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Real Options Valuation of Wind Energy Investments in Norway and Sweden: A Case Study

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Problem description

This thesis examines investments in renewable energy under uncertainty. The opportunity to invest in a project eligible to receive green certificates, in the Norwegian-Swedish market, is evaluated from the perspectives of both a Norwegian and Swedish investor. The investment decisions are analyzed using a case study of a wind energy project, where the focus is on how regulatory differences in the green certificate scheme impact the investment opportunities. The real options approach is used to model the investments, where a time-dependent model with uncertain electricity and green certificate prices is developed. The option values and investment thresholds are estimated using the least-squares Monte Carlo simulation approach. In an extension to the model, the possibility of jumps in the green-certificate price is incorporated, where the investors can learn about the likelihood of a jump.

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Preface

This thesis is submitted as the concluding part of our Master of Science degrees in Industrial Economics and Technology Management, with degree specialization in Financial Engineering, at the Norwegian University of Science and Technology (NTNU).

We would like to thank our supervisors, Associate Professor Verena Hagspiel and Postdoctoral Fellow Maria Lavrutich, for their thorough guidance, valuable feedback and helpful comments throughout the work on this thesis.

We would also like to thank Knut-Harald Bakke, Head of Industrial Portfolio and Projects at Hydro Energi AS, for his helpful suggestions for the case study. Further, we would like to thank Professor Stein-Erik Fleten for his valuable input regarding the Monte Carlo approach, and for providing relevant market data.

Fredrik Finjord and Marius Tangen

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Abstract

We use the real options approach to examine the investment opportunity of a private investor, both from a Norwegian and Swedish perspective, that holds an option to invest in a renewable energy project. Norway and Sweden have implemented a common green certificate subsidy scheme, where the regulations differ between the countries. We evaluate the option values and optimal investment behaviour using a wind energy case study, and analyze how investments are affected by uncertain prices and the regulatory differences.

We find that, in 2017, the investment opportunities of both investors are similar, and the regulations have a low effect on the option values and investment behaviour. At the start of 2021, there is a strong incentive to invest for both investors, as subsidies will decrease if delaying the investment. At the end of 2021, a Norwegian investor faces a deadline, and will invest for a larger range of prices. The Swedish investor, however, has a considerably more valuable investment opportunity. Further, we find that the investment opportunities are highly sensitive to the drift and volatility of the electricity price, in addition to the discount rate. When considering a Swedish policy extension to the support scheme, we find that this will delay investments in Sweden considerably.

Furthermore, we find that a possible collapse in the green certificate price, reduces the values of the options. This causes the investors to delay investments for a longer period, where the Swedish investor is most affected. Being able to learn about the likelihood of a price collapse leads to a small increase in the values of the options. This increases the likelihood of investments before the Norwegian deadline for both investors. We further find that the possibility to learn is slightly more valuable for the Norwegian investor.

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Sammendrag

Vi bruker en realopsjonsmodell for å undersøke investeringsmuligheten til en privat investor, fra både et norsk og svensk perspektiv, som har muligheten til å investere i et prosjekt innenfor den fornybare energisektoren. Norge og Sverige har innført grønne sertifikater som en felles støtteordning, hvor reguleringene er forskjellige i de ulike landene. Vi evaluerer opsjonsverdiene og den optimale investeringsadferden ved bruk av et eksempelstudie av et vindkraftprosjekt, og analyserer hvordan investeringene er påvirket av usikre priser og ulike reguleringer.

Vi finner at investeringsmulighetene for begge investorene er like i 2017, og at reguleringene har en liten effekt på opsjonsverdiene og investeringsadferden. I begynnelsen av 2021 er det et sterkt insentiv for begge investorene til å investere, siden man mottar subsidier for en kortere periode hvis investeringen utsettes. Den norske investoren må investere innen 2021 for å være berettiget til å motta sertifikater, og vil derfor være villig til å investere for lavere priser. Den Svenske investoren har derimot en betraktelig mer verdifull investeringsmulighet. Videre finner vi at en svensk utvidelse av støtteordningen vil utsette investeringer betraktelig i Sverige. Resultatene viser at investeringsmulighetene er veldig sensitiv til driften og volatiliteten av elektrisitetsprisen, i tillegg til diskonteringsrenten.

Vi finner at en mulig kollaps i sertifikatprisen vil redusere verdien av opsjonene. Dette fører til at investorene utsetter investeringene for en lengre periode, hvor den svenske investoren er mest påvirket. Muligheten til å lære om sannsynligheten for et priskollaps fører til en liten økning i opsjonsverdiene, og øker sannsynligheten for investeringer før den norske fristen, for begge investorene. Videre finner vi at muligheten for å lære er litt mer verdifull for den norske investoren.

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Contents

1 Introduction

There is a large commitment worldwide to increase the production from renewable energy sources (REN21 (2016)). At the end of 2015, at least 173 countries had targets for an increase in the share of renewables in the energy mix (REN21 (2016)). In particular, the European Union has a goal of covering 20% of the energy demand from renewable sources by 2020 (European Commission (2017)). To increase the competitiveness of renewable energy projects, and to attract sufficient investments, governments have implemented support policies. These support policies can be divided in two main categories, where the government either sets a target for new renewable production (quantity-driven), or a subsidy level (price-driven). A green certificate scheme is an example of a quantity-driven support policy, where the price of the certificates is decided by the market. Norway and Sweden, who are committed to the EU goal to increase renewable production, have implemented a green certificate scheme where the certificates from both countries are traded on a common market. When to invest in a project eligible to receive certificates is an important topic for investors, as the policy regulations provide an incentive to invest in the near future.

In this thesis, we analyze investments in renewable energy production in Norway and Sweden, where we use a case study of a wind energy project. We consider a price taking, profit maximizing investor, with the option to invest in a renewable energy project eligible to receive green certificates. The future electricity and green certificate prices are uncertain, and the investor must decide the optimal time to invest in the project. We analyze the investment decision from the perspective of both a Norwegian and Swedish investor, and will first consider how the regulatory differences in the green certificate market affect the investment opportunities. Furthermore, we consider how a regulatory change, where the Swedish subsidy scheme is extended, affects the investors. Finally, we will examine the effect of a possible collapse in the green certificate price, and how learning about the likelihood of the price collapse, affect the investors.

An investment in a renewable energy project has large up-front costs, which are to a large extent irreversible. In addition, the revenues generated are highly dependent on the electricity and green certificate prices over the lifetime of the project, and since the future prices are uncertain, the investment is exposed to considerable market risk (Fernandes et al. (2011)). The possibility to delay an investment opportunity with these characteristics creates an additional value of flexibility, as the investor can wait for more information before undertaking an irreversible investment decision. The commonly used traditional net present value (NPV) approach, which treats an investment as a now or never decision, fails to capture the dynamic nature of the investment problem. As a result, the NPV approach disregards the value of flexibility, and, therefore, does not account for uncertainty correctly. According to Dixit and Pindyck (1994), it is more suitable to treat investments under uncertainty as financial options, using the real options approach. In reality, managerial flexibility implies that the investment can be undertaken at any time, where an irreversible cost is paid to receive the profit streams generated by the project. Therefore, the investment opportunity resembles a time-dependent American call option.

According to Fernandes et al. (2011), the real options theory is widely used to evaluate investment opportunities in the energy sector, but is still limited in the field of renewable energy sources. Bøckman et al. (2008) consider optimal capacity choice and investment timing in a renewable energy project. They present a model to evaluate small hydropower plants, which they apply to the Norwegian market. The profitability of a plant is modelled using a stochastic contribution margin. The capacity choice is calculated by simulating the physical properties of the plants using key figures and historical data. For each project, they find the threshold price where it is optimal to invest.

The analysis of the effect of various subsidy schemes on investment behaviour has received considerable attention in recent literature on real options. Kitzing et al. (2017) evaluate a wind energy project under different support schemes using a real options model. They include different correlated factors in one stochastic process to model the gross margin. The investment threshold and optimal capacity is then found for a offshore wind energy case study in the Baltic sea. They find that there is a difference in profit margins and project size when evaluating the various subsidy schemes, where green certificates may lead to a higher profit margin and capacity. They also find that when including an upper limit constraint on the capacity, the investment is hastened.

Adkins and Paxson (2016) use a real options approach to derive the optimal investment timing for a renewable energy project with a subsidy. Different subsidy schemes are evaluated, where the subsidy is proportional to a stochastic price and/or a stochastic quantity. The occurrences of a sudden introduction or retraction of a subsidy are modelled by a Poisson process. Adkins and Paxson (2016) find that the type of subsidy scheme has a large impact on the optimal time of investment, where a retractable subsidy gives the strongest incentive for early investments.

Boomsma and Linnerud (2015) consider how investors respond to market and policy risk, where they consider several renewable support schemes. Market risk refers to the uncertainty in electricity and green certificate prices, while policy risk relates to a possible change of the subsidy scheme, which is modelled as a Poisson process. Correlation between electricity and green certificate prices results in risk diversification, which speeds up investments. They find that the possibility of a retroactive termination of the subsidy scheme encourages later investments, while a non-retroactive termination encourages earlier investments. They find that in order to achieve optimal division of risk between policymakers and investors, the investors should be shielded against some risk.

Fleten et al. (2016) consider perpetual investment opportunities in hydropower projects before green certificates were introduced in Norway. They use a real options model to find the implied level of subsidies in each project, and investigate whether the investors base their decisions on the traditional net present value approach or the real options approach, by conducting interviews. Even though the investors claimed to use the NPV criterion, their decisions were consistent with the real options approach. Their analysis shows that investors follow real options thinking, but the option values are not quantified.

Closest to our work is Boomsma et al. (2012), who examine investment behaviour under different policy support schemes using a case study of a wind energy project in Norway. They employ a real options approach to analyze the optimal investment timing and capacity choice, with steel price, electricity price and subsidy price as the sources of uncertainty. The policy schemes they examine are feed-in tariffs and renewable energy certificates. In addition, they analyze the case where the support scheme employed can change with time, using Markov switching. Boomsma et al. (2012) find that both the timing and capacity choice differ with the various support schemes. Implementing a feed-in tariff encourages an earlier investment, while certificate trading encourages a larger project capacity.

In our thesis, we model investments in renewable energy in Norway and Sweden as an American option with a time-dependent value function. This is because the duration subsidies will be received, depends on the time of investment. In addition, we consider a project with a finite-lifetime. Close to this work is Gryglewicz et al. (2008), who study the effects of uncertainty on finite-life projects. They find that uncertainty in some cases accelerates investments for finite-life projects. They test the robustness of this finding, and also consider the case with finite option life, where they get the same result.

In most cases, there is no closed form solution to options with time-dependent values, and numerical methods must be used (Moreno and Navas (2003)). There is extensive literature on numerical methods to calculate the value of American options. Schwartz (1977) evaluates American options for discrete times and discrete stock prices, by approximating the partial derivatives in the Black-Scholes equation using finite differences. The boundary conditions at the investment deadline of the option are known, and the option value is calculated for a range of stock prices by backwards iterations. Cox et al. (1979) introduce a model where the underlying stochastic process starts at a given value, and follows a binomial process. The value of the option is then derived by iterating backwards using arbitrage arguments, i.e. risk neutral valuation. Boyle (1977) uses Monte Carlo simulation to estimate the value of an European option. This is done by simulating a series of stock price trajectories, which is used to determine the distribution of terminal option values. He finds that this is a simple and flexible method. For example, the underlying variables can follow different types of stochastic processes, and a jump process can easily be incorporated into the model.

In this thesis, we follow the approach by Longstaff and Schwartz (2001) to estimate the value of the time-dependent option. Longstaff and Schwartz (2001) propose the least-squares Monte Carlo method, to approximate the value of an American option numerically. In the algorithm, the conditional expectation of the continuation value is approximated using least-squares regression. They find that in order to get a good approximation, the number of basis functions in the regression can be increased to the point where the value no longer increases. Further, they find that this method is advantageous when considering an option with multiple factors and American-exercise features, as other numerical methods become impractical. Clément et al. (2002) analyze the convergence of the least-squares Monte Carlo algorithm. They find that the approximation for conditional expectation will approach the true value when the number of basis functions goes to infinity. Furthermore, when the number of basis functions are finite, there is almost sure convergence of the algorithm. Moreno and Navas (2003) analyze how robust the least-squares Monte Carlo method is to the choice of basis functions used in the regression. They find that increasing the number of basis functions can both increase and decrease the option value, contrary to Longstaff and Schwartz (2001), who find that increasing the basis functions increases the option value. In addition, they find that in some cases, computational problems can occur when the number of terms gets high. For an American put option, they find that the results are very robust to both the choice and number of basis functions, however, for more complex options, the option value is more sensitive to the basis functions.

In this thesis, we base our model on the real options literature, in particular articles regarding investments in renewable energy with subsidies, where our solution approach is based on the literature on least-squares Monte Carlo simulation. Our contribution is three-fold. First, we add to the literature on renewable energy projects and green certificates. We consider a case study of the option to invest in a wind energy project, where we compare the investment opportunities of a Norwegian and Swedish investor. Our work differs from earlier literature[1], in that we explicitly account for the limited time of the policy scheme and country specific regulations. To the best of our knowledge, we are the first to use a real options model to consider how the end of the policy scheme, and regulatory differences between Norway and Sweden, impacts investors. We find that the Swedish investor has a higher option value, and that the Norwegian investor is likely to invest earlier than the Swedish investor.

Second, we add to the literature on the real options approach, by considering a multi-dimensional option with a time-dependent value, and where the project has a finite lifetime. Related literature is Gryglewicz et al. (2008), who consider a one-dimensional investment opportunity with finite option and project lifetime. An option with a finite lifetime has a time-dependent value function, and therefore, has characteristics similar to the option considered in this thesis. When considering two-dimensional real options, there is a large body of literature on perpetual options, e.g. Boomsma and Linnerud (2015) who calculate the investment threshold of an option analytically and the value at this threshold. Boomsma et al. (2012) consider a three-dimensional perpetual investment opportunity under different support schemes. When there is no analytic solution, they have evaluated the option numerically under more realistic assumptions, with a finite option and project lifetime. Our work differs from their numerical procedure, by considering a perpetual option with a complex time-dependent value function, where changes occurs at given dates. We find that neglecting the time-dependent features of the model, can have a large impact on investment behaviour and option values.

[1] See e.g. Fleten et al. (2016) and Boomsma et al. (2012).

Third, we have developed an algorithm to solve the real options model, using least squares Monte Carlo simulation. The programming code for the simulation is a large part of our contribution. The advantage of using this method is flexibility. Our Monte Carlo model captures the complexity caused by the regulations of the policy scheme, and allows for learning curve effects in the investment cost and correlated stochastic variables. We also use the model to incorporate price jumps, and the possibility to learn about the likelihood of a jump.

This thesis is organized as follows. Section 2 presents background on the electricity and the green certificate market. Section 3 formulates our real options model and solution approach. Section 4 quantifies the parameters used in the case study. Section 5 discusses the results of the case study and compares the Norwegian and Swedish investment opportunities. Section 6 concludes.

2 Background

In this section, we present background information relevant for evaluating the investment opportunities of the Norwegian and Swedish investors. First, we present the Nordic electricity market, before we discuss the key features of the Norwegian/Swedish green certificate scheme.

2.1 Electricity market

Power in the Nordic region is traded on Nord Pool, which is a deregulated and free market, where the price is determined by supply and demand. Nord Pool is an European power market, and offers, among other, intraday and day-ahead trading. Most trading occurs at the day-ahead markets. Buyers are typically utilities, and sellers are the electricity producers. Buyers and sellers assess how much energy they want to trade hourly the next day, and for which price. This is then ordered through Nord Pool, which calculates an hourly price and announces it to the market. Trades are then settled based on the market prices, and the contracts delivered the next day (Nord Pool (2017a)). Figure 2.1 shows weekly electricity spot prices from 2013-2017.

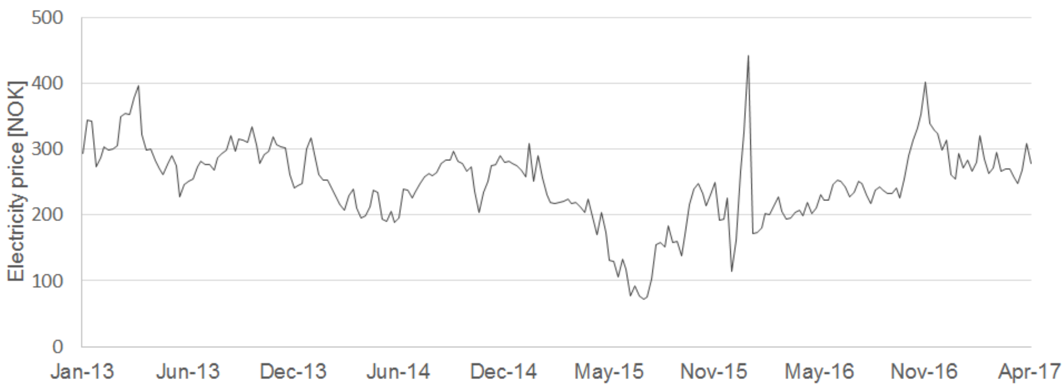


Figure 2.1: Historical prices of electricity per MWh, in the period 2013 to 2017 (Nord Pool (2017b)).

