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# Controlling Risk in Maritime Fleet Renewal

A Stochastic Programming Approach

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Industrial Economics and Technology Management

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# Problem description

The maritime fleet renewal problem (MFRP) is a continuous strategic problem for a shipping company, with decisions that might involve major capital investments. The purpose of this thesis is to propose a model for the maritime fleet renewal problem and discuss the risk of insolvency.

# Preface

This master's thesis is the result of the final work for my MSc. in Industrial Economics and Technology Management, with specialization in Managerial Economics and Operations Research. The thesis is a continuation of my work for the specialization project done in the fall semester 2016.

This thesis examines the effects of cash flow control on optimal fleet renewal plans based on data from a major liner shipping company.

I would like to thank my supervisor Professor Kjetil Fagerholt, for great guidance and help throughout the semester. In addition, I would like to express my gratitude to my co-supervisors, Giovanni Pantuso and Xin Wang, especially with the good guidance regarding the implementation of the models. Finally, I would like to acknowledge Michal Kaut (SINTEF) for allowing the use of his scenario generation code, which has been a valuable contribution to my work.

Trondheim, June $11^{\text{th}}$

Jørgen Skålnes

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# Abstract

This thesis considers the Maritime Fleet Renewal Problem (MFRP) including the risk of insolvency. The MFRP is a strategic problem with a planning horizon of several years. The literature review shows that relatively little research has been done for this problem, and that no one has treated risk explicitly.

Two stochastic models are developed to explicitly control the cash flow in order to reduce the risk of insolvency. One of the models, the cash flow control model, maximizes profit while limiting the worst case cash flow. The other model, the Conditonal Value-at-Risk (CVaR) model, uses CVaR as a risk measure. The CVaR model maximizes profit while limiting the expected minimum cash flow of the worst scenarios.

The computational study examines the effects of controlling the worst case cash flow for 12 distinct cases and demonstrates the impact on the expected profit. The CVaR model is solved for one of these cases, demonstrating how the decision maker can set a suitable risk level. The results from both models are based on a two-stage implementation, and a three-stage implementation is tested confirming that the two-stage simplification is reasonable.

The results from the computational study show that the strategies for improving the worst case cash flow are dependent on the situation. This demonstrates how complex the MFRP is, and how valuable such models can be to adapt the correct strategy for a given situation. In addition, this study shows that for increasing cash flow limits the expected loss in profit increases slowly at first, but then increasing almost exponentially. This is an important finding giving new insight for operations research considering strategic maritime problems. These results show that in many situations it is possible to reduce the risk of cash flow insolvency, with a limited loss in expected profit. Such models will hopefully help the decision makers determine a suitable trade-off between expected profit and risk.

# Sammendrag

Denne avhandlingen vurderer det maritime flåtefornyelsesproblemet (MFFP) inkludert risikoen for insolvens. MFFP er et strategisk problem med en planleggingshorisont som strekker seg over flere år. Litteraturstudien viser at relativt lite forskning har blitt gjort på dette problemet, og ingen har behandlet risiko eksplisitt.

To stokastiske modeller er utviklet for å eksplisitt kontrollere kontantstrømmen for å redusere risikoen for insolvens. En av modellene, kontantstrømmodellen, maksimerer profitt samtidig som den begrenser kontantstrømmen i det verste scenariet. Den andre modellen, "Conditional Value-at-Risk" (CVaR) modellen, bruker CVaR som et mål på risiko. CVaR-modellen maksimerer profitt samtidig som den begrenser den forventede minste konstantstrømmen i de verste scenariene.

Beregningsstudien undersøker effekten av å kontrollere kontantstrømmen i det verste scenariet for 12 ulike instanser, og demonstrerer påvirkningen på forventet profitt. CVaR-modellen er løst for en av instansene, og viser hvordan beslutningstakeren kan finne et passende risikonivå. Resultatene fra begge modellene er basert på en to-stegs implementering, og en tre-stegs implementering er testet for å bekrefte at to-stegs forenklingen er fornuftig.

Resultatene fra beregningsstudien viser at strategiene for å forbedre kontantstrømmen i det verste scenariet er avhengig av situasjonen. Dette demonstrerer hvor komplekst MFFP er, og hvor verdifulle slike modeller kan være for å tilpasse den riktige strategien til en gitt situasjon. I tillegg viser studien at for økende kontantstrømgrense øker det forventede tapet av profitt først sakte, men så nesten eksponensielt. Dette er et viktig funn som gir ny innsikt til optimeringsfeltet innen strategiske maritime problemer. Disse resultatene viser at det i mange situasjoner er mulig å redusere risikoen for insolvens med begrensede tap i profitt. Slike modeller vil forhåpentligvis hjelpe beslutningstakere til finne en passende avveining mellom forventet profitt og risiko.

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Chapter 1

# Introduction

According to the United Nations Conference on Trade and Development (UNCTAD) (2016), over 80 % of global trade by volume was transported by sea in 2015. As Figure 1.1 shows, there has been an enormous increase in seaborne trade over the last decades following the development in Gross Domestic Product (GDP). The seaborne trade has increased from transporting 2605 millions tons in 1970 to more than 10 billion tons in 2015 (Asariotis et al., 2016). Even though the seaborne trade has been increasing steadily over the last decades, except for the financial crisis in 2008, the earnings in the shipping industry has been more volatile. Stopford (2009) writes that the shipping industry is cyclic and volatile of nature, varying between peaks and troughs. A peak is typically recognized by undersupply where the available capacity cannot cover all the demand, which results in rising freight rates, increasing orders of new ships etc. On the other hand, a trough is typically a situation where there is oversupply, which results in hard competition between shipping companies, resulting in lower freight rates leading to a tough financial situation. Stopford (2009) identified three kinds of cycles, namely long term cycles lasting around 60 years, short business cycles lasting from 3 to 12 years from peak to peak, and seasonal cycles that are variations within a year. Despite the knowledge of this cyclic nature, they have been hard to predict, leading to many decisions being made under high uncertainty.

The shipping industry normally has a long time horizon when it comes to acquisition and disposal of ships. A new ship typically has a lead time of one to four years, i.e. the time from ordering to delivery. The lifetime of a ship is usually considered to be around 30 years, which makes it clear that the investment decisions require planning ahead for a long time. Taking into account the high uncertainty of the future, this is a hard task for the shipping companies.

Lawrence (1972) classifies shipping operations into three different modes, namely *industrial*, *tramp* and *liner* shipping. In *industrial* shipping, the owner of the goods transported operates his own fleet of ships and aims at minimizing transportation cost. In *tramp* shipping the ships operate where there are products to transport, i.e. on customers' callings like a taxi service. In addition to have capacity tied to long-term contracts, they seek to increase profits by choosing optional spot cargoes. Finally, in *liner* shipping the ships sail according to a fixed public schedule, and the profit is therefore influenced by the schedule. In this thesis the data used in the computational study is based on a major liner shipping company operating in the Roll-on/Roll-off (RORO) segment. The RORO shipping segment includes the shipment of rolling equipment, e.g. cars, trucks and tractors.

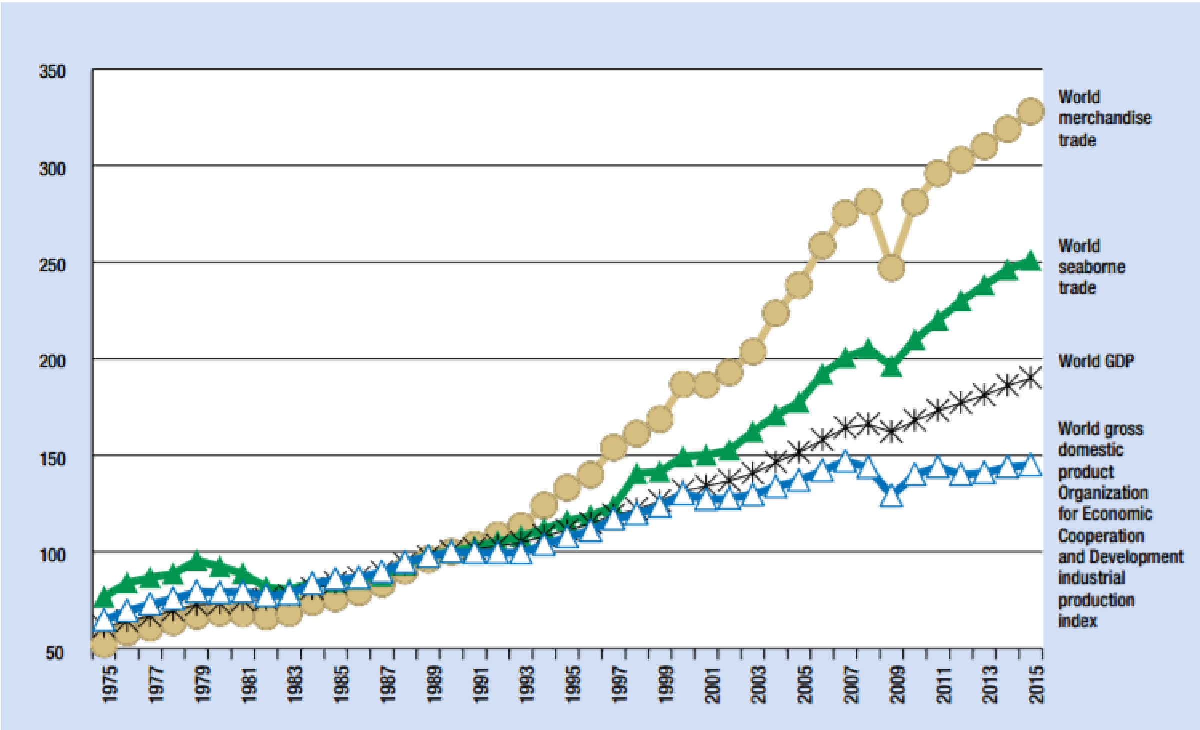



***Figure 1.1:*** *The OECD industiral production index and indieces for world merchandise trade, world seaborn trade and world GDP(1990=100). Note that indices are calculated based on GDP and merchandise trade in dollars and seaborn trade in metric tons (Asariotis et al., 2016).*

Moreover, the decisions a shipping company has to make are often categorized into three different levels, i.e. *operational*, *tactical* and *strategic* decisions. The difference between these categories is the length of the planning horizon for the decisions. *Operational* decisions are on a "day-to-day" level of planning, for instance what speed the ship should travel at. *Tactical* decisions cover problems of longer time horizon, e.g. which ships to service which trades, and fleet size and mix problems for short planning horizons. *Strategic* decisions consider planning horizons that can span up to several years, such as fleet size and mix problems with long planning horizons.

Not knowing future demand, freight rates, fuel prices, ship prices etc. makes determining the fleet size and mix for a shipping company difficult. Under high uncertainty the shipping company has to decide how many and what kind of ships to buy or sell in the second hand market, to charter in or out, to put on lay-up, to scrap and how many and what kind of new ships to order. When this fleet size and mix problem has an initial fleet and the planning horizon spans over several years the problem is often referred to as the Maritime Fleet Renewal Problem (MFRP). Using the categorization mentioned before this can be viewed as a strategic problem.

Over the last decades the MFRP received little attention, but has gotten increasing interest from researchers in recent years. For a long time there only existed deterministic models solving the MFRP, but as seen above the uncertainty is high in this problem and one might end up with poor solutions using models that do not consider this uncertainty. Recently, several models considering uncertainty have been developed and proven to give significantly better solutions.

Looking at the trends in the market today there are several signs of a trough. There is overcapacity and the freight rates are low (The Economist, 2016). In 2015 the world fleet grew by 3,5% which is the lowest growth rate since 2003, but still higher than the 2,5 % increase in demand (Asariotis et al., 2016). Looking at the dry bulk market it is clear that the shipping companies struggle. Figure 1.2 shows how the bankruptcy filings are increasing again after the financial crisis in 2008. Note that the last column only is for the first quarter of 2016. Moreover, Figure 1.3 shows how the majority of the companies in 2015 reported negative earnings before interest, taxes, depreciation and amortization (EBITDA). EBITDA is an indicator of a company's financial performance, and is often used as a proxy for the earning potential of a business (Investopedia, 2017). In addition, two thirds reported a negative operating cash flow in the same year (Hellenic Shipping News, 2016). It is reasonable to think that the situation is similar also for other shipping segments, and this situation naturally brings the discussion on the MFRP over to risk control and how risk can be dealt with in the MFRP. For this thesis the probability distribution for the uncertain parameters are assumed to be known, and risk is used to refer to the potential negative effects of the uncertainty.

A shipping company faces a wide range of risks, but this thesis focuses on the risk of insolvency, which ultimately can lead to bankruptcy. Insolvency is the state of being unable to pay the money owed on time. There are two forms of insolvency, namely cash flow insolvency and balance-sheet insolvency. Cash flow insolvency is when the company has enough assets to pay what is owed, but not enough liquid assets. This is usually solved by selling some of the illiquid assets paying the obligations later and with a penalty cost. For instance, a shipping company could be forced to sell a ship in order to cover upcoming bills. Even though this avoids immediate bankruptcy it will most likely damage the future earning potential, because the demand covered by the sold ship now has to be covered by more expensive means such as charters. Balance-sheet insolvency is when the company does not have enough assets to pay their debts, but may have enough liquid assets to pay the upcoming bills. In this case it is illegal in most countries for the company to pay these bills unless it favours all their creditors.

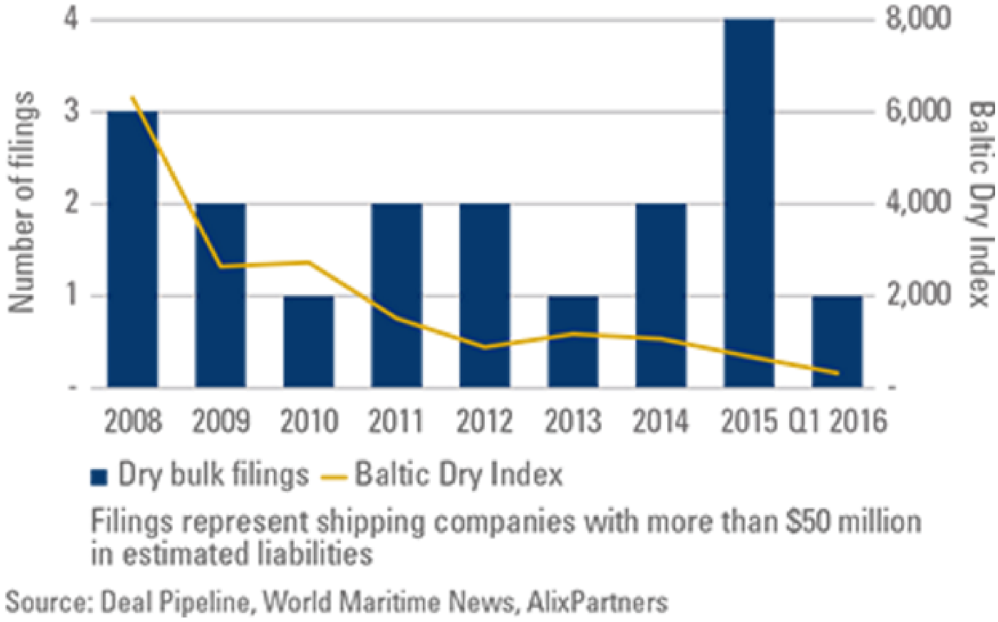



***Figure 1.2:*** *Bankruptcy filings against the Baltic Dry Index (Hellenic Shipping News, 2016).*

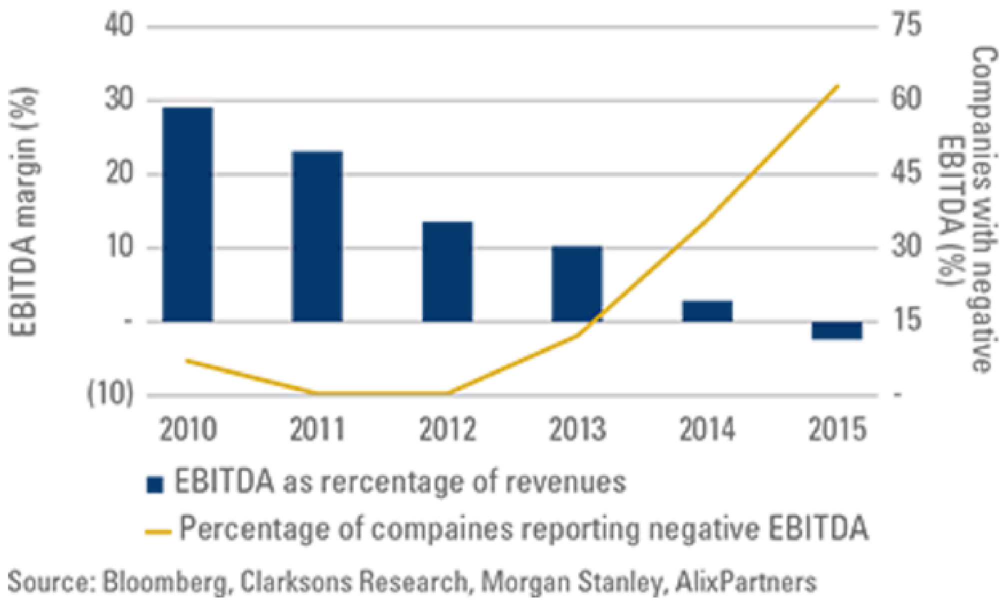



***Figure 1.3:*** *Development of EBITDA from 2010 to 2015 (Hellenic Shipping News, 2016).*

Up to now, there exist no models treating the risk of insolvency in the MFRP. Even though a solution that maximises profit has been found there is no guarantee that the solution is feasible in terms of liquidity for the company. If the company experiences a year that gives them a cash flow lower than what they are able to withstand, there is an immediate risk of insolvency even though the decisions provide a total profit at the end of the planning period.

Balance-sheet insolvency is usually a result of negative profits for a long time, and is therefore easier to anticipate than the cash flow insolvency. Cash flow insolvency is more likely to be a result from short term fluctuations in for instance demand. A sudden drop in demand in one period could lead to cash flow insolvency in this period, but not necessarily balance-sheet insolvency.

Therefore, this thesis suggests two new models explicitly controlling the cash flows. It aims at demonstrating the benefits of using these models and how they can be a valuable tool to reduce the risk of insolvency.

Chapter 2 presents relevant literature for the MFRP, relevant theory on risk measures and how to model uncertainty. In Chapter 3 a detailed description of the MFRP is presented. Chapter 4 presents the models used in this thesis and compares two alternative formulations, namely scenario and node formulation. Furthermore, in Chapter 5 the results of the computational study are presented and discussed. Finally, Chapter 6 presents the concluding remarks and possible directions for future research to pursue.

# Chapter 2

# Literature Review

Since this thesis builds directly on Skålnes (2016) it is found necessary to repeat most of the literature review from this report for the sake of completeness. In addition, further literature is added, especially regarding risk measures. This chapter starts by presenting relevant theory in Section 2.1 to better understand the MFRP and the work done in this thesis. Then the relevant literature on the MFRP is presented in Section 2.2, which is based on the survey made by Pantuso et al. (2014). They give an extensive overview of the relevant literature mainly on the Maritime Fleet Size and Mix Problem (MFSMP), but also on the MFRP. Additional literature is found using Oria and Google Scholar. Finally, theory and literature regarding risk measures are presented in Section 2.3.

## 2.1 Modelling Uncertainty

The MFRP is highly influenced by uncertainty as mentioned in Chapter 1, and therefore it is found necessary to cover some theory on how uncertainty is modelled. Two popular methods of modelling uncertainty are robust optimization and stochastic programming. Since a stochastic model is chosen for this thesis, stochastic programming is explained in more detail than robust optimization.

Very briefly explained, robust optimization uses a worst-case estimate of the uncertain parameters in a deterministic model. However, this often leads to an overly conservative solution, and Bertsimas and Sim (2003) provide a method which allow for adjustments of the robustness. In this way different levels of conservatism can be set and thus provide the decision maker with the option of comparing solutions of different degrees of robustness and conservatism. Robust optimization is often used when feasibility is required for all realizations of the uncertain parameters. In the MFRP there exist options so the solutions will always stay feasible, e.g. charters to cover peaks in demand. Having these charter options, the company will always be able to cover demand and the solution will always be feasible. Thus, stochastic programming is used in this thesis.

Stochastic programming is a topic within mathematical programming (Kall and Wallace, 1994). In other words, it is mathematical programming where some of the parameters are random variables. There are several types of objective functions in stochastic programming, but normally it aims at optimizing the expected value of the objective function given uncertain data. The uncertain parameters are often represented through scenarios. This is done to be able to discretize the distribution function. A scenario is a complete realization of the set of random variables during a time horizon. Here it is important to separate between stages and periods. A period is a unit of time in the problem, i.e. days, weeks, months etc. depending on the problem. Stages are any period where a decision is made and where new information arrives. An example of a scenario tree is given in Figure 2.1, namely a graphical representation of the different scenarios. Each node in the graph represents a state, i.e. the decision maker has a given set of information. Where the node branches, there exists uncertainty and the decision maker has to make a decision without knowing the future development. In stage 1 the uncertain parameters can take on two different realizations represented by the nodes in period 2. Here the decision maker receives more information and can make corrective decisions (recourse actions) from the previous stage. However, uncertainty still exists so he has to make a decision under uncertainty here as well, namely in stage 2. In stage 3 all parameters become certain and the decision maker can make the necessary recourse actions. Each path from the root node to the leaf node is a *scenario*, marked scenario 1, 2, 3 and 4 in Figure 2.1.

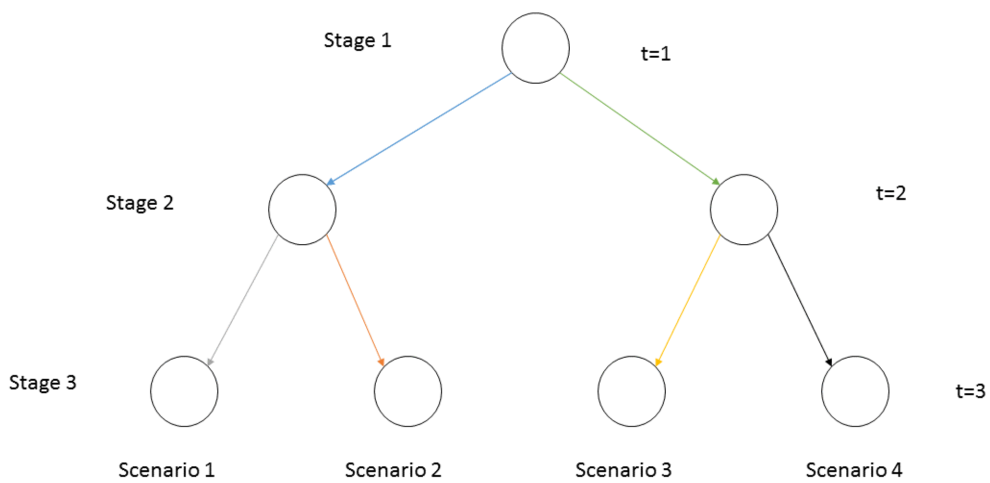



***Figure 2.1:*** *Example of a scenairo tree.*

A stochastic programming model can be formulated in different ways, depending on the problem and preferences. Sen and Higle (1999) present several ways of formulating a stochastic model. Focusing on multi-stage recourse models it is especially two formulations that is natural to consider, namely scenario formulation and node formulation. The difference between these two formulations is the way the scenario tree is represented in the model. In a scenario formulation each node is represented by a time period and a scenario, while in a node formulation the node is directly addressed in the model. This difference is illustrated in Figure 2.2. The advantage with a scenario formulation is that the model becomes more intuitive and easier to understand. However, as Figure 2.2 illustrates, it needs more variables and constraints to represent the same scenario tree. In addition, a node formulation does not require non-anticipativity constraints, i.e. constraints that make sure every decision is equal in the scenarios that shares the same history. For instance, the scenarios 1, 2 and 3 all share the same history in period 1, which means that for this period all the decisions made has to be equal for these scenarios.

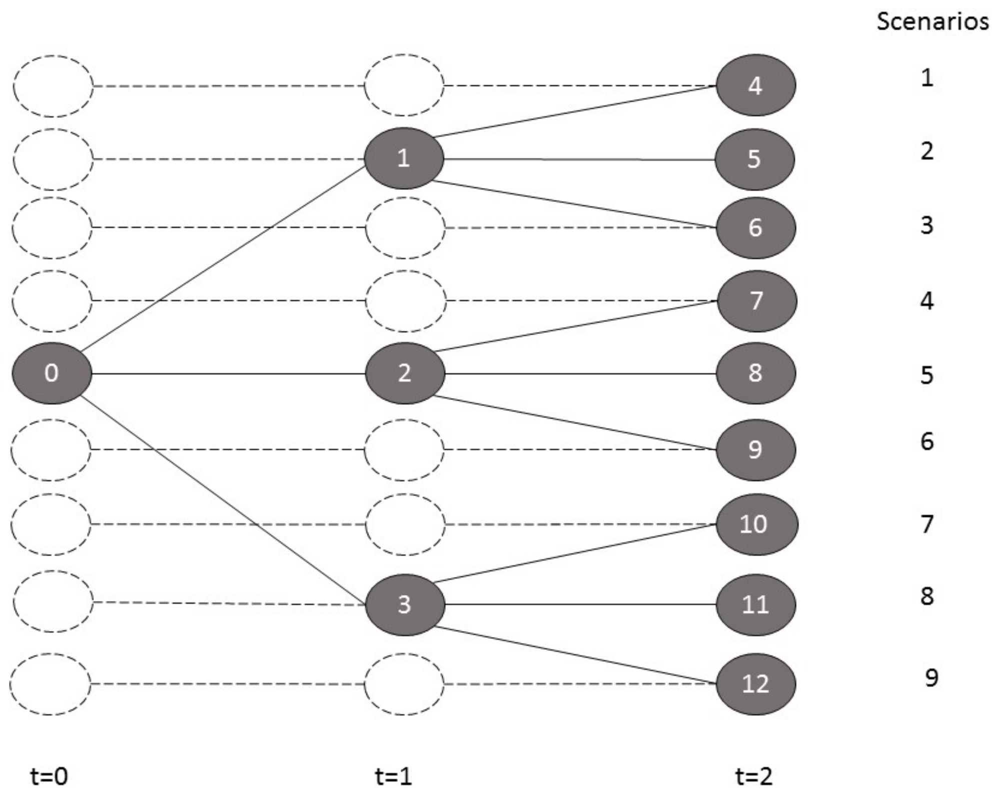



*Figure 2.2: The colored nodes represent the node formulation while all the nodes represent the scenario formulation*

There exists a wide range of scenario generation methods in order to generate the scenario tree used in the stochastic programming model. Details regarding such methods are not explained here, but the method used for this thesis is explained in Section 5.2. A desired property of the scenario tree is to represent the true probability distribution accurately, or at least provide solutions that are also good for the true distribution (Kaut and Wallace, 2003). Kaut and Wallace (2003) formulate minimal requirements that should be imposed on a scenario generation method before it can be used to solve stochastic programming models. They write that there are at least two such requirements a scenario generation method must satisfy. The first one is stability, meaning that generating several trees with the same input data should give the same optimal value of the objective function. The other requirement is that the scenario tree should not introduce any bias compared to the true solution. This means that the optimal solution for a given scenario tree should also be optimal for the original problem having the true distribution. However, testing for this is in most cases practically impossible, because it requires solving the model for the true continuous process. There exist approximations doing this, but the reader is referred to Kaut and Wallace (2003) for more details.

The stability testing is easier to perform, or at least parts of it. Kaut and Wallace (2003) define two stability tests, namely in-sample stability and out-of-sample stability. Using the notation from Kaut and Wallace (2003), let $\tilde{\varepsilon}$ be the stochastic parameters and $\breve{\varepsilon}$ be the discrete stochastic parameters. Assume $K$ scenario trees $\breve{\varepsilon}_k$ are generated for a given stochastic process. Solving the optimization problem for each tree obtaining optimal solutions $x_k^*, k = 1, ..., K$, in-sample stability can be formulated as

$$F(x_k^*; \breve{\varepsilon}_k) \approx F(x_l^*; \breve{\varepsilon}_l) \qquad k, l \in 1, ..., K,$$

where $F(x_k^*; \breve{\varepsilon}_k)$ is the objective function value for an optimal solution $x_k^*$ with scenario tree $\breve{\varepsilon}_k$. Thus, the solutions are in-sample stable if the optimal objective function value for each tree is approximately equal.

The out-of-sample stability can be formulated as

$$F(x_k^*; \tilde{\varepsilon}) \approx F(x_l^*; \tilde{\varepsilon}) \qquad k, l \in 1, ..., K,$$

where $F(x_k^*; \tilde{\varepsilon})$ is the objective function value calculated for the given stochastic process $\tilde{\varepsilon}$ (the true probability distribution) with the optimal solution from scenario tree $k$. Thus, the out-of-sample stability test requires an evaluation of the true objective function, i.e. the objective function using the true continuous distribution. The solutions are out-of-sample stable if all solutions from each scenario tree are inserted into the true objective function and result in approximately equal values. However, in many cases it is impossible to evaluate the true objective function, but generating a large scenario tree can work as an approximation of the true distribution. The size of this tree depends on the specific case.

## 2.2 Relevant literature regarding the Maritime Fleet Renewal Problem

Nicholson and Pullen (1971) study the problem of scaling down a fleet due to major technological change over a long time horizon, 10 years in their case. The objective is to determine the mix of owned and chartered ships during the planning period. Charters are used in case of premature sales of ships. The problem is solved using a heuristic to schedule the sales of ships, while a dynamic programming model is used to find the optimal number of chartered in ships. In addition, their model uses deterministic data.

Jin and Kite-Powell (2000) study a replacement problem using optimal control theory. They aim at optimizing vessel utilization and replacement schedules were the fleet is allowed to vary from period to period. Moreover, they consider a deterministic case, only adjusting the fleet by ordering new ships and scrapping old ships. An interesting finding from this study is that fleet replacement and operation are joint decisions, i.e. they influence each other. This strengthens the motivation for discussing the modelling of deployment decisions in Section 4.1.

Xie et al. (2000) study the development of a fleet over time satisfying a given demand. They use a linear programming model to solve the deployment problems and then a dynamic programming model is used to determine the development of the fleet. However, they use deterministic data, only adjusting the fleet by purchasing new ships and putting ships on lay-up. They show that their algorithm keeps the integrality property, but has the computational advantage of a linear program.

A similar algorithm is developed by Meng and Wang (2011), but they use a scenario-based dynamic programming model consisting of a number of integer linear programming formulations for each planning period. Experts provide some scenarios of the fleet size and mix, consisting of types and numbers of ships, based on their experience and available budget. Given a specific scenario for a specific period with estimated demand, the deployment plan is made by solving the integer linear programming model.

Alvarez et al. (2011) study the multi-period fleet sizing and deployment problem, looking at a wide range of options to acquire or dispose of ships. They include purchases and sales of ships, chartering in and out, putting ships on lay-up and scrapping of ships. However, they do not distinguish between ordering of new ships and ships bought in the second hand market. In order to deal with uncertainty Alvarez et al. (2011) follow the method of Bertsimas and Sim (2003) in order to transform a mixed integer problem to a robust counterpart.

Bakkehaug et al. (2014) and Pantuso et al. (2016) address uncertainty in another way. They solve the maritime fleet renewal problem using a stochastic model minimizing the cost of a shipping company, where revenues are given in negative terms. Here the uncertainty is explicitly handled through scenarios and the expected cost is minimized, showing that their models perform better than the deterministic version. The two models differ on the modelling assumptions and choices. For instance, Pantuso et al. (2016) consider the lead time on ordering new ships, while Bakkehaug et al. (2014) do not account for this. The main difference however, is that the deployment part is simplified in Bakkehaug et al. (2014).

Moreover, Patricksson et al. (2015) extend the MFRP to include regional limitations in the form of emission control areas. They use a stochastic model minimizing expected total cost, only considering the fuel price to be uncertain. They also include speed optimization, where reducing/increasing the speed can contribute to reducing/increasing the capacity of the fleet. Furthermore, they show that in a market with emission limitations their model leads to a better solution than traditional MFRP models.

Mørch et al. (2016) present a stochastic MFRP model that maximizes profit. However, they realised that the decisions made in the profit maximizing model were more aggressive than what the decision makers are comfortable with, i.e. the decision makers are risk-averse and the model does not account for this risk-aversion. Hence, they propose a different objective function, namely a rate of return measure. This turned out to give solutions more coherent with the risk preferences of the decision makers, but at the cost of expected profit. However, this approach does not treat risk explicitly, but the solutions implicitly turn out to be more risk-averse than a profit maximising model.

Arslan and Papageorgiou (2017) study the bulk ship fleet renewal problem considering uncertain demand and charter cost. They introduce a multi-stage stochastic programming model solving it in a rolling horizon fashion, only considering time charter in the here-and-now decisions and voyage charter as the recourse actions. Thus, they solve the fleet renewal problem on a tactical level as opposed to the strategic level of the papers covered so far.

To summarize, it is quite clear that the MFRP received relatively little attention over the years. However, the interest has increased the recent years resulting in several models addressing uncertain parameters of the problem. Still, no one has yet explicitly assessed the risk of insolvency in the MFRP. Even though the expected profit is maximized or the expected cost is minimized and give good results for a given planning horizon, no existing model control the cash flow in the MFRP. This results in the possibility of volatile cash flows, which in the worst case can result in liquidity problems for the company. Thus, this thesis suggests two stochastic models controlling the cash flow in order to reduce the risk of insolvency. Alvarez et al. (2011) do address this case to some extent. However, they only limit the amount available for investing in ships, not the net cash flow. Only limiting the investment budget can still result in high volatility on the cash flow even though the optimal profit is found.

