Reliability Analysis of Wake-induced Collision of Flexible Risers

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Abstract

Collision between risers is an important design and operational concern, especially in deep water since the probability of collision tends to increase as the riser length increases. Riser collision is due to the joint effects of many processes, i.e. environmental loads, hydrodynamic interference and surface floater motions and the most of them are stochastic processes. This paper provides an approach for estimating the failure probability of riser collision by considering these joint effects and their stochastic nature. Firstly, a procedure for establishing the distribution function of the extreme minimum relative distance between two risers is introduced based on simulation tools and statistical data. Numerical simulation is performed to compute the minimum distance between risers for a given duration. Repeated simulations are applied so that the extreme value distribution can be established. Secondly, reliability analysis is performed by considering the uncertainties of input parameters related to environmental loads and riser system. The collision probability is calculated based on both the First/Second Order Reliability Method and the Monte Carlo simulation techniques.

Keywords: riser collision, minimum distance, wake interference, waves and current, collision probability, reliability analysis

Nomenclature

α Scale parameter

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μ Location parameter

ν Kinematic viscosity

ρ Water density

β Reliability index

h Water depth

H_s Significant wave height

T_p Peak period

L Longitudinal distance between the centers of the two cylinders

D Cylinder diameter

V_0 Free stream incoming flow velocity

V_d Deficit velocity

$V(x, y)$ Local velocity in wake field

V_{rel} Relative velocity

V_c Current velocity

$C_D(x, y)$ Drag coefficient of downstream cylinder

$C_L(x, y)$ Lift coefficient of downstream cylinder

C_{Dd0} Reference drag coefficient of downstream cylinder

C_{Du0} Reference drag coefficient of upstream cylinder

$F_D(x, y)$ Drag force per unit length of downstream cylinder

$F_L(x, y)$ Lift force per unit length of downstream cylinder

$\Phi(\cdot)$ Cumulative distribution function of the standard Gaussian distribution

$g(\cdot)$ Limit state function

$\hat{n}$ Sample number

$\hat{N}$ Sample size

x_{min} Minimum distance between two segments

x_{em} Extreme minimum relative distance

$Y_1, ..., Y_n$ Random variables

$U_1, ...U_n$ Random variables in U-space

P_f Failure probability

$F_X(x)$ Cumulative distribution function(cdf)

$f_X(x)$ Probability density function(pdf)

A Platform horizontal offset

σ_L Large standard deviation

σ_S Small standard deviation

1. Introduction

Collision between flexible risers becomes increasingly more important as the offshore industry moves to deeper water since the risk of collision between adjacent risers tends to increase with the riser length. Due to practical considerations, risers are commonly arranged as clusters with relatively small spacing and riser collision is more likely to occur for such compact arrangements. However, riser collision is unlikely to lead to a direct failure, one potential risk is that it can be the onset of fatigue failure from a long term point of view.

The collision analysis for flexible risers is a challenge since it involves many complicated issues, i.e. the non-linearity of the geometry, the statistical nature of environment loads, variation in marine growth and hydrodynamic interference. When adjacent risers are close enough, the flow field around them will be changed, involving complex interactions between the shear layer, vortices, wakes, and Karman vortex streets. Especially, when the downstream riser is near to, partly within, or fully submerged in the wake of the upstream riser, wake interference occurs. The downstream riser will experience a reduced drag force when it is in the wake field generated by the upstream riser, leading to a high likehood of collision. During the last three decades, much effort has been made on investigating the wake interference based on a combination of theory, measurements and observations of the disturbed flow. Huse (1996) and Huse et al. (1998, 2000) proposed a two-dimensional (2D) wake model to describe the velocity in the wake field. Blevins (2005) extended this model by adding a lift force. In practice, the interference analysis of a 3D riser system can be performed using 2D strip theory. During the last decades, some experiments (Brika and Laneville, 1999; Hover et al., 2004; Hover and Triantafyllou, 2001) studied the hydrodynamic interference between a stationary leading cylinder and a flexible downstream cylinder. Some researchers (Huera-Huarte and Bearman, 2011; Huera-Huarte and Gharib, 2011; Huang et al., 2011; Huang and Herfjord, 2013) also investigated the riser behaviour when both risers are flexible. A literature review on this topic can be found in Fu et al. (2015).

Time domain analysis is usually applied for riser collision problem due to its complexity. Many researchers have put effort into estimating the riser collision probability. Duggal and Niedzwecki (1994) established a statistical model to estimate the probability for riser collision subjected to random waves. They considered the relative distance between two vertical risers for a given location as a random process so that the collision probability problem was equivalent to a process of crossing a threshold value. The applicability of this model depends on an accurate estimation of the first four moments of the relative distance; the mean value, standard deviation, skewness and kurtosis. These moments could be estimated based on either experimental data or numerical simulations. Leira et al. (2002) proposed a new procedure in order to consider the combined effect of current and surface platform motion, by which the most 'critical' conditions can be identified from a set of load cases. For each load case, the hydrodynamic load calculation is based on Computational Fluid Dynamics (CFD). He and Low (2012) proposed an efficient procedure for predicting the collision probability by using a limited number of nonlinear dynamic analyses. All these studies are based on the 'Collision Not Allowed' design strategy, see e.g. DNV-RP-F203 (2009). For this design strategy, the general philosophy is that riser collision is not allowed under normal operation, extreme conditions as well as identified accidental scenarios. The problem is then reduced to determination of the probability distribution of the distance between the risers above a given threshold value. Another design strategy is 'Collision Allowed', for which infrequent collisions may be allowed in temporary, accidental and extreme conditions. Hence, assessment of structural interaction will in addition be required. He and Low (2014) investigated the collision probability by taking both local and global analysis into account.