As seen in the figure, power prices are volatile. There are several factors that impact the electricity price, e.g. oil and gas prices, politics and economic growth (Fantoft (2014)). One of the most important factors is weather, where Sweden is mostly depending on wind for power production, while Norway is depending on rain for hydropower production. There are also seasonal variations in the prices, where demand and supply varies throughout the year, e.g. the demand for heating is higher in the winter.

2.2 Green certificate market

In addition to selling electricity, a main source of income for renewable energy projects is green certificates. In 2012, the common Swedish and Norwegian green certificate market was established, with the objective to increase renewable production and contribute to the countries' renewable energy goals. The common market is based on the Swedish green certificate market, established in 2003. The purpose of a having a joint market, is to increase renewable energy production in a more cost-effective way, by directing investments to the most advantageous projects (NVE (2016)). Of a total increase in renewable energy production of 28.4 TWh by 2020, Norway will finance 13.2 TWh, while Sweden will finance 15.2 TWh. Projects in both countries that started production in a specified period before 1. January 2012 are part of a transition scheme, where the production is not a part of the goal of 28.4 TWh (Elsertifikatloven (2011)). Projects in the transition scheme are eligible to receive green certificates for a reduced period. Certificates can be transferred and used in both Norway and Sweden, irrespectively of where they were issued.

The price of green certificates is decided by supply and demand. The total supply of green certificates in a given year, are the issued certificates that year, where one certificate is issed for each MWh of electricity produced, in addition to an accumulated surplus of certificates from previous years. The demand for green certificates is implicitly decided by the Norwegian and Swedish government through each country's green certificate quota curve. Energy consumers with a quota obligation, e.g. energy retailers, must purchase an amount of green certificates which corresponds to the electricity consumption multiplied by the quota for a given year. Every year, the 1st of April, the green certificates needed to fulfill the quota obligation are cancelled from the certificate accounts of the energy retailers (Elsertifikatloven (2011)). The quota curve is illustrated in Figure 2.2,

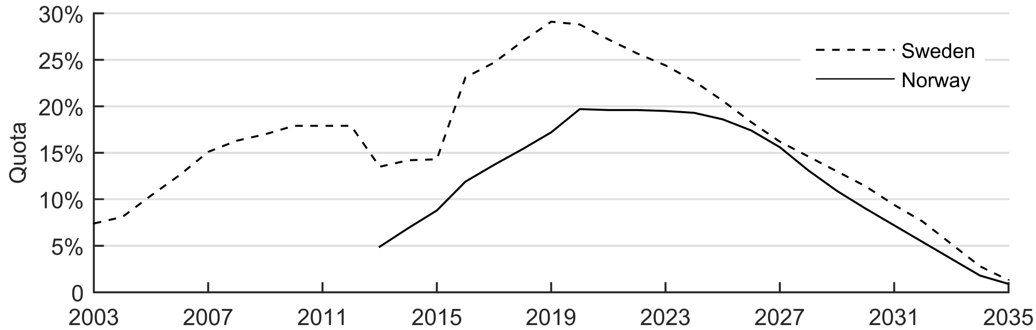


Figure 2.2: Quota curve for Norway and Sweden (NVE (2016)).

and is based on the renewable production in the transition scheme, forecasted renewable production and forecasted electricity consumption where there is a quota obligation. The Swedish quota is higher than the Norwegian quota, which is a consequence of Sweden financing a larger share of the new production, in addition to having more certificates issued as part of the transition scheme. The future production and consumption of electricity are uncertain, as well as the supply and demand of green certificates. If the forecasted estimates used as basis for the quota curve differ from the realized values, there will be a change in the surplus of certificates. If more are issued than cancelled in a year, the total surplus of green certificates will increase. Figure 2.3 shows the issued and cancelled green certificates, in addition to the accumulated surplus since 2003, while Figure 2.4 shows the prices.

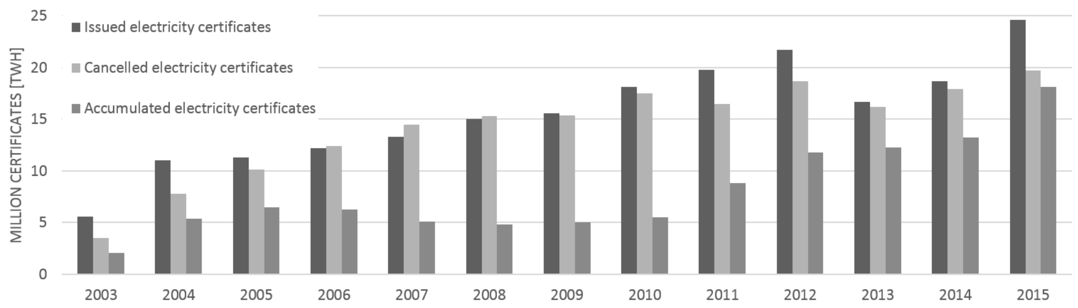


Figure 2.3: Historical demand and supply of green certificates (NVE (2016)).

Figure 2.4: Historical prices of green certificates per MWh in the period 2005 to 2017 (SKM (2017)).

When the surplus is increasing (decreasing), a negative (positive) pressure is put on the prices of green certificates ((NVE, 2016)). This can be observed from 2006 to 2008, where the surplus of green certificates was decreasing, and as illustrated in figure 2.4, the prices increased by about 100% in the corresponding period. In the period 2010 to 2015, more green certificates were issued than cancelled, and the surplus was increasing. In the corresponding period, the prices have had a negative trend.

In the current agreement between Norway and Sweden, the subsidy scheme and green certificate market will end in 2035. Energy producers that are eligible for green certificates will receive these from the start of production, for a maximum of 15 years. The regulations of the policy scheme differ in the two countries, where there are different constrains on when production must have started, in order to receive subsidies. The Norwegian investor has an investment deadline, and must invest by 31. December 2021 to receive green certificates (Elsertifikatloven (2011)). In contrast, the Swedish investor will receive green certificates regardless of the time of investment, but only until 31. December 2035 (Elsertifikatloven (2011)). This is illustrated in Figure 2.5, which shows the duration certificates will be received when investing before and after the Norwegian deadline. This difference, and how it effects the investors, is one of the main focus areas of this thesis.

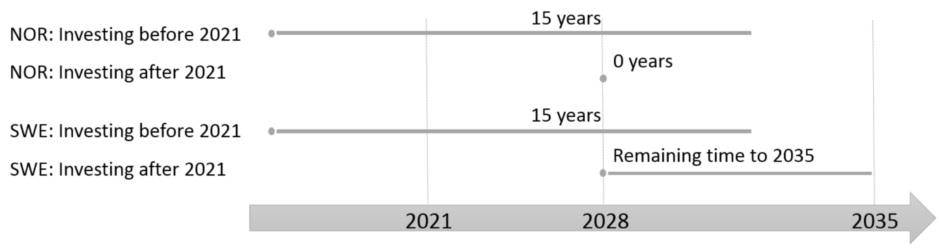


Figure 2.5: The duration subsidies will be received if investing before and after 2021.

The green certificate market is regulated in close cooperation between Norway and Sweden. In accordance with the agreement, progress reviews should be performed on a regular basis. In the first progress review, in 2015, the renewable energy goal was increased with 2 TWh. The quota curve was adjusted, based on previous estimation errors and updated forecasts for electricity consumption (NVE (2016)). In addition, to prevent projects from loosing the right to receive certificates if delayed, the Norwegian deadline was extended by one year, from the end of 2020 to the end of 2021 (Prop. 97 L (2014-2015)). In 2017, the second progress review is planned in order to implement and change potential regulations from 1. January 2018. One of the topics is the possible extension of the Swedish subsidy scheme, and how this would affect the green certificate market. The current proposal to the Swedish Riksdag, is to extend the policy scheme by 10 years, until 31. December 2045 (The Swedish Government (2017)). In addition, it is proposed that a constraint should be implemented, which limits new production eligible to receive certificates. It is not specified which type of constraint, but an example from the proposal is a time constraint similar to the Norwegian deadline (The Swedish Government (2017)).

3 Model

In this chapter, we present a model to evaluate the opportunity to invest in a renewable energy project in Norway and Sweden. We analyze the optimal investment strategy from the perspective of a risk-neutral and profit maximizing investor. In particular, we determine the value of the option to invest, and its optimal exercise timing. We will first consider a model for a perpetual option, where it is assumed that the opportunity to invest and receive green certificates will last forever, which is a common assumption in the literature on renewable energy investments [1]. Second, we consider a time-dependent perpetual option, which incorporates the coming investment deadline and the finite lifetime of the subsidy scheme. By considering both cases, we are able to analyze how the time-dependency influences the investment opportunity. Finally, we present the numerical solution approach.

3.1 Perpetual subsidy scheme

Similarly to Boomsma and Linnerud (2015) and Fleten et al. (2016), we consider a model for a perpetual option, where it is assumed that the opportunity to invest and receive green certificates will last forever. We replicate their semi-analytical solution, where for a given electricity price, the investment threshold for the subsidy and the corresponding option value, can be calculated.

The revenues of a renewable energy project is dependent on the electricity price, denoted E_t, and green certificate price, denoted S_t. As explained in Section 2, the future electricity and certificate prices are uncertain. Therefore, we model the prices using stochastic processes. Both the green certificate and electricity prices have been modelled as geometric Brownian motions by, among others, Fleten et al. (2016), Boomsma et al. (2012) and Boomsma and Linnerud (2015). Similarly, we allow the green certificate and electricity prices to evolve according to (3.1) and (3.2), respectively.

$$dS = \mu_S S dt + \sigma_S S dW_S, \tag{3.1}$$

$$dE = \mu_E E dt + \sigma_E E dW_E, \tag{3.2}$$

where μ_S and μ_E denote the drifts, and σ_S and σ_E denote the volatilities of the green certificate price and electricity price, respectively. Geometric Brownian motions capture the

[1] See e.g. Fleten et al. (2016)

long term development of the prices, which is reasonable in our case, since we consider a long-term investment. In addition, the historical prices in Norway and Sweden have been positive, and geometric Brownian motions, unlike several other processes, does not allow for negative realizations. We assume that the two processes are independent. Later, when we consider the time-dependent option, we relax this assumption.

We choose a profit function of a general form, where the profit stream is dependent on the electricity and green certificate prices, the fixed and variable costs, denoted C_F and C_V, and the quantity, q, produced. We assume that the electricity from a project is produced in the period from the moment of investment, denoted τ, and for the expected lifetime of the project, denoted T_L. Green certificates will be issued to a production facility for a maximum number of years, denoted T_S, after production has begun. We assume the production quantity and both variable and fixed costs to be constants. Therefore, the instantaneous profit stream of the project is given by

$$\pi(S_t, E_t) = \begin{cases} (E_t + S_t - C_V)q - C_F & t \leq \tau + T_S, \\ (E_t - C_V)q - C_F & t > \tau + T_S, \end{cases} \tag{3.3}$$

where time is denoted by t, and τ denotes the time of investment. The value of the project at the time of investment is the expected net present value of the profit stream. Let $V(S_t, E_t)$ denote the value of the project. Integrating the profit stream in equation (3.3), in the period from $t = \tau$ to $t = \tau + T_L$ yields

$$\begin{aligned} V(S_t, E_t) &= \mathbf{E}\Big[\int_{\tau}^{\tau+T_L} \pi(S_t, E_t) e^{-\rho(t-\tau)}\, dt\Big], \\ &= \mathbf{E}\Big[\int_{\tau}^{\tau+T_S} ((E_t + S_t - C_V)q - C_F) e^{-\rho(t-\tau)}\, dt + \int_{\tau+T_S}^{\tau+T_L} ((E_t - C_V)q - C_F) e^{-\rho(t-\tau)}\, dt\Big], \\ &= q(r_S S_\tau + r_E E_\tau) - C, \end{aligned} \tag{3.4}$$

where $r_S = \frac{1}{\rho-\mu_S}(1 - e^{(\mu_S-\rho)T_S})$, $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V + C_F}{\rho}(1 - e^{-\rho T_L})$. The constants r_S and r_E represent the discount factors for green certificate and electricity prices, respectively. The discount factors have the same form, where the differences are the growth rates of the prices, and the duration in which revenues are received. In addition, S_t and E_t are independent variables, thus, the revenue from selling electricity is independent from the revenue received from subsidies. See Appendix A.1 for a more detailed derivation.

The real option can be exercised at any time, where an irreversible cost, denoted by I, is paid to receive the profit stream in equation (3.3). The investment cost is assumed to be constant, similarly to e.g. Boomsma and Linnerud (2015) and Fleten et al. (2016)[2]. When considering the time-dependent option, we let the investment cost decrease with a learning rate.

Let $F(S,E)$ denote the value of the option to invest in Norway or Sweden. The optimal stopping problem of the firm is to find the time of investment, denoted T, that maximizes the value of the investment opportunity,

$$F(S,E) = \max_{T} \mathbf{E}[e^{-\rho T}(V(S,E) - I)], \tag{3.5}$$

where ρ denotes the discount rate. The solution space of this problem can be split into a stopping and continuation region. In the stopping region, it is optimal to invest immediately, and the value of the investment opportunity is therefore equal to the net present value, $V(S,E) - I$. The stopping region and continuation region are separated by an investment threshold, where the option value is equal to the instrinsic value. In the continuation region, the optimal strategy is to delay the investment, and we model the investment decision using the dynamic programming approach. This breaks the investment problem into two sub-problems, the immediate decision, and the consequences of all future decisions (Dixit and Pindyck (1994)). This is expressed using the Bellman equation, where the value of the investment option at any time is given by

$$\rho F(S,E) = \pi + \frac{1}{dt}\mathbf{E}[dF(S,E)], \tag{3.6}$$

which states that the return the investor must have in order to hold the option, is equal to the instantaneous profit plus the change in option value per time. Using Ito's lemma to derive the value of $dF(S,E)$ in the continuation region, provides the following partial differential equation,

$$\frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F}{\partial S^2} + \frac{1}{2}\sigma_E^2 E^2 \frac{\partial^2 F}{\partial E^2} + \mu_S S \frac{\partial F}{\partial S} + \mu_E E \frac{\partial F}{\partial E} - \rho F = 0. \tag{3.7}$$

Assuming a form $F = AS^{\beta_S} E^{\beta_E}$, we get the following fundamental quadratic,

$$Q(\beta_S, \beta_E) = \frac{1}{2}\Big(\sigma_S^2 \beta_S(\beta_S - 1) + \sigma_E^2 \beta_E(\beta_E - 1)\Big) + \mu_S \sigma_S + \mu_e \sigma_E - \rho. \tag{3.8}$$

[2] Making the investment cost stochastic would drastically increase the complexity of the model, while providing little additional insight to the aspects we want to analyze.

The investment threshold can then be expressed by

$$S^* = \frac{\beta_S}{\beta_E + \beta_S - 1} \frac{I + C}{r_S}, \tag{3.9}$$

$$E^* = \frac{\beta_E}{\beta_E + \beta_S - 1} \frac{I + C}{r_E}, \tag{3.10}$$

where β_E and β_S can be calculated from equation (3.8), (3.9) and (3.10) for a given E or S. See Appendix B for a detailed derivation. The value matching condition states that the option value is equal to the intrinsic value at the investment threshold, thus, the option value at the investment threshold is expressed by $V(S^*, E^*) - I$. In what follows, we consider the time-dependent perpetual option, where we discuss how the model is different when incorporating the policy regulations.

3.2 Time-dependent subsidy scheme

An investment strategy is expected to differ between the Norwegian and Swedish investor when considering a renewable energy project, as a consequence of different national regulations. As the Norwegian investor has a deadline for the project to be eligible to receive green certificates, the intrinsic value is time-dependent. The intrinsic value is also time-dependent for the Swedish investor, since the investor will receive subsidies for a shorter duration when investing closer to the end of the policy scheme. Therefore, modelling the options using the perpetual subsidy scheme can yield misleading results. When the intrinsic value depends on the time of investment, the time derivative of the option value is non-zero, and there is no analytical solution to the resulting partial differential equation (Dixit and Pindyck (1994)). Therefore, the investment problem must be solved numerically. This, in addition, gives us more fexibility to relax several assumptions. In what follows, we will first derive a baseline model, where we incorporate the time-dependent factors of the policy scheme. Further, we extend the model, and allow for the possibility of jumps in the prices, both with and without Bayesian learning. Lastly, we present the algorithm used in the Monte Carlo simulation.

3.2.1 Baseline model

The subsidy scheme and the green certificate market is set to end at a given date, denoted T_E. Green certificates will not be issued after this date, and therefore, the revenues from subsidies are zero after T_E, for both Norwegian and Swedish projects. Renewable energy projects in Norway will only be eligible to receive green certificates, if the electricity production has started before a deadline, denoted T_{DN}. We assume production starts at the moment of investment[3], and therefore, a Norwegian project will be eligible to receive green certificates if $\tau \leq T_{DN}$.

