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Stochastic multistage bidding optimisation for a Nordic hydro power producer in the post-spot markets

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Problem description

Most of the power traded on the Nordic power exchange Nord Pool is traded in the day-ahead market. However, with decreasing margins in the day-head market and an increased amount of intermittent energy sources that require more real time balancing of power, the willingness to participate in alternative markets increases. The intraday market, Elbas, and the balancing market allow trading of energy shortly before the production hour. This thesis describes bidding in the post-spot markets for a price-taking Nordic hydro power producer.

The resulting problem is a multistage stochastic optimisation problem with continuous and binary variables. An internal rolling horizon approach is applied to describe how the uncertainty decreases closer to the hour of operation. In each hour, the bidding problem decides how the producer should bid in the post-spot markets to maximise profit. It is desired to reveal the most profitable post-spot trading strategy to decide if one market is more profitable, or if the producer will benefit from using both. Scenarios are generated to describe the market uncertainties. Thus, discrete price and volume scenarios are generated based on historical market uncertainties. This thesis investigates if a comprehensive model of the bidding problem with updated hourly forecasts can reveal an optimal bidding strategy in the post-spot markets. The main focus is to quantify the value of Elbas bidding.

Preface

This master thesis is written within the field of Applied Economics and Operations Research at the Department of Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). The thesis is motivated by the complexity and increased importance of bidding in sequential short-term electricity markets.

We would like to thank our supervisor Post Doc Gro Klæboe for her valuable guidance. We have appreciated the short response time, motivating feedback and her genuine interest in the subject, which has led to interesting discussions. We would also thank Professor Stein-Erik Fleten at NTNU for making the work with this thesis possible.

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Abstract

As a result of an increased focus on climate policy and renewable energy sources, the share of intermittent power in the Nordic power system is growing. With a less predictable production, the interest for trading on the post-spot markets evolves. Producers desire to trade closer to the hour of operation in order to minimise uncertainty of production and demand. This thesis investigates how a Norwegian hydro power producer can maximise daily profit by trading on the post-spot markets. Bidding on the intraday market, Elbas, and the balancing market is modelled as a multistage stochastic mixed integer problem (SMIP) with continuous and binary variables. A comprehensive modelling framework is developed in order to model the market uncertainties. The framework creates realistic demand scenarios for the two markets and implements the model with an internal rolling horizon approach to describe how the bidding problem develops with a decreasing time horizon. The modelling framework is applied to a case study with realistic input data.

A thorough market analysis of Elbas based on exclusive order depth data show that the Norwegian intraday market has very low liquidity. The results show that Elbas premiums normally are higher than the corresponding balancing market premiums, but the probability of a bid being accepted on Elbas is low. As a result of this, considering Elbas bidding does not impact the profit of the hydro power producer significantly. The test cases with higher available production capacity do however benefit more from Elbas trading. With the current market liquidity, Elbas bidding should be considered if it can be done efficiently. The profitability is however considered to increase with more market participants.

Sammendrag

Som et resultat av et økt fokus på klimapolitikk og mer bruk av fornybare energikilder stiger andelen uregulerbar kraft i det nordiske kraftsystemet. Av dette følger en mindre forutsigbar produksjon og interessen for å handle kraft etter at elspotmarkedet har stengt øker. Produsenter ønsker å handle nærmere produksjonstimen for å minimere ubalanse og usikkerhet forbundet med uregulerbare energikilder. Denne masteravhandlingen undersøker hvordan en norsk vannkraftprodusent kan maksimere profitt ved å handle på post-spotmarkedene. Budgivning på intradagmarkedet, Elbas, og regulerkraftmarkedet er modellert som et flerstegs stokastisk blandet heltallsproblem med kontinuerlige og binære variabler. Et omfattende rammeverk er utviklet for å modellere usikkerheten i markedene. Dette rammeverket genererer realistiske etterspørselsscenarioer som deretter implementeres med en endelig rullende horisont for å beskrive utviklingen i budproblemet med en avtagende tidshorisont. Modellen er deretter testet på et realistisk casestudium.

En omfattende markedsanalyse av Elbas basert på eksklusiv ordredybdedata viser at det norske intradagmarkedet har veldig lav likviditet. Resultatene fra teststudiet viser at forskjellen mellom post-spotpris og elspotpris normalt er høyere for Elbas enn i regulerkraftmarkedet. Derimot er sannsynligheten for at Elbasbud blir besvart lav. Som et resultat av dette, vil ikke budgivning på Elbas øke den totale profitten for en vannkraftprodusent nevneverdig. Casestudiet viser at datoer med høyere ledig produksjonskapasitet oppnår høyere profitt av å by til Elbas enn datoer med lavere tilgjengelig kapasitet. Med den nåværende lave likviditeten bør budgivning vurderes hvis det kan gjøres effektivt. Derimot er lønnsomheten av å by på Elbas er forventet å øke med flere deltakere i markedet.

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Nomenclature

Indices:

s	Scenario
h	Hour
i	Generator
j	Reservoir
f	Production segment
k	Demand segment
l	End water value cut
w	Watercourse

Sets:

$\mathcal{S}$	Set of scenarios s
$\mathcal{H}$	Set of production hours h
$\mathcal{H}^b$	Set of bid hours h
$\mathcal{I}$	Set of generators i
$\mathcal{J}$	Set of reservoirs j
$\mathcal{F}$	Set of production segments f used in the approximation of the production function
$\mathcal{K}$	Set of segments k used in the approximation of the Elbas demand function
$\mathcal{L}$	Set of end water value cuts l
$\mathcal{W}$	Set of watercourses w

Parameters:

Pr_s	Probability of scenario $s \in \mathcal{S}$
A_{if}	Intercept of production function for production segment $f \in \mathcal{F}$ for generator $i \in \mathcal{I}$
B_{if}	Slope of production function for segment $f \in \mathcal{F}$ for generator $i \in \mathcal{I}$

C_i	Start-up costs for generator $i \in \mathcal{I}$
D_i	Maximum discharge for generator $i \in \mathcal{I}$
E^{BM}_{sh}	Available demand in the balancing market in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
E^{E}_{k}	Upper volume limit for Elbas bid volume in segment $k \in \mathcal{K}$
F_{lw}	Future income in watercourse $w \in \mathcal{W}$ for cut $l \in \mathcal{L}$
I_{hj}	Inflow in reservoir $j \in \mathcal{J}$ in hour $h \in \mathcal{H}$
P^{Spot}_{h}	Cleared spot price in hour $h \in \mathcal{H}$
Q^{max}_{i}	Maximum production level of generator $i \in \mathcal{I}$
Q^{min}_{i}	Minimum production level of generator $i \in \mathcal{I}$
U^{0}_{hi}	1 if generator $i \in \mathcal{I}$ in hour $h \in \mathcal{H}$ is turned on in the initial state, 0 if it is turned off
V^{0}_{sj}	Initial water volume in reservoir $j \in \mathcal{J}$ for scenario $s \in \mathcal{S}$
V_{jl}	Evaluated reservoir level for cut $l \in \mathcal{L}$ for reservoir $j \in \mathcal{J}$
W^{0}_{w}	Initial water value in watercourse $w \in \mathcal{W}$
W_{jl}	Marginal water value for cut $l \in \mathcal{L}$ for reservoir $j \in \mathcal{J}$
W^{0}_{j}	Initial water value in reservoir $j \in \mathcal{J}$
X_h	Production committed to the day-ahead market in hour $h \in \mathcal{H}$
X^{E}_{sh}	Volume sold or bought on Elbas for previous bid hours in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
η_j	Energy equivalent in [MWh/m^3] in reservoir $j \in \mathcal{J}$
Π_{sk}	Acceptance share of Elbas bid in segment $k \in \mathcal{K}$ for scenario $s \in \mathcal{S}$
ρ^{E}_{shk}	Accepted Elbas bid price in segment $k \in \mathcal{K}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
ρ^{BM}_{sh}	Balancing market price in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$

Variables:

c_{shi}	Induced start-up cost for generator $i \in \mathcal{I}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
d_{shi}	Discharge by generator $i \in \mathcal{I}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
q_{shi}	Net production needed to deliver all commitments for generator $i \in \mathcal{I}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
s_{shj}	Spill from reservoir $j \in \mathcal{J}$ in hour hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
u_{shi}	1 if generator $i \in \mathcal{I}$ is committed in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$, 0 otherwise

v_{shj}	Reservoir volume in reservoir $j \in \mathcal{J}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
w_{sw}	Approximated end water value in watercourse $w \in \mathcal{W}$ for scenario $s \in \mathcal{S}$
x_{sh}^{BM}	Volume committed to the balancing market in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
x_{shk}^{E}	Volume bid to Elbas in segment $k \in \mathcal{K}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$

Indicator matrices:

Γ_{ij}	Explains the connection between reservoir $j \in \mathcal{J}$ and generator $i \in \mathcal{I}$. 1 if generator draws from reservoir, -1 if generator spills into reservoir and 0 if there is no connection
$\Lambda_{jj'}$	Explains the connection between reservoir $j' \in \mathcal{J}$ and reservoir $j \in \mathcal{J}$. 1 if spill is from j' to j, -1 if spill is into j' from j or if the reservoir j' spills out of the system, and 0 if there is no connection between the reservoirs

Chapter 1

Introduction

In a recent report from the Nordic transmission system operators, Statnett, Svenska Kraftnät, Fingrid and Energinet.dk, it is concluded that the Nordic power system is changing (Statnett, 2016). This change is driven by climate policy, which stimulates the development of more renewable energy sources, technological developments and a future European framework for markets, operation and planning. The report outlines the future challenges and opportunities for the Nordic power system, which can be summarised as the closure of thermal plants, an increased share of wind power production, decommission of Swedish nuclear power plants and increased interconnector capacity between the Nordic markets and the continental European power markets. In addition, the Nordic power exchange Nord Pool is expanding into seven Central European markets during the fourth quarter of 2017 (Nord Pool, 2017).

These changes will influence the existing power markets, and new strategies are necessary for the producers to maintain profitability. The Nordic power market is a successfully deregulated and competitive market, where different mechanisms have been established to cover the continuously increasing flexibility in electricity trading. Due to the uncertainty in power generation and consumption, power trading happens closer to the hour of operation. Most of the power is still sold in the day-ahead market, but with an increased share of intermittent power sources in the system, the need for post-spot markets to trade imbalance power is increasing.

Hydro power producers have a distinctive position in the power markets with the ability to store potential energy in large water reservoirs. This facilitates a flexible power generation which can be adapted to fit the current situation in the power system. As the only existing large-scale storage technology to this date, hydro power producers can profit from the imbalances of other producers and thus benefit from an increase in renewable intermittent energy sources.

Over the past years, bidding strategies in coordinated power markets have been a topic of some interest. For producers with the opportunity to decide when to produce, strategies have been developed in order to maximise profit in the sequential markets. Stochastic mathematical models including the uncertainty of power prices and volumes have been successfully implemented to develop bidding strategies. The majority of power is traded in the day-ahead market, but as the demand for post-spot trading is considered to increase in the following years, producers have the opportunity to get a competitive advantage by optimising their post-spot bidding.

The Nordic intraday market, Elbas, is a post-spot market characterised by low liquidity and low volumes traded. It is therefore questionable whether it is profitable for a producer to consider bidding in this market, or if the profit from the established balancing market is larger. Elbas trading requires active participation, and the trade-off between expected increased profit and the labour cost must be evaluated. There has been little research concerning Elbas trading up to this date. This thesis focuses on thorough modelling of Elbas based on historical order depth describing all accepted and declined Elbas bids. This is considered to give a more realistic presentation of the market uncertainty and liquidity than what has been presented before.

The alternative to Elbas trading is to bid regulating power in the balancing market and to be activated by the transmission system operators. A quantification of the value of considering Elbas as an alternative to the balancing market is presented in this thesis. This is done by developing a multistage stochastic mixed integer problem to maximise the profit of the hydro power producer. In order to model an entire day of bidding and to update the market uncertainties closer to the production hour, a rolling horizon approach is applied. Because the quality of the stochastic model is dependent on high quality scenarios to capture the uncertainty, an effort is put into modelling Elbas and the balancing market.

The implemented model is tested on data for a hydro power plant owned by Hydro ASA in NO2. The modelled power plant consists of two aggregated reservoirs in cascade and three generators. Hydro ASA is a global aluminium company with operations and activities throughout the value chain, and also Norway's second largest hydro power producer with an installed capacity of 10 TWh (Hydro, 2017). An empirical study is performed for 12 different days throughout 2016.

This thesis is structured in 9 chapters. Chapter 2 describes background information about the Nordic electricity market and its participants. Here, the different markets for day-ahead trading, intraday trading and balancing power are presented, and an introduction to hydro power production and technical considerations for modelling a hydro power plant is given. In Chapter 3, optimisation theory and its relevance to electricity market modelling is presented. The stochastic mixed integer problem is introduced, and the problem presented in this thesis is related to previous research concerning stochastic optimisation in electricity markets, sequential market bidding and electricity market forecasting. An in-depth study of Elbas based on historical trades and exclusive order depth data from Nord Pool is presented in Chapter 4. In Chapter 5, the problem is described and put into context before simplifying assumptions are presented. Further, the mathematical model and its notation are introduced. Chapter 6 is devoted to a comprehensive description of the scenario generation procedure which terminates with a description of the rolling horizon approach. The mathematical model is tested for a case study of a Norwegian hydro power plant located in NO2 and operated by Hydro ASA, and a description of the case study and relevant input data is given in Chapter 7. The results of applying the mathematical model to the case data is presented in the computational study in Chapter 8. This chapter is also devoted to stability testing of the scenario tree. Finally, concluding remarks and future research are presented in Chapter 9.

Chapter 2

An overview of the Nordic electricity markets and hydro power production

Electricity is a unique commodity due to several specific features. It is produced and consumed in a continuous flow, and there must always be a balance between supply and demand to avoid blackouts or breakdown of the system. In practice, this means that electricity is consumed at the same time it is generated. The physical properties of electricity make it extremely hard to store in larger quantities after production, however the technology of battery storage is increasing. Existing alternatives for power storage are for example large hydro reservoirs which store potential energy. Further, the consumption of electricity varies with characteristic patterns during day/night, a week or a year, but daily variations are to a large extent influenced by outdoor temperature and other uncertain parameters. Due to the need for an instant balance in the electricity market, the price mechanism is not able to work fast enough to balance production and consumption in real-time. The consequence of this is that electricity pricing always must be done before or after real-time, and this must be handled by specific power markets (Wangensteen, 2012).

This section gives a brief introduction to the different Nordic power markets and how they interact. This thesis focuses on the post-spot markets, thus the intraday market Elbas and the balancing market (BM) are presented in further detail. With an increased share of intermittent, renewable energy sources the existing power markets may play a different role in the future (Statnett, 2016). Because of the ability to store power, hydro reservoirs can contribute to the real-time balancing of power with an increased share of intermittent energy sources in the power system. This thesis will investigate how a hydro power producer can maximise profit from post-spot trading, and therefore some features of hydro power production and modelling are introduced.

2.1 The Nordic electricity market

The Nordic restructuring of the electricity market started in Norway in 1990 with the introduction of the Energy Act to improve the efficiency of the power market and create a more flexible use of power (Statnett, 2014). Norway and the Norwegian Water Resources and Energy Administration wanted an integrated Nordic power market, and in 1996 a joint Norwegian/Swedish power exchange was established. In the following years, Finland and Denmark joined the power exchange in 1998 and 2000 respectively, resulting in an open Nordic market. In the Nordic countries the ownership has been dominated by the state, county and municipality, something which makes the power industry different from other industries (Nord Pool, 2016a).

The Nordic countries are dependent on various sources of power production, such as wind, hydro, thermal and nuclear power. The Norwegian market is dominated by hydro power, whereas nuclear and thermal power also are important sources in Sweden and Finland. Denmark is on the other hand the Nordic country with the largest share of wind power production. The use of different energy sources results in a variety of challenges and needs considering power trading, especially because power is distinguished between adjustable and intermittent production. Energy sources which can be regulated in the Nordic market are typically hydro or thermal power, whereas intermittent energy sources consist of wind and solar power. The share of solar power is low in the Nordic market, however it is relevant when considering import from Germany. Producers using adjustable energy sources can typically increase or decrease the production through manual operation, whereas intermittent energy sources depend on external factors. The use of intermittent power gives a shorter planning horizon and larger uncertainties in production. Thus, a continuous growth of intermittent, renewable power makes it difficult to predict production. This generates demand for alternative markets to the day-ahead market where producers can offer their products (Wangensteen, 2012).

About 70 % of the physical trade in the Nordic countries is traded on the power exchange Nord Pool, and the amount is increasing every year. During the fourth quarter of 2017, Nord Pool will be operational as the Nominated Electricity Market Operator (NEMO) for 15 European countries, resulting in a vast market expansion compared to today (Nord Pool, 2017). At Nord Pool the producers and suppliers can trade physical power in different markets: the spot market and the intraday market, Elbas. In addition, regulating power is traded in the balancing markets operated by transmission system operators (TSOs). Table 2.1 gives an overview of the current Nordic electricity markets and trading routines. This thesis focuses on physical trading, but financial trading is also possible. The time line in Figure 2.1 illustrates the different markets and their time windows, and the different markets are introduced briefly in the next sections.

Table 2.1: Overview of the Nordic electricity markets.

Market place	**Physical trade**	**Financial trade**
Nord Pool	Elspot Elbas	
Transmission System Operators (TSOs)	Primary reserve (FNR and FDR) Secondary reserve (FRR) Tertiary reserve (Balancing market)	
Nasdaq OMX Commodities		Futures Forwards Options Contracts for difference (CfD)
Bilateral	Full delivery Load factor contracts Spot (cap and floor), etc.	Forwards Options, etc.

Towards 2025, the Nordic TSOs assume that the main changes in the power system will involve closure of thermal plants, an increased share of wind power, decommission of Swedish nuclear power and an increased capacity between the Nordic power system and the continental European system. These changes will complicate the procedure of forecasting and planning the power system. In addition, a more complex system will require new measures from the TSOs and market participants in order to maintain security. One issue is for example how an increased share of intermittent power should be handled in the markets to maintain the investment climate and profitability, as well as keeping the conventional generation profitable (Statnett, 2016). It will also be interesting to follow the Nord Pool expansion and its effect on the Nordic power markets.

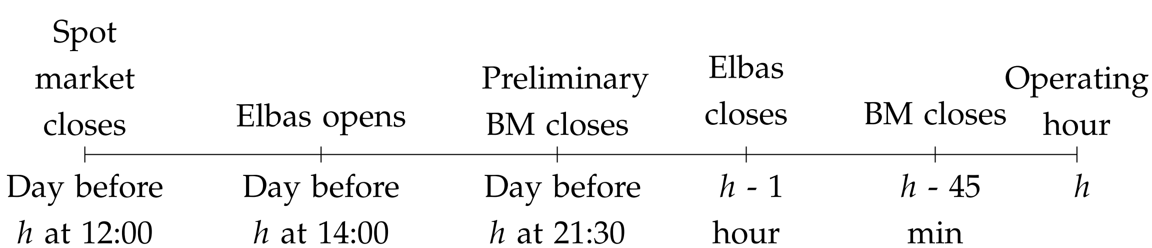


Figure 2.1: Trading time line of the Nordic electricity markets.

2.2 The day-ahead market

The main area for power trading is the day-ahead market. This is handled by the elspot day-ahead market at Nord Pool. In this thesis, the terms day-ahead market and spot market will be used interchangeably to describe the same market. In the spot market, hourly contracts are committed to for physical power exchange the following day. To establish an equilibrium between supply and demand, the spot market is an auction based exchange. The total traded buy and sell volume was 782 TWh in 2016. The countries participating in this trade include Norway, Sweden, Finland, Denmark, Estonia, Lithuania and Latvia (Nord Pool, 2016a). Prior to 12:00 the day before physical power exchange, producers and consumers have to submit their bids covering every hour of the following day. Nord Pool then calculates the system price based on an advanced algorithm, however it is simplified as the crossing price between the buy and sell prices offered in the market. Each producer submit their bids for a given volume and price, where the price is the marginal cost of generation. Intermittent power, like wind or solar typically have low marginal costs, whereas sources like thermal power have higher costs associated with production. The trades are settled after the market clearing price has been calculated, and the physical delivery of power begins at 00:00 the following day.

Figure 2.2: Nord Pool: Bidding areas in the Nordic power market.

The Nordic countries are divided into bidding areas as can be seen in Figure 2.2. Because the available capacity in each area and for the transmission lines between areas varies, different prices may occur in the distinct bidding areas. The price differences are supposed to regulate the demand, to avoid overloading the constraints in the grid and reflect the market conditions. When bottlenecks occur between two bidding areas, the supply of power will always move from the area with low price towards the higher price areas. This also reflects the principle of an open market, where the power moves towards the area where demand is highest, and thus prices are high (Wangensteen, 2012).

2.3 The intraday market - Elbas

Although the spot market is traded with the intention of balancing production and load, the system is continuously affected by events that disturb this balance. Examples of this are change in demand behaviour due to unforeseen weather conditions or how the electricity supply changes if a line falls out. An increasingly important factor describing imbalances after the day-ahead market is settled is the occurrence of intermittent energy sources in the power system, such as wind and solar. For these energy sources, there is uncertainty in production due to weather conditions. Because of limited storage possibilities, the energy must be sold when it is produced, regardless if it was committed to the day-ahead market to begin with.

Intraday trading in the Nordic power market is made possible through Elbas. This is the multilateral market for continuous hourly adjustments of the day-ahead commitments. So far Nord Pool has launched Elbas as the intraday market in the Nordic/Baltic regions, Germany and the UK, with Sweden and Finland being the largest participants with the longest intraday traditions in the Nordic market. Elbas is however a small market with a low degree of liquidity. As an example, only 10.2 TWh was traded on Elbas from 01.01.2016 - 31.12.2016. This equals 1.3 % of the total volume traded on Nord Pool day-ahead in the same time period. Elbas is further coupled to the continental European intraday market operated by EPEX through the cross-border intraday market project XBID (EPEX, 2017).

Elbas prices are defined as pay-as-bid for all transactions. Thus, the intraday prices may vary a lot for the same products during the trading period. Participants can position their imbalances further after the day-ahead market clearing before the balancing market measures are taken by the TSOs. As shown in Figure 2.1, the trading at Elbas opens at 14:00 the day before operation and closes one hour before the production hour. It is possible to offer products for 15 minutes, 30 minutes, one hour or block bids. The bids are either sell or buy bids and exist of a volume in MWh and a price in EUR/MWh for a specific hour. The TSOs publish their transmission system capacities to Elbas every day. When available capacity exists, it is possible to trade between areas. If there is no available capacity, the Elbas bids are not shown for the areas affected by the congestion (Nord Pool, 2016b).

2.4 The balancing power market

The real-time power imbalances in the power system are based on frequency, not on price or committed orders. Momentary imbalances are handled through the primary and secondary reserves, which are spinning generator reserves to quickly adjust deviations in frequency. Primary and secondary reserves are not discussed any further in this thesis. To avoid local bottlenecks or larger imbalances in the system, tertiary reserves are used. These reserves are also known as regulating power, and they are manually activated by the TSOs to release secondary reserves (Grande et al., 2008). The balancing power market is the main regulating power market for all the Nordic countries. Both production resources and consumption resources can be offered. The different states of balancing depends on whether there is an energy surplus or an energy deficit in the system. When consumption is larger than production, upward balancing is required (BM↑). This means that the TSOs require the producers to increase production and sell more power than planned. If consumption is lower than production, downward balancing is required (BM↓). The producers are then required to decrease production. Producers with a surplus production compared to their committed production are called long, whereas producers unable to produce their committed amount of power are called short. Large costs occur if demand can not be satisfied, resulting in partial blackouts or load shedding. This is therefore avoided as far as possible.

Participants can register their preliminary BM-bids to the TSOs until 21:30 the day before the operating hour. New bids or corrections of bids can be registered until 45 minutes before the hour of operation. It is necessary that the reserves used in the submitted bids have an activation time of maximum 15 minutes. This is to ensure safety of delivery. After the production hour, the BM-price is determined based on the regulations carried out in the Nordic power market. Hydro power producers place bids in the balancing market to offer to regulate their planned production. Only the activated bids get paid according to their production of balancing power and the outcome of the balancing market. For each hour, both an upward regulating price and a downward regulating price is determined. Bids are valid for one or several hours and bid prices must receive an integer value when divided by 5. Further, the lowest and highest bid prices are respectively 5 NOK/MWh over the system price for BM↑ and 5 NOK/MWh below the system price for BM↓. The lowest bid quantity is 10 MW (Statnett, 2015).

Table 2.2: Imbalance prices according to a one-price mechanism.

	Upward regulation	Downward regulation
Production deficit	Pay BM↑ price	Pay BM↓ price
Production surplus	Receive BM↑ price	Receive BM↓ price

Table 2.3: Imbalance prices according to a two-price mechanism.

	Upward regulation	Downward regulation
Production deficit	Pay BM↑ price	Pay spot price
Production surplus	Receive spot price	Receive BM↓ price

The bids are handled in real-time by the TSOs to ensure a balance between supply and demand. The bids are rated in merit order, and the upward or downward regulating price is determined by the marginal regulating price in the main regulation direction. Only the activated bids are compensated, and all activated participants receive the resulting balancing market price. Participants that fail to deliver their commitments in the day-ahead market are penalised according to their imbalances. Imbalances from planned production can be reported by nominating system data to the TSOs before the production hour. These participants face imbalance prices according to a one-price mechanism shown in Table 2.2. For imbalances that are not reported, participants face prices according to the two-price mechanism in Table 2.3. In this thesis, it is assumed that the producer is an active participant bidding in the balancing market. The penalty costs are therefore not considered.

2.5 Hydro power production

Hydro power plants utilise the potential energy of stored water and the mechanical energy of falling water to generate power. It is a mature and cost-competitive renewable energy source, which accounted for over 16.6 % of the total electricity production worldwide in 2015 (REN21, 2016). According to Statkraft (2009), hydro power is the power producing mechanism with the lowest emissions, highest efficiency and longest lifespan. Opposed to wind, solar and other renewable energy sources, hydro power has the ability to store water in reservoirs and produce later. Storing water in a reservoir provides the flexibility to generate electricity on demand, and reduces the dependence on uncertain variation of inflows. This helps stabilising the fluctuations between supply and demand, which becomes an increasing problem with the increased share of intermittent energy in the power system. Furthermore, the operation of large water reservoirs also contributes to control of water supply, flood and drought control. IAE (2012) classifies hydro power plants into three categories: run-of-river, reservoir and pumped storage. This thesis will focus on a hydro power plant with reservoirs in a cascading river system.

2.5.1 Operation of a hydro power plant

The turbines in the power plant convert the mechanical energy in the water into energy, which is further converted to electricity in generators. Equation (2.1) shows how the power output of the discharged water depends on the water density, ρ [kg/m^3], the gravitational constant, g [m/s^2], the discharge, Q [m^3/s], the head level, H [m], and the overall efficiency of the power plant, η_a (Kjølle, 1980). Thus, the total output power is limited by the discharge level, which is limited by the topology of the power plant.

$$P = \rho g Q H \eta_a \tag{2.1}$$

The power output further depends on the head level, which is the difference between the water level in upstream and downstream reservoirs. Because the relationship between power output, discharge and head is non-linear, an optimisation model considering head level effects is difficult to solve (Klæboe, 2015). In order to have linear and easily computed models, the head is usually ignored. Both Klæboe (2015) and Catalão et al. (2010) state that a non-linear model including head-effects is more accurate and describes the hydro power generation characteristics better than a linear model. However, for hydro power plants with large storage capacity, Catalão et al. (2010) have found head-effects to be negligible. When the head is neglected, the power output is dominated by the discharge. Kjølle (1980) describes the power output as a concave function of discharge alone, with the point of maximum efficiency denoted as the best point. An optimal operation at the best point means that energy is only delivered in the production process.

Due to the fluctuations of the market demand and prices during the day, the hydro power units regulate production in order to maximise profit by generating during high-price periods and saving water when the prices are low. This pattern of operation causes increased wear and tear on the operation unit and increases the need for maintenance. In addition, starting or stopping hydro power units generate costs due to water losses, efficiency degradation of equipment and potential lost production due to unexpected breakdowns or unsuccessful starts (Bakken and Bjørkvoll, 2002). Some of the costs are directly linked to a start-up, while others are the results of continuous wear during operation hours. As a result of this, Bakken and Bjørkvoll (2002) suggest two approaches to model the costs of starting up a unit, one where the cost is proportional to the number of starts, and another which is increasing with the number of operating hours.

2.5.2 The water value

The direct variable cost of hydro power production is very low and normally neglected in scheduling problems. Instead of the cost of operation being related to the production in itself, it is based on the opportunity cost of producing hydro power. Because production is limited by the amount of available water, producing now means that the water can not be used in future production. The value of the water is in reality a function of future expected load, market prices and inflow. Thus, the cost of operation is regarded as the change in the expected marginal revenue of the energy stored in all reservoirs, also known as the water value (Doorman, 2016).

When bidding hydro power to the market, the water values are deciding which price the producer should demand and how much water to dispatch. Due to different reservoir sizes and time of operation, the planning horizon spans from short term scheduling on an hourly basis to long term scheduling several years in the future. The producer must create a hydro power planning problem that weighs the profit today against possible future profits, as well as taking into account the spring floods and dryer periods of the year. The water value is increasing when the reservoir levels decrease, this means that the opportunity cost for the water increase when the resource becomes more scarce (Klæboe, 2015).

It is not possible to run one simulation for a time horizon spanning from the next hour and five years ahead, thus the scheduling is divided into phases. Doorman (2016) defines the phases as long term scheduling (1 - 5 years), seasonal scheduling (3 - 18 months), short term scheduling (1 - 2 weeks) and detailed simulation (1 - 12 weeks). The different scheduling phases must be coupled at suitable stages to ensure the flow of information and preferences for all phases. This is realised through reservoir levels, which can be presented through price coupling, volume coupling or penalty functions. According to Fosso and Belsnes (2004), the power production in a market-based system should be price dependent, and thus coupling through price is the preferred method in this thesis.

