Coordination of Hydro and Wind Power in a Transmission Constrained Area using SDDP

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***Abstract*—In this work we use a production scheduling model based on Stochastic Dual Dynamic Programming (SDDP) to investigate the effects of coordinating hydro and wind power production in a transmission-constrained area. A case study is performed on an aggregated representation of a region in western Norway for a future situation with a significant integration of wind power. Two strategies for hydro reservoir utilization are developed using the SDDP model, one only considering hydro power and one considering both hydro and wind power. These two strategies are then tested in an out-of-sample simulator with equal conditions so they can be compared properly.**

Results from the case study show that coordination results in a lower reservoir level which reduces spillage and wind curtailment significantly. Coordination increases the export of energy out of the system by increasing the transmission on intermediate levels. Revenue is moved from storable hydro power to run-of-river hydro and wind power as coordination moves storable hydro power production to less profitable periods to reduce spillage and curtailment.

***Index Terms*—Hydroelectric power generation, Power system simulation, Wind energy integration.**

I. INTRODUCTION

As a part of the initiative to reduce the effects of climate change by moving towards an increased share of energy from renewable sources, the European Union has passed the Energy Directive which states clear targets for renewable energy development the next decades. As a part of the European Economic Area (EEA) Norway is affected by EU politics and has currently committed to increase the production of renewable energy by 13,2 TWh within 2020 [1]. The main tool to increase the production of renewable energy is the green-certificate market shared by Norway and Sweden which is expected to increase investments in hydro and wind power.

The number of wind farms in Norway is low compared to the available resources and as the cost of wind power production has dropped significantly the latest years [2] the interest in wind power has increased. Many wind farm projects have received permits from the authorities but most of these projects remain undeveloped due to low power prices and poor transmission capacity. Recently, the largest onshore wind power project in Europe was pronounced by the Norwegian central coast, summing up to 1000 MW of installed wind power capacity. Following this investment, a significant investment in the transmission grid is due. The grid surrounding these rural coastal areas is often quite weak and large investments in the transmission grid are necessary to be able to benefit from the good wind resources.

In [3] the problem of phasing in wind power in the Norwegian power system is studied for a case in Northern-Norway. The study shows that development of even moderate amounts of the available wind resources will cause a significant drop in the power price, significant transmission congestions, increased marginal transmission costs and increased amounts of spillage. The study concludes that development of transmission capacity should be internalised in the wind power projects and that such an internalization would make many projects in rural areas, with weak grid connections, less cost efficient such that other projects should be prioritized instead.

Currently the Norwegian power production portfolio consists of about 97 % hydro power with a total storage capacity of 84 TWh [4]. Wind power could be faster and more cheaply integrated by taking advantage of the characteristics of storable hydro power. Coordination of hydro and wind power may reduce the need for grid investments by adjusting the hydro power production to utilize the transmission lines more optimally. The advantages and disadvantages of such a coordination needs to be investigated further to assess the socio-economic benefits of such a solution.

Several papers have been published on coordination of hydro and wind power for short-term scheduling. In [5], [6] a short-term scheduling algorithm for coordination of hydro and wind power is developed for bidding in the spot market, the algorithm is applied in a case study and compared to hydro power scheduling without considering wind power. In these papers the hydro power plants and wind power farms are assumed owned by different utilities, hydro power is assumed to have priority on the transmission lines and wind power would be curtailed when the lines are congested. The main conclusions from these papers are that coordination between the hydro power utility and wind power utility is mutually beneficial, reduces wind energy curtailment and improves the utilization of the transmission lines.

In [7] the short-term scheduling algorithm from [5], [6] is expanded to include the regulating market. Furthermore [8] proposes a scheme for splitting the extra value caused by the coordination. All the previous papers focuses on short-term planning and uses fixed end-of-week reservoir volumes in the algorithms and thus purposely neglecting the possible long term effects of the coordination.