## 2.3 Risk measures

To be able to assess the risk of insolvency in maritime fleet renewal there is a need to have some kind of risk measure. This section therefore presents the relevant theory on risk measures, first defining what a risk measure is before considering which risk measure is suitable to use in this thesis.

Szegö (2002) defines a risk measure to be a correspondence $\rho$ between the space $X$ of random variables and a non-negative real number $R$, i.e. the functional $\rho : X \longrightarrow R$. In addition an acceptable risk measure has to satisfy the following four properties:

1. Positive homogeneity: $\rho(\lambda x) = \lambda\rho(x)$ for all random variables $x$ and all positive real numbers $\lambda$.

2. Subadditivity: $\rho(x + y) \leq \rho(x) + \rho(y)$ for all random variables $x$ and $y$.

3. Monotonicity: $x \leq y$ implies $\rho(x) \leq \rho(y)$ for all random variables $x$ and $y$.

4. Transitional invariance: $\rho(x + \alpha r_0) = \rho(x) - \alpha$ for all random variables $x$, real numbers $\alpha$ and risk-free rates $r_0$

Any risk measure lacking these properties may lead to inconsistencies between the measured risk and the actual risk. Szegö (2002) writes that a measure often used as a risk measure, Value-at-Risk (VaR), in general does not satisfy these conditions and in particular the subadditivity. VaR of the random variable $X$ with cumulative distribution function $F_X(z) = P\{X \leq z\}$ and with confidence level $\alpha \in (0, 1)$ is defined in Equation (2.1).

$$\text{VaR}_\alpha(X) = min\{z|F_X(z) \geq \alpha\} \tag{2.1}$$

In the case where $X$ means loss and $\alpha = 0.95$, the loss will with a $95\%$ confidence not exceed VaR. For a graphical representation see Figure 2.3. Moreover, since VaR in general turns out to not satisfy the properties mentioned (Szegö, 2002), VaR is considered to be unsuitable as a risk measure for this thesis.

A risk measure that do have these properties is Conditional Value-at-Risk (CVaR), which is related to VaR. CVaR is the expected value of the distribution exceeding $\text{VaR}_\alpha$. See Figure 2.3 for a graphical representation. The general definition of CVaR for a random variable $X$ with confidence level $\alpha \in (0, 1)$ is the mean of the generalized $\alpha$-tail distribution $F_X^\alpha(z)$ shown in Equation (2.2) (Sarykalin et al., 2008).

$$\text{CVaR}_\alpha(X) = \int_{-\infty}^{\infty} z dF_X^\alpha(z), \tag{2.2}$$

where

$$F_X^\alpha(z) = \begin{cases} 0, & \text{when } z < \text{VaR}_\alpha(X), \\ \dfrac{F_X(z) - \alpha}{1 - \alpha}, & \text{when } z \geq \text{VaR}_\alpha(X) \end{cases}$$

For instance, if VaR has the confidence level $\alpha = 0.95$, there is a $5\%$ probability that the consequences of the realisation of the random variable $X$ will not be worse than VaR. Furthermore, the value of CVaR having the same confidence level is the expected mean of the $\alpha$-tail distribution, which is based on the distribution between "VaR" and "Max Consequence" marked in Figure 2.3. Relating Equation (2.2) to Figure 2.3, $z$ is the realisations of the random variable $X$ represented by the values on the $x$-axis labelled "Consequence". The $y$-axis labelled "Frequency" represents the discrete probability distribution of the random variable $X$. Recall that $F_X(z)$ is the cumulative probability distribution and is therefore the integral of the probability distribution represented in the figure. Moreover, the mean of the $\alpha$-tail distribution, CVaR, will always lie between "VaR" and "Max Consequence".

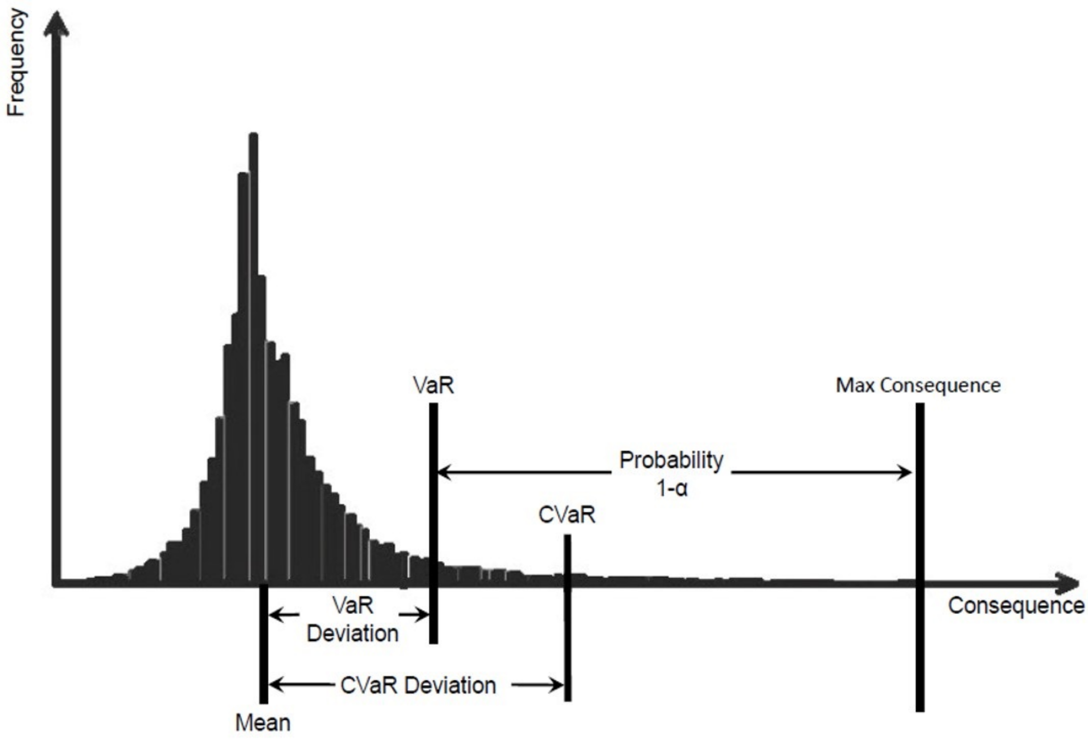



***Figure 2.3:*** *Relation between VaR and CVaR (Sarykalin et al., 2008)*

Moreover, Szegö (2002) writes that VaR is non-convex and therefore impossible to use in optimization problems. CVaR on the other hand can be replaced by a simpler auxiliary function $F_\alpha$ defined in Equation (2.3) (Sarykalin et al., 2008). Let $\alpha$ be the confidence level, $\zeta$ an auxiliary variable and let $f(x, y)$ be some random loss function which depends on the decision vector $x$ and the vector $y$ of random variables. Let $[f(x, y) - \zeta]^+$ be the positive part of $[f(x, y) - \zeta]$, meaning $[f(x, y) - \zeta]^+ = max\{0, [f(x, y) - \zeta]\}$.

$$F_\alpha(x, \zeta) = \zeta + \frac{1}{1-\alpha} E\{[f(x, y) - \zeta]^+\} \tag{2.3}$$

Uryasev and Rockafellar (2001) show that $F_\alpha(x, \zeta)$ is convex w.r.t. $\zeta$, and also convex w.r.t. $x$ if $f(x, y)$ is convex and the probability distribution does not depend on $x$. Moreover, they show that $\zeta$ is equal to VaR when $F_\alpha(x, \zeta)$ in Equation (2.3) is minimized and solved to optimality.

When $Y$ is a discrete probability space with elements $y_k$ with probabilities $p_k$, $k = 1, ..., N$ where $N$ is the number of random variables, Equation (2.3) translates into Equation (2.4).

$$F_\alpha(x, \zeta) = \zeta + \frac{1}{1-\alpha} \sum_{k=1}^{N} p_k [f(x, y_k) - \zeta]^+ \tag{2.4}$$

In optimization problems, CVaR can enter into the objective function, constraints or both. For any selection of confidence level $\alpha$ and loss tolerance $\omega$, the CVaR-constraints in the problem in Equation (2.5), can be replaced by the system of inequalities in Equations (2.6)-(2.8) (Sarykalin et al., 2008). Here, $\zeta$ is a free variable being equal to VaR when solved to optimality. Moreover, $\eta_k$ is an artificial variable needed to linearise CVaR.

$$\begin{aligned} &\min_{x \in X} \; g(x) \qquad\qquad (2.5) \\ \text{s.t.} \quad &\text{CVaR}_\alpha(x) \leq \omega \end{aligned}$$

$$f(x, y_k) - \zeta - \eta_k \leq 0, \qquad k = 1, ..., N, \tag{2.6}$$

$$\eta_k \geq 0, \tag{2.7}$$

$$\zeta + \frac{1}{1-\alpha} \sum_{k=1}^{N} p_k \eta_k \leq \omega \tag{2.8}$$

For more details regarding this replacement or the use of CVaR in the objective function see Sarykalin et al. (2008) or Uryasev and Rockafellar (2001). Szegö (2002) mentions other risk measures that also have the deired properties of a risk measure. However, CVaR is used as a risk measure in this thesis, because it can be easily linearised and CVaR has been successfully used in other stochastic programming models. For example, Gu et al. (2016) successfully used the system of inequalities presented here to linearise the CVaR constraints in their work regarding maritime bunker management having stochastic fuel prices.

Chapter 3

# Problem description

The maritime fleet renewal problem (MFRP) is a version of the maritime fleet size and mix problem (MFSMP). The MFSMP is the problem of finding the optimal size and composition of the fleet, i.e. deciding how many and what kind of ships the fleet should consist of. The term MFRP is used when the company has an initial fleet and they can change the composition of the fleet over a longer time horizon (Pantuso et al., 2014). Thus, the MFRP is the problem of deciding how many and what kind of ships the company should add or dispose of, at what time they should do so, in order to satisfy demand and maximize profit.

These decisions are made in a highly volatile and cyclic market (Stopford, 2009). Furthermore, the earning rate of a ship can increase or decrease by a factor of 50 in six months at a turning point in the shipping market (Alvarez et al., 2011). This shows that the decisions are made under high uncertainty.

The demand the company has to cover is given by the trades the company is committed to. Figure 4.1 illustrates an example of a trade between a set of pick-up ports in North-East Asia and a set of delivery ports in Europe. The trades are typically classified as contracted trades and optional trades. A contracted trade is a trade that has been agreed upon before the planning horizon begins, while an optional trade is a trade which the company can enter during the planning horizon. The demand on each trade is usually not explicitly determined before the planning horizon, but given as a fraction of the customer's production. Thus, the quantity of goods transported can vary significantly from period to period on both contracted and optional trades.

Another important decision that has to be considered in the MFRP in order to obtain a good solution is to decide how to deploy the ships that will be in the fleet during the planning period. Without a good consideration of the deployment of the ships the solution to the MFRP might not be optimal due to wrong assumptions regarding the need of ships. Note that the deployment

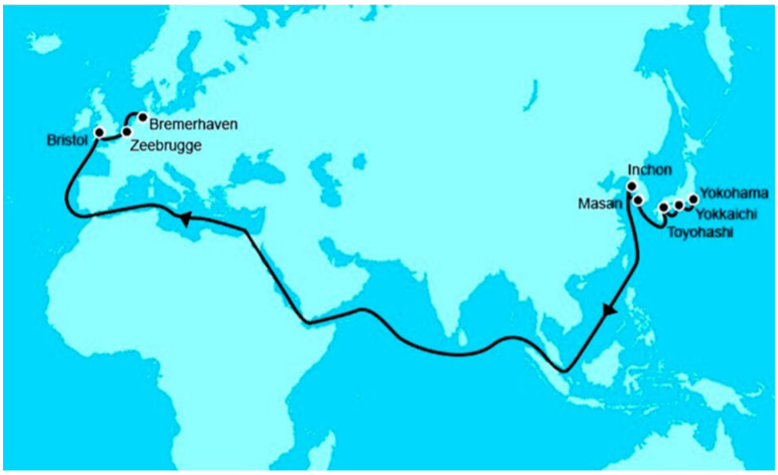



***Figure 3.1:*** *Example of a trade transporting products from North-East Asia to Europe (Pantuso et al., 2016).*

does not have to be optimal in order to obtain a good solution to the MFRP, but it has to be of such a quality that the error of estimating the number of ships needed are within a certain limit. This limit will vary from company to company and there is always a trade off between computational time and the quality of the solution.

There are several ways a company can add or remove ships from the fleet. They can order new ships, buy them in the second hand market, charter in and out ships, sell ships and scrap ships.

Ordering of new ships gives the company a possibility of tailoring their order so that the ships fulfil the company's needs to the highest degree possible. However, there exists a lead time on such an order which can typically vary from one to four years. The lead time is the time from the order is placed until the ship is delivered to the shipping company. Moreover, the company can buy the ships in the second hand market. The main advantage here is that there is not any lead time to get the ship, except for the possible transportation from one geographical area to another.

Another way to cover the demand is to charter in ships. Stopford (2009) distinguishes between three kind of charters, i.e. *voyage charter*, *time charter* and *bare-boat charter*. Under a *voyage charter*, the charterer pays the shipowner to carry a specific cargo on a specific ship at a negotiated price per tonnage cargo. This is also referred to as *space charter* in this thesis. The shipowner covers all the costs related to the shipping, i.e. capital costs, operating costs, port costs etc. Under a *time charter* the charterer, i.e. the shipping company, hires a ship, complete with crew, for a fee per day, month or year. The shipowner covers the capital costs and the fixed operating costs, but the charterer covers the remaining costs such as fuel and port costs. Under a *bare-boat charter* the shipowner hires out the ship without crew or any operational responsibility. The shipowner only covers the capital costs and the charterer covers the rest.

Capital cost is related to the amount paid for a ship and how it is financed, i.e. through debt or equity. Financing by debt will give interest costs, and financing through equity will result in an expected return on capital from investors. Operating costs are normally classified as variable or fixed operating costs. Variable operating costs are all costs related to the use of the ship, such as fuel consumption, port and canal fees. Fixed operating costs are costs which occur regardless of the use, such as wages to the crew, maintenance, insurance, repairs and administrative activities. It can be argued that wages to the crew is dependent on the use, i.e. if the ship is on lay-up or not, but assuming the ship is operating normally the wages will be fixed and do not depend on the speed the ship travels by or which and how many ports it visits. Depending on the time perspective, insurance, maintenance and repairs could be seen as variable costs, because they will depend on what cargo is transported, area of service etc., but on a day to day perspective these costs can be considered to be fixed. However, in this problem they are considered to be variable, because ships can be put on lay up.

To reduce tonnage the shipping company can charter out ships they already own. Other ways to reduce tonnage would be to dispose of ships, and here the shipping company has several options. One option is to sell ships in the second hand market. Another option is to dispose of their ships by scrapping them. This is usually done when the ship reaches the age of expected lifetime. In this case, the ship is sold to a scrap dealer that uses the metal in the ship for any desired purpose. The value of scrapping a ship will therefore usually be the value of the steel in the ship. It is also worth mentioning that the shipowner sometimes scraps the ship earlier than this when he encounters falling demand, or makes investments to prolong the lifetime when he encounters increasing demand.

Furthermore, the company has the option of lay-up. In order to save operating expenses the ship can be put on lay-up, i.e. it lays anchored somewhere with a minimum of crew, fuel consumption etc. A ship on lay-up has reduced the operating costs to a minimum, standing by to quickly be operative again if needed.

To control the risk of insolvency it is important to describe the cash flows correctly. Usually, the payments of ships purchased or ordered are assumed to be paid in a lump sum. However, Stopford (2009) writes that the payments are usually made in stages, for instance $10\%$ when the contract is signed and the rest at equally big instalments for specified milestones throughout the construction period. However, in most cases the ship is financed through loans or equity from investors. In the case where the company finances the purchase by a loan, the actual cash flow is the instalments paid to the bank, and not the instalments to the shipyard. In the cases where the ship is financed by equity it can also be viewed in the same way, namely that the company pays instalments in form of dividends to the shareholders after the purchase of the ship. This gives a more realistic description of the actual cash flows regarding the purchases and orders of ships.

To summarize, the shipping company has a given initial fleet in the beginning of the planning period to service a given number of trades. These trades can be optional and/or contracted, and they have to decide how many of the optional trades they want to service. For each trade there can be a requirement on the number of services each year, i.e. a frequency constraint. Furthermore, for every year in the planning period, in order to satisfy demand, they have to decide how many new ships to order, how many ships to buy in the second hand market, how many ships to charter in or out and how many ships to scrap, sell in the second hand market and put on lay-up.

Each ship has a capacity to carry a certain amount of a certain product, and it can be able to carry a given number of different product types. A ship has a given speed that will determine the travel times between the given trades. This together with the demand will be the basis for determining the fleet composition. This thesis proposes a cash flow control model demonstrating how a shipping company can use such a model to improve their worst case cash flow. In addition, it proposes a CVaR model to control the expected cash flow in the worst scenarios.

Chapter 4

# Mathematical models

This chapter presents two new mathematical models for the MFRP, namely a cash flow control model and a CVaR model. There are mainly two improvements made from existing models such as Mørch et al. (2016) and Pantuso et al. (2016). Firstly, the payments on ships purchased are modelled as instalments rather than lump sums. Secondly, the models includes cash flow control, to control the risk of insolvency. The cash flow control model uses hard constraints controlling the worst case cash flow, while the CVaR model control the expected cash flow in the $(1-\alpha)$ worst scenarios. From Section 2.3, recall that $\alpha$ is the confidence level having a value in the interval $(0,1)$. The models are presented using a scenario formulation, but in accordance with Bakkehaug and Eidem (2011) it is later indicated that a node formulation performs better regarding computational time and is therefore used for the implementation. Thus, the scenario formulation is solely used for readability and to ease the understanding of the model.

First, the modelling assumptions are described in Section 4.1. Section 4.2 presents the mathematical formulation for the cash flow control model and Section 4.3 presents the mathematical formulation for the CVaR model. Finally, Section 4.4 compares the scenario formulation to a node formulation justifying why a node formulation is used for the implementation.

## 4.1 Modelling assumptions

In this section the modelling assumptions are presented. Consistent with Mørch et al. (2016) the variables for orders of new ships, ships scrapped, second hand sales and purchases are assumed to be integer. The variables for ships put on lay-up, and ships chartered in or out are allowed to be fractional. This is done in order to represent the situation where a ship is put on lay-up or chartered in or out for only parts of the period.

In accordance with Pantuso et al. (2016) and Mørch et al. (2016) the deployment decisions are modelled by means of *loops*. A *loop* consists of one or more trades being serviced on one sailing, which forms a closed circle. A trade consists of two geographical regions, one origin and one destination. At the origin there can be one or more pick-up ports and at the destination there can be one or more delivery ports. See Figure 4.1 for an illustration. The demand at each pick-up port is aggregated into a total demand for the trade and delivered at the delivery ports. For a loop servicing one trade the ship will travel with the products from the origin to the destination and back to the origin in ballast.

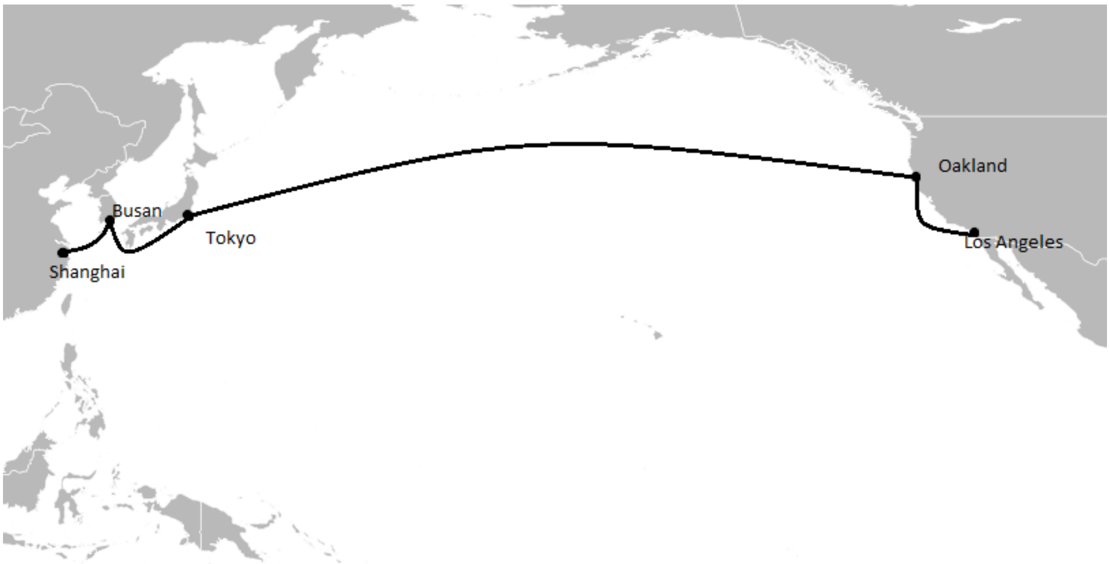



***Figure 4.1:*** *Example of a trade between Asia and North-America with three pick-up ports and two delivery ports. (Mørch, 2014)*

However, it could be the case that the destination region of one trade serves as an origin region for a second trade, i.e. after delivering the products at the destination region the ship may pick up new products at the same destination region. Note that this does not necessarily mean the same port, but the same destination region. Therefore, the ship might have to travel in ballast from a delivery port to a pick-up port in the same destination region. This second trade could be included in the loop, and it would save the ship from sailing in ballast between these trades if they were on two seperate loops. Figure 4.2 illustrates this, where trade TR2 has the same destination as the origin of trade TR3, and they are both served by one loop. The more trades that are allowed on one loop, the bigger is the potential for good routing of the ships. The cardinality, i.e. the number of trades on each loop, is chosen to be maximum two. Pantuso et al. (2016) show that larger cardinality does not improve the solution much, but results in a large increase to the computational time.

Moreover, the model does not include ballast sailing between loops which results in an optimistic estimate of the sailing time. This means that after a ship has finished a loop it can get assigned to a loop that starts in a different region than where the ship finished the previous loop, without taking into account this inter-loop sailing. This might be necessary in order to obtain

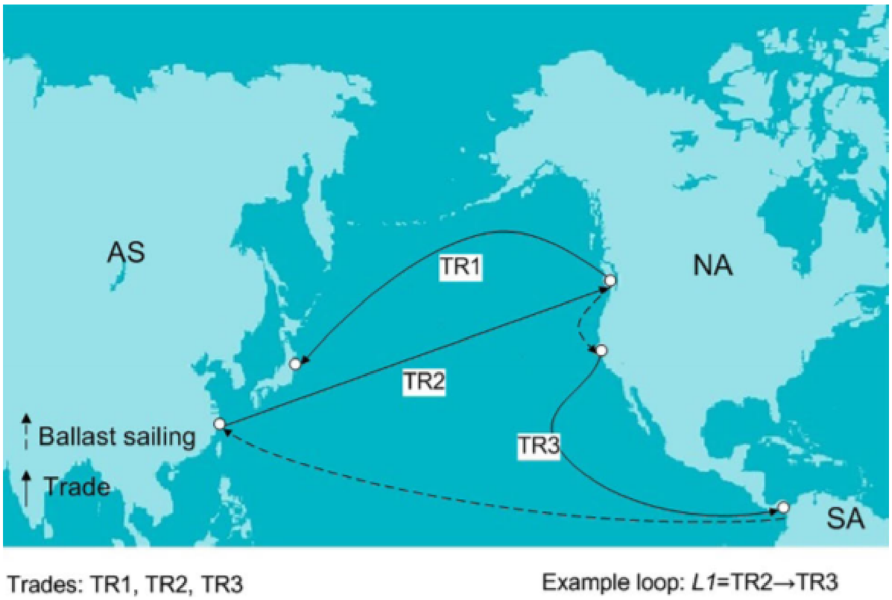



***Figure 4.2:*** *Example of three trades and a loop servicing two of them (Pantuso et al., 2016)*

a feasible solution. However, every loop has a ballast sailing from the last trade of the loop to the origin port of the first trade of the loop. If the next loop starts in another port than the origin port of the previous loop, an actual ship routing decision might want the ship to travel directly to the origin port of the next loop. In this case there is a pessimistic estimate of the sailing time. It is assumed that these optimistic and pessimistic estimates balance each other out and gives a realistic estimate of the tonnage required to satisfy demand. The variables for the number of loops can take on fractional values in order to represent a loop started in the current period and finishing in the next. In accordance with Mørch et al. (2016), once an optional trade is serviced it has to be serviced for the remaining periods in the planning horizon.

The capacities of a ship consist of the individual capacity of each product and a total capacity. The total capacity of a ship is usually smaller than the sum of the individual capacities. Normally, the amount of one product can affect the capacities of the other products, but for simplicity this characteristic is omitted here, and both the individual and total capacities are assumed to be fixed.

The payments regarding the orders of new ships and purchases of second-hand ships are modelled as $M$ instalments. This means that once a ship is ordered/purchased the company pays a fixed number of instalments in the future periods. For simplicity the number of instalments are assumed to be equal for all ships and paid once every period. Setting $M$ to $1$ represents paying a ship with a lump sum up front. Setting it equal to the lead time, it will typically represent the case described by Stopford (2009) mentioned in Chapter 3. Any other $M$ will typically represent the case where the company pays instalments to a bank or in form of dividends to investors.

Moreover, period $0$ is set to be the period before the planning period begins. This means that no operating activities are included in this period, but ships can be bought, ordered, sold and scrapped at the end of this period in order to prepare for the future. Due to this modelling choice a budget $B$ for ordering and purchasing new and second hand ships are included to allow investments in this period. This budget will typically be the profit the company made in period $0$ plus any cash reserve they have for investments. Investments in future periods are assumed to be enabled by retained profit in the respective periods.

## 4.2 Cash flow control model

This section presents the scenario formulation of the MFRP with cash flow control. First, the notation is described and then the mathematical model is presented in detail using a stochastic scenario formulation. A complete summary of this model can be found in Appendix A.

Let $\mathcal{T}$ be the set of time periods in the planning horizon,indexed by $t$, where $|\mathcal{T}|$ is the number of periods. Let $\mathcal{S}$ be the set of scenarios indexed by $s$. Furthermore, let $\mathcal{S}_{ts}^{NA}$ be the set of all scenarios that are connected to scenario $s$ in period $t$, meaning that the decisions made in scenario $s$ and period $t$ must be the same for all scenarios in this set. Let $\mathcal{K}$ be the set of products, indexed by $k$. Let $\mathcal{V}_t$ be the set of ship types existing in the market in period $t$, indexed by $v$. Note that for the rest of this thesis it is important to distinguish between the terms *ship class* and *ship type*. A *ship class* is defined as all ships with the same physical characteristics, e.g. weight, capacity, sailing speed etc., while a *ship type* is defined as all ships having the same physical characteristics and the same production year (age). In this way all the information regarding the lifetime of a ship is handled in this set, i.e. when the ship reaches the expected lifetime it leaves this set. Thus, age does not need to be handled directly in the model and makes the model easier to read and implement.

Moreover, let $\bar{T}_v^L$ be the lead time of a ship of type $v$, i.e. the time from an order is placed until the ship is delivered. Let $\mathcal{V}_t^N$ be the set of a new ship types existing in the marked in period $t$, indexed by $v$. These are the ships ordered in period $t - \bar{T}_v^L$ and delivered in period $t$. A negative number for the time of ordering means that the ship was ordered in the previous planning period and to be delivered in period $t$. $\mathcal{V}_t^{IN}$ is the set of ship types where the company pays instalments in period $t$, indexed by $v$. Let $\mathcal{N}_t$ be the set of available trades in period $t$, indexed by $i$. The subsets $\mathcal{N}_t^C$ and $\mathcal{N}_t^O$ are the sets of the contracted trades and the optional trades, respectively, indexed by $i$. Let $\mathcal{R}_t$ be the set of loops available for sailing in period $t$, indexed by $r$. The subset $\mathcal{R}_{vt}$ is the set of loops that can be sailed by ship $v$ in period $t$, indexed by $r$. Let $\mathcal{R}_{ivt}$ be the set of loops servicing trade $i$ that can be sailed by a ship of type $v$ in period $t$.

Furthermore, let $P_s$ be the probability of scenario $s$ to occur. Let $R_{vs}^{SV}$ be the sunset value of a ship of type $v$ in scenario $s$. For a period $t$ and scenario $s$, let $R_{its}^D$ be the revenue of transporting one unit of a product on trade $i$, let $R^{SE}_{vts}$ be the revenue (price) of selling a ship of type $v$ in the second hand market, let $R^{SC}_{vts}$ be the scrapping value of a ship of type $v$, let $R^{LU}_{vts}$ be the lay-up savings for one period for a ship of type $v$ and let $R^{CO}_{vts}$ be the charter out revenue of a ship of type $v$.

Moreover, for a period $t$ and scenario $s$, let $C^{CI}_{vts}$ be the charter in cost of a ship of type $v$, let $C^{OP}_{vts}$ be the operating cost for a ship of type $v$, let $C^{TR}_{vrts}$ be the cost for a ship of type $v$ to perform a loop $r$ and let $C^{SP}_{ikts}$ be the space charter cost of transporting one unit of product $k$ on trade $i$.

For a ship of type $v$ in period $t$ and scenario $s$, let $\overline{CI}_{vts}$ be the limit on the number of ships available for charter in and let $\overline{CO}_{vts}$ be the limit on the number of ships possible to charter out. Moreover, for a ship of type $v$ in period $t$ and scenario $s$, let $\overline{SE}_{vts}$ be the limit on the number of ships possible to sell in the second hand market, and let $\overline{SH}_{vts}$ be the limit on the number of ships available for purchase in the second hand market. These limits reflect the size of the second hand market.

For a period $t$ and scenario $s$, let $\overline{CI}_{ts}$ be the limit on the total number of ships that can be chartered in for one period, and let $\overline{CO}_{ts}$ be the limit on the total number of ships that can be chartered out for one period. Furthermore, for a period $t$ and a scenario $s$, let $\overline{SE}_{ts}$ be the limit on the total number of ships that can be sold in the second hand market, and let $\overline{SH}_{ts}$ be the limit on the total number of ships that can be bought in the second hand market.

Let $Q_{vk}$ be the capacity of a product $k$ on a ship of type $v$, and let $Q_v$ be the total capacity on a ship of type $v$. Let $Z_{vr}$ be the time a ship of type $v$ needs to perform a loop $r$, and let $Z_v$ be the total time available to a ship of type $v$ in one period. Moreover, let $D_{ikts}$ be the demand on trade $i$ and product $k$, in period $t$ and scenario $s$. Let $F_{it}$ be the frequency requirement on trade $i$ in period $t$, i.e. the number of times a trade has to be serviced during one period.

The number of ships of type $v$ ordered in the previous planning period, delivered at the beginning of period $t$, is denoted $Y^{NB}_{vt}$. Let $Y^{IP}_{v}$ be the initial fleet of ships of type $v$, i.e. the number of ships at the beginning of the planning period. Furthermore, let $C^{IN}_{vt'ts}$ be the instalment paid in period $t$ and scenario $s$, on a ship of type $v$ ordered in period $t'$. Let $C^{SH}_{vt'ts}$ be the instalments paid in period $t$ and scenario $s$, on a ship of type $v$ purchased in period $t'$. $C^{IN}_{vt}$ represents the instalment paid for a ship of type $v$ in period $t$ for the ships in the initial fleet, i.e. before the planning period begins. Note that this parameter is not stochastic, because the ship has already been bought and thus the instalments are already determined. Let $M$ be the number of instalments.

For a ship of type $v$ in period $t$ and scenario $s$, let $y^{SC}_{vts}$ be the number of ships scrapped, let $y^{SE}_{vts}$ and $y^{SH}_{vts}$ be the number of ships sold and bought in the second hand market, respectively. Let $y^{NB}_{vt's}$ be the number of ships of type $v$ ordered in period $t'$ and scenario $s$. For a ship of type $v$ in period $t$ and scenario $s$, let $y^{P}_{vts}$ be the number of ships in the fleet, let $l_{vts}$ be the number of ships put on lay-up, and let $h^{I}_{vts}$ and $h^{O}_{vts}$ be the number of ships chartered in and out, respectively. Moreover, let $x_{vrts}$ be the number of loop $r$ performed by a ship of type $v$ in period $t$ and scenario $s$. The amount of product $k$ delivered by space charter to trade $i$ in period $t$ and scenario $s$ is denoted $n_{ikts}$. Let $\delta_{its}$ be set to $1$, if the company chooses to service trade $i$ in period $t$ and scenario $s$, and $0$ otherwise.