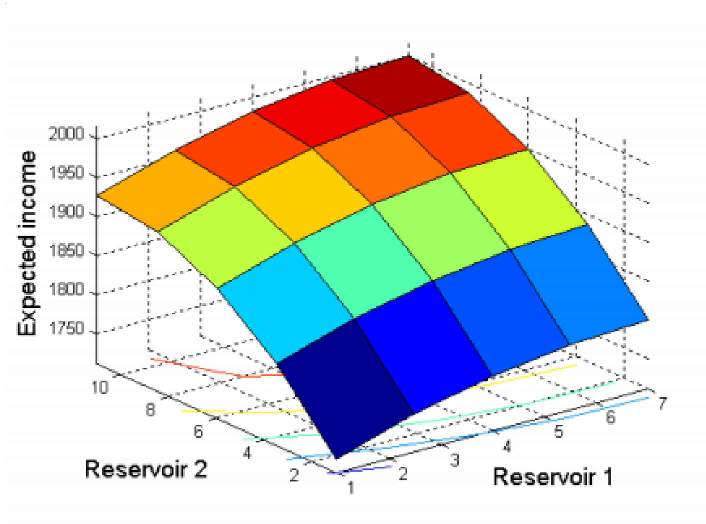


Figure 2.3: Linearised profit function for two reservoirs described with 16 water value cuts (Doorman, 2016).

In the seasonal model, water values for all reservoirs are calculated with initial and end period reservoir levels as input parameters. Then the water value is found as the shadow price of the reservoir balance constraint in the profit optimisation problem for each reservoir. With increased reservoir level increases the risk of spillage, which will turn the water value into zero. Thus, the future profit as a function of the reservoir level is a concave function, resulting in the marginal water value being positive, but decreasing. A hydro power system usually has several reservoirs in cascade, hence the water value in one reservoir depends on the water value in all other reservoirs in the system. Therefore, the price coupling should be based on multidimensional water value functions which often are presented as cutting planes. An example of this can be seen in Figure 2.3.

2.6 Summary

The Nordic power system is characterised by a large share of hydro power, with thermal and nuclear power still playing an important part. Due to climate change and increased focus on environmental politics, the sources of generation are changing from fossil sources towards more renewable production. This causes more development in both new and existing power segments, in order to include intermittent power sources. An increase in installed intermittent power over the following years, with more wind and solar energy in the Nordic grids, will increase the demand for short term trading. Hours where regulation is necessary will occur more frequently, demanding effective and profitable markets for balancing and intraday power. In addition, structural changes and larger transmission capacities to the European continent will give the Nordic countries new markets to trade in and new competition. It is reasonable to believe that market participants in the intraday and balancing markets will increase corresponding to more intermittent power generation and increased transmission capacity. With an increase in market participants, the liquidity will increase and the markets will become more attractive. A hydro power producer has the ability to store water as potential energy in water reservoirs, and this energy can play an important part in balancing the power system. As long as the hydro power producer receives a better price than the marginal value of the water stored in the reservoirs, it will be beneficial to produce and contribute to power system balance.

Chapter 3

Optimisation theory in modelling of electricity markets

This chapter presents an overview of existing literature which is relevant for the modelled problem. This thesis presents a multistage stochastic mixed integer problem that aims to solve sequential bidding in Elbas and the balancing market with uncertain Elbas demand as well as uncertain balancing market prices and volumes. A theoretical introduction of the stochastic mixed integer problem is first given with a detailed description of the multistage type. After the theoretical framework is described, a more detailed presentation is given of the problem specific topics. This thesis is compared to existing literature concerning the bidding problem for power production. More specific, the problem modelled in this thesis is linked to previous stochastic optimisation problems in electricity markets. Then, it is shown how the bidding problem in sequential power markets presented in this thesis differs from previous studies. Finally, a summary of existing forecasting models for post-spot prices and volumes is given. In this chapter, the focus is primarily on hydro power scheduling due to the relevance to this thesis.

3.1 Stochastic mixed integer programming

Optimisation models where some of the data included in the objective value or the constraints are uncertain is called stochastic programs. In many practical situations where decisions are made consecutively in time, several decision stages occur. A stage is in this setting defined as a moment in time where a decision is made based on new available information. Thus, a multistage problem is a problem where new information is obtained between two subsequent decisions, and the following decision takes this new information into account combined with historical information (Prékopa, 1995). In this thesis, a multistage stochastic program is formulated. The decision variables are defined for each stage, depending on when the uncertain parameters are revealed. In most cases the first stage decision is not subject to uncertainty, however this thesis models stochastic first stage variables as well. To describe the set of possible future outcomes, one often define a scenario tree which branches for each stage. The scenario tree thus describes the possible outcomes when adapting to new information.

The decision variables in the multistage stochastic program can be defined as both continuous or integer, or a combination of these two. When the problem contains integer variables, it is characterised as a stochastic mixed integer problem (SMIP). A formulation of the multistage SMIP as given by (Guglielmo and Suvrajeet, 2004) is defined below.

A finite-horizon decision process is defined for the time periods $h \in \mathcal{H}$. Information is given by a discrete time stochastic process $\{\xi_{h=1}^{|\mathcal{H}|}\}$ defined for some probability space. Decisions are based on the information available at that time and therefore depending on the previous decisions and outcomes of the random variables. The vector of all decisions made from stage 1 to stage h is denoted $\underline{x}_t = (x_1, ..., x_h)$ and the vector of random variable outcomes is given as $\underline{\xi}_t = (\xi_1, ..., \xi_h)$. A prototypical definition of the problem is then given by:

$$\max\{c_1(\xi_1)x_1 + Q_1(x_1) : W_1 x_1 \geq h_1(\xi_1), x_1 \in \mathbf{X}_1\}, \tag{3.1}$$

where the recourse decision for $h = 1, ..., (|\mathcal{H}| - 1)$ with $Q_{|\mathcal{H}|} = 0$ is given as:

$$Q_t(\underline{x}_t) = E_{\widetilde{\xi}_{h+1}|\underline{\xi}_t} \max\{c_{h+1}(\widetilde{\xi}_{h+1})x_{h+1} + Q_{h+1}(\underline{x}_{t+1}) : \tag{3.2}$$

$$T_{h+1}(\underline{\widetilde{\xi}}_{h+1})\underline{x}_h + W_{h+1}x_{h+1} \geq h_{h+1}(\underline{\xi}_{h+1}), \tag{3.3}$$

$$x_{h+1} \in \mathbf{X}_{h+1}\} \tag{3.4}$$

It is assumed that for all realisations of ξ and time stages, the matrices and vectors $T_h(\underline{\xi}_h)$, W_t, $c_t(\xi_t)$ and $h_t(\underline{\xi}_t)$ are rational with conformable dimensions. In this thesis it is assumed that the random vector ξ has a finite probability distribution, and it is therefore possible to represent the uncertainty by scenarios. A scenario is defined as a realisation of the random variable given by $(c(\xi), h(\xi), T(\xi))$.

When evaluating each scenario separately, the deterministic equivalent of the problem can be defined. For a set of scenarios $\mathcal{S}$, a vector of decisions for each scenario $x(\xi_s) = (x_1(\xi_s), ..., x_{\mathcal{H}}(\xi_s))$ and a probability for each scenario Pr_s, the multistage SMIP can be reformulated to a large MIP given below:

$$\min \sum_{s \in \mathcal{S}} Pr_s [\sum_{h=1}^{\mathcal{H}} c_h(\xi_{sh}) x_h(\xi_s)] : \tag{3.5}$$

$$x_h(\xi_s) \in \Omega_s \quad h \in \mathcal{H} \tag{3.6}$$

$$x_h(\xi_s) = x_h(\xi_\zeta), \quad s \in \mathcal{S}_\zeta, h \in \mathcal{H}_\zeta, \zeta \in \mathcal{Z} \tag{3.7}$$

Here Ω_s represents the feasible solutions of the decision variables for scenario s, and Equation (3.7) is the formulation of the nonanticipativity constraints. They state that all scenarios with the same information up to a new stage, must make the same decision in all previous stages. For a nonanticipativity set, $\mathcal{Z}$, a set of information, ζ, exists. For every ζ the same decision must be made. In the above constraint, $x_h(\xi_\zeta)$ is the decision made with the information $\zeta \in \mathcal{Z}$ for scenarios $s \in \mathcal{S}_\zeta$ and time period $h \in \mathcal{H}_\zeta$.

3.2 Stochastic optimisation in electricity markets

Due to the uncertain properties of electricity production and demand, it is desired to model the electricity market uncertainties by describing different scenarios. Stochastic programming is often used as a method for models including uncertainty, and the power markets are no exception. This thesis models uncertain demand in the post-spot markets, and the optimisation problem is modelled as a stochastic program. An application example of stochastic programming in electricity markets is presented by Fleten and Kristoffersen (2007). They present a hydro power producer participating in the day-ahead market. The problem is modelled with a mixed integer linear two-stage stochastic model for determining optimal bidding strategies taking uncertain prices into account. Similar to Fleten and Kristoffersen (2007), this thesis uses a stochastic mixed integer program with time series price forecasts to generate uncertain scenarios. However, instead of modelling a two-stage problem for the day-ahead market, this thesis presents a multistage problem for multi-markets. An extension of the two-stage problem is presented by Fleten et al. (2011), who model a multistage stochastic mixed integer linear programming model for hydro power production. A similar model is presented in this thesis, but the stages in this thesis are short-term and depending on bid hours, whereas Fleten et al. (2011) model long-term stages to combine with short-term hydro power planning.

Aasgard et al. (2014) are looking at a multi-reservoir hydro power system and use a successive stochastic optimisation model to create a linear model for short-term hydro power scheduling with the inflow modelled as an uncertain parameter. Instead of modelling uncertain inflow and focusing on optimal scheduling of the reservoirs as Aasgard et al. (2014), this thesis models the uncertainty in post-spot bids. The majority of literature combining hydro power scheduling and stochastic programming focuses on day-ahead bidding dependent on reservoir control. This thesis models the reservoir constraints and value of water, however the uncertainty in hydro power scheduling and reservoir levels is assumed negligible because the post-spot volumes are small.

Stochastic programming is widely used to solve bidding problems in electricity markets, however the problems typically focus on single-market modelling of the day-ahead market or hydro power scheduling. Problems solely considering intraday and balancing markets have not been well represented in the literature, but an introductory study has been performed by Engmark and Sandven (2016) who model a stochastic two-stage linear model for optimal post-spot trading for a wind power producer. Opposed to the work presented in this thesis, the problem modelled has no flexibility due to the intermittent nature of wind power production, hence the only strategic decision concerns which post-spot market is assumed most profitable. In this thesis, the producer has the option to restrain production in order to produce in a more profitable hour. The problem is thus extended to include technical constraints regarding hydro power production, multistage decision variables with time dependency and a rolling horizon approach to model the uncertainty with a decreasing number of bid hours.

3.3 Bidding in sequential markets

With an increased number of electricity markets, multi-market bidding and development of strategies for bidding in sequential markets have become topics of increased interest. This thesis will thus contribute to the existing literature based on optimisation theory. Some articles only consider the issue of modelling and coordinating the markets, whereas others compare the profitability of considering the sequential markets. This thesis models bidding in the post-spot markets Elbas and the balancing market. The demand in both markets is modelled in order to develop strategic decisions for hydro power producers after the day-ahead market is closed. It is expected that the importance of coordinated bidding in sequential markets will increase with more installed intermittent power, and this thesis investigates the influence of a greater demand on the post-spot profitability.

Optimisation models describing sequential markets must take into account that the different markets are settled at different times and that the demand and products vary. One way to describe this is by multistage stochastic programs. Ugedo et al. (2006) use a stochastic optimisation model to obtain the distribution of produced resources among the sequential markets day-ahead, post-spot and ancillary services. The main focus of the article is to model the markets and their bid curves, and unlike this thesis there is no comparison between the profitability of bidding in the different markets.

In this thesis, the value of considering Elbas bidding is quantified as done by Faria and Fleten (2011). Faria and Fleten (2011) develop a model for a hydro power producer bidding in the day-ahead market when taking the possibility of trading energy on Elbas into account. A two-stage stochastic mixed integer program is used to describe the optimisation problem, where the first stage problem involves bidding into the day-ahead market and the second stage includes trading on Elbas and real-time hydro power production. This article is one of few articles that takes Elbas into account. The article is however written before Norway joined Elbas and therefore Sweden is used as reference for scenario generation before the model is fitted into the Norwegian market. The main difference between the article and the problem described in this thesis is the focus on post-spot bidding, with a more complex scenario generation method including a rolling horizon to account for the bid hours. The market modelling of Elbas is also more detailed in this thesis and based on order-depth, not just historical trades. Faria and Fleten (2011) introduce and compare the value of including Elbas to the results of not taking Elbas trading into account. For a price-taker, medium-sized producer there is no significant profit of considering Elbas when trading in the day-ahead market. In this thesis, the value of considering Elbas is compared to the value of only trading on the balancing market after the day-ahead market is closed.

Scharff and Amelin (2016) also consider Elbas. However, the article does not model the market, but only analyse market behaviour and trends. The article concludes that further research should tackle the question on how to model the market, which is a problem this thesis is contributing to solve. Engmark and Sandven (2016) perform a thorough study on Elbas market behaviour in order to model demand, and solves a sequential model for trading on the balancing market and Elbas. It is concluded that Elbas prices usually are higher than balancing market prices, and for an inflexible wind power producer it will be profitable to consider Elbas trading for selling surplus production. Because of the price differences, it will be profitable to trade production deficit in the balancing market in most of the cases. However, the Elbas market modelling in this paper depends on historical trades, which gives a false picture of the actual demand. This thesis provides an improved model of Elbas prices and demand, which gives a more realistic picture of the value of considering Elbas.

The majority of literature concerning sequential bidding include the day-ahead market and typically the balancing market, in contrast to the markets modelled in this thesis. For example is the potential of coordinated bidding in the day-ahead market and the balancing market analysed by Boomsma et al. (2014). The relation between bid volume and volume dispatched for a producer using hydro and thermal energy sources is evaluated. The clearing prices and dispatched volumes are unknown at the time of bidding, and the sequential behaviour is included by taking into account that the spot market prices are known, but balancing prices are unknown at the time of bidding into the balancing market. In this thesis, Elbas decisions are taken before the balancing market decisions, thus volumes sold at Elbas are considered known before adjusting bids to the balancing market. Boomsma et al. (2014) evaluate different pricing-mechanisms for the balancing market, and the article concludes that there is no incentive to enter and bid into the balancing market under a one-price mechanism, but under two-price settlement there is a significant gain to enter and coordinate bidding with the spot market. In this case, thermal power which has a larger operational cost than hydro power is included. The producer modelled in this thesis is considered an active participant in the balancing market, and it is assumed that the imbalances are avoided.

3.4 Forecasting of electricity prices and volumes

At the time of planning and bidding, the accepted prices in the different markets are uncertain and can thus be regarded as stochastic parameters. To solve stochastic optimisation problems, scenario trees are typically generated to form input data for the models. Statistical studies describing the complexity of electricity markets are presented by Weron (2007), who evaluates different forecasting techniques and modelling of electricity load and prices. It is illustrated how most power market data can be described by time series, which are based on the assumption that successive values in the data file represent consecutive measures at equally spaced time intervals. This is accurate for the balancing market data forecasted in this thesis, however the low liquidity of Elbas makes it difficult to apply time series analysis. An overview of the different forecasting methods available is presented by Weron (2014), who evaluates different models used in forecasting electricity prices. Multi-agent models, fundamental methods, reduced-form models, statistical approaches and computational intelligence techniques are presented as possible alternatives. This thesis forecasts balancing market premiums using time series models with Markov probabilities, balancing volumes using fitted probability distributions and Elbas demand curves using regression models and discretised probability distributions, and is thus a contribution to the reduced-form models and statistical approaches.

With the increasing share of intermittent energy in the market, more flexible methods are needed to predict prices and demand. Time series analysis is usually applied to forecast the data, and several studies have investigated different forecasting techniques of prices in the day-ahead market, Elbas and the balancing market. To generate realistic market price scenarios, Faria and Fleten (2011) use ARMAX and GARCH techniques where both the uncertainties of the day-ahead prices and Elbas prices are considered. Due to the low liquidity of Elbas and a more detailed information about the market demand given by the order depth, time series analysis is not considered sufficient for Elbas forecasting in this thesis. Instead, demand curves are modelled based on historical bids with a custom probability distribution for a bid being accepted based on volume and price. This is the first time to our notice that Elbas demand is modelled based on order depth rather than historical accepted bids.

Olsson and Söder (2008) model the real-time balancing power market using combined seasonal auto regressive integrated moving average (SARIMA) and discrete Markov processes. This is done to be able to generate price series in periods where no demand exists. This thesis forecasts balancing market premiums using an auto regressive moving average (ARMA) model because no seasonality can be detected and the time series are stationary. Even though it is possible to generate prices for periods without demand, the method does not handle the auto-correlation on both sides of the missing values. Therefore, this thesis applies the method described by Erdogan et al. (2005). To resample the time series with missing values, an algorithm is designed which uses an extended AR(1) model to fill in the defined intervals with zero premium.

Jaehnert et al. (2009) provide a long term statistical model of the balancing market based on the regulating volume with observed correlations between balancing market premium and regulating volume. The model has both a deterministic and a stochastic part, where the stochastic part is modelled as an error term describing the difference between market data and the deterministic part. For the data used in this thesis, the correlation between balancing market premium and balancing volumes has decreased compared to the study of Jaehnert et al. (2009) and is therefore not included. The balancing volume is therefore forecasted separately using the same generalised extreme value distribution as Jaehnert et al. (2009). Klæboe et al. (2015) present a benchmark study of four different models for predicting the balancing volume, which conclude that the volumes are random and difficult to forecast. Because the volume is also dependent on the regulating state, this must be considered when predicting volume and premium scenarios simultaneously. In this thesis, the forecasted balancing volume serves as an upper demand limit in the market. It is therefore assumed that the forecasting model presented by Jaehnert et al. (2009) is suitable to describe the distribution of regulating volumes.

Time series models should be updated for each hour where new information is received. The problem formulated in this thesis is modelling a time horizon from Elbas opens for bidding until the end of the following day. For each bid hour the model is run, new information is received about the balancing premiums. This requires an updated forecast. Similar to Beraldi et al. (2011) this is done by dynamically solving the stochastic model in a rolling horizon fashion by iteratively considering more and more recent information and a planning horizon of decreasing length. Opposed to Beraldi et al. (2011) who model an electricity consumer in the day-ahead market, this thesis models a power producer in the post-spot markets, thus the uncertain and dynamic parameters are different. Further studies combining rolling horizon techniques and stochastic programming have been done by Devine et al. (2016), who apply rolling horizon heuristics to the natural gas market. Applications to energy planning have been performed by Champion and Gabriel (2017), who show that the rolling horizon approach is performing better than the fixed-horizon, multistage model. The problem in this thesis is not compared to a fixed-horizon model due to the problem size.

3.5 Summary

This chapter aims to give an overview of relevant literature used as inspiration and supervision for the problem modelled in this thesis. In this thesis, a multistage SMIP is solved for a hydro power producer in the post-spot electricity market. The uncertain parameters of the balancing market have been well studied, and the scenario generation procedure in this thesis is inspired by, and to some extent reproducing, previous research. Elbas is a market with low liquidity and scarce available data, and previous forecasting studies are not considered sufficient. This thesis creates a new regression model describing the order depth, and thus the actual market demand. To our notice, this has not been done before. However, due to the low market liquidity, the modelled parameters are also modified in a sensitivity analysis in order to evaluate how the market may develop. Due to the different deadlines for submitting bids in each market and the different forecasting methods, a rolling horizon approach is applied to the stochastic problem in order to update information with new bid hours. Up to this date, no studies have been found which model multi-market electricity bidding problems using a rolling horizon approach for a multistage SMIP.

Chapter 4

Elbas market behaviour in NO2

As described in Section 2.3, Elbas makes it possible to trade energy close to the production hour. This way the participants in the electricity market can reduce their expected imbalances or optimise production schedules. Producers with adjustable production can trade energy when it is profitable by either reducing or increasing production according to the original plan. This type of flexibility can increase the power producer's profit, and a greater understanding of Elbas can give a competitive advantage. Elbas has not been well studied in the literature, probably due to its rather short life span and low liquidity. Scharff and Amelin (2016) have analysed Elbas and present a main overview of the intraday trades. The contribution in this chapter is a thorough analysis based on their work, but with focus on bidding area NO2. Scharff and Amelin (2016) conclude that further research should tackle the question on how to model the continuous intraday market, and that is one of the main purposes of this thesis.

Further Elbas studies have been performed by Engmark and Sandven (2016), who evaluate factors affecting the prices and demand in Elbas. Examples of these factors are spot prices, wind imbalances, consumption imbalances and the behaviour of the balancing market. The research from this report shows that Elbas to some degree correlates with the spot market, but is often depending on unforeseen events. No significant correlation was found between Elbas and the other market drivers, thus a different way of modelling the market is required. The following chapter is based on previous research done by Scharff and Amelin (2016) and Engmark and Sandven (2016) with extended analyses.

4.1 Historical overview of Elbas in NO2

The Norwegian Elbas market is characterised by few trades and low liquidity. Other Scandinavian countries such as Sweden and Finland have a larger share of market participants and a greater amount of traded volumes. The European intraday market with Germany in the lead is also superior to the Norwegian market. An important aspect in order to benefit from Elbas trading, is to understand the liquidity and characteristics of the market. Due to the generally low Elbas liquidity in Norway, Elbas trading in NO2 is depending on the option to trade with the more developed intraday markets in Scandinavia and Europe. To trade with other bid areas, available transmission capacity is important and can be limiting. It is interesting to investigate whether the available transmission capacity actually is a bottleneck, or if bids simply are not placed or accepted.

Table 4.1: Percentage share of Elbas trades in NO2 based on transmission capacity and trade behaviour.

	Capacity + trade	No capacity + internal trade	Capacity + no trade	No capacity + no trade
Buy	22.18 %	0.14 %	77.17 %	0.52 %
Sell	23.32 %	0.16 %	75.67 %	0.85 %

Table 4.2: Percentage share of available transmission capacity to neighbouring areas.

	NO1	NO5	DK1	NL
Buy	97.3 %	78.3 %	91.1 %	86.2 %
Sell	92.9 %	90.0 %	58.8 %	15.4 %

Table 4.1 shows the percentage share of Elbas trades based on status of transmission capacity and trade status in the time period 01.01.15 to 31.12.16. Internal trade is the state with no transmission capacity, but still internal Elbas trading between market participants inside NO2. The most frequent state is observed as the state with transmission capacity and no trade, and this emphasises that transmission capacity to the nearest bidding areas probably is not the limiting factor. Table 4.2 shows the percentage occurrence of transmission capacity to the neighboring bidding areas for all hours. It can be seen that the transmission capacity to more liquid intraday markets in Denmark and the Netherlands is typically more limited than to the Norwegian areas. This table also highlights that

transmission capacity for buy volumes occurs more often than transmission capacity for sell volumes, indicating that transmission capacity out of NO2 is less available.

As the transmission capacity does not seem to be the reason for low Elbas liquidity, trading behaviour is investigated and important participants are analysed. To identify valuable bidding areas, one can identify the areas with the highest number of trading incidents. This is presented in Figure 4.1, which represents the number of all completed buy and sell trades between NO2 and other bidding areas from 2015 to 2016. Based on the analysis, trading is most likely to happen between NO2 and DK1, however trading with SE3 and NO5 is also usual as seen in Figure 4.1. It seems that the limited transmission capacity between DK1 and NO2 from Table 4.2 has small impact. As mentioned in Chapter 2, plans have been made to increase the transmission capacity between Norway and the European continent. For example is the HVDC transmission line NorGer between Norway and Germany under development. It is reasonable to believe that intraday trading with continental Europe will increase with increased transmission capacity, although available transmission capacity is not considered the main issue for low Elbas liquidity in NO2 for now.

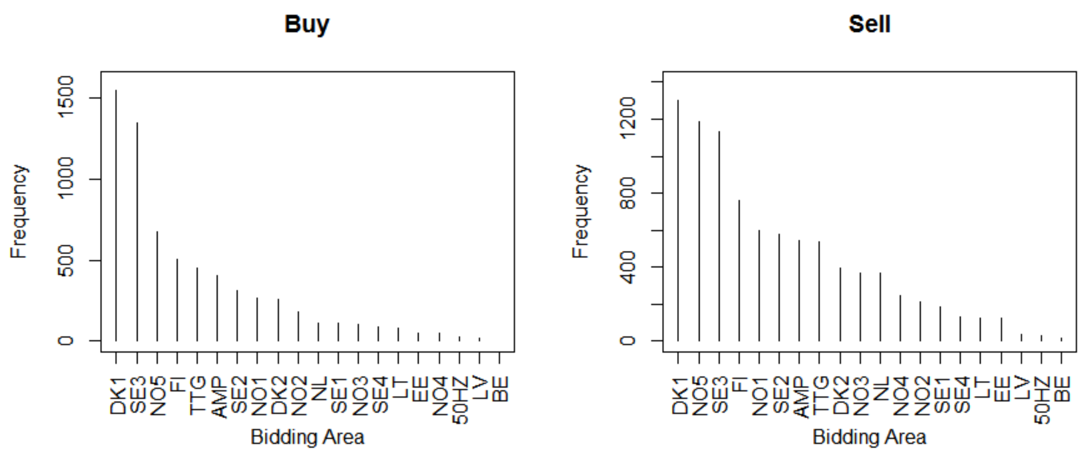


Figure 4.1: Number of trades between bidding areas and NO2, time period 2015 - 2016.

Other features of trading between NO2 and other bidding areas are average price and volume per trade. From the results found in Appendix A, it can be seen that the average trade price is similar for all bidding areas. Thus, no area seems to be more accepting of higher prices. The average volumes are however more volatile, and especially the volumes traded with SE3 are a lot higher. Because SE3 is also a frequent customer, it can be concluded that it is the most valuable bidding area for NO2 to trade with based on historical data. This can be explained by an Elbas market maker agreement with Swedish power producers which are committed to quote both buy and sell orders every day (Nord Pool, 2015).

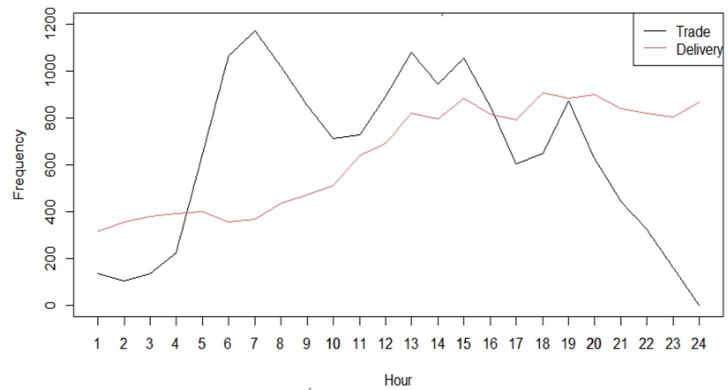


Figure 4.2: Frequency of trades based on trade hour and production hour 2015 - 2016.

Because Elbas trading is done manually, it is interesting to look at when trades typically are completed. There will be a trade-off between the labour cost of an employee who trades at Elbas and the actual benefit from Elbas trades. Figure 4.2 shows the number of trades depending on hour of trade and delivery hour. Most trades are completed during office hours and the number of trades during the night is low. It does not seem beneficial to hire an employee for Elbas trading during the night, as the number of performed trades is close to zero. The figure also shows that an increasing number of Elbas trades are delivered during the day.

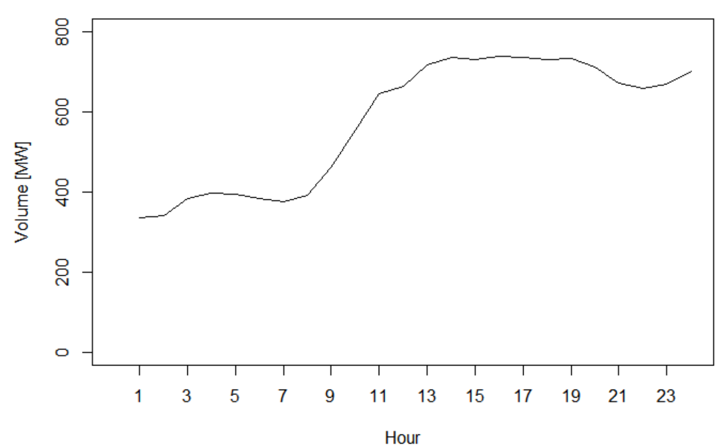


Figure 4.3: Average total traded volume dependent on production hour in 2016.

Although the number of trades does not represent how much volume is traded, it is reasonable to believe that the traded volume correlates with the frequency of trades. This is also supported by Figure 4.3, which shows that the average total volume increases during the day. Thus, it is concluded that Elbas trades differ from a typical load curve with peaks in the morning and evening. The trend of increasing bids during the day and the hours with higher number of trades, can however be seen in context with the lead time in Figure 4.4. Lead time is the time between the bid is published and the production hour where the physical power is delivered. Most trades have a lead time between four and nine hours and can therefore explain why the production hour frequency increases during the day. Figure 4.4 therefore highlights the customers' need for trading close to the production hour, because most trades at Elbas have a lead time less than nine hours compared to the possible lead time of 35 hours in the spot market. It can however be noted that EPEX has reduced the intraday closure time to 30 minutes before production hour. If Nord Pool follows this trend the lead time will most likely decrease in the future and trading even closer to production hour will be possible (EPEX, 2015).

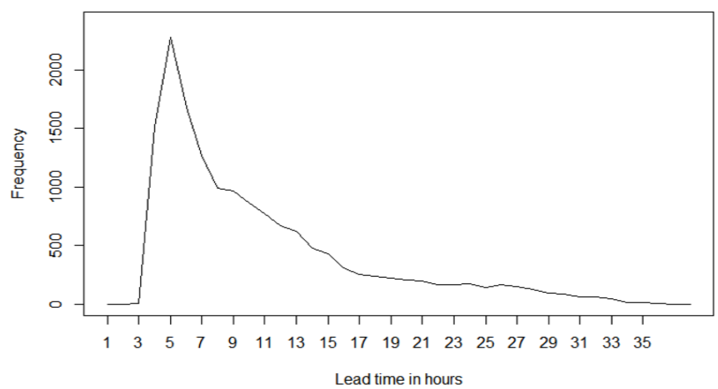


Figure 4.4: Lead time: Time between trade hour and delivered production for 2015 - 2016.