Coordination of hydro and wind power has previously also been studied for long- and medium-term scheduling. In [9] and [10] an algorithm for coordination of hydro and wind power is outlined which uses a predefined reservoir strategy for hydro power as input. Depending on which control-strategy is chosen, adjust hydro power production or curtail wind power, the original hydro power strategy is modified or used as is when the system operation is simulated. [10] includes a comprehensive case-study for a region of the power system in Northern-Norway, the study shows that coordination between hydro and wind power increases the amount of wind power that can be integrated into the system by increasing the utilization of the transmission lines and reducing the spillage. As both of the previously mentioned medium-term studies uses the same simulation method and gives positive results in regards to wind power integration it is interesting to see how these results holds up compared to a more formal optimization method.

A modern state-of-the-art method to solve long- and medium-term optimization problems is stochastic dual dynamic programming (SDDP), this method was introduced for power system applications in [11] and can be used for a wide range of problems. Some of the main advantages with the SDDP algorithm compared to more traditional methods as e.g. the water-value method, based on stochastic dynamic programming (SDP) [12][13], is that it allows for many state variables and more detailed modelling of the power system while keeping the problem tractable and capturing the dynamic effects between different reservoirs.

In [14], [15] a SDDP-model is developed and used for medium-term hydro power scheduling of multi-reservoir systems, combining SDDP and SDP with a Markov-chain for stochastic representation of the spot price. A similar model including a linear grid model is used in [16] to model a future case of the Icelandic power system with wind power, pumped-storage hydro power and a cable to the UK. [16] shows that using a fine time resolution and including an internal grid model with linearised power flow equations is important to obtain a realistic solution.

The main objective of this paper is to provide a basic analysis of the effects regarding coordination of hydro and wind power and thus a one-area model where the internal transmission grid is neglected and all the power plants in the area is aggregated into one storable hydro power plant, one run-of-river hydro power plant and one wind power farm serves as a sufficient model.

This paper is organized as follows; Section II gives a brief introduction to the most important features of the SDDP-algorithm, other basic modelling features are explained and the LP-formulation for the model is presented. A case study is outlined and results are presented in Section III. Finally, the main conclusions and suggestions for further work are given in Section IV.

II. METHOD AND MODELLING

A. LP-problem

The LP-problem for the power system in time-stage t is formulated in Equation (1) to (8) and consist of the objective function (1), reservoir balances (2), energy balance (3), discharge limits (4), reservoir capacity limits (5), import and export limit (6), (7) and cuts (8).

The objective function in (1) maximizes energy sales from the area under the assumption that the local load has to be covered, this is the same as optimizing the production and can be shown by substituting the energy balance (3) in to the objective function.

$$\hat{\alpha}_t(v_t, z_t) = \max\{\pi_t^S e_t^S - \pi_t^P e_t^P - \pi^R r_t - \sum_{i\in\mathcal{U}} \pi^T \varpi_{ti} + \hat{\alpha}_{t+1}\} \tag{1}$$

s.t.

$$v_{ti} + q_{ti} + s_{ti} - \varpi_{ti} = v_{(t-1)i} + I_{ti}(z_{(t-1)i}) \qquad \forall i \in \mathcal{U} \tag{2}$$

$$\sum_{i\in\mathcal{U}} q_{ti} - e_t^S + e_t^P + r_t = D_t \tag{3}$$

$$0 \le q_{ti} \le Q_i^{max} \qquad \forall i \in \mathcal{U} \tag{4}$$

$$0 \le v_{ti} \le V_i^{max} \qquad \forall i \in \mathcal{U} \tag{5}$$

$$0 \le e_t^S \le T_t^{S,max} \tag{6}$$

$$0 \le e_t^P \le T_t^{P,max} \tag{7}$$

$$\hat{\alpha}_{t+1} \le \alpha_{t+1}^* + \sum_{i\in\mathcal{U}} \pi_{(t+1)i}^j (v_{ti} - v_{ti}^*) + \sum_{i\in\mathcal{U}} \mu_{(t+1)i}^j (z_{ti} - z_{ti}^*) \qquad \forall j \in \mathcal{H}_i \tag{8}$$