Let $\bar{F}$ be the worst case cash flow allowed, i.e. no period and scenario are allowed to have a worse cash flow than this limit. Let $B$ be the budget available for ordering and purchasing ships in period $0$. Finally, to improve readability let $f^{I}_{ts}$ and $f^{O}_{ts}$ represent the expressions for cash inflow and outflow, respectively. The cash inflow $f^{I}_{n}$ consist of revenue from contracted and optional trades, revenue from scrapping ships, selling ships in the second hand market, chartering out ships and savings from putting ships on lay-up. The cash outflow $f^{O}_{n}$ consist of the instalments paid on new ships ordered and ships purchased in the second hand market, chartering in cost, space charter cost, operating cost and sailing cost. The cash inflow and outflow are summarized below:

$$
\begin{aligned}
f^{I}_{ts} = {} & \sum_{i\in\mathcal{N}^{O}_{t}}\sum_{k\in\mathcal{K}} R^{D}_{its}D_{ikts}\delta_{its} + \sum_{i\in\mathcal{N}^{C}_{t}}\sum_{k\in\mathcal{K}} R^{D}_{its}D_{ikts} \\
& + \sum_{v\in\mathcal{V}_t}(R^{CO}_{vts}h^{O}_{vts} + R^{SE}_{vts}y^{SE}_{vts} + R^{LU}_{vts}l_{vts} + R^{SC}_{vts}y^{SC}_{vts}), && t\in\mathcal{T}\setminus\{0\}, s\in\mathcal{S} \\
f^{O}_{ts} = {} & \sum_{v\in\mathcal{V}^{IN}_{t}} C^{IN}_{vts}Y^{IP}_{v} + \sum_{t-M\le t'\le t}\ \sum_{v\in\mathcal{V}^{IN}_{t}}(C^{IN}_{vt'ts}y^{NB}_{vt's} + C^{SH}_{vt'ts}y^{SH}_{vt's}) \\
& + \sum_{i\in\mathcal{N}^{C}_{t}}\sum_{k\in\mathcal{K}} C^{SP}_{ikts}n_{ikts} + \sum_{v\in\mathcal{V}_t}(C^{OP}_{vts}y^{P}_{vts} + C^{CI}_{vts}h^{I}_{vts} + \sum_{r\in\mathcal{R}_{vt}} C^{TR}_{vrts}x_{vrts}), && t\in\mathcal{T}\setminus\{0\}, s\in\mathcal{S}
\end{aligned}
$$

The mathematical model is presented and described in detail below.

**Objective function**

$$max\ z = \sum_{s \in \mathcal{S}} P_s \Big( \sum_{t \in \mathcal{T}, t>0} \big( \sum_{i \in \mathcal{N}_t^O} \sum_{k \in \mathcal{K}} R_{its}^D D_{ikts} \delta_{its} \tag{4.1a}$$

$$+ \sum_{i \in \mathcal{N}_t^C} \sum_{k \in \mathcal{K}} (R_{its}^D D_{ikts} - C_{ikts}^{SP} n_{ikts}) \tag{4.1b}$$

$$- \sum_{v \in \mathcal{V}_t} (C_{vts}^{OP} y_{vts}^P + C_{vts}^{CI} h_{vts}^I - R_{vts}^{CO} h_{vts}^O \tag{4.1c}$$

$$+ \sum_{r \in \mathcal{R}_{vt}} C_{vrts}^{TR} x_{vrts} - R_{vts}^{LU} l_{vts}) \big) \tag{4.1d}$$

$$- \sum_{t \leq M-1} \sum_{v \in \mathcal{V}_t^{IN}} C_{vt}^{IN} Y_v^{IP} \tag{4.1e}$$

$$- \sum_{t \in \mathcal{T}} \sum_{t-M \leq t' \leq t} \sum_{v \in \mathcal{V}_t^{IN}} (C_{vt'ts}^{IN} y_{vt's}^{NB} + C_{vt'ts}^{SH} y_{vt's}^{SH}) \tag{4.1f}$$

$$+ \sum_{v \in \mathcal{V}_{\bar{T}}} (R_{vs}^{SV} y_{v\bar{T}s}^P - \sum_{t=\bar{T}}^{t'+M} \sum_{t' \in \mathcal{T}} C_{vt'ts}^{IN} y_{vt's}^{NB} + C_{vt'ts}^{SH} y_{vt's}^{SH}) \tag{4.1g}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{v \in \mathcal{V}_t} (R_{vts}^{SC} y_{vts}^{SC} + R_{vts}^{SE} y_{vts}^{SE}) \Big) \tag{4.1h}$$

The objective function maximizes profit using a stochastic scenario formulation. The term (4.1a) represents the revenue from optional trades. The term (4.1b) gives the revenue from contracted trades minus the space charter cost. Moreover, the term (4.1c) represents the operating cost, charter in cost and charter out revenue. The sailing costs minus the lay-up revenues are given in term (4.1d). The term (4.1e) sums up the instalments that still has to be paid on ships in the initial fleet. Next, term (4.1f) sums up the instalments that has to be paid in the current planning period on the new and second hand ships ordered and purchased in the current planning period. The term (4.1g) gives the sunset value minus the sum of the instalments that has to be paid after the planning period on new and second hand ships ordered and purchased in the current planning period. Finally, term (4.1h) represents the revenue from scrapping ships and selling ships in the second hand market.

**Demand constraints**

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_{vk} x_{vrts} + n_{ikts} \geq D_{ikts}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^C, k \in \mathcal{K}, s \in \mathcal{S}, \tag{4.2}$$

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_{vk} x_{vrts} \geq D_{ikts} \delta_{its}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^O, k \in \mathcal{K}, s \in \mathcal{S}, \tag{4.3}$$

The demand constraints make sure the demand is satisfied for all products $k$ on every trade $i$, in all periods $t$ and scenarios $s$. Note however, that $t = 0$ is before the planning period begins, and that from $t = 1$ the operating decisions has to be taken. Constraints (4.2) ensure that the demand is satisfied for all contracted trades $i$, and constraints (4.3) make sure that the demand is satisfied for all optional trades $i$. The number of loops required to satisfy the demand is determined in the time constraints (4.8).

**Capacity constraints**

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_v x_{vrts} + \sum_{k \in \mathcal{K}} n_{ikts} \geq \sum_{k \in \mathcal{K}} D_{ikts}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^C, s \in \mathcal{S}, \quad (4.4)$$

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_v x_{vrts} \geq \sum_{k \in \mathcal{K}} D_{ikts} \delta_{its}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^O, s \in \mathcal{S}, \quad (4.5)$$

The capacity constraints make sure that the total capacity is sufficient to cover the demand for all trades $i$, in all periods $t$ (except for $t = 0$) and scenarios $s$. Constraints (4.4) make sure that the total capacity, i.e. the number of loops performed by the owned fleet and the chartered in ships plus the amount of goods transported by space charters, is sufficient to satisfy demand on all contracted trades $i$. Constraints (4.5) make sure that the total capacity is sufficient to cover demand for all optional trades $i$ serviced by the company. Note that space charter is not available to cover the demand on optional trades.

**Frequency constraints**

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} x_{vrts} \geq F_{it}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^C, s \in \mathcal{S}, \quad (4.6)$$

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} x_{vrts} \geq F_{it} \delta_{its}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^O, s \in \mathcal{S}, \quad (4.7)$$

The frequency constraints make sure that the frequency requirements are fulfilled, i.e. the given trade is serviced at least as many times as required for all trades $i$ in each period $t$ and scenario $s$. Constraints (4.6) ensure that the frequency requirement on the contracted trades are fulfilled, while constraints (4.7) make sure they are fulfilled for the optional trades.

**Time constraints**

$$\sum_{r \in \mathcal{R}_{vt}} Z_{rv} x_{vrts} \leq Z_v (y_{vts}^P + h_{vts}^I - h_{vts}^O - l_{vts}), \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \quad (4.8)$$

The time constraints (4.8) make sure there are enough ships available to service the number of loops required to satisfy demand. The ships available are the ships owned by the company plus the ships chartered in minus the ships chartered out minus the ships put on lay-up.

#### Optional trades constraints

$$\delta_{its} \leq \delta_{i,t+1,s} \qquad t \in \mathcal{T} \setminus \{0, \bar{T}\}, i \in \mathcal{N}_t^O, s \in \mathcal{S}, \qquad (4.9)$$

The optional trades constraints (4.9) ensure that when the company choose to service an optional trade it is serviced for the rest of the planning period.

#### Pool constraints

$$y^P_{v0s} = Y^{IP}_v \qquad v \in \mathcal{V}_0, s \in \mathcal{S}, \qquad (4.10)$$

$$y^P_{vts} = y^P_{v,t-1,s} - y^{SC}_{v,t-1,s} + y^{SH}_{v,t-1,s} - y^{SE}_{v,t-1,s} \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t \setminus \mathcal{V}^N_t, s \in \mathcal{S}, \qquad (4.11)$$

$$y^P_{vts} = y^{NB}_{v,t-\bar{T}_v{}^L,s} \qquad t \in \mathcal{T} : t \geq \bar{T}_v{}^L, v \in \mathcal{V}^N_t, s \in \mathcal{S}, \qquad (4.12)$$

$$y^P_{vts} = Y^{NB}_{vt} \qquad t \in \mathcal{T} : t < \bar{T}_v{}^L, v \in \mathcal{V}^N_t, s \in \mathcal{S}, \qquad (4.13)$$

$$y^P_{vts} \geq l_{vts} - h^I_{vts} + h^O_{vts} \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \qquad (4.14)$$

$$y^P_{vts} = y^{SC}_{vts} \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t \setminus \mathcal{V}_{t+1}, s \in \mathcal{S}, \qquad (4.15)$$

The pool constraints ensure that the pool variables $y^P_{vts}$, the scrapping variables $yvts^{SC}$ and the new buildings variable $y^{NB}_{vts}$ take the correct values. The pool variables are dependent on decisions made regarding ordering and scrapping in the previous periods. Constraints (4.10) make sure that the pool of ships in $t = 0$ is equal to the initial fleet. Constraints (4.11) represent the pool balance which makes sure that the pool of ships depend on the pool in the previous period and the decisions made in that period. Note that the scrapping of ships happen at the end of the period. Constraints (4.12) ensure that the ships ordered in $t - \bar{T}^l_v$ become part of the pool at time of delivery $t$. Constraints (4.13) make sure that the ships ordered prior to the planning period become part of the pool at time of delivery $t$. Constraints (4.14) ensure that the ships put on lay up are less than the number of ships in the pool plus the ships chartered in minus the ships chartered out. Finally, constraints (4.15) make sure that the ships that reaches their lifetime in period $t + 1$ is scrapped at the end of period $t$.

**Charters and second hand constraints**

$$y_{vts}^{SH} \leq \overline{SH}_{vts}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.16}$$

$$y_{vts}^{SE} \leq \overline{SE}_{vts}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.17}$$

$$h_{vts}^{I} \leq \overline{CI}_{vts}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.18}$$

$$h_{vts}^{O} \leq \overline{CO}_{vts}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.19}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}_t^N} y_{vts}^{SH} \leq \overline{SH}_{ts}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, s \in \mathcal{S}, \tag{4.20}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}_t^N} y_{vts}^{SE} \leq \overline{SE}_{ts}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, s \in \mathcal{S}, \tag{4.21}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}_t^N} h_{vts}^{I} \leq \overline{CI}_{ts}, \qquad t \in \mathcal{T} \setminus \{0\}, s \in \mathcal{S}, \tag{4.22}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}_t^N} h_{vts}^{O} \leq \overline{CO}_{ts}, \qquad t \in \mathcal{T} \setminus \{0\}, s \in \mathcal{S}, \tag{4.23}$$

The charters and second hand constraints make sure that the charter and second hand market decisions do not exceed limitations in the market, i.e. how many ships are available for a given ship type $v$ at a given period $t$ and scenario $s$. For every ship of type $v$ in period $t$ and scenario $s$, constraints (4.16)-(4.19) limit the numbers of ships available for buying, selling, chartering in and out respectively. Furthermore, for every period $t$ and scenario $s$, constraints (4.20)-(4.23) limit the total number of ships that are available for buying, selling, chartering in and out, respectively.

**Cash flow constraints**

Skålnes (2016) used a ratio between the cash inflow and cash outflow to control the cash flow. However, in this thesis the scenarios are generated with a higher degree of uncertainty resulting in a higher spread between the data. This could result in a higher variance of the cash flow leading to undesirable solutions.

This is best shown through an example. If, for instance, the cash inflow and outflow in one period and scenario are $10$ and $20$ respectively, and $150$ and $200$ in another, the ratios are $0.5$ and $0.75$ respectively. Increasing the lowest allowed ratio to $0.6$ could lead to both the mentioned scenarios to have a ratio of $0.6$ with the following cash flows as an example: $12$ and $20$ for the low case and $120$ and $200$ for the high case. This means that the $2$ units improvement of the cash flow in the low case come at a cost of $-30$ units in the high case. This results in a worse cash flow for the company in absolute terms which is a solution undesirable to them. Therefore, this thesis will use an absolute measure for the cash flow to avoid this problem. Constraints (4.24) and (4.25) are the cash flow constraints using an absolute measure for the cash flow.

$$\sum_{v \in \mathcal{V}_0} (R^{SE}_{v0s} y^{SE}_{v0s} + R^{SC}_{v0s} y^{SC}_{v0s}) - \sum_{v \in \mathcal{V}^{IN}_0} (C^{IN}_{v00s} y^{NB}_{v0s} + C^{SH}_{v00s} y^{SH}_{v0s}) + B \geq \bar{F}, \qquad s \in \mathcal{S}, \quad (4.24)$$

$$f^I_{ts} - f^O_{ts} \geq \bar{F}, \qquad t \in \mathcal{T} \setminus \{0\}, s \in \mathcal{S}, \quad (4.25)$$

Constraints (4.24) make sure that the cash flow in period $0$ plus the budget for ordering and purchasing ships, stay above a given limit $\bar{F}$. Note that the operating revenues and expenses are not included here, because period $0$ is before the planning period begins. Moreover, constraints (4.25) make sure that the cash flow in every period $t$ and scenario $s$ in the remaining periods stay above the given cash flow limit $\bar{F}$. For instance, if $\bar{F}$ is set to $10$ units the cash inflow has to be $10$ units greater than the cash outflow at every period $t$ and scenario $s$.

**Non-anticipativity constraints**

$$y^{SC}_{vts} = y^{SC}_{vt\bar{s}}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.26)$$

$$y^{NB}_{vts} = y^{NB}_{vt\bar{s}}, \qquad t \in \mathcal{T} : t \leq \bar{T} - \bar{T}_v^L, v \in \mathcal{V}^N_t, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.27)$$

$$y^{SH}_{vts} = y^{SH}_{vt\bar{s}}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.28)$$

$$y^{SE}_{vts} = y^{SE}_{vt\bar{s}}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.29)$$

$$h^I_{vts} = h^I_{vt\bar{s}}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.30)$$

$$h^O_{vts} = h^I_{vt\bar{s}}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.31)$$

$$l_{vts} = l_{it\bar{s}}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.32)$$

$$x_{vrts} = x_{vrt\bar{s}}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, r \in \mathcal{R}_{vt}, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.33)$$

$$n_{ikts} = n_{ikt\bar{s}}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^C_t, k \in \mathcal{K}, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.34)$$

$$\delta_{its} = \delta_{it\bar{s}}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^O_t, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \quad (4.35)$$

The non-anticipativity constraints (4.26)-(4.35) make sure that future information is not used at a point in time when it is not yet available. This means that in a period $t$, the same decisions have to be made in scenario $s$ and all the scenarios which share the same history, represented by the set $S^{NA}_{ts}$. This is illustrated in Figure 2.2 in Section 2.1. In Figure 2.2 all the scenarios in period $0$ share the same history and therefore the same decisions have to be made in every scenario in this period.

#### Convexity and integer constraints

$$y_{vts}^{NB} \in \mathbb{Z}^+, \qquad t \in \mathcal{T} : t \leq \bar{T} - \bar{T}_v^L, v \in \mathcal{V}_{t+T^L}^N, s \in \mathcal{S}, \tag{4.36}$$
$$y_{vts}^{SC} \in \mathbb{Z}^+, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.37}$$
$$y_{vts}^{SH} \in \mathbb{Z}^+, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}\ v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.38}$$
$$y_{vts}^{SE} \in \mathbb{Z}^+, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}\ v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.39}$$
$$y_{vts}^{P} \in \mathbb{R}^+, \qquad t \in \mathcal{T}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.40}$$
$$h_{vts}^{I} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.41}$$
$$h_{vts}^{O} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.42}$$
$$l_{vts} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{4.43}$$
$$x_{vrts} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, r \in \mathcal{R}_{vt}, s \in \mathcal{S}, \tag{4.44}$$
$$n_{ikts} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^C, k \in \mathcal{K}, s \in \mathcal{S}, \tag{4.45}$$
$$\delta_{its} \in \{0,1\}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^O, s \in \mathcal{S} \tag{4.46}$$

The convexity and integer constraints ensure that the variables are of the correct type and take on values according to this. Constraints (4.36)-(4.39) make sure that the variables for new buildings, scrapping, buying and selling in the second hand market only can take on integer and non-negative values. Moreover, constraints (4.40)-(4.45) ensure that the variables for the pool of ships, charters, ships on lay up, number of loops and quantity of products carried by space charters only take on real and non-negative values. Note that the variables for the pool of ships, $y_{vts}^P$, implicitly becomes an integer from the pool constraints and therefore do not need to have an explicit integer requirement. Furthermore, note that in practical terms the variables in constraints (4.40)-(4.45) are integer, but the possible fractional value indicate the situation where for instance a loop is stared in one period and ended in another, or where a ship only is chartered in for parts of the period. Finally, constraints (4.46) restrict the optional trade variables to be binary.

To summarize, this model maximizes expected profit returning the most robust solution for a given cash flow limit, meaning that the cash flows in all periods and scenarios are required to be better than the given limit. Note that even though uncertainty is modelled differently from stochastic programming to robust optimization the intention behind using robust optimization is usually to ensure feasibility. This model can ensure feasibility in all scenarios in terms of avoiding insolvency, assuming there is a room for improving the cash flow. Thus, this model has some of the benefits from robust optimization within a stochastic programming framework.

## 4.3 Conditional Value-at-Risk model

This section presents the Conditional Value-at-Risk (CVaR) model. It is based on the cash flow control model presented in Section 4.2, replacing the cash flow constraints with CVaR constraints. First, some additional notation is required. Let $\mathcal{T}^F$ be the set of periods in the first stage, i.e. when the parameters are deterministic. Then, let $\mathcal{T}^S$ be the set of periods under uncertainty, i.e. all periods after the first stage.

Using the same notation as in Section 2.3, let $\alpha$ be the confidence level, and let $\zeta$ and $\eta_{ts}$ be the artificial variables used in the CVaR constraints. The artificial variable $\zeta$ is not scenario dependent, and in an optimal solution it will represent the VaR value. The artificial variables $\eta_{ts}$ are defined for each period $t$ and scenario $s$ making these variables scenario dependent. They will represent the negative margin that the actual cash flow is short of VaR. This means they represent the cash flows exceeding VaR minus the VaR value. This becomes more clear once the constraints are presented. Finally, let $\bar{F}_\alpha$ be the minimum allowed expected cash flow under confidence level $\alpha$.

Since CVaR is a probabilistic measure the CVaR constraints are only imposed in the uncertain periods. For the first stage periods the cash flow constraints from the cash flow control model still applies. For completeness they are presented here and denoted “hard cash flow constraints”. Having defined the necessary notation, the constraints replacing the cash flow constraints are presented:

**Hard cash flow constraints**

$$\sum_{v \in \mathcal{V}_0} (R^{SE}_{v0s} y^{SE}_{v0s} + R^{SC}_{v0s} y^{SC}_{v0s}) - \sum_{v \in \mathcal{V}_0^{IN}} (C^{IN}_{v00s} y^{NB}_{v0s} + C^{SH}_{v00s} y^{SH}_{v0s}) + B \geq \bar{F}_\alpha, \qquad s \in \mathcal{S}, \quad (4.47)$$

$$f^I_{ts} - f^O_{ts} \geq \bar{F}_\alpha, \qquad t \in \mathcal{T}^F \setminus \{0\}, s \in \mathcal{S}, \quad (4.48)$$

Note that constraints (4.47) and (4.48) are identical to the cash flow constraints in Section 4.2, except for the minimum allowed expected cash flow $\bar{F}_\alpha$ and that they only are enforced in the first stage. However, the deterministic nature of the first stage makes $\bar{F}_\alpha$ equal to $\bar{F}$ for these constraints.

**CVaR constraints**

$$\zeta + \frac{1}{1-\alpha}\sum_{s\in\mathcal{S}} P_s \eta_{ts} \geq \bar{F}_\alpha, \qquad t \in \mathcal{T}^S, \tag{4.49}$$

$$\eta_{ts} \leq f^I_{ts} - f^O_{ts} - \zeta, \qquad t \in \mathcal{T}^S, s \in \mathcal{S}, \tag{4.50}$$

$$\eta_{ts} = \eta_{t\bar{s}}, \qquad t \in \mathcal{T}^S, s \in \mathcal{S}, \bar{s} \in \mathcal{S}^{NA}_{ts}, \tag{4.51}$$

$$\eta_{ts} \in \mathbb{R}^-, \qquad t \in \mathcal{T}^S, s \in \mathcal{S} \tag{4.52}$$

Constraints (4.49), (4.50) and (4.52) are based on the system of inequalities (2.6)-(2.8) presented in Section 2.3. In Section 2.3 the CVaR constraints are defined using a loss function, but in this case the CVaR constraints aim at limiting the cash flow. This requires some modifications to the system of inequalities in Section 2.3, namely multiplying the artificial variables by $-1$, in order to express the CVaR constraints in terms of cash flows. The artificial variables $\eta_{ts}$ are now defined for only non-positive values instead of non-negative. Notice in constraints (4.50) that when the cash flow in one period and scenario is lower than $\zeta$, i.e. the cash flow is short of VaR, the artificial variable $\eta_{ts}$ becomes negative and is included in constraints (4.49). Constraints (4.51) are non-anticipativity constraints for the artificial variables $\eta_{ts}$.

Furthermore, it is worth noticing that choosing a confidence level so high that the $(1-\alpha)*100\%$ worst scenarios only consist of one scenario, the CVaR model translates to the cash flow control model. Thus, the cash flow control model presented in Section 4.2 is a special case of the CVaR model when the confidence level is sufficiently high. For instance, having $100$ scenarios and a confidence level of $0.99$ the expected cash flow of the $(1-0.99)*100\%$ worst scenarios simplifies to the cash flow of the worst scenario, which is the same as the cash flow control model.

Even though this relation exists the models are presented separately for two reasons. The cash flow constraints are easier to comprehend than the CVaR constraints, and from a manager's point of view it is easier to use a model he or she understands. Secondly, when using the cash flow control model there is no need to adjust the confidence level every time the number of scenarios change in order to get the most robust solution. Robust in this context simply refers to the solution where the cash flow in all scenarios stays above the cash flow limit, and is not to be confused with robust optimization.

## 4.4 Comparing the scenario formulation to a node formulation

This section compares the scenario formulation in the cash flow control model to a node formulation, justifying why a node formulation is chosen for the implementation. This comparison focuses mainly on the different number of variables and constraints. The most apparent difference is that the node formulation does not require any non-anticiapativity constraints which immediately reduces the number of constraints compared to the scenario formulation. For simplicity both the cash flow constraints and the non-anticipativity constraints are left out from the expressions and calculations in this section.

To convert the scenario formulation to a node formulation some new notation is needed. The symbols for the sets, parameters and variables are the same as for the scenario formulation, except for the indexes. The node formulation use index $n$ instead of $ts$, so for example $\delta_{its}$ becomes $\delta_n$. However, some additional notation is needed. Let $\mathcal{L}$ be the set of nodes, where the subset $\mathcal{L}_t$ is the set of nodes in period $t$. Let $a(n, t')$ be the ancestor node of node $n$ in the scenario tree in period $t'$, with $a(n, t-1)$ written as $a(n)$. Note that there are no need for non-anticipativity constraints using this formulation. The mathematical formulation is quite similar to the scenario formulation, but a complete presentation of the node formulation can be found in Appendix B.

Starting with the scenario formulation the number of variables $NV^S$ can be expressed as:

$$\begin{aligned} NV^S =&|\mathcal{S}|\big( \sum_{t\in\mathcal{T}:t\le\bar{T}-\bar{T}^L_v} |\mathcal{V}^N_t| + \sum_{t\in\mathcal{T}\setminus\{0\}} (3|\mathcal{V}_t| + |\mathcal{N}^C_t||\mathcal{K}| + |\mathcal{N}^O_t|) + \sum_{t\in\mathcal{T}}|\mathcal{V}_t| \\ &+ \sum_{t\in\mathcal{T}\setminus\{\bar{T}\}} 3|\mathcal{V}_t| + \sum_{t\in\mathcal{T}\setminus\{0\}}\sum_{v\in\mathcal{V}_t}|\mathcal{R}_{vt}|\big) \end{aligned} \tag{4.53}$$

Simplifying having constant size of $\mathcal{V}^N_t, \mathcal{V}_t, \mathcal{R}_{vt}, \mathcal{N}^C_t$ and $\mathcal{N}^O_t$ this expression simplifies to:

$$\begin{aligned} NV^S =&|\mathcal{S}|\big((|\mathcal{T}| - \bar{T}^L_v)|\mathcal{V}^N_t| \\ &+(|\mathcal{T}|-1)(6|\mathcal{V}_t| + |\mathcal{R}_{vt}||\mathcal{V}_t| + |\mathcal{N}^C_t||\mathcal{K}| + |\mathcal{N}^O_t|) + |\mathcal{T}||\mathcal{V}_t|\big) \end{aligned} \tag{4.54}$$

For the node formulation the number of variables $NV^N$ can be expressed as:

$$
\begin{aligned}
NV^N = &\sum_{t \in \mathcal{T}: t \leq \bar{T} - \bar{T}_v^L} |\mathcal{L}_t||\mathcal{V}_t^N| + \sum_{t \in \mathcal{T} \setminus \{0\}} |\mathcal{L}_t|(3|\mathcal{V}_t| + |\mathcal{N}_t^C||\mathcal{K}| + |\mathcal{N}_t^O|) \\
&+ \sum_{t \in \mathcal{T}} |\mathcal{L}_t||\mathcal{V}_t| + \sum_{t \in \mathcal{T} \setminus \{\bar{T}\}} 3|\mathcal{L}_t||\mathcal{V}_t| + \sum_{t \in \mathcal{T} \setminus \{0\}} \sum_{v \in \mathcal{V}_t} |\mathcal{L}_t||\mathcal{R}_{vt}| \qquad (4.55)
\end{aligned}
$$

For at two-stage scenario tree where the nodes branch in period $b$, i.e. there are one node in period $b - 1$ and several nodes in period $b$, Expression (4.55) simplifies to the following, assuming constant size of $(\mathcal{L}_t, t \in \mathcal{T} : t \geq b)$, $\mathcal{V}_t^N, \mathcal{V}_t, \mathcal{R}_{vt}, \mathcal{N}_t^C$ and $\mathcal{N}_t^O$:

$$
\begin{aligned}
NV^N = &b(|\mathcal{V}_t^N| + 4|\mathcal{V}_t|) + (b-1)(3|\mathcal{V}_t| + |\mathcal{R}_{vt}||\mathcal{V}_t| + |\mathcal{N}_t^C||\mathcal{K}| + |\mathcal{N}_t^O|) + |\mathcal{L}_t|\big((|\mathcal{T}| - \bar{T}_v^L - b)|\mathcal{V}_t^N| \\
&+ (|\mathcal{T}| - b)(4|\mathcal{V}_t| + |\mathcal{R}_{vt}||\mathcal{V}_t| + |\mathcal{N}_t^C||\mathcal{K}| + |\mathcal{N}_t^O|) + 3(|\mathcal{T}| - b - 1)|\mathcal{V}_t|\big) \qquad (4.56)
\end{aligned}
$$

Ignoring non-anticipativity and convexity constraints the number of constraints $NC^S$ for the scenario formulation can be expressed as:

$$
\begin{aligned}
NC^S = &|\mathcal{S}|\Big( \sum_{t \in \mathcal{T} \setminus \{0\}} (|\mathcal{N}_t^C||\mathcal{K}| + |\mathcal{N}_t^O||\mathcal{K}| + 2|\mathcal{N}_t^C| + 2|\mathcal{N}_t^O| + |\mathcal{V}_t \setminus \mathcal{V}_t^N| + 3|\mathcal{V}_t| + 2) \\
&+ \sum_{t \in \mathcal{T} \setminus \{0, \bar{T}\}} |\mathcal{N}_t^O| + \sum_{t \in \mathcal{T}} (|\mathcal{V}_t^N| + |\mathcal{V}_t|) + \sum_{t \in \mathcal{T} \setminus \{\bar{T}\}} (2|\mathcal{V}_t| + 2 + |\mathcal{V}_t \setminus \mathcal{V}_{t+1}|)\Big) \qquad (4.57)
\end{aligned}
$$

Let $\mathcal{V}_t^N, \mathcal{V}_t, \mathcal{N}_t^C$ and $\mathcal{N}_t^O$ be constant. Then Expression 4.57 can be simplified to:

$$
\begin{aligned}
NC^S = &|\mathcal{S}|\big((|\mathcal{T}| - 1)(|\mathcal{N}_t^C||\mathcal{K}| + |\mathcal{N}_t^O||\mathcal{K}| + 2|\mathcal{N}_t^C| + 2|\mathcal{N}_t^O| + |\mathcal{V}_t \setminus \mathcal{V}_t^N| + 5|\mathcal{V}_t| + 4 + |\mathcal{V}_t \setminus \mathcal{V}_{t+1}|) \\
&+ (|\mathcal{T}| - 2)|\mathcal{N}_t^O| + |\mathcal{T}|(|\mathcal{V}_t^N| + |\mathcal{V}_t|)\big) \qquad (4.58)
\end{aligned}
$$

Furthermore, ignoring the convexity constraints the number of constraints $NC^N$ for the node formulation can be expressed as:

$$NC^N = \sum_{t\in\mathcal{T}\setminus\{0\}} |\mathcal{L}_t|(|\mathcal{N}_t^C||\mathcal{K}| + |\mathcal{N}_t^O||\mathcal{K}| + 2|\mathcal{N}_t^C| + 2|\mathcal{N}_t^O| + |\mathcal{V}_t \setminus \mathcal{V}_t^N| + 3|\mathcal{V}_t| + 2)$$
$$+ \sum_{t\in\mathcal{T}\setminus\{0,1\}} |\mathcal{N}_t^O||\mathcal{L}_t| + \sum_{t\in\mathcal{T}}|\mathcal{L}_t|(|\mathcal{V}_t^N| + |\mathcal{V}_t|) + \sum_{t\in\mathcal{T}\setminus\{\bar{T}\}} |\mathcal{L}_t|(2|\mathcal{V}_t| + 2 + |\mathcal{V}_t \setminus \mathcal{V}_{t+1}|) \tag{4.59}$$

For at two-stage scenario tree where the nodes branch in period $b$, i.e. there are one node in period $b-1$ and several nodes in period $b$, Expression (4.59) simplifies to the following, assuming constant size of $(\mathcal{L}_t, t \in \mathcal{T} : t \geq b)$, $\mathcal{V}_t^N, \mathcal{V}_t, \mathcal{R}_{vt}, \mathcal{N}_t^C$ and $\mathcal{N}_t^O$:

$$\begin{aligned} NC^N =&(b-1)(|\mathcal{N}_t^C||\mathcal{K}| + |\mathcal{N}_t^O||\mathcal{K}| + 2|\mathcal{N}_t^C| + 3|\mathcal{N}_t^O| + |\mathcal{V}_t \setminus \mathcal{V}_t^N| + 3|\mathcal{V}_t| + 2) \\ &+b(|\mathcal{V}_t^N| + 3|\mathcal{V}_t| + 2 + |\mathcal{V}_t \setminus \mathcal{V}_{t+1}|) \\ &+|\mathcal{L}_t|\big((|\mathcal{T}| - b)(|\mathcal{N}_t^C||\mathcal{K}| + |\mathcal{N}_t^O||\mathcal{K}| + 2|\mathcal{N}_t^C| + 2|\mathcal{N}_t^O| + |\mathcal{V}_t \setminus \mathcal{V}_t^N| + 4|\mathcal{V}_t| + |\mathcal{V}_t^N| + 2) \\ &+(|\mathcal{T}| - b - 1)(2|\mathcal{V}_t| + |\mathcal{N}_t^O| + 2 + |\mathcal{V}_t \setminus \mathcal{V}_{t+1}|)\big) \end{aligned} \tag{4.60}$$

To compare the node formulation to the scenario formulation a small instance is constructed to show the difference between the two formulations. The values of the sets and parameters can be found in Table 4.2. Note that $\mathcal{V}_t \setminus \mathcal{V}_{t+1} = 2$ means that 2 ship types are scrapped in each period. Furthermore, note that this instance solely is constructed to demonstrate the difference between the two formulations and not as a realistic case.

***Table 4.1:*** *Comparison of number of variables and constraints.*

| | Scenario formulation | Node formulation |
|---|---|---|
| Variables | 20 660 | 12 072 |
| Constraints | 14 580 | 8412 |

Table 4.1 presents the number of variables and constraints for both formulations. For the scenario formulation, the number of variables and the number of constraints are calculated using Equation (4.54) and (4.58), respectively. For the node formulation the numbers of variables and constraints are calculated using Equation (4.56) and (4.59), respectively.

The number of variables and constraints are drastically reduced using the node formulation rather than the scenario formulation. Recall that the non-anticipativity constraints are not included in the calculations which make the real difference even greater. Furhtermore, the convexity constraints will be equal to the number of variables, and adding these to the number of constraints the difference is further enhanced. In addition, Bakkehaug and Eidem (2011) demonstrate that the node formulation outperforms the scenario formulation with respect to computational time for their model. These are two strong indications that the node formulation will perform better for the cash flow control model as well. Therefore, the node formulation is used for the implementation and to perform the computational study.