Because the Elbas price is defined as the spot price added to a positive or negative premium, the correlation between Elbas price and spot price as presented by Engmark and Sandven (2016) is not surprising. However, the correlation varies between the system price and the area price, as can be seen in Figure 4.5. Similar to the spot price, the average Elbas price is also defined as a system price and an area price. Figure 4.5 shows the average price depending on production hour for the system price, spot price in NO2, Elbas price in NO2 and system Elbas price. The different prices follow the same trend throughout the day, with higher price for the load peaks. The system price is as expected higher than the NO2 spot price, and the same correlation applies to the Elbas system price and Elbas NO2 price. In addition, the Elbas price is higher than the associated area or system price for all hours except during the night. This is another indicator that Elbas trading outside office hours may not be not profitable for now.

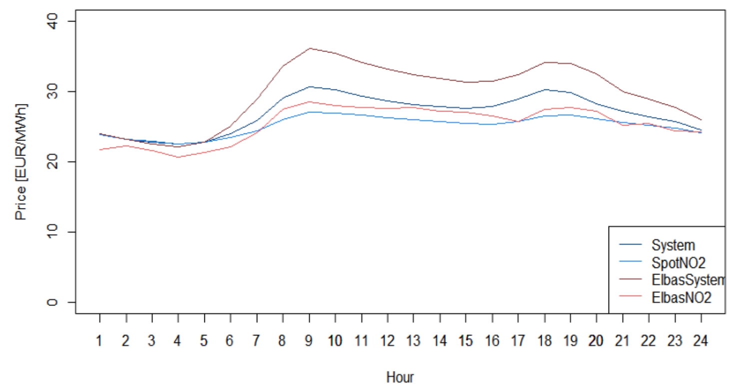


Figure 4.5: Average hourly Elbas price and spot price for NO2 and the system.

It can be seen that the Elbas price is higher than the associated system price. However, the Elbas price in NO2 is lower than the system Elbas price. The obvious explanation for this is that the system price is higher, and following is the Elbas price higher. Because Elbas is influenced by unforeseen events like extreme drought, rain, spot prices, generator failure or maintenance and other factors, the price is typically pushed above the spot price due to urgent demand. It will however be interesting to follow the development of the NO2 price if trades with the continental European market increase. The European intraday market is more liquid and developed than Elbas, and European producers may have larger incentives to trade on Elbas due to more intermittent power. It is therefore likely that increased transmission capacity to Europe and increased trading will further increase the Elbas price in NO2 closer to the Elbas system price. Further, the price is considered to be influenced by other market drivers to a greater extent than today. European intraday prices are correlated with external factors such as consumption imbalance, which are easier to predict than unforeseen events (EPEX, 2013).

The analyses performed in this section give an overview of the Elbas situation in NO2 in 2015 - 2016. It must be emphasised that NO2 is a small market participant and the analyses presented here describes aggregated data over two years. The main findings are nevertheless that transmission capacity rarely is a limiting constraint for Elbas trading and that trading outside of office hours is not considered profitable. Although SE3 trades larger volumes with NO2 than other bidding areas, it is difficult to describe the underlying structure of Elbas bidding from NO2 and the true demand. As a result of this, the order depth is obtained and analysed further in the following section.

4.2 Analyses of the total order depth

The above analyses have been performed based on accepted Elbas bids. However, due to the low liquidity of the market and infrequent trades, it is assumed that the complexity of the market is not captured. Equilibrium prices based on merit order as in the spot market and balancing market are not presentable for Elbas due to the low liquidity and the occurrence of hours with no bids at all. The continuous Elbas trading implies that trades can be settled whenever a market participant accepts an offer of another market participant. Therefore, prices vary from trade to trade. This is significantly different from the day-ahead market where all hourly prices are settled simultaneously (Scharff and Amelin, 2016). The bids are not regulated by regular power market mechanisms, but depend on single bid volumes and prices and the probability of someone accepting the bid. A majority of the bids are not accepted, and it is assumed that being able to analyse and anticipate the Elbas bids gives an opportunity to execute market power. The order depth, which contains all bids submitted to Elbas is received from Nord Pool with data from February 2017, and the following analyses depend on this. In the received data set the different orders have different statuses depending on the outcome of the trade. The statuses defined as *expired*, *open*, *validity_expired* and *open_remaining* are orders that have not been fulfilled, where *filled* and *matched* are orders that have been answered and completed. Bids that are *cancelled*, *changed* and *inactivated* have been withdrawn by the bidder.

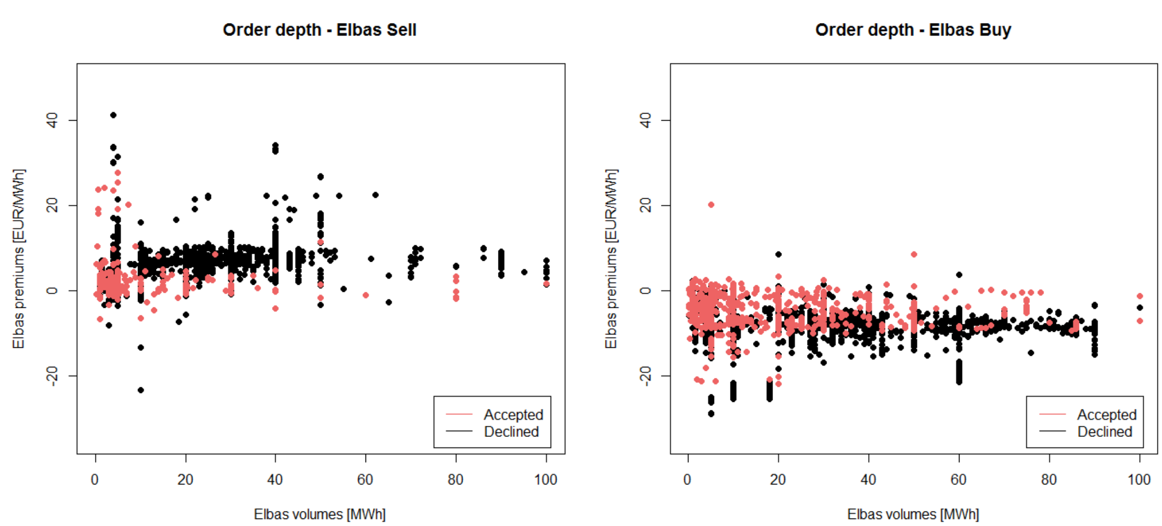


Figure 4.6: Order depth for Elbas NO2, February 2017.

Figure 4.6 pictures the entire Elbas order depth for Elbas sell and Elbas buy bids for one month, and shows the trade-off between premium and volume for accepted and declined orders. The premium is the difference between the spot NO2 price and the Elbas price for the order. It can be seen that orders with price close to spot price is more often answered than the ones with higher premiums. Although the data set contains few data points in order to observe any clear trends, it is obviously a correlation between volume and premium for the bids. Typical demand curves with a linear relationship between premium and volume seem to be an alternative to model the correlation due to the decreasing volumes with increased premiums.

When the producer sell to Elbas, a bid is committed for typically a higher price than spot price, and vice versa for the case of buying at Elbas. The producer wants to maximise profit by earning more than the spot price from post-spot trading. However, the market participant who answers the bids also try to maximise profit and is interested in paying less than spot price. This results in bids close to zero premium having a larger probability of being accepted. In the case where premiums or volumes are very high, it is reasonable to believe that the producer has committed a bid with the intention of being lucky, and that someone desperately needed the product due to unforeseen events. However, it can be seen from Table 4.3 that most of the bids are not answered, thus the probability of abnormal bids being accepted is low.

Table 4.3: Percentage share for status of order depth.

	Accepted	Declined	Withdrawn
Buy	13.5 %	63.6 %	22.9 %
Sell	8.3 %	67.7 %	24.0 %

Because the density of the bids is not shown in Figure 4.6, the bids are analysed to understand where most bids are placed. The results in Table 4.5 is obtained by dividing the data into three parts based on premium value. All bids are included in the analysis, which computes the $1/3$ and $2/3$ highest premiums and investigates how many bids are present within each category of low, medium and high premiums. For the buy premiums, the high premiums are defined as the premiums with the highest negative value. The limits of the premium values are shown in Table 4.4 for premium δ. Further, to check if there is a correlation between bid density and acceptance of bids, the density of accepted bids within each category is also computed. The percentage value is based on the total number of bids for all computations and therefore reflects the probability of a bid being accepted within that category compared to all bids.

Table 4.4: Classification of Elbas premium in high, medium and low sections.

	Sell [EUR/MWh]	Buy [EUR/MWh]
High	$\delta > 10.997$	$\delta < -12.707$
Medium	$10.997 > \delta > 2.743$	$-12.707 < \delta < -4.533$
Low	$\delta < 2.743$	$\delta > -4.533$

From the results in Table 4.5 it can be seen that most bids have a premium with a medium value, and extreme premium values are therefore considered more unlikely to happen. This supports the theory that extreme bids mostly occur for unforeseen events. However, looking at the accepted bids, it can be seen that although less bids are submitted with a low sell premium, the ones who are submitted are more likely to be accepted. High premiums are more likely to be submitted, but are rarely accepted compared to low premiums. From the medium premiums where most bids are submitted, but very few bids are accepted, the low liquidity of Elbas is confirmed once more. It seems that the highest demand exists for bids with low premium.

Table 4.5: Density of Elbas bids based on premium sections.

	High		**Medium**		**Low**	
	Sell	Buy	Sell	Buy	Sell	Buy
All bids	5.35 %	3.89 %	77.49 %	83.80 %	17.16 %	1.88 %
Accepted	0.36 %	0.33 %	2.44 %	7.64 %	5.71 %	1.18 %
Share of accepted	6.73 %	8.48 %	3.15 %	9.12 %	32.42 %	62.77 %

The key findings from the order depth analyses is that the probability of a bid being accepted at Elbas is low, and there are many more bids that are committed than actually accepted. Further, it can be seen that the relationship between Elbas premium and volume can resemble linear demand curves where the premium and volume are proportional. The probability of a bid being accepted decreases with larger premium and volume. Based on the number of bids that are declined, it seems like there is enough potential for Elbas trading, the question is only how to generate bids that will be accepted. This question will be further studied in Chapter 6. Considering the amount of bids with low premium that is not accepted, it looks like there is also a large potential to answer existing Elbas bids.

Chapter 5

Problem formulation

This chapter presents the description and mathematical formulation of the deterministic equivalent of a multistage stochastic mixed integer problem which maximises profit for a hydro power producer in the post-spot power markets. The post-spot markets are defined as the intraday market, Elbas, and the balancing power market operated by the TSOs. First, a description of the problem is provided. The different markets and the technical constraints of the power plant are described in order to understand the mathematical requirements. To reduce complexity and computational time, some assumptions are presented to simplify the problem. Finally, the mathematical problem is presented and described in detail.

5.1 Optimal post-spot trading for a hydro power producer

After the day-ahead market is closed and the production plan of the hydro power producer is settled, the producer still has the ability to increase profit by trading in the post-spot markets. Because the power demand and actual load in the power system is uncertain, the producer has the possibility to sell or buy additional production in order to maximise total profit. The producer can therefore benefit from imbalances in the market by offering surplus or deficit production. Hence, a strategic aspect for the producer is to evaluate if obligations to the day-ahead market should be delivered by production from own reservoirs, or if it is more profitable to save the water and buy committed production in the post-spot markets. The hydro power producer is only interested in using excess water from the reservoir if the alternative cost of water and the eventual start-up costs are lower than the expected income.

Post-spot bidding is possible for each hour after Elbas opens. The uncertainty in demand, load and prices are assumed to decrease when the bid hour is closer to the production hour. Elbas opens two hours after the day-ahead market closes, at 14:00 the day before production, and bids must be placed at least one hour before the production hour. For the balancing market, bids can be placed or corrected up to 45 minutes before the production hour for the producer to receive a balancing market price according to the one-price mechanism. If there is no balancing event, nor will there be demand for balancing volumes. A decision is made each bid hour for trading on Elbas compared to the expected balancing market demand. The total time horizon for post-spot bidding is considered from Elbas opens until the bid hour before the last production hour. This yields a time horizon from 14:00 the day before production until 22:15 the day of production, resulting in 33 hours of post-spot bidding. An overview of the post-spot bidding structure can be seen in Figure 5.1.

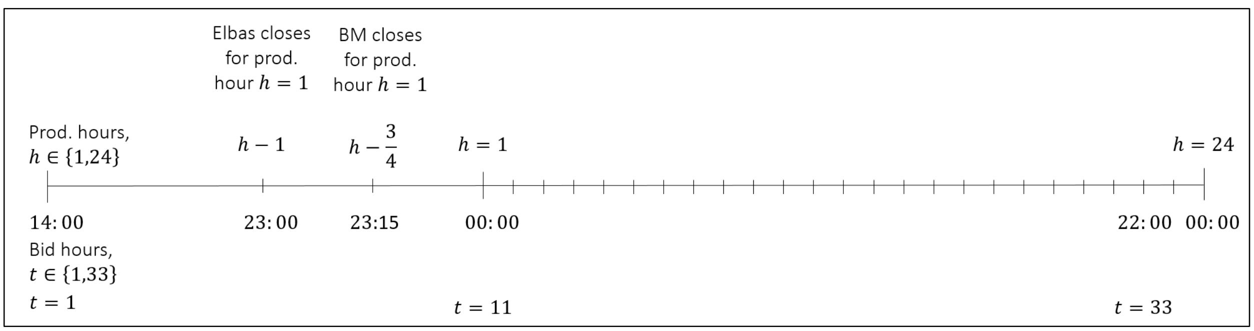


Figure 5.1: Time line of the post-spot bidding problem, for production hour one. Post-spot bidding is possible for all remaining production hours.

In order to trade at the post-spot markets, the producer is depending on existing demand in Elbas and the balancing markets. To decide how much to produce for a given hour, the net committed production to the day-ahead market, Elbas and the balancing market must be covered. The technical limitations of the hydro power plant must be considered to be sure that committed production can be delivered. The generators have upper and lower limits on how much power they can produce, and the amount of power delivered is correlated with the water discharge from the generator's production discharge curve. The reservoir levels are changing with inflow, discharge and spill for every production hour, and the alternative cost of water is defined as the total water value of the volume difference. It can therefore be profitable for the producer to buy power from the post-spot markets in order to avoid using water.

Since Elbas is a relatively new market with low liquidity, the market modelling depends on revealing good correlations and predictions to be able to determine the market's future behaviour. A main issue in this paper is to model these correlations and derive reasonable assumptions to predict the uncertainty in Elbas. To solve the optimisation problem, it is desired to have a linear model to describe which prices and volumes that are accepted when bidding at Elbas. The Elbas bids are modelled as demand curves with proportional Elbas premium and volume. Thus, a larger premium yields a lower volume. The chance of a bid being accepted is depending on the premium and the volume. However, it is also possible that only parts of the volume is accepted for a given bid premium.

The volume sold or bought on the balancing market is dependent on the balancing state and available volume after Elbas bidding. The producer bids available capacity based on marginal cost of production, and the actual delivery is depending on how much the TSO require and the resulting balancing market price. The balancing price is a result of a merit order rating and therefore uncertain. Time series models can be used to forecast the price and describe the uncertainty. The different scenarios are thus depending on the market state: whether the market is upward regulated, downward regulated, or if there is no regulation state defined at all. The balancing price is defined as the spot price with a premium added depending on the market state. For an upward regulated market the premium is positive, whereas for a downward regulated market the premium is negative.

5.2 Assumptions

The mathematical model describes the Norwegian post-spot markets, and the markets are therefore modelled as described in Chapter 2. However, to avoid a high level of complexity and reduce computational time some assumptions are necessary. Assumptions related specifically to the case study is presented in Chapter 7.

Elbas is modelled with demand curves and an acceptance share of placed bids. The available transmission capacity is therefore not modelled separately, but included in the total acceptance share. It is assumed that block bids have no liquidity on Elbas, and only hourly bids are taken into account. Either is the entire submitted bid accepted, or only parts of it is taken depending on the acceptance share. It is possible to buy or sell volumes at Elbas for each bid hour, but it is not possible to buy and sell volumes in the same bid hour. However, it is possible to have both buy and sell commitments at Elbas for the same production hour, as long as the bids are placed for different bid hours.

The producer updates the balancing market bids 45 minutes before the production hour depending on the most recent balancing market price forecast. The state is defined as either upward or downward regulated for one hour, and in hours with no regulation, the balancing market volume is also zero. It is assumed that the balancing volumes are part of a bigger portfolio for the producer, thus the minimum limit of 10 MW is considered covered. The producer is regulated according to the final bid, thus all power bid the hour before operation is delivered in the following hour. It is assumed that the producer avoids imbalances of own production, thus no penalty costs for imbalance production are considered.

It is not possible to speculate in the post-spot markets. For the case where no generators are running, it is not possible to buy commitments on Elbas for the purpose of delivering upward regulated power later. If there is no production in an hour, it is not allowed to bid in the post-spot markets without starting up a generator. This is to avoid situations where commitments in the post-spot markets can not be covered by own production.

Some of the complexity of the hydro power modelling can be simplified without affecting the results notably. The change in head level as well as the travel time between the reservoirs are therefore neglected. The alternative cost of using water is depending on the production level and thus the change in water level. The production level determines the amount of discharge. Because the spot commitments must be included in the production decision, the change in water volume is also depending on spot commitments. Since the alternative cost of water used for the spot commitments is included in the objective, the income of spot is also added to the objective value.

5.3 Mathematical model

The mathematical model describes a multistage stochastic mixed integer problem (SMIP). Figure 5.2 shows the problem structure, where the scenarios represent different realisations of Elbas bids and demand, balancing market prices and production. All decisions are dependent on scenario, thus there is no decision in the root node. The first stage decision is to decide the Elbas bid volume based on a scenario dependent Elbas price. Because the premium, and thus the price, is described by a linear demand curve which is dependent on the bid volume, the demand curve is discretised into segments with a maximum bid volume each. This is to get a linear relationship between the price and volume which is multiplied to get Elbas profit in the mathematical model. Then, the acceptance shares of the different bid segments are revealed, and the volume sold or bought on Elbas is decided based on how much is bid into each segment and their associated acceptance shares. The total accepted bid volume for each scenario is thus the second stage decision by simple recourse.

After the Elbas volumes are decided, the producer has the possibility to bid to the balancing market. This makes the balancing market volume the third stage decision. The bid volume depends on an uncertain balancing market price. After all commitments are settled, the total production is decided. For every bid hour, the scenario structure is repeated. To handle the updated information when bidding closer to the hour of operation, the problem is formulated using a rolling horizon approach. This is described in detail in Section 6.5. Generation of BM-prices and -volumes as well as modelling Elbas prices and -volumes are described in detail in Chapter 6. The complete mathematical model with associated nomenclature can be seen in Appendix B

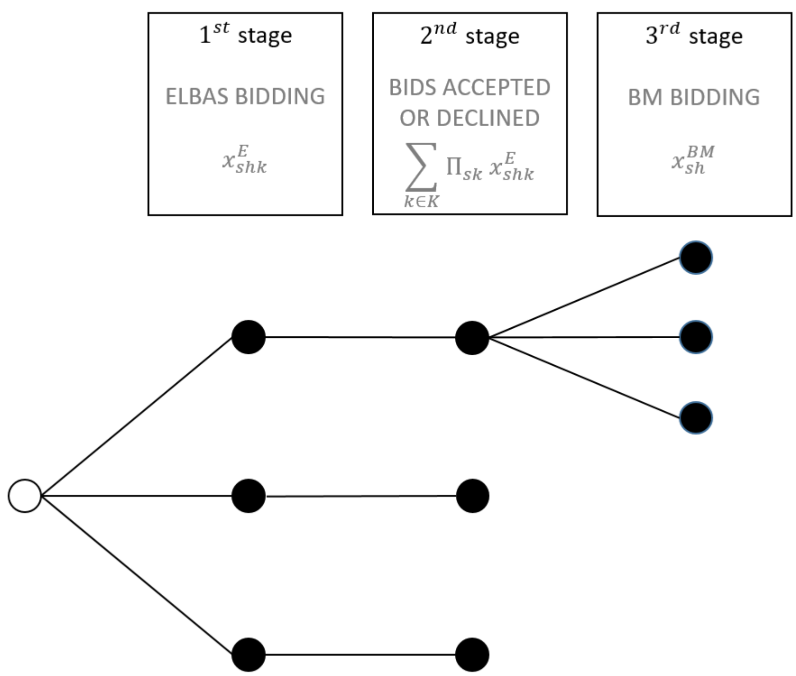


Figure 5.2: Scenario tree illustrating the problem structure and decision variables.

The deterministic equivalent of the objective function of the SMIP given by (5.1) maximises profit for all scenarios $s \in \mathcal{S}$ and production hours $h \in H$. The income from Elbas is given by the Elbas price, ρ^E_{shk}, and the volume sold, $\sum_{k \in \mathcal{K}} \Pi_{sk} x^E_{shk}$, for the volume bid in each segment $k \in \mathcal{K}$. Here, Π_{sk} is the acceptance share. In the balancing market, the profit from selling or buying at the balancing market is the balancing price, ρ^{BM}_{sh}, and the volume, x_{sh}^{BM}. The production cost is given by the change in water value, w_{sw}, and the induced start-up cost, c_{shi}. Because the total use of water for a production hour is included in the objective, the day-ahead profit must also be included.

$$\max \sum_{s\in\mathcal{S}} \sum_{h\in\mathcal{H}} Pr_s [\sum_{k\in\mathcal{K}} \Pi_{sk} \rho_{shk}^{E} x_{shk}^{E} + \rho_{sh}^{BM} x_{sh}^{BM}) \\ - \sum_{w\in\mathcal{W}} (W_w^0 - w_{sw}) - \sum_{h\in\mathcal{H}} \sum_{i\in\mathcal{I}} c_{shi}] + \sum_{h\in\mathcal{H}} P_h^{Spot} X_h \quad (5.1)$$

Every Elbas scenario is described by a demand curve which describes the relationship between the bid volume and bid premium. Because the premium is a function of the volume, the objective value becomes non-linear. To avoid this, the demand curve is discretised into segments which describes fixed volume intervals. The premium can therefore be decided for each segment volume, which is not the same as the bid volume used in the objective. In order to obtain a linear model, the Elbas bid volumes, x_{shk}^{E}, are therefore discretised and bounded by an upper demand volume, E_k^E, for each volume segment, $k \in \mathcal{K}$. This can be seen in (5.2). The segment volume then defines the bid price, ρ_{shk}^{E}, which is calculated according to Section 6.1, where a detailed description of the Elbas modelling is provided.

$$|x_{shk}^{E}| \leq |E_k^E| \qquad s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K} \quad (5.2)$$

The BM-volume is limited by a stochastic volume for each production hour, $h \in \mathcal{H}$, and scenario, $s \in \mathcal{S}$, as shown in (5.3). The volume is limited by the forecasted demand for regulating power and defined based on regulating state. The volume can be both positive and negative, and a detailed description of the generation of volume scenarios is given in Section 6.3.

$$|x_{sh}^{BM}| \leq |E_{sh}^{BM}| \qquad s \in \mathcal{S}, h \in \mathcal{H} \quad (5.3)$$

The delivered obligations committed in the different markets must be covered by the net production, q_{shi}. For buy volumes it is possible to reduce production as long as delivery obligations are fulfilled. The resulting production balance is given by (5.4). The total volumes consist of the volume already committed to the day-ahead market, X_h, including the sold or bought Elbas volumes and the BM-volumes. This includes the Elbas volumes that have been traded in each scenario for previous bid hours, X_{sh}^{E}. Since only a percentage of volumes placed on Elbas is answered due to the acceptance share, a bid constraint is necessary to make sure that the volume bid to Elbas does not exceed the production limitations. Constraint (5.5) makes sure that the sum of Elbas bids for all bid hours, BM-volumes and spot commitments do not exceed the total maximum generation for each generator, Q_i^{max}.

$$\sum_{k\in\mathcal{K}} \Pi_{sk} x^E_{shk} + X^E_{sh} + x^{BM}_{sh} + X_h = \sum_{i\in\mathcal{I}} q_{shi} \qquad s\in\mathcal{S}, h\in\mathcal{H} \tag{5.4}$$

$$\sum_{k\in\mathcal{K}} x^E_{shk} + X^E_{sh} + x^{BM}_{sh} + X_h \leq \sum_{i\in\mathcal{I}} Q_i^{max} \qquad s\in\mathcal{S}, h\in\mathcal{H} \tag{5.5}$$

The net production can not exceed the fabricated capacity limits, and each generator, $i \in \mathcal{I}$, therefore has bounds on maximum and minimum production given by (5.7) and (5.6). The binary variable, u_{shi} is 1 if a generator is running and 0 otherwise.

$$q_{shi} \leq Q_i^{max} u_{shi} \qquad s\in\mathcal{S}, h\in\mathcal{H}, i\in\mathcal{I} \tag{5.6}$$

$$q_{shi} \geq Q_i^{min} u_{shi} \qquad s\in\mathcal{S}, h\in\mathcal{H}, i\in\mathcal{I} \tag{5.7}$$

Change in reservoir level is described by the reservoir constraint given by (5.8). The reservoir volume, v_{shj}, increases with inflow, I_{hj}, and the decrease is determined by the discharge, d_{shi}, and spill, s_{shi}. For cascaded reservoirs, the discharge and spill for up-stream reservoirs affect the reservoirs downstream. Γ_{ij} explains the connection between reservoir $j \in \mathcal{J}$ and generator $i \in \mathcal{I}$. The indicator matrix equals 1 if the generator draws water from the reservoir, -1 if the generator discharges into the reservoir and 0 if there is no connection. $\Lambda_{jj'}$ explains the connection between two reservoirs, and is 1 if reservoir j' spills into reservoir j, -1 if the opposite happens or the spill is out of the system and 0 if there is no connection. The reservoir balance in the first hour is described by (5.9). V^0_{sj} is the initial volume in reservoir $j \in \mathcal{J}$. For the rolling horizon approach, the initial reservoir level is updated for each bid hour and is therefore dependent on scenario. The reservoir level also has upper and lower bounds.

$$v_{shj} - v_{s(h-1)j} = I_{hj} - \sum_{i\in\mathcal{I}} \Gamma_{ij} d_{shi} + \sum_{j'\in\mathcal{J}} \Lambda_{jj'} s_{shj'} \qquad s\in\mathcal{S}, h\in\mathcal{H}\backslash 1, j\in\mathcal{J} \tag{5.8}$$

$$v_{s1j} - V^0_{sj} = I_{1j} - \sum_{i\in\mathcal{I}} \Gamma_{ij} d_{s1i} + \sum_{j'\in\mathcal{J}} \Lambda_{jj'} s_{s1j'} \qquad s\in\mathcal{S}, j\in\mathcal{J} \tag{5.9}$$

The production discharge curve (P-Q curve) and maximum discharge is given by (5.10) and (5.11). For each generator correlation points between volume discharge and power produced are approximated as piece-wise linear functions to keep the problem linear. This curve describes the operational settings of the generator that the net production must follow, in which A_{if} is the constant and B_{if} is the slope of the production function for segment $f \in \mathcal{F}$ and generator $i \in \mathcal{I}$. The P-Q curves are described in detail in Section 7.1.

The maximum discharge, D_i, limits the discharge for each generator, d_{shi}, as described by (5.11).

$$q_{shi} \leq A_{if} u_{shi} + B_{if} d_{shi} \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I}, f \in \mathcal{F} \tag{5.10}$$

$$d_{shi} \leq D_i u_{shi} \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \tag{5.11}$$

Start-up costs, c_{shi}, are included in the objective function when units are started up. If the generator is started from one hour to the next, c_{shi}, is assigned the start-up cost, C_i. Because the generator may be running already due to spot commitments, the original generator state in hour $h \in \mathcal{H}$ is indicated by U^0_{hi}. The start-up cost is given by (5.12) and (5.13) for the first hour.

$$c_{shi} \geq C_i(u_{shi} - \max[u_{s(h-1)i}, U^0_{hi}]) \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \tag{5.12}$$

$$c_{s1i} \geq C_i(u_{s1i} - U^0_{1i}) \qquad s \in \mathcal{S}, i \in \mathcal{I} \tag{5.13}$$

The water value describes the future expected income of a marginal amount of water in the reservoir. For each watercourse, $w \in \mathcal{W}$, the future income, F_{lw}, and the marginal water value, W_{jl}, for each reservoir, $j \in \mathcal{J}$, are calculated for $l \in \mathcal{L}$ different reference reservoir levels, V_{jl}. Each reference level, $l \in \mathcal{L}$, is also known as a cut. End water value in the short-term model is approximated by (5.14). The future income and reference reservoir level are parameters from the seasonal scheduling model and $v_{s|H|j}$ is the reservoir level at the end of the planning period. The producer described in the following case study does not operate with cut files for water values, thus a simplification is done by keeping the water values constant.

$$w_{sw} \leq F_{lw} - \sum_{j \in \mathcal{J}^w} W_{jl}(V_{jl} - v_{s|H|j}) \qquad s \in \mathcal{S}, l \in \mathcal{L}, w \in \mathcal{W} \tag{5.14}$$

In addition, nonanticipativity constraints are necessary to ensure that decisions only are made based on available information (Birge and Louveaux, 2011). The constraints link the equal stage decisions together, and make sure that all decisions made on later stages are based on the same information. Because the problem is formulated as a deterministic equivalent with a finite number of scenarios, it is necessary to include these constraints which can be seen in (5.15).

$$\omega_{sh} = \omega_{\zeta h} \qquad s \in \mathcal{S}_\zeta, h \in \mathcal{H}_\zeta, \zeta \in \mathcal{Z} \tag{5.15}$$

For a nonanticipativity set, $\mathcal{Z}$, a set of information, ζ, exists. For every ζ the same decision must be made. In the above constraint, $\omega_{\zeta h}$ is the decision made with the information

$\zeta \in \mathcal{Z}$ for scenarios $s \in \mathcal{S}_\zeta$ and time period $h \in \mathcal{H}_\zeta$. Thus, for all decisions made in the branching of Figure 5.2, the value of the decision leading to this branching must be equal. The total number of nonanticipativity sets in this problem is therefore two. The first stage nonanticipativity set makes sure that the Elbas bid volumes are fixed in all the first stage nodes and the following branches, and the second stage nonanticipativity set fixes the total accepted Elbas volume in each second stage node and the following branches. This makes sure that the third stage variables in each scenario are decided based on available information for the entire path through the scenario tree. It is therefore ensured that the volumes bid to the balancing market in each scenario are depending on the Elbas bid volumes and accepted Elbas volumes.