In the reservoir balance for a unit i discharge, q_{ti}, and spillage, s_{ti}, is treated as outgoing energy while inflow, $I_{ti}(z_{(t-1)i})$, and tank water, ϖ_{ti}, is ingoing energy. Tank water has to be included due to the VAR(1) model used for modelling the inflow. Negative inflow can occur when the seasonal inflow-profile is at its lowest, if the reservoir levels are low the problem might become infeasible due to a negative right-hand-side of the reservoir balance. Use of tank water is penalized(π^T) in the objective function as it doesn't represents a real system state.

In the energy balance curtailed load, r_t, discharge and purchased, e_t^P, energy is energy into the system while sold, e_t^S, and locally consumed, D_t, energy is energy out of the system. Load curtailment is penalized, π^R, by the value of lost load in the objective function as it has a high socio-economic cost and is very undesirable. There is a small price difference in the objective function between the price to sell, π_t^S, and purchase, π_t^P, energy to avoid selling and purchasing at the same time.

Transmission is represented by a constant maximum transmission capacity when calculating the strategy for coordination and in the simulator for both cases. When not coordinating, a transmission capacity profile is used in the strategy calculations to account for the reduced transmission capacity from the hydro power point-of-view due to wind power production. The transmission capacity profile is calculated by subtracting the expected wind power production from the transmission capacity limit.

B. Inflow modelling

The stochastic parameters in SDDP must be stage-wise independent, inflow and wind typically exhibit serial correlation which can be accounted for by state-space enlargement. Both wind and inflow have a seasonal pattern, this is first extracted by normalizing the series. The normalized series are modelled using a vector auto-regressive model of order one, VAR(1) [17]. The multivariate time series comprise reservoir inflow, run-of-river (RoR) inflow and wind. The VAR(1) model is formulated in Equation 9, where $\boldsymbol{\phi}$ is the auto-regressive coefficient matrix for the inflow and wind data and $\boldsymbol{\epsilon}_t$ is a random error.

$$\mathbf{z}_t = \boldsymbol{\phi}\mathbf{z}_{t-1} + \boldsymbol{\epsilon}_t \tag{9}$$

The expression for the inflow of inflow series j in time-stage t, I_{tj}, is shown in Equation 10. Where $\boldsymbol{\phi}_j$ is row j of the autoregressive coefficient matrix.

$$I_{tj}(\mathbf{z}_{t-1}) = \mathbf{z}_t\sigma_{tj} + \mu_{tj} = (\boldsymbol{\phi}_j\mathbf{z}_{t-1} + \epsilon_{tj})\sigma_{tj} + \mu_{tj} \tag{10}$$

As seen from Equation 10 the inflow in a given week, I_t, is dependent on the inflow in the previous week through the inflow state variables, $\mathbf{z}_{t-1}$, while the stochastic noise parameter, ϵ_t, is independent of previous weeks inflow. The noise distribution is discretizised into a number of branches using a fast forward scenario reduction algorithm described in [18].

C. Stochastic Dual Dynamic Programming

Hydro power scheduling and many other real-life applications are often significantly affected by uncertainty as future values of parameters are hard to predict e.g. spot price, inflow and wind speed. By using a discrete description of the state-variables the stochastic problem can be solved to optimality using Stochastic Dynamic Programming (SDP). A problem using SDP is that the problem grows exponentially with the number of state-variables. The exponential growth in problem size causes the solution-time to become too large for problems with detailed Mixed Integer Program (MIP) modelling.