In the time-dependent model, we will relax the assumption of independent stochastic processes, such that

$$\mathbf{E}[dW_S dW_E] = \rho_{SE} dt, \tag{3.11}$$

where ρ_{SE} denotes the correlation between the prices. The profit and payoff functions now incorporate the constraints related to the time of investment. Thus, the instantaneous profit function for the Norwegian investor is

$$\pi_N(S_t, E_t, t) = \begin{cases} (E_t + S_t - C_v)q - C_F, & t \leq T_E \ \wedge \ \tau \leq T_{DN} \ \wedge \ t < \tau + T_S, \\ (E_t - C_v)q - C_F, & t > T_E \ \vee \ \tau > T_{DN} \ \vee \ t > \tau + T_S. \end{cases} \tag{3.12}$$

The Norwegian investor receives subsidies if investing before T_{DN}, and if the investor has received subsidies for less than T_S years. The investor will not receive subsidies after the policy scheme has ended, $t \geq T_E$. The only difference between the Norwegian and Swedish investors is that $T_{DN} = T_E$ for the Swedish investor, whose instantaneous profit function is given by

$$\pi_S(S_t, E_t, t) = \begin{cases} (E_t + S_t - C_v)q - C_F, & t \leq T_E \ \wedge \ t < \tau + T_S, \\ (E_t - C_v)q - C_F, & t > T_E \ \vee \ t > \tau + T_S. \end{cases} \tag{3.13}$$

The Swedish investor receives subsidies if investing before T_E and if the investor has received subsidies for less than T_S years.

Using these profit functions, the intrinsic values of the projects can be calculated. Let $V_N(S_t, E_t, t)$ and $V_S(S_t, E_t, t)$ denote the intrinsic values of the Norwegian and Swedish projects, respectively. By integrating the profit function of the Norwegian investor for the lifetime of the project, we get

$$V_N(S_t, E_t, \tau) = \begin{cases} q(r_S(\tau)S_\tau + r_E E_\tau) - C, & \tau \leq T_{DN}, \\ r_E q E_\tau - C, & \tau > T_{DN}, \end{cases} \tag{3.14}$$

[3] This assumption will not affect the insight from the results, as a potential building time easily can be incorporated in the investment deadline.

where $r_S(\tau) = \frac{1}{\rho-\mu_S}(1 - e^{min\{T_E-\tau,T_S\}(\mu_S-\rho)})$, $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1 - e^{-\rho T_L})$. If investing for $t \leq T_E - T_S$, the intrinsic value is the same as in the case where the subsidy scheme is perpetual. For investments in the period $T_E - T_S < t < T_{DN}$, subsidies will be received for a reduced duration. When $\tau > T_{DN}$, the project does not receive any subsidies, and $r_S = 0$. For the Swedish investment project, we have the following intrinsic value,

$$V_S(S_t, E_t, \tau) = \begin{cases} q(r_S(\tau)S_\tau + r_E E_\tau) - C, & \tau \leq T_E, \\ r_E q E_\tau - C, & \tau > T_E, \end{cases} \tag{3.15}$$

where $r_S(\tau) = \frac{1}{\rho-\mu_S}(1 - e^{min\{T_E-\tau,T_S\}(\mu_S-\rho)})$, $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1 - e^{-\rho T_L})$. The present value of the revenues from subsidies depends on the time of investment. As the time until T_E decreases, r_S goes to 0. If the investment is undertaken before T_{DN}, the intrinsic values of both investors are equal. This is also the case when $t > T_E$, as neither of the investors will receive subsidies. See Appendix A.2 for derivations.

Furthermore, in the time-dependent model, we let the investment cost, I_t, decrease with time due to learning effects. The learning rate, λ, is a measure of how much the investment cost decreases, as the cumulated capacity doubles. The yearly learning rate, λ_Y, is calculated based on forecasted values of production. The investment cost can, therefore, be expressed by $I_t = I_0 e^{-\lambda_Y t}$. Let $F_N(S_t, E_t, t)$ and $F_S(S_t, E_t, t)$ denote the value of the option to invest in Norway and Sweden, respectively. The investors solve the following optimal stopping problems,

$$F_N(S_t, E_t, T) = \max_T \mathbf{E}[e^{-\rho T}(V_N(S_t, E_t, T) - I_t)], \tag{3.16}$$

$$F_S(S_t, E_t, T) = \max_T \mathbf{E}[e^{-\rho T}(V_S(S_t, E_t, T) - I_t)], \tag{3.17}$$

where T represents the optimal time of investment. These optimal stopping problems can not be solved analytically, and we will, therefore, use numerical methods to obtain a solution.

3.2.2 Model extension

In 2014, the head of Statkraft warned about the possibility of a collapse in the certificate prices (Øyvind Lie (2014a)). Also, at the same time, both BKK and DNB feared a price drop (Øyvind Lie (2014b)). The uncertainty about possible price drops is a topic of great interest for investors, as many of them are relying on green certificates for projects to be profitable.

In this section, we therefore incorporate the possibility of discrete jumps in the green certificate price into the model. We let the jumps occur at random times following a Poisson process, given by

$$dq = \begin{cases} 0, & \text{with probability} \quad 1-\Lambda dt, \\ 1, & \text{with probability} \quad \Lambda dt, \end{cases} \tag{3.18}$$

where Λ represents the mean arrival rate of the jumps. The green certificate price then follows a geometric Brownian motion combined with a Poisson jump process,

$$dS = \mu_S E dt + \sigma_S E dW_S - \phi S dq, \tag{3.19}$$

where ϕS is the size of the jump. The process for the electricity price remains as in equation (3.2). The jumps are assumed uncorrelated to the returns of the prices.

The possibility of a jump will lower the expected green certificate price, and hence, decrease the values of the Norwegian and Swedish projects. Dixit and Pindyck (1994) find that when there is a risk of a jump, the expected change of the stochastic process decreases by $\Lambda\phi$. Hence, the expected development of the green certificate price is

$$\mathbf{E}[S(t)] = S_\tau e^{(\mu_S - \lambda\phi)(t-\tau)}. \tag{3.20}$$

Therefore, using the same approach as in Appendix A.1, the intrinsic values of the Norwegian and Swedish projects can be expressed by equation (3.21) and (3.22), respectively.

$$V_{JN}(S_t, E_t, \tau) = \begin{cases} q(r_{JS}(\tau)S_\tau + r_E E_\tau) - C, & \tau \le T_{DN}, \\ r_E q E_\tau - C, & \tau > T_{DN}, \end{cases} \tag{3.21}$$

$$V_{JS}(S_t, E_t, \tau) = \begin{cases} q(r_{JS}(\tau)S_\tau + r_E E_\tau) - C, & \tau \le T_E, \\ r_E q E_\tau - C, & \tau > T_E, \end{cases} \tag{3.22}$$

where $r_{JS}(\tau) = \frac{1}{\rho+\Lambda\phi-\mu_S}(1 - e^{min\{T_E-\tau, T_S\}(\mu_S-\rho-\Lambda\phi)})$, $r_E = \frac{1}{\rho-\mu_E}(1-e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1-e^{-\rho T_L})$. The solution approach is equivalent to the baseline model, where the difference is the price process of the green certificates, in addition to the intrinsic values. For additional details, see Appendix E.1.

There are speculations about how the price of green certificates will develop, and if a collapse in the price is likely. By observing signals from the government and other institutions, the investors can learn about the likelihood of a price collapse. These signals can, for example, be investment decisions of large projects. Just a few large investments are needed to balance the demand in Norway before the deadline, T_{DN}, and therefore, the prices can drop if investment decisions are taken by large investors (Barstad (2017)). In addition, signals regarding the policy scheme, and in particular a possible extension, is crucial information for investors. In the pending proposal regarding the Swedish policy extension, new production above the goal of 28.4 TWh in Norway, should be part of the Swedish extended goal of 18 TWh. This is not yet decided, and if the proposal is rejected, or revised, there can be a collapse in green certificate prices (Barstad (2017)).

Following the approach by Thijssen et al. (2004), we implement Bayesian learning into the model, where investors can update their belief about the likelihood of a price drop by receiving signals. The signals arrive at discrete times following a Poisson process,

$$dn(t) = \begin{cases} 0 & \text{with probability} \quad 1 - \Lambda_S dt, \\ 1 & \text{with probability} \quad \Lambda_S dt, \end{cases} \tag{3.23}$$

where n denotes the number of signals, and Λ_S is the mean arrival rate of the signals. Depending on the state of the world, there is either a high or a low probability of a jump. The jump intensities for the high and low probability cases, are denoted Λ_H and Λ_L, respectively. Let P_0 denote the initial likelihood of being in a good state, where the jump intensity is low. By receiving signals, the investors update this probability. The likelihood of a signal indicating a high or low jump intensity depends on the true state of the market. This is illustrated in Table 3.1, where ω denotes the reliability of the signals.

Table 3.1: Reliability of a signal

	True probability is low	True probability is high
Signal indicates low probability	ω	1-ω
Signal indicates high probability	1-ω	ω

Let k denote the number of signals indicating a low jump intensity in excess of signals indicating a high jump intensity. Then the probability of being in a good state is given by

$$p(k) = \frac{\lambda^k}{\lambda^k + \zeta(1-\lambda)^k} \tag{3.24}$$

where $\zeta = (1-p_0)/p_0$. Let V_L and V_H denote the payoff of the project when the jump intensity is Λ_L and Λ_H, respectively. Then the intrinsic value of the project is the conditional expected payoff, given by

$$V(S_t, E_t, \tau) = p(k)V_L(S_t, E_t, \tau) + (1 - p(k))V_H(S_t, E_t, \tau). \tag{3.25}$$

This is solved numerically using Monte Carlo simulation. For details regarding the solution algorithm, see Appendix E.2.

3.2.3 Numerical solution approach

Valuing real options with American-exercise features can be challenging. In the perpetual case, there are several numerical approximation methods that can be employed, but many of these are impractical to implement in a multi-factor setting (Longstaff and Schwartz (2001); Rodrigues and Rocha Armada (2006)). In this thesis, we have chosen to implement the least-squares Monte Carlo simulation approach to value the options. The advantage of this method is the possibility to incorporate all the critical features of the model, such as multiple factors, time dependency and American-exercise features. Also, the introduction of additional stochastic variables increase the computational time linearly (Rodrigues and Rocha Armada (2006)), which makes it easier to increase dimensionality in further extensions.

To calculate the value of the option using Monte Carlo simulation, the investment problem must be reduced to a finite number of sub-problems. This is achieved using the Bellman equation,

$$F(S, E, t) = max\Big\{V(S, E, t) - I(t), \frac{1}{1+\rho dt}\mathbf{E}[dF(S + dS, E + dE, t + dt)|S, E, t]\Big\}, \tag{3.26}$$

which states that the value of the option is the maximum of the immediate exercise value and the expected value if delaying the investment. To be able to solve the problem numerically, time is discretized in N_{time} steps. The time between each step is $\Delta t = \frac{T_D}{N_{time}}$, such that $t = 0, \Delta t, 2\Delta t, ..., T_D$, where T_D denotes the time at the end of the simulation. Price paths for a large number of scenarios, denoted i, are simulated, where $i = 1, ..., N_{Paths}$. For each scenario, there is a path for both the green certificate price, denoted S_{ti}, and the electricity price, denoted E_{ti}. To simulate the price paths, equation (3.1) and (3.2) must be discretized. To avoid discretization errors when simulating the price paths, Ito's Lemma is used to transform these equations, which gives

$$S_{(t+\Delta t)i} = S_{ti} e^{(\mu_S - \frac{1}{2}\sigma_S^2)\Delta t + \epsilon_1 \sigma_S \sqrt{\Delta t}}, \tag{3.27}$$

$$E_{(t+\Delta t)i} = E_{ti} e^{(\mu_E - \frac{1}{2}\sigma_E^2)\Delta t + \epsilon_2 \sigma_E \sqrt{\Delta t}}, \tag{3.28}$$

where $\epsilon_1 \sim N(0,1)$ and $\epsilon_2 \sim \rho\epsilon_1 + \sqrt{1-\rho^2}N(0,1)$ (Brandimarte (2014)). See Appendix C for details. At the investment deadline, there is no value of delaying the investment. Hence, the value of the option is the net present value of the investment opportunity,

$$F_i(S_{T_D i}, E_{T_D i}, T_D) = max\{0, V_c(S_{T_D i}, E_{T_D i}, T_D) - I_{T_D}\}, \tag{3.29}$$

where V_c represents V_N and V_S for the Norwegian and Swedish case, respectively.

Following the least-squares Monte Carlo approach by Longstaff and Schwartz (2001), we estimate the value of the option by iteration from the investment deadline to the first time step. At each time step, the investors will decide to either exercise the option or wait. The optimal strategy is to exercise the option if the immediate expected payoff from exercising the option, is larger than the expected future payoff if delaying the investment (Huynh et al. (2011)). The immediate expected payoff is the profit from exercising the option, i.e. $V_c(S_{ti}, E_{ti}, t) - I_t$. The continuation value is calculated using least squares regression, where it is assumed that the value of the option can be expressed as a linear combination of a set of basis functions, denoted by $\phi_b(S_t, E_t, t)$, where $b = 1, ..., B$ is the number of basis functions. Further, let α_{bt} denote the regression coefficients, then, the value of waiting can be expressed by

$$F(S_t, E_t, t) = \sum_{b=1}^{B} \alpha_{bt}\phi_b(S_{ti}, E_{ti}, t). \tag{3.30}$$

For each time step, the regression coefficients are calculated based on the future expected cash flows, denoted CF_i, where the option is exercised at $t = \tau_i$. Thus, for a given time step, the present value of the cash flows for a given scenario can be calculated by

$$Y_{ti} = CF_i e^{-\rho(\tau_i - t)dt}. \tag{3.31}$$

Only paths where the option is in the money are used in the regression, as this increases the efficiency of the approach, and decreases the computations needed (Longstaff and Schwartz

(2001)). Let the in-the-money paths be denoted by M_t. Then, for a given t, the regression coefficients can be represented by

$$\begin{aligned} & min \quad \sum_i e_{ti}^2, \\ & s.t. \quad Y_{ti} = \sum_{b=1}^{B} \alpha_{bt} \phi_b(S_t, E_t, t) + e_{ti}, \qquad \forall\ i \in M_t, \end{aligned} \tag{3.32}$$

where e_{ti} is the residual for path i at time t (Brandimarte (2014)). At each step, if it is optimal to exercise the option for a scenario, the exercise time, τ_i and the cash flows, CF_i, for that scenario are updated. At $t = 0$, the continuation value is calculated by taking the average of the discounted cash flows for all scenarios. The value of the option at $t = 0$ is, therefore,

$$F(S_0, E_0, 0) = max\left\{0, V_c(S_0, E_0, 0) - I_0, \frac{\sum_{i=1}^{N_P} Y_{ti}}{N_P}\right\}. \tag{3.33}$$

The investment threshold, at $t = 0$, is found iteratively by first calculating the value of waiting for some initial prices, E_0 and S_0. If the value of waiting is higher than the value of immediate exercise, it is not optimal to invest for these prices. Then, by stepwise increasing S_0 while keeping E_0 constant, the investment threshold, S^* and $E^* = E_0$, is where the intrinsic value is equal to the value of waiting. This can be repeated for a large number of initial values for the electricity price, to approximate a two dimensional investment boundary.

The computational time and results of the Monte Carlo simulation are highly dependent on the model parameters, specifically the number of simulated paths, i, the number of time steps, N_{time}, and the basis functions, ϕ_b. These parameters must, therefore, be calibrated. The semi-analytical solution derived in Section 3.1 is used as reference in the calibration. We found that when calculating only one option value, 300,000 paths and 500 time steps provide a sufficient estimation of the option value for an acceptable running time. When performing a simulation where several option values are calculated, e.g. investment threshold and sensitivity analysis, the computational time is significantly longer. Therefore, 100,000 paths and 500 time steps are chosen for these simulations.

There are different types of polynomials that can be used in the regression, and as proposed by Longstaff and Schwartz (2001), we tested weighted Hermite polynomials and weighted Laguerre polynomials. In addition, we have tested simple power functions, e.g. x, x^2, x^3. The polynomials were tested by comparing the numerical results to the semi-analytical solution, where the criteria used were convergence speed, precision and accuracy. There were only minor differences using the various polynomials. This is consistent with the findings by Moreno and Navas (2003), who find that the least-squares method is quite robust to the choice of basis functions. However, we found the Laguerre polynomials to be slightly better across all criteria. We have, therefore, chosen to use the weighted Laguerre polynomials in the regression, which can be calculated by

$$L_n(x) = e^{-x/2} \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x}), \tag{3.34}$$

where the first Laguerre polynomials are

$$\begin{aligned} L_0 &= e^{-x/2}, \\ L_1 &= e^{-x/2}\ (-x+1), \\ L_2 &= e^{-x/2}\ \tfrac{1}{2}(x^2 - 4x + 2), \\ L_3 &= e^{-x/2}\ \tfrac{1}{6}(-x^3 + 9x^2 - 18x + 6). \end{aligned} \tag{3.35}$$

Based on the literature, intuition and testing, we found a combination of 33 basis functions, that provide a sufficient accuracy of the calculated option values. These basis functions consist of four expressions used as variables in the Laguerre polynomials up to the 8th degree, in addition to a constant. The chosen variables are the electricity price, E, the green certificate price, S, the sum of the prices weighted by the discount factors, $r_E E + r_S S$, and the product of the prices, $E \times S$. We found that increasing the degrees of polynomials further, did not make improvements to the results. Additional details regaring the calibration is provided in Appendix D.