5.3.1 Without water value cuts

Since the producer modelled in the case study does not preserve cut files for the water values, the objective function applied in the model is reformulated to a objective with constant water values. Without the cuts the alternative cost for using water is then decided by the constant water value and the amount of water used. The water used during a time period is the difference in reservoir volume between two time periods. The water value is originally stated as [NOK/MWh], however multiplied with an energy equivalent, η_j, and an exchange rate the notation becomes [EUR/m^3]. Because the change in reservoir volume also depends on discharge due to day-ahead commitments, the spot profit is included. With no water value cuts, (5.14) from the formulation above is no longer needed. The new objective function is given by (5.16).

$$\begin{aligned}\max \sum_{s\in\mathcal{S}} Pr_s[\sum_{h\in\mathcal{H}} (\sum_{k\in\mathcal{K}} \Pi_{sk}\rho^E_{shk}x^E_{shk} + \rho^{BM}_{sh}x^{BM}_{sh}) \\ - \sum_{j\in\mathcal{J}} W^0_j \eta_j (\sum_{h\in\mathcal{H}\backslash 1} (v_{s(h-1)j} - v_{shj}) + V^0_{sj} - v_{s1j}) \\ - \sum_{h\in\mathcal{H}}\sum_{i\in\mathcal{I}} c_{shi}] + \sum_{h\in\mathcal{H}} P^{Spot}_h X_h \quad (5.16)\end{aligned}$$

The case study presented in the following chapters contains only one watercourse with two reservoirs. Therefore is the parameter W^0_w, initial water value in watercourse $w \in \mathcal{W}$ changed to W^0_j, initial water value in reservoir $j \in \mathcal{J}$, in the objective. The change in reservoir volume for all hours, including the initial hour, is multiplied with the constant water value. On the basis of constant water values, the mathematical model lack incentives to keep water in the reservoirs. Therefore, additional assumptions and limitations on reservoir parameters are presented in Chapter 7.

Chapter 6

Market modelling and scenario generation

This chapter presents approaches to generate scenarios reflecting the uncertainty in the post-spot markets. First, Elbas is modelled based on historical order depth. The bids are modelled as demand curves described by the relationship between premium and volume, discretised into volume segments to obtain a linear problem. For a given bid volume and premium, acceptance shares are defined to model how much of the bid volume that is accepted. Further, the balancing market is forecasted using time series analysis and balancing market volumes are fitted to generalised extreme value distributions. Algorithms are defined for both markets to generate scenarios before the rolling horizon model is finalised. A complete overview of the scenario generation procedures' interaction with the programming problem can be seen in Figure 6.1. The procedure is repeated for each day in the case study presented in Chapter 7.

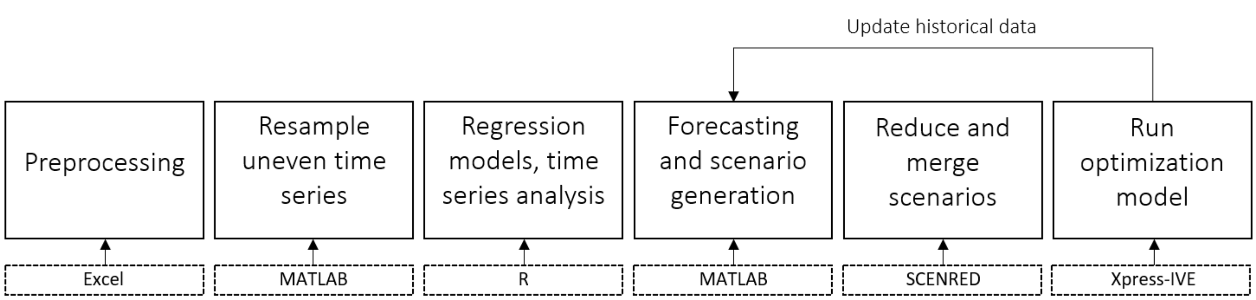


Figure 6.1: Flowchart of the scenario generation procedure.

6.1 Modelling price and volume for Elbas

Elbas is a market with low liquidity and few trades as described in Chapter 4. Previous work presented by Engmark and Sandven (2016) has shown that correlations between potential market drivers and their influence on Elbas are negligible. Hence, it is considered that factors such as consumption imbalance, wind imbalance and trends in the balancing market are insufficient to describe expected behaviour of Elbas. It can thus be assumed that the Elbas market in Norway is often driven by unforeseen events, such as generator outages, maintenance and problems with bilateral contracts, which are difficult to predict. This being said, it can be observed that Germany has a well-functioning and more liquid intraday market, which is influenced by more fundamental market drivers. It is reasonable to believe that the market behaviour will become similar to that of Germany with increased liquidity.

In this paper, an extension of the work done by Engmark and Sandven (2016) is presented. Because of the low market liquidity, using the average Elbas price in a model is not considered sufficient. This is a result of the scarce and varying bid volumes and premiums at Elbas with no identifiable pattern, which is possible due to the continuous market. As described in Chapter 4, the order depth at Elbas has been obtained from Nord Pool. The data contains all NO2 Elbas bids for February 2017, both accepted and declined, and is not publicly available. Thus, the data used for analysis in this paper is more detailed than for previous studies, although the accessible data only represent one month. In this section, linear regression models are fitted to the order depth data to create demand curves. All bids, both accepted and declined, are considered in the process. The demand curves are later used in the scenario generation, where they are associated with acceptance shares for the bids.

6.1.1 Preprocessing of the data

Electricity prices are known to exhibit high volatility with unanticipated price changes known as spikes. However, methods used in regression analysis tend to be very sensitive to extreme observations and outliers. It is therefore recommended to filter the extreme observations prior to fitting of the regression models in order to obtain more appropriate estimates (Trück et al., 2007). From the Elbas premiums shown in Figure 4.6, it can be seen that some premiums are much higher or lower than the rest of the bid premiums. Several methods can be used to detect and define which bids are considered spikes. Weron (2014) mentions recursive filters, variable price thresholds, fixed price change thresholds, regime-switching classification and wavelet filtering. The spike filters can either modify or remove the outliers before calibration of the model parameters, however the spikes are usually replaced by a chosen threshold value, neighbouring values or values for a similar day.

There is no clear definition of a price spike in the literature, although a spike obviously is a value exceeding a certain threshold for a short time period. Trück et al. (2007) mention fixed price thresholds, variable price change thresholds and prediction intervals as methods to identify spikes. Different definitions and techniques may lead to various results, however a common method described by Trück et al. (2007) is to identify spikes as an increase in absolute value more than three times the standard deviation above or below the mean value. Applying this method to the historical Elbas premiums removes all spikes. For the sell bids, 2 % of the data was identified as spikes, whereas 3.9 % of the premiums for the buy bids described extreme values. In this thesis, the Elbas premiums are not analysed as time series and thus the spike premiums were replaced by the value of the threshold. The resulting spike removed data can be seen in Figure 6.2.

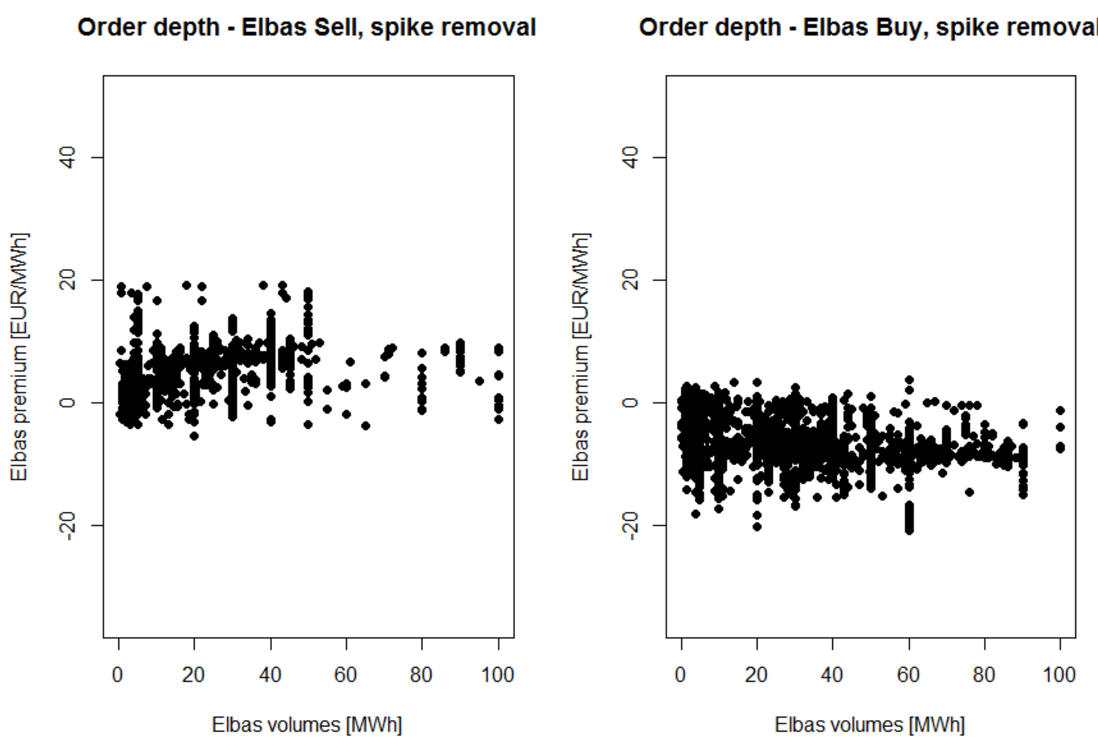


Figure 6.2: Elbas order depth after spike removal in NO2, February 2017.

6.1.2 Data separation

The intention of this thesis is to model the expected premium and volume for Elbas bids. Due to the low market liquidity and the high number of hours where no bids are present, it is concluded that a time series model would result in large amounts of missing values and be a poor basis for modelling. The idea is rather to identify the expected correlation between Elbas premium and volume through demand curves for the historical bids. However, the bid points are clustered and not immediately associated with an appropriate model. Multivariate models have also been tested and considered non-satisfactory. A suggestion in order to obtain data which is easier to fit to regression models, is to separate the data into buckets with certain features before applying regression models.

The idea is to divide the bids into reasonable buckets which can be identified in the scenario generation process. The separated data can then be analysed and modelled individually, before merging the different models into one comprehensive model describing all the bids. There is a trade-off between the level of detail described by an increasing number of buckets, and the increased computational time associated with an increased level of detail. In this thesis, two methods for separating the data are described. Both methods manage to fit regression models sufficiently with a reasonable amount of buckets. The separation is done based on two different approaches of sorting the bids described below.

- **Method 1:** In the first method, the bids are sorted from highest to lowest based on the product of the Elbas premium and volume. The data is then separated into five buckets. Thus the bid points in the highest buckets have the highest product value of associated bid premium and volume, whereas the bid points in the lowest bucket have the correspondingly lowest product value.

- **Method 2:** In the second method, three demand scenarios are defined based on the size of the premium and volume - high, medium and low. A demand scenario is thus defined as $d \subseteq \mathcal{S}$. The high scenario is defined as all bid points lying above the line drawn between the $2/3$ highest premium and $2/3$ highest volume. The medium demand scenario consists of all bid points between the high demand line and the low demand line, which is drawn between the $1/3$ highest premium and $1/3$ highest volume, whereas the bid points building the low demand scenario lies below the low demand line. For the buy bids, the highest level is defined as the lowest premium, and will thus be opposite to the sell bids

An illustration of the separated data with associated trend lines can be seen in Appendix C. To evaluate which of the two separation methods that gives the best representation of the original data, an analysis is performed on the generated regression models described in the next section. Only one separation method is chosen for the final scenario generation.

6.1.3 Linear modelling of the Elbas premium and volume

In order to model demand curves which describe the relationship between Elbas premium and volume for each bucket, a linear regression model is applied. In Equation (6.1) $\hat{y}$ is the predicted Elbas premium, β_0 and β_1 are the intercept and slope parameters respectively, x is the predictor volume and ϵ is the error term.

$$\hat{y} = \beta_0 + \beta_1 x + \epsilon \tag{6.1}$$

For the two data separation methods presented in the previous section, the entire data set of Elbas orders is described by five and three linear regression lines respectively. For each data bucket, a linear regression model is obtained with significant regression parameters. Further, the null hypothesis of the F-statistic is rejected for all regression models, illustrating that there is a statistically reliable relationship between the response variable and the predictor. All regression models yield relatively low values for R^2, but are in this case considered acceptable due to the results of the F-statistics and parameter significance. Only one of the data separation methods is used in the actual scenario generation algorithm, and thus the two methods are compared based on their in-sample forecasting errors. The available data set is considered too small to restrain data points for out-of sample testing. Due to the different number of regression lines for each method, the expected values of the root mean square error (RMS) and mean absolute deviation (MAD) are computed. The RMS represents the sample standard deviation of the residuals, whereas the MAD represents the size of the forecast error in units. The main difference between the two error measures is that the MAD punishes errors linearly in comparison to the RMS which punishes larger errors more. The results can be seen in Table 6.1.

Table 6.1: Comparison of forecast errors for the two data separation methods used for Elbas forecasting.

	Method 1		**Method 2**	
	Sell	Buy	Sell	Buy
MAD [EUR/MWh]	3.05	2.65	1.51	1.41
RMS [EUR/MWh]	1.81	2.25	1.97	1.79

Another condition for a linear regression model is that the residuals are independent, normally distributed random variables with zero mean and constant variance. Analysing the residuals of the different methods, it can be seen that this is only the case for Method 2. Thus, based on the forecast errors and the distribution of the residuals, Method 2 is chosen for modelling the Elbas demand curves. The resulting parameters for the linear regression models have been computed using $\mathcal{R}$ software, and can be seen in Table 6.2. Both the buy and sell trades are modelled, resulting in a total of six regression models. For each regression model, an error term is drawn randomly from the historical residual distribution.

Table 6.2: Regression parameters for the Elbas demand curves.

	High		**Medium**		**Low**	
	Sell	Buy	Sell	Buy	Sell	Buy
β_0	10.2334	-12.7255	5.9847	-8.0483	0.7284	-1.7618
β_1	-0.0457	0.0535	-0.0401	0.0359	-0.0566	0.0352

6.1.4 Modelling the market uncertainties

So far, the data set has not considered the acceptance share of placed bids. As presented in Chapter 4, the majority of Elbas bids are not answered. No probability distribution can be identified to fit the data, and therefore a custom probability distribution is created in order to determine the acceptance share for placed Elbas bids. Because the Elbas premium is a function of the volume, the Elbas profit becomes non-linear. To avoid this, the demand curve is discretised into volume segments, and for each segment a premium can be defined. The volume axis is therefore divided into eleven parts of 10 MWh each ranging from 0 MWh to 110 MWh. This results in 33 different segments which are defined by a demand scenario and a bid volume. The historical probability of a bid being accepted given a demand scenario, $d \subseteq \mathcal{S}$, and a volume segment, $k \in \mathcal{K}$, is defined as the acceptance share given by Π_{dk} shown in the matrices below. The discretised bids based on demand scenario and segment can be seen in Figure 6.3, which also shows the historical accepted bids within each segment.

$$\Pi_{dk}^{Sell} = \begin{bmatrix} 0.36\% & 0.00\% & 0.04\% & 0.00\% & 0.04\% & 0.04\% & 0.00\% & 0.00\% & 0.22\% & 0.00\% & 0.07\% \\ 1.82\% & 0.36\% & 0.55\% & 0.44\% & 0.07\% & 0.04\% & 0.04\% & 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ 3.02\% & 0.62\% & 0.65\% & 0.07\% & 0.04\% & 0.04\% & 0.00\% & 0.00\% & 0.00\% & 0.00\% & 0.00\% \end{bmatrix}$$

$$\Pi_{dk}^{Buy} = \begin{bmatrix} 0.28\% & 0.12\% & 0.07\% & 0.02\% & 0.11\% & 0.53\% & 0.21\% & 0.05\% & 0.11\% & 0.00\% & 0.02\% \\ 3.49\% & 0.72\% & 1.65\% & 0.79\% & 0.40\% & 0.74\% & 0.05\% & 0.32\% & 0.00\% & 0.00\% & 0.04\% \\ 2.41\% & 0.90\% & 0.56\% & 0.28\% & 0.07\% & 0.05\% & 0.00\% & 0.00\% & 0.00\% & 0.00\% & 0.00\% \end{bmatrix}$$

The acceptance share is low in each segment. This is according to the analyses in Chapter 4, and clearly demonstrates the low market liquidity. The size of the volume segments is decided based on the hydro power plant modelled in the case study. The power plant is relatively small, and the available generation capacity is therefore often limited for intraday trading. The small segment volumes thus limits the bid volumes.

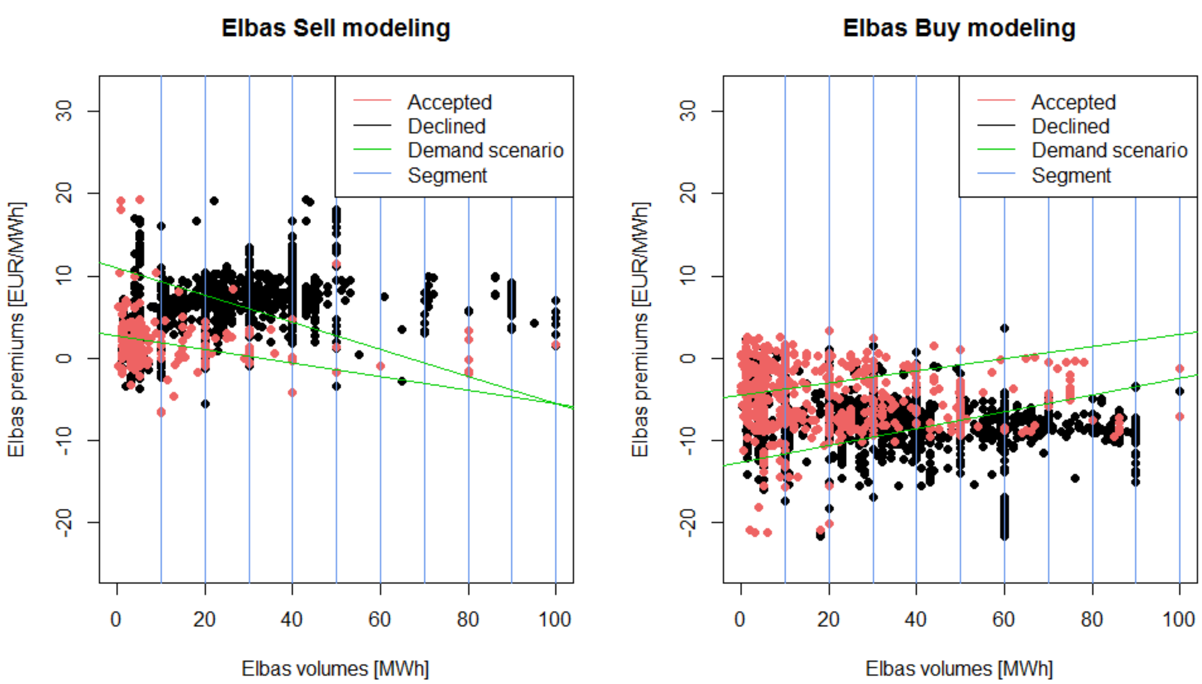


Figure 6.3: Demand scenarios and segments for Elbas discretisation with associated bid status.

The continuous trading at Elbas implies that trades can be settled whenever a market participant accepts a bid from another participant (Scharff and Amelin, 2016). Prices vary from trade to trade, and all participants have the possibility to submit bids for a chosen

Elbas price. However, if a producer exercises market power on Elbas, it is assumed that the entire bid volume is accepted. The Elbas modelling in this thesis intend to describe bids with a volume and premium combination based on historical demand, however the acceptance shares limit the accepted volume. Because the producer modelled has a small market share and is considered a price-taker, the acceptance shares limit the possibility for the producer to exercise market power. It is concluded that the producer has no market power on Elbas.

6.2 Forecasting the balancing market premium

Over the past few years, several studies have focused on forecasting the balancing market. Different variations of time series analysis have been used to forecast the data, like different types of auto regressive (AR) models by Klæboe et al. (2015), a seasonal auto regressive moving average (SARIMA) model by Olsson and Söder (2008) and an auto regressive moving average with exogenous input (ARMAX) model by Boomsma et al. (2014). In this section, the balancing premiums are forecasted using an auto regressive moving average (ARMA) model, based on the work by Engmark and Sandven (2016). The forecasting model is later used in order to generate price scenarios to be used in the stochastic model.

6.2.1 Preprocessing of the data

In this thesis, the balancing market premium is forecasted using time series analysis based on the work done by Engmark and Sandven (2016). The premiums for both BM↑ and BM↓ are subject to large auto-correlations between consecutive hours. The balancing market premium, δ_{mh}, in production hour $h \in H$ and balancing market $m \in M$, where $m = 2$ denotes BM↑ and $m = 3$ describes BM↓, is defined as the difference between the BM-price, ρ_{mh}, and the spot price, ρ_h^{spot}, for upward balancing and the difference between spot price and BM-price for downward balancing.

$$\delta_{mh} = \begin{cases} \rho_{2h} - \rho_h^{spot} & \text{if m = 2} \\ \rho_h^{spot} - \rho_{3h} & \text{if m = 3} \\ \text{Not defined} & \text{Otherwise} \end{cases} \qquad h \in \mathcal{H} \tag{6.2}$$

Before fitting the data to a time series model, some preprocessing is necessary. For example a variance-stabilising transformation is performed in order to reduce the variance of the data. This transformation requires non-negative values, however the data contains zero values when there is no premium. Instead of omitting these hours, something which will result in auto-correlations based on wrong time periods, the missing value problem is solved based on a solution presented by Erdogan et al. (2005). The process used is created for analysing irregularly sampled time series and the goal is to resample irregular time series into regular time series. This is done by using an extension of an AR(1) model, which is an auto regressive model. To resample the time series with missing values, an algorithm is designed which uses an extended AR(1) model to fill in the defined intervals with zero premium. The missing value problem is solved simultaneously as the variance-stabilising transformation. Thus, the logarithmic value of the balancing premiums are computed for the resampled time series. The logarithmic transformation is a special case of the Box-Cox transformation and contributes to normalise the data and stabilise the variance to the extent necessary (Sakia, 1992). Further, the mean value of the logarithmic resampled time series is subtracted from each data point to ensure zero mean of the time series.

6.2.2 ARMA parameters

The data used for creating the time series model is hourly data from the spot market and balancing market in NO2 in the time period 01.01.2016 to 31.12.2016. The data is obtained from Nord Pool, and both prices and volumes are used. The hourly prices for the balancing market and the spot market form the basis for the price forecast, whereas the volumes are used to determine the balancing state. In hours with both upward and downward balancing, the state is defined by the largest balancing volume. This is a reasonable simplification because the hours with both up- and downward regulating volumes count for less than 0.7 % of the hours with balancing volumes. Historical Nord Pool data show that the market is upward regulated in 21.99 % of the hours, downward regulated in 29.08 % of the hours and that there is no regulation in 48.91 % of the hours. The balancing states are then used to define probability matrices for transition between each state for each hour of the day. This is later used to define the Markov process in the scenario generation. The statistical software program $\mathcal{R}$ is used to determine the forecasting parameters.

The regression model is described by an auto regressive polynomial of order p, which can be expressed as follows:

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p \tag{6.3}$$

In addition, the model has a moving average polynomial of order q, which is defined as:

$$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + ... + \theta_q B^q \tag{6.4}$$

In the above equations, B is defined as the back shift operator, $B : B\nu_h = \nu_{h-1}$. This operator is used to produce the previous element in the time series. Having defined all the important elements, the ARMA(p, q) model can finally be defined in Equation (6.5).

$$\phi(B)(\nu^{\mu}_{mh}) = c + \theta(B)\omega_{mh} \tag{6.5}$$

Where ν^{μ}_{mh} and ω_{mh} are the input parameters for each market, thus the transformed and resampled market premium and the market residuals respectively.

The stationarity of the test data was tested using the Augmented Dicky-Fuller test. The null hypothesis to prove that the time series are stationary can not be rejected, and the test thus indicate stationarity. Because no linear trend is present in the evaluated data and the seasonality is not taken into account, the model of choice is an ARMA(p, q) model to forecast balancing premiums in BM↑ and BM↓. The terms of the model, p and q denote the auto regressive terms and the moving average terms respectively. The value of these terms are chosen for each model by analysing the auto-correlation function (ACF) and partial auto-correlation function (PACF) and minimising Akaike's Information Criterion (AIC) as described by Shumway and Stoffer (2006). In addition, the residuals are analysed to ensure white noise. For BM↑, $p = 1$ and $q = 2$ are chosen and for BM ↓, $p = 2$ and $q = 3$ are chosen. For the models, the ACF and PACF for the residuals are analysed graphically to conclude that they are white noise. This is also supported by the Box-Ljung test, which is used to test if the residuals are independently distributed and not auto-correlated (Guthrie, 2013). The test concludes that the models are within acceptable limits (Olsson and Söder, 2008).

Table 6.3: Model parameters for the log-transformed and resampled BM-premiums.

	c	μ^{ν}_{m}	ϕ_1	ϕ_2	θ_1	θ_2	θ_3	MSE
δ_{2h}	-0.0148	1.1467	0.9532	NA	0.0236	0.0405	NA	0.0911
δ_{3h}	-0.0003	1.4195	1.8762	-0.8777	-1.0093	0.0221	0.0188	0.1227

6.2.3 Evaluation of ARMA model compared to historical data

To evaluate the performance of the forecasting model, it is important to use genuine forecasts. Thus, it is invalid to look at how well the model fits the historical data or the training set used to create the model. The accuracy can only be determined by evaluating how well the model performs on test data that has not been used previously. Thus, the distributions and forecasting errors of the model are compared to data not used to create the model. This is an out-of-sample test of the model fit. The model is also compared to a naive forecasting model which uses the BM-premium two hours earlier as the hourly forecast.

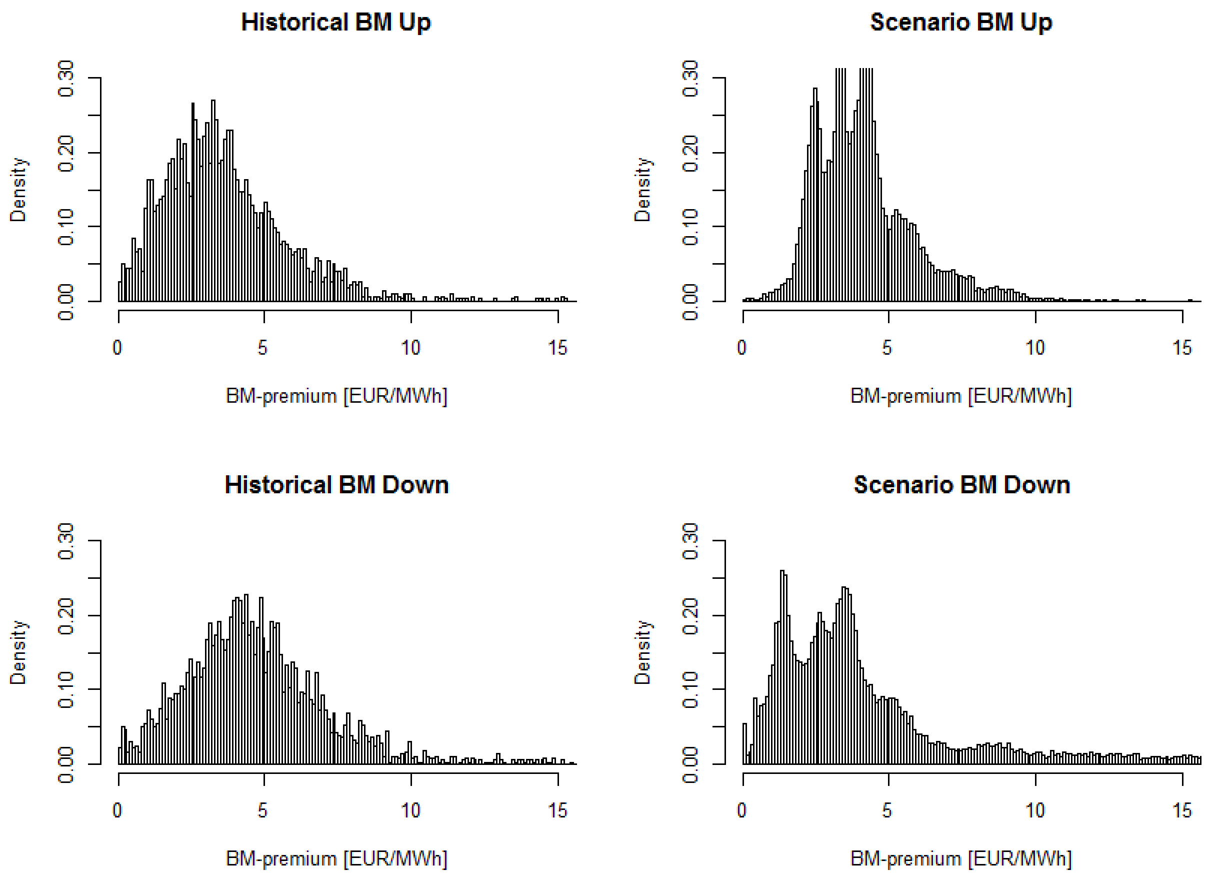


Figure 6.4: Distributions of BM-premiums for the historical data and test data

Looking at the distributions of the historical BM-premiums and the scenario generated data in Figure 6.4, it can be seen that the distributions look similar. The plots of the cumulative distribution functions in Figure 6.5 also undermines this statement. The data used is the hourly BM-premiums for 2016 as historical data, and this is compared to accumulated forecasts for the first week of 2017. The forecasts are accumulated for 50 different simulations in order to get a similar number of data points compared to the training data.

