LPV model reference control for fixed-wing UAVs ⋆

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Abstract: This paper proposes a linear parameter varying (LPV) model reference-based control for fixed-wing unmanned aerial vehicles (UAVs), which achieves agile and high performance tracking objectives in extended flight envelopes, e.g. when near stall or deep stall flight conditions are considered. Each of the considered control loops (yaw, pitch and airspeed) delivers an error model that can be reshaped into a quasi-LPV form through an appropriate choice of the scheduling variables. The quasi-LPV error models are suitable for designing error feedback controllers using linear matrix inequalities (LMIs), which are derived within the quadratic Lyapunov framework. Simulation results are used to show the effectiveness of the proposed approach.

Keywords: LPV systems; Unmanned aerial vehicles (UAVs); Model reference control.

1. INTRODUCTION

In the recent decades, the use of unmanned aerial vehicles (UAVs) for military and civilian applications has increased rapidly, since UAVs can be applied in situations where manned missions would introduce challenges, such as monitoring of inaccessible locations or hazardous areas. Several control techniques based on linearised models obtained under the assumption of low angle-of-attach (AOA) have been proposed, e.g. (Espinoza et al., 2014).

The AOA is an important parameter for describing the aerodynamics of a UAV. When it increases beyond a certain angle, the boundary layer of the air flow separates from the airfoil of the wing and creates a turbulent wake behind the wing, which causes a reduction in the lift on the wing and an increase of the drag (Kermode, 2006, Bertin and Cummings, 2014). The AOA for which this phenomenon occurs, i.e. the AOA for which the flow separates, is referred to as *stall angle*.

Usually, near stall or deep stall (AOA higher than the stall angle) flight conditions are to be avoided. However, there are some situations where it is desirable that the UAV's AOA goes near or beyond the stall angle (deep stall). For example, to recover a UAV in a small space without a runaway, a recovery net can be used. In this case, it is desirable to ease the impact by minimizing the speed at which the UAV meets the landing target. Deep stall landing lets the UAV decrease its altitude while at the same time reducing its speed (Mathisen et al., 2015, 2016). In other situations, severe turbulence or agile manoeuvres can cause the airfoils to enter into stall conditions. In these situations, the aerodynamics are unsteady, nonlinear and sensitive to small changes in flight conditions, so they cannot be described by linear models (Shields and Mohseni, 2011). Hence, the need for developing alternative frameworks for UAV control arises.

During the last decades, the linear parameter varying (LPV) paradigm (Shamma, 2012) has been applied successfully to many applications (Rotondo et al., 2013) and validated by several experiments and high-fidelity simulations (Natesan et al., 2006, Hoffmann and Werner, 2015). The main advantage of this approach is that, by embedding the system's nonlinearities in the varying parameters, nonlinear systems can be controlled using an extension of linear techniques. When trajectory tracking is desired, a possible solution relies on the use of a reference model that generates the desired trajectory (Abdullah and Zribi, 2009, Rotondo et al., 2015a,b).

This paper proposes an LPV model reference-based controller for fixed-wing UAVs that is able to cope with the highly nonlinear dynamics and achieve tracking objectives also when near stall or deep stall flight conditions are considered. Simulation results are used to show the effectiveness of the proposed approach.

2. DYNAMIC MODEL OF THE FIXED-WING UAV

Let us consider the 6-DOF aircraft nonlinear model (Beard and McLain, 2012), consisting of three equations for the relative velocity components (u, v, w), three equations for the attitude (ϕ, θ, ψ) and three equations for the angular rates (p, q, r) [1] :

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[1] In the following, the dependence of the variables on time t is omitted to ease the notation.

$$\dot{u} = rv - qw - g\sin\theta + (\mathscr{A}_x + \mathscr{T})/m \quad (1)$$
$$\dot{v} = pw - ru + g\cos\theta\sin\phi + \mathscr{A}_y/m \quad (2)$$
$$\dot{w} = qu - pv + g\cos\theta\cos\phi + \mathscr{A}_z/m \quad (3)$$
$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \quad (4)$$
$$\dot{\theta} = q\cos\phi - r\sin\phi \quad (5)$$
$$\dot{\psi} = q\sin\phi\sec\theta + r\cos\phi\sec\theta \quad (6)$$
$$\dot{p} = \Gamma_1 pq - \Gamma_2 qr + \mathscr{M}_p \quad (7)$$
$$\dot{q} = \Gamma_5 pr - \Gamma_6\left(p^2 - r^2\right) + \mathscr{M}_q \quad (8)$$
$$\dot{r} = \Gamma_7 pq - \Gamma_1 qr + \mathscr{M}_r \quad (9)$$