According to Longstaff and Schwartz (2001), the problem must be renormalized when using weighted Laguerre polynomials. This is because these polynomials can lead to computational issues, since there are exponential terms, e.g. if the price is 1500, $L_n(1500)$ would be rounded to 0 in Matlab. We therefore renormalized the problem by applying a scaling factor to the costs and the initial prices. The scaling is used when making all computations, and the final results are scaled back by the same factor. We got the best results when scaling the problem by the investment cost, which is consistent with Longstaff and Schwartz (2001). We implemented the code in Matlab, see Appendix F for more information.

4 Case Study

Both Norway and Sweden have significant unexploited potential for wind power, which is one of the energy sectors where investors are receiving green certificates, and where we can expect new investments in the near future. We therefore consider a case study of a wind energy project to analyze the Norwegian and Swedish investment opportunities. In this section, we quantify the parameters which will be used as the baseline case in the analysis. The parameter values are mainly based on estimates from agency reports[1] and scientific contributions[2], and are chosen in cooperation with Knut-Harald Bakke, Head of Industrial Portfolio and Projects at Hydro Energi AS. The parameters used in the baseline case are summarized in Table 4.1.

Table 4.1: Numerical values used in the case study

Parameters	Benchmark value
E_0 - Initial electricity price	250 NOK/MWh
S_0 - Initial subsidy price	138 NOK/MWh
μ_E - Drift of electricity price	2.5 %
μ_S - Drift of subsidy price	2.5 %
σ_E - Volatility of electricity price	15.5%
σ_S - Volatility of subsidy price	16.3%
ρ_{SE} - Correlation	5.1%
I - Investment cost	10,000 NOK/kW
λ_Y - Yearly reduction of investment cost	0.58%
C_V - Variable costs	0.14 NOK/kWh
C_F - Fixed costs	0 NOK/kWh
Q - Capacity	35 MW
ϵ - Capacity factor	40%
q - Production quantity	122,640 MWh/year
ρ - Discount rate	6 %
T_0 - Start year	2017
T_L - Project lifetime	20 years
T_S - Maximum duration of subsidies	15 years
T_{DN} - Norwegian deadline	2021
T_E - End of policy scheme	2035

Wind energy investments typically have a long-term planning horizon, where the certified lifetime of most wind turbines are $T_L = 20$ years (DNV (2016); NVE (2015)).

As explained in detail in Section 3.1, the electricity and green certificate prices are uncertain, and evolve according to geometric Brownian motions. Therefore, we first need to

[1] See e.g. NVE (2015), IRENA (2014) and IRENA (2016)

[2] See e.g. Boomsma and Linnerud (2015) and Fleten et al. (2007)

quantify the initial values, the drift and the long term volatility of these processes. In the baseline case, the initial prices of electricity and green certificates are set to the average daily price of 2016 to focus on long term development, rather than short term deviations and seasonal variations in the prices. This gives an initial electricity price of $E_0 = 250$ NOK/MWh, and an initial green certificate price of $S_0 = 138$ NOK/MWh. Similarly to Boomsma and Linnerud (2015), we assume that the prices are expected to increase with the general price level, and set the drift of both price processes to the inflation rate, $\mu_E = \mu_S = 2.5\%$.

There is no consensus in the literature on which method to use when estimating the volatility of the electricity or green certificate prices. As a result, there are large variations in the volatility estimates in the Norwegian/Swedish market. When estimating the long-term volatility for geometric Brownian motions, forward contracts are often used, e.g. as in Fleten et al. (2007) and Boomsma and Linnerud (2015). Following their approach, we use historical average weekly three-year-forward contracts to estimate the volatility of electricity and green certificate prices[3]. In the period 2007-2016, we estimate the annual volatilities of the electricity and green certificate prices to be $\sigma_E = 15.5\%$ and $\sigma_S = 16.3\%$, respectively. As explained in Section 2, both green certificate and electricity prices are affected by the production from renewable energy projects and the demand for electricity, and it is likely that the prices are correlated. For the same time period and data as the volatility estimates, we calculate a correlation of $\rho_{SE} = 5,1\%$[4]

We will now quantify the parameters of the wind energy project, more specifically the capacity parameters and project costs. Naturally, the project characteristics vary a lot depending on the park location. The goal of our model is to study the regulatory differences in the policy scheme. To isolate this effect, and not project specific characteristics, we compare a similar problem in Norway and Sweden. As a candidate project, we consider an average Norwegian wind park.

Based on a report by NVE (2017), the average Norwegian wind park has 15 turbines, where each wind turbine produces 2.3 MW. Therefore, an average project size of $Q = 35$ MW is chosen as the capacity for the case study. This represents the maximum capacity of a wind turbine. The produced electricity is then calculated from the maximum capacity and the capacity factor, denoted ϵ[5]. The capacity factor is a measure of the relationship between

[3] The data for electricity forward prices was obtained from NASDAQ Commodities using Montel, while the data for the green certificate forward prices was obtained from SKM (2017).

[4] Boomsma and Linnerud (2015). find a correlation of 4% in the period 2005-2015, using the same method.

[5] $q = 8760\,Q\epsilon$, where 8760 is the number of hours in one year and ϵ is the capacity factor

the actual power produced and the maximum possible power produced. We assume ϵ to be constant throughout the wind turbine lifetime[6]. In Norway, by 2014, a capacity factor of 37.5% was achieved by the best wind power projects (NVE (2015)). By assuming a small increase by 2017, we set $\epsilon = 40\%$.

The cost of the wind turbines, and other investment costs, are the main expenditure of a project. The investment cost depends on the characteristics of the projects, where an empirical study by NVE (2015) found a spread of approximately 15% from the average. There are many reasons for the spread, e.g. Wiser and Bolinger (2016), IRENA (2014) and NVE (2015) find that the installation cost is dependent on the location of the wind park, as the cost of shipping and installing turbines varies. The investment cost for wind power projects are often expressed as a cost per installed capacity, i.e. investment cost increases linearly with the capacity[7]. One of the few contributions where this assumption is relaxed is Wiser and Bolinger (2016), who find that the investment cost per installed capacity is larger when the project or turbine size is small. However, they find that the investment cost increase close to linearly with capacity when the project size is above 5 MW and the turbine size is above 1 MW. Since we consider a project with a relatively large capacity of 35 MW, and turbine size of 2.3 MW, it is reasonable to assume a linear relation between the investment cost and capacity. Vindportalen (2017) finds that a wind power project typically has a total investment cost of 10,000-12,000 kr/kW. This is consistent with NVE (2015), who finds that the average investment cost of wind turbines in Norway between 2011-2013 was 12,005 kr/kW, but expects the cost to decrease to approximately 10,000 kr/kW due to learning effects by 2017. Therefore, we also set $I = 10,000$ NOK/kW. In addition, NVE (2015) forecasts the expected learning rate for the investment cost to be 10% in the period 2014-2020, and 7% in the period 2020-2035. IRENA (2016) considers the historical average global learning rate, and finds that in the period 1983-2014, it has been 7%. Accordingly, we assume the investment cost to be decreasing with a learning rate of $\lambda = 7\%$. NVE (2015) estimates the global capacity of wind energy in 2012 to be 282 GW, and forcasts capacity to be 612 GW and 1130 GW in 2020 and 2035, respectively. Using this data, we estimate the yearly growth rate of the global capacity of wind energy to be 5.8%, using exponential regression. Consequently, with the learning rate of 7 %, the investment cost is estimated to decrease with a yearly rate of 0.58%.

[6] We neglect short term and seasonal variations in the capacity factor, since we focus on a long-term investment

[7] See e.g. NVE (2015) and IRENA (2014)

Various projects have different service agreements, giving a large range of fixed and variable costs (NVE (2015)). The service agreements typically have a long contract period, and we assume constant operational costs throughout the project lifetime. Vindportalen (2017) estimates a range of 0.10-0.15 NOK/kWh, while NVE (2015) assumes a cost of 0.15 NOK/kWh. Based on these estimates, we have set a total operational cost of $C_V = 0.14$ NOK/kWh, which includes all variable and fixed costs.

The discount rate used to calculate the project value is specific to individual projects and investors (NVE (2015)). IRENA (2016) finds that the average discount rate of renewable energy investments globally is 10%. NVE (2015) considers Norwegian wind energy investments, and assumes discount rates of 4% and 6%. Based on this, we set the discount rate to $\rho = 6\%$.

Lastly, we provide a summary of the parameters related to the policy scheme, which was discussed in detail in Section 2. The start date of the case study is set to the beginning of 2017. After investing, subsidies will be received for a maximum period of $T_S = 15$ years. The parameter values for the deadlines represent the 31st of December that year. In the original agreement between Norway and Sweden, the end of the policy scheme is set to $T_E = 2035$. The Swedish investor will receive green certificates regardless of the time of investment, while the Norwegian investor must invest before the Norwegian deadline at $T_{DN} = 2021$.

5 Results

In this section, we analyze the investment opportunities using Monte Carlo simulation. First, we analyze the effect of time-dependency on the option value and investment behaviour. Further, we compare the Norwegian and Swedish investment opportunities, and perform a sensitivity analysis. Finally, we assess the effect of a Swedish policy extension and the possibility of jumps in the green certificate price, with and without learning.

5.1 The effect of time-dependency on investment behaviour

In what follows, we assess the effect of an investment deadline to receive green certificates. The initial time is set to $t = 2017$, and we compare an option with a perpetual subsidy scheme, to the cases where there is a deadline, such that investments must be made before $t = 2018$, $t = 2022$ and $t = 2032$. We use the parameter values outlined in Section 4, where we set $\rho_{SE} = 0$ and $\lambda_Y = 0$, to isolate the effect of the time-dependency on the option values[1].

In the case with a perpetual subsidy scheme, the value of the investment opportunity is 190 MNOK. The impact of a deadline on the option value is illustrated in Figure 5.1.

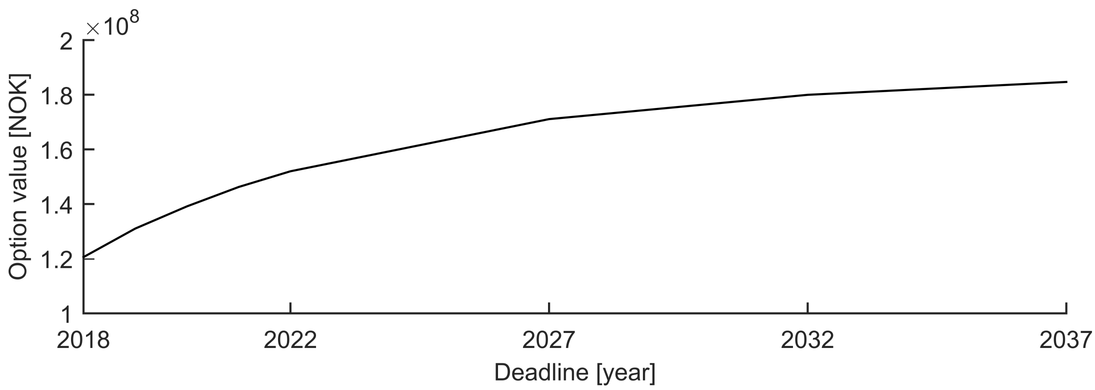


Figure 5.1: $F(E_0, S_0, t)$ as a function of deadline for the set of parameters, E_0 = 250 NOK/MWh, S_0 = 138 NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=0%, λ_Y=0%.

The option value is more sensitive when the time until the deadline is lower. A deadline of 15 years, in 2032, has a relatively low effect on the option value, which is reduced by 5% to 180 MNOK, compared to the case of a perpetual subsidy scheme. In contrast, a deadline of 1 year reduces the value by 36%, to 121 MNOK. Thus, a time-constraint has a considerable effect on the investment opportunity when the time until the deadline is relatively low. Therefore, it

[1] We want to keep the development of the electricity price equal for all deadlines, thus, $\rho_{SE} = 0$

is important to incorporate time-dependency when evaluating the Norwegian and Swedish investment opportunities.

The effect of a deadline on investment behaviour is analyzed by comparing the investment thresholds of the options, which is illustrated in Figure 5.2.

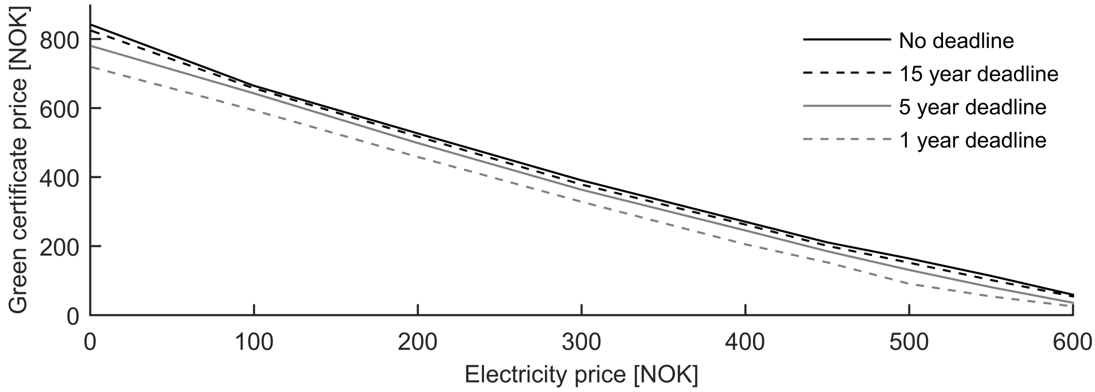


Figure 5.2: E^* and S^* for four perpetual options with different deadlines, for the set of parameters, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=0%, λ_Y=0%.

The sensitivity of the investment threshold is larger when the time until the deadline is lower. A deadline of 15 years, gives approximately the same investment threshold as a prepetual subsidy scheme. Having a deadline after 5 years, lowers the investment threshold by approximately 28 NOK for most electricity prices, compared to the perpetual subsidy scheme. In contrast, a deadline after 1 year, lowers the threshold by approximately 65 NOK. Hence, having a time constraint has a relatively low effect on the investment behaviour if the time until the deadline is long, but should be considered when the deadline is in the near future.

5.2 Comparison of the Norwegian and Swedish investors

In this section, we analyze the results of the baseline case outlined in Section 4, and compare the investment opportunities of the Norwegian and Swedish investors. We will first compare the intrinsic values of the projects, and analyze how they change with time. Then, we will do the same analysis for the option values and investment thresholds, and perform a sensitivity analysis. Further, we will consider the impact of the Swedish policy extension, before incorporating the possibility of drops in the green certificate price, both with, and without, learning.

5.2.1 Baseline model

The intrinsic values of the Norwegian and Swedish projects, $V_N(E_0, S_0, t)$ and $V_S(E_0, S_0, t)$, are illustrated in Figure 5.3 as a function of time.

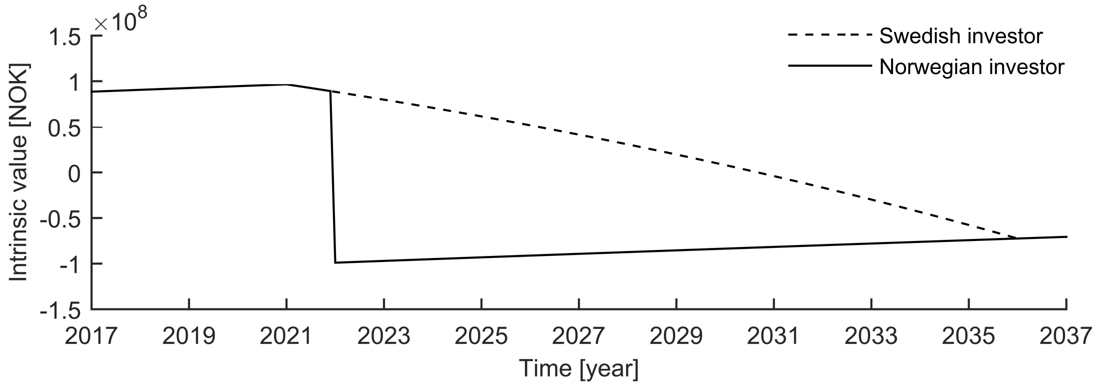


Figure 5.3: $V_N(E_0, S_0, t)$ and $V_S(E_0, S_0, t)$ as a function of time, t, for the set of parameters, E_0 = 250 NOK/MWh, S_0 = 138 NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%.

Initially, the intrinsic values of both investors are 89 MNOK. There are two factors which influence these values. The first is the learning effect, which gradually reduces the investment cost for the whole investment period. This increases the intrinsic values of the projects, and is the dominant effect for both investors in the periods $2017 \leq t < 2021$ and $t \geq 2036$. The second factor is the decreasing duration of subsidies the investors will receive for later investments, which decreases the intrinsic values of the projects. In the period $2021 \leq t < 2022$, the second factor has a larger impact on the projects than the first factor. Hence, the intrinsic values are decreasing. The intrinsic values differ in the period $2022 \leq t < 2036$, which is a result of the policy regulations. In this period, the Norwegian project is no longer eligible to receive certificates, and therefore, the value drops from 88 MNOK to -99 MNOK the moment after T_{DN}. In contrast, the Swedish intrinsic value has a steady decrease in the corresponding period. After the end of the policy scheme in 2035, the intrinsic values of both investors are equal. In summary, the policy regulations lead to considerable differences in the intrinsic values in the period $2022 \leq t < 2036$.