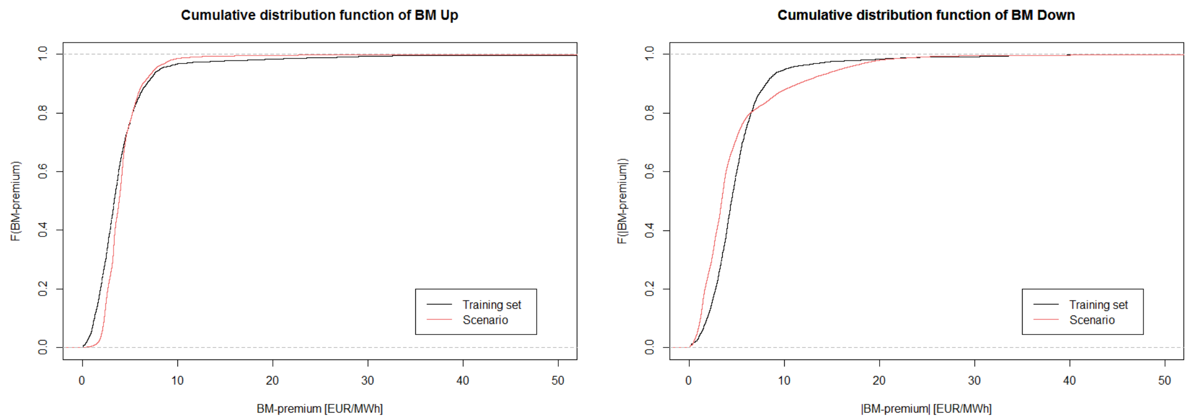


Figure 6.5: Cumulative distribution function of historical data and test data for BM↑ and BM↓. The absolute value of the BM↓-premiums is presented.

To evaluate the forecast accuracy, a naive model is used in comparison with the ARMA models. The naive model forecasts the hourly value by setting it equal to the historical premium two hours earlier. The MAD is calculated to measure the fit of the forecast, or in other words the size of the forecast error in units. Another model benchmark is the bias, which is simply the arithmetic mean of the errors. Depending on the sign and size of the mean, one can tell if the forecast is on average too low or too high, but one would of course prefer it to be zero. The comparison between the ARMA model and the naive model can be seen in Table 6.4. The out-of-sample tests have been performed on data from 01.05.15 - 30.06.15 and 01.01.17 - 28.02.17.

Table 6.4: Comparison of the ARMA-model and a naive model for the out-of-sample test data.

	2015		**2017**	
	ARMA	Naive	ARMA	Naive
MAD [EUR/MWh]	5.62	3.25	3.86	1.64
Std. deviation [EUR/MWh]	5.25	5.80	11.32	3.55
Bias [EUR/MWh]	0.31	-0.01	-0.35	-0.37

It can be seen that the naive model performs better than the time series model when the chosen model benchmarks are used. Compared to the historical data, the time series models are more volatile and have larger outliers, something which can influence the performance measures. Klæboe et al. (2015) have shown that point-forecasting balancing premiums is a difficult task due to the volatile data, and this is also the case for this forecast. A suggested method to stabilise the data is to remove outliers before fitting the models. However, the purpose of this forecast is to generate scenarios describing the uncertainty in the balancing market to use in a stochastic optimisation problem, and it is therefore more reasonable to look at the spread of the generated scenarios and how it captures the market uncertainty rather than the accuracy of point forecasts. It is therefore assumed that the ARMA models are suitable for generating scenarios, as long as the resulting scenario trees are stable and describe the market uncertainty and distribution. This is supported by Figure 6.6, which shows that most of the historical premiums are captured by the 25/75 percentile.

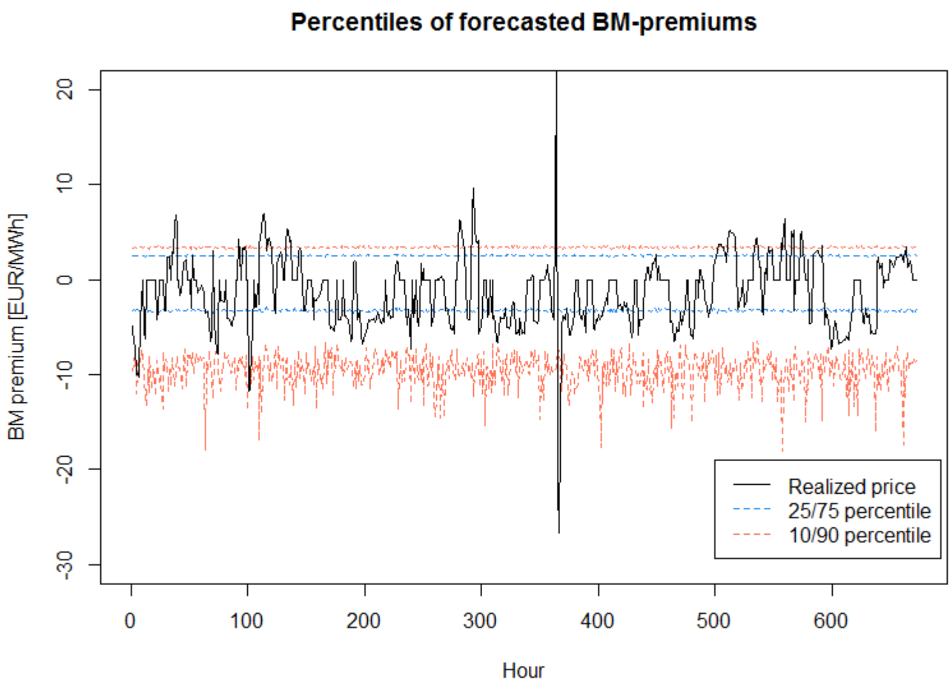


Figure 6.6: Resulting scenarios for balancing market premiums 01.01.17 - 01.02.17.

6.3 Forecasting of the balancing market volume

Klæboe et al. (2015) present a benchmark study of four different models for predicting the balancing volume. Similar to the balancing premiums, it is concluded that the volumes are random and difficult to forecast. In this thesis, the forecasted balancing volume serves as an upper demand limit in the market. It is therefore assumed that the forecasting model presented by Jaehnert et al. (2009) is sufficient to describe the distribution of regulating volumes. As in the article, a generalised extreme value (GEV) distribution is revealed to best describe the historical distribution of the volumes seen in Figure 6.8. Although Klæboe et al. (2015) conclude that a SARMA-model performs best for balancing volume forecasts, the purpose in this thesis is to describe the balancing market uncertainty for small total volumes, and therefore a less accurate model is accepted. The balancing volumes are considered small due to the size of the hydro power plant modelled in the case study.

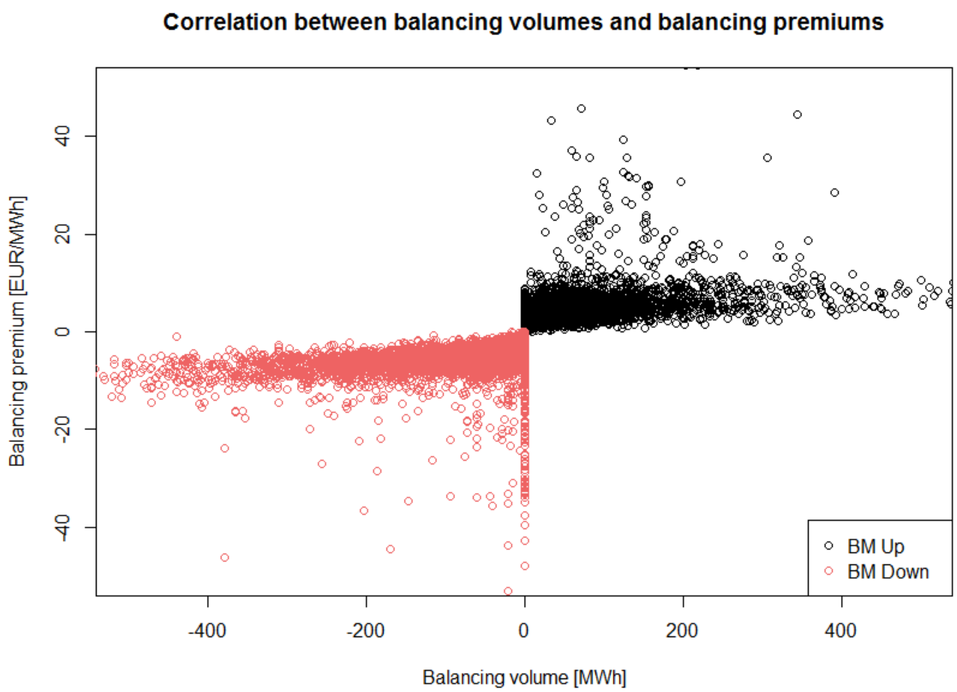


Figure 6.7: Correlation between BM-premiums and BM-volumes in NO2, 01.01.15 - 31.12.16.

Except for the balancing market state, the correlation between balancing premiums and balancing volumes are not considered any further. It can be seen that the correlation has decreased from 0.78 in the time period 2003 - 2007 as presented by Jaehnert et al. (2009) to 0.49 for the data from 2015 - 2016 shown in Figure 6.7. Further, Olsson and Söder (2008) show that the percentiles of the balancing price depending on different balancing volume scenarios have no considerable difference. Combined with the observation that the probability for very high balancing volumes is low, as can be seen in Figure 6.8, and that the balancing premium difference is small when the balancing volumes are small, as can be seen in Figure 6.7, it is concluded that the impact of the balancing volume on the balancing premium can be neglected. Thus, the balancing volumes are drawn independently of state or balancing premium from two separate generalised extreme value distributions.

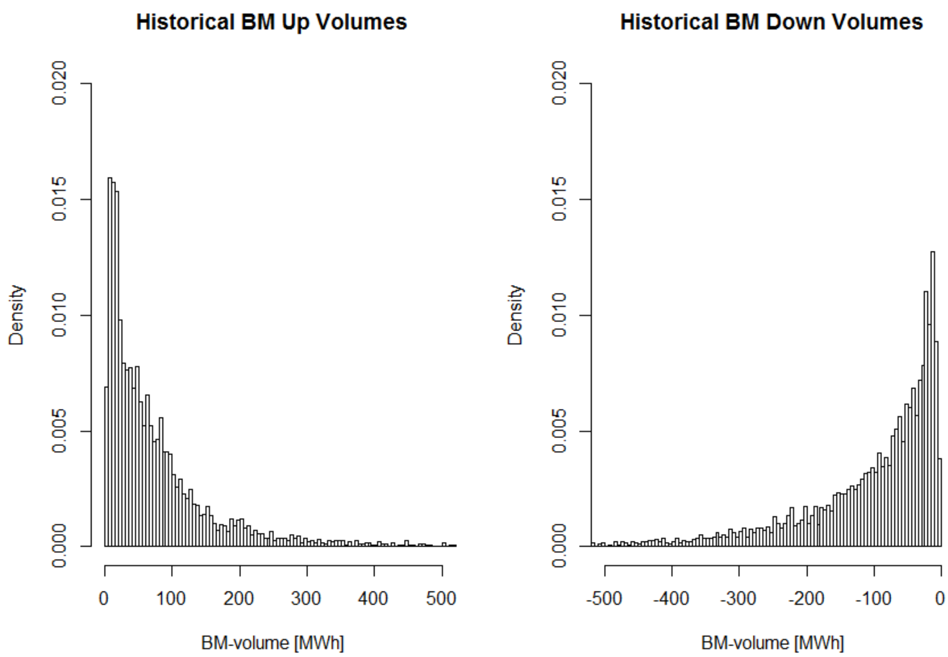


Figure 6.8: Distributions of the historical BM-volumes in NO2.

6.4 Scenario generation and resulting scenario tree

The forecasting models are being used to generate scenarios for the BM-premiums, Elbas demand curves and limits for the BM-volumes. The scenarios are generated using the numerical programming language Matlab®. The two markets are considered independent of each other, thus scenarios are generated for each market separately before they are merged into one large scenario tree. $|\mathcal{S}^E|$ scenarios are generated for Elbas and $|\mathcal{S}^{BM}|$ are generated for BM. Different numbers of Elbas scenarios and BM-scenarios are combined and the stability of the scenario tree is tested for the mathematical model in Section 8.2.

6.4.1 Generation of Elbas scenarios

Each Elbas scenario contains a demand curve describing the relationship between Elbas premium and Elbas volume as described in 6.1.3. Because the regression is not depending on hour of the day or have any auto regressive terms, it is assumed that the Elbas scenarios can be computed once for each production hour. This is unlike the BM-scenarios where information is updated closer to the hour of production. The Elbas scenarios are therefore computed once for each scenario and each hour of operation.

The slope, β_{1dh}, of the demand curve depends on the demand scenario $d \subseteq \mathcal{S}^E$. The total number of scenarios within each demand scenario is decided based on historical percentage occurrence within each segment. The intercept, β_{0dh}, will however be different for each scenario and vary with the residual term, ϵ_{dh}. Since it is possible to place bids for a production hour in several bid hours, the total Elbas volume delivered in a production hour is the sum of all Elbas volumes sold or bought in previous bid hours. This is handled by the rolling horizon framework described later in this chapter. Further, the discretisation of the bid volumes described in Section 6.1.4 gives the subscript $k \in \mathcal{K}$. For the high demand scenario, $d \in \mathcal{S}^H$, medium demand scenario, $d \in \mathcal{S}^M$, and low demand scenario, $d \in \mathcal{S}^L$, the regression line in Equation (6.6) is computed. The sum of all demand scenarios equal the total number of Elbas scenarios. Algorithm 1 is used to create the Elbas scenarios.

$$\rho^E_{dhk} = \beta_{0dh} + \beta_{1dh} x^E_{dhk} + \epsilon_{sh} \qquad s \in \mathcal{S}^E, d \subseteq \mathcal{S}^E, h \in \mathcal{H}, k \in \mathcal{K} \tag{6.6}$$

Algorithm 1: Elbas scenario generation algorithm

Result: Demand curves for each Elbas scenario
Data: Regression parameters, total hours $|\mathcal{H}|$, total number of each demand scenario $|\mathcal{S}^H|$, $|\mathcal{S}^M|$, $|\mathcal{S}^L|$
for $h \in \mathcal{H}$ **do**
  for $d \in \mathcal{S}^H$ **do**
    Initialize regression parameters from Table 6.2;
    Set $\beta_{0dh} := \beta_{0d}$;
    Draw residual, ϵ_{dh}, randomly from historical residuals;
    Set $\beta_{1dh} := \beta_{0d} + \epsilon_{dh}$;
  for $d \in \mathcal{S}^M$ **do**
    Initialize regression parameters from Table 6.2;
    Set $\beta_{0dh} := \beta_{0d}$;
    Draw residual, ϵ_{dh}, randomly from historical residuals;
    Set $\beta_{1dh} := \beta_{0d} + \epsilon_{dh}$;
  for $d \in \mathcal{S}^L$ **do**
    Initialize regression parameters from Table 6.2;
    Set $\beta_{0dh} := \beta_{0d}$;
    Draw residual, ϵ_{dh}, randomly from historical residuals;
    Set $\beta_{1dh} := \beta_{0d} + \epsilon_{dh}$;

6.4.2 Generation of BM-scenarios

The BM-scenarios are generated based on the ARMA forecasting model and the GEV-distributions. For each scenario $b \subseteq \mathcal{S}^{BM}$ and each hour, a balancing state and a premium is defined. The balancing state decides the market regulation, which determines if the BM-volumes have a positive upper bound for upward regulation or a negative lower bound for downward regulation. The bounds are thus represented by E^{BM}_{msh} for $m \in \{2,3\}$. Because the power plant modelled in the case study has low installed capacity compared to the associated bidding area, the volumes are multiplied with a factor of 0.1 to represent a more realistic demand. Due to the auto regressive and moving average terms in the model, the forecast uncertainty decreases closer to the hour of operation. It is therefore necessary to update the BM-scenarios in each hour of operation for each bid hour. This is because new information is available when the bid hour is closer to the hour of operation. However, BM-trading is only considered realised in the following hour and the premium and volumes are therefore not depending on bid hour. For each production hour, $h \in \mathcal{H}$, and each BM-scenario, $b \in \mathcal{S}^{BM}$, the transformed BM-premium, ν^{μ}_{mbh}, is calculated using Equation (6.7) and Equation (6.8).

$$\nu^{\mu}_{2bh} = \phi^{\nu 2}_1 \nu^{\mu}_{2b(h-1)} + \theta^{\nu 2}_1 \omega_{2b(h-1)} + \theta^{\nu 2}_2 \omega_{2b(h-2)} + c_2 \qquad h \in \mathcal{H}, b \in \mathcal{S}^{BM} \tag{6.7}$$

$$\begin{aligned}\nu^{\mu}_{3bh} = \phi^{\nu 3}_1 \nu^{\mu}_{3b(h-1)} + \phi^{\nu 3}_2 \nu^{\mu}_{3b(h-2)} + \theta^{\nu 3}_1 \omega_{3b(h-1)} \\ + \theta^{\nu 3}_2 \omega_{3b(h-2)} + \theta^{\nu 3}_3 \omega_{3b(h-3)} + c_3 \qquad h \in \mathcal{H}, b \in \mathcal{S}^{BM}\end{aligned} \tag{6.8}$$

The premiums are transformed back to ordinary value by using Equation (6.9). Here, the residuals ω_{msh} are drawn randomly from the time series' residuals, also known as the white noise, and $\frac{MSE}{2}$ is added in order to create unbiased scenarios. MSE is the mean square error of the residuals (Newman, 1993). Further, to ensure that the properties of the premiums are as shown in Equation (6.2), the premiums for BM↓ are defined to be negative.

$$\delta_{mbh} = \pm exp(\nu^{\mu}_{mbh} + \mu^{\nu}_m + \omega_{mbh} + \frac{MSE}{2}) \qquad m \in \mathcal{M}, h \in \mathcal{H}, b \in \mathcal{S}^{BM} \tag{6.9}$$

In order to ensure that the uncertainty of the balancing market is properly represented, a large set of scenarios is computed to begin with. In this thesis, the number is set to 3000 for the initial scenario generation. However, there is a trade-off between the number of scenarios and computational time, thus the scenarios are further reduced to $|\mathcal{S}^{BM}|$ scenarios. Scenario reduction is executed in SCENRED, which uses a scenario reduction algorithm to determine a scenario subset and assign optimal probabilities to the preserved scenarios. The library in SCENRED contains two algorithms to reduce the scenarios and control the reduction and returns a reduced scenario set with associated probabilities to implement in further modelling. Algorithm 2 is used to create the scenario tree for one bid hour.

Algorithm 2: Balancing market scenario generation algorithm

```
Result: Premiums and upper/lower volume limit for each BM-scenario
Data: Total number of original BM-scenarios, total number of BM-scenarios |S^BM|,
      total hours |H|, case data for BM
for h ∈ H do
    for b ∈ 3000 do
        Determine the BM-state ẽ_bh for the actual hour using the transition
          probabilities in Appendix D;
        for m ∈ {2,3} do
            Calculate the BM-premiums using Equation (6.7), (6.8) and (6.9);
            if ẽ_bh = m;
             then
                Set δ_mbh equal to calculated BM-premium;
                Draw the volume limit E^BM_mbh from the GEV-distributions;
            else
                Set δ_mbh:= 0;
                Set E^BM_mbh := 0;
    Reduce the number of BM-scenarios to |S^BM| using SCENRED;
```

6.5 Implementation of the rolling horizon approach

Due to the lag in price discovery, where the realised BM-prices are not known before the hour after operation, there will never be perfect information about the balancing market when bidding in the post-spot markets. The balancing market forecasting begins when the model starts to run as Elbas opens at 14:00 the day before operation. The idea is that the closer to the hour of operation, the less uncertain is the balancing market scenarios. The Elbas demand curves have however been modelled independent of hour of operation and are therefore possible to generate further apart from the production hour. Since there is no significant correlation between Elbas and the balancing market as described by Engmark and Sandven (2016), these two markets are modelled independently. Whereas the Elbas scenarios can be generated before the day of operation, the scenarios of the balancing market premiums should be updated as soon as new information about realised prices is revealed. Thus, the optimisation model runs using an internal rolling horizon approach. The term internal rolling horizon refers to a problem with a fixed horizon, but where more information about the previous hours is fixed as the problem is solved. An example of this can be seen in Figure 6.9. Here, it is illustrated how the Elbas scenarios are generated for the entire problem horizon to begin with, whereas the balancing market scenarios are generated for each hour.

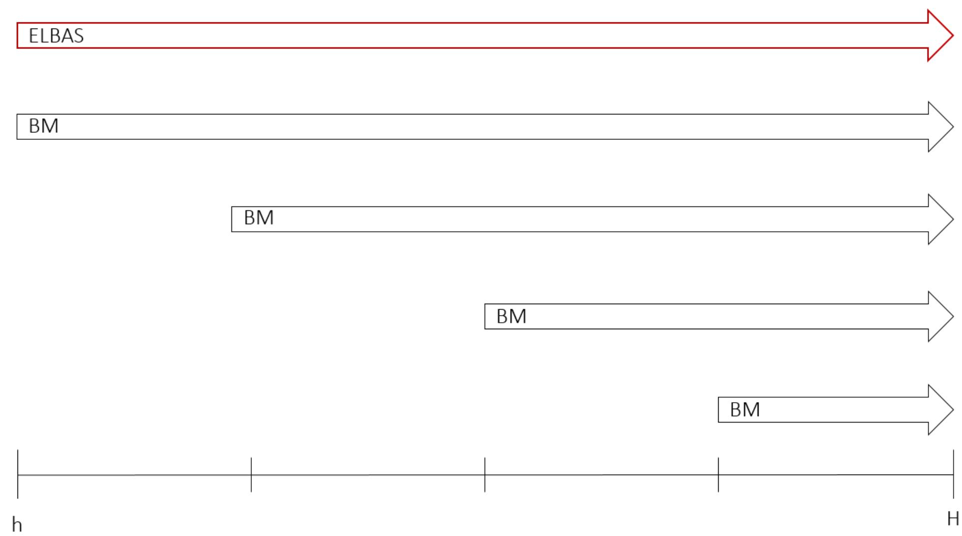


Figure 6.9: Illustration of the internal rolling horizon approach. Elbas scenarios are generated once, but BM-scenarios are updated for each model run.

The total scenario tree is thus updated for each bid hour. The Elbas scenarios are kept constant except the time horizon which is shortened for each model run. The balancing market scenarios are updated with new information for each bid hour, and the total scenario tree is thus decreasing. For a given number of Elbas scenarios, $|\mathcal{S}^E|$, a given number of BM-scenarios, $|\mathcal{S}^{BM}|$ is generated. The total number of scenarios in the scenario tree for each bid hour is therefore given by Equation (6.10).

$$|\mathcal{S}| = |\mathcal{S}^E||\mathcal{S}^{BM}| \tag{6.10}$$

Because the model is run for 33 bid hours starting the day before production, the only new information the first ten hours is the updated information about balancing market premium. After the production begins further information about production and water level must be updated. The rolling horizon model must thus update the time horizon which is decreasing with one hour for each run after the first ten hours has passed. Initial water level is updated based on the production and discharge in the previous hour. Because it is possible to submit several Elbas bids for the same hour of production, but for different bid hours, the total volume sold or bought at Elbas for a production hour must be updated for each model run. Further, the generator status is updated to keep track of eventual generators starting up. The total mathematical problem for an entire day of Elbas bidding is described by Constraints (5.2)-(5.15) and the objective presented by (5.16) in Chapter 5 which is solved for a total of 33 bid hours with updated input data.

Algorithm 3: Complete rolling horizon model

```
for h ∈ H do
    Generate Elbas scenarios using Algorithm 1;
for i ∈ H^b do
    Generate BM-scenarios using Algorithm 2;
    Merge Elbas and BM-scenarios into one scenario tree;
    for h ∈ H do
        Update dynamic input files (BM-scenarios, initial reservoir, generator status,
          total Elbas volumes);
        if |H| < 24 then
            Update time horizon of Elbas scenarios;
            Update all input files to match time horizon;
        Run optimisation model;
        Update BM-premium and Elbas bid volumes;
```

Chapter 7

Case description

The case study presented in this thesis is a short term bidding problem for a hydro power producer. The purpose is to decide the most profitable trade-off between trading post-spot volumes on Elbas or placing bids for trading in the balancing market. In this case study, a watercourse owned by Norsk Hydro ASA located in bidding area NO2 is modelled. This chapter gives a presentation of the case which is later used for a computational study of the model described in Chapter 5. Data considering hydro power production and technical parameters of the generators and reservoirs are received from Hydro ASA and Powel AS. Market data containing hourly prices and volumes from the day-ahead market, Elbas and the balancing market are obtained from Nord Pool. In order to display seasonal variations in planning, the first Wednesday of each month in 2016 is chosen for modelling.

7.1 Case description

The watercourse in this case study consists of two reservoirs in cascade, each connected to a power station. The watercourse is illustrated in Figure 7.1, where the real life watercourse is presented to the left. A simplification of the watercourse is presented to the right in the figure. To reduce complexity, the reservoirs are simplified as described in the following section.

The upper reservoir, denoted $j = 1$, is large and thus has more flexibility than the lower reservoir, denoted $j = 2$. The lower volume of the second reservoir implies a greater risk of spilling than for the upper reservoir. The upper reservoir is simplified in the model and is an aggregation of the three upper reservoirs as shown to the left in Figure 7.1. The technical data in the simplified model for the upper reservoir is calculated as the volume weighted average of the individual technical data in the three upper reservoirs for the original watercourse. This is done to decrease the number of reservoirs included in the mathematical model. The simplification is justified because the difference in the marginal water values for the three separated reservoirs is small. For the lower reservoir, an aggregated reservoir is computed which includes a small intake reservoir.

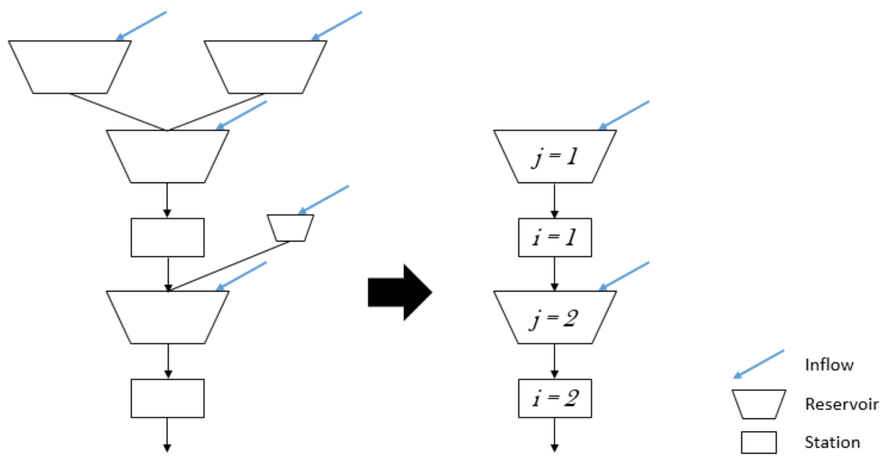


Figure 7.1: Illustration of the original (left) and simplified (right) watercourse.

As described in Chapter 5, Hydro ASA does not have available data for water value cuts in 2016 and a constant water value for each day is therefore implemented. The constant water value does not change with reservoir level, and thus some additional restrictions has to be applied to spill and discharge volumes in order to avoid large unnecessary use of water. However, because the change in reservoir level for each day usually is low, especially for the large reservoir, it is considered acceptable to use constant water values. In the watercourse, the water value for the aggregated upper reservoir is derived as the volume weighted average of the water value for the three individual aggregated reservoirs. Table 7.1 and Table 7.2 show a technical overview of the watercourse for the generators and reservoirs in respective order.

Because the case study describes the first Wednesday of every month in 2016, the reservoir and water values vary depending on seasonal effects like snow melting and change in temperature and rainfall. The highest inflow can be found in the summer months June, July and August, however December also has a lot of inflow. In June, the high initial reservoir level in the lower reservoir combined with large inflow results in a water value of zero.

Table 7.1: Overview of the technical parameters for the generators in the watercourse.

Generator	P_{min} [MW]	P_{max} [MW]	Q_{max} [m^3/h]
G1 Upper reservoir ($i = 1$)	15	45	57600
G1 Lower Reservoir ($i = 2$)	29	76	57600
G2 Lower Reservoir ($i = 3$)	29	80	57600

Table 7.2: Overview of technical parameters for the reservoirs the first Wednesday in every month in 2016.

	Reservoir $j = 1$			Reservoir $j = 2$		
	Initial water volume V_{hj} [Mm^3]	Average inflow I_{hj} [m^3/h]	Constant water value W_j [NOK/MWh]	Initial water volume V_{hj} [Mm^3]	Average inflow I_{hj} [m^3/h]	Constant water value W_j [NOK/MWh]
06.01.16	163.1	2759.0	111.5	0.92	3175.8	175.5
03.02.16	137.0	7148.8	89.5	0.82	7048.1	167.2
02.03.16	112.4	4188.0	70.3	0.81	1289.9	160.5
06.04.16	77.7	3124.9	58.2	1.14	19777.1	182.6
04.05.16	45.1	1456.7	40.5	1.13	9538.4	202.5
01.06.16	67.0	70251.3	61.9	1.72	122960.0	0
06.07.16	127.6	16906.2	138.6	0.93	20162.8	156.1
03.08.16	134.7	18937.2	157.8	0.92	26639.0	178.2
07.09.16	137.5	10884.4	161.4	0.92	16146.4	190.8
05.10.16	142.8	7260.6	214.8	0.42	11512.4	241.9
02.11.16	132.3	9457.7	286.4	1.33	11546.7	347.3
07.12.16	113.6	16083.0	173.7	0.21	65808.3	257.0

An important aspect in Table 7.1 is the relationship between the power output and discharge of the power stations. The relationship between power production from the power station and discharge through the turbine is the operating conditions. This correlation is plotted as a production discharge curve (P-Q curve). These curves, one for each generator, are non-linear and need to be linearly approximated in order to be included in the mathematical problem described in Chapter 5. Linear approximations of the three generators are presented in Figure 7.2. Here, the linear approximations are defined as piece-wise linear curves with three production segments for each generator describing the operating conditions. Each generator is modelled independently and the situation where generators run simultaneously is not considered.