Another way of solving a stochastic problem is by formulating it as a linear problem. Stochastic linear problems are usually decomposed by time-stage or scenario, when decomposing by time-stage a scenario-tree is created due to the discrete stochastic data. Solving a multi-stage stochastic linear problem to optimality involves solving all the nodes in the scenario-tree which grows exponentially with the number of time-stages. The exponential growth in problem size causes the solution-time to become too large for problems with a detailed time-resolution or a long time-horizon.

Stochastic Dual Dynamic Programming is a statistically based approximation method for reducing the solution time of the multi-stage stochastic problem. The scenario-tree is constructed using stage-wise independent data i.e. all realizations of the uncertain data has the same discrete probability distribution for all nodes within a time-stage. Constructing the scenario-tree by using stage-wise independent data enables Sample Average Approximation (SAA) to be used solving the problem by sampling different paths through the scenario-tree. According to the SAA-theorem the solution of the SDDP method converges towards the optimal solution as the number of samples increases.

A common formulation of the linear problems in multistage stochastic programs is derived using dual theory and Benders decomposition where cuts are created to provide an approximate description of the future profit function[19]. An advantage with SDDP is that the stage-wise independent property of the uncertain data allows cuts to be shared amongst all nodes within a time-stage and thus provide a better description of the future cost function.

The SDDP algorithm uses an iterative process with a forward and backward run, in the forward run the LP-problem is solved for all time-stages and solutions are obtained using the current cuts to describe the future value function. In the backward run the solution of the state variables obtained in the forward run is used to create more cuts which is added to the LP-problems to improve the description of the future value function [20]. A more comprehensive explanations of the SDDP-method are found in e.g. [19] or [20].

D. Initial Reservoir Level and End-Cuts

The strategy is calculated over a period of three years with equal demand and prices each year. As it is normal for the reservoirs to be emptied before the spring flood or flooding in the autumn for most of the cases the first and third year provides a decoupling from the values in each end of the planning period for the strategy in the second year. When studying long-term effects of coordination, the focus is on the second year as it is least affected by the initial reservoir level and end cuts while calculations on revenue is performed for the whole planning period.

E. Simulator

The strategies obtained from the strategy calculations are compared in an out-of-sample simulator, the simulator is similar to the forward run in the SDDP-algorithm for the coordination case but with pre-sampled inflow and wind series. In the simulator the inflow and wind series are pre-sampled using a continuous description of the noise distribution which provides better samples more similar to the underlying data.

To keep an acceptable calculation time the strategy calculations uses a lower number of discrete noise levels and data samples as the main driver of the calculation time is the backward runs where the cuts, i.e. the reservoir strategy, is calculated. The simulator uses a significantly higher number of inflow and wind samples compared to the strategy model which further helps providing a better representation of the possible system states in the scenario-tree.

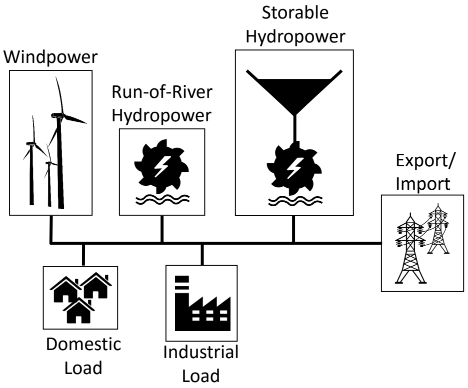

Fig. 1. Illustration of the power system model.

III. CASE STUDY

A. System modelling

A one-area model is created to represent the grid-constrained area as illustrated in Figure 1, internal transmission-lines are neglected and power plants are aggregated. The power plants are aggregated based on the following categories, storable hydro power, run-of-river (RoR) hydro power and wind power. The local load is represented by a deterministic load series based on the average load, while the transmission-lines are modelled as restrictions on the amount of power that can be exchanged with the external power market. To focus on the stochastic properties of inflow and wind a deterministic power-price is used for power sales to, or purchases from, the external market.