where g is the gravitational acceleration, m is the vehicle mass, $\mathscr{A}_i$ are aerodynamical forces (lift and drag), $\mathscr{T}$ is the propulsion thrust force, $\mathscr{M}_i$ are aerodynamical torques, and Γ_i are coefficients obtained as combinations of the main inertia coefficients J_x, J_y, J_z and J_{xz}. More specifically:

$$\mathscr{T} = \frac{\rho S_{prop} C_{prop}}{2}\left[k_m^2\delta_t^2 - V_a^2\right] \quad (10)$$
$$\mathscr{A}_x = \frac{\rho V_a^2 S}{2}\left[C_X(\alpha) + C_{X_q}(\alpha)\frac{cq}{2V_a} + C_{X_{\delta_e}}(\alpha)\delta_e\right] \quad (11)$$
$$\mathscr{A}_y = \frac{\rho V_a^2 S}{2}\left[C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{bp}{2V_a} + C_{Y_r}\frac{br}{2V_a} + C_{Y_{\delta_a}}\delta_a\right] \quad (12)$$
$$\mathscr{A}_z = \frac{\rho V_a^2 S}{2}\left[C_Z(\alpha) + C_{Z_q}(\alpha)\frac{cq}{2V_a} + C_{Z_{\delta_e}}(\alpha)\delta_e\right] \quad (13)$$
$$\mathscr{M}_p = \frac{\rho V_a^2 Sb}{2}\left[C_{p_0} + C_{p_\beta}\beta + C_{p_p}\frac{bp}{2V_a} + C_{p_r}\frac{br}{2V_a} + C_{p_{\delta_a}}\delta_a\right] \quad (14)$$
$$\mathscr{M}_q = \frac{\rho V_a^2 Sc}{2J_y}\left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{cq}{2V_a} + C_{m_{\delta_e}}\delta_e\right] \quad (15)$$
$$\mathscr{M}_r = \frac{\rho V_a^2 Sb}{2}\left[C_{r_0} + C_{r_\beta}\beta + C_{r_p}\frac{bp}{2V_a} + C_{r_r}\frac{br}{2V_a} + C_{r_{\delta_a}}\delta_a\right] \quad (16)$$

where ρ is the air density, S_{prop} is the area swept out by the propeller, k_m is the constant that specifies the efficiency of the motor, S is the wing surface area, c is the mean aerodynamic chord, b is the wingspan of the aircraft and:

$$C_X(\alpha) = -C_D(\alpha)\cos\alpha + C_L(\alpha)\sin\alpha \quad (17)$$
$$C_Z(\alpha) = -C_D(\alpha)\sin\alpha - C_L(\alpha)\cos\alpha \quad (18)$$

The inputs entering the system are the thrust command δ_t, the elevator deflection δ_e and the aileron deflection δ_a. It is assumed that no rudder command δ_r is available. Notice that this assumption is not restrictive, since the presence of a rudder command would increase the controllability of the UAV, and would require only a slight modification of the UAV's model and control algorithm.

The non-dimensional coefficients C_i are usually referred to as *stability* and *control derivatives*. Some of them are nonlinear functions of the angle-of-attack α, defined as:

$$\alpha = \arctan(w/u) \quad (19)$$

V_a and β are the total airspeed and the sideslip angle, respectively, defined as:

$$V_a = \sqrt{u^2 + v^2 + w^2} \quad (20)$$
$$\beta = \arcsin(v/V_a) \quad (21)$$

The velocity dynamics, i.e. (1)-(3), is affected by the wind, that can be expressed as the additional unknown input:

$$\begin{pmatrix} \dot{\nu}_u \\ \dot{\nu}_v \\ \dot{\nu}_w \end{pmatrix} = -R(\phi,\theta,\psi)\begin{pmatrix} \dot{\nu}_N \\ \dot{\nu}_E \\ \dot{\nu}_D \end{pmatrix} \quad (22)$$

where the matrix $R(\phi,\theta,\psi)$ represents the rotation from inertial to body frame, and $\dot{\nu} = (\dot{\nu}_N, \dot{\nu}_E, \dot{\nu}_D)$ is the wind acceleration expressed in the inertial frame (mainly due to turbulence).

It is assumed that the UAV's state is available for feedback and scheduling purposes, using sensor readings coming from an autopilot and Kalman filter-based estimations, see Beard and McLain (2012).

3. MODEL REFERENCE LPV CONTROL OF THE FIXED-WING UAV

In order to control the UAV, three different control loops are considered, following the suggestion provided by Liu et al. (2015): yaw control loop, pitch control loop and airspeed control loop. The goal of the reference model (Abdullah and Zribi, 2009) is twofold: (i) to generate the desired trajectory by feeding it with an appropriate reference input (feedforward action) and (ii) to obtain an error model that is linear with respect to the error variables and the incremental input (feedback action).