The three production segments are evaluated to give an accurate description, but at the same time avoid an unnecessary increase of the computational time of the model. This is a trade-off between modelling accuracy and the fact that more production segments will increase the number of constraints and problem complexity. The production segments are chosen such that one intersects the best point, one intersects the point of maximum production and one point intersects in the middle of the other two points. The result is that the approximation have a corner point near the optimal production point. This is beneficial because optimisation problems have optimal solutions in the corner points of the convex solution space. Binary variables increase the run time of the bidding problem and are thus preferably avoided. Therefore, the linear approximation intersects in the origin. A result of this, is that the approximation is worse close to minimum discharge. It is however considered that the value of a shorter solution time is high compared to the added value of a more accurate approximation of the lower discharge points.

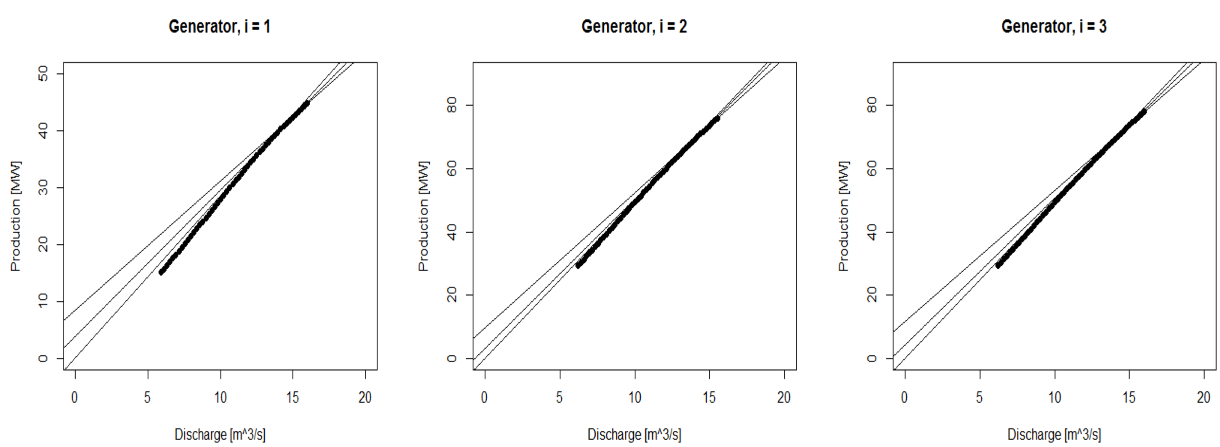


Figure 7.2: Piece-wise linear approximation of P-Q curves for each generator.

The available volumes for post-spot trading and the expected price is dependent on the day-ahead commitments. The different dates have varying spot prices and volume commitments, which will result in different demand for using the post-spot markets. Although the spot commitments typically vary throughout the day, the daily trend can be seen in the average value of the spot price and volume presented in Table 7.3.

Table 7.3: Average spot commitments for the first Wednesday in every month in 2016.

Date	Avg. spot volume [MWh/h]	Avg. spot price [EUR/MWh]
06.01.16	130.43	25.71
03.02.16	119.26	17.69
02.03.16	106.40	21.84
06.04.16	156.85	20.87
04.05.16	161.13	25.34
01.06.16	148.00	22.66
06.07.16	176.17	20.05
03.08.16	166.13	22.98
07.09.16	156.31	22.78
05.10.16	52.14	26.18
02.11.16	74.02	37.05
07.12.16	137.75	31.48

It can be seen that for April - September, the average committed volumes to spot are getting close to the maximum available generation of the power plant. Further, the average spot price is typically higher for October - January, where May is an exception from the other spring/summer months with higher average price. The profitability of post-spot trading may be influenced by the different spot commitments, and this is analysed further in Chapter 8.

7.1.1 Operational assumptions for each day

Because the water value is constant for each day, certain restrictions and assumptions are necessary to obtain feasible solutions of the optimisation problem. With a constant water value, the model has no incentive to keep water in the upstream reservoir when there is a higher water value in the downstream reservoir. Because a cost is associated with the water value multiplied with the change in reservoir level, the model will typically spill as much as possible from the higher reservoir to the lower reservoir. The positive change in the lower reservoir results in a larger income than the cost of using water in the upper reservoir. Due to different inflow, spot commitments and production for the different days, a custom fitting of parameters such as minimum reservoir volume has been performed. A specified reservoir limit at the end of the day is also implemented to avoid emptying the reservoirs. In addition, the possibility to spill is limited in cases where this seems appropriate.

The low Elbas liquidity and the limited BM-volumes result in low additional production to the day-ahead volumes. It is therefore reasonable to believe that the majority of spill planning and reservoir control is related to seasonal- and day-ahead planning, and it is therefore not considered in this problem. For the dates in January, February, March, April, May, July, August and September it is possible to produce without spilling. Since it is not preferred to increase the objective value just by spilling into the lower reservoir with higher water value, the spill is restrained to zero.

In June the lower reservoir has a risk of flooding. This is because the reservoir is full and the maximum discharge is less than the inflow. To avoid overflow in the lower reservoir the model allows spill from the lower reservoir. Only the water that will cause the reservoir to overflow is allowed to spill.

The water level in the downstream reservoir is low in October. In order to have enough water in the downstream reservoir to keep spot commitments and have the ability to trade in the post-spot markets, it is necessary to spill from the upstream reservoir. This spill is limited by the volume difference between the inflow and discharge from the lower reservoir. Thus, the upper reservoir is allowed to spill enough to avoid that the lower reservoir level decreases.

In November there are no generators running the first seven hours due to no spot commitments. In these hours the model does not allow trading between the post-spot markets if the generators are not running. Therefore, the volume bid to the post-spot markets must be above minimum production in order to force generator start-ups. This is done to avoid Elbas commitments based on uncertain balancing market scenarios and avoid market speculation without having the security of production. Further, spill is not allowed due to normal inflow and reservoir levels. The inflow increases in December, and the model allows to spill the exceed volume from the downstream reservoir.

Chapter 8

Computational study

This chapter presents the computational results of solving the multistage SMIP. First, an introduction is given to the hardware and software used to solve the problem. Then a brief overview of the problem size and computational time is given, in order to understand the limitations of the mathematical program. The stability of the scenario tree is then tested in order to decide the number of scenarios to implement. Based on in-sample and out-of-sample testing of the scenario tree, the number of scenarios which result in stable solutions and at the same time reduce the problem size is chosen. Finally, the computational results of applying the SMIP to the case study is presented. The value of post-spot trading is quantified for a hydro power producer in NO2.

The multi-stage model of the deterministic equivalent presented in Chapter 5 is implemented with Mosel modelling language and solved with Xpress-IVE Version 1.24.08. Tests are run on a 64-bit Windows 10 PC with 3.40 GHz Intel® Core™ i7-6700 CPUs and 32 GB RAM. The rolling horizon model is run from Matlab®, which includes the entire framework of the problem from scenario generation to execution of the optimisation problem and updating the dynamic parameters. When solving large-scale SMIP models, Xpress first find the solution of the LP relaxed problem before using heuristics and root cuts, and finally branch and bound in order to find the best integer solution. Large problems with several binary variables are difficult to solve to optimality within reasonable time, and feasible solutions found before optimality is reached can therefore be stated with a corresponding duality gap. The duality gap is the percentage difference between the lower feasible bound (LB) and the upper feasible bound (UB) shown in Equation (8.1).

$$\text{Duality gap} = \frac{LB - UB}{LB} * 100\% \tag{8.1}$$

8.1 Problem size

The model is implemented with a defined number of Elbas and BM-scenarios for each bid hour. As can be seen in Table 8.1, the problem contains a large number of variables and constraints, with both continuous and binary variables. In Table 8.2, the computational time is presented for a varying number of BM-scenarios and a constant number of 10 Elbas scenarios. The problem size and computational time are increasing with the total number of scenarios, as a result of all the constraints being scenario dependent. Because the modelled problem has a practical application, a low computational time is important. This can be ensured by applying a maximum run time or an acceptable duality gap when solving the optimisation problem. Problems of a certain size often have some sort of redundancy among the constraints and variables. Xpress IVE tries to identify these instances when the solution process begins in order to reduce the problem size. It can be seen from results in Table 8.1 that presolve reduces the number of constraints and variables, resulting in a simplified problem matrix. However, the percentage decrease after presolve does not depend on the number of scenarios, and presolve will therefore not handle the increase in problem size due to more stochastic scenarios.

The problem size is defined for the first bid hour in the rolling horizon framework, thus for a total of 24 production hours. Because the problem implemented in the rolling horizon model is solved for 33 bid hours, the properties shown in Table 8.1 and Table 8.2 is applied each time the optimisation problem is solved. However, because the time horizon decreases for additional bid hours, so does the problem size. Thus, the properties presented are the maximum values for each optimisation problem solved in the rolling horizon model. Due to the practical application of the model, which is intended to find a solution for each bid hour, the maximum computational time is set to 3600 seconds. It can be seen that this is sufficient to solve the largest scenario tree to optimality.

Table 8.1: Problem size before and after presolve for a varying number of BM-scenarios, with 10 Elbas scenarios each.

	Before presolve			**After presolve**			
# BM-scen.	# Const.	# Var.	# Bin. var.	# Const.	Δ Const.	# Var.	Δ Var.
10	223352	98400	12000	128264	-42.57 %	23824	-75.74 %
30	670152	295200	36000	384769	-42.58 %	71630	-75.74 %
50	1116952	492000	60000	641096	-42.60 %	119077	-75.80 %

Table 8.2: Computational properties for a varying number of BM-scenarios, with 10 Elbas scenarios each.

# BM-scen.	Computational time [s]	Duality gap
10	113.7	0 %
30	738.8	0.84 %
50	2636.7	0.93 %

8.2 Stability of the scenario tree

An important feature of a stochastic programming problem is the stability of the scenario tree. Testing the quality of the scenario generation method is crucial when quantifying how good the scenario tree is and further the accuracy of the resulting stochastic problem. Having a low computational time is essential for this kind of problem where the model runs close to the time of decision. Thus, the preferable scenario tree is small, and still satisfies stability measures. A stable model guarantees that the sampling error applied to the scenario generation has an insignificant influence on the results. All theory about stability testing is based on the work of Kaut and Wallace (2007).

The main goal of the scenario generation is to have a small optimality gap, $e_f(G, \hat{G})$. Evaluation of the scenario generation method can be done by investigating the size of the optimality gap, which is the difference between the value of the objective function at the optimal solution of the true problem and the approximated problem. One possible way to test this gap is to build a reference tree and use the tree as an approximation of the true objective value. This reference tree should be as big as possible to make sure that the tree is stable. The solution time of solving the big tree does not matter since it only has to be solved once. The optimality gap is given by Equation (8.2).

$$\begin{aligned} e_f(G, \hat{G}) &= F(\text{argmin}_{x \in \mathcal{X}} \hat{F}(x)) - F(\text{argmin}_{x \in \mathcal{X}} F(x)) \\ &= F(\text{argmin}_{x \in \mathcal{X}} \hat{F}(x)) - \text{min}_x F(x) = F(\hat{x}^*) - z^* \end{aligned} \tag{8.2}$$

In the equation above, G is the distribution which is independent of the decision variable, x, within the feasible region $\mathcal{X}$. Further, $\hat{F}$, $\hat{G}$ and $\hat{x}^*$ are the scenario-based objective function, scenario-based distribution and the optimal solution of the approximated solution respectively. Stability testing of scenario trees and quantification of the tree size is often done by in-sample and out-of-sample stability tests. The results from the stability tests are then used to approximate the number of appropriate scenarios to use in the computational study. The following sections describe the two stability requirements and how they are applied to the scenario tree generated by Algorithm 3. This thesis models the problem for the first Wednesday in every month in 2016. However, because the scenario trees are generated equally for all dates, only the first date, 06.01.16, is used to test the stability requirements.

8.2.1 In-sample stability

If a scenario tree is in-sample stable it is guaranteed that the objective value is approximately equal regardless which scenario tree is used as input. It is called in-sample stability because the solutions are evaluated on the sample tree they came from. This stability only depends on the optimisation model, and therefore it can - and should - always be checked. Kaut and Wallace (2007) define the in-sample stability by Equation (8.3).

$$\hat{F}_k(\hat{x}_k^*) \approx \hat{F}_l(\hat{x}_l^*) \tag{8.3}$$

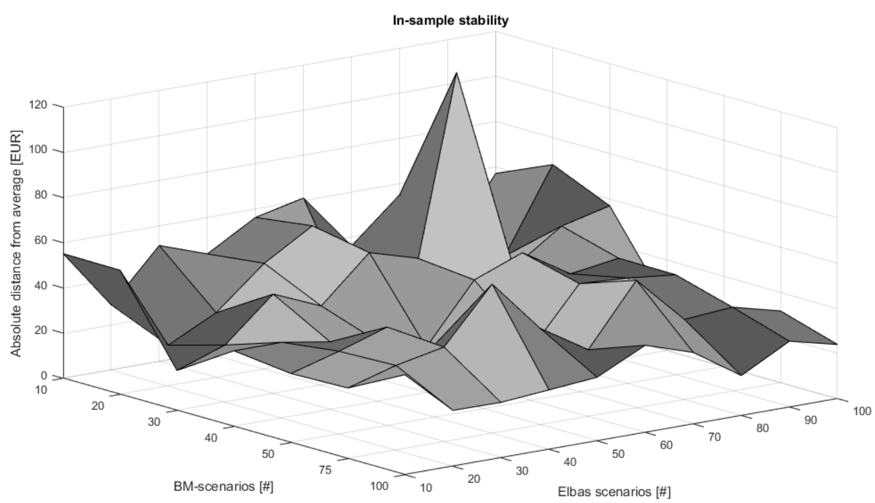


Figure 8.1: In-sample stability testing: Average deviation from average objective for a combination of Elbas and BM-scenarios.

The stability requirement ensures that running the scenario generation and using the generated scenario tree as input in the stochastic programming problem should result in the same objective value if the same procedure is repeated with a different scenario tree. To test in-sample stability the problem is run five times with different scenario trees. Figure 8.1 shows the average absolute distance to the mean value for the five runs, computed for different combinations of Elbas and BM-scenarios. The number of scenarios is changed step-wise for Elbas scenarios from 10-100 and BM-scenarios with values [10, 20, 30, 40, 50, 75, 100]. The goal is to find out whether the Elbas or BM-scenarios influence the stability of the tree most. Thus, the result for each scenario combination over five runs for bid hour one is presented. Because the scenario trees are updated using the same algorithm for each bid hour, the first bid hour is assumed sufficient to create an overview of the relationship between the different markets' influence on the scenario tree.

Figure 8.1 shows that the average distance decreases with increasing BM-scenarios, and that the influence of the number of Elbas scenarios is smaller. For example is the average distance value rather stable for 100 BM-scenarios regardless of the number of Elbas scenarios. A larger amount of Elbas scenarios does not contribute to a lower average distance as long as there are few BM-scenarios. The behaviour of increased Elbas scenarios can be explained by the fact that the Elbas scenarios have a small impact on the objective value because the Elbas volumes are small. Thus, a larger amount of Elbas scenarios will not necessarily lead to a large increase in trades, because the Elbas scenarios reflect the poor market liquidity. An increase in balancing market scenarios does however reduce the average distance as a main trend. Some of the volatility can however be described by the fact that the average is calculated based on only five runs, and single deviations will therefore have a greater impact.

Table 8.3: In-sample stability testing: Coefficient of variance for all scenario combinations for bid hour one.

	E10	**E20**	**E30**	**E40**	**E50**	**E60**	**E70**	**E80**	**E90**	**E100**	**Avg.**
BM10	0.161	0.118	0.209	0.194	0.194	0.212	0.149	0.234	0.108	0.233	0.181
BM20	0.146	0.057	0.079	0.120	0.178	*0.040*	0.054	0.327	0.093	0.207	0.130
BM30	0.045	0.083	0.120	0.095	0.138	0.138	0.089	0.107	0.149	0.167	0.113
BM40	0.090	0.087	0.083	0.066	0.073	0.116	0.142	0.100	0.127	0.064	0.095
BM50	0.092	0.093	0.106	0.091	0.065	0.058	0.120	0.124	0.147	0.047	0.094
BM75	0.072	0.081	0.113	0.138	0.106	0.082	0.148	0.085	0.108	0.098	0.103
BM100	0.085	0.059	0.054	0.062	0.065	0.105	0.076	0.049	0.089	0.070	**0.071**
Avg.	0.099	**0.083**	0.109	0.109	0.117	0.107	0.111	0.147	0.117	0.127	

When considering computational time it is preferable with fewer scenarios and there will be a trade-off between the amount of Elbas and BM-scenarios that can be included because the total number of scenarios is the product of the two. Table 8.3 presents the coefficient of variance (CV) for all scenario combinations. The CV is a measure of the dispersion of the objective value around the mean value and is calculated by dividing the standard deviation by the mean. The single combination with the *lowest* CV is given for 60 Elbas scenarios and 20 BM-scenarios. However, due to the uncertainty of a single combination being influenced by single scenario tree deviations, the **average** value is calculated for all scenario combinations keeping one market constant. It can be seen that the average value is lowest for 20 Elbas scenarios and 100 BM-scenarios. Further, the difference between the highest and lowest average CV is 0.064 and 0.110 for Elbas and BM respectively, where the CV is decreasing with increased number of BM-scenarios, but actually increasing with an increased number of Elbas scenarios.

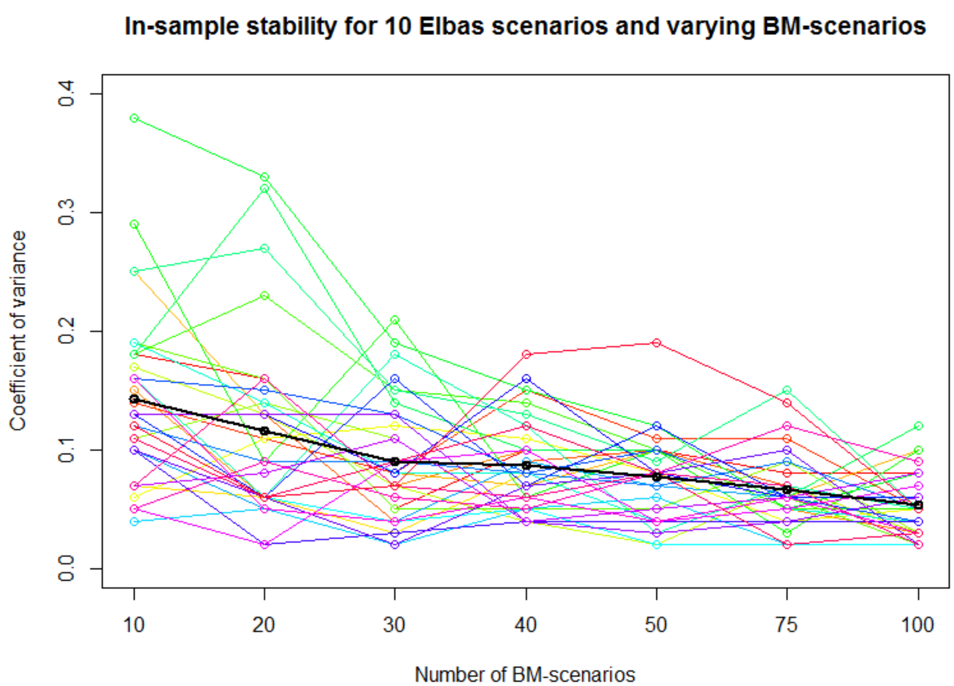


Figure 8.2: In-sample stability testing: Coefficient of variance for all bid hours for 10 Elbas scenarios and varying BM-scenarios.

It is difficult to conclude if the Elbas scenarios will become stable for a much greater amount of scenarios because this is beyond the model's computational limitations. Based on Figure 8.1 and Table 8.3 it can be concluded that within the possible combinations, the number of BM-scenarios influence the stability the most, and that the average distance to the mean become less volatile after about 30 BM-scenarios regardless of the number of Elbas scenarios. Because of the challenge of deciding the stable amount of Elbas scenarios, it is concluded that it is better to keep the scenarios low because increasing them gives no certainty for stability. To reduce the size of the model, it is therefore decided to keep the number of Elbas scenarios constant at 10 in the remaining stability tests.

Figure 8.2 shows the coefficient of variance plotted for all 33 bid hours for all combinations of BM-scenarios when keeping the number of Elbas scenarios constant at 10. It can be seen that there are large variations in CV between the bid hours, and especially the green curves which refer to the bid hours in the beginning of the day of production are volatile. Once again this can be be explained by the marginal error of only five runs for each bid hour and scenario combination. Further, the first production hours are also the hours with largest variations in objective value because the dynamic updates of input parameters such as reservoir level begins. Nevertheless, the overall trend for all bid hours is that the coefficient of variance decreases with increased number of BM-scenarios. From the average value which is the bold black line, it can be seen that the overall decrease in CV slows down above 30 BM-scenarios.

Table 8.4: In-sample stability testing: Average relative mean deviation and coefficient of variance for all bid hours.

BM-scenarios (10 Elbas scenarios)	**Relative mean deviation [%]**	**Coefficient of variance [%]**
10	10.84	14.33
20	8.81	11.60
30	6.86	8.99
40	6.63	8.66
50	5.78	7.67
75	4.98	6.63
100	4.02	5.31

A summary of the in-sample stability testing for bid hour one can be found in Table 8.4. This table presents the relative mean deviation and CV for different BM-scenarios. Due to the variations between the different bid hours, the values in this table are the average values for all bid hours. Because a low number of scenarios is preferred, the lowest number of scenarios where the scenario tree is considered in-sample stable is chosen. Based on the above analyses and the values of Table 8.4, it is concluded that at least 30 BM-scenarios is necessary to produce an in-sample stable scenario tree. Because the objective value from the in-sample stability testing is low, a CV of about 9 % is concluded to be sufficient. In comparison to the objective value of the problem solved in the case study, a dispersion of the scenarios of 9 % is not considered to influence the results significantly.

8.2.2 Out-of-sample stability

The idea of out-of-sample stability is to generate several scenario trees and solve the stochastic programming problem with each tree, to ensure that the scenario generation has not created any incorrect stability. The objective value obtained for each tree should be approximately the same as the value of the true objective function. The stability is evaluated on a different sample than the one used for finding the solution, hence the term "out-of-sample". The definition used by Kaut and Wallace (2007) is as follows:

> Generate K scenario trees $\hat{\xi}_k$, solve the optimisation problem with each one of them, and obtain optimal solutions $\hat{x}_k^*$, $k = 1...K$. The stability requirement is written:

$$F(\hat{x}_k^*) \approx F(\hat{x}_l^*) \tag{8.4}$$

$$F(\text{argmin}_{x\in\mathcal{X}}\hat{F}_k(x)) \approx F(\text{argmin}_{x\in\mathcal{X}}\hat{F}_l(x)) \tag{8.5}$$

$$e_f(G, \hat{G}_k) \approx e_f(G, \hat{G}_l) \tag{8.6}$$

In this definition the true objective function $F(x)$ must be evaluated. This is a problem, because the actual objective value is hard to find and in most cases not given. If the true value can not be obtained, a weaker form of out-of-sample stability can be used. This method says:

> If we have two scenarios trees $\hat{\xi}_k$ and $\hat{\xi}_l$ and their respective optimal solution $\hat{x}_k^*$ and $\hat{x}_l^*$, then a stable method implies

$$\hat{F}_k(\hat{x}_k^*) \approx \hat{F}_k(\hat{x}_l^*) \tag{8.7}$$

$$\hat{F}_l(\hat{x}_l^*) \approx \hat{F}_l(\hat{x}_k^*) \tag{8.8}$$

$$\hat{F}_k(\hat{x}_l^*) \approx \hat{F}_l(\hat{x}_k^*) \tag{8.9}$$

This thesis models a multistage problem, which makes the out-of-sample stability test more complicated. It is no longer possible to evaluate the true objective value, $F(x)$ for a value x, because the vector of possible solutions corresponds to the multistage scenario tree which does not coincide with the two-stage scenario tree. Thus, a solution based on one tree cannot simply be evaluated in another scenario tree because decision variables exist in other nodes than the root node. However, this can be solved by implementing only the first stage decision and re-run the model with the fixed first stage decision and an updated scenario tree. This means that the first stage solutions from one scenario tree must be run on another tree, and vice versa. The method is then out-of-sample stable if the objective values are approximately equal.

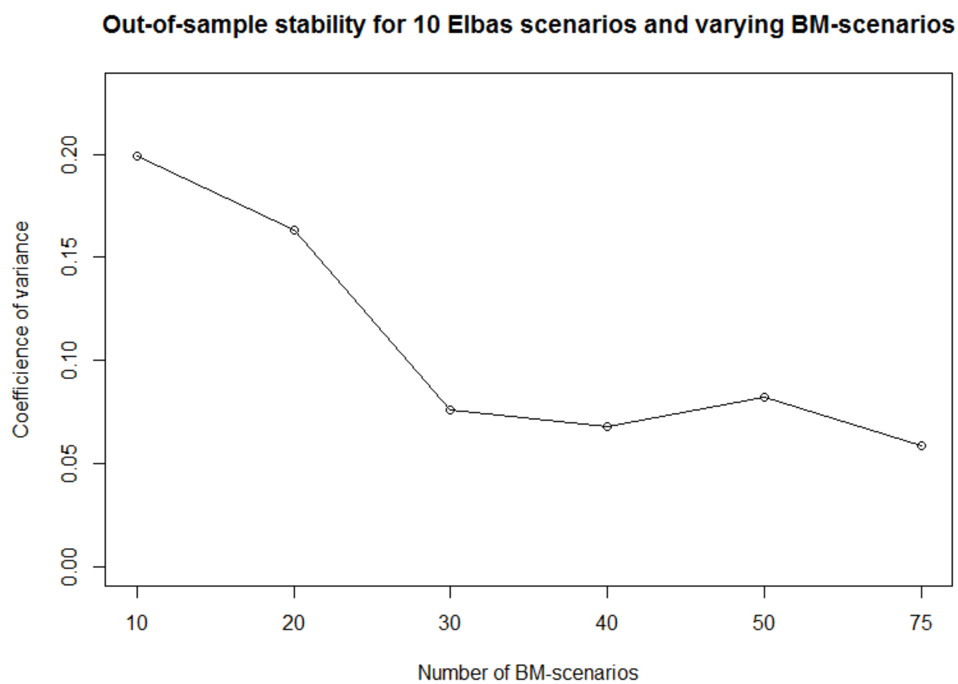


Figure 8.3: Out-of-sample stability testing: Coefficient of variance for bid hour one.

In the multistage structure of this problem, the first stage decision is the volume bid to Elbas. After the volume is bid, the acceptance share for each bid is revealed and actual volumes sold or bought are known. When the Elbas sell or buy volumes are known, the third stage decision variable of volume bid to the balancing market can be decided. Because the BM-volumes are bid after Elbas is settled and therefore depend on accepted Elbas volumes, which further depend on actual bid volumes, the first-stage bid volumes are fixed for each scenario. The fixed bid volumes with associated Elbas prices are then used to solve the problem for updated scenario trees.

It is not possible to obtain a large enough reference tree due to the problem size. Thus, the out-of-sample test must be done by comparing the objective values of different scenario trees with fixed first stage solutions. The stability in this problem is therefore tested by varying the number of BM-scenarios from 10-75 with a fixed number of 10 Elbas scenarios. Because the scenario generation procedure is equal for each bid hour, it is considered sufficient to test the stability for only one bid hour in the rolling horizon model. Thus, to reduce the computational time of the stability testing, the out-of-sample stability is only tested for bid hour one.

For each total number of scenarios, two scenario trees are generated and the optimisation problem is solved for the first bid hour. The first stage decision variables are then fixed, and applied to the other scenario tree. The problem solved in the out-of-sample test therefore depends on the acceptance shares and the uncertain BM-scenarios. This is repeated five times for each scenario combination before the average value is presented. The CV and relative mean deviation for the out-of-sample stability testing are presented in Table 8.5.

Table 8.5: Out-of-sample stability testing: Average relative mean deviation and coefficient of variance for bid hour one.

BM-scenarios (10 Elbas scenarios)	**Relative mean deviation [%]**	**Coefficient of variance [%]**
10	17.15	19.90
20	12.60	16.35
30	6.53	7.57
40	8.23	6.81
50	7.11	8.22
75	4.99	5.84

Having an out-of-sample stable solution means that the real performance of the solution does not depend on the scenario tree that is being used. As with the in-sample testing, the number of test runs per scenario combination is considered a source of uncertainty in the results. However, looking at Figure 8.3, the trend of increasing the BM-scenarios is obvious. For the number of BM-scenarios below 30, it is concluded that the scenario tree can not be considered stable due to large variability. Due to large computational time the test is not performed for more than 75 scenarios, but it is reasonable to believe that the trend seen in Figure 8.3 will continue with an increase in BM-scenarios. It is concluded that the CV stabilises around 7 %. The percentage is justified and concluded to be sufficient because the objective value from out-of-sample stability testing is low. A dispersion around the objective value of 7 % will not influence the computational results significantly. Based on the stabilisation of the CV and relative mean distance in Table 8.5, it is concluded that the scenario tree can be considered out-of-sample stable for above 30 BM-scenarios.

It is important to have both out-of-sample and in-sample stability, and not only one of the requirements. This is because the out-of-sample stability states that the solution is stable and does not depend on which scenario tree is used, and in-sample stability states that the solution is approximately equal for all scenario trees. On the basis of the out-of-sample and in-sample stability tests it is concluded that a scenario tree with 30 BM scenarios and 10 Elbas scenarios is considered stable and therefore used for further computational studies.

8.3 Empirical results and discussion

The multistage SMIP is applied to the case study presented in Chapter 7 in order to analyse the value of post-spot trading. It is assumed that the producer acts risk neutral in Elbas and the balancing market, because the expected production has been bid to the day-ahead market. The producer is considered a price taker with no market power, as presented in Section 6.1.4. To observe seasonal variations in the results and to get a representative selection of the data, the rolling horizon framework is solved for the twelve days presented. In every bid hour, the optimisation problem decides how much to bid to Elbas to what price, how much of the Elbas bids that are accepted, and how much to bid to the balancing market for the remaining production hours. All bid hours for all days have been solved with a duality gap less than 2.0 % for a maximum computational time of 3600 s.