***Table 4.2:*** *Instance description.*

| Set/Parameter | Size |
|---|---|
| $\bar{T}_v^L$ | 2 |
| $\mathcal{T}$ | 6 |
| $\mathcal{V}_t$ | 10 |
| $\mathcal{V}_t \setminus \mathcal{V}_{t+1}$ | 2 |
| $\mathcal{V}_t \setminus \mathcal{V}_t^N$ | 8 |
| $\mathcal{V}_t^N$ | 2 |
| $\mathcal{N}_t^C$ | 10 |
| $\mathcal{N}_t^O$ | 3 |
| $\mathcal{R}_{vt}$ | 10 |
| $\mathcal{K}$ | 3 |
| $b$ | 3 |
| $\mathcal{S}$ | 20 |
| $\mathcal{L}_t$ | 20 |

Chapter 5

# Computational study

In this chapter the computational study is presented. It is based on data from a major shipping company in the Roll-on/Roll-off (RORO) shipping market. This is a market that uses highly specialized ships to cover demand, resulting in a small market for charters and second hand buying and selling. Therefore, these options are excluded from the model. In addition, this results in a problem with fewer recourse possibilities, which stresses the importance of controlling cash flows. Recall that recourse actions are the options the company has to recover from a bad first-stage decision. The models are implemented as two-stage models except for the cash flow control model which also is implemented as a three-stage model. However, the computations are mainly done on the two-stage model, and the three-stage model is used to demonstrate that the two-stage simplification is reasonable.

Furthermore, the models are implemented using C++ as programming language with CPLEX callable libraries. These libraries are provided by IBM ILOG CPLEX Optimization Studio Version 12.6.1. All runs of the implementation were performed on a computer running Windows 10 Pro 64-bit operating system, having an Intel ® Core ™i7-4500U CPU @ 1.8 GHz (2.4 Ghz) and 8 GB RAM.

The scope of this computational study is to illustrate the effects of cash flow control both using cash flow constraints and CVaR constraints, to understand how different levels of conservatism impact the expected profit, and discuss how this can be used to reduce the risk of insolvency. Both Pantuso et al. (2016) and Mørch et al. (2016) demonstrated a significant value of the stochastic solution (VSS) for the MFRP, and this computational study is therefore not including this topic. The computational study is based on 12 distinct cases, constructed from three different instances with four different versions.

Section 5.1 presents the instances used for this computational study. Section 5.2 explains the scenario generation and the versions of the instances, and Section 5.3 presents the cash flow effects for the cash flow control model for the four versions of the large instance. Section 5.4 presents the cash flow effects for the same model for the four versions of the medium and small instances. In Section 5.5 the results of the CVaR model using different confidence levels $\alpha$ are presented. Section 5.6 illustrates the difference between a two and three-stage case indicating that the two-stage simplification made for the implementation is reasonable.

## 5.1 Test instances

This section presents the three instances which the 12 distinct cases are built upon. All three instances have a planning horizon of five years. There are 18 ship types available for all instances. However, the number of trades and the size and composition of the initial fleet vary. The demand for each product on each trade in year 1 is shown in Table 5.1 for each instance. The symbol "C" indicates that the trade is a contracted trade, while the "O" indicates an optional trade. The symbol "-" indicates that the trade is not included in the given instance. There exists demand for three product types, namely cars, High & Heavy vehicles (HH) and Break-Bulk (BB) products. HH are cargo that cannot be stowed in car decks due to their height or weight, and BB are goods shipped loose in the hold of the ship which does not fall into the other categories. Demand and capacity are given in the unit RT43, which is a standard unit in RORO shipping. RT43 is a unit of size in square metres designed to be the equivalent of one car (based on a 1967 model of a Toyota Corona) including required stowage surrounding space ( 1 RT43 $= 7.38975 m^2$).

The large instance has $14$ trades representing a total demand of approximately $2.9$ millions $RT43$ units. The medium instance consists of nine trades covering approximately $65\%$ of the demand in the large instance, while the small instance consists of five trades covering approximately $30\%$ of the demand in the large instance. The optional trades hold approximately $10\%$ of the total demand in each instance.

Table 5.2 presents the initial fleet for each ship class and the respective capacity. The total capacity for a ship class is set equal to the maximum individual capacity of the same ship class. Recall that a ship type is a ship class with a specific production year as explained in Section 4.2. This is why there are $18$ ship types for each instance, but only seven ship classes.

The parameters used for the instances are based on data from Mørch et al. (2016) which is based on a real case from a major liner shipping company. The demand, sailing cost, operating cost, lay-up savings, scrapping value and new building price are based on data from the case company. The lead time $\bar{T}_v^L$, the time from ordering a new ship until it is delivered, is set to two years. The unit revenue is set as a profit margin on top of the sailing cost for the ship type with the worst fuel efficiency on each trade. The sailing cost is determined by fuel prices, fuel consumption and distance travelled. Scrapping value is determined by the lightweight tonnage

***Table 5.1:*** *Demand on each trade and product.*

| Trades | BB | Car | HH | Small | Medium | Large |
|---|---|---|---|---|---|---|
| AFEU | 0 | 19 200 | 0 | O | C | C |
| ASCE | 2 166 | 119 397 | 44 120 | - | - | C |
| ASEU | 5 761 | 435 213 | 77 046 | C | C | C |
| ASNE | 36 845 | 35 331 | 66 606 | C | O | C |
| ASNW | 1 939 | 60 158 | 7 400 | O | O | O |
| EUNAOC | 26 198 | 297 688 | 123 779 | - | C | C |
| EUNE | 20 075 | 469 379 | 67 853 | - | - | C |
| EUNW | 7 425 | 89 405 | 21 427 | - | C | C |
| EUOC | 16 776 | 266 855 | 55 474 | - | C | C |
| NAAS | 3 251 | 24 818 | 10 434 | - | - | O |
| NAEU | 14 115 | 140 508 | 53 928 | - | - | O |
| NAME | 4 476 | 103 048 | 14 200 | - | C | C |
| NASA | 3 404 | 41 036 | 24 419 | - | - | C |
| SANA | 2 689 | 97 999 | 29 239 | C | C | C |

***Table 5.2:*** *The initial fleet size and composition displaying the capacities for each product $k$.*

| | Initial fleet | | | Capacity | | | |
|---|---|---|---|---|---|---|---|
| Ship class | Small | Medium | Large | Cars | HH | BB | Total |
| PCTC1 | 2 | 3 | 5 | 4975 | 2200 | 300 | 4975 |
| PCTC2 | 5 | 10 | 15 | 6800 | 2500 | 300 | 6800 |
| PCTC3 | 2 | 9 | 12 | 5450 | 2200 | 900 | 5450 |
| PCTC4 | 1 | 5 | 6 | 6150 | 1800 | 200 | 6150 |
| LCTC1 | 5 | 8 | 9 | 6000 | 2000 | 1500 | 6000 |
| RORO1 | 1 | 2 | 4 | 4853 | 3100 | 1500 | 4853 |
| RORO2 | 0 | 0 | 0 | 5660 | 4000 | 2200 | 5660 |
| SUM | 16 | 37 | 51 | | | | |

of the ship and the scrap price. The main value of a scrapped ship comes from the steel value, and the scrap price is therefore assumed to be equal and perfectly correlated to the steel price.

The instalments paid in each period are detemined by the new building price, the repayment time and the interest rates offered by banks or as expected return on investments from investors. Stopford (2009) writes that a normal repayment time is $2-8$ years, and is therefore set to five years for the instances in this thesis. Moreover, Stopford (2009) writes that the interest rate on loans regarding ship financing generally are quoted at a margin over London Interbank Offered Rate (LIBOR). The spread of this margin typically range from $0.6\%$ to $2.0\%$ (Stopford, 2009). $1.25\%$ is chosen as a margin on top of the LIBOR, and using the 1 year LIBOR rate of $1.75\%$ (Bankrate, 2017) results in an interest rate of $3.0\%$.

Furthermore, the ship values in the instances are estimated using linear depreciation based on the new building cost and an expected lifetime of $30$ years. This is consistent with Stopford (2009) who identified a linear depreciation of the ship value with respect to age for the Panamax bulk carriers sold in the first nine months of $2002$. However, preliminary tests showed that using the ship value in the last year as the sunset value resulted in unrealistic solutions where the company drastically over-invests and puts close to half the fleet on lay-up. This kind of behaviour is assumed to be of no interest to a shipping company and therefore the sunset value is set to be $70\%$ of the ship value in the last period. This level is found to be sufficient for preventing such over-investments, while providing the desired modelling characteristics sunset values are intended to have, namely maintaining a realistic fleet at the end of the planning horizon.

The space charter price is set to be $2000$ USD per unit of goods transported by space charter. This can be interpreted as a penalty cost for not being able to satisfy demand on a given trade. Thus, the parameter is assumed to be deterministic. As suggested in Stopford (2009), all input values are properly discounted using a discount factor of $12\%$.

The demand, sailing cost, scrapping value and the new building price are implemented as stochastic parameters, while the remaining parameters are kept deterministic. The budget for ordering ships in year $0$ is set to be $5\%$ of the contracted revenue in period $1$, assuming similar revenues in period $0$ and $1$. A real company would know this budget with certainty, but for this thesis an estimate has to be made due to lack of data.

## 5.2 Scenario generation

This section describes how the scenario generation is performed. The scenarios in this thesis are generated using scenario generation with distribution functions and correlations (Høyland et al., 2003).

A scenario represents the complete realisation of all the random variables. The random variables used in this thesis are the relative incremental change in:

- Demand on each trade and product
- New building price
- Fuel price
- Steel price

The relative incremental change is the relative magnitude and direction of change from one period to the next. For instance, let the realisation of the random variable for the demand on a given trade $i$ and product $k$ be noted $\bar{r}_{ikn}$ at node $n$ in the scenario tree. Then, the parameter $D_{ikn}$ will have the following value at node $n$: $D_{ikn} = D_{ika(n)}(1 + \bar{r}_n)$. Note how the parameter at node $n$ depend on the value at the ancestor node in the previous period and the realisation of the relative incremental change for the respective random variable. The large instance has $14$ trades and three products resulting in the following number of random variables:

$(14 * 3) + 1 + 1 + 1 = 45.$

Figure 5.1 illustrates a possible realisation of a random variable for a two-stage problem with five scenarios. Up to year 2 the random variable takes the value of the initial state, but in the second stage, i.e. years $3$, $4$ and $5$, the random variable can take the value of the realisation in any of the five scenarios. Note that a realisation of the random variable consists of a realisation in all three periods in the second stage, which can be equal or different from period to period.

The reason relative incremental changes are used instead of absolute values is to prevent unrealistically rapid changes in the parameters. For instance, if the random variable for the demand on a given trade and product were absolute and could range from $50$ to $150$ units the realisation could change from $50$ to $150$ in one period which would mean an increase of $200\%$. If instead the random variable for the demand on the same trade and product represents the relative incremental change, the absolute value of the realisation can still range between $50$ and $150$, but the one-period change is limited. This results in a smoother and more realistic development for the parameters from period to period.

All random variables in this thesis are assumed to have a triangular distribution with lower limit of $-0.5$, upper limit of $0.5$ and mode of $0$. In coherence with Pantuso et al. (2017), the random

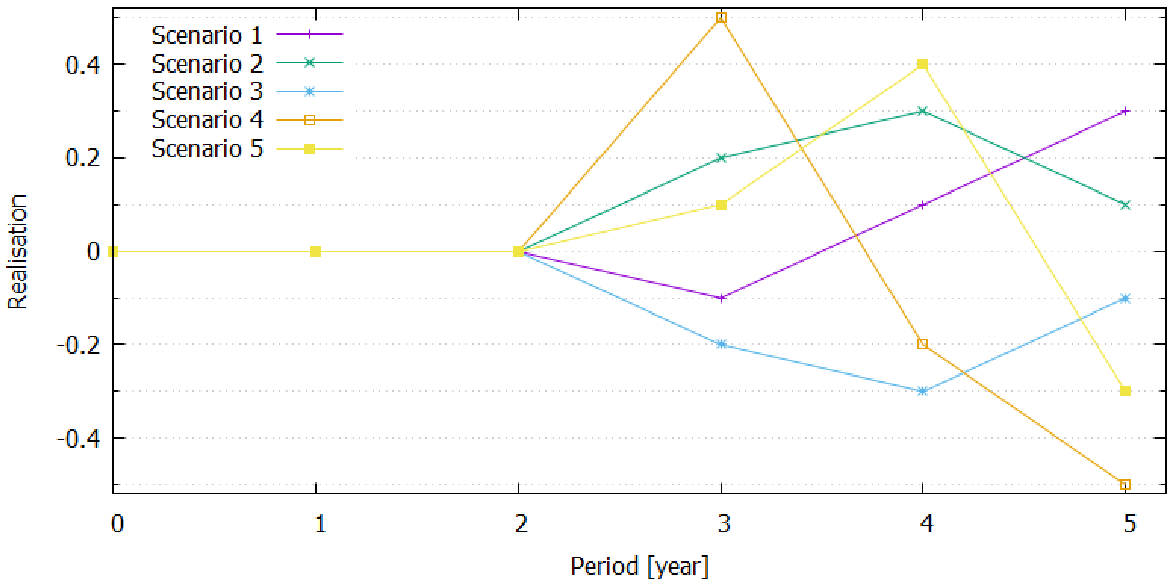



***Figure 5.1:*** *Realisation of a random variable. Illustrating a possible realisation of one random variable with five scenarios in a two-stage problem, branching in period* 3*.*

variables for the demand and the new building price are assumed to have the same underlying drivers and thus a strong correlation between themselves. However, the fuel price and the steel price are affected by a much bigger market than the shipping market itself. Therefore, they are assumed to be weakly correlated between themselves and to the demand and the new building price. The strong and weak correlation is assumed to have a correlation coefficient of $0.7$ and $0.2$ respectively. The correlated correlation matrix is shown in Table 5.3. This table shows the correlation between all the random variables, where the first columns and rows represent the incremental change in demand for every trade and product. Then, the column "New build" represents the incremental change in the new building price, the column "Fuel" represents the incremental change in the fuel price and finally the column "Steel" represents the incremental change in the steel price.

To examine how the effects of cash flow control change with different situations the instances are solved using one uncorrelated and one correlated correlation matrix. The uncorrelated correlation matrix has the same structure as the correlated correlation matrix in Table 5.3, but all the numbers off the diagonal are $0$. Using these two matrices makes it possible to see how the cash flow control behaves in one correlated and one uncorrelated world.

***Table 5.3:*** *Correlated correlation matrix.*

| | | Trade 1 | | | Trade 2 | ··· | Trade N | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Car | HH | BB | Car | ··· | BB | New build | Fuel | Steel |
| Trade 1 | Car | 1 | 0.7 | 0.7 | 0.7 | ··· | 0.7 | 0.7 | 0.2 | 0.2 |
| | HH | | 1 | 0.7 | 0.7 | ··· | 0.7 | 0.7 | 0.2 | 0.2 |
| | BB | | | 1 | 0.7 | ··· | 0.7 | 0.7 | 0.2 | 0.2 |
| Trade 2 | Car | | | | 1 | ··· | 0.7 | 0.7 | 0.2 | 0.2 |
| ⋮ | ⋮ | | | | | ⋱ | ⋮ | ⋮ | ⋮ | ⋮ |
| Trade N | BB | | | | | | 1 | 0.7 | 0.2 | 0.2 |
| | New build | | | | | | | 1 | 0.2 | 0.2 |
| | Fuel | | | | | | | | 1 | 0.2 |
| | Steel | | | | | | | | | 1 |

Besides solving the cash flow control model for the correlated and uncorrelated correlation matrix, the instances are solved with the normal space charter price of $2000$ USD and with a $50\%$ reduction in the space charter price. This results in the four versions for each instance:

- Correlated correlation matrix with normal space charter price
- Correlated correlation matrix with reduced space charter price
- Uncorrelated correlation matrix with normal space charter price
- Uncorrelated correlation matrix with reduced space charter price

To determine the numbers of scenarios used on the instances in the computational study an in-sample stability test is performed. The cash flow control model is solved without cash flow constraints for $20$ scenario trees and then the maximum relative difference between the smallest and highest expected profit is calculated:

$$\text{Maximum relative difference} = \frac{\text{Maximum exp. profit} - \text{Minimum exp. profit}}{\text{Average exp. profit}}$$

Recall from Section 2.1 that the scenario generation is in-sample stable if the objective function value is approximately equal for all scenario trees. Thus, the relative maximum difference is used to measure this, because it makes it possible to assess whether it is true for all scenario trees.

Table 5.4 shows the results of running the in-sample stability test for the large instance with uncorrelated correlation matrix and normal space charter price. The correlated correlation matrix is not used for this test, because the scenario generation algorithm requires more than $50$ scenarios to complete the generation for this matrix. To get the widest spread of scenarios the uncorrelated correlation matrix is therefore used. For any version of the large instance the computer runs out of memory before it can reach optimality, but with an optimality gap of $1\%$ the computational time spans from a couple of minutes for $50$ scenarios to $1 - 2$ hours for $400$ scenarios. All solutions presented in this thesis are therefore solved using a $1\%$ optimality gap.

From Table 5.4 it can be observed that when increasing the numbers of scenarios the scenario trees become more in-sample stable. However, considering the $1\%$ optimality gap the differences are considered to be small enough to be in-sample stable for $100$ scenarios and higher.

Moreover, for $100$ scenarios the computational time stays within $5 - 10$ minutes for $1\%$ optimality gap, compared to $1 - 2$ hours for $400$ scenarios. Therefore $100$ scenarios are used for the computational study in this thesis. However, when including the cash flow constraints the computational time rapidly increases. For the tightest formulation, i.e. the highest cash flow limit possible without reaching infeasible solutions, the computational time can be as much as $4 - 6$ hours. Due to time limitations the in-sample stability test is not performed for increasing cash flow limits. However, the model performed very well for the large instance without cash flow constraints and it is therefore assumed to be in-sample stable for increasing cash flow limits and the other instances.

***Table 5.4:*** *In-sample stability test for the large instance with uncorrelated correlation matrix.*

| Number of scenarios | 50 | 100 | 200 | 400 |
|---|---|---|---|---|
| Maximum relative difference | $1.70\%$ | $1.09\%$ | $0.77\%$ | $0.55\%$ |

The out-of-sample test is not performed in this thesis, due to the challenges of evaluating the true objective function. In addition, the scope of this computational study is not to evaluate every aspect of the scenario generation method. Thus, the in-sample test is found to be sufficient to indicate some degree of stability.

To summarize, all versions of the instances are solved using a two-stage model with $100$ scenarios branching in period $3$ as illustrated in Figure 5.1, unless stated otherwise.

## 5.3 Effects of cash flow control for the large instance

In this section the effects of cash flow control for the large instance are presented, solving the cash flow control model for increasing cash flow limits $\bar{F}$. Recall that all instances are solved with $1\%$ optimality gap due to computational considerations. This means that the objective function value, i.e. the expected profit, for all solutions are within $1\%$ of the best bound. Some solutions will therefore have an apparent improvement in expected profit for increasing cash flow limit $\bar{F}$. This happens because the solver terminates at any optimality gap below $1\%$, resulting in some solutions being closer to optimality than others.

Furthermore, this section consists of one subsection for each version, i.e. Section 5.3.1-5.3.4. Each subsection presents the solutions for the cash flow control model with increasing cash flow limits ranging from the solution without cash flow constraints to the highest cash flow limit possible without reaching infeasibility. Finally, Section 5.3.5 sums up the findings for the large instance discussing the managerial insights made from these solutions.

### 5.3.1 Correlated correlation matrix and normal space charter price

The solution of the cash flow control model for the large instance with correlated correlation matrix and normal space charter price is shown in Table 5.5. The column “New builds” gives the average number of ships ordered at the end of the period denoted in column “t”. The transition from the first stage to the second stage is marked with the horizontal line separating periods $2$ and $3$. “Scrappings” gives the number of ships scrapped at the end of the period, and ”Pool” gives the number of ships in the fleet in the beginning of the period. Recall that new ships are delivered with a lead time of two years in the beginning of the delivery period. Thus, the “pool” in period $2$ is determined by the initial fleet, the ships scrapped at the end of periods $0$ and $1$, and the ships ordered at the end of period $0$, namely $50 = 51 - 3 - 6 + 8$. “Lay-up” gives the number of ships put on lay up in the given period, and “Space charter” gives the number of units transported by space charter. “Optional trades” gives the number of optional trades serviced in the given period. Recall that once an optional trade is serviced it has to be serviced for the rest of the planning horizon, resulting in non-decreasing numbers in this column.

Note that for this version of the instance, the space charter cost is relatively high compared to owning and operating a ship. Thus the solution suggests to prepare for the high demand scenarios in the first stage and scrap the ships not needed when arriving to the second stage. In addition, the optional trades are used to increase the total demand when the demand is low, and they are not serviced at all in the high demand scenarios. The ships scrapped in the first stage are old ships that reach their lifetime during the planning horizon. Furthermore, a significant number of ships are kept on lay up in period 4 and 5 while using space charter. However, the space charter used in these periods are in the scenarios with the highest demand and where no

*Table 5.5: Solution of the large instance with correlated correlation matrix and normal space charter price without cash flow constraints.*

| t | New builds | Scrappings | Pool | Lay-up | Space charter | Optional trades |
|---|---|---|---|---|---|---|
| 0 | 8 | 3 | 51 | 0.00 | 0 | 0 |
| 1 | 10 | 6 | 48 | 0.10 | 0 | 0 |
| 2 | 11 | 0 | 50 | 0.27 | 0 | 0 |
| 3 | 1.99 | 15.43 | 60 | 9.07 | 362 | 1.28 |
| 4 | 0 | 3.66 | 55.57 | 3.40 | 6 634 | 1.56 |
| 5 | 0 | 0 | 53.9 | 0.43 | 2 928 | 1.82 |

ships are kept on lay up. The ships kept on lay up in the low demand scenarios prepares for future increase in demand.

When solving the model with cash flow constraints, the cash flow limit is increased starting with the limit where the constraints are not yet binding. The starting limit is equivalent to solving the model without cash flow constraints. Then the limit is increased through an iteration process and solved repeatedly until infeasibility is reached. The results from this process can be seen in Table 5.6, where the first row represents the solution without cash flow constraints which is a summary of the results presented in Table 5.5. The columns “First” and “Second” show the sum of the first stage decisions and the second stage decisions for the given variables, except for the “Optional” column, which shows the average number of optional trades serviced. Table 5.5 and 5.7 shows more detailed results of the first and last row in Table 5.6. Thus, the second stage values for the first row in Table 5.6 are the sums of the corresponding values in periods $3$, $4$ and $5$ in Table 5.5.

The column “Exp. Profit” shows the expected profit for each cash flow limit $\bar{F}$. Note that the expected profit increases from the third row to the fourth row, due to the $1\%$ optimality tolerance imposed. If solved to optimality the expected profit will be non-increasing with increasing cash flow limit. The column “New builds” shows the number of ships ordered in the first and second stage, while the column “Scrappings” shows the number of ships scrapped in the first and second stage. The columns “Lay-up” and “Space” show the number of ships put on lay-up and number of units transported by space charter in the first and second stage, respectively. Finally, the column “Optional” shows the average of the number of optional trades serviced in the first and second stage.

To increase the worst case cash flow it is mainly three observations from Table 5.6 that is worth emphasising. Reducing the number of ships ordered in the first stage clearly reduces the worst

***Table 5.6:*** *Solutions for the large instance with correlated correlation matrix and normal space charter price for increasing cash flow limits $\bar{F}$.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -66.3 | 1 109.4 | 29 | 1.99 | 9 | 19.09 | 0.37 | 12.9 | 0 | 9 924 | 0 | 1.55 |
| -59.7 | 1 109.0 | 29 | 2 | 9 | 19.19 | 0.37 | 12.72 | 0 | 13 989 | 0 | 1.46 |
| -53.1 | 1 108.5 | 29 | 1.98 | 9 | 19.16 | 0.37 | 12.84 | 0 | 10 913 | 0 | 1.55 |
| -46.4 | 1 110.1 | 29 | 2.08 | 9 | 19.28 | 0.37 | 12.61 | 0 | 12 416 | 0 | 1.47 |
| -39.8 | 1 109.3 | 29 | 2.02 | 9 | 19.16 | 0.37 | 12.72 | 0 | 12 912 | 0 | 1.48 |
| -33.2 | 1 104.9 | 26 | 2.39 | 8 | 17.85 | 0.41 | 12.56 | 0 | 26 709 | 0 | 1.42 |
| -26.5 | 1 094.2 | 26 | 2.41 | 9 | 16.77 | 2.37 | 12.63 | 0 | 35 091 | 0 | 1.46 |
| -19.9 | 1 078.6 | 26 | 2.39 | 9 | 16.86 | 4.36 | 12.69 | 0 | 34 769 | 0 | 1.46 |
| -13.3 | 1 065.4 | 26 | 2.37 | 9 | 16.71 | 6.34 | 12.84 | 0 | 38 723 | 0 | 1.52 |
| -6.6 | 1 038.2 | 24 | 2.7 | 6 | 18.16 | 10.39 | 12.64 | 0 | 28 493 | 0 | 1.44 |
| 0.0 | 955.7 | 23 | 2.67 | 5 | 18.34 | 12.19 | 15.05 | 42 367 | 36 051 | 0 | 1.54 |

case cash flow. Delaying scrapping decisions provide the same effect, and a more subtle way of reducing the worst case cash flow is to change the timing of the orders of new ships in the first stage. This last observation is not that clear, but it can be seen in Table 5.7, which presents a more detailed result of the solution with cash flow limit $\bar{F} = 0$. Comparing Table 5.7 to Table 5.5 it can be observed that the number of ships ordered in period $0$ is doubled in order to reduce the cash flow in period $5$. Recall that the ships are paid with five instalments over five years, meaning that a ship ordered in period $0$ is paid in periods $0$, $1$, $2$, $3$ and $4$, while a ship ordered in period $1$ is paid in periods $1$, $2$, $3$, $4$ and $5$. A consequence of ordering the ships earlier and delaying scrapping decisions is that ships get delivered earlier and are scrapped later resulting in more ships put on lay up in the first stage. Another result of changing the first stage decisions is that the use of space charter increases. The three observations mentioned might occur separately or as a combination, making the task of reducing the cash flow very complex.

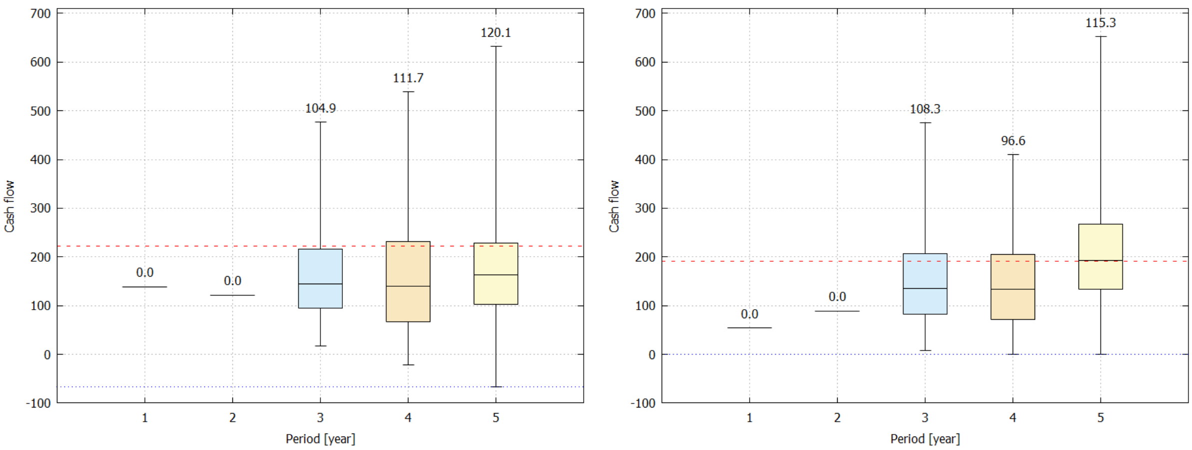
Having a closer look at Table 5.7, it is clear that all three observations mentioned occur at the highest cash flow limit. The total number of ships are reduced, reducing the cash outflow from instalments. The timing of orders has shifted towards period $0$ reducing the cash outflow in period $5$, and the scrapping of old ships are delayed making the need for new ships smaller. A result from reducing the fleet is the increased use of space charter, increasing the cash outflow. Note that for this particular case the number of ships scrapped are increased in period $0$, thus increasing the budget for investments, allowing for a greater shift of cash flows from the uncertain and potentially bad period $5$ to the certain period $0$. This results in a reduction of the fleet

*Table 5.7:* *Solution for the large instance with correlated correlation matrix and normal space charter price with the tightest cash flow constraints ($\bar{F} = 0$).*

| t | New builds | Scrappings | Pool | Lay-up | Space charter | Optional trades |
|---|---|---|---|---|---|---|
| 0 | 16 | 4 | 51 | 0 | 0 | 0 |
| 1 | 1 | 1 | 47 | 0 | 42 367 | 0 |
| 2 | 6 | 0 | 62 | 12.19 | 0 | 0 |
| 3 | 2.67 | 14.02 | 63 | 11.16 | 1 430 | 1.28 |
| 4 | 0 | 4.32 | 54.98 | 3.46 | 26 034 | 1.54 |
| 5 | 0 | 0 | 53.33 | 0.44 | 8 587 | 1.79 |

in period $1$ worsening the cash flow in this period with increased use of space charter. Note also that even though the fleet is increased in periods $2$ and $3$ the fleet composition has changed resulting in a higher use of space charter in period $3$. This illustrates how complex the cash flow effects can be from changing the first stage decisions.

Figure 5.2 shows the cash flow development for each period and scenario as a box plot comparing the solution of the model without cash flow constraints (Figure 5.2a) and the solution of the model with the tightest cash flow constraints (Figure 5.2b). These solutions correspond to $\bar{F} = -66.3$ and $\bar{F} = 0$ in Table 5.6. The red dashed line represents the annualised expected profit, while the blue dotted line indicates the worst case cash flow. The lower end of the box represents the first quartile, the upper end of the box represents the third quartile, and the line inside the box represents the median. The ends of the whiskers represent the minimum and maximum of all the data. This means that $50\%$ of the scenarios are located inside the box, while $25\%$ is located on each side of the box between the ends of the box and the ends of the whiskers. Note how the worst case cash flow, i.e. the bottom whisker in year $5$, is improved at the cost of expected profit loss and a significant reduction of the cash flow in periods $2$ and $3$. In addition, the upside in year $4$ is reduced significantly, which can be seen as a downward shift of the box, including the median, and the top whisker. Furthermore, it is interesting to see that the uncertainty is decreased in period $4$ and $5$, but increased in period $3$, seen by the change in standard deviation.




***(a)*** *Without cash flow constraints* ***(b)*** *With cash flow constraints ($\bar{F} = 0$)*

***Figure 5.2:*** *The cash flow development for the large instance with correlated correlation matrix and normal space charter price. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*

### 5.3.2 Correlated correlation matrix and reduced space charter price

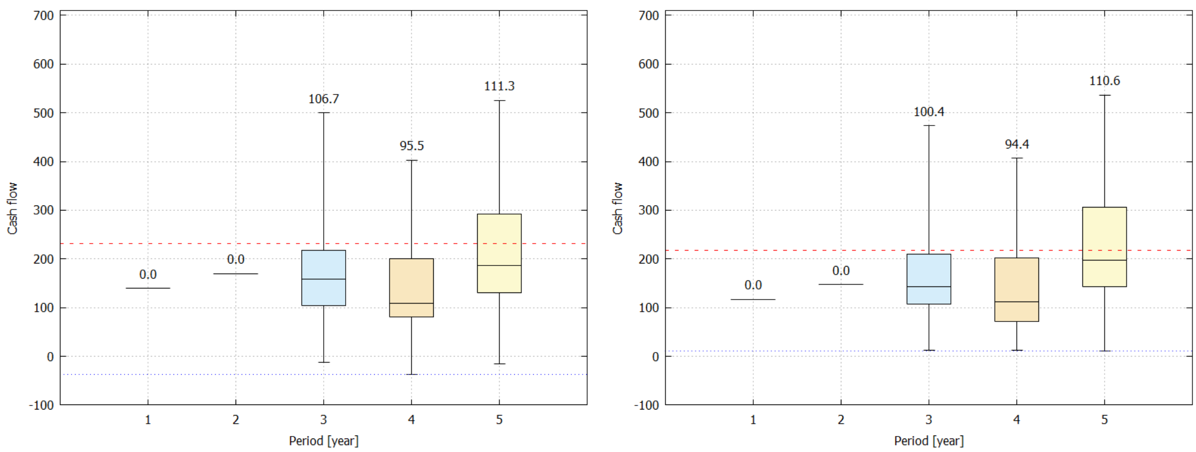
In this section the large instance with correlated correlation matrix and reduced space charter price is tested in order to examine what effects lower penalty cost for unfulfilled demand have on the cash flow. The solutions of the cash flow control model for this version of the large instance are shown in Table 5.8. Space charter is used to a much higher degree where the solution prepares for lower demand scenarios than in the version with normal space charter price. Note how the solutions up to $\bar{F} = -15.0$ are approximately indifferent. This indicates that relatively small adjustments to the solution without cash flow constraints can make a good improvement of the worst case cash flow without the cost of loss in expected profit. Moreover, the three observations made in Section 5.3.1 can also be found here, namely that reducing the orders of new ships in the first stage, delaying scrapping decisions and changing the timing of orders lead to improved worst case cash flow. In addition, a fourth observation can be made here. The optional trades are used more actively where for three of the solutions the company commits to one optional trade in period 2 in the first stage.