However, most of the above mentioned methods did not pay much attention to the wake effect when the downstream riser is located in the wake field generated by the upstream one. It should be noted that for a long flexible cylinder damping is large enough to avoid significant VIV (Vortex Induced Vibration). Therefore the focus of this paper is on the collision between flexible risers induced by the wake effect. As for the collision probability, most of the previous research considered only the statistical nature of wave loads. In fact, the uncertainties associated with riser collision analysis are due to many aspects, i.e. the current velocity, wake interference and riser diameter due to marine growth. The purpose of the present paper is to propose a method for estimation of collision probability between two flexible risers by accounting for the uncertainties of important parameters affecting the riser response. The paper focuses on the 'Collision Not Allowed' design strategy. The challenge is accordingly to find the cumulative distribution function (cdf) of the extreme minimum distance between two risers by considering the wake effect. In the present study, the Riflex code (Fylling et al., 1995) is applied to calculate the relative position of the Finite Element nodes. Furthermore, a reliability analysis of the riser collision is conducted when the uncertainties of the environmental loads and riser system are taken into account. This is achieved by using the FORM/SORM based on Taylor series expansion.

2. Theoretical Background

In the following subsections, the interference model, the stochastic theory and the reliability analysis method which have been used in the present study, will be briefly described.

2.1. Interference Model

When the downstream cylinder is within the wake generated by the upstream cylinder, it experiences a reduced drag force and a lift force. The drag force is reduced due to the reduced mean current velocity in the wake. The lift force is a result of the anti-symmetry flow when the cylinder is located outside the wake centerline. The drag and lift forces are related to the gap to diameter ratio (L/D) and the Reynolds number ($R_e = V_0 D/\nu$). Here L is the longitudinal distance between the centers of the two cylinders, as shown in Figure 1; D is the cylinder diameter; V_0 is the free stream velocity; ν is the kinematic viscosity of the fluid. The wake field behind a bluff body is usually modeled as a deficit velocity V_d relative to the incoming flow V_0,

$$V(x, y) = V_0 - V_d(x, y) \tag{1}$$

where $V(x, y)$ is the local velocity in the wake field. The origin of the local coordinate system is located at the center of the upstream cylinder, with the x-axis in the incoming flow direction, and the y-axis in the transverse direction. Note that the flow is assumed to be 2D, i.e. the out of plane components of the flow field and vertical displacements are neglected.

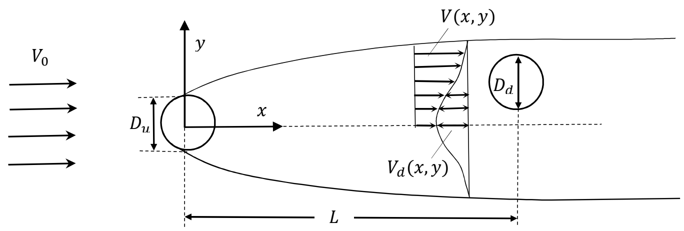

Figure 1: Coordinate system for description of drag- and lift-forces on downstream riser (DNV-RP-F203, 2009).

Huse wake model: Huse (1996) proposed the following formulation to model the downstream velocity reduction:

$$V_d(x, y) = k_2 V_0 \left(\frac{C_{Du} D_u}{x_s}\right)^{\frac{1}{2}} exp\left(-k_3 \left(\frac{y}{b}\right)^2\right) \tag{2}$$

where $x_s = x + 4D_u/C_{Du}$ and $b = k_1(C_{Du} D_u x_s)^{\frac{1}{2}}$; C_{Du} is the drag coefficient of the upstream cylinder based on the free stream velocity V_0 and upstream cylinder diameter D_u; k_1, k_2 and k_3 are Huse's constants which for a bare cylinder are $k_1 = 1$, $k_2 = 0.25$ and $k_3 = 0.693$.

Blevins wake model: Blevins (2005) proposed a similar formulation with different constants and adding a lift force, as given in Equation 3,

$$
\begin{aligned}
C_D(x,y) &= C_{Dd0}\left\{1 - a_1\left(C_{Du0}(\frac{D_u}{x})^{\frac{1}{2}}\right) exp\left(\frac{-a_2 y^2}{C_{Du0} D_u x}\right)\right\}^2 \\
C_L(x,y) &= a_3 \frac{dC_D}{d(y/D_d)}
\end{aligned}
\tag{3}
$$

where $C_D(x,y)$ is the drag coefficient for the downstream cylinder subjected to the local incoming flow velocity $V(x,y)$; $C_L(x,y)$ is the downstream cylinder lift coefficient subjected to the local velocity $V(x,y)$; D_u and D_d indicate the upstream and downstream cylinder diameters, respectively; C_{Du0} and C_{Dd0} indicate the reference drag coefficient for the upstream and downstream cylinders, respectively. The lift coefficient is anti-symmetric about the wake, and it becomes negative when $y < 0$. If the risers are in tandem arrangement (y=0), the lift force is zero, and Equation 3 reduces to:

$$
\begin{aligned}
C_D(x) &= C_{Dd0}\left\{1 - \left(C_{Du0}\frac{D_u}{x}\right)^{\frac{1}{2}}\right\}^2 \\
C_L &= 0
\end{aligned}
\tag{4}
$$

According to the Blevins wake model, the drag and lift forces per unit length can be expressed as:

$$
\begin{aligned}
F_D(x,y) &= \frac{1}{2}\rho {V_{rel}}^2 D C_D(x,y) \\
F_L(x,y) &= \frac{1}{2}\rho {V_{rel}}^2 D C_L(x,y)
\end{aligned}
\tag{5}
$$

Here ρ is the water density and V_{rel} is the relative velocity. In the following, Blevins model is applied in order to emphasize the presence of the lift force. For the upstream cylinder, Bokaian and Geoola (1985) have shown that the fluid interaction has no influence on the upstream cylinder if the relative distance is more than about 2 to 3 diameters.