The different days have varying input data such as reservoir level, inflow and spot commitments which influence the results. Table 8.6 gives an overview of the properties of the model solution. The table shows the percentage share of the Elbas trades which are sell or buy trades, how often the Elbas sell price is higher than the BM-price and how often Elbas buy price is lower than BM-price. It can be seen that April, July, August and September only submit buy bids, whereas only sell bids are considered for October. Further, it can be concluded that the Elbas premium is usually higher than the balancing market premium.

Table 8.6: Percentage occurrence for features of post-spot trading: relationship between Elbas bid types and post-spot prices.

Date	Elbas sell bids	Elbas buy bids	Elbas sell > BM	Elbas buy < BM
06.01.16	59.26 %	41.30 %	80.00 %	95.48 %
03.02.16	11.67 %	87.41 %	89.75 %	98.10 %
02.03.16	60.93 %	85.25 %	86.19 %	96.27 %
06.04.16	0 %	100 %	85.85 %	97.12 %
04.05.16	13.70 %	86.30 %	86.19 %	96.54 %
01.06.16	81.30 %	17.79 %	89.50 %	97.38 %
06.07.16	0 %	100 %	84.06 %	97.21 %
03.08.16	0 %	100 %	85.85 %	97.12 %
07.09.16	0 %	100 %	86.53 %	96.73 %
05.10.16	100 %	0 %	87.77 %	96.46 %
02.11.16	63.52 %	36.48 %	86.80 %	95.84 %
07.12.16	99.07 %	0.74 %	87.81 %	96.85 %

Regardless whether the producer wants to sell or buy volumes, it can be seen that the Elbas price normally is more profitable than the balancing market price. It seems that Elbas participants are willing to accept higher bid premiums, simultaneously as the BM-premium is low. It is however a trade-off between the probability of an Elbas bid being accepted, and the probability that the balancing market is regulated. From the scenario generation and Elbas analyses, it can be concluded that the overall probability of an Elbas sell bid being accepted is 8.30 %, whereas the probability that BM is upward regulated is 21.99 %. Similarly, the probability of an Elbas buy bid being accepted is 13.50 % compared to the probability of downward regulation which is 29.08 %. However, there is still a 48.91 % chance that the balancing market is not regulated. It can be assumed that Elbas trading always is possible as long as the price is low enough. This would imply that Elbas sell volumes always are accepted as long as the premium is low enough, and that Elbas buy volumes always are accepted for a high enough premium. This assumption is however dependent on active participants in the market. For the low liquidity and few participants in the Norwegian Elbas market, the assumption is therefore not considered valid.

8.3.1 Numerical analysis of the results

In order to get a quantitative understanding of the results, one day is chosen for more in-depth analyses. The 1st of June is chosen for the reason that it is a date with both sell and buy volumes on Elbas. Further, because the water value is 0 in the lower reservoir, and the generator in direct connection to the upstream reservoir is not in operation, it is easier to quantify how much the water value in the reservoirs influence the objective value. In June, there will be a positive contribution to the objective from inflow in the upper reservoir as long as the upper generator does not discharge into the lower reservoir. This must not be mistaken as intraday income. An illustration of this is given in Table 8.7.

Table 8.7: Computational results for the first bid hour for 01.06.16. Objective value and post-spot profit.

	Objective [EUR]	Post-spot profit [EUR]	Profit [%]
Objective	8424.21	479.09	5.69 %

The results in Table 8.7 show the objective value of solving the optimisation problem without spot profit. The model does however not separate between water used for day-ahead commitments and water used for post-spot trading, and the inflow in both reservoirs have been included in order to model the change in reservoir volumes. This results in a positive difference in reservoir level for the upper reservoir because no water is being used. This increase in reservoir volume is treated as an income in the objective value. Because the post-spot volumes are produced with water from the lower reservoir, there is no contribution to the objective value from water used for post-spot trading, due to zero water value. Therefore, the profit presented in Table 8.7 is the expected income from post-spot trading in the first bid hour. The value includes expected profit from the balancing market for all production hours, whereas the contribution from Elbas volumes represent actual accepted bids. It should be noticed that the post-spot profit represents under 6 % of the total objective value, which indicates that the contribution to the total hydro power production scheduling is low.

The uncertainty of the balancing market forecast decreases with bid hours closer to the hour of operation. Because the inflow is a constant parameter for each hour, the contribution to the objective value from filling the upper reservoir is also constant. However, the total contribution of the water value can differ between scenarios if for example the upper generator is started. The changes in the objective are therefore influenced by the scenarios, and an overview of how the stability of the objective value develops throughout the bid hours is presented in Figure 8.4. It can be seen that the spread decreases with decreasing bid hours as expected. The objective value is also decreasing for each bid hour after the production begins. This trend is also expected because the total time-horizon of the problem decreases.

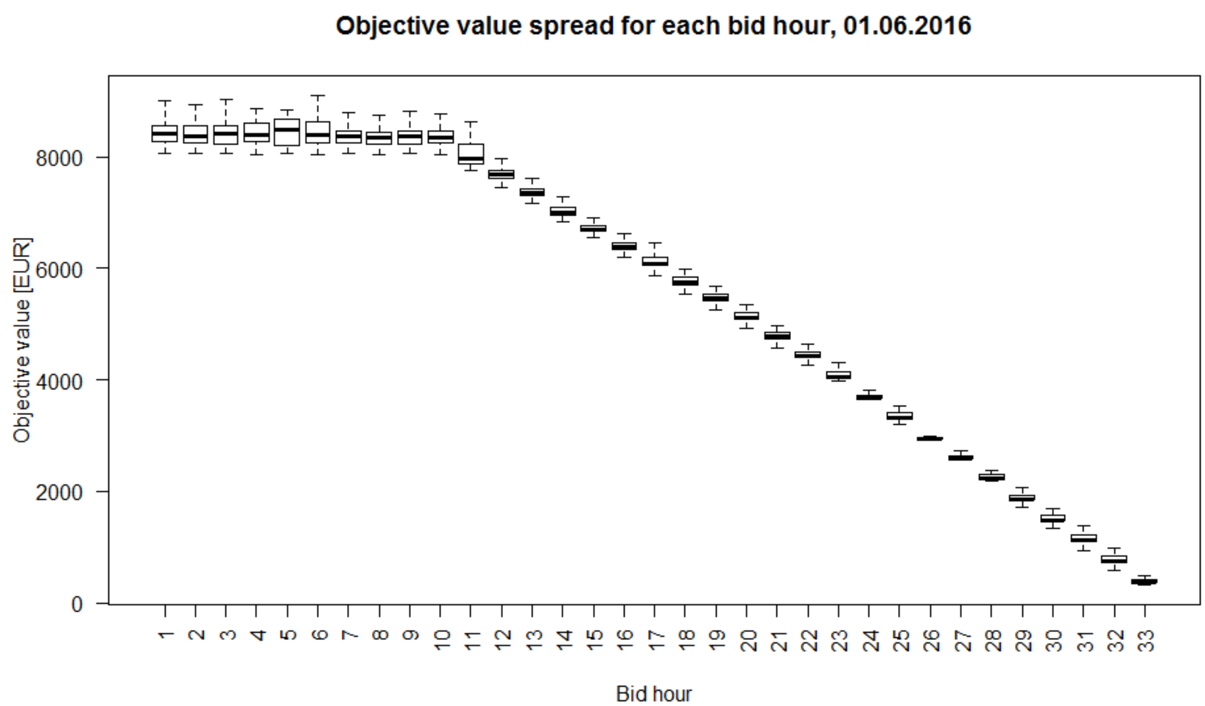


Figure 8.4: Variations in the objective value for each bid hour for 01.06.16.

To illustrate how the acceptance share influences the bid volumes, the average expected volumes bid and the average expected volumes accepted in every production hour is presented in Figure 8.5. The average value represents the expected average of all bid hours in the current production hour for all scenarios. This is the average value for 10 bid hours in production hour one, and for 33 bid hours in production hour 24. Both sell and buy bids are submitted during the day, however the number of sell bids exceed the number of buy bids. As can be seen in Figure 8.5, the acceptance share modifies the bid volumes depending on the price and volume of the bid. The upper plots show the volume bid to Elbas, whereas the lower plots show the accepted volumes. It can be seen that although 47 MWh is bid to sell on Elbas, the acceptance share reduces the actual sold volume to 0.2 MWh. Although the acceptance shares are equal for all scenarios, and therefore can give a misleading picture of one single Elbas bid, they reflect the total market liquidity and the producer's lack of market power. It is assumed that the profit obtained from the small accepted bid volumes gives a realistic picture of the Elbas profitability. In this case, the total average expected sell volume for one day of production is 3.78 MWh. This equals 1380 MWh for one year of Elbas bidding, or 26.5 MWh every week. For a price-taker, small-sized producer, this is concluded to be a representative volume.

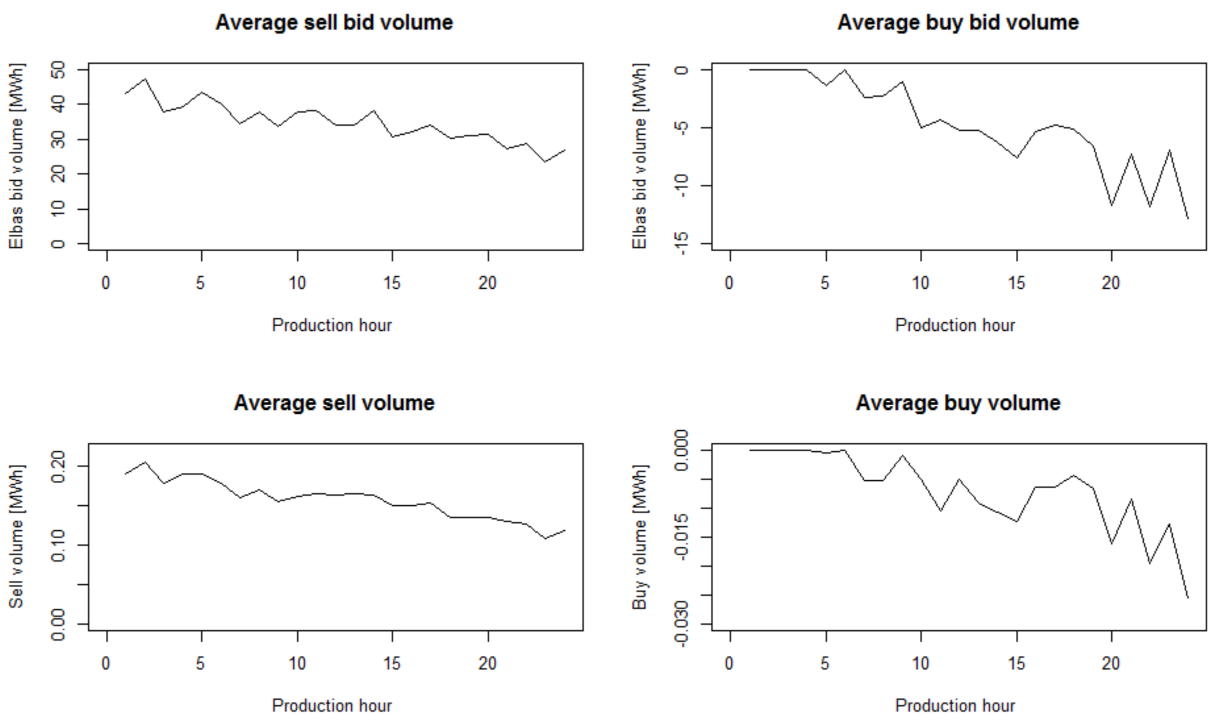


Figure 8.5: Average Elbas bid volumes and accepted bid volumes for each production hour for 01.06.16.

8.3.2 Value of considering Elbas

To measure the effect of considering Elbas trading after the spot market is closed, the problem is solved without the option of Elbas trading. This means that only BM-trading is included in the problem in addition to the already committed spot volumes. The results can be seen in Table 8.8 where the Elbas volume is fixed to zero when considering spot and the balancing market. Using this approach it can be evaluated to which extent trading on Elbas increases the objective value. The value of considering Elbas is therefore given as the percentage difference of the following two problems:

1. The optimal expected profit of solving the model including Elbas trades, thus the mathematical model described in Section 5.3.

2. The optimal expected profit of solving the model described in Section 5.3 with Elbas volumes fixed to zero.

The value of considering Elbas trading can then be described as:

$$\Delta(\%) = [(1.) - (2.)]/(2.) * 100 \tag{8.10}$$

Table 8.8: Objective value for the first bid hour in the analysis, including the value of Elbas.

	Obj. value [EUR] Spot + BM	Obj. value [EUR] Spot + BM + Elbas	Δ [EUR] Including Elbas	Δ [%] Including Elbas
06.01.16	75 221.3	75 383.3	162.0	0.215 %
03.02.16	49 846.4	49 969.7	123.3	0.247 %
02.03.16	59 268.2	59 365.8	97.6	0.165 %
06.04.16	71 292.9	71 311.4	18.5	0.026 %
04.05.16	82 057.1	82 181.1	124	0.151 %
01.06.16	89 022.2	89 118.4	96.2	0.108 %
06.07.16	65 372.4	65 472.2	99.8	0.153 %
03.08.16	78 420.8	78 519.4	98.6	0.126 %
07.09.16	67 828.4	67 973.9	145.5	0.215 %
05.10.16	25 240.2	25 343.8	103.6	0.410 %
02.11.16	65 873.3	66 021.2	147.9	0.225 %
07.12.16	143 652	143 819	167	0.116 %

As shown in Table 8.8 there is a marginal increase of profit when including the possibility to trade on Elbas for every day in the case study. However, the value of Elbas is quantified with an average percentage of 0.180 % or 115.3 EUR. For a hydro power producer with an income in the size of several thousands a day, the contribution is poor. However, the option is profitable although to a low degree, and the operational resources and labour costs associated with Elbas trading should therefore be evaluated against the gain. As seen in Table 8.6, the days in April, July, August and September where only buy volumes are traded have a lower average value of including Elbas. The average gain for these days is only 0.130 %, compared to the day in October with only sell bids which has a gain of 0.410%. This also indicates that it is more profitable to sell additional volumes on Elbas than to buy Elbas volumes to restrain production. The problem is run for the whole day, but only the first bid hour is presented here.

A more thorough correlation study has been performed to identify characteristics of the days where the Elbas gain is higher. Input parameters as initial reservoir level, inflow, spot prices and volumes, as well as the associated Elbas and BM-results have been analysed to identify common features. The parameters that were considered most important are presented in Figure 8.6. The dates are split into three groups identified by the value of considering Elbas: the dates with high value are defined as January, February, September, October and November with colour code green. The dates with medium value are defined as March, May, June, July, August and December with colour code yellow, and finally April is defined to have a low value and represented with colour code red.

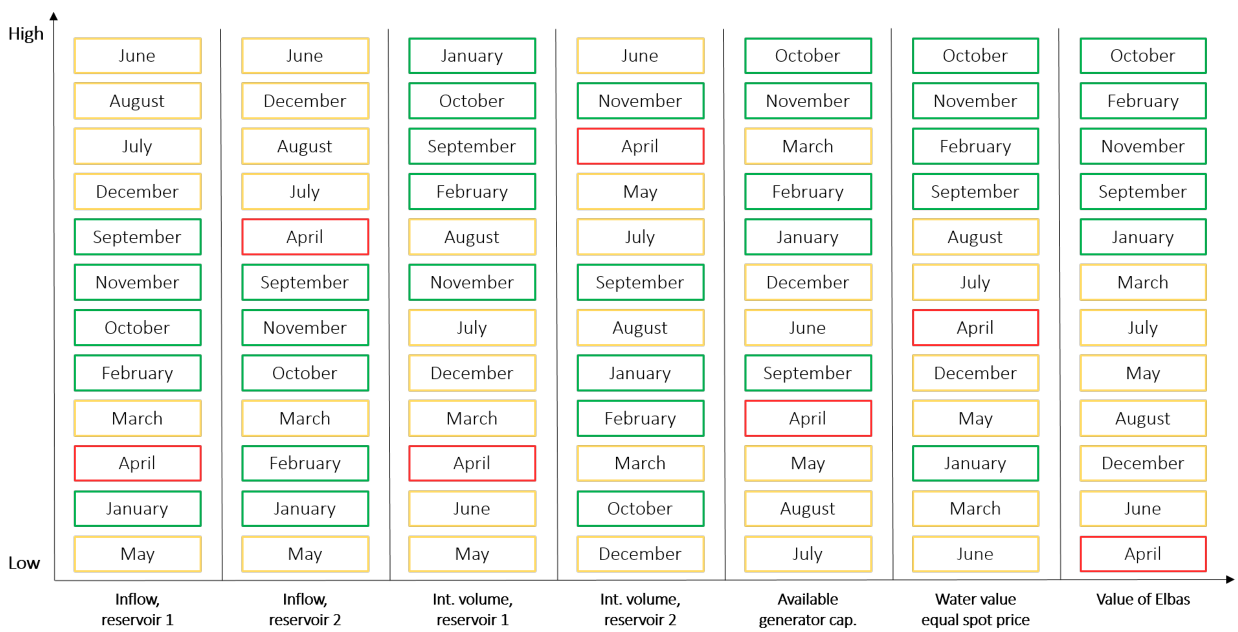


Figure 8.6: Ranking of monthly characteristics that may affect the value of Elbas.

The factors that are not presented in the figure include BM-price, water values, Elbas and BM-volumes and the relationship between Elbas and BM-prices. No obvious correlation can be found between these characteristics and the value of Elbas trading. When comparing Elbas and BM-trades in the different dates it is concluded that these markets to a large extent operate without considering each other (Engmark and Sandven, 2016). If the balancing market is regulated and the BM-price is profitable, the regulation bids will be submitted independent of Elbas bidding. An explanation for this is that the Elbas volumes are so low that they do not affect the availability of producing BM-volumes.

Figure 8.6 presents a ranking system from high to low for the average daily parameters inflow in both reservoirs, initial volume in both reservoirs, available generator capacity and the equality between water values and spot prices. The dates where the water value is close to the spot price are defined with a high equality. It can be seen that the dates with high value of including Elbas typically have available volume for post-spot bidding. This is represented by higher initial water volume in the reservoirs and high available generator capacity. The increased value of Elbas trading is a result of the relationship between Elbas prices and spot prices, where the Elbas premium normally gives a more profitable price for Elbas volumes than for spot commitments. This way, days with more available capacity for intraday trading gets a higher income for the Elbas volumes than for the same amount of spot volumes. In addition, when the producer has the ability to bid larger volumes this leads to a larger share of accepted volumes, which results in an additional income.

It can further be seen that when spot price and water value are similar, there is a correlation with high value of Elbas. An assumption is that when the water value is close to spot price, typically less volumes are committed to the day-ahead market. Thus, by placing the unused capacity on Elbas, the producer can increase profit from a higher Elbas price. Further, it can be seen that the days with large inflow have a lower gain from Elbas trading. This can once again be explained by available generator capacity. It is assumed that the day-ahead volumes are planned according to the prediction of large inflow, and the production is thus higher for days with large inflow. To summarise the results from Figure 8.6, it can be concluded that the factors influencing the value of Elbas have in common that they facilitate available generator capacity. Available capacity can further be linked to reservoir level, inflow and spot price, but it is assumed that the crucial factor for Elbas profitability is the possibility to produce extra intraday volumes. This is explained by the fact that Elbas prices usually are more profitable than the spot price.

The modelled hydro power plant is a small participant in the post-spot markets, and the gain from bidding at Elbas is only a fraction of the total profit for the producer. It would have been interesting to investigate how the profitability develops for a producer with a larger production portfolio. A producer with several power plants and larger generation capacity will have the ability to coordinate bidding between plants and therefore bid larger volumes to Elbas. This will increase the profitability. It is assumed that different power plants and reservoirs have different technical parameters and restrictions for the reservoirs and generators, something which can lead to different optimal bidding solutions. This means that the different power plants can place bids with various volume and price, and therefore cover a bigger range of the demand curve. Thus, the Elbas bids are assumed to be more differentiated for a larger production portfolio. Because the acceptance shares change for different volume/premium combinations, it is reasonable that the total acceptance share can increase. This is because the limit Elbas orders make it possible to only accept parts of the total bid. If the producer can bid larger volumes to a lower price, as is described by the demand curves, it is more likely that a part of the bid is accepted compared to a bid with equal accepted volume but higher price. A producer with the possibility to differentiate the bid portfolio is therefore assumed to have an increased value of Elbas.

8.3.3 Sensitivity analysis of increased liquidity

The predicted development of the Nordic power system, where a higher share of intermittent power will increase the demand for trading closer to production hour, is considered to increase intraday trading. It can therefore be assumed that the Elbas liquidity will increase in the years to come. It is desired to perform a theoretical study which show how the value of Elbas can develop with increased market liquidity.

To model Elbas in the future with fictive liquidity, the acceptance shares Π_{dk}^{Sell} and Π_{dk}^{Buy} from Section 6.1.4 are increased to simulate a market with higher demand and more participants. Table 8.9 shows a model of a future Elbas market where Π_{dk}^{Sell} and Π_{dk}^{Buy} are multiplied with the factors 10 and 50 to estimate different outcomes of future demand in Elbas. The input parameters and BM-forecast are however kept equal to the present model, and the only model sensitivity is therefore the increased acceptance shares. Because of the originally low acceptance shares, the scaling factors are chosen relatively high. The resulting analysis is performed with one date from every season to identify characteristics for the different results. Acceptance shares with zero value in Section 6.1.4 are estimated as the average of the neighbouring volume segments. For the cases where some acceptance shares exceed 100 %, which can happen for the low premium and low volume segments with scaling factor 50, the acceptance shares are fixed to 100 %. It is considered reasonable that all bids with low volumes close to the spot price are accepted in an intraday market with large liquidity.

Table 8.9 shows that the value of including Elbas will increase with a more liquid Elbas market. It can however be seen that although the acceptance share is multiplied with a factor, the increased profit from Elbas trading is not proportional to the scaling factor.

The ranking between the days, describing which day has the largest value of Elbas, is equal to the original case for the scenario with scaling factor 10. However, for the scenario with scaling factor 50, December has a larger value of including Elbas than August. This can be explained by the available generator capacity. In August, the available generator capacity is lower than in December. With a scaling factor of 50, the generator capacity becomes a limiting factor for intraday trading in August. December, which has available generator capacity, therefore has a greater value of Elbas because of the option to produce more. Because the value of Elbas is limited by the possibility to sell or buy volumes, it is reasonable that the gain is not proportional to the scaling factors. Larger scaling factors give the opportunity to buy or sell larger volumes, but the bid volumes are still limited by available capacity.

Table 8.9: Objective value and the percentage value of Elbas for two increased liquidity scenarios.

	Spot+BM [EUR]	Including Elbas [EUR]		Δ %	
		10*Π_{dk}	50*Π_{dk}	10*Π_{dk}	50*Π_{dk}
03.02.16	49 846.4	51 136.6	54 119.5	2.588 %	8.573 %
04.05.16	82 057.1	83 015.9	86 253.2	1.168 %	5.114 %
03.08.16	78 420.8	79 196.8	81 692.9	0.990 %	4.172 %
07.12.16	143 652	144 994	150 226	0.934 %	4.576 %

In order to realise the future liquidity outcomes, factors such as government regulations and amount of installed intermittent power will have a huge impact. It is also reasonable that the liquidity will increase with more transmission capacity to the continental European market which has larger intraday traditions. As mentioned in Section 4.1 some Swedish power producers are committed to act as market makers and quote both buy and sell orders at Elbas. By introducing this type of agreement in Norway, the Elbas liquidity is forced to increase. If the amount of intermittent power increases faster than the Elbas liquidity, the need for balancing market regulation by the TSO will increase. The alternative is to have enough participants and high enough demand in Elbas, so that the producers of intermittent power can handle the imbalances on their own by Elbas trading.

Chapter 9

Concluding remarks

In this thesis a model for optimal bidding in the post-spot electricity markets, Elbas and the balancing market has been solved. The purpose is to investigate alternatives for a hydro power producer to maximise profit after the day-ahead market has been settled. The problem is modelled as a multistage stochastic mixed integer problem with continuous and mixed integer variables and scenario dependent stages. A comprehensive modelling framework has been developed and implemented in order to create realistic scenarios describing the uncertainty in the two markets. This framework consists of a thorough market analysis of Elbas to model demand scenarios, as well as forecasting of the balancing market. The scenario tree is generated and implemented in a rolling horizon framework to describe the development of the bidding problem with a decreasing time horizon.

Elbas has been modelled based on historical order depth obtained exclusively from Nord Pool for this thesis. This thesis provides a valuable contribution to Elbas modelling and the understanding of the total market liquidity. The bids are modelled as demand curves described by the relationship between bid premium and volume, discretised into maximum bid volume segments to obtain a linear problem. For a given bid volume and premium, acceptance shares are defined to model how much of the bid volume that is accepted. By using order depth to model Elbas demand, a more realistic approach to the actual market mechanism is presented. This is to our knowledge the first time order depth has been used to model Elbas.

In order to model the total time horizon of the post-spot bidding, where the uncertainty in the balancing market forecast is reduced closer to the hour of production, a rolling horizon approach is implemented. The model updates information for each bid hour to ensure that post-spot bidding always is performed with the smallest uncertainty possible. In addition, it is possible to obtain information of previously traded Elbas bids for following bid hours. An analysis of the spread of the objective value for decreasing bid hours shows that the rolling horizon model successfully contribute to lower uncertainty with a decreased time horizon.

The mathematical model has been run smoothly for a set of historical test cases, and the computational time and duality gap are sufficient for bidding situations. Because the model is run for every bid hour, a computational time under one hour is preferable. All tests were solved within 3600 seconds with a duality gap less than 2.0 %. The value of bidding in both post-spot markets has been compared to the option of only bidding in the balancing market. Although the value of Elbas bidding is positive for all days in the case study, the average value of the quantified gain is 0.180 %. Because the increased profit is marginal, the resources used to participate in Elbas trading should be evaluated against the gain. It can be seen that days with more flexibility in production due to high reservoir levels or available production capacity have a higher value of including Elbas. It is concluded that selling additional volumes on Elbas is more profitable than to buy Elbas volumes to restrain production.

Comparing the generated scenarios and the optimal bid solutions, it can be seen that the Elbas prices normally is more profitable than the BM-price. This is the main reason that Elbas trading is profitable. However, due to the low market liquidity, the accepted volumes are small and the profit contribution from Elbas is lower than expected. As a contribution to the case study, a simulation of a more liquid intraday market is carried out by increasing the acceptance shares. The results show that the value of Elbas increases with increased liquidity, and the limitation in a more liquid market is rather available production capacity.

For a hydro power producer operating in Elbas as of today, it will be profitable to consider Elbas trading if the bidding procedure is efficient and the producer has available production capacity. Although the liquidity is low, bids are occasionally accepted to a price that normally is more profitable than the spot price. An alternative is to consider automatic bidding by using a computer to place bids depending on specified seasonal data and reservoir parameters. There is also a large amount of Elbas bids that are not answered, and it is therefore potential in answering existing bids.

An extension of the work presented in this thesis is to obtain order depth for several months in order to model the Elbas demand depending on production hour. In Chapter 4 it was shown that the volume and prices vary during a day of production, but the amount of data received for this thesis was not sufficient to derive hourly demand. By defining the Elbas demand for each production hour, an even more precise model can be derived. Order depth for more developed intraday markets can also be analysed in order to obtain more realistic acceptance shares. Further, it is interesting to analyse the difference between Elbas and for example the German intraday market to identify how Elbas liquidity can increase in the future.

Bibliography

Aasgard, E. K., Andersen, G. S., Fleten, S.-E., and Haugstvedt, D. (2014). Evaluating a stochastic-programming-based bidding model for a multireservoir system. *IEEE Transaction on Power Systems*, 29:1748–1757.

Bakken, B. and Bjørkvoll, T. (2002). Hydropower unit start-up costs. *Technical Report TR A5351, Trondheim: SINTEF Energiforskning*.

Beraldi, P., Violi, A., Scordino, N., and Sorrentino, N. (2011). Short-term electricity procurement: A rolling horizon stochastic programming approach. *Applied Mathematical Modelling*, 35:3980–3990.

Birge, R. and Louveaux, F. (2011). *Introduction to Stochastic Programming*. Springer, 1. edition.

Boomsma, T., Juul, N., and Fleten, S.-E. (2014). Bidding in sequential electricity markets: The nordic case. *European Journal of Operational Research*, 283:797–809.

Catalão, J., Mariano, S., Mendes, V., and Ferreira, L. (2010). Nonlinear optimization method for short-term hydro scheduling considering head-dependency. *European Transactions on Electrical Power*, 20:172–183.

Champion, B. R. and Gabriel, S. A. (2017). A rolling horizon approach for stochastic mixed complementarity problems with endogenous learning: Application to natural gas markets. *Energy and Buildings*, 135:338–349.

Devine, M. T., Gabriel, S. A., and Moryadee, S. (2016). A rolling horizon approach for stochastic mixed complementarity problems with endogenous learning: Application to natural gas markets. *Computers and Operations Research*, 68:1–15.

Doorman, G. L. (2016). *Hydro Power Scheduling*. Department of Electrical Power Engineering, NTNU, 1. edition.

Engmark, E. and Sandven, H. (2016). Optimal post-spot bidding for a wind power producer in the nordic power market.

EPEX (2013). Power for today, power for tomorrow. https://www.epexspot.com/document/25343/EPEX%20SpOT's%20presentation, Last access 01.02.2017.

EPEX (2015). Products intraday continuous. https://www.epexspot.com/en/product-info/intradaycontinuous/intraday_lead_time, Last access 29.05.2017.

EPEX (2017). Xbid: Cross-border intraday market project. http://www.epexspot.com/en/market-coupling/xbid_cross_border_intraday_market_project, Last access: 07.06.2017.

Erdogan, E., Ma, S., Beygelzimer, A., and Rish, I. (2005). Statistical models for unequally spaced time series. In *Proceedings of the 2005 SIAM International Conference on Data Mining*, pages 626–630.

Faria, E. and Fleten, S.-E. (2011). Day-ahead market bidding for a nordic hydropower producer: taking the elbas market into account. *Computational Management Science*, 8:75–101.

Fleten, S.-E., Haugstveit, D., Steinsbø, J.-A., Belsnes, M., and Fleischmann, F. (2011). Bidding hydropower generation: Integrating short- and long-term scheduling. *MPRA*, 44450.

Fleten, S.-E. and Kristoffersen, T. (2007). *European Journal of Operational Research*, 181:916–928.