***Table 5.8:*** *Solutions for the large instance with correlated correlation matrix and* $50\%$ *reduction in space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -37.5 | 1 159.5 | 18 | 2.85 | 9 | 10.4 | 0.37 | 12.06 | 0 | 108 158 | 0 | 1.53 |
| -33.8 | 1 159.1 | 18 | 2.86 | 9 | 10.41 | 0.27 | 12.09 | 7 151 | 106 945 | 0 | 1.5 |
| -30.0 | 1 164.1 | 18 | 2.86 | 9 | 10.4 | 0.1 | 12.04 | 2 503 | 118 934 | 0.33 | 1.72 |
| -26.3 | 1 159.1 | 17 | 3.03 | 8 | 10.8 | 0.1 | 11.81 | 4 183 | 123 844 | 0.33 | 1.71 |
| -22.5 | 1 164.0 | 18 | 2.79 | 9 | 10.42 | 0.1 | 11.99 | 2 503 | 121 887 | 0.33 | 1.72 |
| -18.8 | 1 156.8 | 18 | 2.69 | 9 | 10.38 | 0.1 | 12.13 | 18 203 | 107 839 | 0 | 1.44 |
| -15.0 | 1 160.8 | 17 | 2.87 | 9 | 9.75 | 0.27 | 11.38 | 7 151 | 131 583 | 0 | 1.44 |
| -11.3 | 1 155.8 | 16 | 3 | 9 | 9.23 | 0.45 | 10.61 | 0 | 166 679 | 0 | 1.42 |
| -7.5 | 1 154.1 | 16 | 2.99 | 8 | 10.2 | 1.37 | 11.13 | 0 | 141 487 | 0 | 1.42 |
| -3.8 | 1 143.0 | 15 | 3.22 | 8 | 9.65 | 2.37 | 10.45 | 0 | 176 547 | 0 | 1.42 |
| 0.0 | 1 135.9 | 17 | 2.87 | 9 | 10.03 | 4.36 | 11.18 | 0 | 141 195 | 0 | 1.42 |
| 3.8 | 1 126.2 | 15 | 3.08 | 8 | 9.69 | 4.44 | 10.39 | 0 | 180 343 | 0 | 1.37 |
| 7.5 | 1 115.9 | 17 | 2.73 | 9 | 9.94 | 6.24 | 11.28 | 31 434 | 141 142 | 0 | 1.42 |
| 11.3 | 1 085.8 | 15 | 3.31 | 7 | 11.02 | 8.24 | 10.74 | 43 629 | 160 107 | 0 | 1.31 |

Most solutions in Table 5.8 are combinations of these four observations. For $\bar{F} = -11.3$ the number of ships ordered are reduced, but the time of purchase is not changed, i.e the same number of ships are ordered in period $0$, and the scrapping decisions are kept unchanged. For $\bar{F} = -3.8$ the worst case cash flow is improved by reducing the number of ships ordered in the first stage, making the orders earlier in the first stage, and delaying scrapping of one ship. For $\bar{F} = 0$ the number of ships ordered are only reduced by one ship, ordering them earlier in the first stage, but not delaying any scrapping decisions. This shows how combinations of the four observations made can affect the worst case cash flow, and further illustrates how complex this task is.

Figure 5.3 shows the cash flow development for each period and scenario as a box plot comparing the solutions of the model without cash flow constraints and the model with the tightest cash flow constraints. Here the worst case cash flow is improved at the cost of expected profit, but the cash flows in the first stage periods are not affected much. In addition, it can be seen that the uncertainty is reduced in all second stage periods, seen by the reduction in standard deviation.




***(a)** Without cash flow constraints*        ***(b)** With cash flow constraints ($\bar{F} = 11.3$)*

***Figure 5.3:** The cash flow development for the large instance with correlated correlation matrix and $50\%$ reduction in space charter price. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*

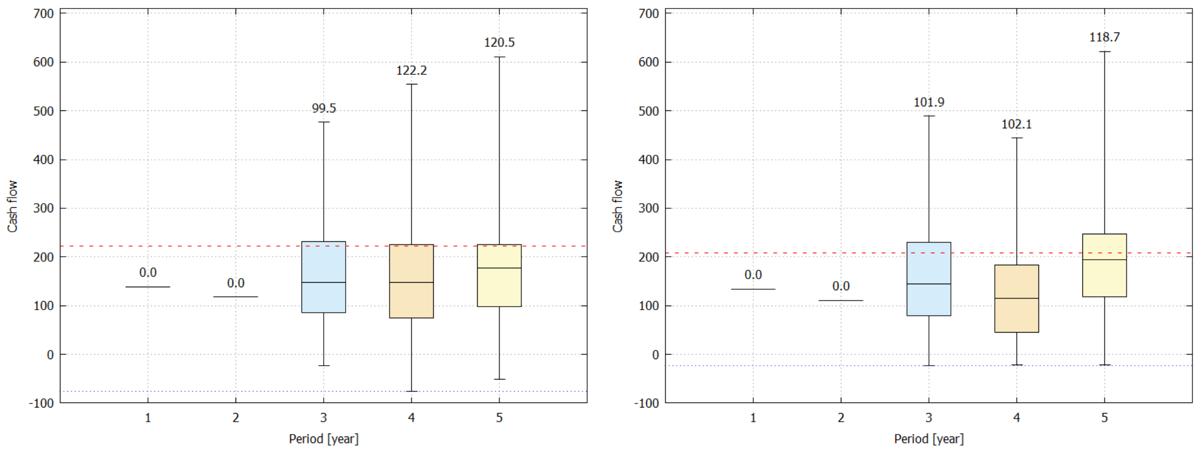
### 5.3.3 Uncorrelated correlation matrix and normal space charter price

The solutions of the cash flow control model for the large instance with uncorrelated correlation matrix and normal space charter price are presented in Table 5.9. The observations made for this version are similar to the correlated version, but there are some interesting differences. The highest possible cash flow limit is $\bar{F} = -22.6$ for the uncorrelated version, while it is $\bar{F} = 0$ for the correlated version. For this instance, it indicates that the cash flow control model has a greater potential of benefit in a correlated world than in an uncorrelated world.

***Table 5.9:*** *Solutions for the large instance with uncorrelated correlation matrix for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -75.3 | 1 109.7 | 30 | 2.47 | 9 | 19.88 | 0.37 | 12.51 | 0 | 3 489 | 0 | 1.4 |
| -67.8 | 1 109.9 | 30 | 2.49 | 9 | 19.68 | 0.37 | 12.88 | 0 | 1 978 | 0 | 1.4 |
| -60.2 | 1 108.9 | 29 | 2.65 | 8 | 20.32 | 0.41 | 11.96 | 0 | 4 510 | 0 | 1.38 |
| -52.7 | 1 108.2 | 29 | 2.7 | 9 | 19.3 | 0.37 | 11.99 | 0 | 14 326 | 0 | 1.44 |
| -45.2 | 1 103.0 | 28 | 3.03 | 8 | 19.41 | 1.37 | 12.37 | 0 | 8 689 | 0 | 1.38 |
| -37.6 | 1 092.4 | 26 | 3.16 | 9 | 17.12 | 0.37 | 11.87 | 0 | 48 836 | 0 | 1.4 |
| -30.1 | 1 073.8 | 26 | 3.35 | 9 | 17.14 | 3.36 | 12.07 | 0 | 43 956 | 0 | 1.47 |
| -22.6 | 1 041.0 | 24 | 3.52 | 7 | 17.67 | 7.35 | 12.77 | 0 | 48 746 | 0 | 1.44 |

Figure 5.4 shows the cash flow development for each period and scenario as a box plot comparing the solution of the cash flow control model without cash flow constraints and the solution of the model with the tightest cash flow constraints. Note how the worst case cash flow, i.e. the bottom whisker in year $4$, is improved at the cost of expected profit loss. In addition, the upside in year $4$ is reduced significantly, which can be seen as a downward shift of the box, including the median, and the top whisker. Furthermore, it is interesting to observe that the uncertainty is decreased in periods $4$ and $5$, but increased in period $3$, seen by the change in standard deviation. This indicates that improving the worst case cash flow and reducing the uncertainty in periods $4$ and $5$ comes at the additional cost of increased uncertainty in period $3$.




***(a)** Without cash flow constraints* ***(b)** With cash flow constraints ($\bar{F} = -22.6$)*

***Figure 5.4:** The cash flow development for the large instance with correlated correlation matrix and normal space charter price. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*

### 5.3.4 Uncorrelated correlation matrix and reduced space charter price

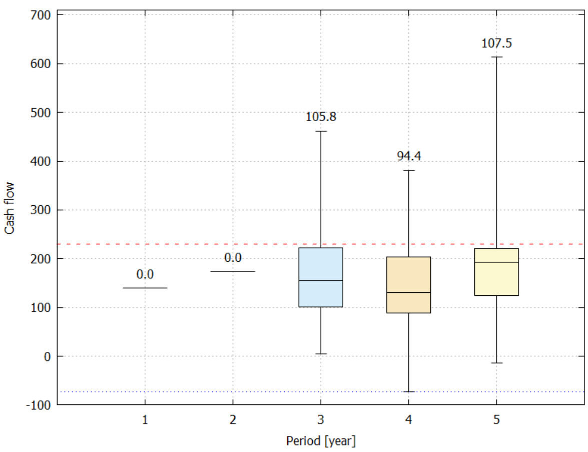
The solutions for the cash flow control model with uncorrelated correlation matrix and reduced space charter price are shown in Table 5.10. Comparing to the normal space charter, the use of space charter has increased drastically while the number of ships ordered is reduced for the solution without cash flow constraints. In addition, the expected profit has increased and the worst case cash flow is better.

The four observations mentioned for the other versions of the large instance can also be found for this version. However, two additional observations are made for the tightest cash flow limit, i.e. the highest possible cash flow limit $\bar{F} = 11.5$. Here the cash flow limit is actually improved by making more aggressive decisions, i.e. ordering and scrapping more ships in the first stage. Most of the ships are ordered in period $0$ reducing the cash outflow in period $5$, and most of the ships are scrapped in the same period improving cash inflow in period $0$. In addition, the demand is increased by committing to one optional trade in the first stage, increasing the cash inflow. Even though these results come from using the uncorrelated correlation matrix, this indicates that there might exist situations where both conservative and aggressive ordering strategies can improve the worst case cash flow. Thus, the complexity of this task is demonstrated once again. Furthermore, the worst case cash flow can be improved significantly without big losses in the expected profit. Note that all the solutions up to the cash flow limit $\bar{F} = -3.8$ are approximately indifferent with respect to the expected profit.

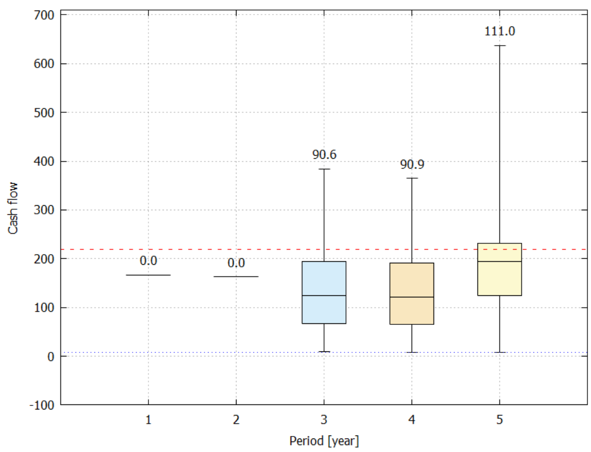
The cash flow developments for the solution of this version without cash flow constraints and the tightest cash flow constraints can be seen in Figure 5.5. Here it can be observed how the cash flow is reduced in periods $1$ and $2$ due to the aggressive ordering and scrapping, while these decisions together with committing to one optional trade prepare for the second stage improving the worst case cash flow here. Note also that the average cash flows in the second stage barely change, seen by the small change of the median. Like for the other instances, the cash flow improvement also comes at the cost of reduced expected profit.

***Table 5.10:*** *Solutions for the large instance with uncorrelated correlation matrix and* 50% *reduction in space charter price for increasing cash flow limits.*

| $\bar{F}$ | Exp. Profit | New builds First | New builds Second | Scrappings First | Scrappings Second | Lay-up First | Lay-up Second | Space First | Space Second | Optional First | Optional Second |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -38.4 | 1 154.4 | 18 | 4.24 | 9 | 10.8 | 0.37 | 12.31 | 0 | 126 199 | 0 | 1.38 |
| -34.5 | 1 154.2 | 18 | 4.46 | 9 | 10.96 | 0.37 | 12.18 | 0 | 122 434 | 0 | 1.38 |
| -30.7 | 1 153.8 | 18 | 4.43 | 9 | 10.95 | 0.37 | 12.2 | 0 | 123 936 | 0 | 1.38 |
| -26.8 | 1 153.8 | 18 | 4.34 | 9 | 10.96 | 0.37 | 12.15 | 0 | 126 118 | 0 | 1.38 |
| -23.0 | 1 153.6 | 18 | 4.28 | 9 | 10.95 | 0.37 | 12.16 | 0 | 127 898 | 0 | 1.38 |
| -19.2 | 1 153.0 | 18 | 4.26 | 9 | 10.97 | 0.37 | 12.17 | 0 | 129 879 | 0 | 1.38 |
| -15.3 | 1 153.8 | 19 | 4.09 | 9 | 11.84 | 0.1 | 11.7 | 2 503 | 134 123 | 0.33 | 1.69 |
| -11.5 | 1 153.9 | 18 | 4.05 | 9 | 11.17 | 0.37 | 11.12 | 0 | 146 538 | 0 | 1.38 |
| -7.7 | 1 147.9 | 18 | 3.98 | 9 | 11.22 | 0.1 | 11.66 | 18 203 | 137 621 | 0 | 1.36 |
| -3.8 | 1 152.2 | 17 | 3.97 | 9 | 10.64 | 0.37 | 11.04 | 0 | 171 784 | 0 | 1.36 |
| 0.0 | 1 142.8 | 17 | 4.14 | 9 | 10.9 | 1.37 | 11.01 | 0 | 174 474 | 0 | 1.33 |
| 3.8 | 1 132.6 | 17 | 3.93 | 9 | 10.73 | 3.36 | 11.01 | 0 | 176 742 | 0 | 1.36 |
| 7.7 | 1 095.5 | 15 | 4.11 | 8 | 11.03 | 2.49 | 7.98 | 0 | 316 925 | 0 | 1.27 |
| 11.5 | 1 043.5 | 22 | 3.51 | 10 | 13.6 | 5.8 | 13.44 | 132 847 | 166 479 | 0.33 | 1.68 |




***(a)*** *Without cash flow constraints*




***(b)*** *With cash flow constraints* ($\bar{F} = 11.5$)

***Figure 5.5:*** *The cash flow development for the large instance with uncorrelated correlation matrix and* 50% *reduction in space charter price. The red dashed line represents the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*

### 5.3.5 Managerial insights

To interpret and visualize the four versions of the large instance, the efficient frontiers are plotted in Figure 5.6. The efficient frontiers are generated based on the relative worst case cash flow and the relative expected profit loss, calculated from the Tables 5.6, 5.8, 5.9 and 5.10. The relative measures are used to easier assess the trade-off between improved worst case cash flow and expected profit loss. The relative expected profit loss is the loss in expected profit from the solution of the cash flow control model without cash flow constraints to a given cash flow limit $\bar{F}$ divided by the expected profit of the solution without cash flow constraints. Since the cash flows are annual the relative cash flow is given in terms of the annual expected profit to give both measures in the same units. Thus, the relative worst case cash flow is the worst case cash flow in a given solution divided by the annualised expected profit of the solution without cash flow constraints. Note that the relative expected profit loss gives the same results regardless of using annualised values or not. The nominal values for the worst case cash flow and the expected profit can be found in the first two columns of Tables 5.6, 5.8, 5.9 and 5.10. Plotting the expected profit loss to the relative worst case cash flow gives the efficient frontiers presented in Figure 5.6. Note that the decreasing relative expected profit loss in parts of the curves are the result of the $1\%$ optimality gap, and not representing the real situation. Thus, in reality the curves are always non-decreasing if the instances are solved to optimality.

From Figure 5.6 it is clear that all versions of the large instance have the same characteristic. They have a given section where the worst case cash flow can be improved without significant loss in expected profit, and a section where the expected profit loss is rapidly increasing as the worst case cash flow improves. Furthermore, for increasing cash flow limits there is a significant difference between the uncorrelated and correlated version with normal space charter price. This illustrates that in a correlated world the benefit of using the cash flow control model is grater than in an uncorrelated world. For example, the relative worst case cash flow of $-10\%$ has an expected profit loss of approximately $6\%$ and $2\%$ for the uncorrelated and correlated version, respectively. In addition, the worst case cash flow can be improved by approximately $15\%$ in terms of the annualised expected profit without any significant loss in expected profit.

However, the difference between a correlated and uncorrelated world is not that clear when having a $50\%$ reduction in space charter price. Even though the efficient frontiers overlap and cross each other, there is a difference of $3\%$ in the relative expected profit for the relative worst case cash flow of $5\%$.

Moreover, these results indicate that regardless of the correlation matrix used and high or low space charter price, the company has a potential of benefiting from using a cash flow control model. To improve the worst case cash flow there has been made six observations, namely reducing or increasing the number of ships ordered, changing the timing of investments, delaying

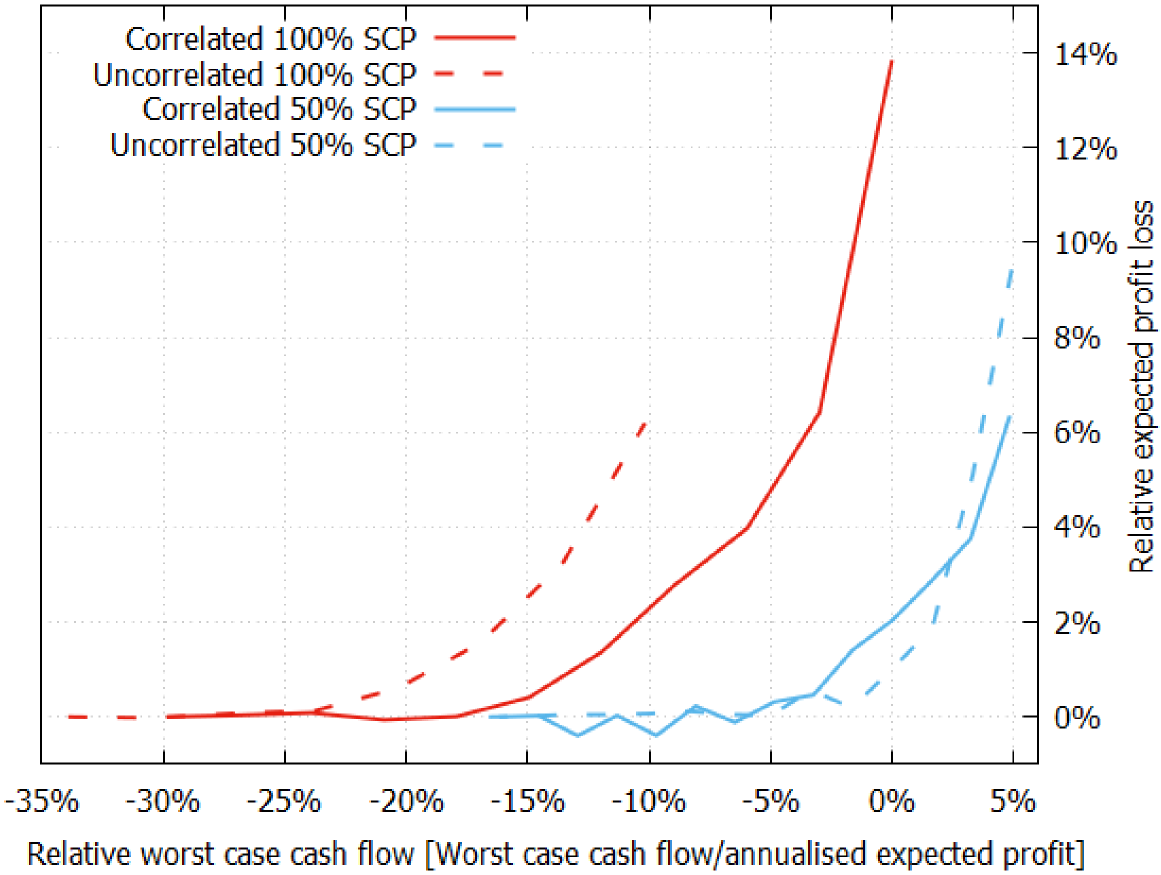



***Figure 5.6:*** *The efficient froniters for the different versions of the large instance. SCP is short for space charter price.*

or increasing scrapping of ships and adjusting demand through optional trades. This makes assessing whether a given strategy actually improves the worst case cash flow or not a very complex task, which makes the cash flow control model a valuable tool for the company.

However, this model says nothing about which of the solutions on the efficient frontier the manager should choose, but offers the manager a range of solutions to choose from. To determine which solution to choose, utility theory (Pindyck and Rubinfeld, 2013) has to be considered. Based on the situation the company faces, the utility of these solutions will be different. In most cases the manager will probably prefer to choose the solution furthest to the right in the flat part of the efficient frontier, because it improves the worst case cash flow with an insignificant reduction in expected profit.

Moreover, if the company is low on cash reserves and thus is facing an immediate risk of cash flow insolvency, the manager will probably choose a solution from some area of the steep part of the efficient frontier. Then the manager can pick a solution that has the worst case cash flow above the limit where the company faces cash flow insolvency, and in that way reduce this risk or eliminate it all together.

On the other hand, if the company has a big cash reserve for the coming planning period, but for some reason the value of their assets (for instance ships) dropped to a level were their liabilities are greater than the asset values, the company would be facing the risk of balance-sheet insolvency. In this situation the manager would probably choose a solution further to the left on the efficient frontier in order to maximize the expected profit, thus increasing the value of the company and reducing the risk of balance-sheet insolvency. However, this comes at the cost of a reduced cash flow in the worst scenario, thus increasing the risk of cash flow insolvency.

A real situation probably consists of a combination of the two examples mentioned, but it illustrates how the cash flow control model can be used as a tool to find the correct trade-off between expected profit and worst case cash flows in order to reduce the risk of insolvency.

## 5.4 Cash flow effects for the medium and small instances

### 5.4.1 Medium instance

The solutions and the cash flow development for the four versions of the medium instance are similar to the corresponding versions of the large instance. The observations made for the large instance are also observed in the medium instance, except for the aggressive solution where ordering more ships and increasing scrapping improves the worst case cash flow. Therefore, these results are not discussed in detail in this section, but can be found in Appendix C and D.

Even though similar observations are made in both the medium and large instance, the efficient frontiers demonstrate the differences that do exist. The efficient frontiers can be seen in Figure 5.7. As for the large instance, it is possible to significantly improve the worst case cash flow in all versions. However, for the correlated matrix with $50\%$ reduction in the space charter price the trade-off between the relative worst case cash flow and the relative expected profit loss is close to linear. Thus, indicating that it might exist situations where there is a small room for improving the worst case cash flow without equivalent losses in expected profit.

Moreover, comparing the versions of the small instance to the large instance, there is an indication that a smaller company has less benefit of using the cash flow control model. This can be seen as the smaller possible improvement of the relative cash flow for all versions except for the uncorrelated version with reduced space charter price. Here the cash flow improvement is greater for the medium instance than the large instance.

For the medium instance with normal space charter price the company will have a greater benefit of the model in a correlated world than in an uncorrelated world, because the relative worst case cash flow can be improved to a higher relative level and at a lower loss in expected profit for the same levels. However, the opposite can be observed for the versions with reduced space charter price.

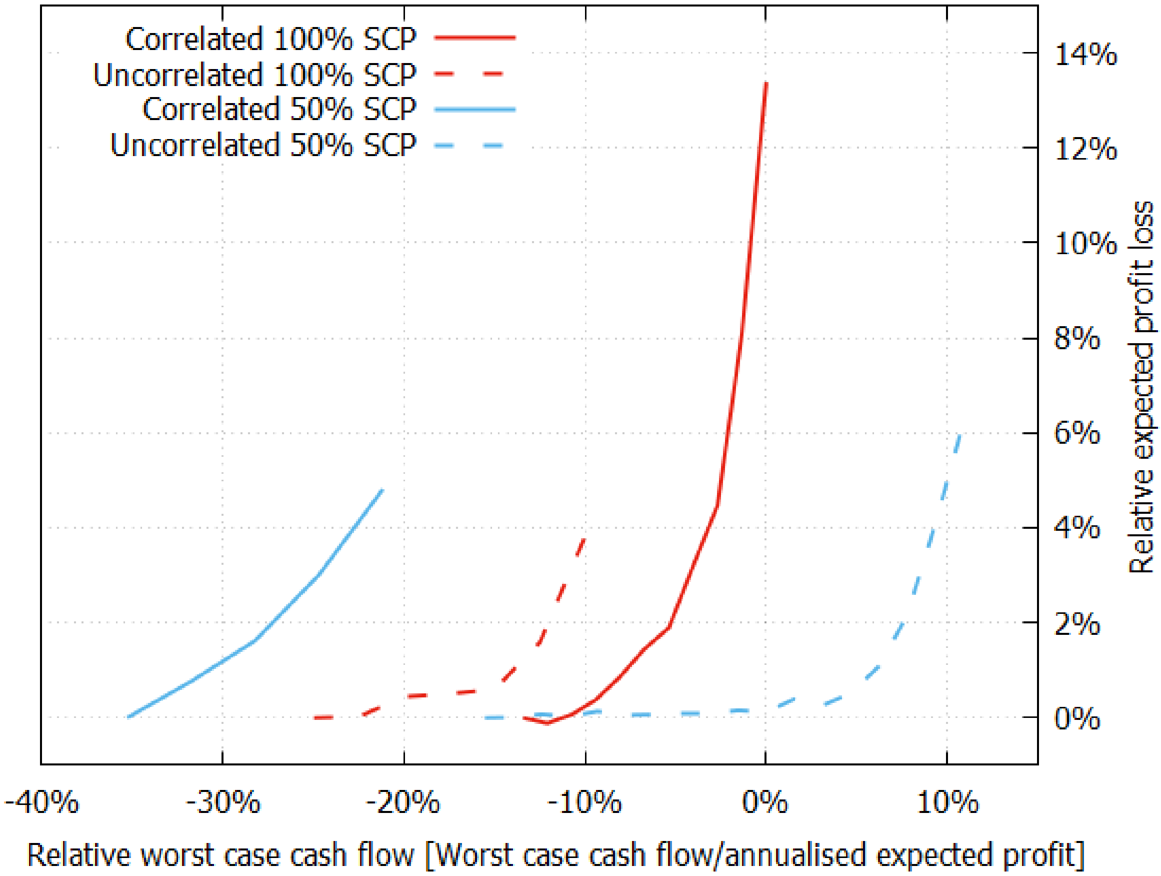



***Figure 5.7:*** *The efficient froniters for the different versions of the medium instance. SCP is short for space charter price.*

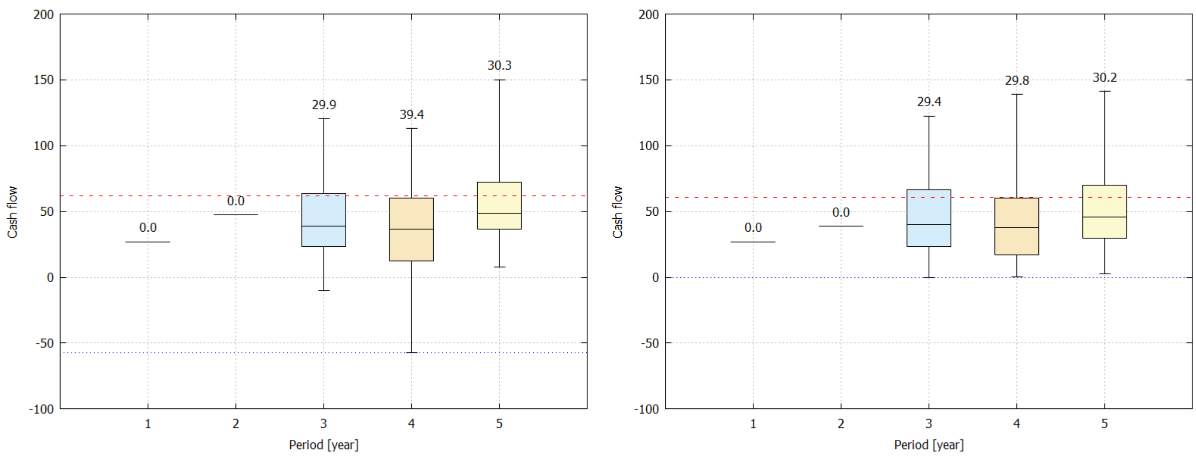
### 5.4.2 Small instance

Similar observations can be made between the correlated and uncorrelated versions with normal space charter price of the small instance and the corresponding versions of the large and medium instance. These results are therefore not discussed in detail here, and can be found in Appendix C and D. However, reducing space charter price by $50\%$ leads to a new observation for ordering new ships for both the uncorrelated and the correlated versions. The results can be seen in Table 5.11 and Table C.7 in Appendix C. The worst case cash flow is here improved by increasing the orders of ships in the first stage. To understand why this is a reasonable strategy the worst case scenarios has to be examined more closely. The scenarios with the worst cash flows are the high demand scenarios where the peak in demand is covered by space charter. The space charter price is still higher than the unit revenue and use of space charter leads to negative cash flows in the high demand scenarios. Increasing the number of ships in the fleet reduces the need for space charter in these scenarios, thus resulting in improved cash flow. However, this comes at the cost of overcapacity in the low demand scenarios having to pay instalments on ships scrapped or kept on lay up in these scenarios. The number of ships ordered in the first stage can therefore be increased until these two effects level each other out.

The cash flow development for the correlated version with reduced space charter price illustrated in Figure 5.8, shows the impact of increasing the number of ships ordered in the first stage from the cash flow control model without cash flow constraints to the highest possible cash flow limit. Note how the cash flow becomes worse in period 2, while the worst case cash flows in the second stage becomes better. In addition, this improvement of the worst case cash flow comes at a very low cost of expected profit. The similar results can be seen for the uncorrelated version with reduced space charter price in Figure D.7 in Appendix D, except for a slightly higher cost of expected profit.

***Table 5.11:*** *Solutions for the small instance with correlated correlation matrix and reduced space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -57.1 | 310.7 | 6 | 1.31 | 3 | 3.96 | 0.12 | 4.24 | 17 005 | 53 534 | 0 | 0.71 |
| -51.4 | 309.7 | 6 | 1.24 | 3 | 3.91 | 0.12 | 4.38 | 17 005 | 56 099 | 0 | 0.7 |
| -45.7 | 309.1 | 7 | 1.09 | 3 | 4.68 | 0.12 | 4.31 | 17 005 | 42 665 | 0 | 0.71 |
| -40.0 | 309.1 | 7 | 1.07 | 3 | 4.69 | 0.12 | 4.33 | 17 005 | 42 856 | 0 | 0.71 |
| -34.3 | 308.4 | 7 | 1.06 | 3 | 4.72 | 0.12 | 4.28 | 17 005 | 43 931 | 0 | 0.66 |
| -28.5 | 307.8 | 7 | 0.99 | 3 | 4.77 | 0.12 | 4.21 | 17 005 | 49 098 | 0 | 0.71 |
| -22.8 | 307.1 | 8 | 0.93 | 3 | 5.48 | 0.12 | 4.33 | 17 005 | 32 275 | 0 | 0.71 |
| -17.1 | 307.3 | 8 | 0.93 | 3 | 5.45 | 0.12 | 4.37 | 17 005 | 31 848 | 0 | 0.72 |
| -11.4 | 306.1 | 8 | 0.89 | 3 | 5.45 | 0.12 | 4.46 | 17 005 | 34 025 | 0 | 0.68 |
| -5.7 | 305.2 | 9 | 0.8 | 3 | 6.11 | 0.12 | 4.48 | 17 005 | 17 955 | 0 | 0.71 |
| 0.0 | 304.4 | 8 | 0.67 | 3 | 5.46 | 0.12 | 4.38 | 17 005 | 43 995 | 0 | 0.71 |




*(a) Without cash flow constraints* *(b) With cash flow constraints ($\bar{F} = 0$)*

***Figure 5.8:*** *The cash flow development for the small instance with correlated correlation matrix and reduced space charter price. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*

For the small instance the versions with reduced space charter price have shown that there could exist situations where the company can improve the worst case cash flow by increasing the number of ships ordered in the first stage. In this situation the efficient frontiers have a different characteristic as well. The efficient frontiers for the four versions are shown in Figure 5.9. Note how the efficient frontier for the correlated version with $50\%$ reduction in space charter price is close to linear for all worst case cash flows. In addition, it is overlapping with the uncorrelated version except for the last part of the graph. Even though they do not have the same characteristic as for the large instance it is clear that also a small company could benefit from using a cash flow control model. For the correlated version with reduction in space charter price, the relative worst case cash flow can be improved by approximately $90\%$ at a cost of $2\%$ in relative expected profit loss.