According to the traditional net present value approach, the initial values of the investment opportunities are 89 MNOK. Since the value is larger than zero, it is optimal to invest in the project immediately. However, this method ignores the possibility to delay the investments. Using the real options approach, the values of the investment opportunities of the Norwegian and Swedish investors are 154 MNOK and 160 MNOK, respectively. The values of the investment opportunities are considerably higher using the real options approach, compared to the net present value approach. By having the possibility to delay the investments, the value of the Norwegian option increases by 73%, or 65 MNOK, while the value of the Swedish option, with no deadline, increases by 80%, or 71 MNOK. The main reason for the additional value, is the possibility to wait for higher electricity and green certificate prices. This opportunity is very valuable, as the options are perpetual, and the prices are relatively volatile. The difference in the values of delaying the investments, between the Swedish and Norwegian investors, is approximately 9%, or 6 MNOK. This is a consequence of the considerably higher intrinsic value of the Swedish project after the Norwegian deadline.

In the case that the projects will be eligible to receive certificates regardless of the time of investment, i.e. the policy scheme has no end and there are no deadlines, the option values are 199 MNOK for both investors. Hence, the investment deadline, and the policy end date, reduce the values of the investment opportunities by 29%, or 45 MNOK, and 24%, or 39 MNOK, for the Norwegian and Swedish investors, respectively. Thus, the value of the investment opportunities are heavily dependent on the regulations of the policy scheme and the possibility to delay the investments.

The value of the Norwegian and Swedish investment opportunities, $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$, change as a function of time, as illustrated in Figure 5.4.

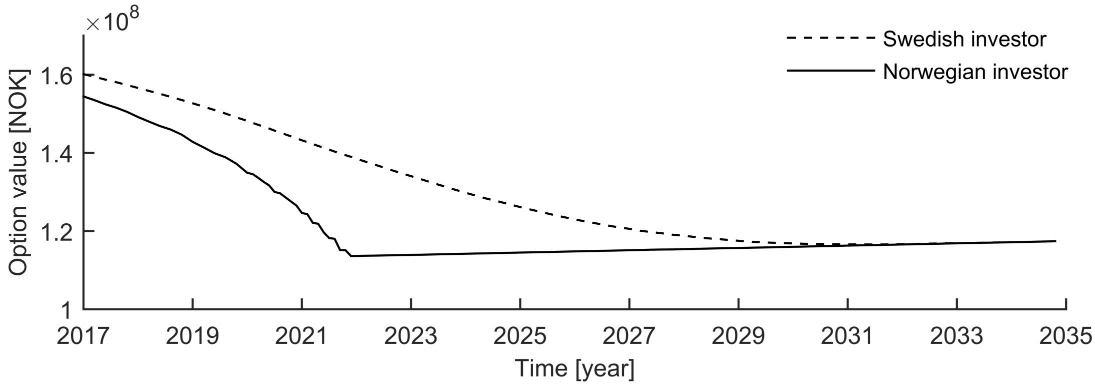


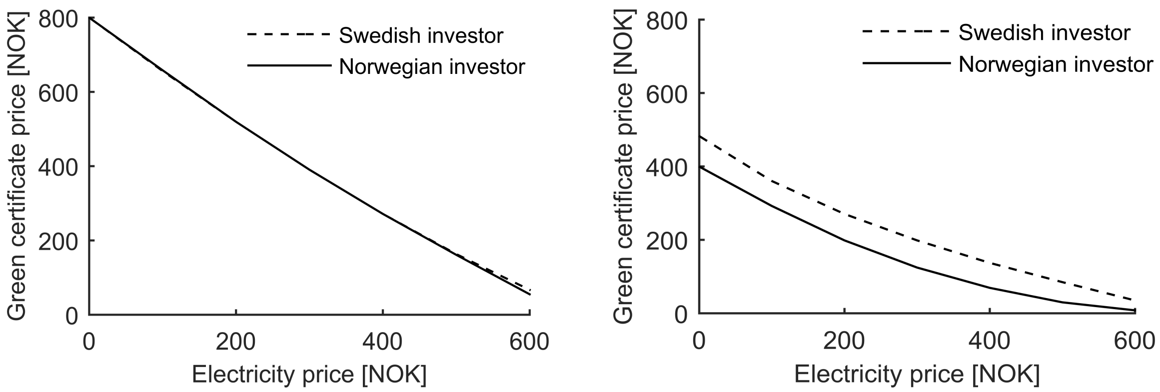
Figure 5.4: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of time, t, for the set of parameters, $E_0 = 250$ NOK/MWh, $S_0 = 138$ NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%.

Before the Norwegian investment deadline, $t \leq 2021$, the value of the Norwegian option decreases faster with time than the Swedish option, in contrast to the intrinsic values, which are equal. The difference between the Norwegian and Swedish option values reaches a maximum at the investment deadline, T_{DN}, where the Swedish investor has a considerably more valuable investment opportunity, worth 139 MNOK, in contrast to the Norwegian option, worth 114 MNOK. This difference is a consequence of the policy regulations, since the

Swedish investor can delay the investment at T_{DN}, and still receive certificates, while the Norwegian investor must invest immediately, or loose the right to receive certificates. The intrinsic value of the Norwegian project is 90 MNOK at T_{DN}, thus, the possibility to delay the investment has an additional value of 24 MNOK. Since the option value is higher than the intrinsic value, the optimal decision is to delay, and invest later with a higher electricity price.

In the period after the investment deadline, the Norwegian option value increases steadily, due to the decreasing investment costs. The value of the Swedish investment opportunity decreases steadily from 2017 to a minimum, approximately 116 MNOK, in the middle of year 2031. Intuitively, a longer waiting period before investing implies a shorter period of receiving subsidies, and consequently, the option value is lower. Around 2031, this effect is outbalanced by the learning effect. The option value will then increase as a consequence of the decreasing investment cost. The difference in the option values decrease after T_{DN}, and the option values are relativity similar for $t > 2031$, where the regulatory differences provide close to zero additional value to the Swedish investor.

We analyze the investment behaviour of the investors by comparing the investment thresholds, E_t^* and S_t^*, which are illustrated in Figure 5.6, for the initial time, $t = T_0$ and the Norwegian deadline, t=T_{DN}.



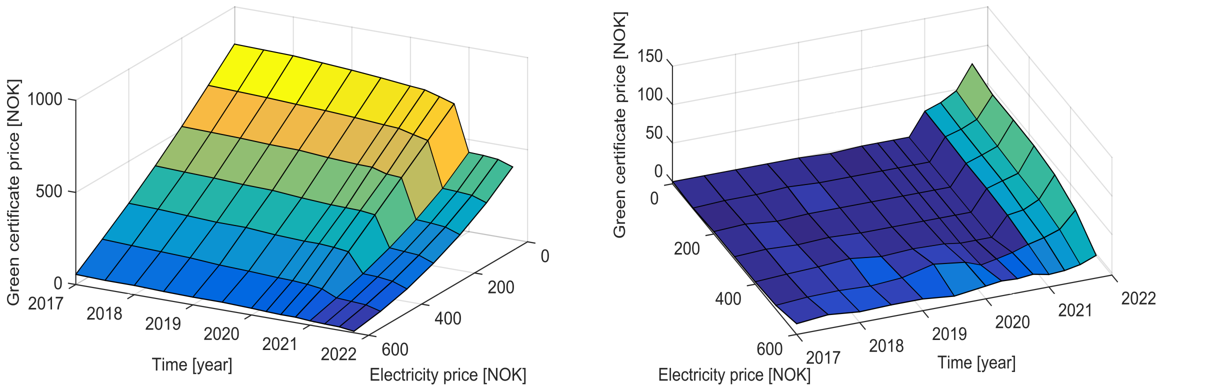
(a) Investment threshold at $t = T_0$.

(b) Investment threshold at the investment deadline, $t = T_{DN}$.

Figure 5.5: Investment threshold, E_t^* and S_t^*, for the set of parameters, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%.

When the electricity price is higher, a lower green certificate price is required for the investment to be profitable, thus, S^* decrease with E^*. At the start of 2017, the investment thresholds of the Norwegian and Swedish investors are very close, where the Swedish boundary is only marginally higher than the Norwegian boundary. This implies that both will invest at approximately the same prices, even though the Swedish investor has a more valuable investment opportunity. It can therefore be concluded that the Norwegian deadline has a low effect on the investment behaviour at $t = T_0$. However, at $t = T_{DN}$, there is a larger difference in the investment thresholds. The Norwegian investor has in that case a significantly lower threshold, with a difference of approximately 100 NOK for most electricity prices. As a consequence, the Norwegian investor will invest for a larger range of certificate prices.

The investment threshold of the Norwegian investor as a function of time is illustrated in Figure 5.6a, and the difference to the Swedish threshold is illustrated in Figure 5.6b.



(a) Investment threshold for the Norwegian investor

(b) Difference in the investment thresholds of the investors

Figure 5.6: Investment threshold, E_t^* and S_t^*, as a function of time, t, for the set of parameters, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%.

Initially, the thresholds of both investors are similar and decrease relatively slowly by time. At the end of 2020, there is a large drop in both thresholds, e.g. for the Norwegian investor for $E_t^* = 200$, the threshold S_t^* decreases by 48 % in 3 months. The main reason for this drop, is that the intrinsic values start declining in 2021 (see Figure 5.3), which makes the possibility to delay the investments less attractive. The difference in the thresholds start to increase during this period, since the difference in regulations has a larger effect on the threshold close to the deadline. In the period from 2021 to 2022, the Norwegian threshold stays relatively constant. This is interesting, since the threshold is expected to decrease towards the deadline. There are two factors influencing the threshold in this period. First, as time is getting closer to the deadline, the option value decreases, as was illustrated in Figure 5.1, which has a negative impact on the threshold. Second, the intrinsic value is decreasing as a consequence of a shorter duration of subsidies, which increases the threshold. The reason for a constant threshold is, therefore, that these two factors have approximately the same impact, and outbalance each other. In the same period, the difference between the investors increases, which is a consequence of the Swedish investor having no deadline. The Swedish investment threshold increases in the period, $2021 \leq t \leq 2035$. This is because the duration certificates will be received decreases, and therefore, a higher certificate price is required for the project to be profitable. In conclusion, the investment behaviour will remain similar before 2021. In 2021, there will be an additional incentive for both investors to exercise their options, as the thresholds drop significantly. In the period 2021-2022, the Norwegian investor will invest for lower prices than the Swedish investor. Given the current prices, investments are not optimal for either of the investors before the Norwegian deadline.

5.2.2 Sensitivity analysis

In this section, we will analyze how a change in key parameters will affect the investment opportunities of both the Norwegian and Swedish investors. Figure 5.7 illustrates the difference between the option values, $F_S(E_t, S_t, t) - F_N(E_t, S_t, t)$, as a function of electricity and green certificate prices, E_0 and S_0, when $t = T_0$.

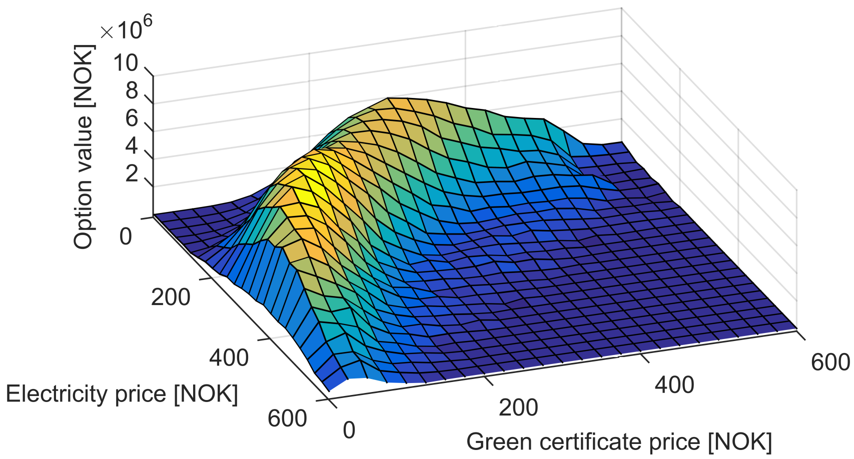


Figure 5.7: Difference in $F_S(E_t, S_t, t)$ and $F_N(E_t, S_t, t)$ as a function of E_0 and S_0, for the set of parameters, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%, t=T_0

When both prices are close to zero, the options are deep out-of-the-money. As a result, the options have a low value, and the difference between the investment opportunities is small. The option values are also similar when the green certificate price is close to zero, regardless of electricity price. This is because the regulatory differences have a lower effect on the option values, when the revenues from subsidies are low. When the prices of electricity and green certificates gradually increase from zero to the break-even prices, the values of the options increase, but at a higher rate for the Swedish investor. When the prices increase further, toward the investment thresholds, the difference between the options decrease. The difference is zero for prices above the investment thresholds of both investors. This is because it is optimal to exercise the options immediately, thus, the option values are equal to the intrinsic values. With the prices in the baseline case, $E_0 = 250$ NOK and $S_0 = 138$ NOK, the difference is approximately 5.7 MNOK, which is around the highest difference possible, when $t = T_0$. The current prices are above the break-even prices, and the difference between the investors will, therefore, increase if the prices decrease, and vice versa.

The option values as a function of the correlation is illustrated in Figure 5.8. A correlation close to $\rho_{SE} = -1$ and $\rho_{SE} = 1$ are not realistic, but for the sake of completeness, we present the whole range.

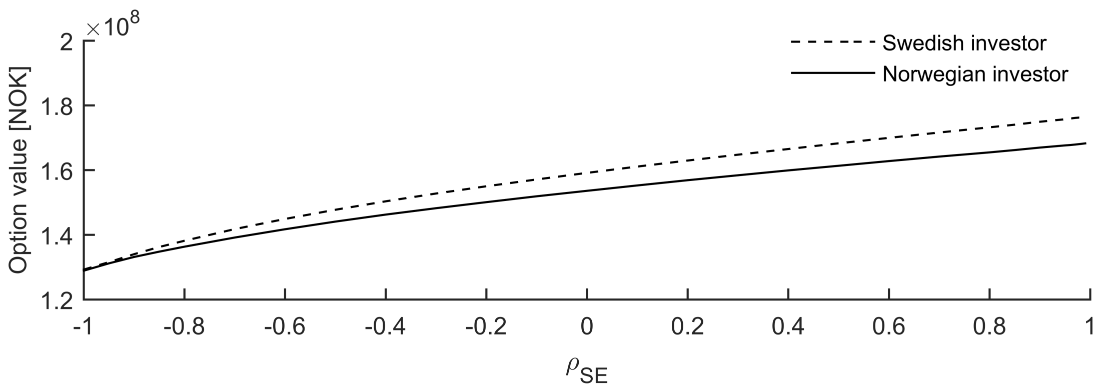
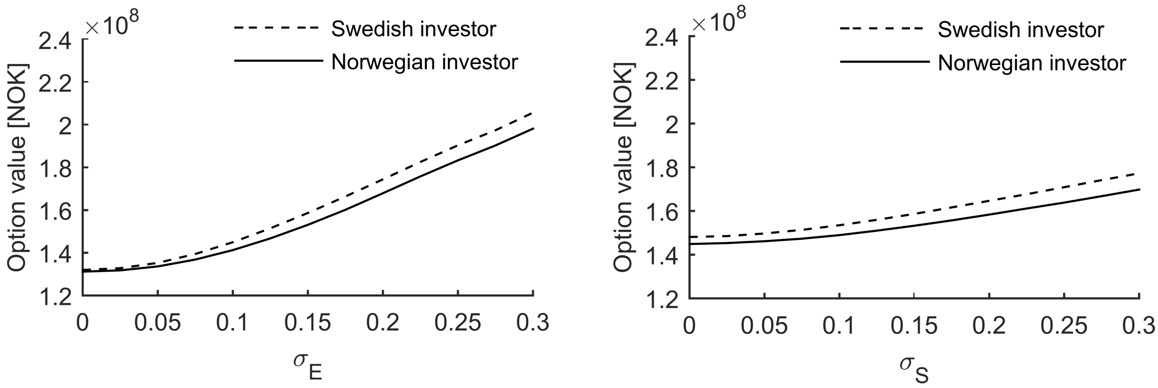


Figure 5.8: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of correlation, ρ_{SE}, for the set of parameters, $E_0 = 250$ NOK/MWh, $S_0 = 138$ NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, $t = T_0$.

The option values of both investors increase with the correlation. This is because increasing the correlation of the prices, also increases the volatilities of the projects. According to the traditional real options theory, a higher volatility increases the value of an option (Dixit and Pindyck (1994)). When increasing the correlation, the value of the Swedish option grows at a higher rate than the Norwegian option. The reason is that the Swedish investor is more affected by the additional volatility, as he is exposed to the risk from green certificates after 2021, contrary to the Norwegian investor. The option values are not very sensitive to the correlation, where an increase of 10 percent points from $\rho_{SE} = 0$ to $\rho_{SE} = 0.1$, would increase the option values by approximately 1.1% and 1.2% for the Norwegian and Swedish investors, respectively.

The sensitivities of the option values to the two volatility parameters are illustrated in Figure 5.9. It is evident that the option values are sensitive to these values, especially σ_E.



(a) Sensitivity to σ_E.