3.1 Yaw control loop

Let us consider the yaw angle subsystem given by Eqs. (6) and (9), and let us define the following reference model:

$$\dot{\psi}_{ref} = q\sin\phi\sec\theta + r_{ref}\cos\phi\sec\theta \quad (23)$$
$$\dot{r}_{ref} = \Gamma_7 pq - \Gamma_1 qr + \mathscr{M}_{r,ref} \quad (24)$$
$$\mathscr{M}_{r,ref} = \frac{\rho V_a^2 Sb}{2}\left[C_{r_0} + C_{r_\beta}\beta + C_{r_p}\frac{bp}{2V_a} + C_{r_r}\frac{br_{ref}}{2V_a} + C_{r_{\delta_a}}\delta_{a,ref}\right] \quad (25)$$

where ψ_{ref} is the reference yaw angle, r_{ref} is the reference angular rate for r, and $\delta_{a,ref}$ is the reference aileron deflection.

By defining the tracking errors $e_\psi = \psi_{ref} - \psi$ and $e_r = r_{ref} - r$, and the new input $\Delta\delta_a = \delta_{a,ref} - \delta_a$, the following error model is obtained:

$$\dot{e}_\psi = \cos\phi\sec\theta e_r \quad (26)$$
$$\dot{e}_r = \frac{\rho V_a Sb^2 C_{r_r}}{4}e_r + \frac{\rho V_a^2 SbC_{r_{\delta_a}}}{2}\Delta\delta_a \quad (27)$$

which can be reshaped into a quasi-LPV form by introducing the scheduling variables $\vartheta_\psi^{(1)} = \cos\phi\sec\theta$, $\vartheta_\psi^{(2)} = V_a$ and $\vartheta_\psi^{(3)} = V_a^2$:

$$\begin{pmatrix} \dot{e}_\psi \\ \dot{e}_r \end{pmatrix} = \begin{pmatrix} 0 & \vartheta_\psi^{(1)} \\ 0 & \frac{\rho Sb^2 C_{r_r}}{4}\vartheta_\psi^{(2)} \end{pmatrix}\begin{pmatrix} e_\psi \\ e_r \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{\rho SbC_{r_{\delta_a}}}{2}\vartheta_\psi^{(3)} \end{pmatrix}\Delta\delta_a \quad (28)$$

The trajectory for the reference yaw angle ψ_{ref} is obtained by fixing $\dot{\psi}_{ref}$, which through (23) becomes a constraint on r_{ref}:

$$r_{ref} = \frac{\dot{\psi}_{ref} - q\sin\phi\sec\theta}{\cos\phi\sec\theta} \quad (29)$$

For the sake of simplicity, we will consider a piecewise constant trajectory for ψ_{ref}, which means $\dot{\psi}_{ref} = \ddot{\psi}_{ref} = 0$ almost always. By taking the derivative of (29) and replacing appropriately the expressions for $\dot{\phi}$, $\dot{\theta}$, $\dot{q}$ and $\dot{r}_{ref}$ given by (4), (5), (8) and (24), the following is obtained:

$$a_{\psi,a}\delta_{a,ref} + a_{\psi,e}\delta_{e,ref} = b_\psi \quad (30)$$

with:

$$a_{\psi,a} = -\frac{\rho V_a^2 Sb}{2}(\cos\phi\sec\theta)^2 C_{r_{\delta_a}}$$

$$a_{\psi,e} = -\frac{\rho V_a^2 Sc}{2J_y}\sin\phi\cos\phi\sec^2\theta C_{m_{\delta_e}}$$

with b_ψ defined in (31) (see top of the next page).

Eq. (30) is a linear equation in the variables $\delta_{a,ref}$ and $\delta_{e,ref}$ which, together with a similar equation provided in Section 3.2, allows calculating the values for the reference inputs that generate the desired trajectory.

Remark 1. Although no roll control loop is considered, it is worth noticing that the yaw control loop has an effect on the roll angle due to couplings. In particular, the transient while the yaw angle is increasing (decreasing) will be associated to positive (negative) values for the roll angle.

3.2 Pitch control loop

Similarly to the yaw case described in Section 3.1, for the pitch angle subsystem given by Eqs. (5) and (8), let us define the following reference model:

$$\dot\theta_{ref} = q_{ref}\cos\phi - r\sin\phi \tag{32}$$

$$\dot q_{ref} = \Gamma_5 pr - \Gamma_6\left(p^2 - r^2\right) + \mathscr{M}_{q,ref} \tag{33}$$

$$\mathscr{M}_{q,ref} = \frac{\rho V_a^2 Sc}{2J_y}\left[C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{cq_{ref}}{2V_a} + C_{m_{\delta_e}}\delta_{e,ref}\right] \tag{34}$$