2.2. Extreme Value Problem

The extreme minimum value of a finite number of independent and identically distributed random variables $X_1,\ X_2,\ ...,\ X_n$ with cdf $F_X(x)$ is the minimum of these random variables $X_e = min\{X_1,\ X_2,\ ...,\ X_n\}$. The distribution of X_e can be written as $F_{X_e}(x_e) = 1 - (1 - F_X(x))^n$. In general the nonlinear response needs to be treated in a semi-empirical manner, e.g. Gumbel, Weibull, or the peaks-over-threshold (POT) methods. In the present work, the extreme value which is given as the extreme minimum distance x_e is computed from time domain simulations. Based on a set of time domain analyses, the cdf of X_e is established by means of the regression line in a Gumbel probability paper, as given in Equation 6.

$$F_{X_e}(x_e) = 1 - exp\left(-exp\left(\alpha(x_e - \mu)\right)\right) \tag{6}$$

where α is the scale parameter, and μ is the location parameter.

2.3. Reliability Analysis Method

The First/Second Order Reliability Method (FORM/SORM) has been considered to be one of the most efficient computational methods for structural reliability. A fundamental problem in structural reliability theory is the computation of the multi-fold probability integral,

$$P_f = P[g(X) < 0] = \iiint_{g(X)<O} f_X(x)\,dx \tag{7}$$

where the integral is taken over the region corresponding to failure, i.e. $g(X) < 0$. $f_X(x)$ is the joint probability density function (pdf) for the n-dimensional vector x of the basic variables. In the following discussion, all the random variables X herein are assumed statistically independent. The methods discussed can be extended to problems with correlated random variables after those variables are converted to independent variables or by using the so-called Rosenblatt transformation. The basic idea of FORM/SORM is to transform the limit state function $g(X) < 0$ to a U-space function $g(U) < 0$, of standard Gaussian distributed variables by introducing a transformation T:

$$\begin{aligned} \Phi(u_i) &= F_X(x_i) \quad i = 1, 2, ..., n \\ x_i &= F_X^{-1}(\Phi(u_i)) \quad i = 1, 2, ..., n \end{aligned} \tag{8}$$

The failure probability is approximated by,

$$P_f = \Phi(-\beta) \tag{9}$$

where $\Phi(\cdot)$ is the standard Gaussian cdf, and β is the reliability index, corresponding to the distance from the origin to the point on the failure function closest to the origin, as illustrated in Figure 2. The minimum distance point on the failure surface is called the 'design point'. Generally, $g(U)$ is a non-linear function which could be linearized at the design point (FORM). This method can be refined by approximating the failure function by a quadratic function instead of a linear one at the same design point, denoted as SORM.

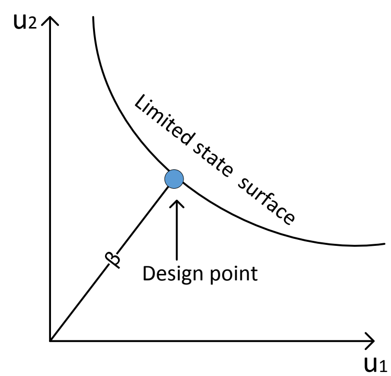

Figure 2: Failure surface and design point.

The FORM/SORM usually can give good approximations for the limit state function, but the accuracy and feasibility decrease with increasing non-linearity of the limit state function and number of non-Gaussian distributed variables. So, Monte Carlo simulation is an alternative method for estimating the failure probability.

In the Monte Carlo simulation a set of random samples are generated numerically according to their cdfs using a random number generator. Then generated samples are substituted into the limit state function $g(\cdot)$. The probability of failure is estimated as:

$$P_f = P[g(\cdot) \leqq 0] \approx \frac{\hat{n}}{\hat{N}} \tag{10}$$

where $\hat{n}$ is the number of samples for which $g(\cdot) \leqq 0$, and $\hat{N}$ is the sample size.

3. Description of the Proposed Approach

In this section, an approach which is applied for calculation of the minimum relative distance between two flexible risers and to estimate the failure probability is described.

3.1. Time Domain Analysis

The time domain analysis is performed by the finite element code Riflex (Fylling et al., 1995) which is specially designed to handle static and dynamic analyses of slender marine structures. For flexible risers, the equilibrium static position should be determined by considering the wake interference. For instance, if risers are in a tandem arrangement, the drag coefficient of the downstream riser, i.e. $C_D(x, y)$ varies along its length due to wake interference, as illustrated in Figure 3. The associated current profile is plotted as well. Riflex provides an option to divide a long slender riser into several segments in order to define different material properties and hydrodynamic coefficients, i.e. added mass, damping ratio and drag coefficient. For each segment, $C_D(x, y)$ is determined based on Blevins' wake model as explained above. The relative distance x in Equation 4 is the average distance between two segments. In order to find the equilibrium static position and the corresponding C_D, an iteration procedure is introduced for each segment as follows:

1. Calculate C_{D_i} according to the initial position of the risers.

2. Perform a static analysis in Riflex and compute the average distance between two segments x_i.

3. Calculate $C_{D_{i+1}}$ according to the new distance x_i.

4. Repeat steps 2-3 until the distance x_i and the corresponding $C_{D_{i+1}}$ satisfy Equation 4.

When computing the dynamic response of interacting risers, the computed C_D using in static analysis is also applied in the dynamic analysis. The uncertainties associated with this simplification will be considered in the reliability analysis in section 5.

For dynamic analyses, a more correct procedure is to update C_D of each segment at each time step since the relative distance varies with time. Patel et al. (2015) provided a map of hydrodynamic coefficients using a combination of empirical formulas based on Blevins' model and numerical interpolation techniques along with experimental towing tank test data and CFD analysis. The map can then be applied in order to calculate user-defined drag and

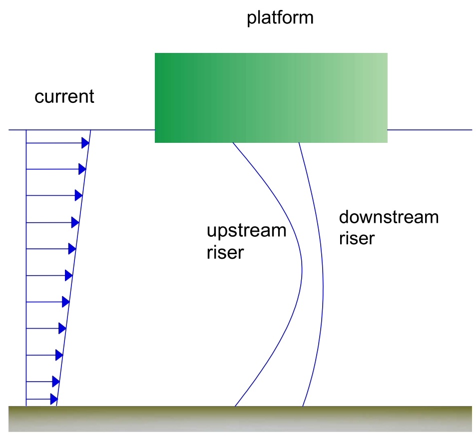

Figure 3: The static profile of risers when subjected to a current.

lift coefficients for riser collision analyses. Herfjord et al. (2002) proposed a similar methodology to study interacting risers. The methodology is based on a pre-established database of hydrodynamic forces acting on the cylinders which is obtained from CFD computations and model tests.