To limit the scope of this analysis and due to the computing resources available a time-step of one week is chosen. The model in this paper is based on many of the same principals as in [14] and [15], but it is also significantly modified to fit to the area model. No considerations are taken with respect to different ownership of power plants or transmission-line priorities as in [5], [6] and [7].

B. Case system

Sogn og Fjordane is a region in western Norway with a lot of potential for development of small hydro power and wind power. It is the region in Norway with the best potential for development of small hydro power and according to the Norwegian Water Resources and Energy Directorate (NVE) the region has about 4.89 TWh of total remaining hydro power resources as of January 2015 [21].

It is also one of the regions in Norway with the best wind power resources according to a wind power resource analysis on behalf of NVE, while the regions in northern Norway are the best [22]. Currently there are only one wind farm in the region with a capacity of 23 MW and a approximate 59 GWh of yearly production, but a lot of permits are given for new projects. About 750 MW of new wind power projects are currently considered and most of them have a final permit.

It's difficult to predict how the system will develop in the future with good accuracy but by using data from [23] and [24] an approximate description of the production portfolio for 2030 can be formulated. Table I includes the estimated power system production capacity and energy for 2011, 2014 and moderate and high predictions for 2030 [23], [24]. The moderate and high prognoses in [24] equals developing about 68% and 92% of the remaining hydro power resources and in Table II these predictions are broken down in categories based on data from [23].

TABLE I
TOTAL POWER AND ENERGY FOR THE POWER SYSTEM.

	2011	2014	2030-Mod	2030-High
Power [MW]	3 743	4 139	5 500	5 900
Energy [GWh]	12 601	13 900	17 500	18 750

TABLE II
PROGNOSIS FOR THE POWER SYSTEM IN 2030.

	Storable Hydro Power		RoR Hydro Power	
	Mod	High	Mod	High
Power[MW]	4 360	4 570	1 140	1 330
Energy[GWh]	13 700	14 320	3 800	4 430
Future Power Share	0.57	0.56	0.43	0.44
Future Energy Share	0.50	0.50	0.50	0.50
Power Increase	30%	36%	199%	249%
Energy Increase	22%	27%	181%	227%

By assuming a future degree of regulation (Reservoir Capacity/Yearly Inflow) at 0.71, same as in 2011[23], the aggregated reservoir capacity is estimated to be 9 730 GWh and 10 170 GWh for the moderate and high case respectively.

Future local load is expected to range between 7000 GWh and 9000 GWh dependent on development of the Energy Intensive Industry [24], a conservative local load estimate at 7000 GWh is used in this analysis. A large transmission capacity at 4000 MW used in this analysis as it provides almost no constraint to the system in the base case with no wind power.

Inflow, wind and demand records for the case study are provided by SINTEF Energy Research and adjusted to fit the energy quantities in the specified cases. Wind records used in this paper are energy records obtained from re-analysis data which is adjusted for regional effects [25]. The inflow and wind energy profiles are shown in Figure 2 for the high 2030 case with 1000 MW installed wind power capacity.

The inflow profiles for storable and RoR hydro power differ significantly from each other as most of the hydro power reservoirs are located at a higher altitude than the RoR hydro power. Due to the high altitude the inflow in the winter is small as most of the precipitation is snow, a significant peak in the inflow-profile occurs as the snow stored throughout the winter melts in the spring.

C. Results

- Hydro (H): Optimization of hydro power production.
- Coord (C): Coordination of hydro and wind power production.