where θ_{ref} is the reference pitch angle, q_{ref} is the reference angular rate for q, and $\delta_{e,ref}$ is the reference elevator deflection (feedforward action). By defining the tracking errors $e_\theta = \theta_{ref} - \theta$ and $e_q = q_{ref} - q$, and the new input $\Delta\delta_e = \delta_{e,ref} - \delta_e$, the following error model is obtained:

$$\dot e_\theta = \cos\phi\, e_q \tag{35}$$

$$\dot e_q = \frac{\rho V_a Sc^2 C_{m_q}}{4J_y}e_q + \frac{\rho V_a^2 Sc C_{m_{\delta_e}}}{2J_y}\Delta\delta_e \tag{36}$$

which can be reshaped into a quasi-LPV form by introducing the scheduling variables $\vartheta_\theta^{(1)} = \cos\phi$, $\vartheta_\theta^{(2)} = V_a$ and $\vartheta_\theta^{(3)} = V_a^2$:

$$\begin{pmatrix}\dot e_\theta\\ \dot e_q\end{pmatrix} = \begin{pmatrix}0 & \vartheta_\theta^{(1)}\\ 0 & \frac{\rho Sc^2 C_{m_q}}{4J_y}\vartheta_\theta^{(2)}\end{pmatrix}\begin{pmatrix}e_\theta\\ e_q\end{pmatrix} + \begin{pmatrix}0\\ \frac{\rho Sc C_{m_{\delta_e}}}{2J_y}\vartheta_\theta^{(3)}\end{pmatrix}\Delta\delta_e \tag{37}$$

The trajectory for the reference yaw angle θ_{ref} is obtained by fixing $\dot\theta_{ref}$, which through (32) becomes a constraint on q_{ref}:

$$q_{ref} = \frac{\dot\theta_{ref} + r\sin\phi}{\cos\phi} \tag{38}$$

Also in this case, for the purpose of calculating the reference inputs, we will consider that $\dot\theta_{ref} = \ddot\theta_{ref} = 0$ (notice that, as shown later in Section 5, the UAV will still be able to track time-varying reference pitch angles). Then, by taking the derivative of (38) and replacing appropriately the expressions for $\dot\phi$, $\dot r$ and $\dot q_{ref}$ given by (4), (9) and (33), the following is obtained:

$$a_{\theta,a}\delta_{a,ref} + a_{\theta,e}\delta_{e,ref} = b_\theta \tag{39}$$

with:

$$a_{\theta,a} = \rho V_a^2 Sb\sin\phi\cos\phi C_{r_{\delta_a}}$$

$$a_{\theta,e} = -\rho V_a^2 Sc\cos^2\phi C_{m_{\delta_e}}/(2J_y)$$

and b_θ defined in (42) (see top of the next page). By solving simultaneously (30) and (39), under the assumption that $\cos\phi \neq 0$, values for $\delta_{a,ref}$ and $\delta_{e,ref}$ can be calculated. Then, the control inputs to be fed to the actuators (aileron and elevator) would be calculated as $\delta_a = \delta_{a,ref} - \Delta\delta_a$ and $\delta_e = \delta_{e,ref} - \Delta\delta_e$ (the computation of $\Delta\delta_a$ and $\Delta\delta_e$ is further discussed in Section 4).

Remark 2. According to Beard and McLain (2012), the evolution of the altitude h is driven by the following differential equation:

$$\dot h = u\sin\theta - v\sin\phi\cos\theta - w\cos\phi\cos\theta \tag{40}$$

By taking into account that $v \approx 0$ and using a small-angle approximation for θ, i.e. $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \theta^2/2$, (40) becomes:

$$\dot h = u\theta - w\cos\phi\left(1 - \frac{\theta^2}{2}\right) \tag{41}$$

Through (41), and using available measurements for u, w and ϕ, it is possible to relate the desired altitude rate with an appropriate trajectory of the reference pitch angle θ_{ref}.

3.3 Airspeed control loop

First of all, let us obtain the equation that describes the dynamical behavior of the airspeed V_a. Taking into account (1)-(3), (20), and the wind effect described by (22), the following is obtained:

$$\begin{aligned}\dot V_a = {} & \left[\left(\mathscr{A}_x + \mathscr{T}\right)u + \mathscr{A}_y v + \mathscr{A}_z w\right]/(mV_a)\\ & + \left[-ug\sin\theta + vg\cos\theta\sin\phi + wg\cos\theta\cos\phi\right]/V_a\\ & + \left[u\dot v_u + v\dot v_v + w\dot v_w\right]/V_a\end{aligned} \tag{43}$$