3.2. Definition of Extreme Value

For each dynamic analysis, it is possible to calculate the minimum distance between two specified line segments x_{min} at all time steps. Figure 4 shows the computed minimum relative distance x_{min} between two specified segments at all time steps. The smallest x_{min} is defined as the extreme minimum relative distance x_{em} which is marked by a red circle in Figure 4. Based on a set of such simulations, the cdf of x_{em} can be established by fitting a set of x_{em} on a Gumbel probability paper, as described in subsection 2.2.

3.3. Seconder Order Reliability Analysis Based on Taylor Expansion

The approach described in subsection 3.2 is used for a fixed set of input parameters describing the environmental loads and the riser system. In reliability analysis, some of these parameters are instead considered as random variables according to their statistical nature. The relationship between the Gumbel parameters α and μ and these variables can be established by the response surface method. This implies that μ and α can be expressed in terms of these variables. Therefore, the distribution function in Equation 6 becomes:

$$F_{X_{em}}(x_{em}) = exp\left(-exp\left(\alpha(Y_1, Y_2,\ ...,\ Y_n)(x_{em} - \mu(Y_1, Y_2,\ ...,\ Y_n))\right)\right) \quad (11)$$

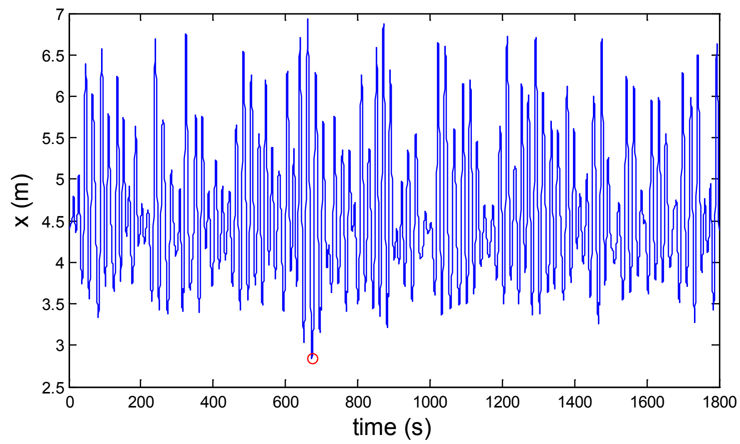

Figure 4: Relative distance x for each time step during one simulation of 30 minutes.

where $Y_1, Y_2, \ ..., \ Y_n$ are random variables related to the environmental loads and the riser system. Now $\mu(Y_1, Y_2, \ ..., \ Y_n)$ and $\alpha(Y_1, Y_2, \ ..., \ Y_n)$ can be calculated based on a smooth analytic function which is computed by fitting a response surface through a number of discrete points. These points are obtained by performing a set of dynamic analyses. The response surface is expressed as a product of polynomials of each random variable. These variables are normalized by dividing with their basecase values. The 'basecase' refers to the sea state and the riser system applied for the analysis. The following set of new variables are introduced:

$$\hat{Y}_1 = \frac{Y_1}{Y_{1,basecase}}; \hat{Y}_2 = \frac{Y_2}{Y_{2,basecase}}; \ ... \ \hat{Y}_n = \frac{Y_n}{Y_{n,basecase}} \tag{12}$$

Subsequently, the scale and shape parameters of the Gumbel distribution can be expressed as

$$\begin{aligned} \alpha &= \sum_{i=1}^{n} a_i \hat{Y}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \hat{Y}_i \hat{Y}_j \\ \mu &= \sum_{i=1}^{n} c_i \hat{Y}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \hat{Y}_i \hat{Y}_j \end{aligned} \tag{13}$$

where a_i, b_{ij}, c_i, d_{ij} are constants.

Eventually, the collision probability problem becomes an extreme value problem by which the probability of x_{em} exceeding a threshold value can be predicted. The limit state function of riser collision can hence be expressed as

$$g(x_{em}, \hat{Y}_{n+1}) = x_{em} - \hat{Y}_{n+1} \tag{14}$$

where x_{em} is the extreme minimum relative distance considered as a random variable; $\hat{Y}_{n+1}$ is the threshold value which can be defined as the sum of the radius of the two risers according to DNV-RP-F203 (2009). The collision probability corresponds to the probability that $g(x_{em}, \hat{Y}_{n+1})$ becomes negative.

Now a standard Gaussian cdf is applied for u_0 to represent the statistical variation of the extreme minimum distance x_{em}, and the following transformation is introduced,

$$F_X(x_{em}) = 1 - exp\{-exp\{\alpha(\hat{Y}_1,\ ...,\ \hat{Y}_n)(x_{em} - \mu(\hat{Y}_1,\ ...,\ \hat{Y}_n))\}\} = \Phi(u_0) \tag{15}$$

This equation can be solved with respect to the variable x_{em}, which subsequently is inserted into the failure function, giving:

$$g(u_0, \hat{Y}_1\ ...,\ \hat{Y}_n, \hat{Y}_{n+1}) = \frac{1}{\alpha(\hat{Y}_1,\ ...,\ \hat{Y}_n)} ln(-ln(1-\Phi(u_0))) + \mu(\hat{Y}_1,\ ...,\ \hat{Y}_n) - \hat{Y}_{n+1} \tag{16}$$