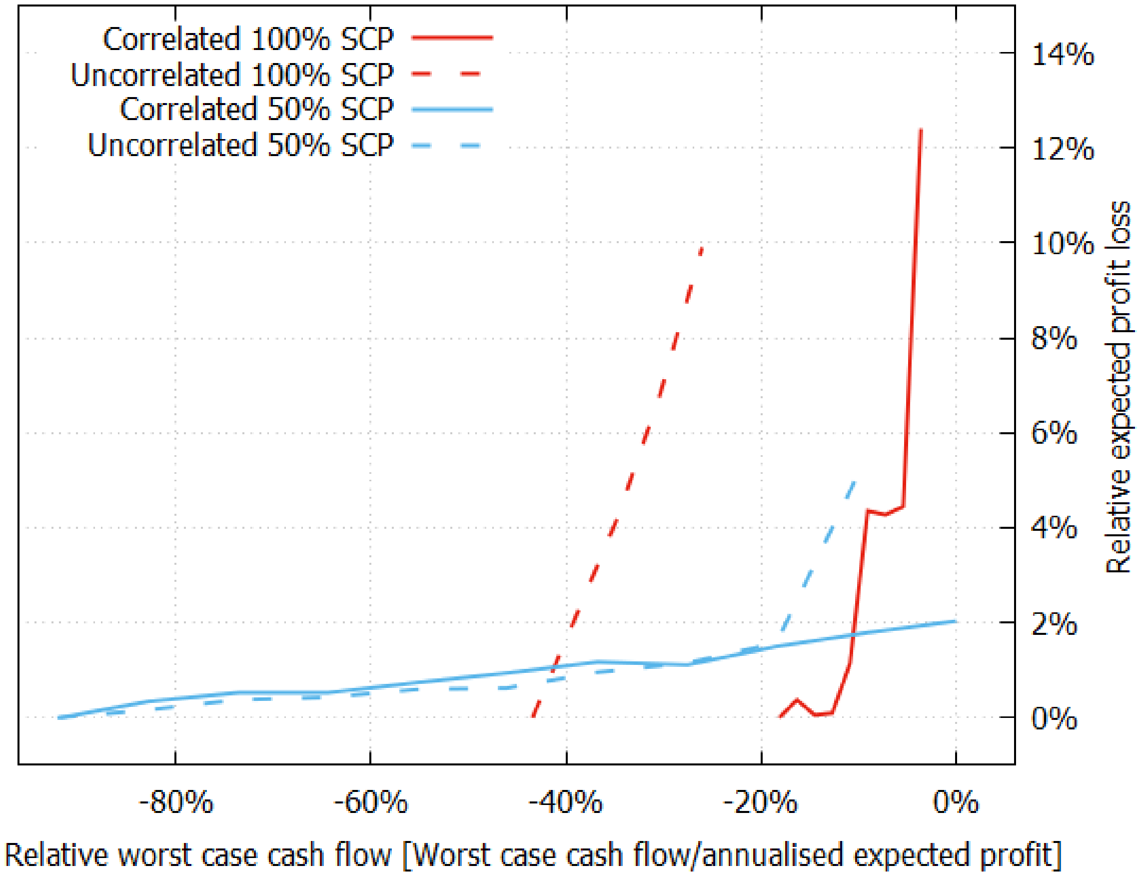



***Figure 5.9:*** *The efficient froniters for the different versions of the small instance. SCP is short for space charter price.*

Furthermore, the uncorrelated version with normal space charter price has a linear efficient frontier, where no improvement of the worst case cash flow can be made without an equivalent loss in expected profit. The correlated version has the efficient frontier that resembles the characteristics of the large instance the most. It has a small section where the worst case cash flow can be improved without any significant cost, but then it behaves as a stepwise function and not as a smooth curve demonstrated in the other instances. This is most likely a result from the size of this instance. Changing any first stage decision in the small instance becomes a much higher relative change than for the large instance. For instance, adding a ship to the initial fleet in the small instance would mean a relative increase of $6.25\%$ of the fleet size. Adding one ship to the large instance would mean an increase of $1.96\%$ in fleet size. This indicates that a change in the solution of the small instance gets a bigger relative impact in the expected profit than the same change in the large instance.

Even though the characteristics of the efficient frontiers in the small instance are different from the corresponding frontiers in the other instances, it is still clear that a company could benefit from using a cash flow control model. In addition, the observation made about increasing number of ordered ships in the first stage, further illustrates the complexity of this decision making process. There is no clear strategy for improving the cash flow and as the 12 cases presented illustrates the strategy depends on the situation. Thus, to come up with the correct solution for the desired worst case cash flow the model needs to be solved, indicating how valuable such a tool can be for a shipping company regardless of the size.

## 5.5 Cash flow effects of the CVaR model for the large instance

This section presents the results from solving the CVaR model with confidence levels of $0.99$, $0.95$ and $0.90$ for the large instance with correlated correlation matrix and normal space charter price. These solutions are compared to the solution of the cash flow control model solved for the same instance. Note that the instance is solved with $100$ scenarios, so the CVaR model solved with confidence level $\alpha = 0.99$ is equivalent to the cash flow control model. Even though these two models are equivalent they have two different set of constraints. This results in the possibility that the solver chooses different paths through the branch and bound algorithm, and with $1\%$ optimality gap this explains the small differences found between the solutions. However, the differences are insignificant, and the solutions for $\alpha = 0.99$ is therefore not presented in detail here, but can be found in Section C.3 in Appendix C.

Table 5.12 presents the solutions for the CVaR model with confidence level of $0.95$. When limiting the expected cash flow of the $5\%$ worst scenarios, the expected profit loss is much lower than for $\alpha = 0.99$ for increasing cash flow limits. This can easily be seen comparing the efficient frontiers in Figure 5.11. Note that the x-axis is different compared to the previous efficient frontiers presented. Instead of relative worst case cash flow the x-axis represents the relative expected cash flow limit which is the minimum expected cash flow allowed $\bar{F}_\alpha$ divided by annualised expected profit. However, for $\alpha = 0.99$ in this particular case, it translates into the worst case cash flow.

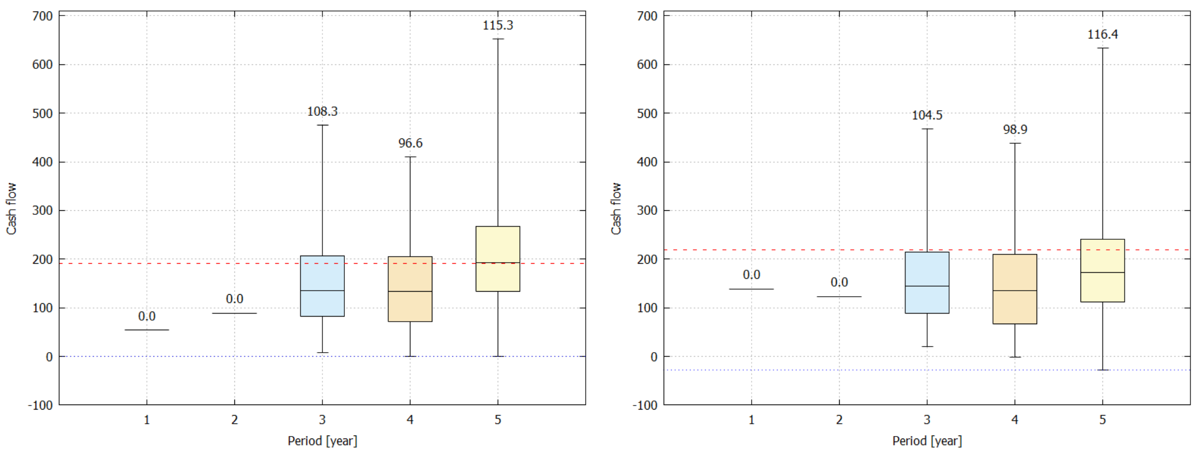
The main observation made from Figure 5.11 is that the increase in cash flow limits results in equal expected profit loss at first, but as the limit increases so does the difference in expected profit loss. This shows that by accepting a $95\%$ confidence of obtaining cash flows above the given limit rather than $99\%$ confidence, a significant improvement of the expected profit can be made. For example, for the relative expected cash flow limit of $0\%$ the relative expected profit can be improved by approximately $12\%$ by accepting a higher risk regarding the worst case cash flow.

Figure 5.10 shows the cash flow development for these two solutions. For $\alpha = 0.95$ the worst case cash flow is $\bar{F}_\alpha = -27.2$, while for $\alpha = 0.99$ it is $0$. In addition, the uncertainty has gotten larger in periods $4$ and $5$ as expected when accepting more risk. This is seen by the increase in standard deviation. However, the increased uncertainty in periods $4$ and $5$ come with a benefit of reduced uncertainty in period $3$.

Increasing the confidence level further to $\alpha = 0.90$ the CVaR constraints never become binding which means that for any $\bar{F}_\alpha$ the solution stays the same, until infeasibility is reached. This explains why the efficient frontier for $\alpha = 0.90$ is flat in Figure 5.11. The decrease in expected

*__Table 5.12:__ Solutions for the large instance with correlated correlation matrix and normal space charter price for the CVaR model with $\alpha = 0.95$.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}_\alpha$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -66.3 | 1109.4 | 29 | 1.99 | 9 | 19.09 | 0.37 | 12.9 | 0 | 9 924 | 0 | 1.55 |
| -59.7 | 1109.1 | 29 | 2 | 9 | 19.16 | 0.37 | 12.85 | 0 | 10 162 | 0 | 1.54 |
| -53.1 | 1109.9 | 30 | 1.99 | 9 | 19.58 | 0.37 | 13.7 | 0 | 454 | 0 | 1.46 |
| -46.4 | 1110.4 | 29 | 1.99 | 9 | 19.1 | 0.37 | 12.8 | 0 | 12 184 | 0 | 1.46 |
| -39.8 | 1109.1 | 29 | 2.12 | 9 | 19.3 | 0.37 | 12.66 | 0 | 12 998 | 0 | 1.46 |
| -33.2 | 1110.0 | 29 | 2.28 | 9 | 19.35 | 0.37 | 12.57 | 0 | 11 082 | 0 | 1.47 |
| -26.5 | 1109.4 | 28 | 2.19 | 8 | 19.34 | 0.41 | 12.62 | 0 | 12 642 | 0 | 1.45 |
| -19.9 | 1108.6 | 29 | 1.99 | 9 | 19.11 | 0.37 | 12.91 | 0 | 10 713 | 0 | 1.55 |
| -13.3 | 1109.1 | 29 | 2.01 | 8 | 20.08 | 0.41 | 12.76 | 0 | 5 811 | 0 | 1.46 |
| -6.6 | 1108.2 | 27 | 2.26 | 9 | 17.58 | 0.37 | 12.66 | 0 | 26 938 | 0 | 1.45 |
| 0 | 1093.7 | 26 | 2.39 | 8 | 17.86 | 2.37 | 12.62 | 0 | 26 128 | 0 | 1.45 |
| 6.6 | 1078.2 | 27 | 2.43 | 9 | 17.91 | 4.41 | 12.71 | 0 | 26 004 | 0 | 1.51 |
| 13.3 | 1033.0 | 23 | 3.05 | 4 | 19.89 | 8.27 | 12.69 | 5 992 | 32 448 | 0 | 1.4 |




***(a)** Cash flow control model / $\alpha = 0.99$ ($\bar{F} = 0$)* ***(b)** CVaR model with $\alpha = 0.95$ ($\bar{F}_\alpha = 0$)*

*__Figure 5.10:__ Comparing the cash flow development between the cash flow control model and the CVaR model with $\alpha = 0.95$ for the large instance with correlated correlation matrix and normal space charter price. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*

profit loss towards the end is due to the 1% optimality gap, and the solutions can be seen in detail in Section C.3 in Appendix D. The fact that the CVaR constraints never become binding means that for this instance solved without cash flow constraints the expected cash flow of the

$10\%$ worst scenarios already are better than 13.3, which is when the CVaR model becomes infeasible for $\alpha = 0.90$ and $\alpha = 0.95$.

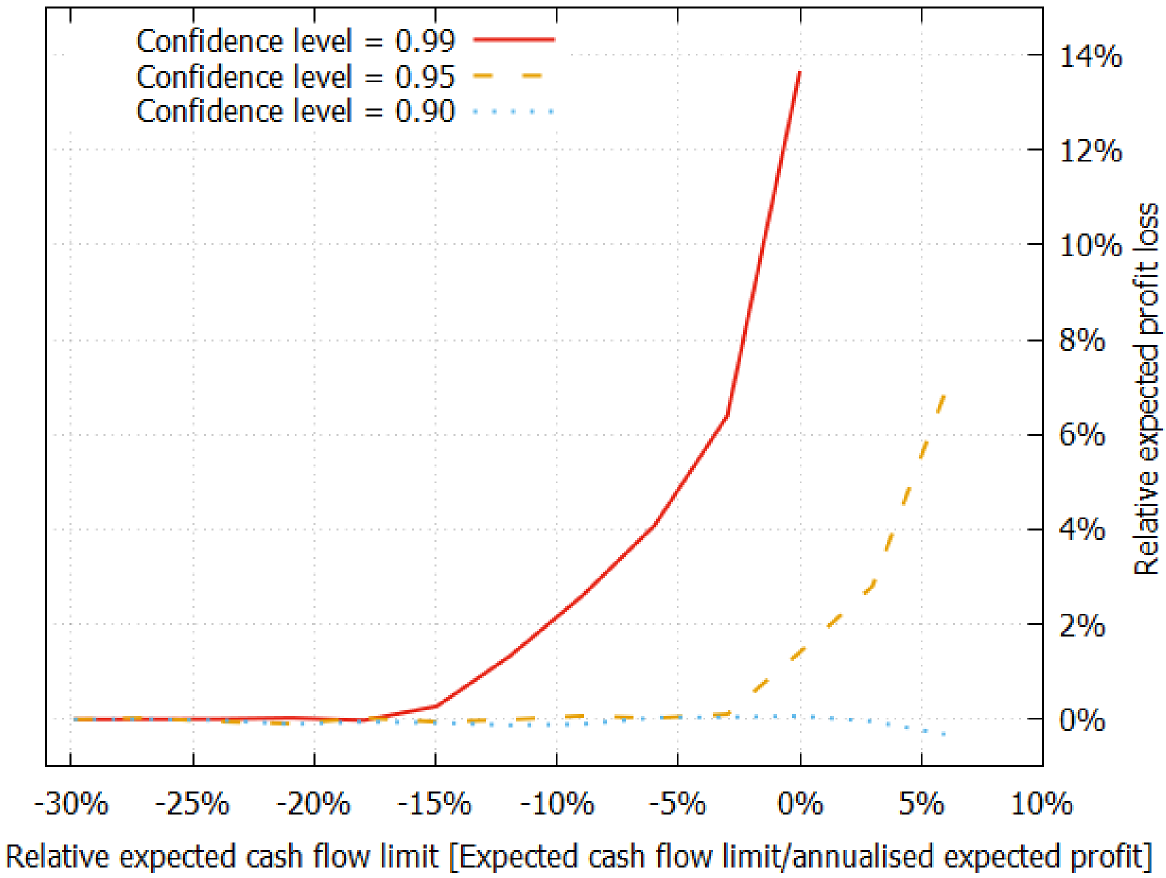



***Figure 5.11:*** *The efficient froniters for the CVaR model solved for the large instance with correlated correlation matrix and normal space charter price*

Section 5.3.5 discusses how to reduce the risk of insolvency using the cash flow control model, which equivalently also applies to the CVaR model solved with the highest confidence level selecting just one scenario, in this case $\alpha = 0.99$. However, using the CVaR model the risk of insolvency can be quantified to some extent. This is best illustrated using an example based on Figure 5.11. Assume the company faces cash flow insolvency if the cash flow becomes negative in any scenario. Then the CVaR model can be solved for $\alpha = 0.99$ obtaining a solution that guarantees the cash flow to be positive for all scenarios, seen at the efficient frontier for $\alpha = 0.99$ with relative expected cash flow limit of $0\%$. Reducing the confidence level leads to an increase in the expected profit at the cost of increased risk. Reducing the confidence level to $\alpha = 0.95$ the company will avoid insolvency with $95\%$ confidence. Moreover, note that the confidence level is not a direct measure of the probability for insolvency to occur. It only states that the expected cash flow limited is including the $(1 - \alpha) * 100\%$ worst scenarios. This however implies that the probability of achieving worse cash flows than the expected cash flow limit is maximum $(1 - \alpha) * 100\%$ and minimum 0. For $\alpha = 0.95$ the maximum number of scenarios that can be worse than the expected cash flow limit are $4$ in this case, because the mean or expected value can never be higher than the values it is computed from. On the other hand, the minimum of scenarios worse than the expected cash flow limit is $0$ if all the five worst scenarios are equal. This shows how the confidence level provides the manager with an upper limit of the probability for an event to occur, in this case scenarios with worse cash flows than the expected cash flow limit.

To determine which confidence level a company should choose depends on their current situation and risk preferences. Like mentioned in Section 5.3.5, the company's utility of a solution might change whether they face the risk of cash flow insolvency or balance-sheet insolvency. In addition, the decision maker's risk preferences also affect the choice of which confidence level to choose. A risk averse decision maker will probably choose a high confidence level, which in this case would be close to $0.99$. A risk neutral decision maker will probably maximize expected profit regardless of the risk, thus not using a binding confidence level at all. A risk seeking decision maker will probably not use a binding confidence level either. In addition, he will probably prefer the solution furthest to the left of the efficient frontier in Figure 5.11, because it is the solution with the highest best case scenarios at the cost of higher risk. Note that the solutions between $-30\%$ and $-20\%$ is close to indifferent with respect to expected profit. Hence, it implies that the higher risk of worse case cash flow has higher best case cash flow, because the expected profit stays the same.

To summarize, the CVaR model provides the decision maker with the possibility of choosing a confidence level which can be interpreted as a maximum limit of the risk relative to the risk faced when solving the cash flow control model. This can serve as a good tool for the company to choose a risk level matching their situation and risk preference.

## 5.6 From a two-stage case to a three-stage case

The instances solved this far in the thesis are solved using a two-stage model, but a multi-stage model would better describe the reality. In the real world a company experience a continuous stream of new information, for instance changing fuel prices from day to day. A multi-stage model will represent this behaviour more accurately. However, a multi-stage model would increase the complexity, making the model harder to solve, and therefore the problem is often simplified to a two-stage problem and solved using a two-stage model. This section will therefore solve the smallest instance as a three-stage model and compare the solution to the two-stage solution, in order to examine whether the two-stage model is a reasonable simplification.

In the three-stage model the scenarios branch in periods $3$ and $5$. An example of a realisation of a random variable with nine scenarios using this scenario tree are shown in Figure 5.12. Every branch node has to branch into at least as many nodes as there are random variables due to requirements in the scenario generation algorithm. Therefore, the instance with the fewest random variables is used to compare a three-stage model to a two-stage model, namely the small instance. The number of branches in each branch period is therefore set to $20$, resulting in the total number of scenarios to be $20 * 20 = 400$. Furthermore, the correlated correlation matrix requires a higher number of scenarios for the scenario algorithm to work, resulting in very long computational times for the three-stage model. Moreover, the version with $50\%$ reduction in space charter price has a wider range where the worst case cash flow can be improved, seen in Figure 5.9. Therefore, the small instance with uncorrelated correlation matrix and reduced space charter price is used for this part of the computational study.

The three-stage solutions for the small instance with uncorrelated correlation matrix and $50\%$ reduction in space charter price can be seen in Table 5.13. Note that ships cannot be ordered or scrapped in the last period, i.e. in the third stage, and are thus only presented with the first and second stage decisions. Comparing these results to the two-stage solution in Table C.7 in Appendix C, the results are similar, but there are some interesting differences. The second stage orders are increased and the ships scrapped in the second stage are reduced. In addition, the optional trades are used more actively. There is a significant increase in ships put on lay-up in the last period as well. This comes from the fact that the company prepares for the third stage and thus has an overcapacity in the low demand scenarios. In the two-stage model this would be avoided, because the company would know the demand in period $5$ with certainty when arriving to period $3$. Therefore they could scrap the excess ships at the end of period $3$ or $4$.

The efficient frontiers for the three and two stage solutions are shown in Figure 5.13. They both have similar characteristics, where there is a section where the worst case cash flow is increased at a small cost in expected profit loss, and a section where the cost is rapidly increasing with worst case cash flow improvement. This indicates that the characteristics found in the efficient

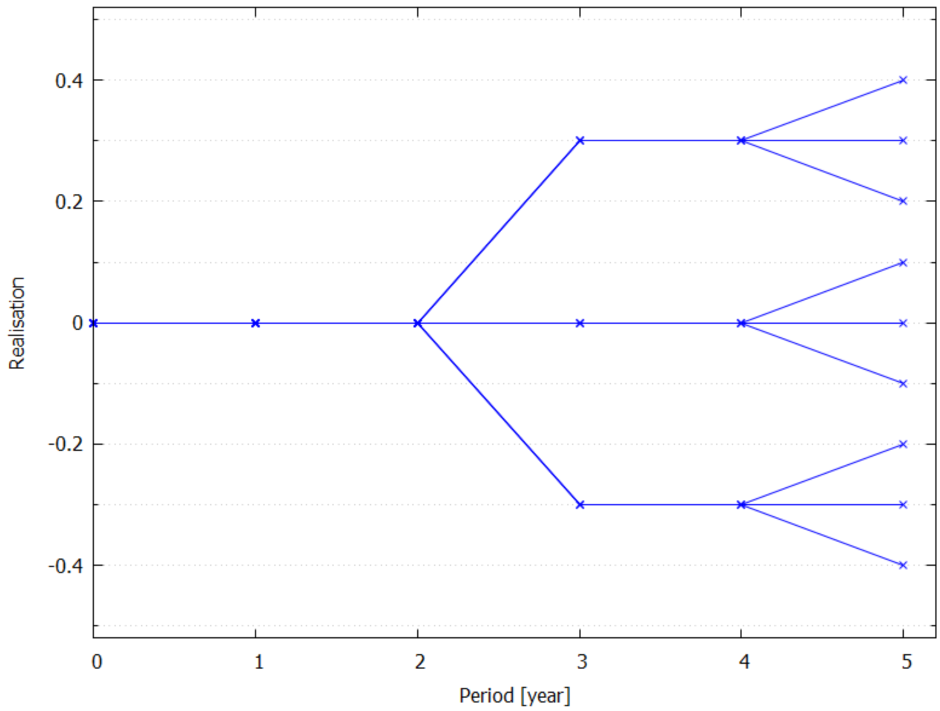



***Figure 5.12:*** *Realisation of one random variable in a three-stage model. Illustrating a possible realisation of one random variable with nine scenarios in a three-stage problem, branching in periods* 3 *and* 5.

frontiers in the two-stage instances are similar also for a three-stage instance.

However, the results do show some differences between the two-stage and three-stage solutions, but they are mainly in the second stage. The first stage decisions are very similar, and for a shipping company it is mainly the here and now decisions that are of the greatest interest. Therefore, these results indicate that a two-stage model with $100$ scenarios work as a reasonable simplification of a three-stage model with $400$ scenarios. The fact that the $100$ scenario two-stage model gives similar first stage solutions as the $400$ scenario three-stage model is an important finding with respect to computational time. This shows that a larger number of scenarios do not add much improvements to the first stage decisions, which makes it possible to solve the model with fewer scenarios saving computational time.

In addition, solving the largest instance with a three-stage model would require minimum $45 * 45 = 2025$ scenarios with the scenario generation algorithm used for this thesis, which was impossible to solve with the computer used for this computational study. Thus, implementing a multi-stage model increases the computational time drastically. To sum up, the two-stage simplification seems to be a good trade-off between computational time and quality of the first stage decisions, at least for the small instance with uncorrelated correlation matrix and reduced space charter price.

***Table 5.13:*** *Three-stage solutions for the small instance with uncorrelated correlation matrix and* $50\%$ *reduction in space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | |
|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | Third |
| -44.1 | 320.6 | 6 | 2.9 | 3 | 3.25 | 0.12 | 4.32 | 2.83 |
| -39.7 | 320.6 | 7 | 2.45 | 3 | 3.65 | 0 | 4.6 | 2.92 |
| -35.3 | 320.6 | 7 | 2.4 | 3 | 3.65 | 0 | 4.6 | 2.87 |
| -30.9 | 321.4 | 7 | 2.45 | 3 | 3.65 | 0.12 | 4.6 | 2.92 |
| -26.5 | 321.4 | 7 | 2.5 | 3 | 3.65 | 0.12 | 4.6 | 2.96 |
| -22.1 | 320.9 | 8 | 2.05 | 3 | 4.25 | 0.12 | 4.68 | 2.9 |
| -17.7 | 319.6 | 8 | 1.75 | 3 | 4.3 | 0.12 | 4.61 | 2.71 |
| -13.2 | 317.5 | 8 | 1.85 | 3 | 4.45 | 0.12 | 4.45 | 2.7 |
| -8.8 | 312.0 | 8 | 1.6 | 3 | 4.3 | 0.87 | 4.51 | 2.64 |
| -4.4 | 287.8 | 8 | 1.75 | 3 | 4.5 | 2.78 | 4.5 | 2.6 |

| | Exp. | Space | | | Optional | | |
|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | Third | First | Second | Third |
| -44.1 | 320.6 | 17 005 | 49 237 | 17 771 | 0 | 0.63 | 0.94 |
| -39.7 | 320.6 | 18 736 | 45 130 | 17 252 | 0.33 | 1 | 1.01 |
| -35.3 | 320.6 | 18 736 | 45 130 | 17 450 | 0.33 | 1 | 1.01 |
| -30.9 | 321.4 | 17 005 | 37 829 | 15 978 | 0 | 0.63 | 0.94 |
| -26.5 | 321.4 | 17 005 | 37 829 | 15 553 | 0 | 0.63 | 0.91 |
| -22.1 | 320.9 | 17 005 | 26 709 | 15 966 | 0 | 0.63 | 0.93 |
| -17.7 | 319.6 | 17 005 | 26 709 | 22 513 | 0 | 0.63 | 0.91 |
| -13.2 | 317.5 | 17 005 | 26 384 | 23 403 | 0 | 0.58 | 0.89 |
| -8.8 | 312.0 | 17 005 | 33 685 | 27 388 | 0.33 | 1 | 1.01 |
| -4.4 | 287.8 | 53 595 | 33 685 | 28 811 | 0.33 | 1 | 1.01 |

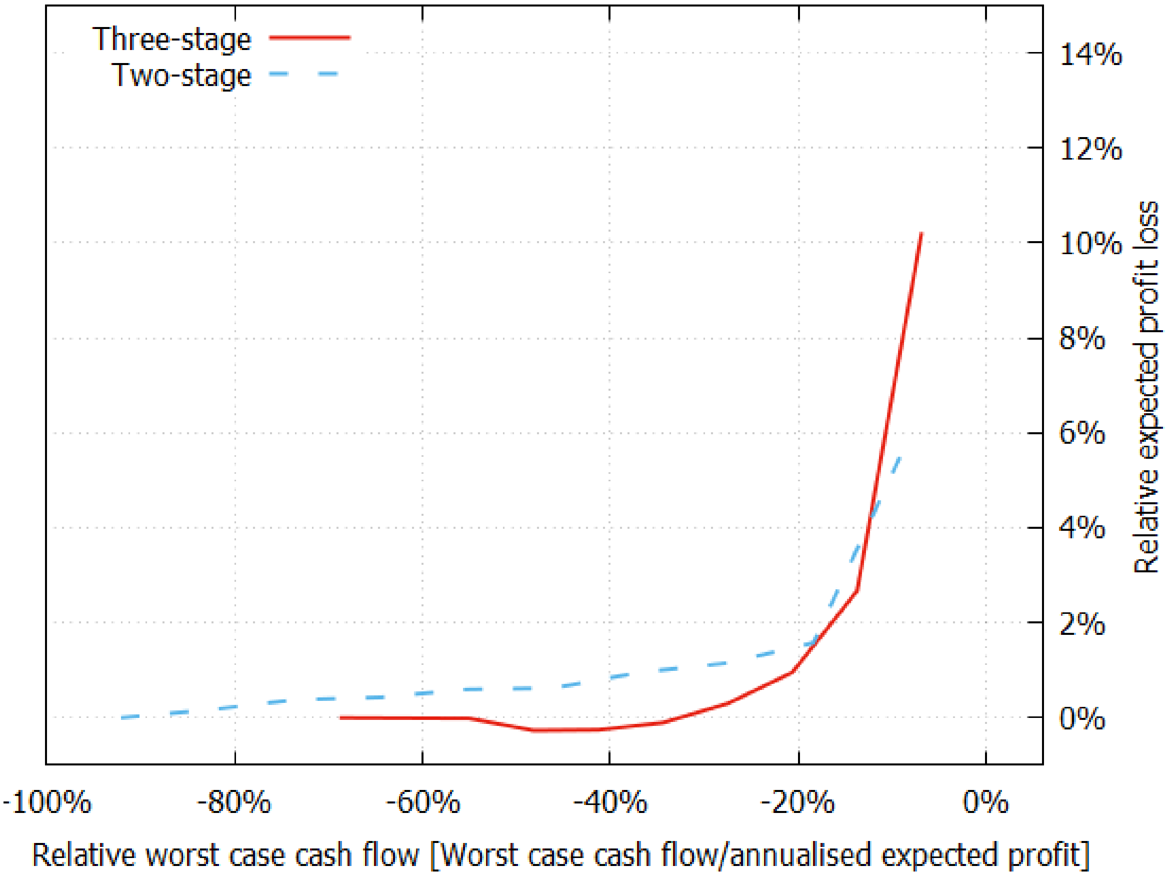



***Figure 5.13:*** *The efficient frontiers for the three and two stage solutions for the small instance with uncorrelated correlation matrix and reduced space charter price.*

Chapter 6

# Concluding remarks and future research

This thesis introduces two new models for solving the Maritime Fleet Renewal Problem (MFRP) focusing on the risk of insolvency. Payments of ships are modelled as instalments rather than lump sums for both models. The cash flow control model maximises profit while limiting the worst case cash flow to a given limit. Moreover, the CVaR model replaces the cash flow constraints in the cash flow control model with CVaR constraints limiting the expected cash flow of the $(1-\alpha) * 100\%$ worst scenarios, where $\alpha$ is a predetermined confidence level. The cash flow control model is proved to be a special case of the CVaR model having such a high confidence level that just one scenario is included in the expected cash flow. The current literature indicates that payments modelled as instalments and the control of cash flows have not been applied to these type of problems before.

This thesis demonstrates that solutions of the cash flow control model for increasing cash flow limits improve the cash flow in the worst scenario. However, this comes at the cost of reduced expected profit. Moreover, this thesis shows that the strategy for improving the worst case cash flow is dependent on the situation. This illustrates how complex the MFRP is and how valuable operations research are for this problem.

Furthermore, solving the CVaR model for a set of confidence levels demonstrates how the company can adjust their risk level according to their risk preference when facing the risk of insolvency. CVaR is chosen as a risk measure for this thesis, but there exist other measures that also might be appropriate. In addition, this thesis tests a three-stage implementation of the cash flow control model indicating that the two-stage simplifications are reasonable.

Regarding future research, there are several directions that can be pursued and some of them are briefly discussed here. One possible direction is to implement the models also including second hand ships and charters to see if this affects the cash flow characteristics identified in this thesis. It would be expected to reduce the need of controlling the cash flows, because the number of recourse actions increase. This is anticipated to reduce the consequences of a first stage decision that turned out to be unfavourable. However, further research is required to verify this expected behaviour.

Another direction to take could be to develop a model including cash flow reserves. For this thesis it is assumed that the decision maker is able to come up with the correct cash flow limit when solving the models. This might not be the case and by including cash flow reserves the necessary cash flow limit could be imposed. In this way allocation of the profit could be included as well. Thus, profits from good years can be kept as a reserve to prepare for future uncertainties.

Focusing on solution algorithms is also a possible direction to pursue. Especially if it is desired to test a multi-stage implementation for larger instances than the three-stage instance that was solved in this thesis. The computer used for the computational study for this thesis never found a solution to the large three-stage instance. An improvement of the solution time here can probably be made improving the algorithms for solving the model, but also on the scenario generation. The main problem regarding the computational times for the large three-stage instance were located to the huge number of scenarios required to generate the scenario tree. Improving the scenario generation making it possible to generate fewer scenarios for the same number of random variables could possibly improve the computational time.

To summarize, this thesis demonstrates how the models presented can help the decision maker find a suitable risk level when facing the risk of insolvency. However, as briefly discussed in this section there are still several topics within the MFRP that can benefit from further research.