Let us define the following reference model for the airspeed control loop:

$$\begin{aligned}\dot V_{a,ref} = {} & \left[\left(\mathscr{A}_x + \mathscr{T}_{ref}\right)u + \mathscr{A}_y v + \mathscr{A}_z w\right]/(mV_a)\\ & + \left[-ug\sin\theta + vg\cos\theta\sin\phi + wg\cos\theta\cos\phi\right]/V_a\end{aligned} \tag{44}$$

$$\mathscr{T}_{ref} = \frac{\rho S_{prop}C_{prop}}{2}\left[k_m^2\delta_{t,ref}^2 - V_{a,ref}^2\right] \tag{45}$$

where $V_{a,ref}$ is the reference airspeed and $\delta_{t,ref}$ is the reference thrust command (feedforward action). By defining the tracking error $e_{V_a} = V_{a,ref} - V_a$, and the new input $\Delta\delta_t = \delta_{t,ref}^2 - \delta_t^2$, the following error model is obtained:

$$\dot e_{V_a} = \frac{\rho S_{prop}C_{prop}u}{2mV_a}\left[k_m^2\Delta\delta_t - e_{V_a}\right] - \left[u\dot v_u + v\dot v_v + w\dot v_w\right]/V_a \tag{46}$$

By neglecting the wind-dependent terms, which are not available for control purposes, and will act as exogenous disturbances that will be rejected through the feedback, it is straightforward to obtain a quasi-LPV model for (46) by defining the scheduling variable $\vartheta_{V_a} = u/V_a$:

$$\dot e_{V_a} = -\frac{\rho S_{prop}C_{prop}}{2m}\vartheta_{V_a}e_{V_a} + \frac{\rho S_{prop}C_{prop}k_m^2}{2m}\vartheta_{V_a}\Delta\delta_t \tag{47}$$

In this case, the computation of the reference thrust command can be performed using (44)-(45) for a given $\dot V_{a,ref}$. Then, $\delta_t^2 = \delta_{t,ref}^2 - \Delta\delta_t$ with $\Delta\delta_t$ calculated using the error feedback controller detailed in the next section.

4. ERROR FEEDBACK CONTROLLER DESIGN USING LPV TECHNIQUES

The error models (28), (37) and (46) all have the following structure:

$$\dot e(t) = A\left(\vartheta(t)\right)e(t) + B\left(\vartheta(t)\right)\Delta\delta(t) \tag{48}$$

$$b_\psi = (\cos\phi\sec\theta)^2\left[\Gamma_7 pq - \Gamma_1 qr + \rho V_a^2 Sb\left(C_{r_0} + C_{r_\beta}\beta + C_{r_p}bp/(2V_a) + C_{r_r}br_{ref}/(2V_a)\right)/2\right] \tag{31}$$
$$+\cos\phi\sec\theta\left\{\left[\Gamma_5 pr - \Gamma_6\left(p^2 - r^2\right) + \rho V_a^2 Sc\left(C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}cq/(2V_a) - C_{m_{\delta_e}}\Delta\delta_e\right)/(2J_y)\right]\sin\phi\sec\theta\right.$$
$$+q\cos\phi\sec\theta\left(p + q\sin\phi\tan\theta + r\cos\phi\tan\theta\right) + q\sin\phi\sec\theta\tan\theta\left(q\cos\phi - r\sin\phi\right)\}$$
$$-q\sin\phi\sec\theta\left\{-\sin\phi\sec\theta\left(p + q\sin\phi\tan\theta + r\cos\phi\tan\theta\right) + \cos\phi\sec\theta\tan\theta\left(q\cos\phi - r\sin\phi\right)\right\}$$

$$b_\theta = \cos^2\phi\left[\Gamma_5 pr - \Gamma_6\left(p^2 - r^2\right) + \rho V_a^2 Sc\left(C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}cq_{ref}/(2V_a)\right)/(2J_y)\right] \tag{42}$$
$$-\cos\phi\left\{\sin\phi\left[\Gamma_7 pq - \Gamma_1 qr + \rho V_a^2 Sb\left(C_{r_0} + C_{r_\beta}\beta + C_{r_p}bp/(2V_a) + C_{r_r}br/(2V_a) - C_{r_{\delta_a}}\Delta\delta_a\right)\right]\right.$$
$$\left.+r\cos\phi\left(p + q\sin\phi\tan\theta + r\cos\phi\tan\theta\right)\right\} - \sin^2\phi\left(p + q\sin\phi\tan\theta + r\cos\phi\tan\theta\right)$$

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where $e \in \mathbb{R}^{n_e}$ is the error vector, $\Delta\delta \in \mathbb{R}^{n_\delta}$ is the input vector, $A(\vartheta(t))$ and $B(\vartheta(t))$ are matrix functions of the vector of varying parameters $\vartheta \in \Theta \subset \mathbb{R}^{n_\vartheta}$.