The limit state function then becomes an explicit expression in terms of the variables $U_0,\ \hat{Y}_1\ ...,\ \hat{Y}_n,\ \hat{Y}_{n+1}$. The next transformation is performed for the non-standard Gaussian variables related to the input parameters $\hat{Y}_1\ ...,\ \hat{Y}_n,\ \hat{Y}_{n+1}$. This implies that the following relations are introduced:

$$F_{\hat{Y}_1}(\hat{Y}_1) = \Phi(u_1); \quad F_{\hat{Y}_2}(\hat{Y}_2) = \Phi(u_2); \quad \cdots \quad F_{\hat{Y}_{n+1}}(\hat{Y}_{n+1}) = \Phi(u_{n+1}) \tag{17}$$

where the cdf for these variables are introduced. By inserting Equation 17 into Equation 16, a highly non-linear limit state function in terms of a set of standard Gaussian variables can be obtained. To identify the failure probability P_f, the minimum distance from the origin in the U-space to the failure function, i.e. the reliability index β, is required. This solution can not be computed analytically due to the highly non-linear failure function. Accordingly, the expression can be expanded in a first or second order Taylor series in each of the variables. Then the β and the corresponding P_f can subsequently be found numerically based on an iterative scheme Melchers (1999).

3.4. Summary of the Proposed Approach

Figure 5 illustrates a flow diagram that summarizes the entire framework for prediction of the riser collision probability. The procedure is composed of two main parts. The first part is a method to estimate the cdf of the extreme minimum distance between the two risers based on a set of dynamic analyses. The wake interference based on Blevins wake model is applied for both the static and dynamic analysis. The second part corresponds to a reliability analysis by taking into account the uncertainties of the input parameters related to the environmental loads and the riser system. This is achieved by using FORM/SORM based on Taylor series expansion.

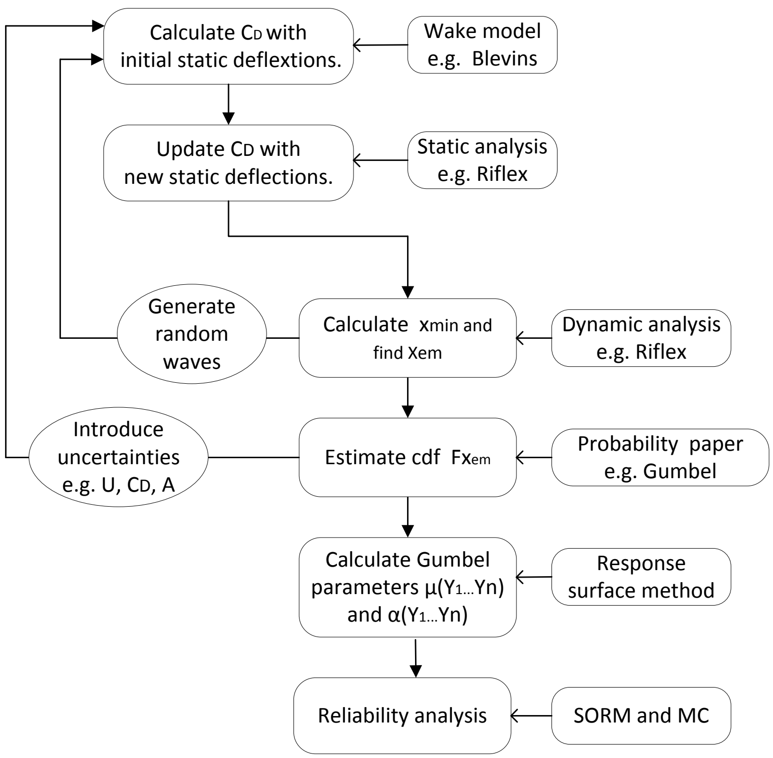

Figure 5: Flowchart of the proposed approach.

4. Basecase study

In this section, the procedure corresponding to the first part, i.e. the estimation of the cdf of the extreme minimum distance between two risers, will be explained through a case study.

4.1. Description of Riser System

Two identical flexible risers are modeled in a steep wave configuration, as shown in Figure 6. The water depth is $h = 100m$. The larger diameter indicates that the risers are covered with buoyancy elements, and the risers are fixed in translation at both the top and the bottom ends. The gap at the top and the bottom ends are both 10m. The main properties of the risers are listed in Table 1. The rigid body vessel motions in six DOF (degrees-of-freedom) are specified through the Response Amplitude Operators (RAOs). For simplicity, only the first-order wave loads are considered in dynamic analyses. The slow drift response due to the second-order wave loads will be considered as a static model uncertainty in reliability analysis.

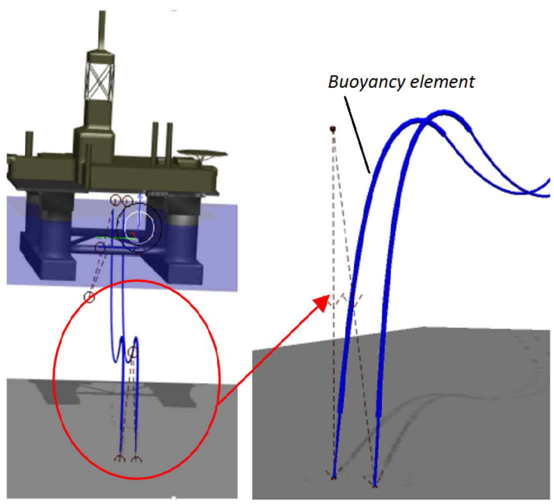

Figure 6: Model of the riser system.

For short-term analyses it is assumed that the most critical response occurs during a design sea state corresponding to a given return period, taken as 100 years, and the sea state is also assumed to be stationary. The JONSWAP spectrum is used to characterize the sea state, with a significant wave height $H_s = 14m$ and a spectral peak period $T_P = 19s$. The current velocity V_c is set to be $1.1m/s$ at the sea surface and to decrease linearly to $0.8m/s$ at the seabed. In order to offer a conservative response, both the incident waves and current are in the direction of the negative y-axis, which is perpendicular to the symmetry plane of the risers. The platform has an limited initial horizontal offset which is limited by the horizontal

Table 1: Properties of the risers.

