around weeks 17 to 33, more variation in trajectories can be observed.

Since M6 has the lowest DoR, it is intuitively the reservoir most likely to spill. Considering this, it makes sense that M6 is drained first, resulting in a rather sharp volume decrease in the initial weeks. Then, as M6 moves close to its lower bound, M7 is gradually emptied. Looking at the forecasted price in Fig. 3, it also seems logical that both reservoirs are emptied during the inital high price, and then start filling up at the very low prices during summer. In summary, the reservoir trajectories seem sensible. The next plot (Fig. 4) illustrates the slopes for discharge and pumping in every (t, τ)-combination for all uncertainty series. They indicate the amount of adjustment of a decision, given the realization of the price error, inflow error in M7 and inflow error in M6, respectively. The greater the magnitude of coefficients, the more the realization of $\delta_{u\tau}$ affects discharge and pumping in t.

In general, the discharge adjustment decisions are positively dependent on both the price and inflow deviation in the current time period. This means that if the realized price or inflow exceeds the



(a) Percentiles for reservoir volumes of M7 **(b)** Percentiles for reservoir volumes of M6

Fig. 2: Reservoir trajectories

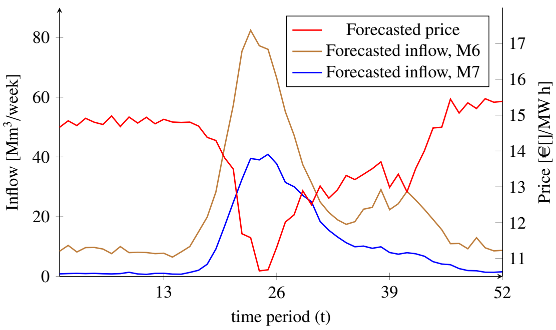


Fig. 3: Forecasted inflows and price

forecast, the discharge is adjusted upwards, generating more power. Intuitively, the strongest dependencies are found in the reservoir's own inflow series. Generation decisions at M7 depend only on its own inflow series, while pumping decisions are entirely independent of both inflow disturbances, only reacting to the price in the previous period. Discharge decisions at M6 (Fig. 4),depend mainly on inflow at its own reservoir, but also on inflow to M7. These observations are reasonable, as M6 is the bottom reservoir, and its reservoir level depends on decisions made at M7. M7 does not need to take M6 into account, and pumping is used mainly as a financial opportunity, rather than as a tool to balance reservoir levels.

The possibility of reducing memory has been mentioned. Results show that there are mainly two periods where long memory matters. First, there are dependencies between decisions around week 30 and realizations of inflow disturbance for up to the 10 previous weeks. The second period is the end of the scheduling horizon, where dependencies on many earlier periods are seen. Common for both periods is a need for a specific reservoir level. Around week 30, the reservoirs are approaching their maximum capacity, so accurate decisions keep the reservoirs from spilling. At the end, one aims for the target volume. Keeping more water than the minimum requirement does not give any benefits, but reduces the total generation, so here, higher accuracy pays off. It seems that the memory in the other time periods can safely be reduced, without affecting the optimal objective value much.

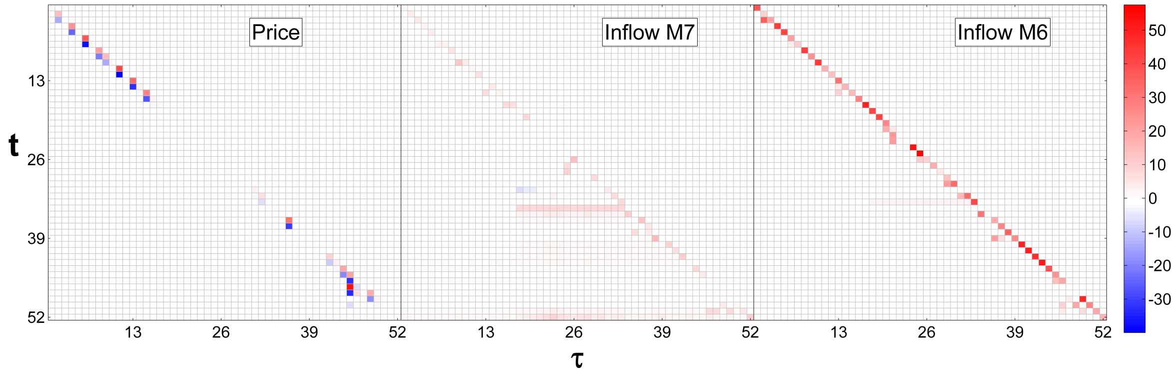


Fig. 4: Optimal slope matrix for discharge from the smaller downstream reservoir, $K^{q}_{M6t u\tau}$

5. Concluding remarks

In this work, the seasonal planning problem for a hydropower producer is considered. Decisions regarding the amount to discharge for generation, pumping and spillage are made in the face of uncertainty in inflow and price. A robust optimization model using LDR is proposed. Recourse decisions are decided by affine functions of the stochastic disturbances, rendering a tractable pure LP. The results from an illustrative example provide credible decisions and reservoir trajectories, the obtained policies are robust with respect to deviations in inflow and price, and the level of discharge is positively dependent on the inflow and price disturbance.

Future research should include testing with larger water systems, and comparison of LDR with alternative solutions methods, both in terms of run time, objective value and schedule. Long memory was only significant in certain periods, suggesting that memory could be limited in others. This will improve LDR's performance, so research on this topic is recommended. Furthermore, quantifying the approximation error will be important to test LDR's applicability in hydropower scheduling.

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