The aim of this section is to design an error-feedback control law:

$$\Delta\delta(t) = K(\vartheta(t))e(t) \tag{49}$$

where $K(\vartheta(t))$ is a matrix function chosen such that the resulting closed-loop error system is stable with poles placed in some desired region of the complex plane regardless from the value of the varying parameter $\vartheta(t)$. Notice that since the yaw, pitch and airspeed control loops are completely decoupled, the stability of each individual loop can be enforced independently.

In this paper, both stability and pole placement are analyzed within the quadratic Lyapunov framework, where the specifications are assured by the use of a single quadratic Lyapunov function. The system (48) with the error-feedback control law (49) is quadratically stable if there exist $X_s \succ 0$ and $K(\vartheta(t))$ such that:

$$He\{A(\vartheta)X_s + B(\vartheta)K(\vartheta)X_s\} \prec 0 \tag{50}$$

$\forall\vartheta \in \Theta$ (Packard and Becker, 1992) where, for a given matrix M, the shorthand notation $He\{M\} \triangleq M + M^T$ is used. On the other hand, pole clustering is based on the results obtained by Chilali and Gahinet (1996), where subsets $\mathscr{D}$ of the complex plane, referred to as *LMI regions*, are defined as:

$$\mathscr{D} = \{s \in \mathbb{C} : f_{\mathscr{D}}(s) < 0\} \tag{51}$$

where $f_{\mathscr{D}}$ is the *characteristic function*, given by:

$$f_{\mathscr{D}}(s) = \alpha + s\beta + \bar{s}\beta^T = [\alpha_{kl} + \beta_{kl}s + \beta_{lk}\bar{s}]_{k,l\in[1,m]} \tag{52}$$

with $\alpha \in \mathbb{S}^{m\times m}$, $\beta \in \mathbb{R}^{m\times m}$ and $\bar{s}$ denoting the complex conjugate of s. Hence, the system (48) with the error-feedback control law (49) is quadratically $\mathscr{D}$-stable (i.e. with poles in $\mathscr{D}$) if there exist $X_{\mathscr{D}} \succ 0$ and $K(\vartheta(t))$ such that:

$$\begin{aligned}[\alpha_{kl}X_{\mathscr{D}} + \beta_{kl}\left(A(\vartheta) + B(\vartheta)K(\vartheta)\right)X_{\mathscr{D}}\\ \left.+\beta_{lk}X_{\mathscr{D}}\left(A(\vartheta) + B(\vartheta)K(\vartheta)\right)^T\right]_{k,l\in[1,m]} \prec 0\end{aligned} \tag{53}$$

The main difficulty with using (50) and (53) is that they impose an infinite number of constraints. In order to reduce this number to finite, a polytopic approximation of (48)-(49) is considered, as follows:

$$\begin{pmatrix} A(\vartheta(t)) \\ K(\vartheta(t)) \end{pmatrix} = \sum_{i=1}^{N_A}\gamma_i(\vartheta(t))\begin{pmatrix} A_i \\ K_i \end{pmatrix} \tag{54}$$

$$B(\vartheta(t)) = \sum_{j=1}^{N_B}\chi_j(\vartheta(t))B_j \tag{55}$$

with $\gamma_i(\vartheta), \beta_j(\vartheta) \geq 0$ and $\sum_{i=1}^{N_A}\gamma_i(\vartheta) = \sum_{j=1}^{N_B}\chi_j(\vartheta) = 1, \forall\vartheta \in \Theta$. Then, using a common Lyapunov matrix $X = X_s = X_{\mathscr{D}} \succ 0$ and through the change of variables $\Gamma_i \triangleq K_iX$, (50) and (53) can be brought to a finite number of linear matrix inequalities (LMIs), as follows:

$$He\left\{A_iX + B_j\Gamma_i\right\} \prec 0 \tag{56}$$

$$\left[\alpha_{kl}X + \beta_{kl}\left(A_iX + B_j\Gamma_i\right) + \beta_{lk}\left(A_iX + B_j\Gamma_i\right)^T\right]_{k,l\in[1,m]} \prec 0 \tag{57}$$

with $i = 1, \ldots, N_A$ and $j = 1, \ldots, N_B$. The set of LMIs (56)-(57) can be solved efficiently using available software, e.g. YALMIP (Löfberg, 2004)/SeDuMi (Sturm, 1999).

5. SIMULATION RESULTS

The LPV model reference control for fixed-wing UAVs described in Section 3 and 4 has been applied to the nonlinear model of a small UAV, namely the Aerosonde UAV, which is described in detail in Beard and McLain (2012).