	Unit	Riser	Buoyancy
Outside diameter	[m]	0.27	0.63
Inside diameter	[m]	0.05	0.05
Mass coefficient	[kg/m]	100	100
EI	[kNm^2]	104	104
Content density	[kN/m^3]	1000	1000
Total length	[m]	110	50

stiffness provided by the mooring system. Totally, there are eight mooring lines and the stiffness for each is $36N/m$. The corresponding horizontal offset is computed to be $5m$ according to the platform geometry.

4.2. Results and Discussion

Each computation of cases with a minimum distance time series $x_{min}(t)$ is time consuming, especially for the large number of elements. However, it should be noted that some elements, for instance the elements of the risers near the seabed, are not likely to clash, and will not contribute to x_{min}. Therefore, the first step is to identify which elements are more likely to clash. Based on several dynamic analyses, it is found that the riser section covered by the buoyancy elements usually gives a critical x_{min}. Therefore, we define the buoyancy elements as a separated segment and calculate x_{min} at all time steps.

Totally 100 30-minute simulations are performed in this work. The simulation time is about 5 hours. For each simulation, x_{min} at all time steps are calculated and x_{em} is found. Figure 7 shows the 100 simulated 30 minutes extremes x_{em} in a Gumbel probability paper with a fitted straight line. According to the 'Collision Not Allowed' design principle, a collision event occurs if $x_{em} \leq 0.63m$. From Figure 7 it appears that most of the x_{em}-values are in a range of $1.2m \leq x_{em} \leq 2m$. The most critical case is $x_{em} = 0.648m$, which is slightly larger than the sum of two risers' radii. Therefore, there is no clash during all 100 simulations for the present simulation length. The estimated Gumbel parameters μ and α are 1.7 and 5.06, respectively, with a regression coefficient $R^2 = 0.989$. The failure probability is $P_f = 4.5 \times 10^{-3}$, with a given riser diameter.

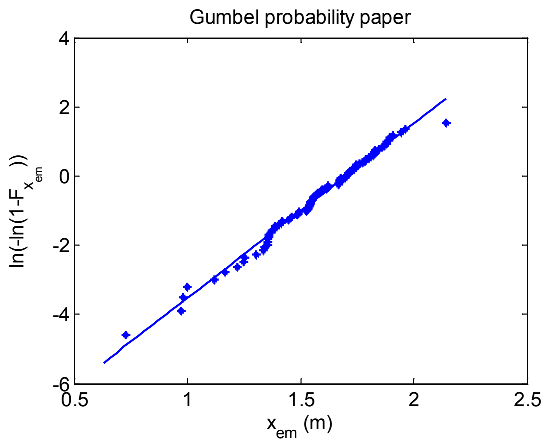

Figure 7: Gumbel probability paper for basecase.

5. Reliability Analysis

5.1. Basic Random Variables

The method used to estimate the location and scale parameters μ and α in section 4 is based on a fixed set of input parameters related to environmental loads and the riser system. In the reliability analysis, some of these input parameters are considered as variables based on DNV-RP-F204 (2005). Fu et al. (2015) performed parametric studies by varying these input parameters to verify how they affect the riser behavior. The following parameters were considered:

Drag coefficient: In the basecase study, C_D for the downstream riser is the mean value along one segment based on its equilibrium static position. This static C_D is also applied for the dynamic analysis. This simplified approach implies a significant uncertainty associated with C_D. In the reliability analysis, a two-sided variation is applied.

Current velocity: The riser response is significantly affected by the current. In practice, the current velocity varies by both time and location. The sensitivity of this is studied by introducing a two-sided variation and keeping the same linear profile.

Platform offset: The platform usually has a horizontal offset due to, for instance, current force and slow drift due to second order wave effects, with a limited value depending on the horizontal stiffness, provided by the mooring lines and/or the positioning system. In practice, the extreme horizontal offset of the platform relative to the connection point of the riser to the sea

Table 2: Summary of random variables applied in reliability analysis.

Variables		Distribution	Mean (Norm.)	Standard deviation σ	
				σ_L	σ_S
$\hat{Y}_1$	$\hat{V}_c$	Gumbel	1	0.1	0.05
$\hat{Y}_2$	$\hat{C}_D$	Lognormal	1	0.1	0.05
$\hat{Y}_3$	$\hat{A}$	Normal	1	0.25	
$\hat{Y}_4$	$\hat{D}$	Normal	1	0.1	

floor should be less than 10% of the water depth. Therefore, two additional horizontal offsets are introduced; $A = 0m$ corresponding to no offset and $A = 10m$ corresponding to the largest offset (Recall that $A = 5m$ for basecase).

Riser diameter: Additionally, the riser diameter D influences the collision criteria through the limit state function. The uncertainty of D mainly comes from the marine growth. The diameter will affect the threshold value directly as seen from Equation 14 above.

Since the V_c applied for the basecase study is the extreme current condition, V_c is assumed to be Gumbel distributed. C_D is assumed to be Lognormal distributed and A is Gaussian distributed, see e.g. DNV-RP-F204 (2005). Moreover, the riser diameter D is assumed to be normal distributed. These random variables are assumed to be statistically independent. Other parameters related to the extreme minimum distance x_{em} are considered to be deterministic. In order to study the effect of the standard deviation of the key variables, a large and a small standard deviation, indicated as σ_L and σ_S, are applied for the variables V_c and C_D. The standard deviations for A and D are constant. A summary of the statistical properties of these variables are given in Table 2, where the normalized variables are used.

5.2. Response Surface Method

Figure 8 compares the resulting linear fitting in Gumbel paper for three variables: V_c, C_D and A by using the values of the large standard deviation σ_L. The diameter D is assumed to be constant. The vertical line indicates the sum of the radii of the risers. The scale parameter α is associated with the slope of the fitting line, and the change of the location parameter μ relative to the basecase is associated with the offset from the origin fitting line (basecase).