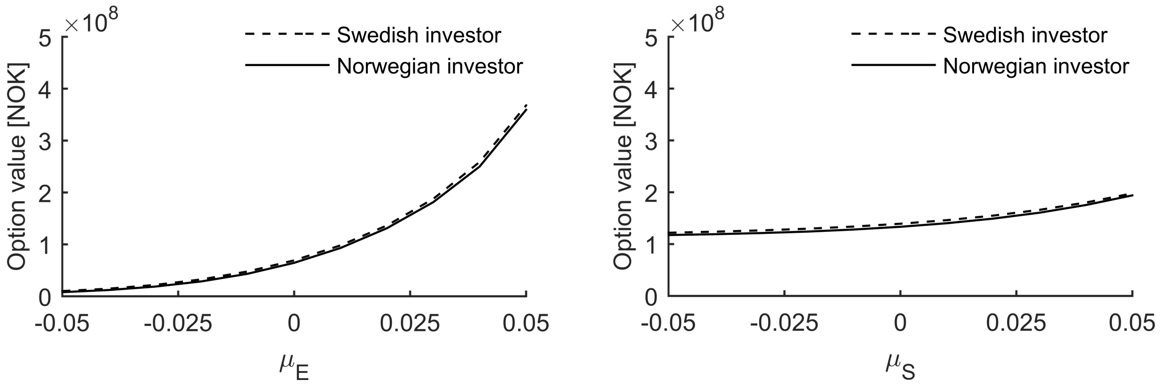
(b) Sensitivity to σ_S

Figure 5.9: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of volatility, σ, for the set of parameters, E_0 = 250 NOK/MWh, S_0 = 138 NOK/MWh, ρ_{SE}=5.1%, μ_E =2.5%, μ_S=2.5 %, $t = T_0$.

Increasing σ_E with 5 percent points from the baseline case, increases the option values by 10% for both investors. In contrast, increasing σ_S with 5 percent points, increases the option values by 3% and 4% for the Norwegian and Swedish investors, respectively. The option values are more sensitive to the volatility of the electricity price, than the volatility of the green certificate price. This can be explained by the intrinsic value being more dependent on the revenues from electricity, which are received for 20 years, while green certificates are received for a maximum of 15 years. In addition, the green certificate market has a finite lifetime, and, therefore, the investors are exposed to the volatility of the electricity price for a longer period. An increase in either of the volatilities increase the option values for both investors, as the value of waiting is higher when the risk is higher.

The relative difference in option values between the Norwegian and Swedish investors increases as either of the volatilities increase. As the volatility of the green certificate price increases, the additional value of flexibility for the Swedish investor grows at a higher rate than the Norwegian investor. This is a consequence of the regulatory differences, where the Swedish investor is affected by the uncertainty in green certificates for a longer period of time, and therefore, benefits more from the increased risk. When the volatility of the electricity price, σ_E, is low, both investors have an incentive to invest before the duration of subsidies starts to decrease after the end of 2020. This makes the effect of the regulatory differences low, and therefore, the option values are similar. As the volatility increases, the added value of flexibility increases at a higher rate for the Swedish investor. This is because the Swedish investor can wait for higher electricity prices, and still receive subsidies when $t < T_E$.

The sensitivities of the option values to the two drift parameters are illustrated in Figure 5.10. The Norwegian and Swedish investment opportunities are approximately equally sensitive to a change in the drift parameters.



(a) Sensitivity to μ_E. (b) Sensitivity to μ_S

Figure 5.10: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of drift, μ, for the set of parameters, E_0 = 250 NOK/MWh, S_0 = 138 NOK/MWh, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%, $t = T_0$.

The option values are most sensitive to the drift of the electricity price, e.g. decreasing the drift by one percent point from the baseline value, decreases the option values by 28%. In contrast, decreasing the drift of the green certificate price by one percent point from the baseline value, decreases the option value by 6%. The option values are highly sensitive to both the drifts and the volatilities of the prices. Since both parameters are difficult to predict, and various investors have different beliefs about the future drifts and/or volatilities, there will be large variations in their estimated option values. This can effect the investment thresholds of investors, and thus, the investment behaviour.

Figure 5.11 illustrates the option values of the Norwegian and Swedish investors as a function of discount rate.

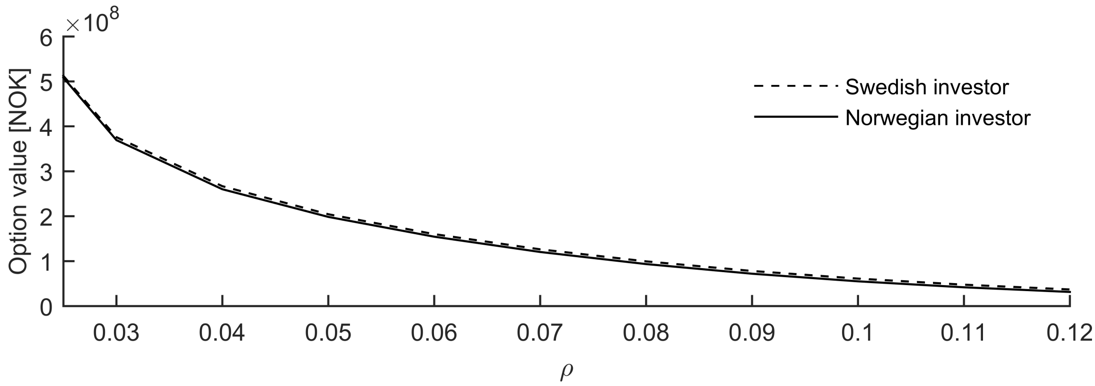


Figure 5.11: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of ρ, for the set of parameters, E_0 = 250 NOK/MWh, S_0 = 138 NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%, $t = T_0$.

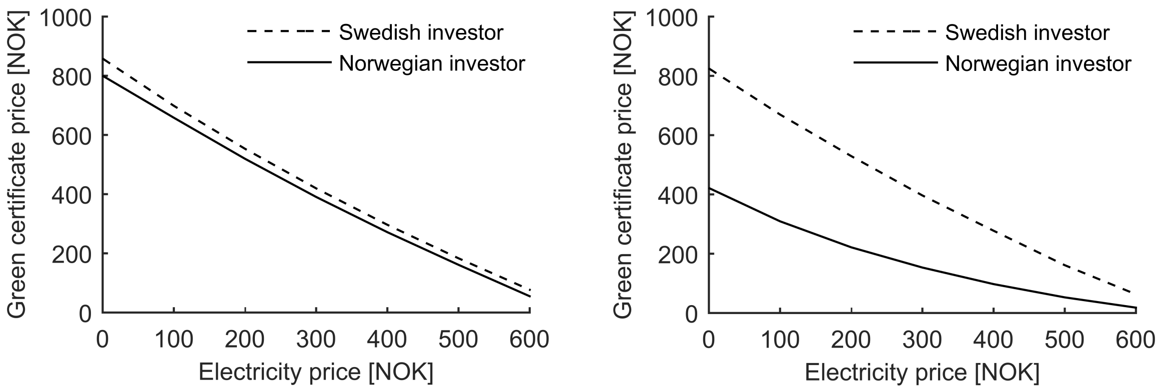
The values of the investment opportunities decrease with the discount rate, where the sensitivities of both investors are approximately the same. The option values are highly dependent on the discount rate, where decreasing the discount rate by one percent point from the baseline value, increases the option value by 28%. This is because with a lower discount rate, the present value of the future revenues are higher. Since the project has a relatively long lifetime of 20 years, this has a large impact on the intrinsic value. In addition, when the difference between the drift and the discount rate decreases, the value of waiting for higher prices increases, thus we get a higher option value. In general, the discount rate will vary between investors and projects, and can therefore contribute to large variations in the estimated values of wind energy investments.

5.2.3 Swedish policy extension

In what follows, we examine the effects of the probable extension of the policy scheme, discussed in detail in Section 2. We analyze the case where the Swedish policy scheme is extended by 10 years, which is consistent with the current proposal. Thus, the Swedish investor will receive certificates for $T_S = \min(15, 2045 - t)$ years if investing before $T_E = 2045$. It is also probable that a time-constraint to limit investments will be implemented. We will, therefore, also consider the case of an investment deadline for the Swedish investor, such that certificates will only be received if investing for $t \leq 2030$.

In the case of an extension, the intrinsic value of the Swedish investor will increase due to lower investment costs until $t = 2030$. This is a considerable difference to the Norwegian investor, where the project will become unprofitable after 2021. The first case, where the Swedish investor receives subsidies regardless of the time of investment, increases the value of the Swedish investment opportunity by 18 %, to 188 MNOK. The second case, where the Swedish investor has a deadline to receive certificates, $T_{DS} = 2030$, increases the option value to 187 MNOK. Therefore, the difference between the Swedish and Norwegian option values has increased from 6 MNOK to 33 or 34 MNOK depending on the case.

We found that the thresholds for the two Swedish cases are equal. Therefore, we will only compare the Swedish investment threshold in the case of no deadline, to the Norwegian threshold. The thresholds are illustrated in Figure 5.13, both at $t = T_0$ and $t = T_{DN}$. The Swedish threshold at T_0 has increased by approximately 30 NOK for $E^* = 250$ NOK, compared to the baseline case in Figure 5.6, as a consequence of the policy extension. There-



(a) Investment threshold at $t = T_0$.

(b) Investment threshold at the investment deadline, $t = T_{DN}$.

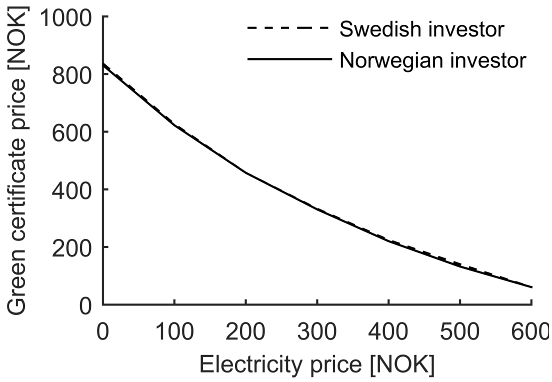
Figure 5.12: Investment threshold, E_t^* and S_t^*, in the case of a Swedish policy extension for the set of parameters, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%.

fore, the difference to the Norwegian investor has increased, in contrast to the baseline case, where the thresholds were similar. The difference between the Norwegian and Swedish thresholds is also considerably larger at $t = T_{DN}$. This is because the Swedish investment threshold will remain at a high level, in contrast to the baseline case, where it decreased in the period $T_0 \leq t \leq T_{DN}$. In summary, the policy extension gives an incentive for the Swedish investor to delay investments for a longer period, compared to the baseline case. As a result, the difference to the Norwegian investor increases considerably.

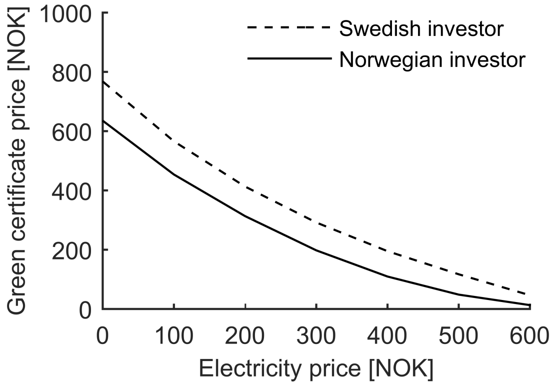
5.2.4 Possibility of a price collapse

In this section, we will analyze how the possibility of drops in the green certificate price influences the option values and investment behaviour. We use the assumptions from the baseline case, and consider the case where the arrival rate of jumps is $\Lambda = 0.156$. Thus, during a time step of $dt = 0.1$, the probability of a jump is 1.56 %. To put this in perspective, the probability of a jump during a year is 15%. We set the jump magnitude to $\phi = 0.5$, which means the price will drop by 50 % in the case of a jump. This is consistent with a drop in prices which occurred from week 2-7 in 2017, see Figure 2.4. Considering this case, the option values of the Norwegian and Swedish investors are 119 MNOK and 123 MNOK, respectively. Thus, the possibility of jumps decreases the option values by 23 %, for both investors. This is a considerable decrease in the option values, and neglecting the possibility of a price collapse, can therefore lead to an overvaluation of the investment opportunities.

The investment thresholds of both investors at $t = T_0$ and $t = T_{DN}$ is illustrated in Figure 5.13. The possibility of a jump causes the investment thresholds of both investors to



(a) Investment threshold at $t = T_0$.



(b) Investment threshold at the investment deadline, $t = T_{DN}$.

Figure 5.13: Investment threshold, E_t^* and S_t^*, when there is a possibility of a jump for the set of parameters, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%, $\Lambda dt = 0.0156$, $\phi = 0.5$.

increase, especially for low electricity prices. For example, at $t = T_0$ for a given electricity price of $E^* = 100$, the required green certificate price for investment, S^*, has increased by 14 NOK for both investors. The effect is, however, much larger at $t = T_{DN}$, where the threshold for the corresponding electricity price has increased by 232 NOK and 278 NOK, for the Norwegian and Swedish investors, respectively. Therefore, the possibility of a jump causes the investors to wait longer before investing, compared to the baseline case.

In what follows, we analyze the sensitivity of the option values to the jump parameters. Figure 5.14 illustrates how the option values of both investors change as a function of the jump magnitude.

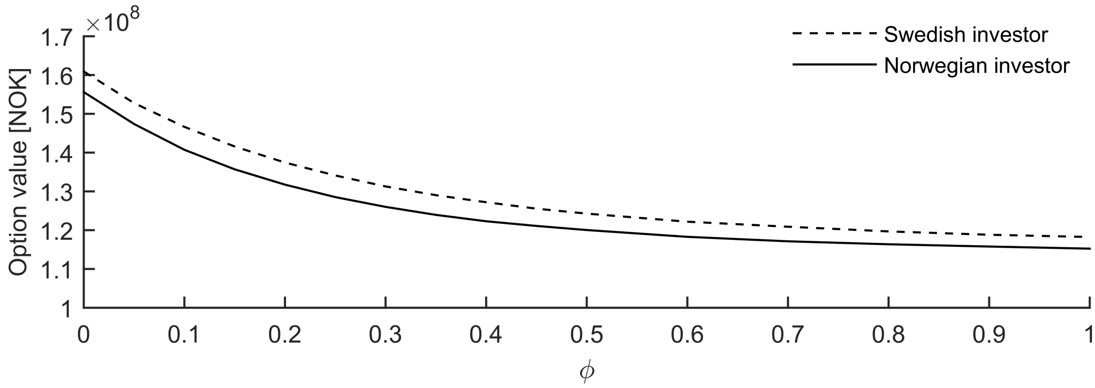


Figure 5.14: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of jump magnitude, ϕ, for the set of parameters, $E_0 = 250$ NOK/MWh, $S_0 = 138$ NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%, $\Lambda dt = 0.0156$.

The option values of both investors decrease with ϕ. This is expected, since when ϕ is higher, the expected future green certificate price is lower. When the jump magnitude is low, the sensitivities of the option values are relatively high, however, the sensitivities decrease with ϕ. The Norwegian investor is slightly more affected by the introduction of price jumps for low values of ϕ. However, for larger jumps, the Swedish investor is more affected. Therefore, the difference between the investors increases for low jump magnitudes, and decreases when the jump magnitude is high.

Figure 5.15 illustrates the option values of both investors as a function of jump intensity.

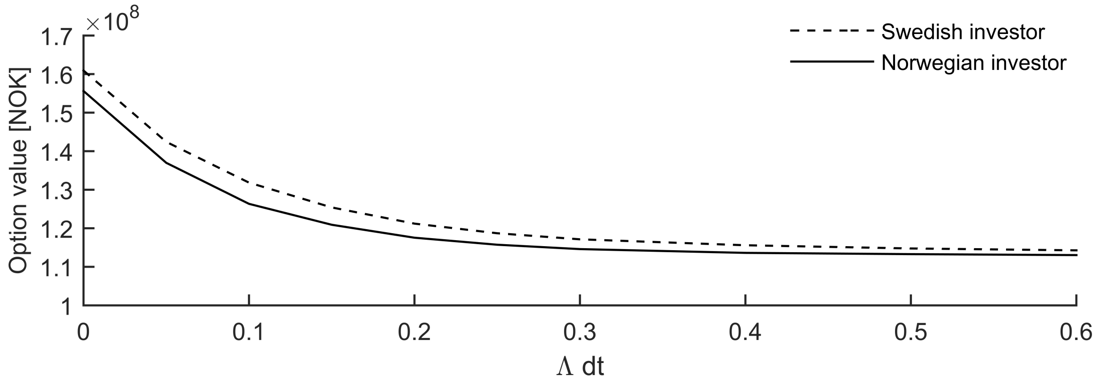


Figure 5.15: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of jump intensity, Λdt, for the set of parameters, $E_0 = 250$ NOK/MWh, $S_0 = 138$ NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%, $\phi = 0.5$.

Similarly to the jump magnitude, increasing the jump intensity lowers the expected future green certificate price, and therefore, the option values decrease with Λdt. We see that the difference between the investors is largest for values of Λdt between 0 and 0.015, where the difference is approximately 5.7 MNOK at its highest. As the jump intensity increases, the expected revenues from green certificates decrease for both investors. Intuitively, when the green certificate price is low, the impact of the regulatory differences decrease. Thus, the difference in the option values decreases with the jump intensity.