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# Appendices

Appendix A

# Scenario formulation

## A.1 Cash flow control model

Sets

| | |
|---|---|
| $\mathcal{T}$ | Set of periods, indexed by $t$ |
| $\mathcal{S}$ | Set of scenarios, indexed by $s$ |
| $\mathcal{S}_{ts}^{NA}$ | Set of all scenarios that are connected to scenario $s$ in period $t$, meaning that decisions made in scenario $s$ in period $t$, must be the same for all scenarios in this set |
| $\mathcal{K}$ | Set of products, indexed by $k$ |
| $\mathcal{V}_t$ | Set of ship types existing in the market in period $t$, indexed by $v$ |
| $\mathcal{V}_t^N$ | Set of new ship types existing in the market in period $t$ |
| $\mathcal{V}_t^{IN}$ | The set of ship types the company pays instalments for in period $t$ |
| $\mathcal{N}_t$ | Set of trades operated in period $t$, indexed by $i$ |
| $\mathcal{N}_t^C$ | Set of contractual trades the shipping company is committed to serve in period $t$, indexed by $i$ |
| $\mathcal{N}_t^O$ | Set of optional trades the shipping company can choose to undertake or not in period $t$, indexed by $i$ |
| $\mathcal{R}_t$ | Set of loops in period $t$, indexed by $r$ |
| $\mathcal{R}_{vt}$ | Set of loops that can be sailed by a ship of type $v$ in period $t$ , indexed by $r$ |

| | |
|---|---|
| $\mathcal{R}_{ivt}$ | The set of loops servicing trade $i$ that can be sailed in period $t$ by a ship of type $v$, indexed by $r$ |

Parameters

| | |
|---|---|
| $P_s$ | The probability for scenario $s$ to take place |
| $R^D_{its}$ | The revenue of transporting one unit of goods on trade $i$, at period $t$, and scenario $s$ |
| $R^{SE}_{vts}$ | The selling price of a ship of type $v$, in period $t$, and scenario $s$ |
| $R^{SC}_{vts}$ | The scrapping value of ship of type $v$, in period $t$, and scenario $s$ |
| $R^{LU}_{vts}$ | The lay-up savings for one period, for ship of type $v$, in period $t$, and scenario $s$ |
| $R^{SV}_{vs}$ | The sunset value of a ship of type $v$, in scenario $s$ |
| $R^{CO}_{vts}$ | The charter out revenue for one period, for ship of type $v$, in period $t$ and scenario $s$ |
| $C^{CI}_{vts}$ | The charter in cost for ship of type $v$, in period $t$ and scenario $s$ |
| $C^{OP}_{vts}$ | The fixed operating cost for ship of type $v$, in period $t$ and scenario $s$ |
| $C^{TR}_{vrts}$ | The cost of performing a loop $r$, for ship of type $v$, in period $t$ and scenario $s$ |
| $C^{SP}_{ikts}$ | The space charter cost for one unit of product $k$ on trade $i$ , in period $t$, and scenario $s$ |
| $\overline{CI}_{vts}$ | The limit on the number of ships of type $v$ available for chartering in in period $t$ under scenario $s$ |
| $\overline{CO}_{vts}$ | The limit on the number of ships of type $v$ available for chartering out in period $t$ under scenario $s$ |
| $\overline{SH}_{vts}$ | The limit on the number of ships of type $v$ available for purchasing in the second hand market in period $t$ under scenario $s$ |
| $\overline{SE}_{vts}$ | The limit on the number of ships of type $v$, that can be sold in the second hand market, in period $t$ under scenario $s$ |
| $\overline{CI}_{ts}$ | The limit of the total number of ships that can be chartered in in period $t$ under scenario $s$ |

| | |
|---|---|
| $\overline{CO}_{ts}$ | The limit of the total number of ships that can be chartered out in period $t$ under scenario $s$ |
| $\overline{SH}_{ts}$ | The limit of the total number of ships that can be bought in the second hand market in period $t$ under scenario $s$ |
| $\overline{SE}_{ts}$ | The limit of the total number of ships that can be sold in the second hand market in period $t$ under scenario $s$ |
| $\bar{T}_v^L$ | The lead time for building a ship of type $v$ |
| $Q_{vk}$ | The total capacity of product $k$ on ship of type $v$ |
| $Q_v$ | The total capacity on ship of type $v$ |
| $Z_{vr}$ | The time a ship of type $v$ needs to perform a loop $r$ |
| $Z_v$ | The total available time in one period for a ship of type $v$ |
| $D_{ikts}$ | The demand on trade $i$ of product $k$ in period $t$ and scenario $s$ |
| $F_{it}$ | The frequency requirement on trade $i$ in period $t$ |
| $Y_{vt}^{NB}$ | The number of ships of type $v$ ordered in the previous planning period, delivered at the start of period $t$ |
| $Y_v^{IP}$ | The initial fleet of ships of type $v$ |
| $C_{vt'ts}^{IN}$ | The instalment paid in period $t$ and scenario $s$ on the ship of type $v$ ordered in period $t'$ and scenario $s$ |
| $C_{vt}^{IN}$ | The instalment paid for a ship of type $v$ in period $t$ for the ships in the initial fleet, i.e before the planning horizon begins |
| $C_{vt'ts}^{SH}$ | The instalment paid in period $t$ and scenario $s$, on a ship of type $v$ bought in the second hand market at time $t'$ and scenario $s$ |
| $M$ | Number of instalments |
| $\bar{F}$ | The cash flow limit, i.e. no period and scenario are allowed to have a worse cash flow than this limit |
| $B$ | The budget available for ordering or purchasing ships in period 0 |

Variables

| | |
|---|---|
| $y_{vt's}^{NB}$ | The number of new buildings ordered of ships of type $v$, in period $t'$, and scenario $s$ |

| | |
|---|---|
| $y^{SH}_{vt's}$ | The number of ships bought in the second hand market of ship of type $v$, in period $t'$ and scenario $s$ |
| $y^{SE}_{vts}$ | The number of ships of type $v$ sold in the second hand market , in period $t$, and scenario $s$ |
| $y^{SC}_{vts}$ | The number of ships of type $v$ scrapped in period $t$ and scenario $s$ |
| $y^{P}_{vts}$ | The number of ships of type $v$ in the pool, in period $t$ and scenario $s$ |
| $h^{I}_{vts}$ | The number of ships of type $v$ chartered in, in period $t$ and scenario $s$ |
| $h^{O}_{vts}$ | The number of ships of type $v$ chartered out, in period $t$ and scenario $s$ |
| $l_{vts}$ | The number of ships of type $v$ on lay-up, in period $t$ and scenario $s$ |
| $x_{vrts}$ | The number of loops $r$ performed by a ship of type $v$, in period $t$ and scenario $s$ |
| $n_{ikts}$ | The amount of product $k$ delivered in period $t$, by space charters on trade $i$ and scenario $s$ |
| $\delta_{its}$ | Set to 1 if the company services optional trade $i$ in period $t$ and scenario $s$, 0 otherwise. |
| $f^{I}_{ts}$ | The cash inflow in period $t$ and scenario $s$ |
| $f^{O}_{ts}$ | The cash outflow in period $t$ and scenario $s$ |

**Cash flow expressions**

$$f^I_{ts} = \sum_{i\in\mathcal{N}^O_t}\sum_{k\in\mathcal{K}} R^D_{its}D_{ikts}\delta_{its} + \sum_{i\in\mathcal{N}^C_t}\sum_{k\in\mathcal{K}} R^D_{its}D_{ikts}$$

$$+ \sum_{v\in\mathcal{V}_t}(R^{CO}_{vts}h^O_{vts} + R^{SE}_{vts}y^{SE}_{vts} + R^{LU}_{vts}l_{vts} + R^{SC}_{vts}y^{SC}_{vts}), \qquad t\in\mathcal{T}\setminus\{0\}, s\in\mathcal{S}$$

$$f^O_{ts} = \sum_{v\in\mathcal{V}^{IN}_t} C^{IN}_{vts}Y^{IP}_v + \sum_{t-M\le t'\le t}\sum_{v\in\mathcal{V}^{IN}_t}(C^{IN}_{vt'ts}y^{NB}_{vt's} + C^{SH}_{vt'ts}y^{SH}_{vt's})$$

$$+ \sum_{i\in\mathcal{N}^C_t}\sum_{k\in\mathcal{K}} C^{SP}_{ikts}n_{ikts} + \sum_{v\in\mathcal{V}_t}(C^{OP}_{vts}y^P_{vts} + C^{CI}_{vts}h^I_{vts} + \sum_{r\in\mathcal{R}_{vt}} C^{TR}_{vrts}x_{vrts}), \qquad t\in\mathcal{T}\setminus\{0\}, s\in\mathcal{S}$$

**Objective function**

$$max\ z = \sum_{s\in\mathcal{S}} P_s \Big( \sum_{t\in\mathcal{T},t>0} \big( \sum_{i\in\mathcal{N}_t^O} \sum_{k\in\mathcal{K}} R_{its}^D D_{ikts}\delta_{its} \tag{A.1a}$$

$$+ \sum_{i\in\mathcal{N}_t^C} \sum_{k\in\mathcal{K}} (R_{its}^D D_{ikts} - C_{ikts}^{SP} n_{ikts}) \tag{A.1b}$$

$$- \sum_{v\in\mathcal{V}_t} (C_{vts}^{OP} y_{vts}^P + C_{vts}^{CI} h_{vts}^I - R_{vts}^{CO} h_{vts}^O \tag{A.1c}$$

$$+ \sum_{r\in\mathcal{R}_{vt}} C_{vrts}^{TR} x_{vrts} - R_{vts}^{LU} l_{vts})\big) \tag{A.1d}$$

$$- \sum_{t\leq M-1} \sum_{v\in\mathcal{V}_t^{IN}} C_{vt}^{IN} Y_v^{IP} \tag{A.1e}$$

$$- \sum_{t\in\mathcal{T}} \sum_{t-M\leq t'\leq t} \sum_{v\in\mathcal{V}_t^{IN}} (C_{vt'ts}^{IN} y_{vt's}^{NB} + C_{vt'ts}^{SH} y_{vt's}^{SH}) \tag{A.1f}$$

$$+ \sum_{v\in\mathcal{V}_{\bar{T}}} (R_{vs}^{SV} y_{v\bar{T}s}^P - \sum_{t=\bar{T}}^{t'+M} \sum_{t'\in\mathcal{T}} C_{vt'ts}^{IN} y_{vt's}^{NB} + C_{vt'ts}^{SH} y_{vt's}^{SH}) \tag{A.1g}$$

$$+ \sum_{t\in\mathcal{T}} \sum_{v\in\mathcal{V}_t} (R_{vts}^{SC} y_{vts}^{SC} + R_{vts}^{SE} y_{vts}^{SE})\Big) \tag{A.1h}$$

**Demand constraints**

$$\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} Q_{vk} x_{vrts} + n_{ikts} \geq D_{ikts}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^C, k\in\mathcal{K}, s\in\mathcal{S}, \tag{A.2}$$

$$\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} Q_{vk} x_{vrts} \geq D_{ikts}\delta_{its}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^O, k\in\mathcal{K}, s\in\mathcal{S}, \tag{A.3}$$

**Capacity constraints**

$$\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} Q_v x_{vrts} + \sum_{k\in\mathcal{K}} n_{ikts} \geq \sum_{k\in\mathcal{K}} D_{ikts}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^C, s\in\mathcal{S}, \tag{A.4}$$

$$\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} Q_v x_{vrts} \geq \sum_{k\in\mathcal{K}} D_{ikts}\delta_{its}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^O, s\in\mathcal{S}, \tag{A.5}$$

**Frequency constraints**

$$\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} x_{vrts} \geq F_{it}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^C, s\in\mathcal{S}, \tag{A.6}$$

$$\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} x_{vrts} \geq F_{it}\delta_{its}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^O, s\in\mathcal{S}, \tag{A.7}$$

### Time constraints

$$\sum_{r \in \mathcal{R}_{vt}} Z_{rv} x_{vrts} \leq Z_v (y^P_{vts} + h^I_{vts} - h^O_{vts} - l_{vts}), \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{A.8}$$

### Optional trades constraints

$$\delta_{its} \leq \delta_{i,t+1,s} \qquad t \in \mathcal{T} \setminus \{0, \bar{T}\}, i \in \mathcal{N}^O_t, s \in \mathcal{S}, \tag{A.9}$$

### Pool constraints

$$y^P_{v0s} = Y^{IP}_v \qquad v \in \mathcal{V}_0, s \in \mathcal{S}, \tag{A.10}$$

$$y^P_{vts} = y^P_{v,t-1,s} - y^{SC}_{v,t-1,s} + y^{SH}_{v,t-1,s} - y^{SE}_{v,t-1,s} \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t \setminus \mathcal{V}^N_t, s \in \mathcal{S}, \tag{A.11}$$

$$y^P_{vts} = y^{NB}_{v,t-\bar{T_v}^L,s} \qquad t \in \mathcal{T} : t \geq \bar{T_v}^L, v \in \mathcal{V}^N_t, s \in \mathcal{S}, \tag{A.12}$$

$$y^P_{vts} = Y^{NB}_{vt} \qquad t \in \mathcal{T} : t < \bar{T_v}^L, v \in \mathcal{V}^N_t, s \in \mathcal{S}, \tag{A.13}$$

$$y^P_{vts} \geq l_{vts} - h^I_{vts} + h^O_{vts} \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{A.14}$$

$$y^P_{vts} = y^{SC}_{vts} \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t \setminus \mathcal{V}_{t+1}, s \in \mathcal{S}, \tag{A.15}$$

### Charters and second hand constraints

$$y^{SH}_{vts} \leq \overline{SH}_{vts}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{A.16}$$

$$y^{SE}_{vts} \leq \overline{SE}_{vts}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{A.17}$$

$$h^I_{vts} \leq \overline{CI}_{vts}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{A.18}$$

$$h^O_{vts} \leq \overline{CO}_{vts}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{A.19}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} y^{SH}_{vts} \leq \overline{SH}_{ts}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, s \in \mathcal{S}, \tag{A.20}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} y^{SE}_{vts} \leq \overline{SE}_{ts}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, s \in \mathcal{S}, \tag{A.21}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} h^I_{vts} \leq \overline{CI}_{ts}, \qquad t \in \mathcal{T} \setminus \{0\}, s \in \mathcal{S}, \tag{A.22}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} h^O_{vts} \leq \overline{CO}_{ts}, \qquad t \in \mathcal{T} \setminus \{0\}, s \in \mathcal{S}, \tag{A.23}$$

**Cash flow constraints**

$$\sum_{v\in\mathcal{V}_0}(R^{SE}_{v0s}y^{SE}_{v0s}+R^{SC}_{v0s}y^{SC}_{v0s})-\sum_{v\in\mathcal{V}_0^{IN}}(C^{IN}_{v00s}y^{NB}_{v0s}+C^{SH}_{v00s}y^{SH}_{v0s})+B\geq\bar{F}, \qquad s\in\mathcal{S}, \tag{A.24}$$

$$f^I_{ts}-f^O_{ts}\geq\bar{F}, \qquad t\in\mathcal{T}\setminus\{0\}, s\in\mathcal{S}, \tag{A.25}$$

**Non-anticipativity constraints**

$$y^{SC}_{vts}=y^{SC}_{vt\bar{s}}, \qquad t\in\mathcal{T}\setminus\{\bar{T}\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.26}$$

$$y^{NB}_{vts}=y^{NB}_{vt\bar{s}}, \qquad t\in\mathcal{T}: t\leq\bar{T}-\bar{T}_v^{\,L}, v\in\mathcal{V}^N_t, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.27}$$

$$y^{SH}_{vts}=y^{SH}_{vt\bar{s}}, \qquad t\in\mathcal{T}\setminus\{\bar{T}\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.28}$$

$$y^{SE}_{vts}=y^{SE}_{vt\bar{s}}, \qquad t\in\mathcal{T}\setminus\{\bar{T}\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.29}$$

$$h^I_{vts}=h^I_{vt\bar{s}}, \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.30}$$

$$h^O_{vts}=h^I_{vt\bar{s}}, \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.31}$$

$$l_{vts}=l_{it\bar{s}}, \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.32}$$

$$x_{vrts}=x_{vrt\bar{s}}, \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, r\in\mathcal{R}_{vt}, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.33}$$

$$n_{ikts}=n_{ikt\bar{s}}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}^C_t, k\in\mathcal{K}, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.34}$$

$$\delta_{its}=\delta_{it\bar{s}}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}^O_t, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.35}$$

$$\tag{A.36}$$

**Convexity and integer constraints**

$$y^{NB}_{vts}\in\mathbb{Z}^+, \qquad t\in\mathcal{T}: t\leq\bar{T}-\bar{T}_v^{\,L}, v\in\mathcal{V}^N_{t+T^L}, s\in\mathcal{S}, \tag{A.37}$$

$$y^{SC}_{vts}\in\mathbb{Z}^+, \qquad t\in\mathcal{T}\setminus\{\bar{T}\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \tag{A.38}$$

$$y^{SH}_{vts}\in\mathbb{Z}^+, \qquad t\in\mathcal{T}\setminus\{\bar{T}\}\ v\in\mathcal{V}_t, s\in\mathcal{S}, \tag{A.39}$$

$$y^{SE}_{vts}\in\mathbb{Z}^+, \qquad t\in\mathcal{T}\setminus\{\bar{T}\}\ v\in\mathcal{V}_t, s\in\mathcal{S}, \tag{A.40}$$

$$y^{P}_{vts}\in\mathbb{R}^+, \qquad t\in\mathcal{T}, v\in\mathcal{V}_t, s\in\mathcal{S}, \tag{A.41}$$

$$h^I_{vts}\in\mathbb{R}^+, \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \tag{A.42}$$

$$h^O_{vts}\in\mathbb{R}^+, \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \tag{A.43}$$

$$l_{vts}\in\mathbb{R}^+, \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \tag{A.44}$$

$$x_{vrts}\in\mathbb{R}^+, \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, r\in\mathcal{R}_{vt}, s\in\mathcal{S}, \tag{A.45}$$

$$n_{ikts}\in\mathbb{R}^+, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}^C_t, k\in\mathcal{K}, s\in\mathcal{S}, \tag{A.46}$$

$$\delta_{its}\in\{0,1\}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}^O_t, s\in\mathcal{S} \tag{A.47}$$

## A.2 Conditional Value-at-Risk model

Replacing the cash flow constraints in the cash flow control model with the following sets, variables and constraints results in the Conditional Value-at-Risk model.

| | |
|---|---|
| **Sets** | |
| $\mathcal{T}^F$ | The set of periods in the first stage |
| $\mathcal{T}^S$ | The set of periods under uncertainty, i.e. all periods after the first stage |
| **Parameters** | |
| $\alpha$ | Confidence level |
| $\bar{F}_\alpha$ | The minimum expected cash flow allowed under confidence level $\alpha$ |
| **Variables** | |
| $\zeta$ | Artificial variable for CVaR constraints |
| $\eta_{ts}$ | Artificial variable for CVaR constraints in period $t$ and scenario $s$ |

**Hard cash flow constraints**

$$\sum_{v\in\mathcal{V}_0}(R^{SE}_{v0s}y^{SE}_{v0s} + R^{SC}_{v0s}y^{SC}_{v0s}) - \sum_{v\in\mathcal{V}_0^{IN}}(C^{IN}_{v00s}y^{NB}_{v0s} + C^{SH}_{v00s}y^{SH}_{v0s}) + B \geq \bar{F}_\alpha, \qquad s\in\mathcal{S}, \tag{A.48}$$

$$f^I_{ts} - f^O_{ts} \geq \bar{F}_\alpha, \qquad t\in\mathcal{T}^F\setminus\{0\}, s\in\mathcal{S}, \tag{A.49}$$

**CVaR constraints:**

$$\zeta + \frac{1}{1-\alpha}\sum_{s\in\mathcal{S}} P_s\eta_{ts} \geq \bar{F}_\alpha, \qquad t\in\mathcal{T}^S, \tag{A.50}$$

$$\eta_{ts} \leq f^I_{ts} - f^O_{ts} - \zeta, \qquad t\in\mathcal{T}^S, s\in\mathcal{S}, \tag{A.51}$$

$$\eta_{ts} = \eta_{t\bar{s}}, \qquad t\in\mathcal{T}^S, s\in\mathcal{S}, \bar{s}\in\mathcal{S}^{NA}_{ts}, \tag{A.52}$$

$$\eta_{ts} \in \mathbb{R}^-, \qquad t\in\mathcal{T}^S, s\in\mathcal{S} \tag{A.53}$$

Appendix B

# Node formulation

## B.1 Cash flow control model

Sets

| | |
|---|---|
| $\mathcal{T}$ | Set of periods, indexed by $t$ |
| $\mathcal{L}$ | Set of nodes, indexed by $n$ |
| $\mathcal{L}_t$ | Set of nodes in a time period $t$, indexed by $n$. $a(n, t')$ is the ancestor node of node $n$ in the scenario tree in period $t'$, with $a(n, t-1)$ written as $a(n)$. |
| $\mathcal{K}$ | Set of products, indexed by $k$ |
| $\mathcal{V}_t$ | Set of ship types existing in the market in period $t$, indexed by $v$ |
| $\mathcal{V}_t^N$ | Set of new ship types existing in the market in period $t$ |
| $\mathcal{V}_t^{IN}$ | The set of ship types the company pays instalments for in period $t$ |
| $\mathcal{N}_t$ | Set of trades operated in period $t$, indexed by $i$ |
| $\mathcal{N}_t^C$ | Set of contractual trades the shipping company is committed to serve in period $t$, indexed by $i$ |
| $\mathcal{N}_t^O$ | Set of optional trades the shipping company can choose to undertake or not in period $t$, indexed by $i$ |
| $\mathcal{R}_t$ | Set of loops in period $t$, indexed by $r$ |
| $\mathcal{R}_{vt}$ | Set of loops that can be sailed by a ship of type $v$ in period $t$ , indexed by $r$ |

| | |
|---|---|
| $\mathcal{R}_{ivt}$ | The set of loops servicing trade $i$ that can be sailed in period $t$ by a ship of type $v$, indexed by $r$ |

Parameters

| | |
|---|---|
| $P_n$ | The probability for node $n$ to occur |
| $R_{in}^{D}$ | The revenue of transporting one unit of goods on trade $i$, at node $n$ |
| $R_{vn}^{SE}$ | The selling price for a ship of type $v$, at node $n$ |
| $R_{vn}^{SC}$ | The scrapping value of a ship of type $v$, at node $n$ |
| $R_{vn}^{LU}$ | The lay-up savings for one period, for a ship of type $v$, at node $n$ |
| $R_{vn}^{SV}$ | The sunset value of a ship of type $v$, at node $n$ |
| $R_{vn}^{CO}$ | The charter out revenue for one period , for a ship of type $v$, at node $n$ |
| $C_{vn}^{CI}$ | The charter in cost for a ship of type $v$, at node $n$ |
| $R_{vn}^{CO}$ | The charter out revenue for a ship of $v$, at node $n$ |
| $C_{vn}^{OP}$ | The fixed operating cost for a ship of type $v$, at node $n$ |
| $C_{vrn}^{TR}$ | The cost of performing a loop $r$, for a ship of type $v$, at node $n$ |
| $C_{ikn}^{SP}$ | The space charter cost for one unit of product $k$ on trade $i$ , at node $n$ |
| $\overline{CI}_{vn}$ | The limit on number of ships of type $v$ available for chartering in at node $n$ |
| $\overline{CO}_{vn}$ | The limit on number of ships of type $v$ available for chartering out at node $n$ |
| $\overline{SH}_{vn}$ | The limit on number of ships of type $v$ available for purchase in the second hand market at node $n$ |
| $\overline{SE}_{vn}$ | The limit on number of ships of type $v$ that can be sold in the second hand market at node $n$ |
| $\overline{CI}_{n}$ | The limit of the total number of ships that can be chartered in at node $n$ |
| $\overline{CO}_{n}$ | The limit of the total number of ships that can be chartered out at node $n$ |
| $\overline{SH}_{n}$ | The limit of the total number of ships that can be bought in the second hand market at node $n$ |

| | |
|---|---|
| $\overline{SE}_n$ | The limit of the total number of ships that can be sold in the second hand market at node $n$ |
| $\bar{T}_v^L$ | The lead time for building a ship of type $v$ |
| $Q_{vk}$ | The total capacity of product $k$ on ship of type $v$ |
| $Q_v$ | The total capacity on ship of type $v$ |
| $Z_{vr}$ | The time a ship of type $v$ needs to perform a loop $r$ |
| $Z_v$ | The total available time in one period for a ship of type $v$ |
| $D_{ikn}$ | The demand on trade $i$ of product $k$ in node $n$ |
| $F_{in}$ | The frequency requirement on trade $i$ in node $n$ |
| $Y_{vn}^{NB}$ | The number of ships of type $v$ ordered in the previous planning period, delivered at node $n$ in the beginning of the time period |
| $Y_v^{IP}$ | The initial fleet of ships of type $v$ |
| $C_{va(n,t')n}^{IN}$ | The instalment paid at node $n$ on a ship of type $v$ ordered at node $a(n,t')$ |
| $C_{vn}^{IN}$ | The instalment paid for a ship of type $v$ at node $n$ for the ships in the initial fleet, i.e before the planning horizon begins |
| $C_{va(n,t')n}^{SH}$ | The instalments paid at node $n$ on a ship of type $v$ bought in the second hand market at node $a(n,t')$ |
| $M$ | Number of instalments |
| $\bar{F}$ | The cash flow limit, i.e. no period and scenario are allowed to have a worse cash flow than this limit |
| $B$ | The budget available for ordering or purchasing ships in period $0$ |
| Variables | |
| $y_{vn}^{NB}$ | The number of new buildings ordered of ship of type $v$, at node $n$ |
| $y_{vn}^{SH}$ | The number of ships of type $v$ bought in the second hand market, at node $n$ |
| $y_{vn}^{SE}$ | The number of ships of type $v$ sold in the second hand market at node $n$ |
| $y_{vn}^{SC}$ | The number of ships of type $v$ scrapped at node $n$ |
| $y_{vn}^{P}$ | The number of ships of type $v$ in the pool, at node $n$ |
| $h_{vn}^{I}$ | The number of ships of type $v$ chartered in, at period $n$ |

| | |
|---|---|
| $h_{vn}^{O}$ | The number of ships of type $v$ chartered out at node $n$ |
| $l_{vn}$ | The number of ships of type $v$ on lay-up, at node $n$ |
| $x_{vrn}$ | The number of loops $r$ performed by a ship of type $v$, at node $n$ |
| $n_{ikn}$ | The amount of product $k$ delivered at node $n$, by space charters on trade $i$ |
| $\delta_{in}$ | Set to 1 if the company services optional trade $i$ at node $n$, 0 otherwise. |
| $f_{n}^{I}$ | The cash inflow at node $n$ |
| $f_{n}^{O}$ | The cash outflow at node $n$ |

**Cash flow expressions**

$$\begin{aligned} f_n^I = & \sum_{i \in \mathcal{N}_t^O} \sum_{k \in \mathcal{K}} R_{in}^D D_{ikn} \delta_{in} + \sum_{i \in \mathcal{N}_t^C} \sum_{k \in \mathcal{K}} R_{in}^D D_{ikn} \\ & + \sum_{v \in \mathcal{V}_t} (R_{vn}^{CO} h_{vn}^O + R_{vn}^{SE} y_{vn}^{SE} + R_{vn}^{LU} l_{vn} + R_{vn}^{SC} y_{vn}^{SC}), \qquad t \in \mathcal{T} \setminus \{0\}, n \in \mathcal{L}_t \\ f_n^O = & \sum_{v \in \mathcal{V}_t^{IN}} C_{vn}^{IN} Y_v^{IP} + \sum_{t-M \le t' \le t} \sum_{v \in \mathcal{V}_t^{IN}} (C_{va(n,t')n}^{IN} y_{va(n,t')n}^{NB} + C_{va(n,t')n}^{SH} y_{va(n,t')n}^{SH}) \\ & + \sum_{i \in \mathcal{N}_t^C} \sum_{k \in \mathcal{K}} C_{ikn}^{SP} n_{ikn} + \sum_{v \in \mathcal{V}_t} (C_{vn}^{OP} y_{vn}^P + C_{vn}^{CI} h_{vn}^I + \sum_{r \in \mathcal{R}_{vt}} C_{vrn}^{TR} x_{vrn}), \qquad t \in \mathcal{T} \setminus \{0\}, n \in \mathcal{L}_t \end{aligned}$$

**Objective function**

$$
\begin{aligned}
max\ z = & \sum_{t\in\mathcal{T}\setminus\{0\}} \sum_{n\in\mathcal{L}_t} P_n\big( \sum_{i\in\mathcal{N}_t^O} \sum_{k\in\mathcal{K}} R_{in}^D D_{ikn}\delta_{in} && \text{(B.1a)}\\
& + \sum_{i\in\mathcal{N}_t^C} \sum_{k\in\mathcal{K}} (R_{in}^D D_{ikn} - C_{ikn}^{SP} n_{ikn}) && \text{(B.1b)}\\
& - \sum_{v\in\mathcal{V}_t} (C_{vn}^{OP} y_{vn}^P + C_{vn}^{CI} h_{vn}^I - R_{vn}^{CO} h_{vn}^O && \text{(B.1c)}\\
& + \sum_{r\in\mathcal{R}_{vt}} C_{vrn}^{TR} x_{vrn} - R_{vn}^{LU} l_{vn})\big) && \text{(B.1d)}\\
& - \sum_{t\leq M-1} \sum_{n\in\mathcal{L}_t} \sum_{v\in\mathcal{V}_t^{IN}} P_n C_{vn}^{IN} Y_v^{IP} && \text{(B.1e)}\\
& - \sum_{t\in\mathcal{T}} \sum_{t-M\leq t'\leq t} \sum_{n\in\mathcal{L}_t} \sum_{v\in\mathcal{V}_t^{IN}} P_n (C_{va(n,t')n}^{IN} y_{va(n,t')}^{NB} + C_{va(n,t')n}^{SH} y_{va(n,t')}^{SH}) && \text{(B.1f)}\\
& + \sum_{v\in\mathcal{V}_{\bar{T}}} ( \sum_{n\in\mathcal{L}_{\bar{T}}} P_n R_{vn}^{SV} y_{vn}^P && \text{(B.1g)}\\
& - \sum_{t=\bar{T}}^{t'+M} \sum_{t'\in\mathcal{T}} \sum_{n\in\mathcal{L}_t} P_n (C_{va(n,t')n}^{IN} y_{va(n,t')}^{NB} + C_{va(n,t')n}^{SH} y_{va(n,t')}^{SH})) && \text{(B.1h)}\\
& + \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{L}_t} \sum_{v\in\mathcal{V}_t} P_n (R_{vn}^{SC} y_{vn}^{SC} + R_{vn}^{SE} y_{vn}^{SE}) && \text{(B.1i)}
\end{aligned}
$$

**Demand constraints**

$$
\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} Q_{vk} x_{vrn} + n_{ikn} \geq D_{ikn}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^C, k\in\mathcal{K}, n\in\mathcal{L}_t, \qquad \text{(B.2)}
$$

$$
\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} Q_{vk} x_{vrn} \geq D_{ikn}\delta_{in}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^O, k\in\mathcal{K}, n\in\mathcal{L}_t, \qquad \text{(B.3)}
$$

**Capacity constraints**

$$
\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} Q_v x_{vrn} + \sum_{k\in\mathcal{K}} n_{ikn} \geq \sum_{k\in\mathcal{K}} D_{ikn}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^C, n\in\mathcal{L}_t, \qquad \text{(B.4)}
$$

$$
\sum_{v\in\mathcal{V}_t} \sum_{r\in\mathcal{R}_{ivt}} Q_v x_{vrn} \geq \sum_{k\in\mathcal{K}} D_{ikn}\delta_{in}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^O, n\in\mathcal{L}_t, \qquad \text{(B.5)}
$$

**Frequency constraints**

$$\sum_{v\in\mathcal{V}_t}\sum_{r\in\mathcal{R}_{ivt}} x_{vrn} \geq F_{in}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^C, n\in\mathcal{L}_t, \tag{B.6}$$

$$\sum_{v\in\mathcal{V}_t}\sum_{r\in\mathcal{R}_{ivt}} x_{vrn} \geq F_{in}\delta_{in}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^O, n\in\mathcal{L}_t, \tag{B.7}$$

**Time constraints**

$$\sum_{r\in\mathcal{R}_{vt}} Z_{rv}x_{vrn} \leq Z_v(y_{vn}^P + h_{vn}^I - h_{vn}^O - l_{vn}), \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, n\in\mathcal{L}_t, \tag{B.8}$$

**Optional trades constraints**

$$\delta_{ia(n)} \leq \delta_{in}, \qquad t\in\mathcal{T}\setminus\{0,1\}, i\in\mathcal{N}_t^O, n\in\mathcal{L}_t, \tag{B.9}$$

**Pool constraints**

$$y_{v0}^P = Y_v^{IP} \qquad v\in\mathcal{V}_0 \tag{B.10}$$

$$y_{vn}^P = y_{v,a(n)}^P - y_{v,a(n)}^{SC} + y_{v,a(n)}^{SH} - y_{v,a(n)}^{SE} \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t\setminus\mathcal{V}_t^N, n\in\mathcal{L}_t, \tag{B.11}$$

$$y_{vn}^P = y_{va(n,t-\bar{T}_v{}^L)}^{NB} \qquad t\in\mathcal{T}: t\geq \bar{T}_v{}^L, v\in\mathcal{V}_t^N, n\in\mathcal{L}_t, \tag{B.12}$$

$$y_{vn}^P = Y_{vn}^{NB} \qquad t\in\mathcal{T}: t< \bar{T}_v{}^L, v\in\mathcal{V}_t^N, n\in\mathcal{L}_t, \tag{B.13}$$

$$y_{vn}^P \geq l_{vn} - h_{vn}^I + h_{vn}^O \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, n\in\mathcal{L}_t, \tag{B.14}$$

$$y_{vn}^P = y_{vn}^{SC} \qquad t\in\mathcal{T}\setminus\{\bar{T}\}, v\in\mathcal{V}_t\setminus\mathcal{V}_{t+1}, n\in\mathcal{L}_t, \tag{B.15}$$

**Charter in constraints**

$$y_{vn}^{SH} \leq \overline{SH}_{vn}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.16}$$

$$y_{vn}^{SE} \leq \overline{SE}_{vn}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.17}$$

$$h_{vn}^{I} \leq \overline{CI}_{vn}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.18}$$

$$h_{vn}^{O} \leq \overline{CO}_{vn}, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.19}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}_t^N} y_{vn}^{SH} \leq \overline{SH}_{n}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, n \in \mathcal{L}_t, \tag{B.20}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}_t^N} y_{vn}^{SE} \leq \overline{SE}_{n}, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, n \in \mathcal{L}_t, \tag{B.21}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}_t^N} h_{vn}^{I} \leq \overline{CI}_{n}, \qquad t \in \mathcal{T} \setminus \{0\}, n \in \mathcal{L}_t, \tag{B.22}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}_t^N} h_{vn}^{O} \leq \overline{CO}_{n}, \qquad t \in \mathcal{T} \setminus \{0\}, n \in \mathcal{L}_t, \tag{B.23}$$