Polytopic representation for the error models (28), (37) and (46) (neglecting the term due to the wind acceleration, which will act as an exogenous disturbance) have been obtained by applying a *bounding box* approach (Sun and Postlethwaite, 1998), considering $\phi, \theta \in [-\pi/3, \pi/3]$ and $V_a \in [10m/s, 30m/s]$. The feedback controllers:

$$\Delta\delta_a(t) = K_a\left(\vartheta_\psi(t)\right)\begin{bmatrix} e_\psi(t) \\ e_r(t) \end{bmatrix} \tag{58}$$

$$\Delta\delta_e(t) = K_e\left(\vartheta_\theta(t)\right)\begin{bmatrix} e_\theta(t) \\ e_q(t) \end{bmatrix} \tag{59}$$

$$\Delta\delta_t = K_t\left(u/V_a\right)e_{V_a} \tag{60}$$

have been designed using the approach described in Section 4, requiring stability and regional pole placement, obtaining the closed-loop pole configuration shown in Fig. 1.

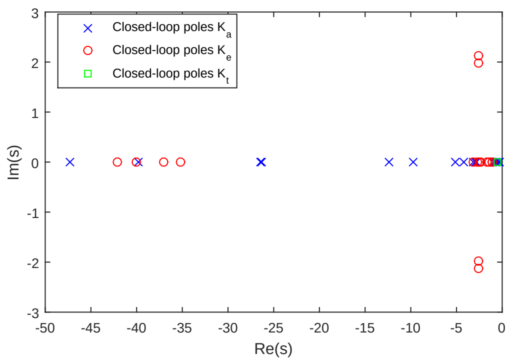

Fig. 1. Closed-loop poles.

The results shown in this paper refer to a simulation which lasts $100\,s$, where the UAV is required to follow a desired trajectory for yaw angle, pitch angle and airspeed. as depicted in red solid line in Figs. 2-4. Notice that the reference for the pitch angle has been calculated to achieve some desired altitude rate, as described in Section 3.2. The sequence of wind gusts velocities simulated in this paper has been obtained using the widely accepted Dryden wind turbulence model (U. S. Department of Defense, 1980), by considering an altitude of $150\,m$, an aircraft speed of $20\,m/s$ and a low-altitude intensity defined by a wind speed of $5\,m/s$, and is shown in Fig. 5. Moreover, in order to test the robustness of the proposed approach against sensor noise, it is assumed that the measured state variables are affected by uniform noise with magnitude $\pm 10\%$ of the measured values.

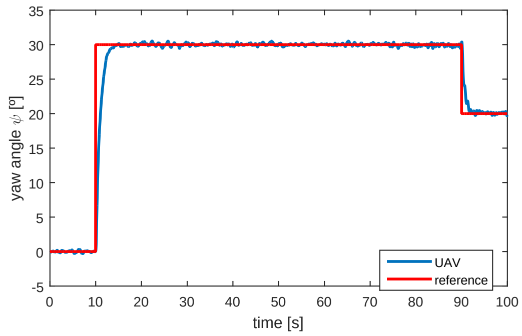

Fig. 2. Yaw angle ψ and reference yaw angle ψ_{ref}.

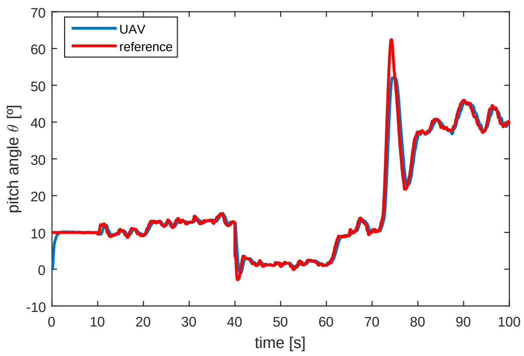

Fig. 3. Pitch angle θ and reference pitch angle θ_{ref}.

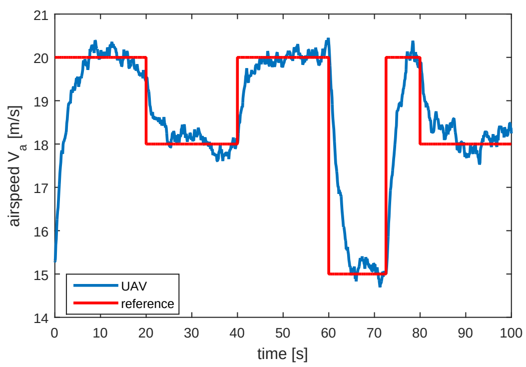

Fig. 4. Airspeed V_a and reference airspeed $V_{a,ref}$.