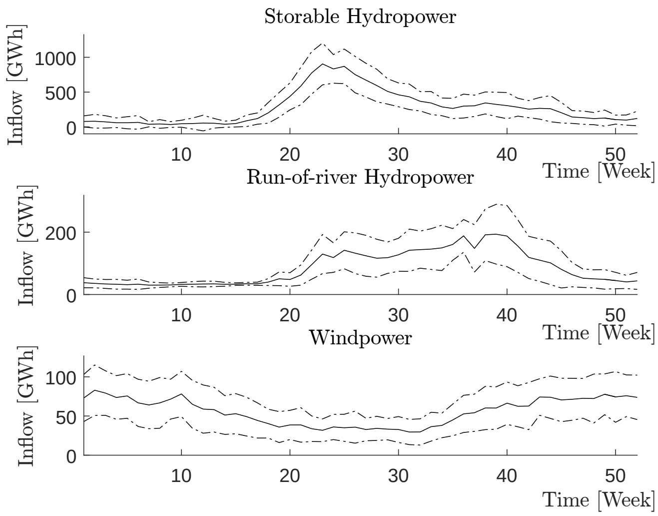

Fig. 2. Aggregated inflow and wind profiles for Sogn og Fjordane, with mean and standard deviations.

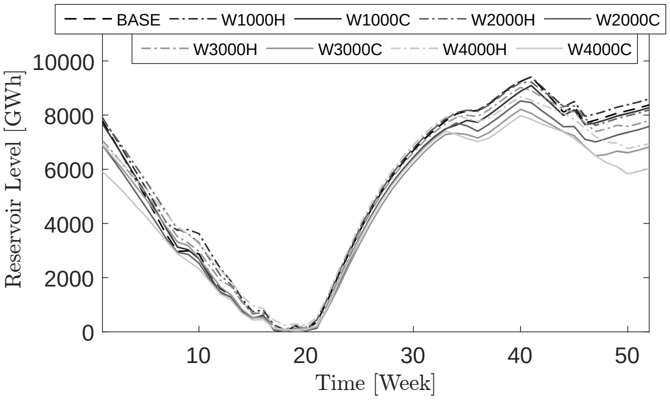

Fig. 3. Reservoir level for storable hydro power.

As shown in Figure 3 the reservoir level is reduced in the winter as more wind power is included regardless of coordination or not. Coordination results in significantly larger reductions in the reservoir level as the amount of wind power increases. A lower reservoir level during the winter allows the storable hydro power to adjust the production dependent on the highly intermittent wind power without risking additional spillage.

Increasing the amounts of wind power in the system decreases the number of hours of storable hydro power generation at high power levels regardless of coordination or not as shown in Figure 4. Coordination results in a small shift in production from high to lower levels and an increase in total production for storable hydro power as the spillage is reduced. The shift in duration is a result of the reduced reservoir level as production is moved to periods with higher RoR hydro power inflow.

Curtailment of RoR hydro power and wind power is a result of spillage from the reservoirs as the marginal cost of generation for storable hydro power also is zero when water spills and the inflow can't be stored for future production. Figure 5 shows how coordination, as a result of the reduced reservoir levels, significantly reduces the amount of wind power curtailment and spillage. The increased production results in a small increase in export on intermediate power levels. This shows how coordination takes advantage of the flexibility of a system with significant amounts of storable hydro power to allow large amounts of wind power to be integrated without increasing the amount of lost energy.

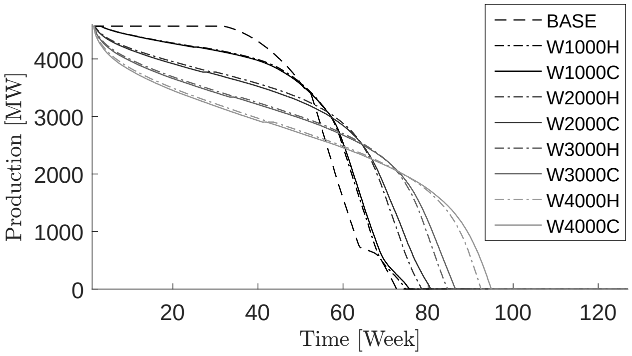

Fig. 4. Duration of storable hydro power production.