Fosso, O. and Belsnes, M. (2004). Short-term hydro scheduling in a liberalized power system. *International Conference on Power System Technology*.

Grande, O., Doorman, G., and Bakken, B. (2008). Exchange of balancing resources between the nordic synchronous system and the netherlands/germany/poland. https://www.sintef.no/globalassets/project/balance_management/tr/tr_a6652_exchange_of_balancing_resources_between_the_nordic_synchronous_system_and_the_netherlands_germany_poland.pdf, Last access 05.02.2017.

Guglielmo, L. and Suvrajeet, S. (2004). A branch-and-price algorithm for multistage stochastic integer programming with application to stochastic batch-sizing problems. *Management Science*, 50(6):786–796.

Guthrie, W. (2013). Box-ljung test. http://www.itl.nist.gov/div898/handbook/pmc/section4/pmc4481.htm, Last access 10.02.2017.

Hydro (2017). About hydro. http://www.hydro.com/en/about-hydro/, Last access 28.05.2017.

IAE (2012). Technology roadmap, hydropower. https://www.iea.org/publications/freepublications/publication/2012_Hydropower_Roadmap.pdf, Last access 14.03.2017.

Jaehnert, S., Farahmand, H., and Doorman, L. (2009). Modelling of prices using the volume in the norwegian regulating power market. In *PowerTech*. IEEE.

Kaut, M. and Wallace, S. W. (2007). Evaluation of scenario-generation methods for stochastic programming. *Pacific Journal of Optimization*, 3(2):257–271.

Kjølle, A. (1980). *Vannkraftmaskiner*. Oslo: Universitetsforlaget, 2. edition.

Klæboe, G. (2015). Stochastic short-term bidding optimizing for hydro power producers. *PhD*.

Klæboe, G., Eriksrud, A., and Fleten, S.-E. (2015). Benchmarking time series based forecasting models for electricity balancing market prices. *Energy Systems*, 6:43–61.

Newman, M. (1993). Regression analysis of log-transformed data: Statistical bias and its correction. *Environmental Toxicology and Chemistry*, 12:1129–1133.

Nord Pool (2015). Market makers elbas. http://www.nordpoolspot.com/globalassets/download-center/intraday/market-makers-elbas.pdf, Last access 30.05.2017.

Nord Pool (2016a). History. http://nordpoolspot.com/About-us/History/, Last access 20.01.2017.

Nord Pool (2016b). Intraday market. http://www.nordpoolspot.com/How-does-it-work/Intraday-market/, Last access 20.09.2016.

Nord Pool (2017). A joint european journey. http://www.nordpoolspot.com/TAS/join-our-markets/a-joint-european-journey/, Last access 01.05.2017.

Olsson, M. and Söder, L. (2008). Modeling real-time balancing power market prices using combined sarima and markov processes. *IEEE Transactions on Power Systems*, 23:443–450.

Prékopa, A. (1995). *Stochastic Programming*. Kluwer Academic Publishers, 1. edition.

REN21 (2016). Renewables 2016, global status report. http://www.ren21.net/wp-content/uploads/2016/06/GSR_2016_Full_Report.pdf, Last access 14.03.2017.

Sakia, R. M. (1992). The box-cox transformation technique: A review. *Journal of the Royal Statistical Society. Series D (The Statistician)*, 41:169–178.

Scharff, R. and Amelin, M. (2016). Trading behaviour on the continuous intraday market elbas. *Energy Policy*, 88:544–557.

Shumway, R. and Stoffer, D. (2006). *Time Series Analysis and Its Applications With R Examples*. Springer Science+Business Media, LLC, 2. edition.

Statkraft (2009). Vannkraft. http://www.statkraft.no/globalassets/old-contains-the-old-folder-structure/documents/no/vannkraft-09-no_tcm10-4585.pdf, Last access 20.02.2017.

Statnett (2014). Legislation. http://www.statnett.no/en/About-Statnett/Ownership/Legislation/, Last access 20.01.2017.

Statnett (2015). Vilkår for anmelding, håndtering av bud og prissetting i regulerkraftmarkedet. http://www.statnett.no/Global/Dokumenter/Kraftsystemet/Markedsinformasjon/RKOM/Vilk%C3%A5r%20for%20RK-markedet%20-%2025%20juni%202015.pdf, Last access 05.02.2017.

Statnett (2016). Challenges and opportunities for the nordic power system. http://www.statnett.no/Global/Dokumenter/Challenges%20and%20opportunities_Report.pdf, Last access 02.02.2017.

Trück, S., Weron, R., and Wolf, R. (2007). Outlier treatment and robust approaches for modeling electricity spot prices. *Proceedings of the 56th Session of the ISI. Available at MPRA: http://mpra.ub.uni-muenchen.de/4711*.

Ugedo, A., Lobato, E., Franco, A., Rouco, L., Fernández-Caro, J., and Chofre, J. (2006). Strategic bidding in sequential electricity markets. *IEE Proceedings - Generation, Transmission and Distribution*, 153:431–442.

Wangensteen, I. (2012). Power system economics: the nordic electricity market.

Weron, R. (2007). *Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach*. John Wiley Sons, 1. edition.

Weron, R. (2014). Electricity price forecasting: A review of the state-of-the-art with a look into the future. *International Journal of Forecasting*, 30:1030–1081.

Appendix A

Elbas analyses

Average traded Elbas volume and price between NO2 and Nord Pool bidding areas.

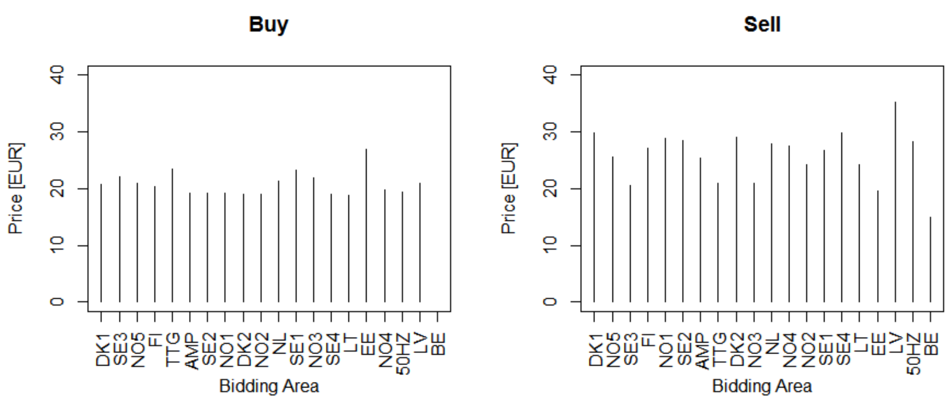


Figure A.1: Average trade price between bidding area and NO2

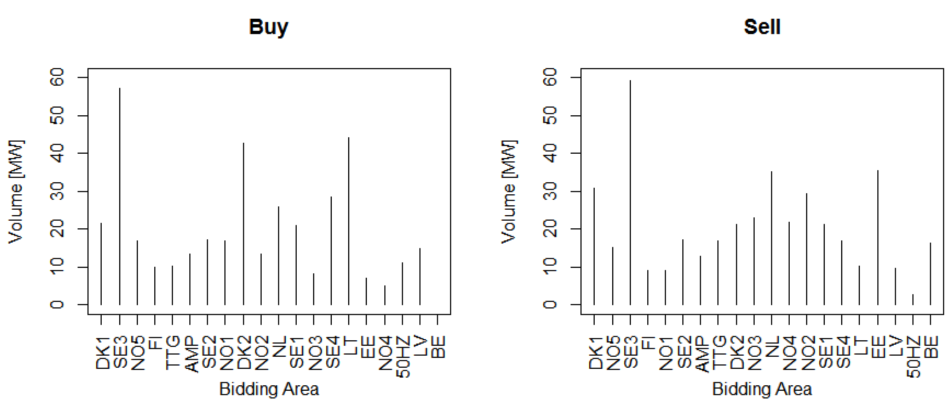


Figure A.2: Average trade volume between bidding area and NO2

Appendix B

Problem formulation

Indices:

s	Scenario
h	Hour
i	Generator
j	Reservoir
f	Production segment
k	Demand segment
l	End water value cut
w	Watercourse

Sets:

$\mathcal{S}$	Set of scenarios s
$\mathcal{H}$	Set of production hours h
$\mathcal{H}^b$	Set of bid hours h
$\mathcal{I}$	Set of generators i
$\mathcal{J}$	Set of reservoirs j
$\mathcal{F}$	Set of production segments f used in the approximation of the production function
$\mathcal{K}$	Set of segments k used in the approximation of the Elbas demand function
$\mathcal{L}$	Set of end water value cuts l
$\mathcal{W}$	Set of watercourses w

Parameters:

Pr_s	Probability of scenario $s \in \mathcal{S}$

A_{if}	Intercept of production function for production segment $f \in \mathcal{F}$ for generator $i \in \mathcal{I}$
B_{if}	Slope of production function for segment $f \in \mathcal{F}$ for generator $i \in \mathcal{I}$
C_i	Start-up costs for generator $i \in \mathcal{I}$
D_i	Maximum discharge for generator $i \in \mathcal{I}$
E^{BM}_{sh}	Available demand in the balancing market in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
E^{E}_{k}	Upper volume limit for Elbas bid volume in segment $k \in \mathcal{K}$
F_{lw}	Future income in watercourse $w \in \mathcal{W}$ for cut $l \in \mathcal{L}$
I_{hj}	Inflow in reservoir $j \in \mathcal{J}$ in hour $h \in \mathcal{H}$
P^{Spot}_{h}	Cleared spot price in hour $h \in \mathcal{H}$
Q^{max}_{i}	Maximum production level of generator $i \in \mathcal{I}$
Q^{min}_{i}	Minimum production level of generator $i \in \mathcal{I}$
U^{0}_{hi}	1 if generator $i \in \mathcal{I}$ in hour $h \in \mathcal{H}$ is turned on in the initial state, 0 if it is turned off
V^{0}_{sj}	Initial water volume in reservoir $j \in \mathcal{J}$ for scenario $s \in \mathcal{S}$
V_{jl}	Evaluated reservoir level for cut $l \in \mathcal{L}$ for reservoir $j \in \mathcal{J}$
W^{0}_{w}	Initial water value in watercourse $w \in \mathcal{W}$
W_{jl}	Marginal water value for cut $l \in \mathcal{L}$ for reservoir $j \in \mathcal{J}$
W^{0}_{j}	Initial water value in reservoir $j \in \mathcal{J}$
X_h	Production committed to the day-ahead market in hour $h \in \mathcal{H}$
X^{E}_{sh}	Volume sold or bought on Elbas for previous bid hours in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
η_j	Energy equivalent in [MWh/m^3] in reservoir $j \in \mathcal{J}$
Π_{sk}	Acceptance share of Elbas bid in segment $k \in \mathcal{K}$ for scenario $s \in \mathcal{S}$
ρ^{E}_{shk}	Accepted Elbas bid price in segment $k \in \mathcal{K}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
ρ^{BM}_{sh}	Balancing market price in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$

Variables:

c_{shi}	Induced start-up cost for generator $i \in \mathcal{I}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
d_{shi}	Discharge by generator $i \in \mathcal{I}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
q_{shi}	Net production needed to deliver all commitments for generator $i \in \mathcal{I}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$

s_{shj}	Spill from reservoir $j \in \mathcal{J}$ in hour hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
u_{shi}	1 if generator $i \in \mathcal{I}$ is committed in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$, 0 otherwise
v_{shj}	Reservoir volume in reservoir $j \in \mathcal{J}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
w_{sw}	Approximated end water value in watercourse $w \in \mathcal{W}$ for scenario $s \in \mathcal{S}$
x_{sh}^{BM}	Volume committed to the balancing market in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$
x_{shk}^{E}	Volume bid to Elbas in segment $k \in \mathcal{K}$ in hour $h \in \mathcal{H}$ for scenario $s \in \mathcal{S}$

Indicator matrices:

Γ_{ij}	Explains the connection between reservoir $j \in \mathcal{J}$ and generator $i \in \mathcal{I}$. 1 if generator draws from reservoir, -1 if generator spills into reservoir and 0 if there is no connection
$\Lambda_{jj'}$	Explains the connection between reservoir $j' \in \mathcal{J}$ and reservoir $j \in \mathcal{J}$. 1 if spill is from j' to j, -1 if spill is into j' from j or if the reservoir j' spills out of the system, and 0 if there is no connection between the reservoirs

Model

$$\max \sum_{s \in \mathcal{S}} \sum_{h \in \mathcal{H}} Pr_s [\sum_{k \in \mathcal{K}} \Pi_{sk} \rho_{shk}^{E} x_{shk}^{E} + \rho_{sh}^{BM} x_{sh}^{BM}) - \sum_{w \in \mathcal{W}} (W_w^0 - w_{sw}) - \sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{I}} c_{shi}] + \sum_{h \in \mathcal{H}} P_h^{Spot} X_h \quad \text{(B.1)}$$

$$|x^E_{shk}| \leq |E^E_k| \qquad s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K} \tag{B.2}$$

$$|x^{BM}_{sh}| \leq |E^{BM}_{sh}| \qquad s \in \mathcal{S}, h \in \mathcal{H} \tag{B.3}$$

$$\sum_{k \in \mathcal{K}} \Pi_{sk} x^E_{shk} + X^E_{sh} + x^{BM}_{sh} + X_h = \sum_{i \in \mathcal{I}} q_{shi} \qquad s \in \mathcal{S}, h \in \mathcal{H} \tag{B.4}$$

$$\sum_{k \in \mathcal{K}} x^E_{shk} + X^E_{sh} + x^{BM}_{sh} + X_h \leq \sum_{i \in \mathcal{I}} Q^{max}_i \qquad s \in \mathcal{S}, h \in \mathcal{H} \tag{B.5}$$

$$q_{shi} \leq Q^{max}_i u_{shi} \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \tag{B.6}$$

$$q_{shi} \geq Q^{min}_i u_{shi} \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \tag{B.7}$$

$$v_{shj} - v_{s(h-1)j} = I_{hj} - \sum_{i \in \mathcal{I}} \Gamma_{ij} d_{shi} + \sum_{j' \in \mathcal{J}} \Lambda_{jj'} s_{shj'} \qquad s \in \mathcal{S}, h \in \mathcal{H} \backslash 1, j \in \mathcal{J} \tag{B.8}$$

$$v_{s1j} - V^0_{sj} = I_{1j} - \sum_{i \in \mathcal{I}} \Gamma_{ij} d_{s1i} + \sum_{j' \in \mathcal{J}} \Lambda_{jj'} s_{s1j'} \qquad s \in \mathcal{S}, j \in \mathcal{J} \tag{B.9}$$

$$q_{shi} \leq A_{if} u_{shi} + B_{if} d_{shi} \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I}, f \in \mathcal{F} \tag{B.10}$$

$$d_{shi} \leq D_i u_{shi} \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \tag{B.11}$$

$$c_{shi} \geq C_i(u_{shi} - \max[u_{s(h-1)i}, U^0_{hi}]) \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \tag{B.12}$$

$$c_{s1i} \geq C_i(u_{s1i} - U^0_{1i}) \qquad s \in \mathcal{S}, i \in \mathcal{I} \tag{B.13}$$

$$w_{sw} \leq F_{lw} - \sum_{j \in \mathcal{J}^w} W_{jl}(V_{jl} - v_{s|H|j}) \qquad s \in \mathcal{S}, l \in \mathcal{L}, w \in \mathcal{W} \tag{B.14}$$

$$\omega_{sh} = \omega_{\zeta h} \qquad s \in \mathcal{S}_\zeta, h \in \mathcal{H}_\zeta, \zeta \in \mathcal{Z} \tag{B.15}$$

B.1 Objective function without water value cuts

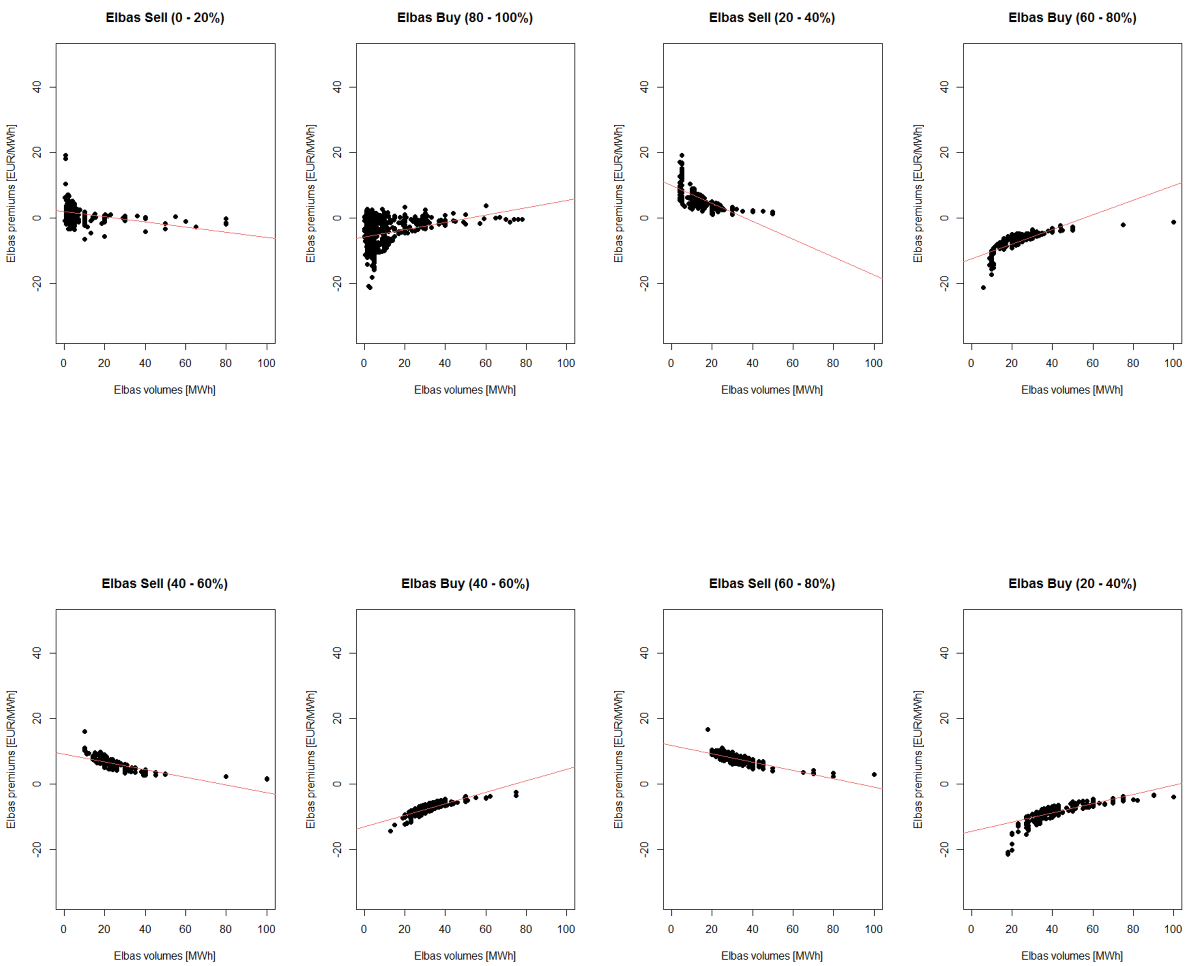
$$\begin{aligned}\max \sum_{s \in \mathcal{S}} Pr_s [\sum_{h \in \mathcal{H}} (\sum_{k \in \mathcal{K}} \Pi_{sk} \rho^E_{shk} x^E_{shk} + \rho^{BM}_{sh} x^{BM}_{sh}) \\ - \sum_{j \in \mathcal{J}} W^0_j \eta_j (\sum_{h \in \mathcal{H} \backslash 1} (v_{s(h-1)j} - v_{shj}) + V^0_{sj} - v_{s1j}) \\ - \sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{I}} c_{shi}] + \sum_{h \in \mathcal{H}} P^{Spot}_h X_h\end{aligned} \tag{B.16}$$

Constraint B.14 is removed.

Appendix C

Elbas data separation

C.1 Method 1, five buckets - product value



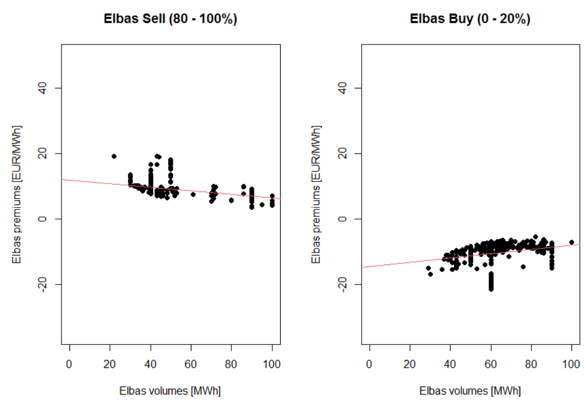


Figure C.1: Data separation for Elbas modelling - Method 1.

C.2 Method 2, three buckets - demand lines

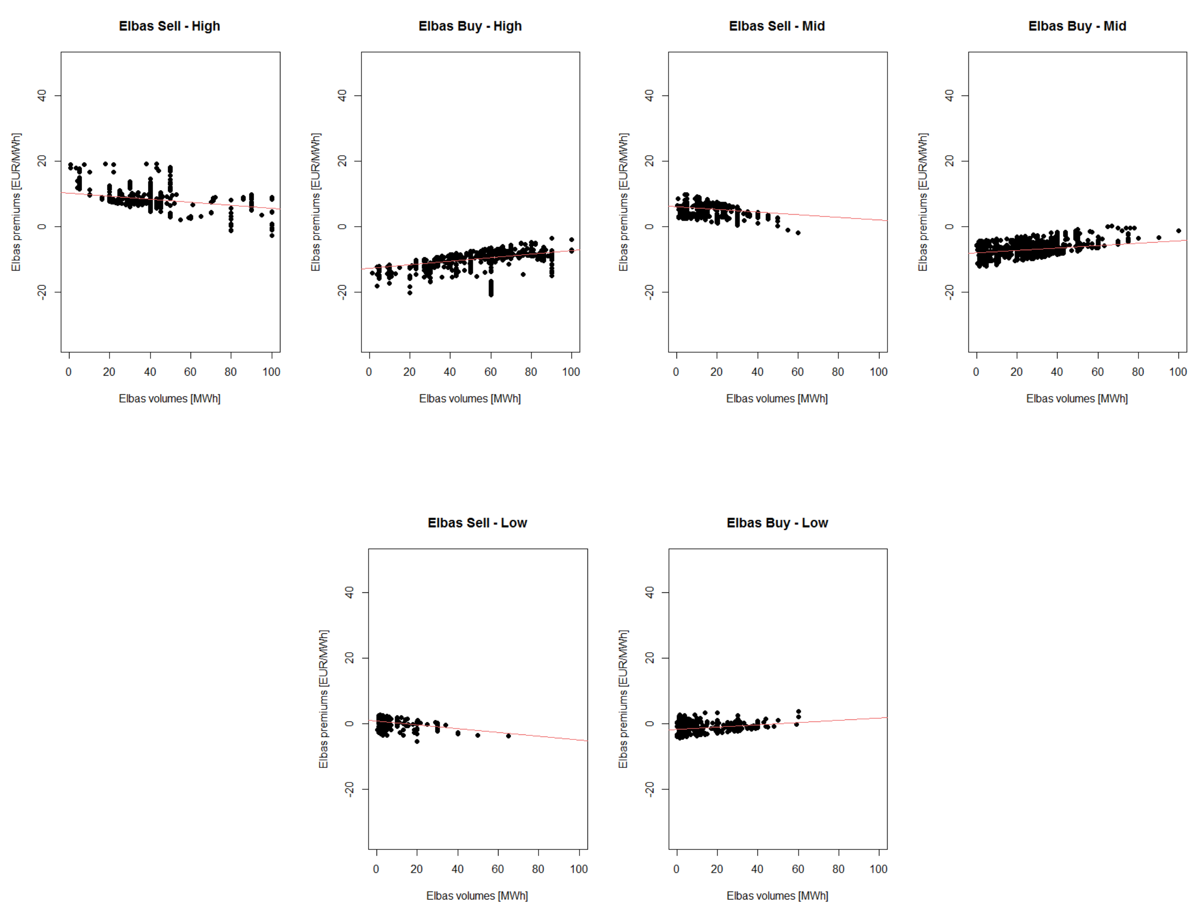


Figure C.2: Data separation for Elbas modelling - Method 2.

Appendix D

Transition probability matrices

Transition probability matrices for transition probability $p_{ij}^{BM_h}$ between regulating states $[i, j] \in \{1, 2, 3\}$ for production hour $h \in \mathcal{H}$. The regulating states are defined as: No regulation = 1, upward regulation = 2, downward regulation = 3.

$$p_{ij}^{BM_1} = \begin{bmatrix} 0.601 & 0.189 & 0.209 \\ 0.522 & 0.268 & 0.210 \\ 0.468 & 0.132 & 0.400 \end{bmatrix} \qquad p_{ij}^{BM_2} = \begin{bmatrix} 0.577 & 0.178 & 0.245 \\ 0.532 & 0.234 & 0.234 \\ 0.446 & 0.176 & 0.377 \end{bmatrix}$$

$$p_{ij}^{BM_3} = \begin{bmatrix} 0.541 & 0.172 & 0.287 \\ 0.503 & 0.282 & 0.215 \\ 0.435 & 0.206 & 0.360 \end{bmatrix} \qquad p_{ij}^{BM_4} = \begin{bmatrix} 0.538 & 0.190 & 0.272 \\ 0.471 & 0.280 & 0.248 \\ 0.423 & 0.209 & 0.367 \end{bmatrix}$$

$$p_{ij}^{BM_5} = \begin{bmatrix} 0.532 & 0.215 & 0.263 \\ 0.500 & 0.216 & 0.285 \\ 0.479 & 0.155 & 0.366 \end{bmatrix} \qquad p_{ij}^{BM_6} = \begin{bmatrix} 0.554 & 0.166 & 0.280 \\ 0.462 & 0.227 & 0.311 \\ 0.450 & 0.128 & 0.421 \end{bmatrix}$$

$$p_{ij}^{BM_7} = \begin{bmatrix} 0.538 & 0.175 & 0.287 \\ 0.478 & 0.217 & 0.304 \\ 0.425 & 0.108 & 0.467 \end{bmatrix} \qquad p_{ij}^{BM_8} = \begin{bmatrix} 0.591 & 0.135 & 0.273 \\ 0.398 & 0.346 & 0.256 \\ 0.410 & 0.162 & 0.427 \end{bmatrix}$$

$$p_{ij}^{BM_9} = \begin{bmatrix} 0.581 & 0.197 & 0.222 \\ 0.381 & 0.347 & 0.273 \\ 0.401 & 0.228 & 0.371 \end{bmatrix} \qquad p_{ij}^{BM_{10}} = \begin{bmatrix} 0.545 & 0.210 & 0.244 \\ 0.491 & 0.289 & 0.220 \\ 0.373 & 0.240 & 0.387 \end{bmatrix}$$

$$p_{ij}^{BM_{11}} = \begin{bmatrix} 0.534 & 0.212 & 0.254 \\ 0.455 & 0.335 & 0.209 \\ 0.357 & 0.276 & 0.367 \end{bmatrix} \qquad p_{ij}^{BM_{12}} = \begin{bmatrix} 0.542 & 0.238 & 0.220 \\ 0.421 & 0.386 & 0.193 \\ 0.401 & 0.209 & 0.390 \end{bmatrix}$$

$$p_{ij}^{BM_{13}} = \begin{bmatrix} 0.529 & 0.241 & 0.230 \\ 0.503 & 0.035 & 0.139 \\ 0.420 & 0.182 & 0.398 \end{bmatrix} \qquad p_{ij}^{BM_{14}} = \begin{bmatrix} 0.535 & 0.242 & 0.223 \\ 0.460 & 0.347 & 0.193 \\ 0.443 & 0.144 & 0.412 \end{bmatrix}$$

$$p_{ij}^{BM_{15}} = \begin{bmatrix} 0.501 & 0.249 & 0.249 \\ 0.448 & 0.320 & 0.233 \\ 0.418 & 0.150 & 0.432 \end{bmatrix} \qquad p_{ij}^{BM_{16}} = \begin{bmatrix} 0.518 & 0.239 & 0.243 \\ 0.421 & 0.333 & 0.246 \\ 0.418 & 0.150 & 0.432 \end{bmatrix}$$

$$p_{ij}^{BM_{17}} = \begin{bmatrix} 0.540 & 0.188 & 0.273 \\ 0.434 & 0.322 & 0.243 \\ 0.415 & 0.166 & 0.419 \end{bmatrix} \qquad p_{ij}^{BM_{18}} = \begin{bmatrix} 0.548 & 0.193 & 0.259 \\ 0.463 & 0.278 & 0.258 \\ 0.377 & 0.182 & 0.442 \end{bmatrix}$$

$$p_{ij}^{BM_{19}} = \begin{bmatrix} 0.522 & 0.240 & 0.237 \\ 0.417 & 0.337 & 0.246 \\ 0.406 & 0.161 & 0.433 \end{bmatrix} \qquad p_{ij}^{BM_{20}} = \begin{bmatrix} 0.528 & 0.258 & 0.214 \\ 0.403 & 0.372 & 0.224 \\ 0.427 & 0.182 & 0.391 \end{bmatrix}$$

$$p_{ij}^{BM_{21}} = \begin{bmatrix} 0.522 & 0.252 & 0.226 \\ 0.452 & 0.328 & 0.220 \\ 0.374 & 0.194 & 0.432 \end{bmatrix} \qquad p_{ij}^{BM_{22}} = \begin{bmatrix} 0.535 & 0.217 & 0.248 \\ 0.497 & 0.284 & 0.219 \\ 0.419 & 0.153 & 0.428 \end{bmatrix}$$

$$p_{ij}^{BM_{23}} = \begin{bmatrix} 0.522 & 0.198 & 0.280 \\ 0.490 & 0.269 & 0.241 \\ 0.468 & 0.154 & 0.377 \end{bmatrix} \qquad p_{ij}^{BM_{24}} = \begin{bmatrix} 0.547 & 0.211 & 0.243 \\ 0.462 & 0.304 & 0.234 \\ 0.495 & 0.158 & 0.347 \end{bmatrix}$$