5.2.5 Possibility to learn about a price collapse

We now include the possibility to learn about the probability of a price drop, by letting the investors receive signals from the government and other institutions. The probability of a price drop is either low or high. We assume that if there is a policy extension, and the quota meets the production, there is a low probability of a price drop, set to 1% during a year. If the policy extension is rejected, and there is an overproduction of certificates, there is a significantly higher probability of a price drop, set to 25% during a year. Thus, with a time step of $dt = 0.1$, the jump intensities are $\Lambda_L = 0.01$ and $\Lambda_H = 0.284$, for the low and high probability cases, respectively. Similarly to the previous section, the drop magnitude is set to $\phi = 0.5$, in both cases. The investor has an initial belief when $t = T_0$, about the likelihood of being in a good state with a low probability of a price drop. We set the initial probability to $P_0 = 0.3$. By receiving signals indicating the likelihood of a jump, the investors update this probability. The reliability of a signal, i.e. the likelihood of a signal reflecting the true state of the world, is set to $\omega = 75\%$. The signals are received at discrete times following a Poisson process. We set the signal intensity to Λ_S=2.06, such that the probability of a signal each time-step is 0.21%, for $dt = 0.1$. To put this in perspective, the investors have a 90% probability of receiving at least one signal during a year.

If there are no signals, and consequently no learning, the values of the investment opportunities reflect the initial beliefs of the investors, and are 119 MNOK and 123 MNOK for the Norwegian and Swedish investors, respectively. If we allow for the possibility to learn, the value of the investment opportunities increase to 123 MNOK and 126 MNOK. Thus, the possibility to learn increases the option values by 4 MNOK and 3 MNOK. For the Norwegian investor, it is especially important to know as much as possible about the probability of a jump before the investment deadline, and the possibility to learn is slightly more beneficial compared to the Swedish investor. In the case of no learning, the probabilities of investing before T_{DN} are 9.5% and 16.5%, for the Norwegian and Swedish investors, respectively. When the investors have the possibility to learn, the probabilities of investing before the deadline increase to 14% and 22%. Thus, the possibility to learn speeds up investments.

Figure 5.16 illustrates the option values as a function of signal intensity.

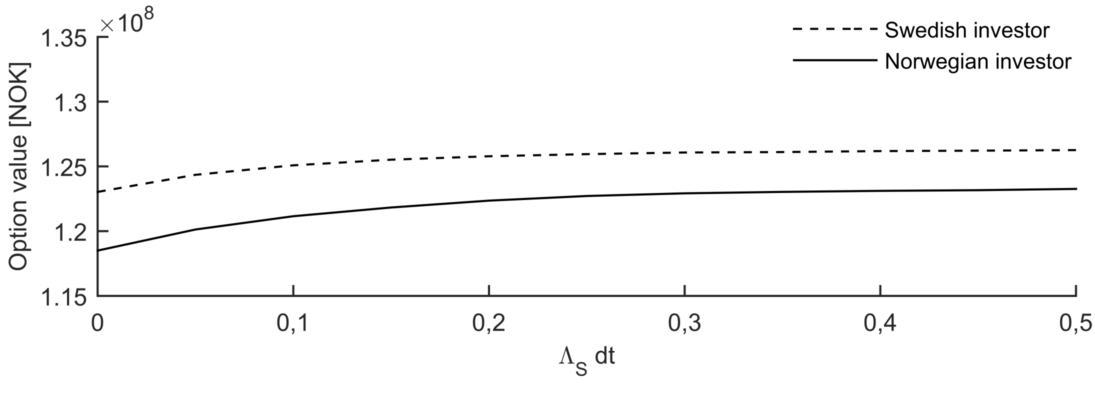


Figure 5.16: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of jump intensity, $\Lambda_S dt$, for the set of parameters, $E_0 = 250$ NOK/MWh, $S_0 = 138$ NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%, $\phi = 0.5$, $P_0 = 0.3$, $t = T_0$, $\Lambda_L = 0.01$, $\Lambda_H = 0.284$.

The option values increase with the signal intensity. This is because more frequent signals allow investors to make more informed decisions. With a signal reliability of 75%, few signals are needed for the investors to be relatively certain about the jump intensity, and additional signals will not provide much additional value. Therefore, the sensitivity of the option values to $\Lambda_S dt$, decreases with the signal intensity. The Norwegian option value grows faster with signal intensity than the Swedish option value, thus, the Norwegian investor has the largest benefit from learning.

Sensitivity to the signal reliability, ω, is illustrated in Figure 5.17. The option values increase with the signal reliability, since the investor can make more informed decisions when fewer signals are needed to learn about the jump intensity.

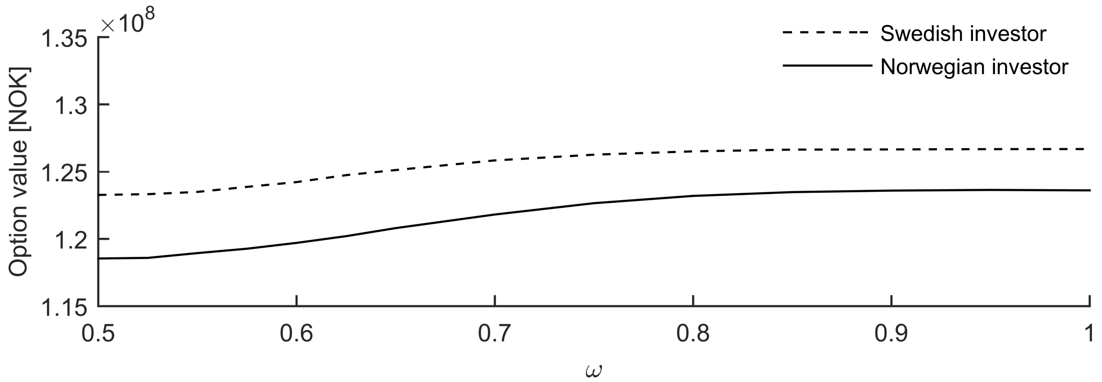


Figure 5.17: $F_N(E_0, S_0, t)$ and $F_S(E_0, S_0, t)$ as a function of ω, for the set of parameters, $E_0 = 250$ NOK/MWh, $S_0 = 138$ NOK/MWh, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=5.1%, $\phi = 0.5$, $P_0 = 0.3$, $t = T_0$, $\Lambda_L = 0.01$, $\Lambda_H = 0.284$, Λ_S=1.1.

When ω is close to 0.5, the signals are independent of the actual jump intensity, and therefore, signals provide no additional information to the investors. The sensitivity of the option values to ω is low when the signal reliability is low, as a large number of signals are needed to make an informed decision. The sensitivity is highest when ω is close to 0.75. When increasing ω further, the sensitivity start to decrease, as few signals are needed to make an informed decision, and a higher signal reliability provides little additional value. Similarly to the signal intensity, increasing the signal reliability also benefits the Norwegian investor more than the Swedish investor.

In conclusion, the possibility to learn about the likelihood of a price collapse in the green certificate price, provides a relatively low increase in the option values of the investors. However, it increases the probability of early investments for both investors. In addition, learning decreases the effect of the regulatory differences between the investors.

6 Conclusions

In this thesis, we develop a real options model to evaluate investments in renewable energy in Norway and Sweden, where the focus is on how regulatory differences in the green certificate market impact investors. Norwegian investors must invest before an upcoming deadline, for projects to be eligible to receive certificates. Swedish investors do not have a deadline, however, certificates are received for a shorter duration for later investments. After the end of the policy scheme, both investors have an equivalent perpetual investment opportunity. We also consider the effect of the probable Swedish policy extension. A case study of a wind power project is used to analyze the investment opportunities, from the perspectives of both a Norwegian and Swedish investor. We model the investment opportunities using a time-dependent real options model, where the electricity and green certificate prices are uncertain. The model also features a finite project lifetime, learning effects in the investment costs and correlation between the prices. In an extension to the model, we incorporate the possibility of a collapse in the prices, and allow for the investors to learn about the likelihood of a price drop, by observing the market and policy announcements.

We first assess the effect of an investment deadline to receive green certificates. Having a deadline lowers the option values and investment thresholds of the investors, where the effect is larger for shorter deadlines. We find that the option values are sensitive to both short and long deadlines, in contrast to the investment thresholds, which are mainly sensitive to deadlines of less than one year. Therefore, a deadline of 5 years, similar to the Norwegian deadline, has a large impact on the option values, but a low impact on the investment thresholds.

When considering the option values in the baseline case, we find that the regulatory differences make the Swedish investment opportunity more valuable than the Norwegian. The difference in option values are initially relatively low, where the Swedish option is 4% more valuable. However, this difference will increase towards the Norwegian deadline, to a maximum of 22%. After 2030, the impact of the policy regulations are negligible, as the option values are approximately equal. The same effect is evident when considering the investment thresholds. At the initial time, the Norwegian deadline has a low effect on investment behaviour, and both investors will invest at approximately the same price levels. An interesting finding is that the deadline is not what mainly affects the thresholds, but rather the declining duration of subsidies for later investments. This causes a significant reduction in the investment thresholds of both investors, and, therefore, provides a strong inventive to invest. At the Norwegian deadline, the difference between the investors increases, and the Norwegian investor will invest for a larger range of prices than the Swedish investor. More specifically, investment is optimal for a green certificate price which is approximately 100 NOK lower than the Swedish investor. Given the current price levels, it is not optimal to invest before the Norwegian deadline for either of the investors.

We found that increasing the volatility or drift of the electricity price, affect the investors more than increasing the volatility or drift of the green certificate price. This is mostly due to investors being more exposed to the uncertainty in the electricity price, as they receive income from selling electricity for 20 years, regardless of the time of investment. In contrast, green certificates are received for a maximum of 15 years depending on the time of investment. We also found that the Swedish investor is more affected by the uncertainty in green certificate prices, as the Swedish investor can receive certificates if investing after the Norwegian deadline. This also explains the effect when increasing the correlation between the prices. The option value of the Swedish investor will increase at a higher rate than the Norwegian investor, as the Swedish investor benefits more from the additional risk. The option values are also highly sensitive to the discount rate, which affects the option values of the investors similarly.

A Swedish policy extension only influences the Swedish investor, where the option value increases by approximately 18%. At the initial time, the difference between the Swedish and Norwegian thresholds is approximately 7%, in contrast to the baseline case, where the thresholds are similar. The Swedish investment threshold stays relatively high as time gets closer to the Norwegian deadline, in contrast to the Norwegian investor. Therefore, the policy extension gives an incentive for the Swedish investor to delay investments for a longer period, which increases the difference between the investors.

Including the possibility of a collapse in the green certificate price has a large impact on the investment opportunities, and reduces the option values by 23% for both investors. Thus, the possibility of a price drop impacts the investors similarly. We found that when the intensity or the size of the jumps are low, the jump has a larger effect on the Norwegian option value, while large jumps or a high jump intensity have a larger effect on the Swedish option value. We also found that the possibility of a collapse in the green certificate price increases the investment thresholds for both investors considerably, thus, the investors are less likely to invest. This effect is especially large for the Swedish investor close to the Norwegian deadline, where the Swedish investor requires a significantly higher price before investing, e.g. the threshold has increased by 206 NOK, or 57%, given an electricity price of 100 NOK.

The possibility to learn about the likelihood of a price collapse in the green certificate price, provides a relatively low increase in the option values of 3% and 2%, for the Norwegian and Swedish investors, respectively. However, it increases the probability of early investments for both investors. The Norwegian investor has a slightly higher benefit from the possibility to learn, thus, the effect of the regulatory differences decreases.

In this study, we consider one specific project, and similar investors in both countries. An interesting direction for further research is to take a macro perspective, and focus on how regulatory differences affect the total capacity of renewable energy in both countries. In addition, it is interesting to consider how different project characteristics influence the results. Another possible extension is to include step-wise investments, where there is a possibility to either expand the project capacity or to extend the lifetime of the project.

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A The intrinsic value of the project

A.1 Perpetual subsidy scheme

The profit stream of the project is given by,

$$\pi(S_t, E_t) = \begin{cases} (E_t + S_t - C_V)q - C_F & t \leq \tau + T_S, \\ (E_t - C_V)q - C_F & t > \tau + T_S. \end{cases} \tag{A.1}$$

The value of the project is the net present value of the profit stream through the lifetime of the project,

$$\begin{aligned} V(S_t, E_t) &= \mathbf{E}\Big[\int_{\tau}^{\tau+T_L} \pi(S_t, E_t)e^{-\rho(t-\tau)}\,dt\Big], \\ &= \mathbf{E}\Big[\int_{\tau}^{\tau+T_S} ((E_t + S_t - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt + \\ &\int_{\tau+T_S}^{\tau+T_L} ((E_t - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt\Big]. \end{aligned} \tag{A.2}$$

The expected values of the stochastic variables, S_t and E_t, which follow geometric Brownian motions, are given by

$$\mathbf{E}[S(t)] = S_\tau e^{\mu_S(t-\tau)}, \quad \mathbf{E}[E(t)] = E_\tau e^{\mu_E(t-\tau)}, \tag{A.3}$$

where τ is the time of investment (Dixit and Pindyck (1994)). By combining equation (A.2) and (A.3), we get

$$\begin{aligned} V(S_t, E_t) = &\int_{\tau}^{\tau+T_S} ((E_\tau e^{\mu_E(t-\tau)} + S_\tau e^{\mu_S(t-\tau)} - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt + \\ &\int_{\tau+T_S}^{\tau+T_L} ((E_\tau e^{\mu_E(t-\tau)} - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt, \\ = &\int_{\tau}^{\tau+T_S} ((qS_\tau e^{\mu_S(t-\tau)}e^{-\rho(t-\tau)}\,dt + \int_{\tau}^{\tau+T_L} ((E_\tau e^{\mu_E(t-\tau)} - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt, \\ = &\int_{\tau}^{\tau+T_S} ((qS_\tau e^{(\mu_S-\rho)(t-\tau)}\,dt + \int_{\tau}^{\tau+T_L} (qE_\tau e^{(\mu_E-\rho)(t-\tau)}\,dt \\ &- \int_{\tau}^{\tau+T_L} (qC_V + C_F)e^{-\rho(t-\tau)}\,dt, \\ = &\frac{qS_\tau}{\mu_S-\rho}(e^{(\mu_S-\rho)T_S} - 1) + \frac{qE_\tau}{\mu_E-\rho}(e^{(\mu_E-\rho)T_L} - 1) - \frac{qC_V+C_F}{-\rho}(e^{-\rho T_L} - 1). \end{aligned} \tag{A.4}$$

By rearranging, the intrinsic value of the project can be expressed by

$$V(S_t, E_t) = q(r_S S_\tau + r_E E_\tau) - C, \tag{A.5}$$

where $r_S = \frac{1}{\rho-\mu_S}(1-e^{(\mu_S-\rho)T_S})$, $r_E = \frac{1}{\rho-\mu_E}(1-e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1-e^{-\rho T_L})$.

A.2 Time-dependent subsidy scheme

A.2.1 Norwegian investor

The instantaneous profit function of the Norwegian investor can be expressed by,

$$\pi_N(S_t, E_t, t) = \begin{cases} (E_t + S_t - C_v)q - C_F & t \leq T_E \ \wedge \ \tau \leq T_{DN} \ \wedge \ t \leq \tau + T_S, \\ (E_t - C_v)q - C_F & t > T_E \ \vee \ \tau > T_{DN} \ \vee \ t > \tau + T_S. \end{cases} \tag{A.6}$$

The Norwegian investor has two regions where he can invest, either before or after the deadline, T_{DN}. If investing in the period $t \leq T_E - T_S$, the investor receives subsidies for a duration of T_S years. However, if investing in the period $T_E - T_S < t \leq T_{DN}$, the investor will receive subsidies for a reduced duration of $(T_E - \tau)$ years. Thus, investments in the region before the deadline yield subsidies for a duration of $min(T_S, T_E - \tau)$ years. The intrinsic value of the investment is the net present value of the profit stream through the lifetime of the project, and we get the following equation,

$$\begin{aligned} V_N(S_t, E_t, t) &= \mathbf{E}\Big[\int_{\tau}^{\tau+T_L} \pi(S_t, E_t, t)e^{-\rho(t-\tau)}\,dt\Big], \\ &= \mathbf{E}\Big[\int_{\tau}^{min(\tau+T_S,T_E)} ((E_t + S_t - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt + \\ &\int_{min(\tau+T_S,T_E)}^{\tau+T_L} ((E_t - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt\Big]. \end{aligned} \tag{A.7}$$

By combining equation (A.3) and (A.7), we get the following equation,

$$\begin{aligned} V_N(S_t, E_t, \tau) = &\int_{\tau}^{min(\tau+T_S,T_E)} ((E_\tau e^{\mu_E(t-\tau)} + S_\tau e^{\mu_S(t-\tau)} - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt + \\ &\int_{min(\tau+T_S,T_E)}^{\tau+T_L} ((E_\tau e^{\mu_E(t-\tau)} - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt, \\ = &\int_{\tau}^{min(\tau+T_S,T_E)} ((qS_\tau e^{\mu_S(t-\tau)}e^{-\rho(t-\tau)}\,dt + \int_{\tau}^{\tau+T_L} ((E_\tau e^{\mu_E(t-\tau)} - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt, \\ = &\int_{\tau}^{min(\tau+T_S,T_E)} ((qS_\tau e^{(\mu_S-\rho)(t-\tau)}\,dt + \int_{\tau}^{\tau+T_L} (qE_\tau e^{(\mu_E-\rho)(t-\tau)}\,dt \\ &- \int_{\tau}^{\tau+T_L} (qC_V + C_F)e^{-\rho(t-\tau)}\,dt, \\ = &\frac{qS_\tau}{\mu_S-\rho}(e^{min(T_E-\tau,T_S)(\mu_S-\rho)} - 1) + \frac{qE_\tau}{\mu_E-\rho}(e^{(\mu_E-\rho)T_L} - 1) - \frac{qC_V+C_F}{-\rho}(e^{-\rho T_L} - 1). \end{aligned} \tag{A.8}$$

By rearranging, the intrinsic value of the project can be expressed by,

$$V_N(S_t, E_t, \tau) = q(r_S(\tau)S_\tau + r_E E_\tau) - C, \tag{A.9}$$

where $r_S(\tau) = \frac{1}{\rho-\mu_S}(1 - e^{min\{T_E-\tau,T_S\}(\mu_S-\rho)})$, $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1 - e^{-\rho T_L})$.