For the current velocity V_c, the values $V_c = 1m/s$, $1.05m/s$, $1.1m/s$,

Table 3: Gumbel distribution parameters and corresponding failure probability for different variables.(The failure probability for the basecase without uncertainties included is 4.50×10^{-3}).

	α		μ		P_f	
	$+\sigma$	$-\sigma$	$+\sigma$	$-\sigma$	$+\sigma$	$-\sigma$
V_c	4.14	7.32	2.55	1.00	6.29×10^{-2}	3.51×10^{-4}
C_D	5.01	6.16	2.58	0.91	5.75×10^{-5}	0.16×10^{-1}
A	4.35	5.21	2.08	1.80	1.80×10^{-3}	2.20×10^{-3}
Basecase	5.05		1.70		4.50×10^{-3}	

$1.15m/s$ and $1.2m/s$ are analyzed, as shown in Figure 8 (a). Generally, it appears that the failure probability P_f (corresponding to the mean value of the riser diameter) increases as V_c increases, since a larger V_c leads to a smaller C_D based on Blevins wake model, resulting in a reduced riser clearance in static analysis. Moreover, it is found that V_c affects the location parameter μ significantly and also has the greatest influence on α. For the drag coefficient C_D, Figure 8 (b) illustrates the effect of C_D a given current velocity $V_c = 1.1m/s$. It shows that P_f (corresponding to the mean value of the riser diameter) increases as C_D decreases. This is because when two downstream risers are located at the same position and experience the same local reduced current, a smaller C_D of the downstream riser leads to smaller clearance in the static analysis, resulting in a higher likelihood of collision in the dynamic analysis. Moreover, it is observed that the location parameter μ is highly sensitive to C_D. Figure 8 (c) shows that both μ and α are less sensitive to the platform horizontal offset A compared with the other variables. The estimated α and μ are reported in Table 3, as well as P_f corresponding to a given riser diameter.

Based on the above sensitivity studies, it is found that V_c and C_D are the two most important factors when computing the response surface. In order to establish a smooth surface for α and μ, more simulations are needed to create discrete points. Additionally, for each V_c, the simulations are analyzed at the discrete points for $\pm\sigma_{\hat{C_D}}$ and $\pm\frac{1}{2}\sigma_{\hat{C_D}}$. Totally 25×100 30-minute simulations are performed including the basecase. The resulting response surfaces are plotted in Figure 9. The x-and y-axis are the normalized variables $\hat{V_C}$ and $\hat{C}_D$, receptively. The red crosses $\times$ represent the analyzed cases. As expected, it appears that both α and μ are strongly sensitive to the variation in $\hat{V_c}$ and

Table 4: Failure probability of riser collision by FORM and SORM.

		σ_L		σ_S	
		FORM	SORM	FORM	SORM
Reliability index	β	1.03	1	1.74	1.68
Failure probability	P_f	1.52×10^{-1}	1.59×10^{-1}	4.12×10^{-2}	4.69×10^{-2}

Table 5: Failure probability of riser collision by using MC simulation.

	σ_L	σ_S
Sample size	100000	100000
Estimated probability	1.64×10^{-1}	4.39×10^{-2}
Standard dev. of Probability	6.95×10^{-4}	6.48×10^{-4}
Coeff of Var. of Probability	0.014	0.015

$\hat{C}_D$.

5.3. Reliability Estimation

Now the reliability of the riser collision is computed by the proposed method based on FORM and SORM. The failure probability P_f and the corresponding reliability index β computed based on both the large and the small standard deviations σ_L and σ_S are summarized in Table 4. Generally, it is observed that P_f obtained from FORM and SORM, irrespective of the standard deviation, are very close, indicating that the failure surface in the U-space is close to a linear surface. Compared with Table 2, it is found by including the uncertainties, the collision probability increases by 2 to 3 orders of magnitude, emphasizing the importance of considering the uncertainties of variables associated with the wake interference, i.e. V_c and C_D, in the riser collision analysis. Moreover, the uncertainty of V_c and C_D is of great significance affecting P_f. By increasing the uncertainty from σ_S to σ_L, P_f increases more than 3 times.

In order to check the results for FORM and SORM, Monte Carlo simulation is performed. A total of 100000 samples are considered and the estimated P_f is given in Table 5. It appears that P_f obtained from the Monte Carlo simulation agrees well with P_f calculated by the SORM.

6. Conclusion and Future Work

This study outlines a method for estimation of the collision probability between two flexible risers by accounting for the wake effect when the risers are close to each other. The proposed method is composed of two main parts.

First, the dynamic response of the risers, represented by the relative minimum distance x_{min}, is taken as a stochastic process due to the randomness of the incoming irregular waves. Based on dynamic analyses, the extreme minimum distance x_{em} is formulated as a variable following the Gumbel distribution. A basecase study is performed to illustrate this procedure.

Secondly, based on sensitive studies, it is found that the some factors of key importance, i.e. the current velocity V_c and reduced drag coefficient C_D, govern the cdf of x_{em} as represented by the scale and location parameters μ and α. During the reliability analysis, V_c and C_D are considered as random variables, where μ and α are expressed as a product of polynomials of these variables based on the response surface method. The failure probability P_f is calculated by applying FORM and SORM numerical algorithms. The results show the importance of considering the uncertainty of the variables in estimating the riser collision probability. By changing the standard deviation of these two variables, it appears that the standard deviation of the variables influences P_f significantly and need to be considered carefully when the corresponding probabilistic models are determined.

In the present study, the reduced drag force is calculated based on Blevins wake model and applied for both static and dynamic analysis. In future work, a more accurate method should be used, so that the drag force can be updated according to its relative position at each time step during the dynamic analysis.