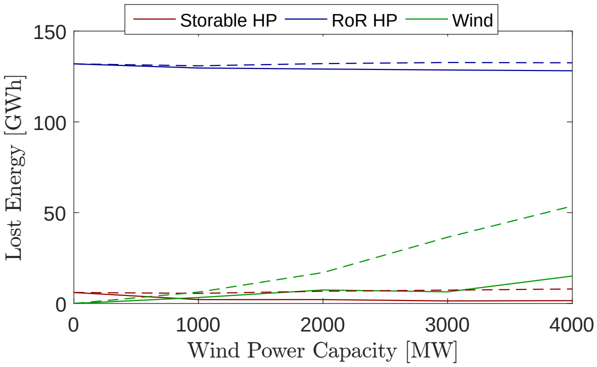

Fig. 5. Lost energy for different level of wind power integration. Hydro (dashed lines) and Coord (solid lines).

Table III shows how the revenue from storable hydro power is reduced for both strategies as more wind power is included in the system. More wind power results in more congestion during the autumn and winter when the prices are high and storable hydro power has to move some production to other periods with lower prices. Differences, defined as percentage difference from Hydro to Coord, shows how coordination shifts revenue from storable hydro power to RoR hydro and wind power as more storable hydro power production is moved to less profitable periods to reduce the reservoir level and energy loss.

As seen from the system profit in Table IV the increase in revenue from RoR hydro and wind power does not completely compensate for the lost revenue from storable hydro power. This is not as expected as previous studies on coordination shows an increased total revenue and might be a result of the

TABLE III

COMPARISON OF TOTAL REVENUE FOR THE PLANNING PERIOD.

	Wind Power [MW]	1000	2000	3000	4000
Hydro	Storable HP [bnNOK]	9.953	9.840	9.666	9.484
	End value water[bnNOK]	1.892	1.904	1.956	1.977
	RoR HP [bnNOK]	2.190	2.190	2.190	2.190
	Wind [bnNOK]	1.763	3.524	5.279	7.033
Coord	Storable HP [bnNOK]	9.967	9.813	9.650	9.450
	End value water [bnNOK]	1.862	1.892	1.930	1.954
	RoR HP [bnNOK]	2.190	2.191	2.191	2.191
	Wind [bnNOK]	1.764	3.527	5.288	7.048
Diff.	Storable HP [%]	0.14	-0.28	-0.17	-0.35
	End value water [%]	-1.58	-0.61	-1.32	-1.18
	RoR HP [%]	0.03	0.04	0.05	0.04
	Wind [%]	0.08	0.10	0.18	0.21

TABLE IV

PROFIT FROM POWER EXCHANGE FOR THE PLANNING PERIOD.

Wind Power [MW]	1000	2000	3000	4000
Hydro [bnNOK]	11.754	13.415	15.049	16.643
Coord [bnNOK]	11.740	13.381	15.018	16.602
Diff [%]	-0.116	-0.258	-0.209	-0.249

inflow model where some tank usage is forced as the reservoir capacity of the RoR hydro and wind power plants are zero. The differences in total profit are small and shows no clear trend as the amount of wind power increases.

IV. CONCLUSION AND FURTHER WORK

Coordination of hydro and wind power results in a lower reservoir level due to the uncertainty associated with wind power. A lower reservoir level contributes to keep the wind power curtailment low as the amounts of wind power in the system increases.

Coordination contributes to shifting storable hydro power production towards lower power levels as production is moved to periods with higher RoR hydro power inflow. The increased production results in a small increase of transmission on intermediate levels.

Revenue is shifted from storable hydro power to RoR hydro power and wind power when coordinating. A small negative result is observed in terms of total system profit but shows no clear trend as more wind power is included.

A. Further Work

Suggestions for further work is:

- Analysis with a more detailed model including internal transmission constraints and a more detailed description of the generation in the area, as in [16].
- Introduce price uncertainty by using a model where price is represented by a Markov-chain and the cut generation in the SDDP-algorithm is supplemented with SDP.

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