The simulation results in Figs. 2-4 (blue solid lines) show that the yaw angle, the pitch angle and the total airspeed, respectively, achieve good tracking performance despite several

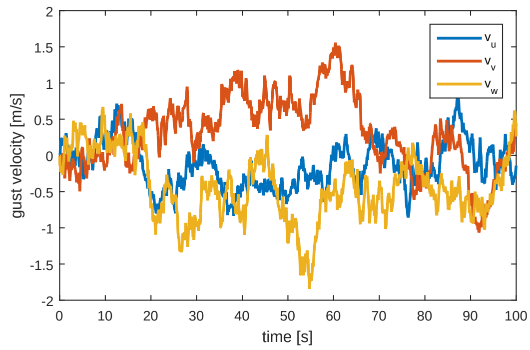

Fig. 5. Gust components of the wind v_u, v_v, v_w.

effects, such as wind, noise and changes in the dynamics due to the nonlinearities. Fig. 6 shows some relevant angular velocities, such as the roll angle ϕ, the AOA α and the sideslip angle β. In particular, it can be seen that after $70\,s$, the UAV is working with an AOA beyond the stall angle, which for the considered UAV is between $23°$ and $24°$. Throughout the simulation, the UAV experiences strong variations of the lift and drag coefficients (see Fig. 7). However, the LPV paradigm is able to take into account these variations and provide guarantees of tracking performance. Finally, the control inputs are illustrated in Fig. 8, showing that for most of the simulation the actuators are working far from their saturations (the linear ranges are $[-1,1]$ for the aileron and elevator commands, and $[0,1]$ for the thrust command), and the altitude response is shown in Fig. 9.

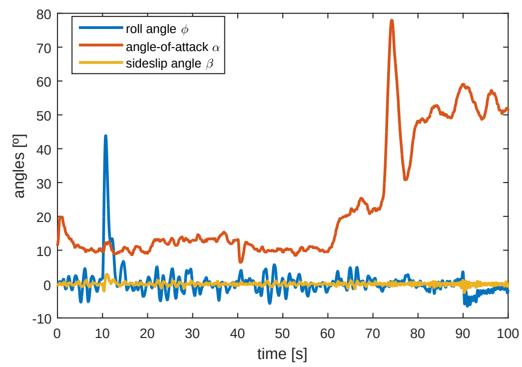

Fig. 6. Roll angle ϕ, AOA α and sideslip angle β.

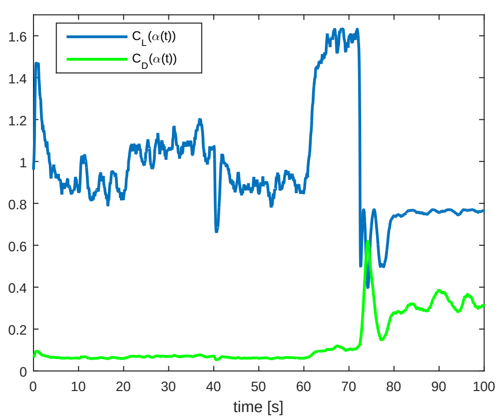

Fig. 7. Lift and drag coefficients.

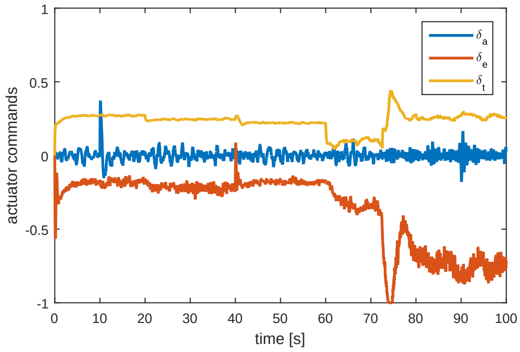

Fig. 8. Control signals.

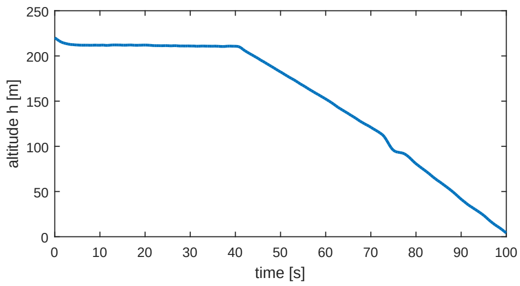

Fig. 9. Altitude h.

6. CONCLUSIONS

In this paper, the problem of controlling the yaw, pitch and total airspeed of a UAV has been solved using an LPV model reference approach. The proposed solution is able to cope with the nonlinearities that arise, e.g. due to near stall or deep stall flight conditions, and relies on the use of a reference model that describes the desired trajectory. The nonlinear error dynamics are brought into a quasi-LPV form that is used for designing an LPV error-feedback controller using LMI-based techniques. The results obtained in simulation have demonstrated the effectiveness of the proposed technique in achieving good tracking performance in spite of changes in the flight conditions.

Future research will aim at increasing the robustness of the proposed approach against additional effects that could hinder the practical implementation on a real set-up, such as model uncertainties and inaccessibility of some of the state variables for feedback and/or scheduling. Also, the design of outer control loops for the tracking of a desired altitude and course angle within the proposed LPV model reference paradigm is an open problem that deserves further investigation.

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