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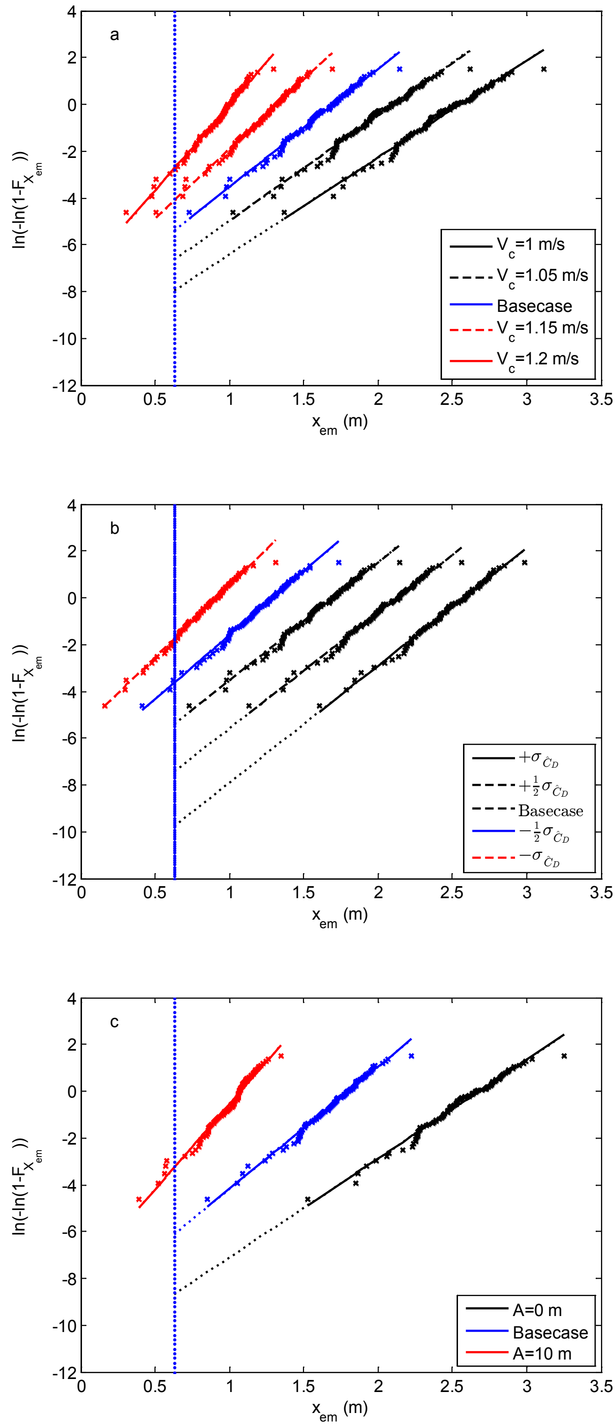

Figure 8: Sensitive studies of Gumbel parameters on the different variables.

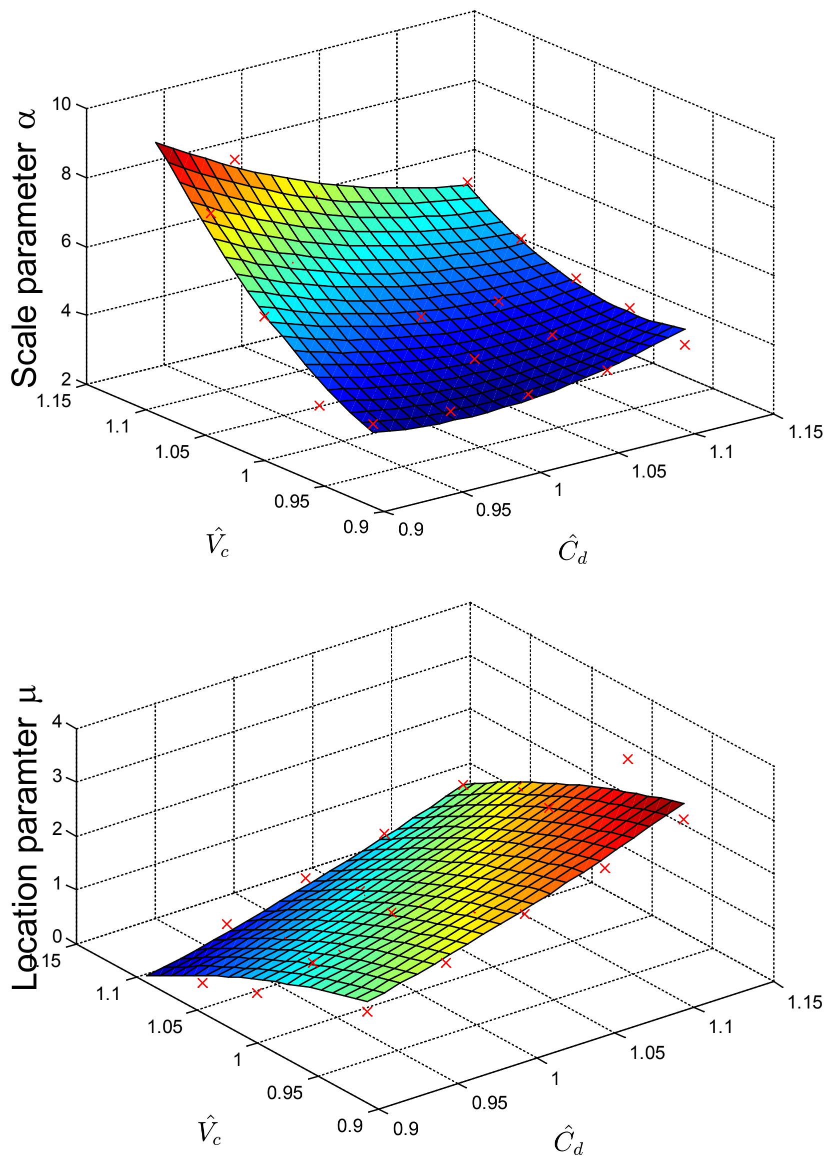

Figure 9: Response surfaces of Gumbel distribution parameters α and μ for the offset $A = 5m$.