If investing after the deadline, $\tau > T_D$, the intrinsic value is given by,

$$\begin{aligned}
V_N(S_t, E_t) &= \mathbf{E}\Big[\int_\tau^{\tau+T_L} \pi(S_t, E_t)e^{-\rho(t-\tau)}\,dt\Big],\\
&= \mathbf{E}\Big[\int_\tau^{\tau+T_L} ((E_t - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt\Big].\\
&= \int_\tau^{\tau+T_L} ((E_\tau e^{\mu_E(t-\tau)} - C_V)q - C_F)e^{-\rho(t-\tau)}\,dt,\\
&= \int_\tau^{\tau+T_L} (qE_\tau e^{(\mu_E-\rho)(t-\tau)}\,dt - \int_\tau^{\tau+T_L} (qC_V + C_F)e^{-\rho(t-\tau)}\,dt,\\
&= \frac{qE_\tau}{\mu_E-\rho}(e^{(\mu_E-\rho)T_L} - 1) - \frac{qC_V+C_F}{-\rho}(e^{-\rho T_L} - 1).
\end{aligned} \tag{A.10}$$

By rearranging, the intrinsic value of the project can be expressed by

$$V_N(S_t, E_t) = q(r_E E_\tau) - C, \tag{A.11}$$

where $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1 - e^{-\rho T_L})$.

We then get the following intrinsic value for the Norwegian investor,

$$V_N(S_t, E_t, \tau) = \begin{cases} q(r_S(\tau)S_\tau + r_E E_\tau) - C & \tau \le T_{DN}, \\ q(r_E E_\tau) - C & \tau > T_{DN}, \end{cases} \tag{A.12}$$

where $r_S(\tau) = \frac{1}{\rho-\mu_S}(1 - e^{min\{T_E-\tau,T_S\}(\mu_S-\rho)})$, $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1 - e^{-\rho T_L})$.

A.2.2 Swedish investor

The instantaneous profit function for the Swedish investor is given by

$$\pi_S(S_t, E_t, t) = \begin{cases} (E_t + S_t - C_v)q - C_F & t \le T_E \ \wedge \ t \le \tau + T_S, \\ (E_t - C_v)q - C_F & t > T_E \ \vee \ t > \tau + T_S. \end{cases} \tag{A.13}$$

The Swedish investor has two regions where he can invest, either before or after the end of the policy scheme, T_E. If investing in the period $t \le T_E$, the Swedish investor will receive subsidies for a duration of $min(T_S, T_E - \tau)$ years. In the region $t \leq T_E$, we get the following intrinsic value,

$$\begin{aligned} V(S_t, E_t, t) &= \mathbf{E}\Big[\int_{\tau}^{\tau+T_L} \pi(S_t, E_t, t) e^{-\rho(t-\tau)}\, dt\Big], \\ &= \mathbf{E}\Big[\int_{\tau}^{min(\tau+T_S, T_E)} ((E_t + S_t - C_V)q - C_F) e^{-\rho(t-\tau)}\, dt + \\ &\int_{min(\tau+T_S,T_E)}^{\tau+T_L} ((E_t - C_V)q - C_F) e^{-\rho(t-\tau)}\, dt\Big]. \end{aligned} \tag{A.14}$$

This value function is equal to the Norwegian value function in the period $t \leq T_{DN}$, see equation (A.8), and can, thus, be expressed by

$$V_S(S_t, E_t, \tau) = q(r_S(\tau)S_\tau + r_E E_\tau) - C, \tag{A.15}$$

where $r_S(\tau) = \frac{1}{\rho-\mu_S}(1 - e^{min\{T_E-\tau, T_S\}(\mu_S-\rho)})$, $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1 - e^{-\rho T_L})$.

If investing after the end of the policy scheme, $\tau > T_E$, the intrinsic value is equal to the Norwegian value function in the period $t > T_{DN}$, see equation (A.10), and can, thus, be expressed by

$$V(S_t, E_t) = q(r_E E_\tau) - C, \tag{A.16}$$

where $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1 - e^{-\rho T_L})$.

We then get the following intrinsic value for the Swedish investor,

$$V_S(S_t, E_t, \tau) = \begin{cases} q(r_S(\tau)S_\tau + r_E E_\tau) - C & \tau \leq T_E, \\ q(r_E E_\tau) - C & \tau > T_E, \end{cases} \tag{A.17}$$

where $r_S(\tau) = \frac{1}{\rho-\mu_S}(1 - e^{min\{T_E-\tau, T_S\}(\mu_S-\rho)})$, $r_E = \frac{1}{\rho-\mu_E}(1 - e^{(\mu_E-\rho)T_L})$, $C = \frac{qC_V+C_F}{\rho}(1 - e^{-\rho T_L})$.

B Semi-analytical solution

The value of the option can be expressed using the Bellmann equation given by

$$\rho F(S,E) = \pi + \frac{1}{dt}\mathbf{E}[dF(S,E)]. \tag{B.1}$$

Using Ito's lemma to derive the value of $dF(S,E,t)$, provides the following partial differential equation,

$$\frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F}{\partial S^2} + \frac{1}{2}\sigma_E^2 E^2 \frac{\partial^2 F}{\partial E^2} + \mu_S S \frac{\partial F}{\partial S} + \mu_E E \frac{\partial F}{\partial E} - \rho F = 0. \tag{B.2}$$

We follow the approach by Boomsma and Linnerud (2015), and assume the form $F(S,E) = AS^{\beta_S}E^{\beta_E}$. We then get the following fundamental quadratic,

$$Q(\beta_S,\beta_E) = \frac{1}{2}\Big(\sigma_S^2\beta_S(\beta_S - 1) + \sigma_E^2\beta_E(\beta_E - 1)\Big) + \mu_S\sigma_S + \mu_e\sigma_E - \rho. \tag{B.3}$$

The boundary conditions are expressed by,

$$A(E^*)^{\beta_E}(S^*)^{\beta_S} = r_E E^* + r_S S^* - (I + C), \tag{B.4}$$

$$A\beta_E(E^*)^{\beta_E - 1}(S^*)^{\beta_S} = r_E, \tag{B.5}$$

$$A\beta_S(E^*)^{\beta_E}(S^*)^{\beta_S - 1} = r_S, \tag{B.6}$$

$$F(V(0,0)) = 0, \tag{B.7}$$

where equation (B.4) is the value matching condition, and equation (B.5) and (B.6) are the smooth pasting conditions. In addition, if both prices are zero, the option value is also zero, hence, we get the boundary condition in equation (B.7). By combining the boundary conditions in equation (B.4), (B.5) and (B.6), we obtain the following expressions for the investment threshold,

$$S^* = \frac{\beta_S}{\beta_E + \beta_S - 1}\frac{I + C}{r_S}, \tag{B.8}$$

$$E^* = \frac{\beta_E}{\beta_E + \beta_S - 1}\frac{I + C}{r_E}. \tag{B.9}$$

To find the threshold for the subsidy price, we must specify the electricity price, E.
Let

$$\eta(E) = \frac{I + C - r_E E}{r_E E}. \tag{B.10}$$

Then, β_E can be calculated from equation (B.3) and (B.9), which gives,

$$\beta_E = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{B.11}$$

where,

$$a = \frac{1}{2}\Big(\sigma_E^2 + \sigma_S^2 \eta^2(E)\Big),$$

$$b = \frac{1}{2}\Big(-\sigma_E^2 + \sigma_S^2 \eta(E)\Big) + \mu_E + \mu_S \eta(E),$$

$$c = \mu_S - \rho.$$

This is used to calculate β_S using equation (B.9). Further, the threshold, S^*, is calculated using equation (B.8). The option value at the boundary can then be expressed by,

$$F(S^*, E^*) = r_E E^* + r_S S^* - C - I = \Big(\frac{\beta_E + \beta_S}{\beta_E + \beta_S - 1}\Big)(I + C) - C - I \tag{B.12}$$

C Price paths

Following the approach by Brandimarte (2014), we transform geometric Brownian motions into discrete price paths. To discretize the price paths, we combine equation (3.1) and Ito's lemma, given by

$$dF = \frac{\partial F}{\partial S} dS + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2 + \frac{\partial F}{\partial t} dt, \tag{C.1}$$

which gives

$$dF = \Big(\frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F}{\partial S^2} + \mu_S S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t}\Big) dt + \sigma_S S \frac{\partial F}{\partial S} dW. \tag{C.2}$$

We set F(S,t)=log(S,t), and derive the following partial differentials,

$$\frac{\partial F}{\partial S} = \frac{1}{S}, \qquad \frac{\partial^2 F}{\partial S^2} = -\frac{1}{S^2}, \qquad \frac{\partial F}{\partial t} = 0. \tag{C.3}$$

Combining equation (C.3) with equation (C.2), we get

$$dF = \Big(-\frac{1}{2}\sigma_S^2 + \mu_S\Big) dt + \sigma_S dW. \tag{C.4}$$

Integrating this equation gives

$$log S(t) = log S(0) + \Big(-\frac{1}{2}\sigma_S^2 + \mu_S\Big) t + \sigma_S W(t). \tag{C.5}$$

Expressing $W(t)$ as $\epsilon\sqrt{t}$, where ϵ is a standard normally distributed random variable. We solve for S(t), and get

$$S(t) = S(0) exp\big[(-\frac{1}{2}\sigma_S^2 + \mu_S) t + \sigma_S \epsilon \sqrt{t}\big]. \tag{C.6}$$

Considering the discrete case, we estimate the price after a small interval Δt by

$$S_{t+\Delta t} = S_t exp\Big[(-\frac{1}{2}\sigma_S^2 + \mu_S)\Delta t + \sigma_S \epsilon \sqrt{\Delta t}\Big]. \tag{C.7}$$

To simulate correlated returns between two geometric Brownian motions, $dW_1 dW_2 = \rho dt$, we let ϵ_2 depend on the realized values of ϵ_1. Let $Z_1, Z_2 \sim N(0,1)$, then, $\epsilon_1 = Z_1$ and $\epsilon_2 = \rho Z_1 + \sqrt{1-\rho^2} Z_2$ (Brandimarte (2014)).

D Calibration

In what follows, we calibrate the parameters of the Monte Carlo simulation, by comparing it to the semi-analytical solution.

Two parameters which have a large impact on the results and the computational time is the number of time steps and the number of simulated price paths. The number of time steps will increase the value of the option, since there are more possible exercise dates. Ideally, the investor should be able to exercise the option at any time, hence, more exercise dates will be a more realistic approximation, and therefore improve the accuracy of the simulation. Increasing the number of price paths reduces the random error of the simulation, and thus improves the accuracy and the precision of the results. Increasing these model parameters will, however, increase the computational time of the simulation, where we observed a close to linear relationship between the running time and the number of price paths or time steps. Therefore, the parameters must be optimized, such that the error of the results are minimized with an acceptable running time.

We calibrate the Monte Carlo simulation by approximating the investment threshold of the semi-analytical solution. We find that simulating a time period of $T_D = 50$ years, gives a good approximation of a perpetual option. Figure D.1 illustrates the investment thresholds of two perpetual options, where one is solved using the semi-analytical approach, and one is approximated using the Monte Carlo simulation approach.

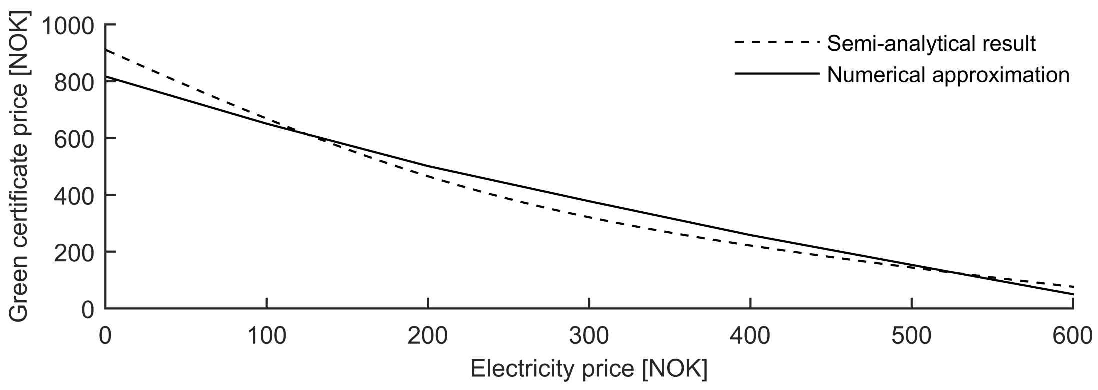

Figure D.1: Investment threshold, S^* and E^*, for the set of parameters, μ_E =2.5%, μ_S=2.5 %, σ_E=15.5%, σ_S=16.3%, ρ_{SE}=0%, $\lambda_Y = 0$%.

The thresholds are highly sensitive to the option values, and since the difference in the thresholds are relatively small, the Monte Carlo simulation provides a sufficient estimation of the option values. The thresholds have the same shape, with a non-linear relationship between the prices. This is a consequence of diversification effects. Since the prices are non-zero and not perfectly correlated, some of the individual risk is diversified in the project. The investment threshold of the semi-analytical solution is lower than the numerical approximation where both prices are high, as the diversification effect is larger for the semi-analytic solution. This is likely a result of the time dependency of the numerical approximation, where the prices will diverge by time, and hence the diversification will decrease by time. Where one of the prices are zero, there is no diversification effect, and it can be observed that the semi-analytical solution has a higher investment threshold than the numerical approximation. This is a consequence of the approximation having a limited number of investment opportunities and a finite-lifetime, which reduces the value of the option, and thus, the investment threshold.

E Solution approach for model extensions

E.1 Possibility of green certificate price collapse

To generate paths for S_t, when there is a possibility of price jumps, we first generate prices following the approach outlined in Section 3.2.3. We then adjust for jumps by drawing random numbers, n_{ti}, for each time-step, t, and price path, i, from a Poisson distribution with mean Λdt. n_{ti} then indicates the number of jumps for a price path and time-step. If a jump occurs, the prices after the jump are adjusted by the factor $(1-\phi)^{n_{ti}}$, where ϕ is the jump magnitude.

E.2 Possibility of green certificate price collapse with learning

When solving the Monte Carlo model with learning, there are two differences to the case where the price can collapse. The first is that price paths have a random jump intensity. The second is that the intrinsic value depend on an additional stochastic process, which represents the good signals in excess of the bad signals. The probability of a price path having a low jump intensity is represented by P_0. Let the jump intensity for the green certificate price in scenario i be denoted by Λ_i. Then, the jump intensities of the price paths are expressed by,

$$\Lambda_i = \begin{cases} \Lambda_L & \text{with probability} \quad P_0, \\ \Lambda_H & \text{with probability} \quad 1-P_0, \end{cases} \qquad \forall\ i \in N_{paths}. \tag{E.1}$$

In each scenario, signals are received at discrete times, following a Poisson process. The signals can either indicate a low or a high jump intensity. Let k_{ti} denote the number of signals indicating a low jump intensity in excess of signals indicating a high jump intensity, for scenario i at time t. Then the probability of path i having a low jump intensity at time t, is represented by

$$p_{ti}(k_{ti}) = \frac{P_0\lambda^{k_{ti}}}{P_0\lambda^{k_{ti}} + (1-P_0)(1-\lambda)^{k_{ti}}}. \tag{E.2}$$

Let $V_L(S_{ti}, E_{ti}, t)$ and $V_H(S_{ti}, E_{ti}, t)$ denote the payoff of a project with a low and high jump intensity, respectively. Then the intrinsic value of the project at the time of investment can be expressed by the conditional expected payoff,

$$V(S_{ti}, E_{ti}, t, k_{ti}) = p_{ti}(k_{ti})V_L(S_{ti}, E_{ti}, t) + (1-p_{ti}(k_{ti}))V_H(S_{ti}, E_{ti}, t). \tag{E.3}$$

F Matlab

The least-squares Monte Carlo model was implemented in Matlab Version 8.5.0.197613 (R2015a), and was run on a Intel(R) Core(TM) i7-4790S. The programming code is available from our supervisor, Maria Lavrutich, upon request.