**Cash flow constraints**

$$\sum_{v \in \mathcal{V}_0} (R_{v0}^{SE} y_{v0}^{SE} + R_{v0}^{SC} y_{v0}^{SC}) - \sum_{v \in \mathcal{V}_0^{IN}} (C_{v00}^{IN} y_{v0}^{NB} + C_{v00}^{SH} y_{v0}^{SH}) + B \geq \bar{F}, \tag{B.24}$$

$$f_n^I - f_n^O \geq \bar{F}, \qquad t \in \mathcal{T} \setminus \{0\}, n \in \mathcal{L}_t, \tag{B.25}$$

**Convexity and integer constraints**

$$y_{vn}^{NB} \in \mathbb{Z}^+, \qquad t \in \mathcal{T} : t \leq \bar{T} - \bar{T}_v^{\,L}, v \in \mathcal{V}_{t+T^L}^N, n \in \mathcal{L}_t, \tag{B.26}$$

$$y_{vn}^{SC} \in \mathbb{Z}^+, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.27}$$

$$y_{vn}^{SH} \in \mathbb{Z}^+, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}\ v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.28}$$

$$y_{vn}^{SE} \in \mathbb{Z}^+, \qquad t \in \mathcal{T} \setminus \{\bar{T}\}\ v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.29}$$

$$y_{vn}^{P} \in \mathbb{R}^+, \qquad t \in \mathcal{T}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.30}$$

$$h_{vn}^{I} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.31}$$

$$h_{vn}^{O} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.32}$$

$$l_{vn} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \tag{B.33}$$

$$x_{vrn} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, r \in \mathcal{R}_{vt}, n \in \mathcal{L}_t, \tag{B.34}$$

$$n_{ikn} \in \mathbb{R}^+, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^C, k \in \mathcal{K}, n \in \mathcal{L}_t \tag{B.35}$$

$$\delta_{in} \in \{0, 1\}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^O, n \in \mathcal{L}_t, \tag{B.36}$$

## B.2 Conditional Value-at-Risk model

Replacing the cash flow constraints in the cash flow control model with the following sets, variables and constraints results in the Conditional Value-at-Risk model.

| | |
|---|---|
| **Sets** | |
| $\mathcal{T}^F$ | The set of periods in the first stage |
| $\mathcal{T}^S$ | The set of periods under uncertainty, i.e. all periods after the first stage |
| **Parameters** | |
| $\alpha$ | Confidence level |
| $\bar{F}_\alpha$ | The minimum expected cash flow allowed under confidence level $\alpha$ |
| **Variables** | |
| $\zeta$ | Artificial variable for CVaR constraints |
| $\eta_n$ | Artificial variable for CVaR constraints at node $n$ |

**Hard cash flow constraints**

$$\sum_{v \in \mathcal{V}_0} (R^{SE}_{v0} y^{SE}_{v0} + R^{SC}_{v0} y^{SC}_{v0}) - \sum_{v \in \mathcal{V}_0^{IN}} (C^{IN}_{v00} y^{NB}_{v0} + C^{SH}_{v00} y^{SH}_{v0}) + B \geq \bar{F}_\alpha, \tag{B.37}$$

$$f^I_n - f^O_n \geq \bar{F}_\alpha, \qquad t \in \mathcal{T}^F \setminus \{0\}, n \in \mathcal{L}_t, \tag{B.38}$$

**CVaR constraints**

$$\zeta + \frac{1}{1-\alpha} \sum_{n \in \mathcal{L}_t} P_n \eta_n \geq \bar{F}_\alpha, \qquad t \in \mathcal{T}^S, \tag{B.39}$$

$$\eta_n \leq f^I_n - f^O_n - \zeta, \qquad t \in \mathcal{T}^S, n \in \mathcal{L}_t, \tag{B.40}$$

$$\eta_n \in \mathbb{R}^-, \qquad t \in \mathcal{T}^S, n \in \mathcal{L}_t \tag{B.41}$$

Appendix C

# Tables

## C.1 Medium instance

***Table C.1:*** *Solutions for the medium instance with correlated correlation matrix and normal space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -23.4 | 872.0 | 22 | 1.58 | 5 | 13.58 | 0.24 | 9.36 | 0 | 11 340 | 0 | 0.73 |
| -21.1 | 873.0 | 22 | 1.57 | 5 | 13.53 | 0.24 | 8.95 | 0 | 11 169 | 0 | 0.73 |
| -18.7 | 871.4 | 21 | 1.68 | 5 | 12.78 | 0.24 | 9.31 | 0 | 19 411 | 0 | 0.73 |
| -16.4 | 868.7 | 20 | 1.82 | 5 | 11.99 | 0.24 | 9.28 | 0 | 28 461 | 0 | 0.73 |
| -14.1 | 864.4 | 20 | 1.84 | 5 | 11.97 | 1.11 | 9.28 | 0 | 26 809 | 0 | 0.73 |
| -11.7 | 859.3 | 21 | 1.72 | 5 | 12.66 | 2.11 | 9.52 | 0 | 17 515 | 0 | 0.73 |
| -9.4 | 855.4 | 20 | 1.91 | 5 | 12 | 2.11 | 9.45 | 0 | 26 019 | 0 | 0.71 |
| -7.0 | 844.0 | 19 | 2.13 | 4 | 12.51 | 3.1 | 8.6 | 0 | 33 916 | 0 | 0.73 |
| -4.7 | 833.0 | 18 | 2.23 | 2 | 13.59 | 5.1 | 8.75 | 0 | 25 294 | 0 | 0.73 |
| -2.3 | 801.5 | 18 | 2.27 | 2 | 13.61 | 5.85 | 8.86 | 22 168 | 24 434 | 0 | 0.72 |
| 0.0 | 755.3 | 18 | 2.2 | 2 | 13.3 | 6.71 | 9.07 | 55 555 | 23 802 | 0 | 0.73 |

***Table C.2:*** *Solutions for the medium instance with correlated correlation matrix and* $50\%$ *reduction in space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -63.5 | 899.7 | 14 | 4.42 | 5 | 8.06 | 0.24 | 8.46 | 0 | 94 002 | 0 | 0.66 |
| -57.2 | 892.8 | 12 | 5.21 | 5 | 7.32 | 0.24 | 5.38 | 0 | 168 709 | 0 | 0.65 |
| -50.8 | 885.1 | 11 | 5.52 | 5 | 6.79 | 1.2 | 5.76 | 0 | 181 021 | 0 | 0.65 |
| -44.5 | 872.6 | 12 | 5.21 | 5 | 7.23 | 4.08 | 6.43 | 0 | 143 805 | 0 | 0.65 |
| -38.1 | 856.4 | 13 | 4.83 | 5 | 7.74 | 6.91 | 7.21 | 0 | 115 393 | 0 | 0.65 |

***Table C.3:*** *Solutions for the medium instance with uncorrelated correlation matrix and normal space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -44.3 | 885.5 | 23 | 2.24 | 5 | 14.29 | 0.24 | 8.58 | 0 | 2 542 | 0 | 0.71 |
| -39.8 | 885.2 | 23 | 2.2 | 5 | 14.3 | 0.24 | 9.09 | 0 | 2 872 | 0 | 0.7 |
| -35.4 | 881.5 | 22 | 2.42 | 5 | 13.64 | 0.24 | 8.94 | 0 | 12 241 | 0 | 0.71 |
| -31.0 | 881.0 | 22 | 2.46 | 5 | 13.7 | 0.24 | 8.84 | 0 | 12 706 | 0 | 0.71 |
| -26.6 | 880.1 | 22 | 2.36 | 5 | 13.73 | 0.24 | 8.84 | 0 | 13 655 | 0 | 0.7 |
| -22.1 | 871.4 | 20 | 2.67 | 5 | 12.13 | 0.24 | 8.94 | 0 | 33 579 | 0 | 0.71 |
| -17.7 | 851.5 | 18 | 3.22 | 3 | 13 | 2.2 | 8.5 | 0 | 38 775 | 0 | 0.68 |

***Table C.4:*** *Solutions for the medium instance with uncorrelated correlation matrix and reduced space charter price increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -28.6 | 921.6 | 14 | 3.17 | 5 | 7.68 | 0.24 | 8.85 | 0 | 71 562 | 0 | 0.72 |
| -25.8 | 921.5 | 14 | 3.12 | 5 | 7.7 | 0.24 | 8.81 | 0 | 72 774 | 0 | 0.72 |
| -22.9 | 920.9 | 14 | 3.21 | 5 | 7.64 | 0.24 | 8.97 | 0 | 71 410 | 0 | 0.72 |
| -20.0 | 921.3 | 14 | 3.08 | 5 | 7.7 | 0.24 | 8.81 | 0 | 74 210 | 0 | 0.72 |
| -17.2 | 920.4 | 14 | 3.18 | 5 | 7.65 | 0.24 | 8.97 | 0 | 74 786 | 0 | 0.72 |
| -14.3 | 921.1 | 14 | 3.03 | 5 | 7.7 | 0.24 | 8.81 | 0 | 76 006 | 0 | 0.72 |
| -11.5 | 921.0 | 14 | 3 | 5 | 7.7 | 0.24 | 8.81 | 0 | 77 083 | 0 | 0.72 |
| -8.6 | 920.9 | 14 | 2.95 | 5 | 7.7 | 0.24 | 8.8 | 0 | 78 520 | 0 | 0.72 |
| -5.7 | 920.9 | 14 | 2.93 | 5 | 7.7 | 0.24 | 8.79 | 0 | 78 879 | 0 | 0.72 |
| -2.9 | 920.2 | 14 | 3.02 | 5 | 7.68 | 0.24 | 8.92 | 0 | 78 344 | 0 | 0.72 |
| 0.0 | 920.5 | 14 | 2.84 | 5 | 7.72 | 0.24 | 8.75 | 0 | 82 168 | 0 | 0.72 |
| 2.9 | 918.0 | 14 | 2.92 | 5 | 8.09 | 0.24 | 8.47 | 0 | 83 833 | 0 | 0.68 |
| 5.7 | 919.2 | 13 | 2.98 | 5 | 7.32 | 0 | 7.57 | 22 824 | 107 749 | 0 | 0.71 |
| 8.6 | 916.8 | 12 | 3.08 | 5 | 6.66 | 0 | 6.93 | 22 824 | 140 725 | 0 | 0.71 |
| 11.5 | 911.7 | 12 | 3.33 | 5 | 7.17 | 0 | 6.66 | 22 824 | 148 247 | 0 | 0.7 |
| 14.3 | 901.8 | 12 | 3.03 | 5 | 6.76 | 0.96 | 6.8 | 56 066 | 143 731 | 0 | 0.71 |
| 17.2 | 884.3 | 11 | 3.46 | 3 | 8.28 | 2.87 | 7.06 | 56 066 | 129 582 | 0 | 0.71 |
| 20.0 | 864.6 | 11 | 3.31 | 3 | 8.36 | 3.86 | 7.14 | 88 669 | 135 948 | 0 | 0.69 |

## C.2 Small instance

***Table C.5:*** *Solutions for the small instance with correlated correlation matrix and normal space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -10.7 | 295.0 | 10 | 0.94 | 3 | 6.83 | 0.26 | 4.62 | 0 | 1 283 | 0 | 0.67 |
| -9.6 | 293.9 | 10 | 0.93 | 3 | 6.96 | 0.26 | 4.51 | 0 | 2 673 | 0 | 0.68 |
| -8.6 | 294.8 | 10 | 0.93 | 3 | 6.83 | 0.26 | 4.72 | 0 | 1 393 | 0 | 0.64 |
| -7.5 | 294.7 | 10 | 0.93 | 3 | 6.88 | 0.26 | 4.56 | 0 | 1 461 | 0 | 0.67 |
| -6.4 | 291.6 | 10 | 0.95 | 3 | 6.88 | 0.26 | 5.22 | 0 | 3 825 | 0 | 0.76 |
| -5.4 | 282.2 | 9 | 1.03 | 2 | 7.15 | 1.26 | 5.22 | 0 | 8 206 | 0 | 0.73 |
| -4.3 | 282.4 | 9 | 1.02 | 2 | 7.17 | 1.26 | 5.16 | 0 | 8 197 | 0 | 0.75 |
| -3.2 | 281.9 | 9 | 1.02 | 2 | 7.13 | 1.26 | 5.21 | 0 | 9 064 | 0 | 0.76 |
| -2.1 | 258.4 | 9 | 1.05 | 2 | 7.19 | 1.87 | 5.1 | 17 005 | 14 116 | 0.33 | 1 |

***Table C.6:*** *Solutions for the small instance with uncorrelated correlation matrix and normal space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -26.1 | 301.0 | 10 | 0.84 | 3 | 6.86 | 0.26 | 4.54 | 0 | 1 554 | 0 | 0.69 |
| -23.5 | 294.7 | 9 | 1.07 | 3 | 6.03 | 0.26 | 4.72 | 0 | 16 223 | 0 | 0.68 |
| -20.9 | 288.3 | 9 | 1.09 | 3 | 6.05 | 1.22 | 5.48 | 0 | 12 602 | 0 | 0.8 |
| -18.3 | 280.5 | 10 | 0.83 | 3 | 6.87 | 3.22 | 4.48 | 0 | 2 226 | 0 | 0.69 |
| -15.7 | 271.2 | 8 | 1.13 | 1 | 7.2 | 4.17 | 5.21 | 0 | 6 745 | 0 | 0.79 |

***Table C.7:*** *Solutions for the small instance with uncorrelated correlation matrix and reduced space charter price for increasing cash flow limits.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -58.2 | 316.0 | 6 | 1.31 | 3 | 3.88 | 0.12 | 4.3 | 17 005 | 50 168 | 0 | 0.72 |
| -52.4 | 315.5 | 6 | 1.37 | 3 | 3.76 | 0.12 | 4.5 | 17 005 | 48 290 | 0 | 0.73 |
| -46.6 | 314.8 | 6 | 1.31 | 3 | 3.92 | 0.12 | 4.28 | 17 005 | 52 764 | 0 | 0.71 |
| -40.8 | 314.6 | 7 | 1.02 | 3 | 4.51 | 0.12 | 4.49 | 17 005 | 38 459 | 0 | 0.73 |
| -34.9 | 314.1 | 7 | 0.98 | 3 | 4.55 | 0.12 | 4.43 | 17 005 | 41 441 | 0 | 0.73 |
| -29.1 | 314.0 | 7 | 0.99 | 3 | 4.47 | 0.12 | 4.56 | 17 005 | 40 273 | 0 | 0.72 |
| -23.3 | 313.0 | 7 | 0.98 | 3 | 4.63 | 0.12 | 4.34 | 17 005 | 44 074 | 0 | 0.71 |
| -17.5 | 312.4 | 7 | 1 | 3 | 4.68 | 0.12 | 4.37 | 17 005 | 45 499 | 0 | 0.67 |
| -11.7 | 311.0 | 8 | 0.86 | 3 | 5.21 | 0.12 | 4.53 | 17 005 | 29 746 | 0 | 0.73 |
| -5.8 | 298.7 | 8 | 0.85 | 3 | 5.26 | 1.82 | 4.55 | 17 005 | 39 552 | 0.33 | 1 |

## C.3 CVaR

*Table C.8: Solutions for the large instance with correlated correlation matrix and normal space charter price for the CVaR model with $\alpha = 0.99$.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}_\alpha$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -66.3 | 1 108.8 | 29 | 1.99 | 9 | 19.09 | 0.37 | 12.9 | 0 | 9 924 | 0 | 1.55 |
| -59.7 | 1 108.7 | 29 | 1.99 | 9 | 19.13 | 0.37 | 12.88 | 0 | 10 605 | 0 | 1.55 |
| -53.1 | 1 108.7 | 29 | 1.99 | 9 | 19.15 | 0.37 | 12.86 | 0 | 10 606 | 0 | 1.55 |
| -46.4 | 1 108.5 | 29 | 2.02 | 8 | 20.16 | 0.41 | 12.65 | 0 | 6 630 | 0 | 1.45 |
| -39.8 | 1 109.0 | 29 | 2 | 9 | 19.24 | 0.37 | 12.66 | 0 | 13 998 | 0 | 1.46 |
| -33.2 | 1 105.7 | 26 | 2.35 | 8 | 17.75 | 0.41 | 12.63 | 0 | 26 284 | 0 | 1.45 |
| -26.5 | 1 094.1 | 26 | 2.4 | 9 | 16.76 | 2.37 | 12.66 | 0 | 35 096 | 0 | 1.46 |
| -19.9 | 1 080.0 | 26 | 2.43 | 9 | 16.82 | 4.36 | 12.73 | 0 | 35 533 | 0 | 1.44 |
| -13.3 | 1 063.4 | 26 | 2.43 | 9 | 17.03 | 6.34 | 12.61 | 0 | 36 572 | 0 | 1.44 |
| -6.6 | 1 037.9 | 24 | 2.73 | 6 | 18.19 | 10.31 | 12.73 | 0 | 30 358 | 0 | 1.43 |
| 0 | 957.2 | 23 | 2.69 | 5 | 18.51 | 12.19 | 14.2 | 42 367 | 40 334 | 0 | 1.52 |

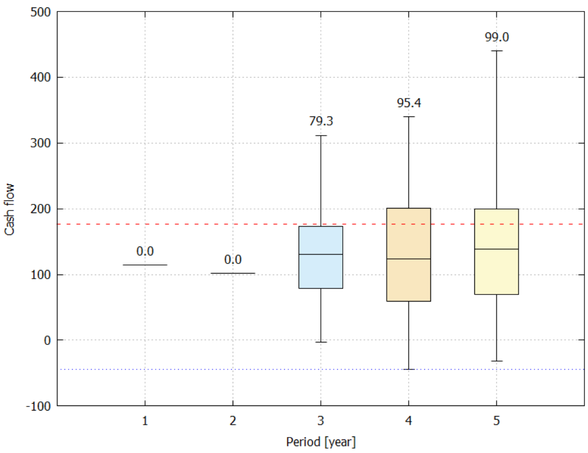
*Table C.9: Solutions for the large instance with correlated correlation matrix and normal space charter price for the CVaR model with $\alpha = 0.90$.*

| | Exp. | New builds | | Scrappings | | Lay-up | | Space | | Optional | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{F}_\alpha$ | Profit | First | Second | First | Second | First | Second | First | Second | First | Second |
| -66.3 | 1 109.1 | 29 | 1.99 | 9 | 19.09 | 0.37 | 12.9 | 0 | 9 924 | 0 | 1.55 |
| -59.7 | 1 109.1 | 29 | 2 | 9 | 19.16 | 0.37 | 12.85 | 0 | 10 162 | 0 | 1.54 |
| -53.1 | 1 109.3 | 30 | 1.99 | 9 | 19.37 | 0.37 | 13.73 | 0 | 2 074 | 0 | 1.46 |
| -46.4 | 1 110.2 | 28 | 2.14 | 8 | 19.25 | 0.41 | 12.74 | 0 | 11 748 | 0 | 1.44 |
| -39.8 | 1 109.7 | 29 | 2.31 | 9 | 19.45 | 0.37 | 12.62 | 0 | 10 240 | 0 | 1.43 |
| -33.2 | 1 109.9 | 28 | 2.21 | 8 | 19.29 | 0.41 | 12.72 | 0 | 11 783 | 0 | 1.46 |
| -26.5 | 1 110.5 | 29 | 2.02 | 9 | 19.14 | 0.37 | 12.76 | 0 | 12 065 | 0 | 1.45 |
| -19.9 | 1 110.2 | 28 | 2.12 | 8 | 19.24 | 0.41 | 12.75 | 0 | 11 857 | 0 | 1.45 |
| -13.3 | 1 108.7 | 29 | 1.99 | 9 | 19.12 | 0.37 | 12.89 | 0 | 10 608 | 0 | 1.55 |
| -6.6 | 1 108.6 | 29 | 2.03 | 9 | 19.24 | 0.37 | 12.68 | 0 | 14 422 | 0 | 1.48 |
| 0 | 1 108.3 | 29 | 1.99 | 9 | 19.13 | 0.37 | 12.88 | 0 | 10 994 | 0 | 1.55 |
| 6.6 | 1 109.6 | 28 | 2.32 | 8 | 19.48 | 0.41 | 12.57 | 0 | 11 387 | 0 | 1.4 |
| 13.3 | 1 112.6 | 29 | 2.12 | 9 | 18.92 | 0.37 | 13.19 | 0 | 7 732 | 0 | 1.48 |

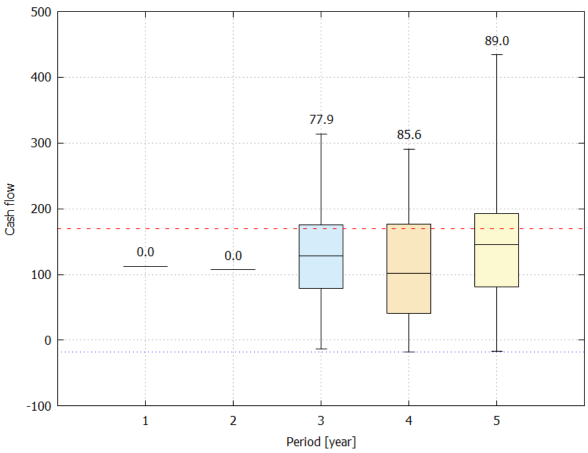
Appendix D

# Box plots

## D.1 Medium instance

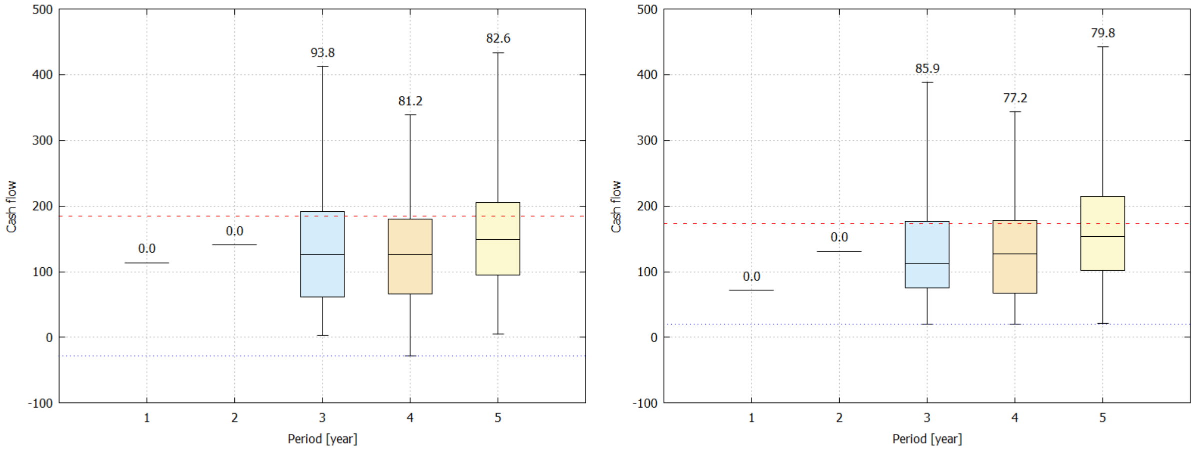



*(a) Without cash flow constraints*



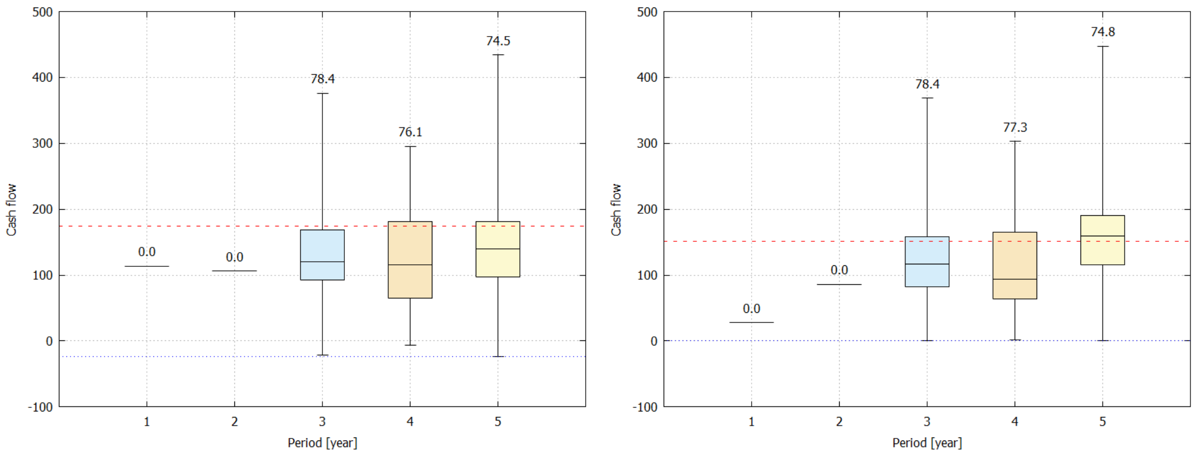

*(b) With cash flow constraints ($\bar{F} = -17.7$)*

***Figure D.1:*** *The cash flow development for the medium instance with uncorrelated correlation matrix. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*



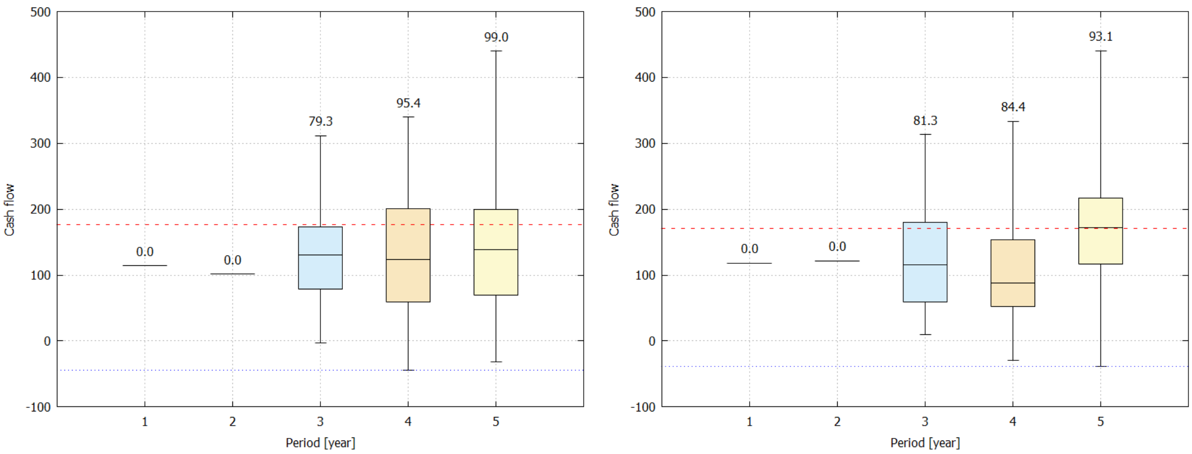

***(a)** Without cash flow constraints* ***(b)** With cash flow constraints ($\bar{F} = 20.0$)*

***Figure D.2:*** *The cash flow development for the medium instance with uncorrelated correlation matrix and reduced space charter price. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*




***(a)** Without cash flow constraints* ***(b)** With cash flow constraints ($\bar{F} = 0.0$)*

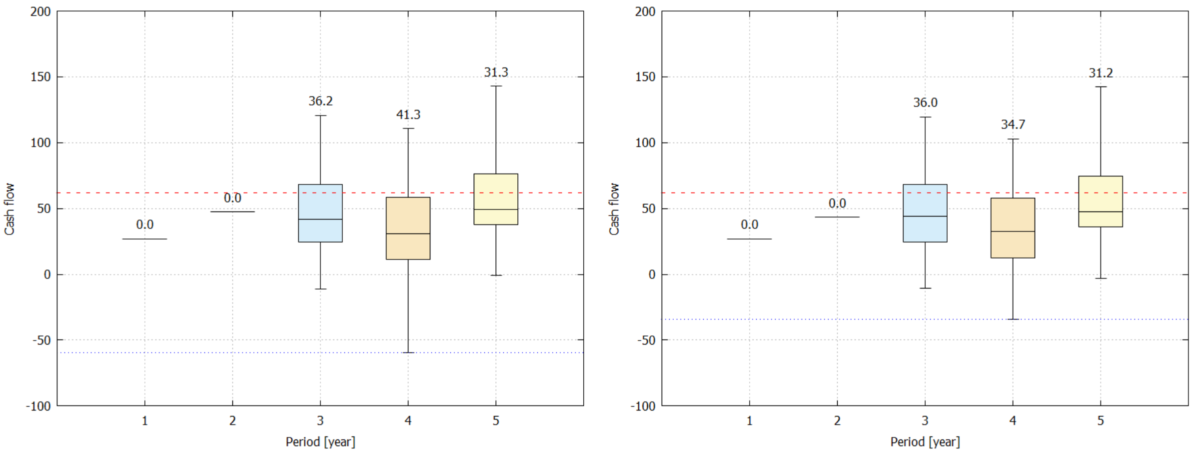
***Figure D.3:*** *The cash flow development for the medium instance with correlated correlation matrix. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*




***(a)*** *Without cash flow constraints* ***(b)*** *With cash flow constraints* ($\bar{F} = -38.1$)

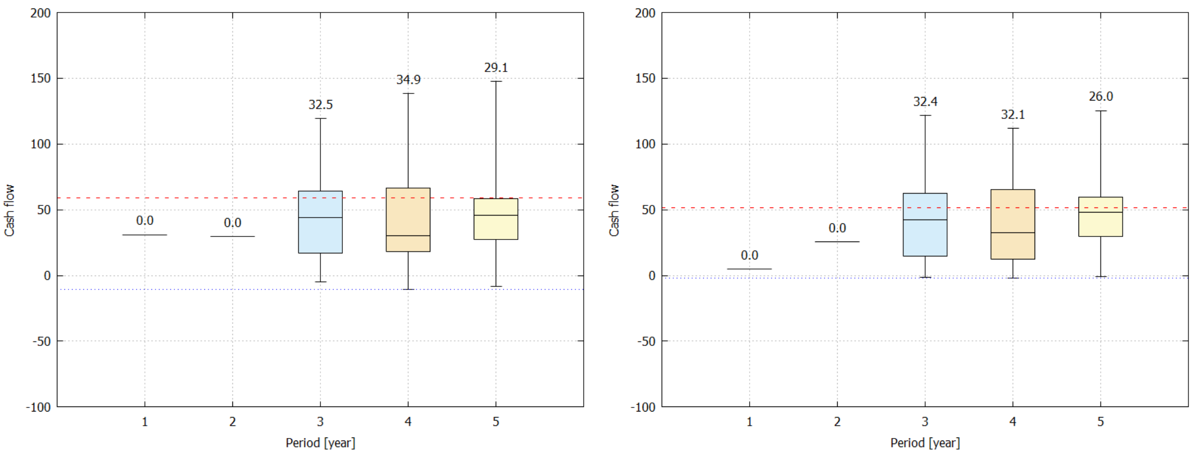
***Figure D.4:*** *The cash flow development for the medium instance with correlated correlation matrix and* $50\%$ *reduction in space charter price. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*

## D.2 Small instance



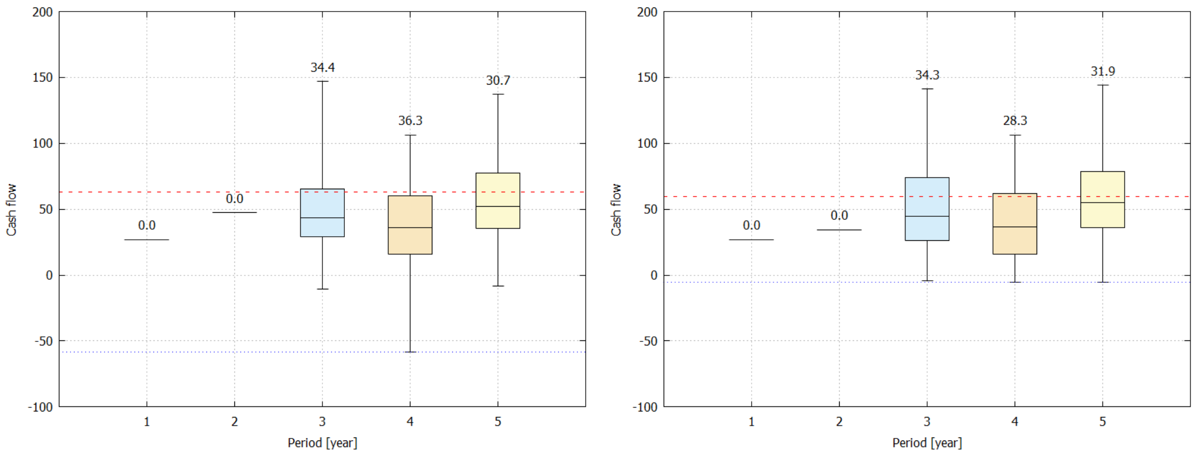

*(a) Without cash flow constraints* *(b) With cash flow constraints ($\bar{F} = -15.7$)*

***Figure D.5:*** *The cash flow development for the small instance with uncorrelated correlation matrix. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*




*(a) Without cash flow constraints* *(b) With cash flow constraints ($\bar{F} = -2.1$)*

***Figure D.6:*** *The cash flow development for the small instance with correlated correlation matrix. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*




***(a)*** *Without cash flow constraints* ***(b)*** *With cash flow constraints ($\bar{F} = -5.8$)*

***Figure D.7:*** *The cash flow development for the small instance with uncorrelated correlation matrix and reduced space charter price. The red dashed line is the annualised expected profit, and the numbers above each whisker is the standard deviation for the given period. The blue dotted line indicates the worst case cash flow.*